Regularization of supersymmetric theories

New results on dimensional reduction

Dominik Stöckinger

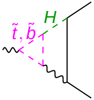
IPPP Durham

Loops & Legs, April 2006

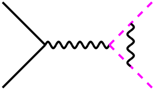
Outline

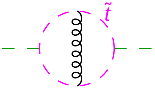
Regularization of SUSY

Motivation: some important observables/calculations. . .

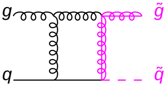
- $(g-2)_\mu$ 

$\rightarrow$ no problem with regularization

- 1-Loop processes 
$\rightarrow$ DRED preserves SUSY!!

- M_h 

$\rightarrow$ DRED SUSY-preserving??

- LHC 

$\rightarrow$ DRED violates factorization!?

Systematic analysis: algebraic renormalization

In principle, we don't have to bother whether a regularization preserves symmetries

Systematic analysis: algebraic renormalization

In practice, life is easier with a symmetry-preserving regularization!

Systematic analysis: algebraic renormalization

In practice, life is easier with a symmetry-preserving regularization!

- counterterms Γ^{ct} also preserve symmetries:

  $g \to g + \delta g$, $m \to m + \delta m$ — "multiplicative renormalization"
- most common situation, often assumed without proof

Outline

Common Regularization Schemes for SM/MSSM

Let's consider now the common regularization schemes for SM/MSSM:

- Dimensional Regularization (DREG) ['t Hooft, Veltman '72]
- Dimensional Reduction (DRED) [Siegel '79]

Properties of DREG/DRED (status Jan. 2005)

DREG:

Dim. Regularization (DREG)

D dimensions
D Gluon/photon-components
4 Gluino/photino-components

DRED:

Dim. Reduction (DRED)

D dimensions
4 Gluon/photon-components
4 Gluino/photino-components

Summary: Properties of DREG and DRED

DREG:	consistent	SUSY-violation	factorization
	$+$	$-$	$+$
DRED:	inconsistent	SUSY (?)	no factorization (?)
	$-$	$+$(?)	$-$(?)

Summary: Properties of DREG and DRED

DREG:	consistent	SUSY-violation	factorization
	$+$	$-$	$+$
DRED:	inconsistent	SUSY (?)	no factorization (?)
	$-$	$+$(?)	$-$(?)

SUSY:

DREG breaks SUSY already e.g. for $m_e(1L) \neq m_{\tilde{e}}(1L)$
DRED preserves SUSY in simple cases, but e.g. M_h: unclear

Summary: Properties of DREG and DRED

DREG:	consistent	SUSY-violation	factorization
	$+$	$-$	$+$
DRED:	inconsistent	SUSY (?)	no factorization (?)
	$-$	$+$(?)	$-$(?)

DRED seems to violate factorization for LHC-processes

Summary: Properties of DREG and DRED

	consistent	SUSY-violation	factorization
DREG:	$+$	$-$	$+$

$\longrightarrow$ no fundamental problem but practical difficulties

	inconsistent	SUSY (?)	no factorization (?)
DRED:	$-$	$+$(?)	$-$(?)

Summary: Properties of DREG and DRED

DREG:	consistent	SUSY-violation	factorization
	$+$	$-$	$+$

$\longrightarrow$ no fundamental problem but practical difficulties

DRED:	inconsistent	SUSY (?)	no factorization (?)
	$-$	$+$(?)	$-$(?)

$\longrightarrow$ fundamental problems but practical advantages

Aims and Preview:

Aims: Solve problems of DRED

Preview:

- not possible to find an ideal regularization
- but fundamental problems of DRED can be solved
- result: DREG and DRED can both be used, and often DRED is easier to apply

Order: 1. Factorization, 2. Consistency, 3. SUSY

Outline

Factorization-problem

Problem: DRED, $m \neq 0$

$$\sigma^{\text{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\ \sigma^{\text{DRED}}(GG \to t\bar{t})$$

$$+\frac{1}{k_2 k_3} K_g\ \sigma^{\text{puzzle}}$$

[Beenakker, Kuijf, van Neerven, Smith '88] [van Neerven, Smith '04] [Beenakker, Höpker, Spira, Zerwas '96]

- One "solution" in practice (unsatisfactory complication): resort to DREG ⇒ SUSY-restoring cts necessary
- Fundamental question: where does the seemingly non-factorizing term σ^{puzzle} come from?

