Dear Editor,
We would like to thank the referees for reviewing this paper and furnishing this report.
We have carefully considered all comments, and we have applied several changes to the original version of the paper to address the issues raised. Detailed responses to all the comments can be found below.
We are at your disposal for any further clarifications and/or additional information.

Best regards

Hannes Jung

Reviewer(s)' Comments to Author (if there no comments below, please check the attachments):
Referee: 1

Comments to the Author
The manuscript presents an updated version of the Monte Carlo event generator CASCADE3. The main new features discussed in the paper are the PB-TMD parton shower and the matching of TMD parton densities to collinear hard matrix elements.

In my opinion, this is a useful study which could be published in EPJC. However, before the paper is accepted, I would like the authors to address the following points.

1. In the introduction, the authors mention the MLM method as an example of the LO merging procedure. However, there is another, equally important scheme, called CKKW, and I believe it would be appropriate to cite it there.
ANSWER: We thank the referee for this comment. Of course, CKKW has to be mentioned.
A reference is included now in the paper.

2. In section 2.1, the authors write that they extend the collinear factorization formula by promoting collinear PDFs to TMDs and then they focus on technical aspect of the matching between kT-dependent PDFs and kT-independent matrix element. I would like to ask the authors to comment on the formal validity of such kT-dependent factorization. To what extent is it proven, and to what extent is this a phenomenological modeling?
ANSWER:
TMD factorization at small k_T is proven for a few processes, like semi-inclusive deep inelastic scattering and Drell Yan production. At small x, k_T factorization is proven also for hadron-hadron scattering for any k_T. For the whole range in x and k_T, and for all flavors the factorization into k_T-dependent parton densities and matrix elements is not formally proven.
However, the approach followed here, is the same approach which is used in Monte Carlo event generators, where the k_T is generated from the parton shower. The difference to using TMDs is that the TMDs are obtained from fits to data, and the parton shower only adds explicitly the partons from the initial cascade, while not changing anymore the kinematics already fixed by using TMDs.

We have added a clarifying sentence in section 2: "TMD factorization is proven for semi-inclusive deep-inelastic scattering, Drell-Yan production in hadron-hadron collisions and e^+e^- annihilation [23–35]. In the high-energy limit (small-x) k_T - factorization has been formulated also in hadronic collisions for processes like heavy flavor or heavy boson (including Higgs) production [14, 36–38], with so-called *unintegrated* parton distribution functions (uPDFs), see *e.g.* Refs. [39–47]."

3. A related point has to do with the result presented in Fig. 1. There, the NLO predictions obtained within collinear factorization are upgraded by replacing the collinear PDF with a TMD. I would like to ask for a comment why such a procedure does not lead to double counting between emissions with kT != 0, which are part of NLO and similar emissions accounted for in the TMD.
ANSWER: In the MCatNLO procedure, which we apply here, the region of soft and collinear emissions are subtracted from the full NLO cross section; this subtraction procedure depends on the type of parton shower used. For the PB-TMDs we use the herwig6 subtraction, since herwig6 uses the same formalism for the parton shower as used in PB TMDs with angular ordering.

There are configurations from the real emission diagrams of NLO and those where an emission is added via the TMD (or the shower), which are similar. We illustrate this in Fig. 1, where we splitt the

