Precision Calculations in the MSSM

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Loops and Legs 2006
Eisenach, April 23 - 28, 2006

- Introduction
- Standard Particles
- Higgs bosons
- SUSY particles

Precision analysis required for

- Indirect tests of the MSSM
  $\longrightarrow$ virtual SUSY effects in precision observables

- Precision studies for SUSY particles
  $\longrightarrow$ determination of masses & couplings
  $\longrightarrow$ reconstruction of model parameters

- Direct versus indirect tests
  $\longrightarrow$ precision observables for precisely measured SUSY parameters
  $\longrightarrow$ consistency check

Processes with external

(i) standard particles

(ii) Higgs bosons, especially light Higgs h^0

(iii) SUSY particles
– the chargino and neutralino sector
– the sfermion sector

(expected) experimental precision

error for	LEP/Tev	Tev/LHC	LC	GigaZ
M_W [MeV]	33	15	15	7
$\sin^2\theta_{\text{eff}}$	0.00017	0.00021		0.000013
m_{top} [GeV]	2.3	2	0.2	0.13
M_{Higgs} [GeV]	–	0.1	0.05	0.05

together with

$\delta M_Z = 2.1$ MeV (LEP)

$\delta G_F/G_F = 1 \cdot 10^{-5}$ (μ lifetime)

Detailed analysis for SPS1a benchmark scenario: potential of LHC (300 fb^{-1}) alone and LHC $+$ LC

	LHC	LHC+LC
$\Delta m_{\tilde{\chi}_1^0}$	4.8	0.05 (input)
$\Delta m_{\tilde{l}_R}$	4.8	0.05 (input)
$\Delta m_{\tilde{\chi}_2^0}$	4.7	0.08
$\Delta m_{\tilde{q}_L}$	8.7	4.9
$\Delta m_{\tilde{q}_R}$	11.8	10.9
$\Delta m_{\tilde{g}}$	8.0	6.4
$\Delta m_{\tilde{b}_1}$	7.5	5.7
$\Delta m_{\tilde{b}_2}$	7.9	6.2
$\Delta m_{\tilde{l}_L}$	5.0	0.2 (input)
$\Delta m_{\tilde{\chi}_4^0}$	5.1	2.23

LHC+LC accuracy limited by LHC jet energy scale resolution

SPS 1a benchmark scenario:

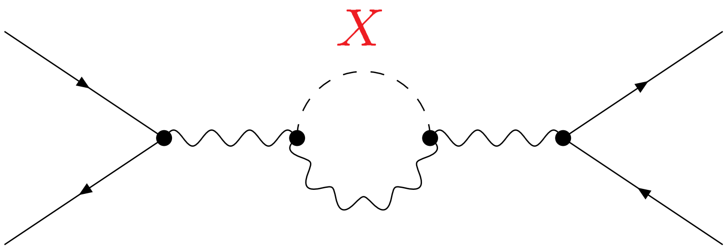
favorable scenario for both LHC and LC

$\Rightarrow$ LC input improves accuracy significantly

Standard Particles

Test of theory at quantum level:

Sensitivity to loop corrections



- μ lifetime: M_W, Δr, G_F

- Z observables: g_V, g_A, $\sin^2\theta_\mathrm{eff}$, Γ_Z, ...

recent review:

Heinemeyer, WH, Weiglein, Phys. Rep. 425 (2006) 265

- new: M_W with 2-loop improvements
  $\mathcal{O}(\alpha\alpha_s,\ \alpha_t^2,\ \alpha_b^2,\ \alpha_t\alpha_b)$
  and complex parameters

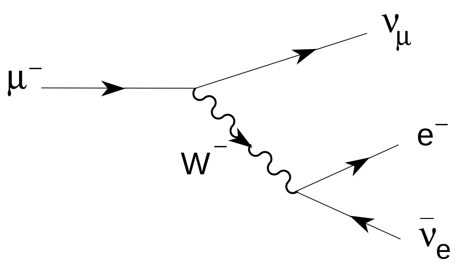
  Heinemeyer, WH, Stöckinger, A. Weber, Weiglein, hep-ph/0604147

$M_W - M_Z$ correlation

Definition of Fermi constant G_F via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1 + \Delta q)$$