Factorization — Conclusions

Main result: $\sigma^{\mathrm{DRED}}(GG \to t\bar{t}G) \to P_{G\to gG}\,\sigma_{Gg} + P_{G\to\phi G}\,\sigma_{G\phi}$

- reconciled DRED and factorization [Signer, DS '05]

Practical consequences

- hadron processes can be computed using DRED

Outline

Where does the inconsistency come from?

DREG: "D-dimensional space" can be consistently defined as a truly ∞-dimensional space with some D-dim characteristics:

[Wilson'73],[Collins]

$$\mu = 0, 1, 2, \ldots \infty, \quad g^{(D)\mu}{}_{\mu} = D$$

DRED: "D-dimensional space" should be a subspace of 4-dim space

$$g^{(4)}{}_{\mu\nu} g^{(D)}{}_{\rho}{}^{\nu} = g^{(D)}{}_{\mu}{}^{\rho} \qquad (*)$$

- "D-dimensional space" or 4-dimensional space alone: no problem
- requirement $(*)$ cannot be satisfied

$\Downarrow$

origin of inconsistency

Way out

"D-dim space" should be ∞-dimensional but subspace of 4-dim space

$\Rightarrow$ Replace ordinary 4-dim space by yet another ∞-dimensional space with some 4-dim characteristics $\longrightarrow$ "quasi-4-dim space"

$$D\text{-dim space} \subset \text{quasi-4-dim space}$$

$$g^{(D)\mu}{}_{\mu} = D, \quad g^{(4)\mu}{}_{\mu} = 4, \quad \mu = 0, 1, 2, \ldots \infty$$

quasi-4-dim space

can be explicitly constructed $\Rightarrow$ no mathematical problems, no inconsistency, unique results for calculations

Practical consequences

- In practice one can forget that the “D-dim” and quasi-4-dim spaces are in reality ∞-dimensional
- Only exception: one cannot rely on index counting or Fierz identities
  - For many SUSY loop calculations, this doesn’t make a difference
- These rules will never lead to inconsistent results

Quantum Action Principle in DRED

Using the consistent formulation of DRED, one can prove the quantum action principle in DRED

$$i\,\delta_{\mathrm{SUSY}}\langle T\phi_1\dots\phi_n\rangle = \langle T\phi_1\dots\phi_n\Delta\rangle$$

Useful to study symmetry-properties of regularizations

Proof has to be carried out for each regularization,

BPHZ [Lowenstein et al '71]
DREG [Breitenlohner, Maison '77]
DRED [DS 2005]

Outline

Problem: SUSY of DRED

- We have seen that DRED preserves SUSY e.g. in the case $m_e = m_{\tilde{e}}$ at the one-loop level
- Does DRED preserve SUSY in general?
- Or at least in cases that are relevant in practice?

DRED preserves SUSY — What does it mean?

SUSY $\Leftrightarrow$ ST-identity $S(\Gamma^{\text{ren}}) = 0$

- defines theory in algebraic renormalization
- combines all identities of the form

$$0 = \delta_{\text{SUSY}} \langle T \phi_1 \ldots \phi_n \rangle$$

DRED preserves SUSY if the ST-identity is already satisfied on the regularized level, $S(\Gamma^{\text{reg}}) = 0$

How much do we know?

Status: many SUSY identities checked in DRED:

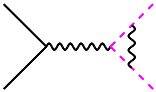
1-Loop Ward identities [Capper,Jones,van Nieuvenhuizen'80]

β-functions [Martin, Vaughn '93] [Jack, Jones, North '96]

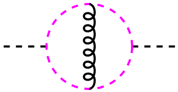
1-Loop S-matrix relation [Beenakker,Höpker,Zerwas'96]

1-Loop Slavnov-Taylor identities [Hollik,Kraus,DS'99] [Hollik,DS'01] [Fischer,Hollik,Roth,DS'03]

- sufficient for one-loop SUSY processes,
  $\Rightarrow$ multiplicative renormalization o.k.
  $\Rightarrow$ no SUSY-restoring counterterms



- but not all identities have been checked
  e.g. two-loop Higgs mass calculations:
  $\Rightarrow$ SUSY-restoring counterterms required?