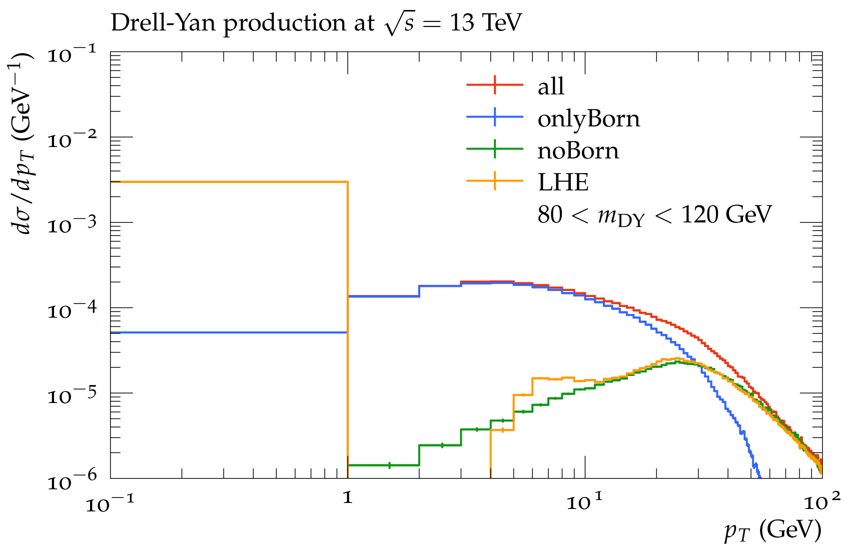


Figure 1: Distribution of dimuons at 13 TeV, separated as Born, noBorn, all (see text)

full NLO process for DY production into a contribution from the “zero emission” part (labelled as Born) and the real emission part (labelled as noBorn). Also shown for comparison is the distribution at LHE level (where the region of subtraction is visible) as well as the full result. Only can see, that in the medium p_T region (around 10-30 GeV) both the “Born” and “noBorn” parts contribute. When both pieces are added, a physical distribution is obtained, labelled as “all”.
The role of SCALUP is to define the region of overlap: if the range of k_T emissions of the “Born” process is not limited by SCALUP, if will extend much more into the region which is described by the real emission diagram “noBorn”.

We have inserted a sentence in Section 2.1 to clarify this:
"The limitation of the transverse momenta coming from the TMD distribution to be smaller than the shower scale SCALUP guarantees that the overlap between emissions from the TMD (and TMD shower) and the real emissions from the matrix element is limited according to the subtraction in the MC@NLO method."

4. I understand that the PB-TMD shower unfolds PB-TMD parton densities, just like the collinear showers unfold collinear PDFs. Both procedures lead to non-trivial pT dependence of various differential distributions. I would like the authors to comment briefly whether their approach has something that the standard (collinear) approach is missing, or whether the method of merging TMDs and PS presented in the paper is necessary in order to generate kT-factorization predictions which would be competitive to those from collinear MCs.
ANSWER: The PB-TMDs are determined with a forward evolution, which makes use of Sudakov form

factors, and is in principle similar to a parton shower. However, a forward evolution is not used in parton shower Monte Carlo event generators because of the low efficiency, while a forward evolution can be used to determine the (TMD) parton density. In the TMD parton shower, as in ordinary initial state parton showers, a backward evolution from the hard scattering towards the proton end is used.

In contrast to an ordinary parton shower, the TMD shower follows directly the behavior of the TMD density. The k_T dependence of the PB TMDs is explicitly included in the equations for the parton shower, shown in eq(9) in the paper draft. Once the k_T from the TMD has been added to the collinear hard process, the TMD shower does induce any further change of the kinematics. This procedure is described in Section 2.1

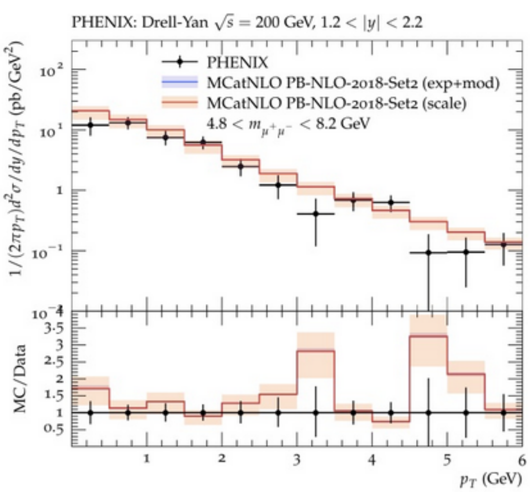
A different approach, which is used in small x, is the k_T-factorization ansatz, where the interacting partons have transverse momenta from the beginning. However, k_T-factorization is only proven for gluon induced processes at small x. CASCADE3 includes processes calculated in k_T-factorization (alternatively one can also use LHE files from processes calculated externally) as well. This procedure is described in Section 2.2.