Δq: QED corrections in Fermi Model,
included in definition



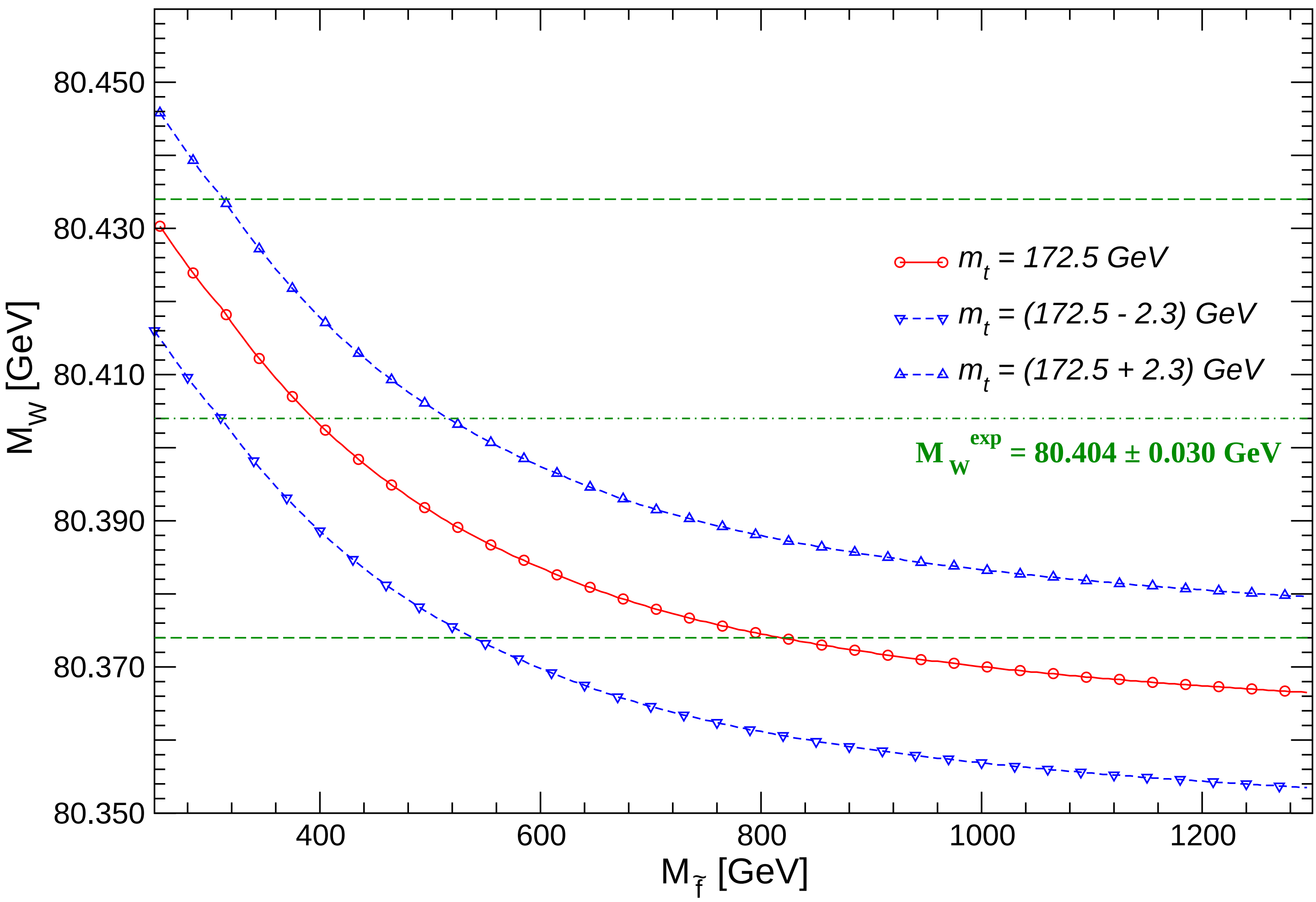
SM/MSSM prediction:

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2\left(1 - M_W^2/M_Z^2\right)}(1 + \Delta r)$$

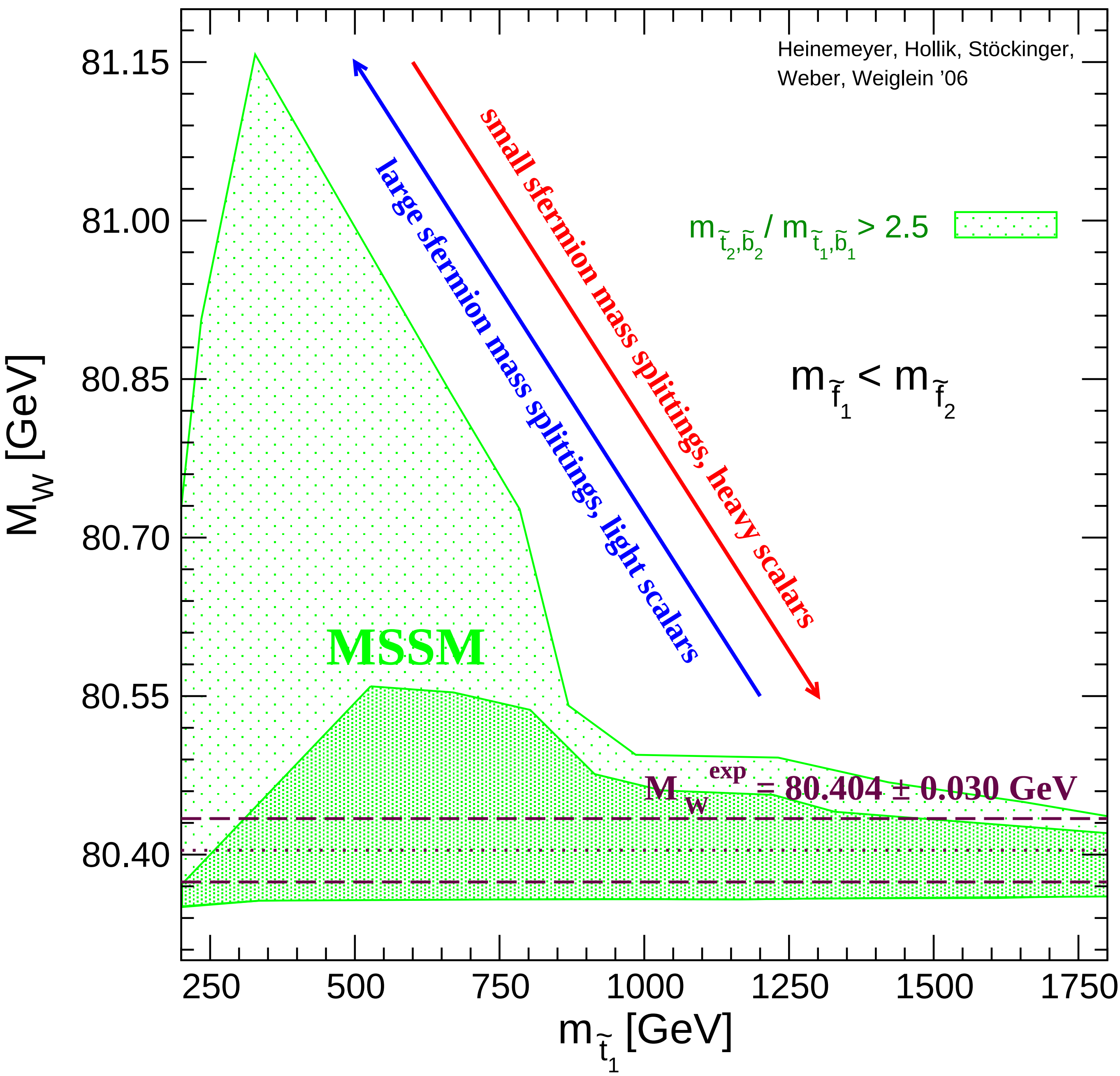
Δr: quantum correction, $\Delta r = \Delta r(m_t, M_H, \ldots)$