Quantum action principle as a tool

So far, all checks have been done by explicitly evaluating all Green functions $\rightarrow$ very tedious, cannot be applied at 2-, 3-Loop

Quantum action principle as a tool

Quantum action principle can be used

STI $\delta_{\text{SUSY}}\langle T\phi_1\ldots\phi_n\rangle = 0$

valid in DRED $\Leftrightarrow$ $\langle T\phi_1\ldots\phi_n\Delta\rangle = 0 \quad \Delta = \delta_{\text{SUSY}}\mathcal{L}$

- Use of qu. action principle in DREG [Breitenlohner, Maison '77]
  $\Rightarrow$ e.g. DREG preserves all QCD Slavnov-Taylor identities at all orders:

$$\delta_{\text{gauge}}\mathcal{L}^{\text{DREG}}_{\text{QCD}} = 0 \quad \Rightarrow \quad \delta_{\text{gauge}}\langle T\phi_1\ldots\phi_n\rangle = 0$$

Quantum action principle as a tool

Quantum action principle can be used

STI $\delta_{\mathrm{SUSY}}\langle T\phi_1\ldots\phi_n\rangle = 0$

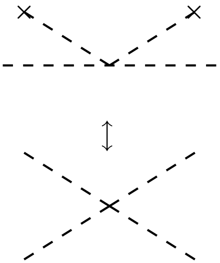
valid in DRED $\Leftrightarrow$ $\langle T\phi_1\ldots\phi_n\Delta\rangle = 0 \quad \Delta = \delta_{\mathrm{SUSY}}\mathcal{L}$

- application here: SUSY of DRED:

$\delta_{\mathrm{SUSY}}\mathcal{L}^{\mathrm{DRED}} = \Delta \neq 0$ gives rise to Feynman rules [DS '05]

  - DRED probably does not preserve all SUSY-identities
- but using the quantum action principle we can check much more complicated symmetry identities

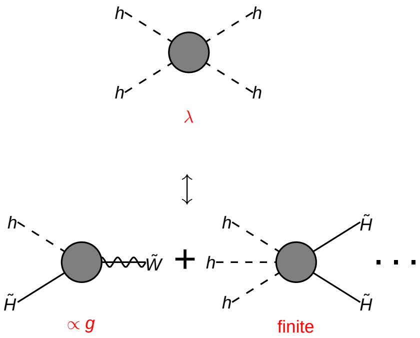
Higgs boson mass and quartic coupling



Higgs mass

- M_h governed by quartic Higgs self coupling λ
- $\lambda \propto g^2$ in SUSY

Quartic coupling and SUSY

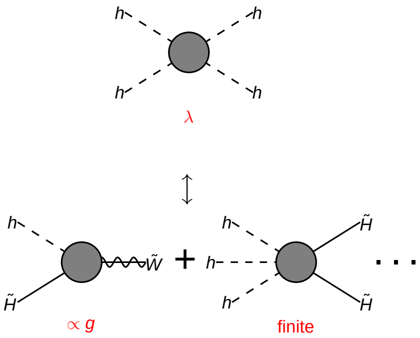


Slavnov-Taylor identity

- expresses $\lambda \propto g^2$
- can be evaluated for two-loop Green functions
- If it is satisfied by DRED $\Leftrightarrow$ multiplicative renormalization o.k.
- Needs to be verified

$$0 \stackrel{?}{=} \delta_{\text{SUSY}} \langle hhh\tilde{H} \rangle$$

Quartic coupling and SUSY

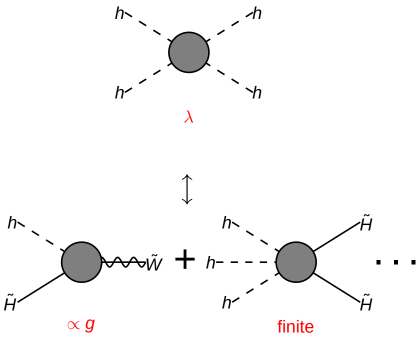


$$0 \stackrel{?}{=} \delta_{\rm SUSY}\langle hhh\tilde{H}\rangle$$

Obstacle:

- Two-loop evaluation:
  - up to 5-point functions
  - very diffi cult
- previously not feasible

Quartic coupling and SUSY



$$0 \stackrel{?}{=} \delta_{\text{SUSY}} \langle hhh\tilde{H} \rangle \equiv \langle \Delta hhh\tilde{H} \rangle$$

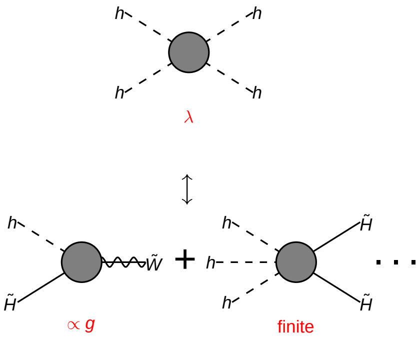
Obstacle:

- Two-loop evaluation:
  - up to 5-point functions
  - very diffi cult
- previously not feasible

Solution:

