Matching Coefficients for α_s in the SM and MSSM

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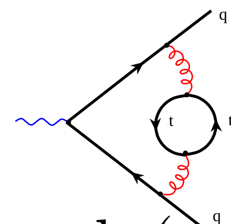
I. Introduction

II. QCD

III. MSSM

IV. Conclusions

Decoupling of heavy particles

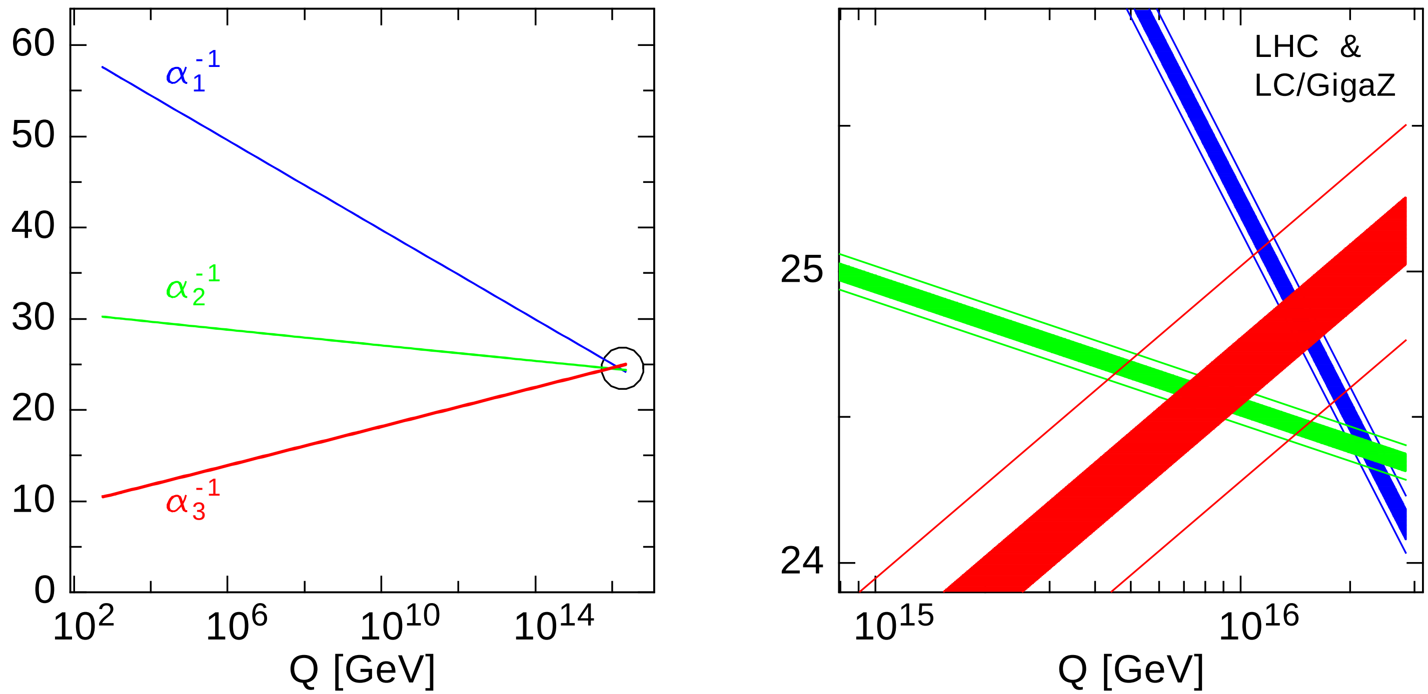
- $\overline{\rm MS}$ scheme: simple, convenient, frequently used, ...
- But: Appelquist-Carazzone-Theorem not applicable:
  decoupling of heavy particles from coupling and masses is not automatic;
  has to be performed "by hand"
- Example: contribution of  to

  $\sigma(e^+e^- \to q\bar{q})$ for $\sqrt{s} \ll m_t \propto \ln(m_t/\mu)$ for $m_t \to \infty$

  use: $\alpha_s^{(6)} = \alpha_s^{(5)} \left(1 + \frac{\alpha_s^{(5)}}{6\pi} \ln \frac{\mu^2}{m_t^2}\right)$

  ⇨ $\sigma(e^+e^- \to q\bar{q})$ expressed in terms of $\alpha_s^{(5)}$ has contribution $\propto 1/m_t^2$ for $m_t \to \infty$
- "decouple top quark from α_s"
- QCD: 2-loop [Bernreuther,Wetzel'81; Larin,van Ritbergen,Vermaseren'94]
  QCD: 3-loop [Chetyrkin,Kniehl,MS'98]
  here: QCD 4-loop and MSSM–SQCD 2-loop

Unification in the MSSM



[Allanach et al.'04]

- precise running of α_s needed
- decoupling of heavy particles

II. Decoupling in QCD

Framework

- QCD with n_f active flavours
- $n_f = n_l + 1$: n_l massless and 1 massive quark
- decoupling constant: $\alpha_s^{0\prime} = \left(\zeta_g^0\right)^2 \alpha_s^0$

Framework

- QCD with n_f active flavours
- $n_f = n_l + 1$: n_l massless and 1 massive quark
- decoupling constant:

$$\alpha_s^{0\prime} = \left(\zeta_g^0\right)^2 \alpha_s^0$$

bare, in effective theory

bare, in full theory

Framework

- QCD with n_f active flavours
- $n_f = n_l + 1$: n_l massless and 1 massive quark
- decoupling constant: $\alpha_s^{0\prime} = \left(\zeta_g^0\right)^2 \alpha_s^0$
- similar for gluon and ghost field:

  $G_\mu^{0\prime,a} = \sqrt{\zeta_3^0}\, G_\mu^{0,a} \quad c^{0\prime,a} = \sqrt{\tilde\zeta_3^0}\, c^{0,a}$
- ζ_g^0 computed from ζ_3^0, $\tilde\zeta_3^0$ and decoupling constant of ghost-gluon vertex ($\tilde\zeta_1^0$): $\zeta_g^0 = \frac{\tilde\zeta_1^0}{\tilde\zeta_3^0 \sqrt{\zeta_3^0}}$

  compare: $Z_g = \frac{\tilde Z_1}{\tilde Z_3 \sqrt{Z_3}}$
- bare ➪ renormalized quantities: as usual

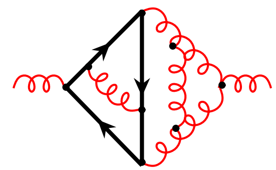
  $\alpha_s^0 = \left(Z_g\right)^2 \alpha_s, \ldots$ ➪ $\zeta_g = \frac{Z_g}{Z_g'} \zeta_g^0$

Framework (2)

$$\zeta_g^0 = \frac{\tilde{\zeta}_1^0}{\tilde{\zeta}_3^0 \sqrt{\zeta_3^0}}$$

4-loop diags.

$$\zeta_3^0 = 1 + \Pi_G^{0h}(0)$$



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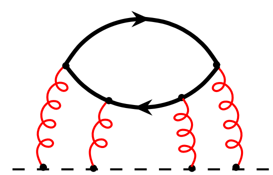
$$\tilde{\zeta}_3^0 = 1 + \Pi_c^{0h}(0)$$



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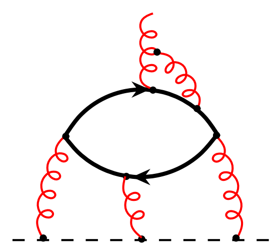
$$\tilde{\zeta}_1^0 = 1 + \Gamma_{G\bar{c}c}^{0h}(0,0)$$



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hard part: external momentum zero; heavy quark present in loop;

n-loop contribution to ζ_g^0 requires n-loop vacuum diagrams

[Chetyrkin,Kniehl,MS'98]