$\rightarrow \quad M_W = M_W(\alpha, G_F, M_Z, m_t, X_{\mathrm{SUSY}})$

$$\Delta r = \Delta r_{\mathrm{SM}} + (\Delta r_{\mathrm{MSSM}} - \Delta r_{\mathrm{SM}})$$



$$\tan\beta = 10, \quad M_A = \mu = M_2 = M_{\tilde{g}} = 300\,\text{GeV}$$



Heinemeyer, Hollik, Stöckinger,
Weber, Weiglein '06
81.15
81.00
80.85
80.70
80.55
80.40
M_W [GeV]
small sfermion mass splittings, heavy scalars
large sfermion mass splittings, light scalars
m_t̃2,b̃2 / m_t̃1,b̃1 > 2.5
m_f̃1 < m_f̃2
MSSM
M_W^exp = 80.404 ± 0.030 GeV
250
500
750
1000
1250
1500
1750
m_t̃1 [GeV]

The Higgs sector of the MSSM

- Two $SU(2) \times U(1)$ doublets: $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$

$$H_i^0 = \frac{v_i + S_i + i\,P_i}{\sqrt{2}} \qquad\qquad \tan\beta = \frac{v_2}{v_1}$$

- The soft SUSY-breaking mass terms for H_1^0 and H_2^0 are responsible for electroweak symmetry breaking (EWSB):

$$\begin{aligned} V_{\rm tree} &= (m_{H_1}^2 + \mu^2)\,|H_1^0|^2 + (m_{H_2}^2 + \mu^2)\,|H_2^0|^2 \\ &+ B\,\left(H_1^0 H_2^0 + \text{h.c.}\right) + \frac{1}{8}(g^2 + g'^2)\,\left(|H_1^0|^2 - |H_2^0|^2\right)^2 \end{aligned}$$

- Five physical states: $h\,,\ H\,,\ A^0\,,\ H^+\,,\ H^-$

- Tree–level mass matrix for the CP–even sector:

$$\left(\mathcal{M}_S^2\right)^{\rm tree} = \begin{pmatrix} m_Z^2\,c_\beta^2 + m_A^2\,s_\beta^2 & -\left(m_Z^2 + m_A^2\right)s_\beta\,c_\beta \\ -\left(m_Z^2 + m_A^2\right)s_\beta\,c_\beta & m_Z^2\,s_\beta^2 + m_A^2\,c_\beta^2 \end{pmatrix}$$

$\rightarrow$ m_h and m_H are predicted in terms of m_Z, m_A and $\tan\beta$

- Tree–level mass relation: $m_h^2 \leq \cos^2 2\beta\, m_Z^2$

dressed Higgs propagators

$$(\Delta_{\text{Higgs}})^{-1} = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_H(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{Hh}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_h(q^2) \end{pmatrix}$$

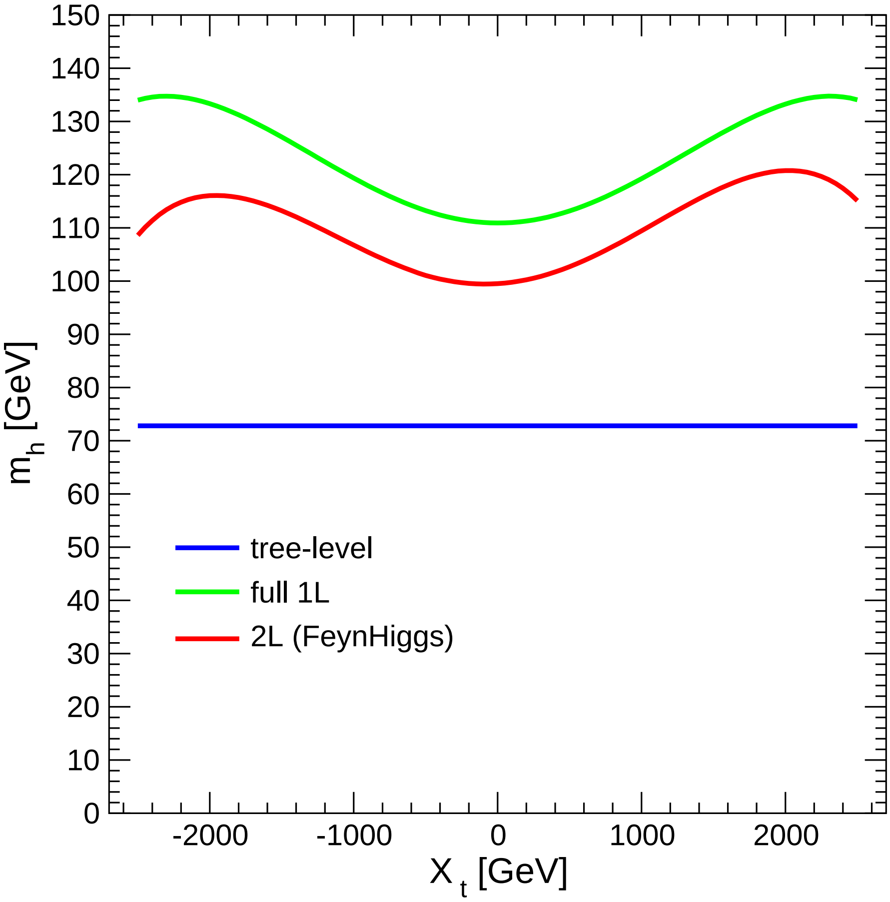
- det = 0 $\quad\longrightarrow\quad m_{h,H}^{\text{pole}}$
- diagonalization $\quad\longrightarrow\quad$ effective couplings (α_{eff})