- Use quantum action principle in DRED

Quartic coupling and SUSY



$$0 \overset{?}{=} \delta_{\text{SUSY}} \langle hhh\tilde{H} \rangle \equiv \langle \Delta hhh\tilde{H} \rangle$$

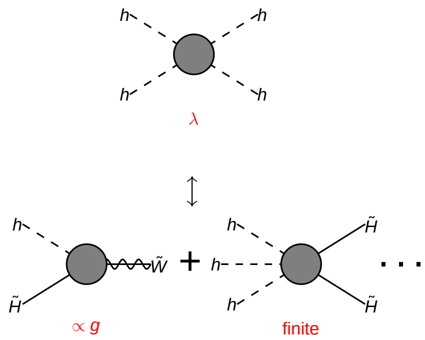
Obstacle:

- Two-loop evaluation:
  - up to 5-point functions
  - very diffi cult
- previously not feasible

Solution:

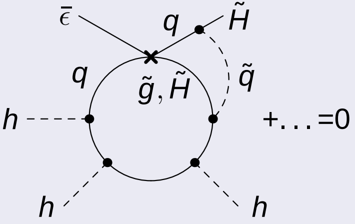
- Use quantum action principle in DRED
- $\delta_{\text{SUSY}} \langle hhh\tilde{H} \rangle = \langle \Delta hhh\tilde{H} \rangle$
- Check of STI much easier

Quartic coupling and SUSY



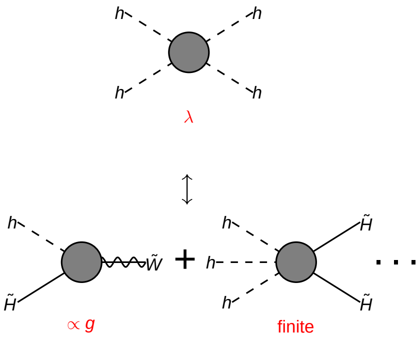
STI valid if

$$\langle \Delta hhh\tilde{H} \rangle = 0 \Leftrightarrow$$



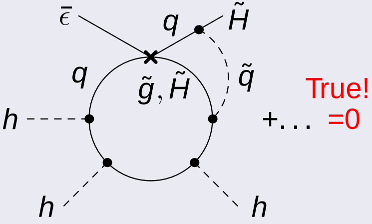
Explicit computation $\Rightarrow$ STI valid in DRED at two-loop level [Hollik, DS '05]

Quartic coupling and SUSY



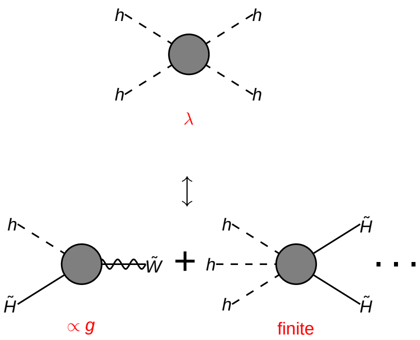
STI valid if

$\langle \Delta hhh\tilde{H} \rangle = 0 \Leftrightarrow$



Explicit computation $\Rightarrow$ STI valid in DRED at two-loop level [Hollik, DS '05]

Quartic coupling and SUSY



Results:

- Two-loop STI valid in DRED (in Yukawa-approximation, $\mathcal{O}(\alpha_{t,b}^2, \alpha_{t,b}\alpha_s)$)
- for M_h-calculation at this order, multiplicative renormalization correct
- Previous calculations sufficient

Explicit computation $\Rightarrow$ STI valid in DRED at two-loop level [Hollik, DS '05]

Summary: Properties of DREG and DRED

DREG:	consistent	SUSY-violation	factorization
	$+$	$-$	$+$
DRED:	consistent	SUSY	factorization
	$+$	$(+)$	$(+)$

Outline

Summary

SUSY quantum effects important e.g. for LHC-predictions

- Renormalizability proven in algebraic, regularization-independent way
- In practice, DREG and DRED can be used, both have advantages and disadvantages
- After recent improvements, DRED is in a good shape and is better suited to SUSY calculations
  - Factorization ok in DRED
  - DRED formulated without inconsistency
  - SUSY of DRED established in important cases

Summary & Outlook

Comparison of DREG and DRED:

- Factorization: holds in DREG and DRED, slightly more complicated in DRED due to different partons g, ϕ

  $\rightarrow$ streamlined prescription for hadron processes in DRED?

- Consistency, quantum action principle: ok in DREG and DRED
- SUSY: DREG breaks SUSY already in simplest cases, DRED preserves SUSY in many cases up to 2-Loop, but not at all orders

  $\rightarrow$ further checks of e.g. RG-running at 3-Loops?