Result

$$
\begin{aligned}
\zeta_g^2 \quad = \quad & 1 + \frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\left(-\frac{1}{6}\ln\frac{\mu^2}{m_h^2}\right) + \left(\frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\right)^2\left(\frac{11}{72} - \frac{11}{24}\ln\frac{\mu^2}{m_h^2} + \frac{1}{36}\ln^2\frac{\mu^2}{m_h^2}\right) \\
& + \left(\frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\right)^3\left[\frac{564731}{124416} - \frac{82043}{27648}\zeta(3) - \frac{955}{576}\ln\frac{\mu^2}{m_h^2} + \frac{53}{576}\ln^2\frac{\mu^2}{m_h^2} - \frac{1}{216}\ln^3\frac{\mu^2}{m_h^2}\right. \\
& \left. + n_l\left(-\frac{2633}{31104} + \frac{67}{576}\ln\frac{\mu^2}{m_h^2} - \frac{1}{36}\ln^2\frac{\mu^2}{m_h^2}\right)\right] + \left(\frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\right)^4\left[\frac{291716893}{6123600}\right. \\
& + \frac{3031309}{1306368}\ln^4 2 - \frac{121}{4320}\ln^5 2 - \frac{3031309}{217728}\zeta(2)\ln^2 2 + \frac{121}{432}\zeta(2)\ln^3 2 - \frac{2362581983}{87091200}\zeta(3) \\
& - \frac{76940219}{2177280}\zeta(4) + \frac{2057}{576}\zeta(4)\ln 2 + \frac{1389}{256}\zeta(5) + \frac{3031309}{54432}a_4 + \frac{121}{36}a_5 - \frac{151369}{2177280}X_6 \\
& + \left(\frac{7391699}{746496} - \frac{2529743}{165888}\zeta(3)\right)\ln\frac{\mu^2}{m_h^2} + \frac{2177}{3456}\ln^2\frac{\mu^2}{m_h^2} - \frac{1883}{10368}\ln^3\frac{\mu^2}{m_h^2} + \frac{1}{1296}\ln^4\frac{\mu^2}{m_h^2} \\
& + n_l\left(-\frac{4770941}{2239488} + \frac{685}{124416}\ln^4 2 - \frac{685}{20736}\zeta(2)\ln^2 2 + \frac{3645913}{995328}\zeta(3)\right. \\
& - \frac{541549}{165888}\zeta(4) + \frac{115}{576}\zeta(5) + \frac{685}{5184}a_4 + \left(-\frac{110341}{373248} + \frac{110779}{82944}\zeta(3)\right)\ln\frac{\mu^2}{m_h^2} \\
& \left. - \frac{1483}{10368}\ln^2\frac{\mu^2}{m_h^2} - \frac{127}{5184}\ln^3\frac{\mu^2}{m_h^2}\right) + n_l^2\left(-\frac{271883}{4478976} + \frac{167}{5184}\zeta(3) + \frac{6865}{186624}\ln\frac{\mu^2}{m_h^2}\right. \\
& \left.\left. - \frac{77}{20736}\ln^2\frac{\mu^2}{m_h^2} + \frac{1}{324}\ln^3\frac{\mu^2}{m_h^2}\right)\right] + \mathcal{O}\left(\left(\frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\right)^5\right)
\end{aligned}
$$

[Schröder,MS'05; Chetyrkin,Kühn,Sturm'05]

Result

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\begin{aligned}
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\end{aligned}
$$

$$X_6 \quad = \quad +1.808879546208334741426364595086952090$$

$$
\begin{aligned}
& - \frac{\cdots}{165888}\zeta(4) + \frac{\cdots}{576}\zeta(5) + \frac{\cdots}{5184}a_4 + \left(-\frac{\cdots}{373248} + \frac{\cdots}{82944}\zeta(3)\right)\ln\frac{\mu^2}{m_h^2} \\
& - \frac{1483}{10368}\ln^2\frac{\mu^2}{m_h^2} - \frac{127}{5184}\ln^3\frac{\mu^2}{m_h^2}\Bigg) + n_l^2\left(-\frac{271883}{4478976} + \frac{167}{5184}\zeta(3) + \frac{6865}{186624}\ln\frac{\mu^2}{m_h^2}\right. \\
& \left. - \frac{77}{20736}\ln^2\frac{\mu^2}{m_h^2} + \frac{1}{324}\ln^3\frac{\mu^2}{m_h^2}\right)\Bigg] + \mathcal{O}\left(\left(\frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\right)^5\right)
\end{aligned}
$$

[Schröder,MS'05; Chetyrkin,Kühn,Sturm'05]

Result

$$\zeta_g^2 = 1 + \frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\left(-\frac{1}{6}\ln\frac{\mu^2}{m_h^2}\right) + \left(\frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\right)^2\left(\frac{11}{72} - \frac{11}{24}\ln\frac{\mu^2}{m_h^2} + \frac{1}{36}\ln^2\frac{\mu^2}{m_h^2}\right)$$

$$+ \left(\frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\right)^3\left[\frac{564731}{124416} - \frac{82043}{27648}\zeta(3) - \frac{955}{576}\ln\frac{\mu^2}{m_h^2} + \frac{53}{576}\ln^2\frac{\mu^2}{m_h^2} - \frac{1}{216}\ln^3\frac{\mu^2}{m_h^2}\right.$$

$$\zeta_g^2 \approx 1 + 0.1528\left(\frac{\alpha_s^{(n_l+1)}(m_h)}{\pi}\right)^2 + (0.9721 - 0.0847\,n_l)\left(\frac{\alpha_s^{(n_l+1)}(m_h)}{\pi}\right)^3$$

$$+ (5.1703 - 1.0099\,n_l - 0.0220\,n_l^2)\left(\frac{\alpha_s^{(n_l+1)}(m_h)}{\pi}\right)^4$$

$$746496 \quad 165888 \quad m_h^2 \quad 3456 \quad m_h^2 \quad 10368 \quad m_h^2 \quad 1296 \quad m_h^2$$

$$+ n_l\left[-\frac{4770941}{2239488} + \frac{685}{124416}\ln^4 2 - \frac{685}{20736}\zeta(2)\ln^2 2 + \frac{3645913}{995328}\zeta(3)\right.$$

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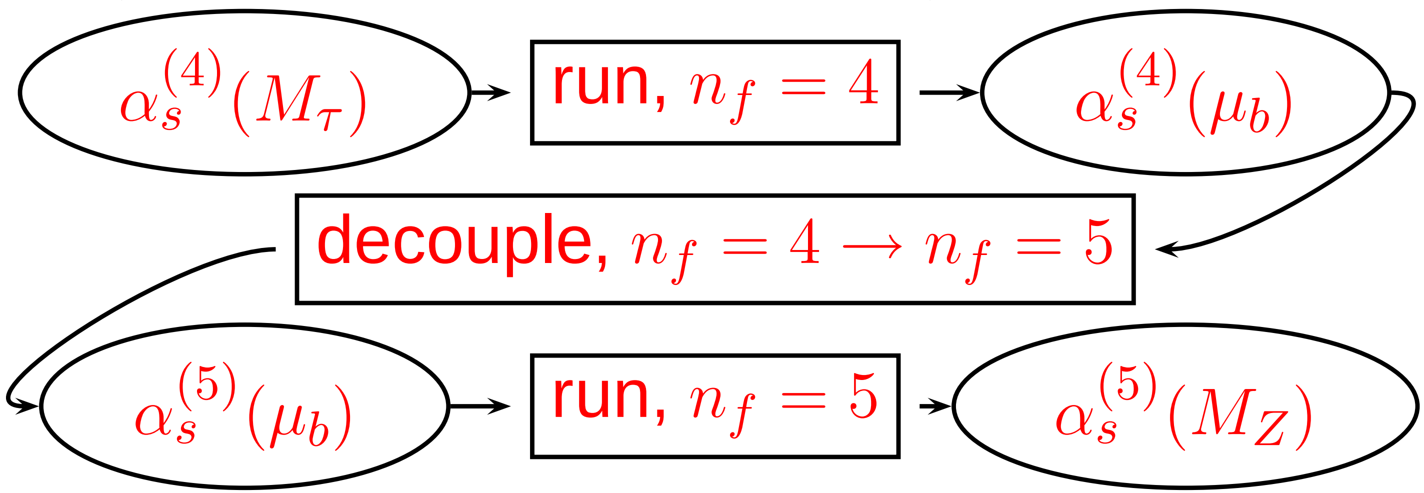
$$\left.- \frac{1483}{10368}\ln^2\frac{\mu^2}{m_h^2} - \frac{127}{5184}\ln^3\frac{\mu^2}{m_h^2}\right] + n_l^2\left[-\frac{271883}{4478976} + \frac{167}{5184}\zeta(3) + \frac{6865}{186624}\ln\frac{\mu^2}{m_h^2}\right.$$

$$\left.\left.- \frac{77}{20736}\ln^2\frac{\mu^2}{m_h^2} + \frac{1}{324}\ln^3\frac{\mu^2}{m_h^2}\right]\right] + \mathcal{O}\left(\left(\frac{\alpha_s^{(n_l+1)}(\mu)}{\pi}\right)^5\right)$$