renormalized self-energies $\hat{\Sigma}$

1-loop: complete

2-loop: QCD corrections $\sim \alpha_s\alpha_t,\ \alpha_s\alpha_b$
Yukawa corrections $\sim \alpha_t^2$
[→ FeynHiggs]

m_{h^0} prediction at different levels of accuracy:



$\tan\beta = 3$, $\quad M_{\tilde{Q}} = M_A = 1$ TeV, $\quad m_{\tilde{g}} = 800$ GeV

X_t : top-squark mixing parameter

$$X_t = A_t - \mu \cot\beta \,, \qquad \mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{t}_L} & m_t X_t \\ m_t X_t & m^2_{\tilde{t}_R} \end{pmatrix}$$

present theoretical uncertainty: $\quad \delta m_h \simeq 4$ GeV

[Degrassi, Heinemeyer, WH, Slavich, Weiglein]

Recent developments:

1. Counterterms at two-loop order

   ST identities valid in dimensional reduction (DR)

   DR scheme consistent with symmetric counterterms

   [WH, Stöckinger]

2. $\mathcal{O}(\alpha_s \alpha_b)$ beyond m_b^{eff} approximation

   $m_b^{\text{eff}} = \frac{m_b}{1+\Delta m_b}$ in α_b Yukawa coupling

   $\Delta m_b =$ non-decoupling SUSY contribution

   $\sim \alpha_s \tan\beta$

   [Heinemeyer, WH, Rzehak, Weiglein]

   small shifts $\sim$ few GeV,
   but stabilizes prediction

3. MSSM with complex parameters at $\mathcal{O}(\alpha_s \alpha_t)$

   tree level: CP conserving Higgs sector

   loop level: CP violation $\leftarrow$ other sectors

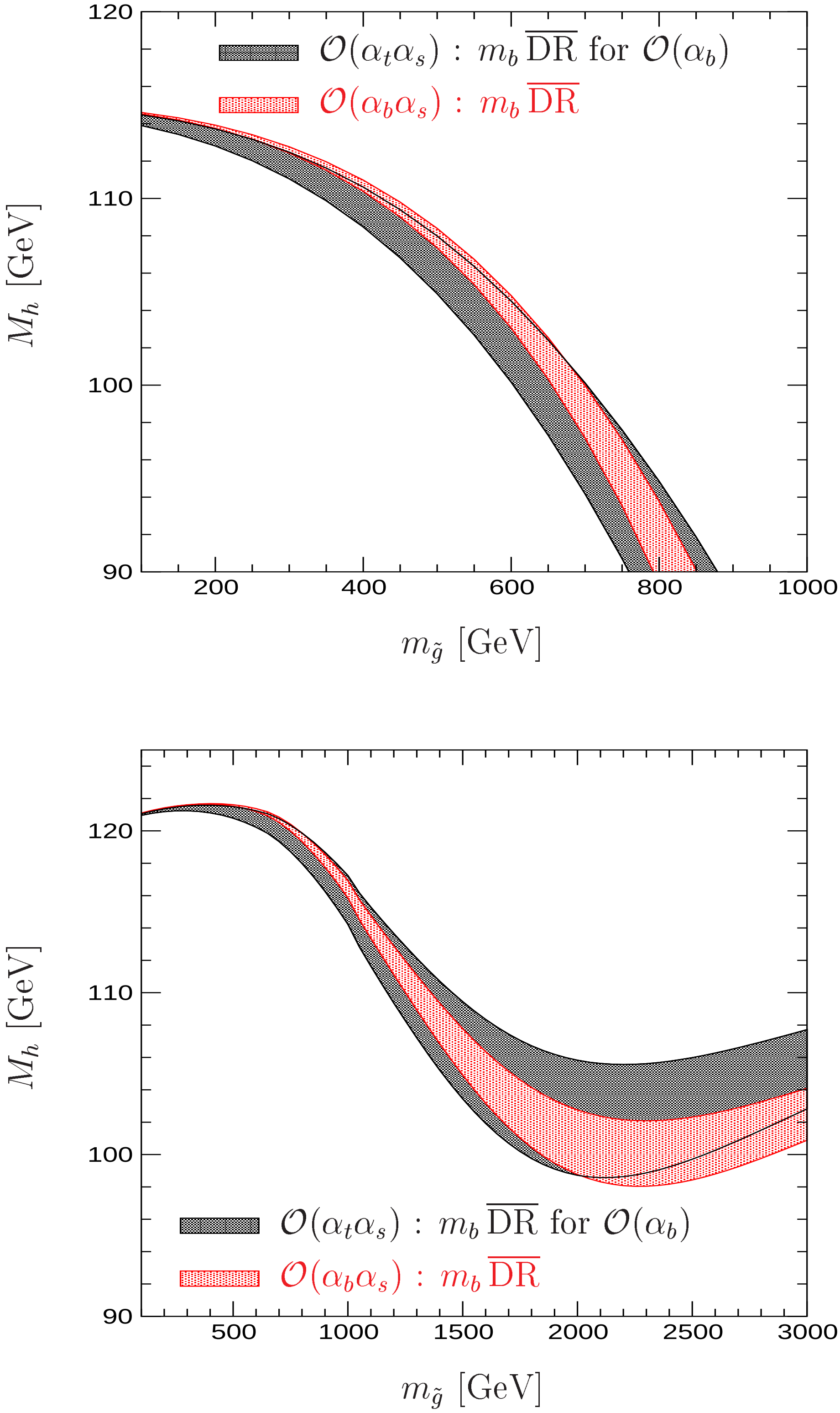
   $$(h, H, A) \rightarrow (h_3, h_2, h_1)$$