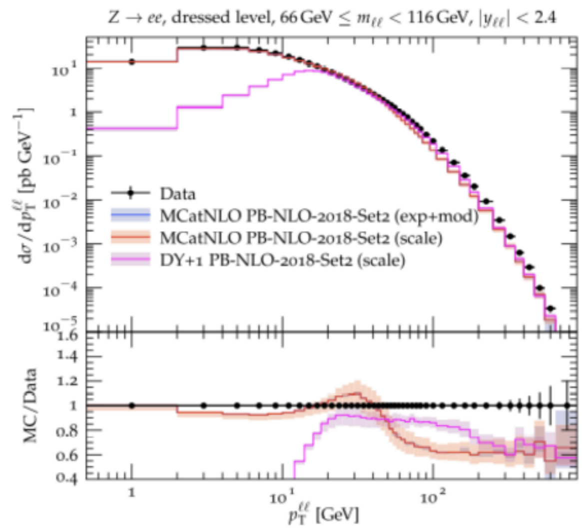
The consistent use of PB-TMDs and PB-TMD shower is shown explicitly in eq.(9), where a relation of the Sudakov form factor used in the parton evolution and the Sudakov form factor used for the backward evolution of the parton shower is given. For ordinary showers and collinear pdfs there is a mismatch coming from the z_M limit in the integral in eq.(9), as pointed out by the paper of Nagy and Soper [76]. This problem is avoided by construction when using PB-pdfs and the PB shower.

We have added a sentence towards the end of Section 3.1:
"A similar relation was also studied in Refs. [76, 77]. In Ref [76] the z_M limit was identified as a source of inconsistency when using conventional showers with standard collinear pdfs; in the PB approach, the same z_M limit is applied in the parton evolution as well as in the PB-shower. "
The clear advantage of applying PB-TMDs and PB-TMD shower is that there are no free parameters left for the shower and the shower uncertainty is entirely given by the uncertainties of the PB-TMDs as determined from the fits.

We have added also a sentence to highlight the advantage of the PB TMD shower at the end of Section 3.1:
"The advantage of using a PB TMD shower is that all the parameters of the parton shower are fixed by the PB TMD and that the uncertainties of the parton shower is given by the uncertainties of the PB TMD which are constrained by the fits of the PB TMD. "

5. In section 4, the authors mention that the parton shower available in CASCADE3 can be consistently used only with PB-TMDs. I would like to ask for a comment how well are the PB-TMDs validated against the existing experimental data, e.g. F2.
ANSWER: The PB-pdfs have been determined from fits to DIS data, as described in Phys Rev D.99.074008 (arXiv 1804.11152). The fits give the same precision as obtained from HERAPDF (which proves consistency, since the same functional form of the input paramterization is used). These PB-TMDs have been then used to predict the Z-boson transverse momentum spectrum at LHC energies (PhysRevD.100.074027, arXiv 1906.00919) as well as the DY transverse momentum spectrum at low DY mass and at low energies (Eur. Phys. J. C 80 (2020) 598, arXiv 2001.06488). The two relevant Figures are repeated here.





From Eur. Phys. J. C 80 (2020) 598, arXiv 2001.06488 at 200 GeV for DY masses between 4.8 and 8.2 GeV.

From PhysRevD.100.074027, arXiv 1906.00919 at 8 TeV for Z-boson mass.

The agreement both in absolute normalization as well as in the shape of the p_T -spectra at low p_T is remarkable (given the complication to describe simultaneously high and low mass DY production as mentioned in Phys Rev D.100.014018 (arXiv 1901.06916). The agreement between measurement and prediction at LHC energies is very good, with small uncertainties coming from the PB TMDs.

We have added a sentence at the end of Section 4:

“In Refs. [85, 86] the transverse momentum distribution of Drell-Yan pairs at low and high mass is compared with measurements, and very good agreement between prediction and measurement is found, illustrating the consistency of the approach.”