[Schröder,MS'05; Chetyrkin,Kühn,Sturm'05]

$\alpha_s^{(4)}(M_\tau)$ ⇨ $\alpha_s^{(5)}(M_Z)$

- Evaluation of $\alpha_s^{(5)}(M_Z)$ from $\alpha_s^{(4)}(M_\tau)$
- cross b quark threshold:
  $n_l = 4, \mu = M_\tau \approx 1.77$ GeV: $m_b \gg \mu$
  $n_l = 5, \mu = M_Z \approx 91.2$ GeV: $m_b \ll \mu$
- 

$\alpha_s^{(4)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$

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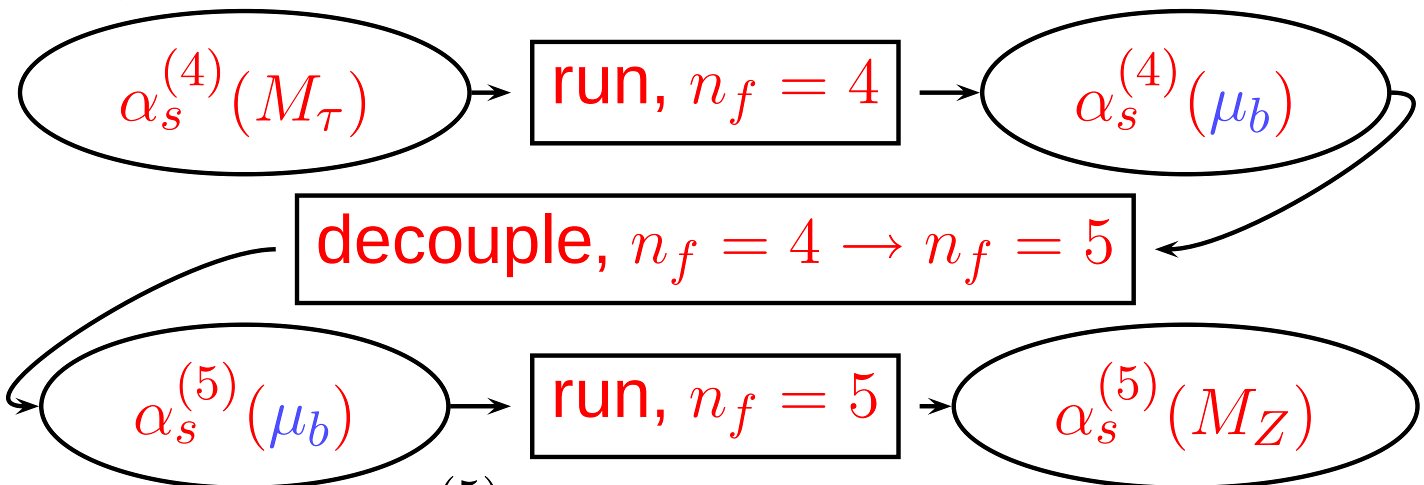
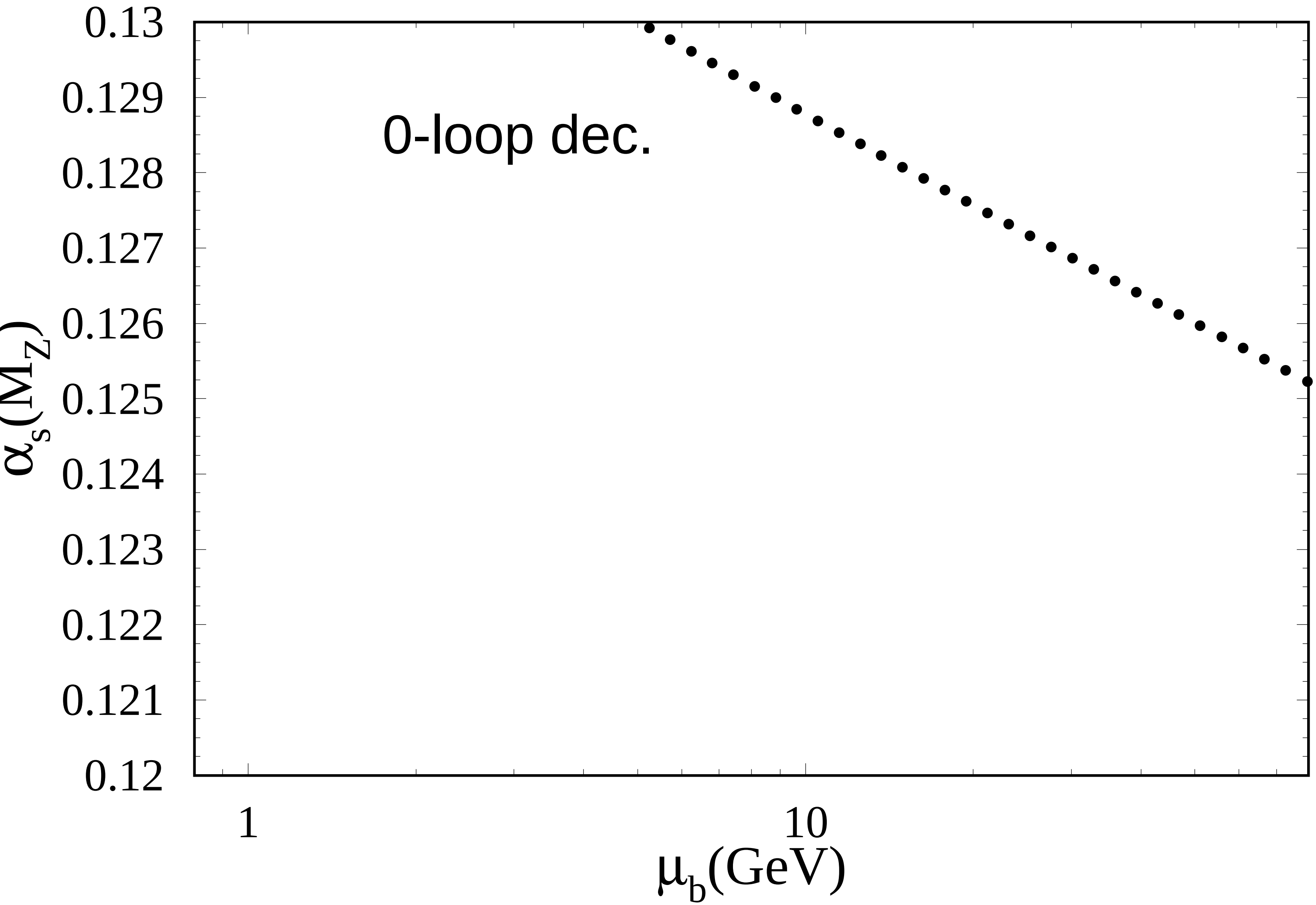
- 

- Dependence of $\alpha_s^{(5)}(M_Z)$ on μ_b
  for 0-, 1-, 2-, 3- and 4-loop decopling
- Note: n-loop decoupling $\Leftrightarrow$ $(n+1)$-loop running
- set 5-loop coefficient of β function to zero

$\alpha_s^{(4)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$



[Schröder,MS'05]