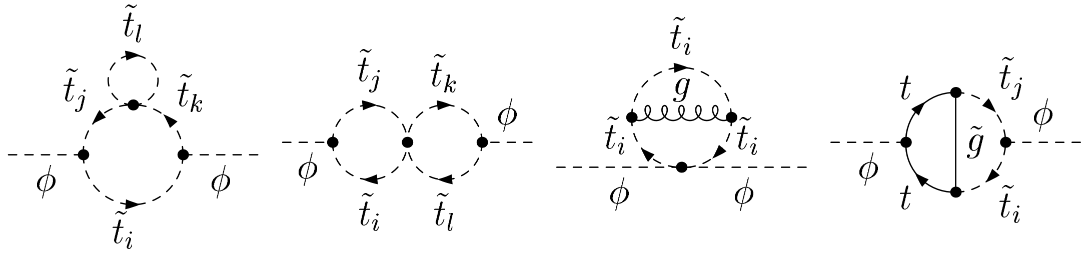
   $$m_3 > m_2 > m_1$$

   [Heinemeyer, WH, Rzehak, Weiglein]

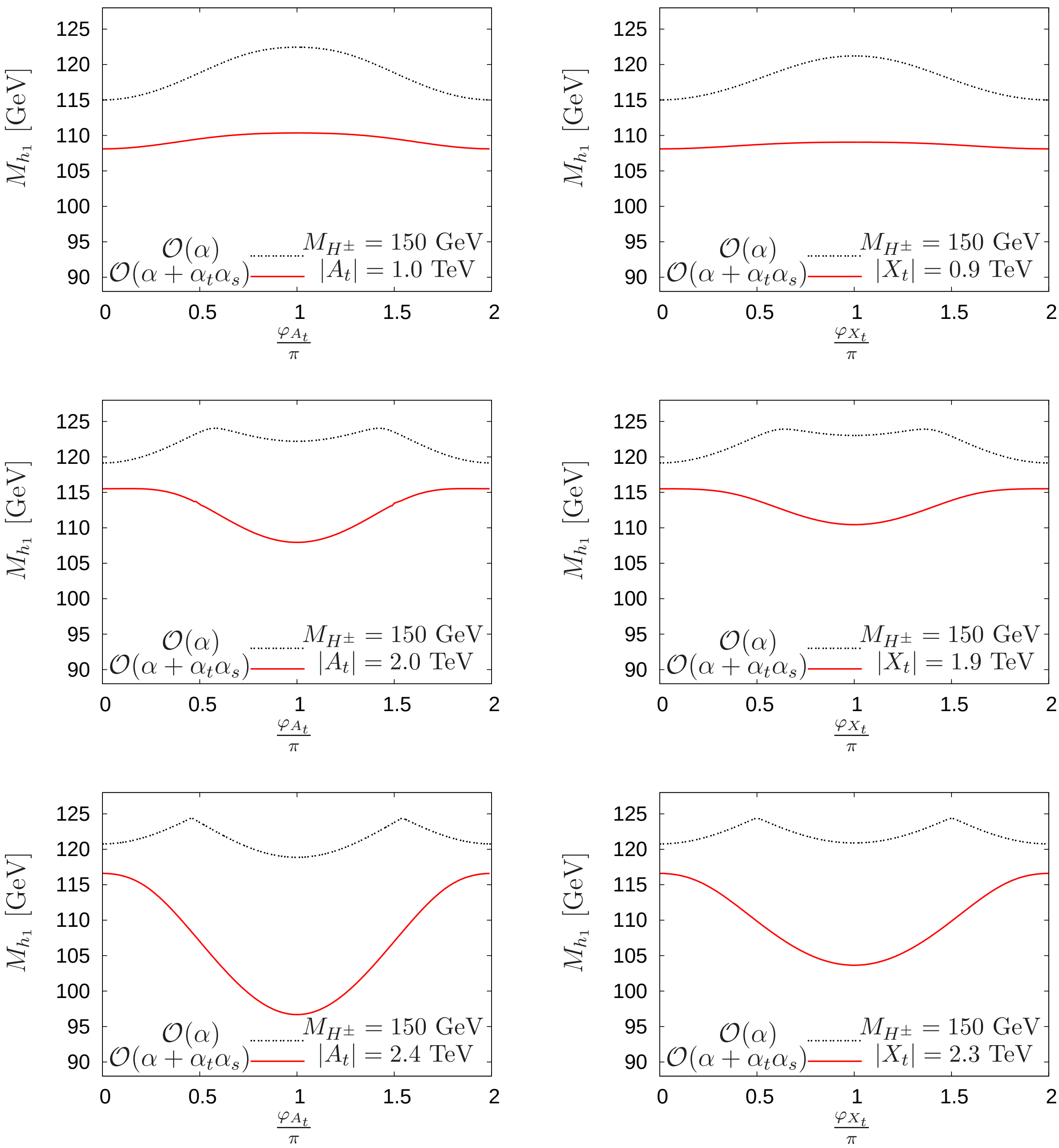
$m_t/2 < \mu^{\overline{\rm DR}} < 2\, m_t$