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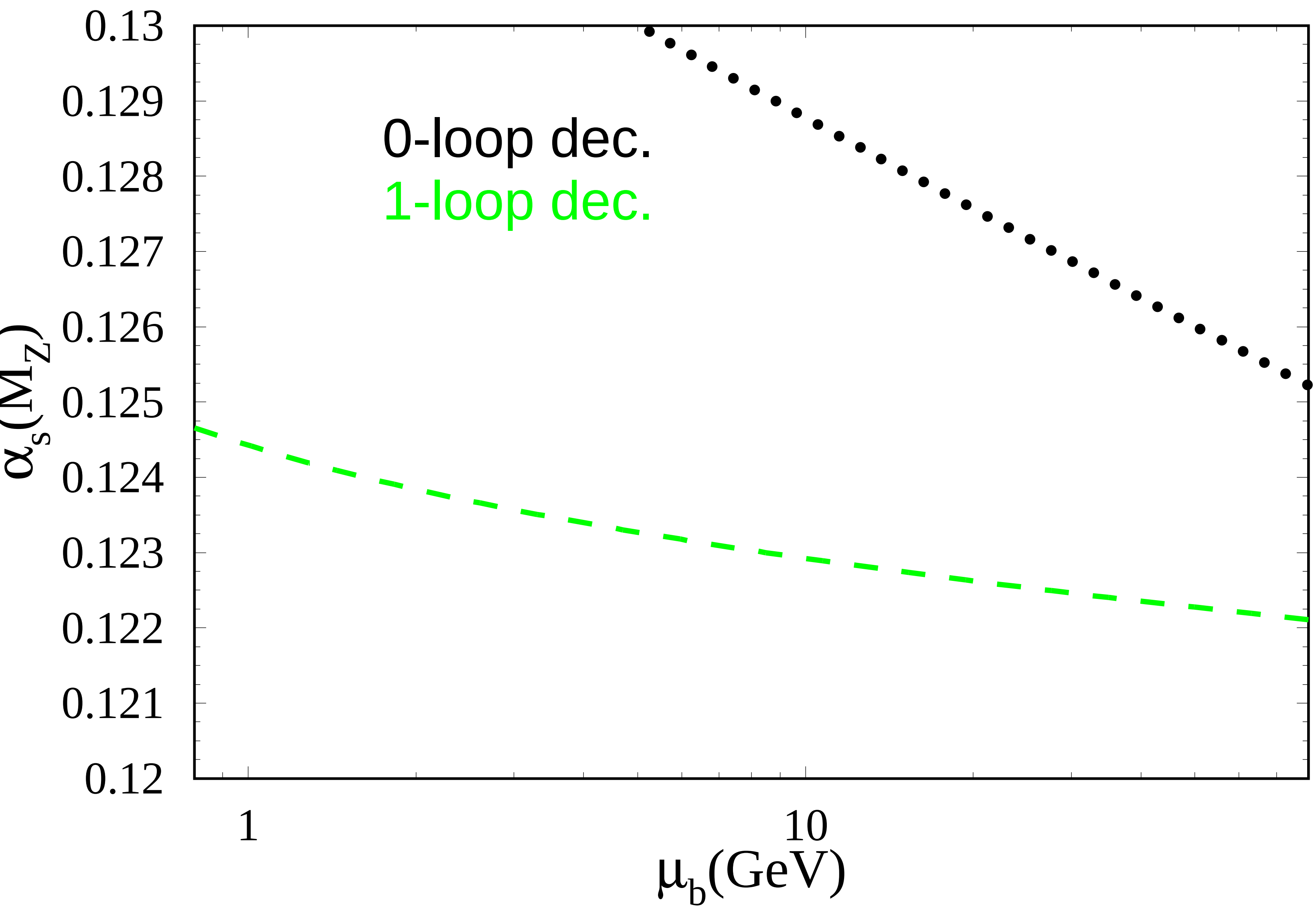

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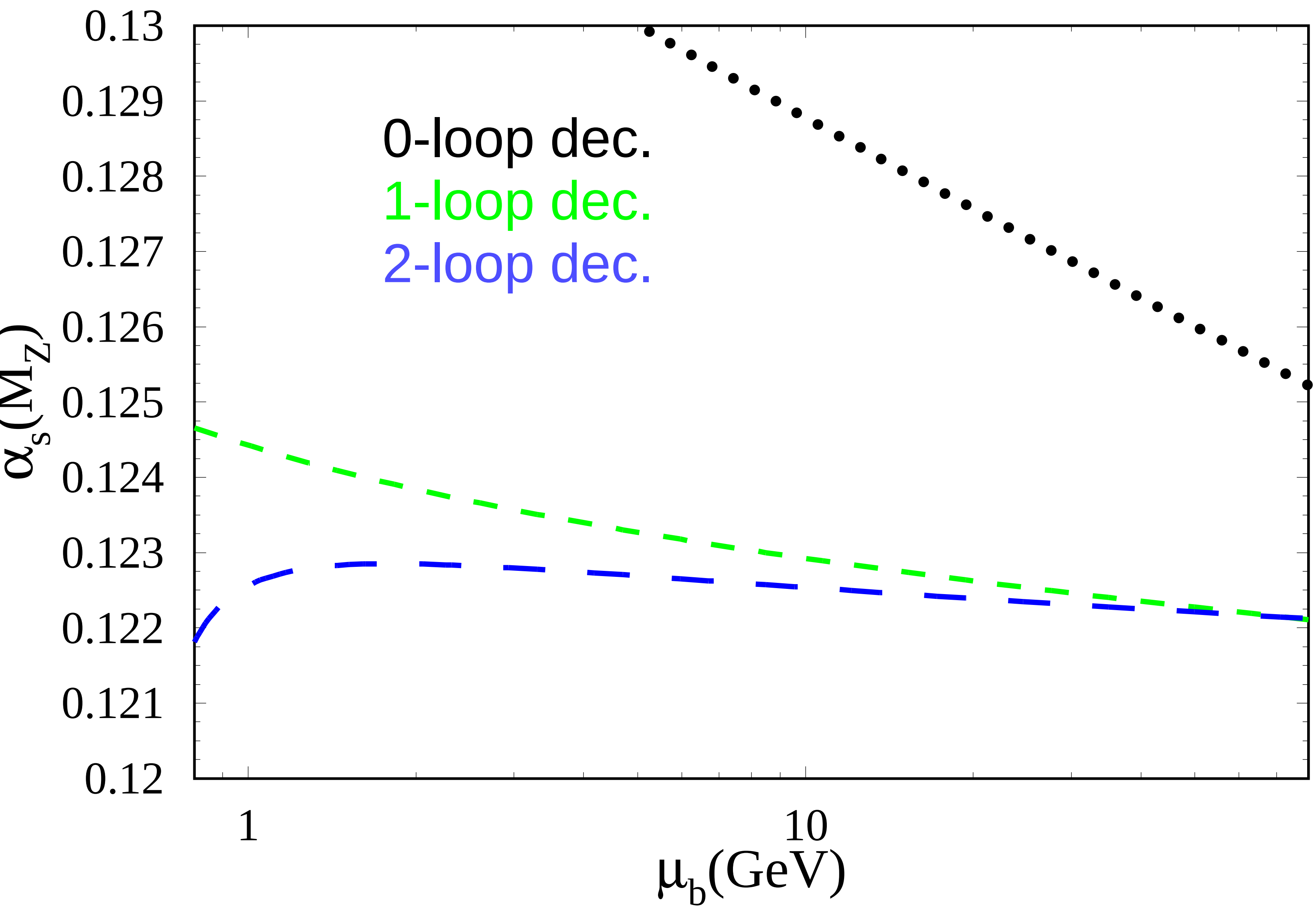

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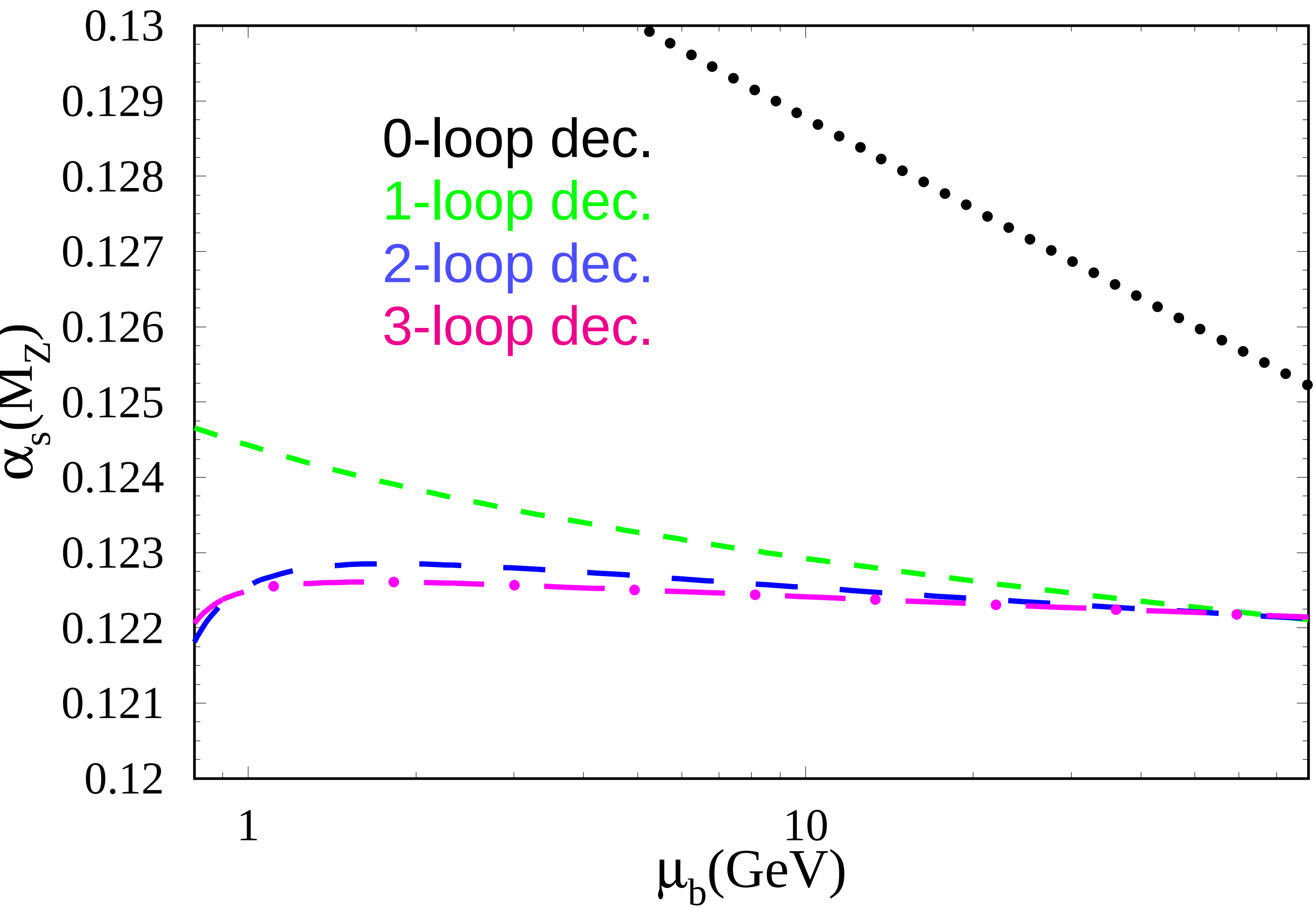
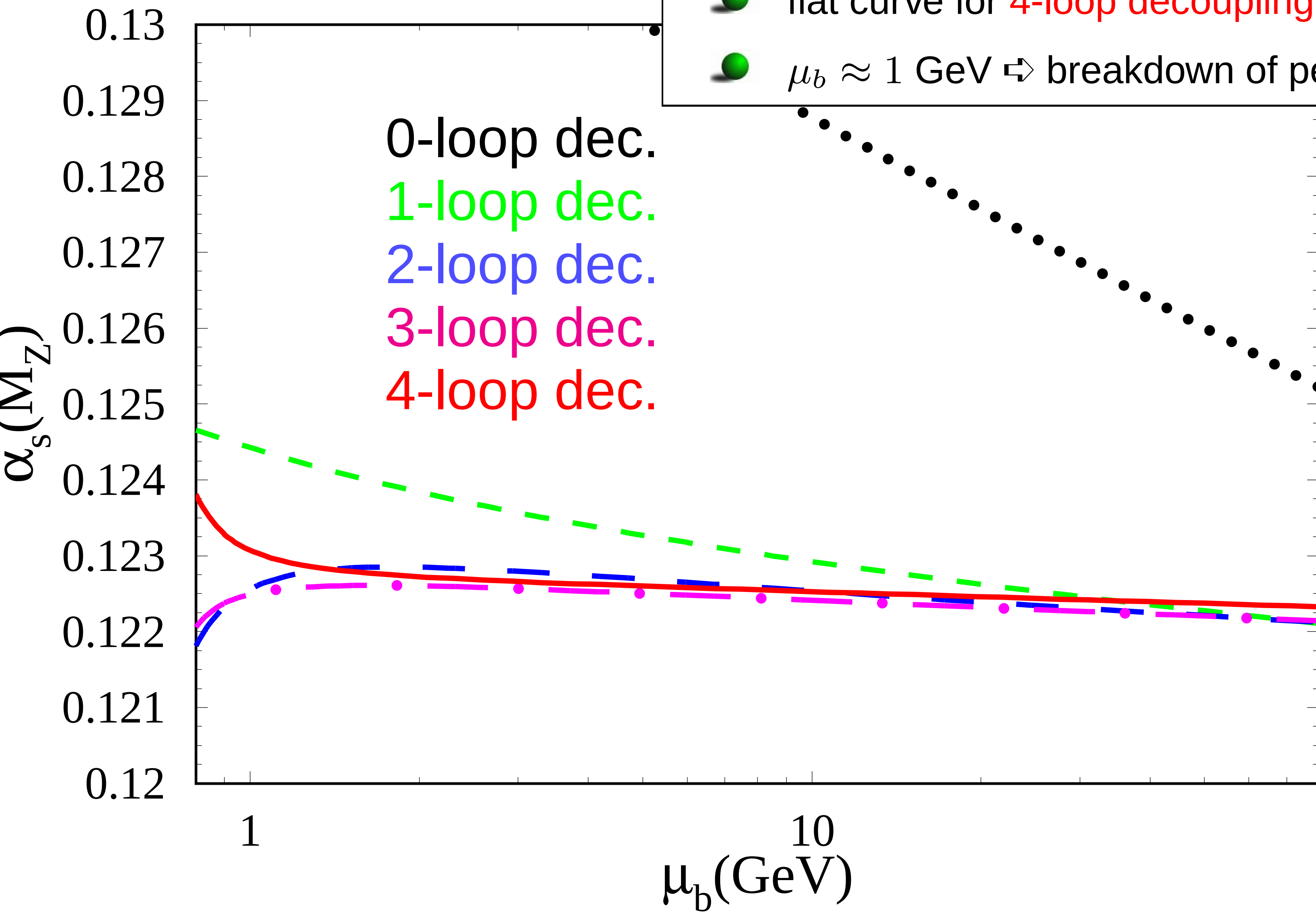
$$\alpha_s^{(4)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$$



[Schröder,MS'05]

$\alpha_s^{(4)}(M_\tau) \Rightarrow \alpha_s^{(5)}(M_Z)$

- flat curve for 4-loop decoupling
- $\mu_b \approx 1$ GeV $\Rightarrow$ breakdown of pert. th. (?)



[Schröder,MS'05]

$\alpha_s^{(3)}(M_\tau)$ ⇨ $\alpha_s^{(5)}(M_Z)$ — numbers

$\alpha_s(M_\tau) = 0.345 \pm 0.004_{\rm exp} \pm 0.008_{\rm th}$ [ALEPH'05; Davier,Höcker,Zhang'05]

3-loop decoupling ⇨

$\alpha_s(M_Z) = 0.1215 \pm 0.0004_{\rm exp} \pm 0.0010_{\rm th} \pm 0.0005_{\rm evol}$

δm_c	0.00020
δm_b	0.00005
μ_c, μ_b	0.00023
trunc. decoupling	0.00026
trunc. running	0.00031

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δm_c	0.00020	
δm_b	0.00005	
μ_c, μ_b	0.00023	⇨ 0.00011
trunc. decoupling	0.00026	⇨ 0.00013
trunc. running	0.00031	

4-loop decoupling ⇨

$\alpha_s(M_Z) = 0.1216 \pm 0.0004_{\rm exp} \pm 0.0010_{\rm th} \pm 0.0004_{\rm evol}$

[Chetyrkin,Kühn,Sturm'05]

ζ_g and the Higgs-gluon coupling

- $\mathcal{L}_{\text{eff}} = -\frac{H}{v} C_1 \mathcal{O}_1$ $\mathcal{O}_1 = \left(G^a_{\mu\nu}\right)^2$: effective operator
  C_1: coefficient function, coupling
- Low-Energy-Theorem (LET): [Chetyrkin,Kniehl,MS‘98]
  $C_1 = -\frac{1}{2} \frac{m_t^2 \partial}{\partial m_t^2} \ln \zeta_g^2$
- C_1: building block for $\Gamma(H \to gg)$and $\sigma(gg \to H)$
- ζ_g known to n loops ➪ C_1 known to $(n+1)$ loops
- result

note: $\zeta_g = \zeta_g(\ln(\mu/m_t))$

$$C_1 \approx -\frac{1}{12} \frac{\alpha_s^{(n_l+1)}(m_t)}{\pi}$$

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$$C_1 \approx -\frac{1}{12} \frac{\alpha_s^{(n_l+1)}(m_t)}{\pi} \left[1 + 2.7500 \frac{\alpha_s^{(n_l+1)}(m_t)}{\pi}\right.$$