$M_A = \begin{cases} 120 \text{ GeV} \\ 700 \text{ GeV} \end{cases}$





phase dependence of m_{h_1}



SUSY particles

- LHC will see SUSY if at low energy scale
- LC and LHC$\oplus$LC for precision studies
- Reconstruction of fundamental SUSY theory and breaking mechanism

from experiment:

$\longrightarrow$ precision analyses of masses and couplings including higher orders

from theory:

$\longrightarrow$ accurate theoretical predictions to match exp. data

$\longrightarrow$ loop contributions Lagrangian param $\leftrightarrow$ observables

$\longrightarrow$ RGEs for extrapolation to high scales

chargino/neutralino sector

complete at one loop [Fritzsche,WH/Eberl,Majerotto,...]
renormalization and mass spectrum
pair production in e^+e^- collisions

sfermion sector

renormalization and mass spectrum
[WH, Rzehak]

$$\begin{pmatrix} m_f^2 + M_L^2 + M_Z^2 c_{2\beta}(I_f^3 - Q_f s_W^2) & m_f(A_f - \mu\kappa) \\ m_f(A_f - \mu\kappa) & m_f^2 + M_{\tilde{f}_R}^2 + M_Z^2 c_{2\beta} Q_f s_W^2 \end{pmatrix}$$

sfermion pair production in e^+e^- collisions
complete at one-loop

[Arhrib, WH]	squarks, sleptons
[Kovarik, Weber, Eberl, Majerotto]	squarks
[Freitas, Miller, von Manteuffel, Zerwas]	sleptons

sfermion decays into fermions and -inos

complete at one-loop
[Guasch, WH, Solà]

Basis for precision calculations

- complete Feynman rules $\longrightarrow$ FeynArts
  [Hahn, Schappacher]

- complete set of counter terms
  automatic generation $\longrightarrow$ FeynArts
  [Fritzsche]

- real photon bremsstrahlung

Renormalization schemes

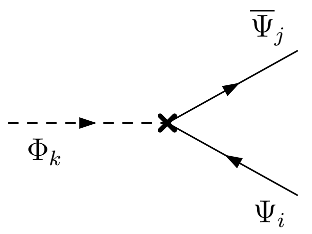
- on-shell scheme:
  renormalization conditions for pole masses
  [WH, Kraus, Roth, Rupp, Sibold, Stöckinger]

- $\overline{\text{DR}}$ scheme:
  CTs = singular parts in dimensional reduction

SUSY parameters different in $\overline{\text{DR}}$ and on-shell

example:

FFS



$$= ie\left[\vec{C}_{\rm FFS}\left(\overline{\Psi}_j, \Psi_i, \Phi_k\right)\cdot\begin{pmatrix}\omega_{\rm L}\\ \omega_{\rm R}\end{pmatrix}\right]$$

- $$\vec{C}_{\rm FFS}\left(\overline{e_{\rm j}}, \tilde{\chi}^0_{\rm i}, \tilde{e}^{\rm s}_{\rm k}\right) = \frac{\delta_{\rm jk}}{2\sqrt{2}\, c_\beta^2\, c_w^3\, M_W^3\, s_w^2}\times$$

$$\left[\begin{array}{c}
-c_\beta\, c_w^2\, M_W^2\, s_w \sum_{n=1}^{4}\left[\delta Z_{\tilde{\chi}^0}\right]_{n\rm i}\left(c_w\, m_{e_{\rm j}}\, N^*_{n3}\, U^{\tilde{e}_{\rm j}*}_{\rm s1} + 2\, c_\beta\, M_W\, s_w\, N^*_{n1}\, U^{\tilde{e}_{\rm j}*}_{\rm s2}\right) - \\
c_\beta\, c_w^2\, M_W^2\, s_w \sum_{n=1}^{2}\left[\delta Z^{\rm k}_{\tilde{e}}\right]_{n\rm s}\left(c_w\, m_{e_{\rm j}}\, N^*_{\rm i3}\, U^{\tilde{e}_{\rm j}*}_{n1} + 2\, c_\beta\, M_W\, s_w\, N^*_{\rm i1}\, U^{\tilde{e}_{\rm j}*}_{n2}\right) - \\
\left(c_w^3\, N^*_{\rm i3}\, U^{\tilde{e}_{\rm j}*}_{\rm s1}\right)\Big\{2\, c_\beta\, M_W^2\, s_w\, \delta m^{\rm e}_{\rm j} + m_{e_{\rm j}}\Big[-2\,\delta c_\beta\, M_W^2\, s_w + c_\beta\, M_W^2\, s_w\,\left[\delta Z^{\rm R}_{\rm e}\right]^*_{\rm jj} + \\
c_\beta\,\left(-\delta M_W^2\, s_w - 2\, M_W^2\,\left(\delta s_w - \delta Z_e\, s_w\right)\right)\Big]\Big\} - \\
\left(2\, c_w^2\,\delta Z_e + 2\,\delta s_w\, s_w + c_w^2\,\left[\delta Z^{\rm R}_{\rm e}\right]^*_{\rm jj}\right)\left(2\, c_\beta^2\, M_W^3\, s_w^2\, N^*_{\rm i1}\, U^{\tilde{e}_{\rm j}*}_{\rm s2}\right) \\
\\
c_\beta\, c_w^2\, M_W^2\, s_w \sum_{n=1}^{4}\left[\delta Z_{\tilde{\chi}^0}\right]^*_{n\rm i}\left(\left(s_w\, N_{n1} + c_w\, N_{n2}\right)\left(c_\beta\, M_W\, U^{\tilde{e}_{\rm j}*}_{\rm s1}\right) - c_w\, m_{e_{\rm j}}\, N_{n3}\, U^{\tilde{e}_{\rm j}*}_{\rm s2}\right) + \\
c_\beta\, c_w^2\, M_W^2\, s_w \sum_{n=1}^{2}\left[\delta Z^{\rm k}_{\tilde{e}}\right]_{n\rm s}\left(\left(s_w\, N_{\rm i1} + c_w\, N_{\rm i2}\right)\left(c_\beta\, M_W\, U^{\tilde{e}_{\rm j}*}_{n1}\right) - c_w\, m_{e_{\rm j}}\, N_{\rm i3}\, U^{\tilde{e}_{\rm j}*}_{n2}\right) - \\
\left(c_w^3\, N_{\rm i3}\, U^{\tilde{e}_{\rm j}*}_{\rm s2}\right)\Big\{2\, c_\beta\, M_W^2\, s_w\, \delta m^{\rm e}_{\rm j} + m_{e_{\rm j}}\Big[-2\,\delta c_\beta\, M_W^2\, s_w + c_\beta\, M_W^2\, s_w\,\left[\delta Z^{\rm L}_{\rm e}\right]^*_{\rm jj} + \\
c_\beta\,\left(-\delta M_W^2\, s_w - 2\, M_W^2\,\left(\delta s_w - \delta Z_e\, s_w\right)\right)\Big]\Big\} + \\
\left(c_\beta^2\, M_W^3\, U^{\tilde{e}_{\rm j}*}_{\rm s1}\right)\Big[\left(2\, c_w^2\,\delta Z_e + 2\,\delta s_w\, s_w + c_w^2\,\left[\delta Z^{\rm L}_{\rm e}\right]^*_{\rm jj}\right)\left(s_w^2\, N_{\rm i1}\right) + \\
\left(s_w\,\left[\delta Z^{\rm L}_{\rm e}\right]^*_{\rm jj} - 2\,\left(\delta s_w - \delta Z_e\, s_w\right)\right)\left(c_w^3\, N_{\rm i2}\right)\Big]
\end{array}\right]$$

SUPERSYMMETRY PARAMETER ANALYSIS

The SPA project is a joint study of theorists and experimentalists working on LHC and Linear Collider phenomenology. The study focuses on the supersymmetric extension of the Standard Model. The main targets are

- High-precision determination of the supersymmetry Lagrange parameters at the electroweak scale
- Extrapolation to a high scale to reconstruct the fundamental parameters and the mechanism for supersymmetry breaking

http://spa.desy.de/spa

P. Zerwas, J. Kalinowski, H.U. Martyn,
W. Hollik, W. Kilian, W. Majerotto,
W. Porod, ...

hep-ph/0511344

SPA CONVENTION

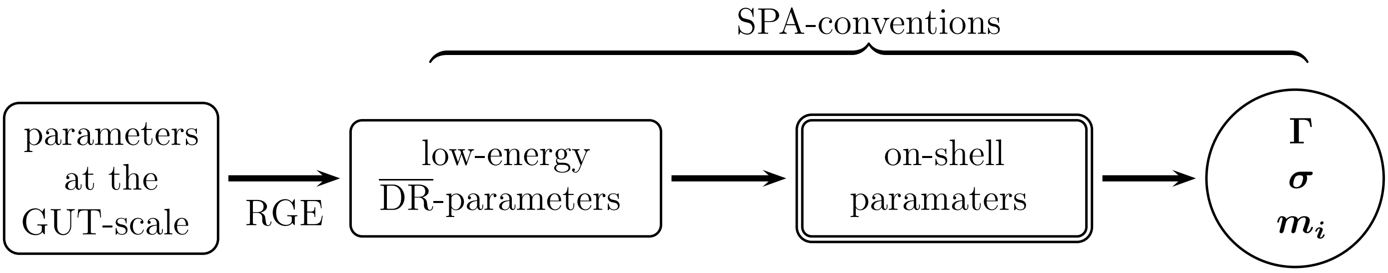
- The masses of the SUSY particles and Higgs bosons are defined as pole masses.