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$$C_1 \approx -\frac{1}{12} \frac{\alpha_s^{(n_l+1)}(m_t)}{\pi} \left[1 + 2.7500 \frac{\alpha_s^{(n_l+1)}(m_t)}{\pi} + (9.7951 - 0.6979\, n_l) \left(\frac{\alpha_s^{(n_l+1)}(m_t)}{\pi}\right)^2\right.$$

ζ_g and the Higgs-gluon coupling

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$$\left. + \left(49.1827 - 7.7743\,n_l - 0.2207\,n_l^2\right)\left(\frac{\alpha_s^{(n_l+1)}(m_t)}{\pi}\right)^3\right.$$

ζ_g and the Higgs-gluon coupling

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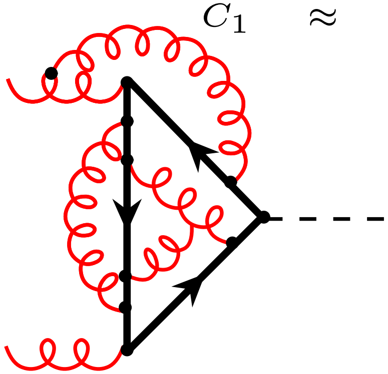


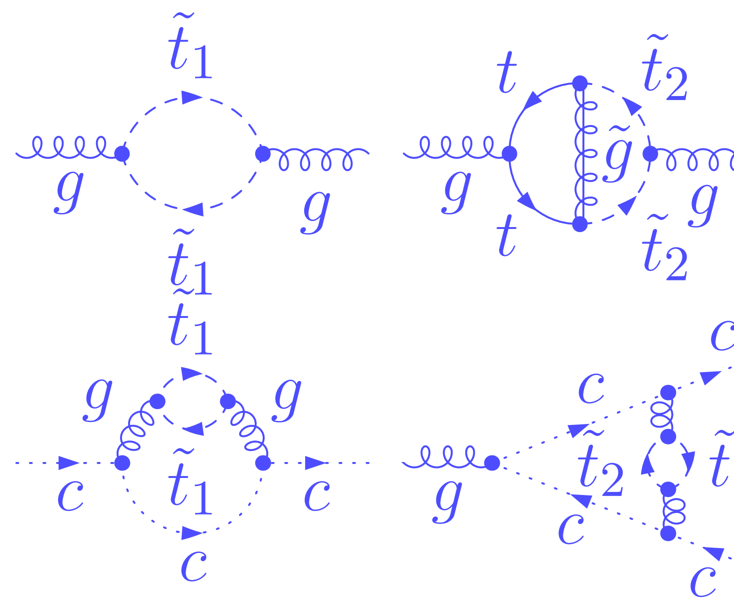
$$C_1 \approx -\frac{1}{12} \frac{\alpha_s^{(n_l+1)}(m_t)}{\pi} \Bigg[1 + 2.7500 \frac{\alpha_s^{(n_l+1)}(m_t)}{\pi} + (9.7951 - 0.6979\, n_l) \left(\frac{\alpha_s^{(n_l+1)}(m_t)}{\pi}\right)^2$$
$$+ \left(49.1827 - 7.7743\, n_l - 0.2207\, n_l^2\right) \left(\frac{\alpha_s^{(n_l+1)}(m_t)}{\pi}\right)^3$$
$$+ \left(-662.5065 + 137.6005\, n_l - 2.5367\, n_l^2 - 0.0775\, n_l^3 + 6\left(\beta_4^{(n_l)} - \beta_4^{(n_l+1)}\right)\right)$$
$$\times \left(\frac{\alpha_s^{(n_l+1)}(m_t)}{\pi}\right)^4 \Bigg]$$

[Schröder,MS’05; Chetyrkin,Kühn,Sturm’05]

II. Decoupling in the MSSM

Framework

- MSSM, SUSY-QCD
- Squarks: $\tilde{u}$, $\tilde{d}$, $\tilde{s}$, $\tilde{c}$, $\tilde{b}$, $\tilde{t}$; Gluino: $\tilde{g}$
- $\overline{\rm DR}$: Dimensional Reduction with minimal subtraction
- $\alpha_s^{\overline{\rm MS}}(M_Z) \leftrightarrow \alpha_s^{\overline{\rm DR}}(M_Z)$ in QCD [Bern,DeFreitas,Dixon,Wong'02]
- 3-loop SUSY β function [Jack,Jones,North'96]
- 2-loop matching [Harlander,Mihaila,MS'05]
- Scenarios:
  - (A) $m_{\tilde{u}}, \ldots, m_{\tilde{b}} \gg m_{\tilde{t}}, m_{\tilde{g}}, m_t \gg m_b$
  - (B) $m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}} \gg m_{\tilde{g}}, m_t \gg m_b$
  - (C) $m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}}, m_{\tilde{g}} \gg m_t \gg m_b$
  - (D) $m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}}, m_{\tilde{g}}, m_t \gg m_b$



- Simplification: $\tilde{M} \gg \tilde{m}, m_t \gg m_b$

Typical result

- $m_{\tilde u}, \ldots, m_{\tilde b} \gg m_{\tilde t}, m_{\tilde g}, m_t \gg m_b$

  $\alpha_s^{(\tilde t,\tilde g,6)} = \left(\zeta_g^{A1}\right)^2 \alpha_s^{(\text{full})}, \alpha_s^{(5)} = \left(\zeta_g^{A2}\right)^2 \alpha_s^{(\tilde t,\tilde g,6)}$

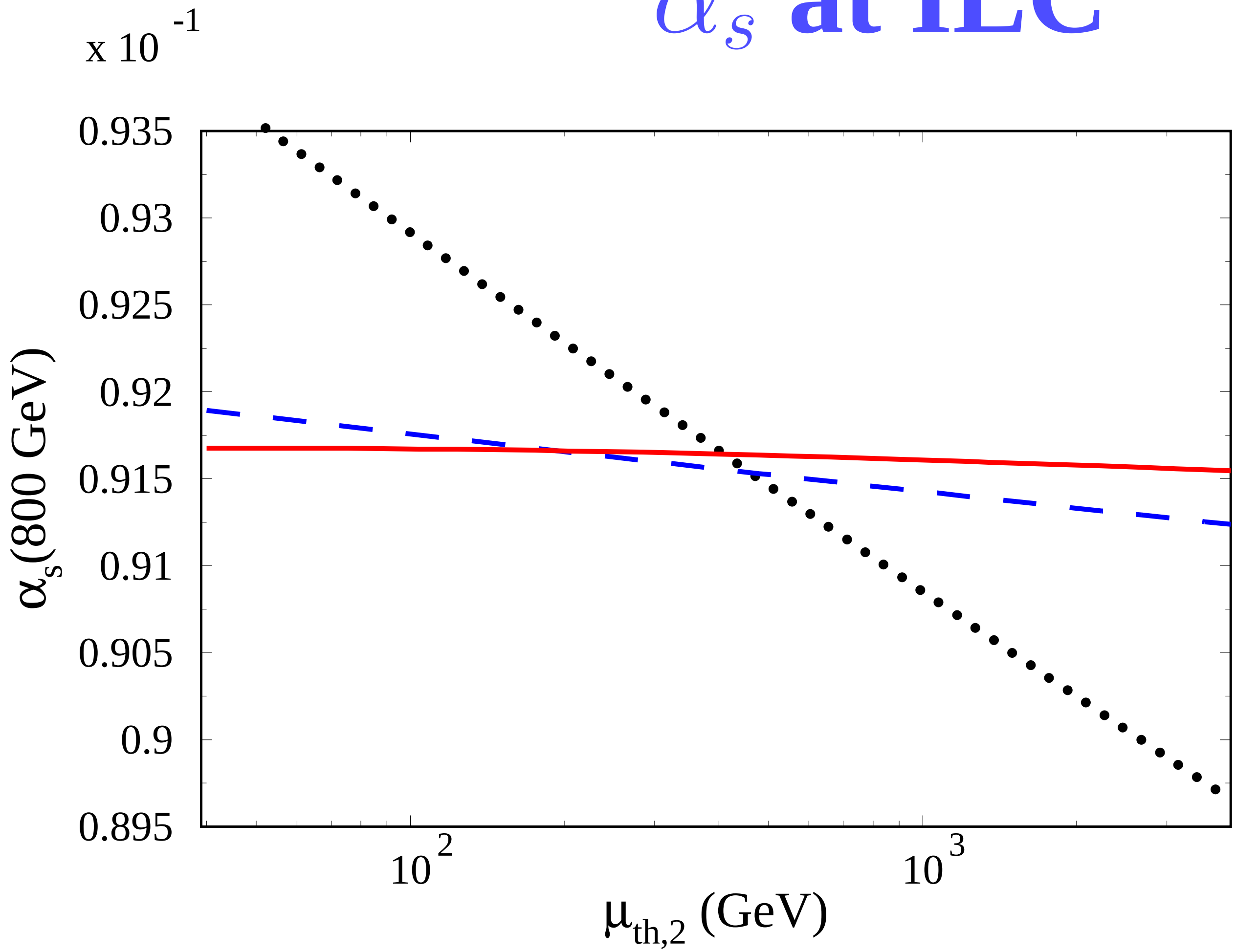
- $$\frac{1}{(\zeta_g^{A1})^2} = 1 + \frac{\alpha_s^{(\tilde t,\tilde g,6)}}{\pi}\frac{n_s}{12}L_{\tilde M} + \left(\frac{\alpha_s^{(\tilde t,\tilde g,6)}}{\pi}\right)^2\left(-\frac{13}{48}n_s - \frac{n_s}{12}L_{\tilde M} + \left(\frac{n_s}{12}L_{\tilde M}\right)^2\right)$$

  $$\begin{aligned}\frac{1}{(\zeta_g^{A2})^2} = 1 &+ \frac{\alpha_s^{(5)}}{\pi}\left(\frac{1}{6}L_t + \frac{7}{12}L_{\tilde m}\right)\\ &+ \left(\frac{\alpha_s^{(5)}}{\pi}\right)^2\left[\frac{265}{96} + \frac{19}{24}L_t + \frac{35}{12}L_{\tilde m} + \left(\frac{1}{6}L_t + \frac{7}{12}L_{\tilde m}\right)^2\right.\\ &+ \left(\frac{m_t}{\tilde m}\right)^2\left(-\frac{5}{48} - \frac{3}{8}L_{t\tilde m}\right) + \frac{7\pi}{36}\left(\frac{m_t}{\tilde m}\right)^3\\ &+ \left.\left(\frac{m_t}{\tilde m}\right)^4\left(-\frac{881}{7200} + \frac{1}{80}L_{t\tilde m}\right) - \frac{7\pi}{288}\left(\frac{m_t}{\tilde m}\right)^5 + \ldots\right]\end{aligned}$$

  [Harlander,Mihaila,MS'05]

- $\tilde m = m_t$ ➪ approximation better than 1%

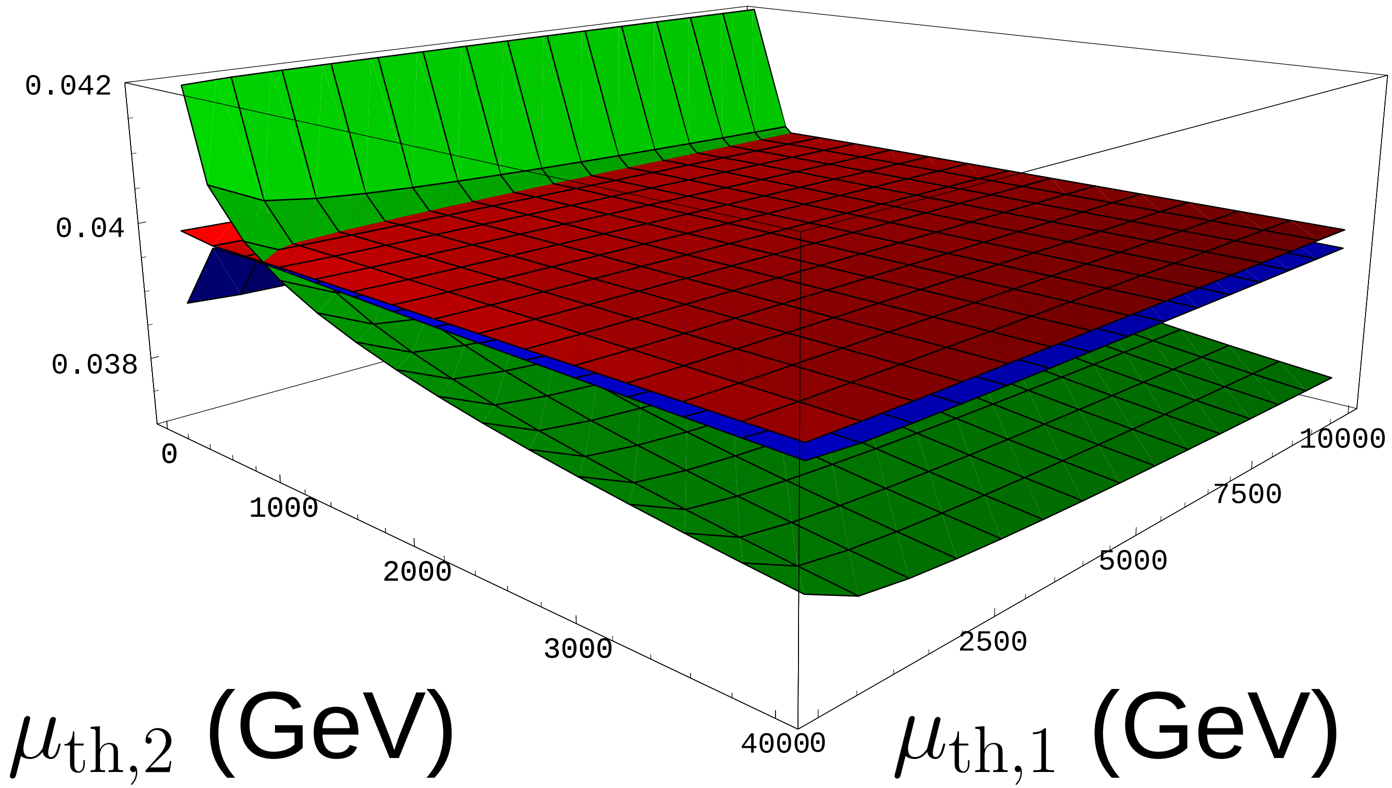
α_s at ILC



running:
1-loop
2-loop
3-loop

- $m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}}, m_{\tilde{g}} \gg m_t \gg m_b$ $\qquad \alpha_s^{(5)} = \left(\zeta_g^{C2}\right)^2 \alpha_s^{(6)}$
- $\alpha_s(M_Z) = 0.120 \pm 0.002$ ⇨ $\alpha_s(800\text{ GeV}) = 0.0915 \pm 0.0012$
- $\mu_{\text{th},2} = 2000$ GeV: $\qquad \delta\alpha_s(\text{3-loop} - \text{2-loop}) \approx 0.0025$

$\alpha_s(\mu_{\rm GUT})$

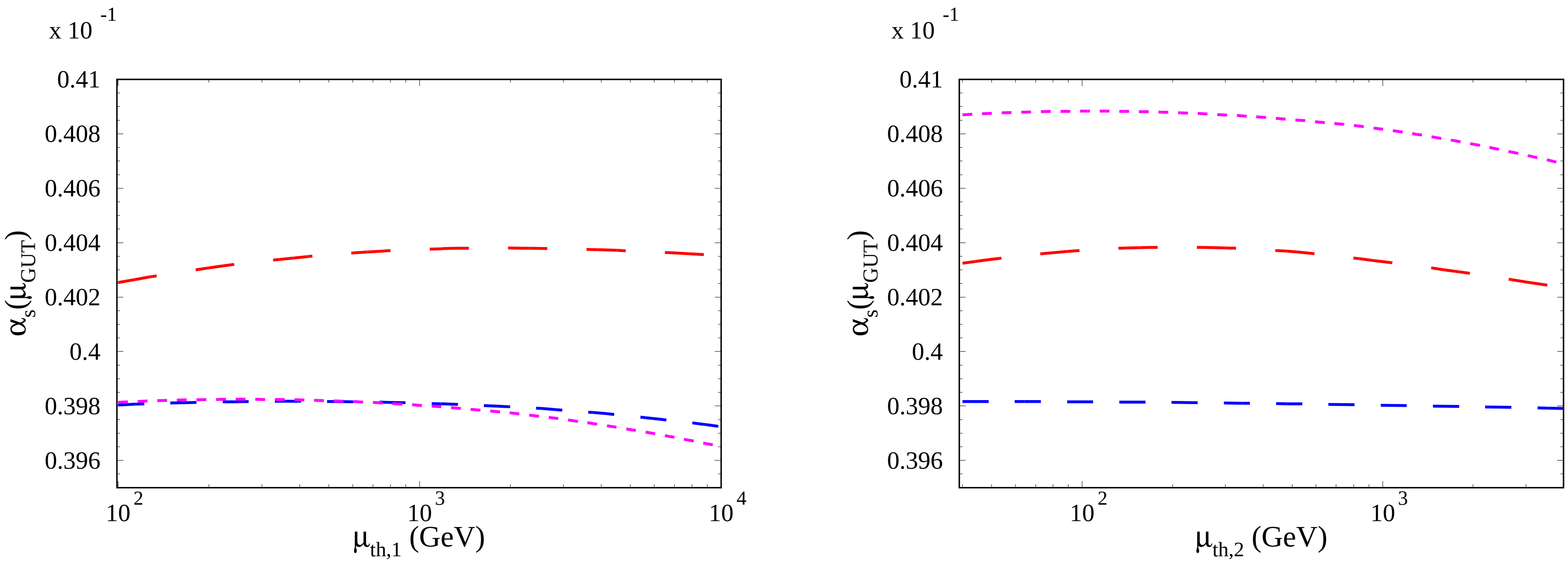


running:
1-loop
2-loop
3-loop

- $m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}}, m_{\tilde{g}} \gg m_t \gg m_b$
- $\alpha_s^{(6)} = \left(\zeta_g^{\rm C1}\right)^2 \alpha_s^{\rm (full)}, \qquad \alpha_s^{(5)} = \left(\zeta_g^{\rm C2}\right)^2 \alpha_s^{(6)}$
- 40 GeV $\le \mu_{\rm th,2} \le$ 4 000 GeV
  100 GeV $\le \mu_{\rm th,1} \le$ 10 000 GeV

$\tilde{m} = 400$ GeV
$\tilde{M} = 1000$ GeV

Comparison of scenarios



- 3-loop curves for:

  $m_{\tilde{u}}, \ldots, m_{\tilde{t}} \gg m_{\tilde{g}}, m_t \gg m_b$

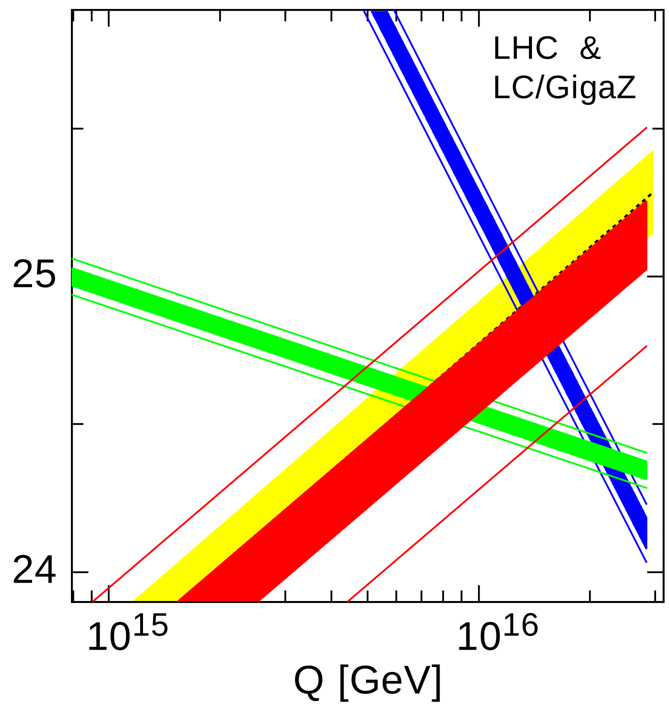
  $m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}}, m_{\tilde{g}} \gg m_t \gg m_b$

  $m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}}, m_{\tilde{g}}, m_t \gg m_b$

- $\alpha_s(\mu_{\mathrm{GUT}}) =$

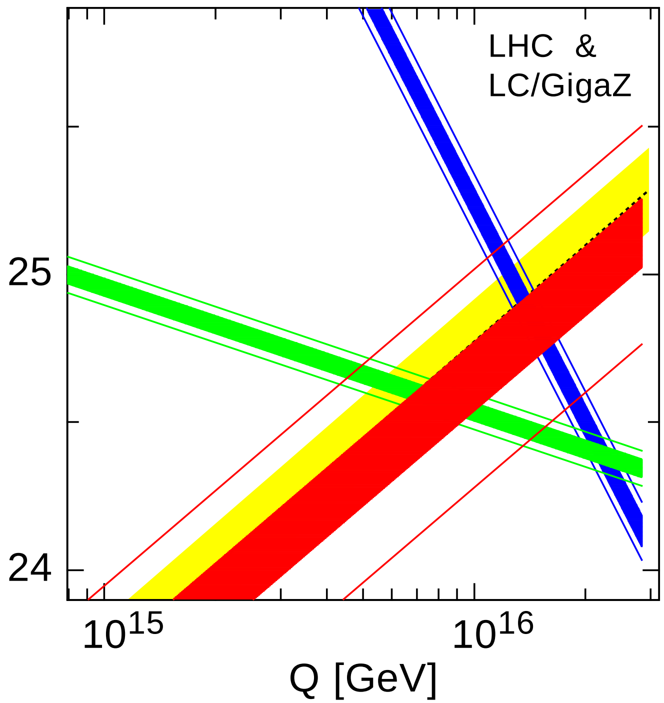
$$0.039\,8 \pm 0.000\,23\Big|_{\delta\alpha_s(M_Z)} \pm 0.001\,00\Big|_{\mathrm{masses}} \pm 0.000\,07\Big|_{\mathrm{th}}$$

Unification in the MSSM

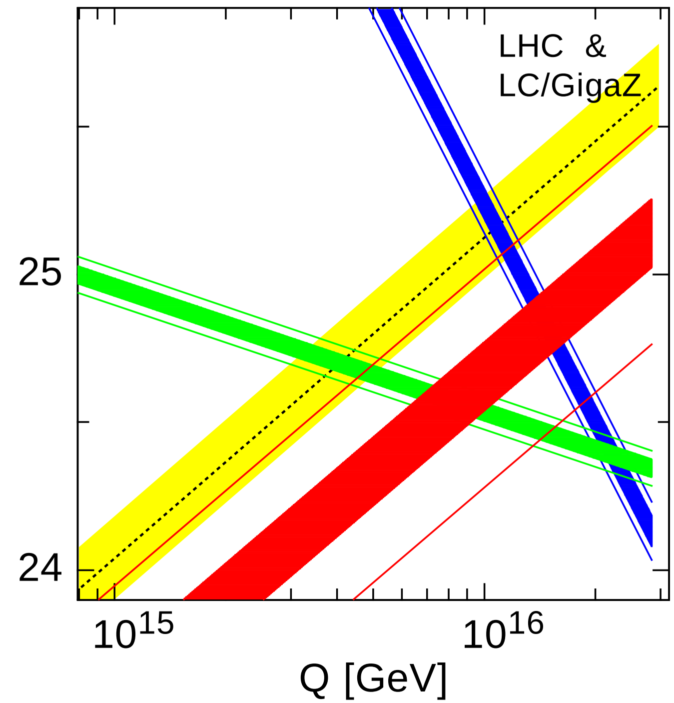


$$m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}} \gg m_{\tilde{g}}, m_t \gg m_b$$

Unification in the MSSM



$m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}} \gg m_{\tilde{g}}, m_t \gg m_b$



$m_{\tilde{u}}, \ldots, m_{\tilde{b}}, m_{\tilde{t}}, m_{\tilde{g}} \gg m_t \gg m_b$

- mass effects are important
- 3-loop running and 2-loop matching needed for precise unification

Conclusions

- QCD
  - 4-loop decoupling relation
  - stable prediction for $\alpha_s(M_Z)$ from $\alpha_s(M_\tau)$
  - Higgs-gluon coupling to 5-loop order
- MSSM
  - 2-loop decoupling $\leftrightarrow$ 3-loop running
  - various scenarios with 2 matching scales
  - significant dependence on SUSY mass pattern
  - influence on unification
  - ➪ needs to be considered in the context of SPA (Supersymmetry Parameter Analysis)