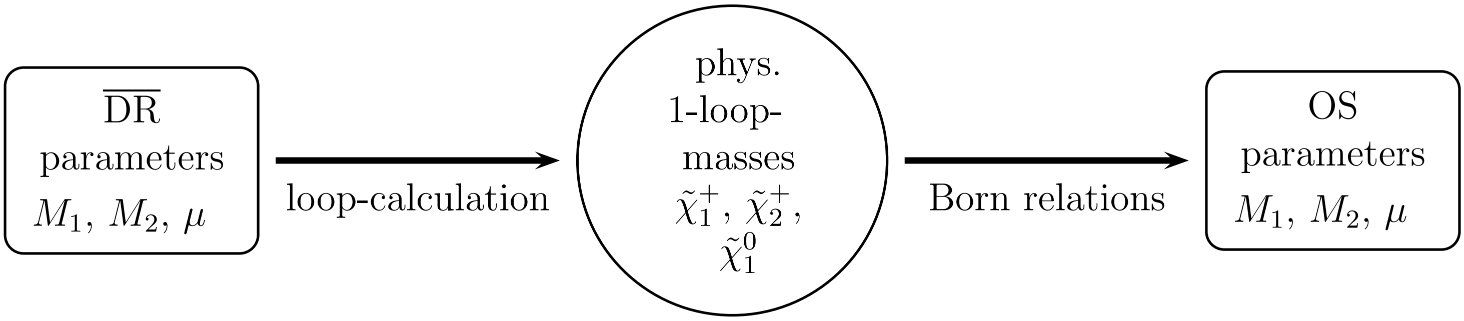
- All SUSY Lagrangian parameters, mass parameters and couplings, including $\tan\beta$, are given in the $\overline{\text{DR}}$ scheme and defined at the scale $\tilde{M} = 1$ TeV.

- Gaugino/higgsino and scalar mass matrices, rotation matrices and the corresponding angles are defined in the $\overline{\text{DR}}$ scheme at $\tilde{M}$, except for the Higgs system in which the mixing matrix is defined in the on-shell scheme, the momentum scale chosen as the light Higgs mass.

- The Standard Model input parameters of the gauge sector are chosen as G_F, α, M_Z and $\alpha_s^{\overline{\text{MS}}}(M_Z)$. All lepton masses are defined on-shell. The t quark mass is defined on-shell; the b, c quark masses are introduced in $\overline{\text{MS}}$ at the scale of the masses themselves while taken at a renormalization scale of 2 GeV for the light u, d, s quarks.

- Decay widths/branching ratios and production cross sections are calculated for the set of parameters specified above.





pole masses $\leftrightarrow$ on-shell MSSM input

$$
\begin{aligned}
M_2^2 + \mu^2 + 2M_W^2 &= m^2_{\tilde\chi_1^+} + m^2_{\tilde\chi_2^+} \\
\left(M_2\,\mu - 2M_W^2 \sin\beta\,\cos\beta\right)^2 &= m^2_{\tilde\chi_1^+}\, m^2_{\tilde\chi_2^+} \quad .
\end{aligned}
$$

$$
\begin{aligned}
M_1 \;=\; & \Big[-M_2\mu M_Z^2 \sin 2\beta + \big[\mu M_Z^2 \sin 2\beta - M_2\big(\mu^2 + M_Z^2 s_W^2\big)\big] m_{\tilde\chi_1^0} \\
& \quad + \big[\mu^2 + M_Z^2\big] m^2_{\tilde\chi_1^0} + M_2 m^3_{\tilde\chi_1^0} - m^4_{\tilde\chi_1^0}\Big] \\
& \times \Big[\mu M_Z^2 c_W^2 \sin 2\beta - M_2\mu^2 + \big[\mu^2 + M_Z^2 c_W^2\big] m_{\tilde\chi_1^0} + M_2 m^2_{\tilde\chi_1^0} - m^3_{\tilde\chi_1^0}\Big]^{-1}
\end{aligned}
$$

$\overline{\text{DR}}$ parameters (SPS1a')

$$
\begin{aligned}
\tan\beta &= 10 \quad ; \quad \mu = 402.87 \text{ GeV} \\
M_{A^0} &= 431.02 \text{ GeV} \; ; \; M_1 = 103.22 \text{ GeV} \\
M_3 &= 572.33 \text{ GeV} \; ; \; M_2 = 193.31 \text{ GeV}
\end{aligned}
$$

$$
\begin{aligned}
A_{u,c} &= -784.7 \text{ GeV} \; ; \; A_t = -535.4 \text{ GeV} \\
A_{d,s} &= -1025.7 \text{ GeV} \; ; \; A_b = -938.5 \text{ GeV} \\
A_{e,\mu} &= -449.0 \text{ GeV} \; ; \; A_\tau = -445.5 \text{ GeV}
\end{aligned}
$$

$$
\begin{aligned}
m_{\tilde{l}}^{1,2} &= 181.3 \text{ GeV} \; ; \; m_{\tilde{l}}^3 = 179.5 \text{ GeV} \\
m_{\tilde{e},\tilde{\mu}} &= 115.6 \text{ GeV} \; ; \; m_{\tilde{\tau}} = 109.8 \text{ GeV} \\
m_{\tilde{q}}^{1,2} &= 526.9 \text{ GeV} \; ; \; m_{\tilde{q}}^3 = 471.3 \text{ GeV} \\
m_{\tilde{u},\tilde{c}} &= 507.7 \text{ GeV} \; ; \; m_{\tilde{t}} = 384.6 \text{ GeV} \\
m_{\tilde{d},\tilde{s}} &= 505.5 \text{ GeV} \; ; \; m_{\tilde{t}} = 501.3 \text{ GeV}
\end{aligned}
$$

on-shell parameters

$$
\begin{aligned}
\tan\beta &= 10 \quad ; \quad \mu = 399.26 \text{ GeV} \\
M_{A^0} &= 431.02 \text{ GeV} \; ; \; M_1 = 100.11 \text{ GeV} \\
M_3 &= 612.85 \text{ GeV} \; ; \; M_2 = 197.55 \text{ GeV}
\end{aligned}
$$

$$
\begin{aligned}
A_{u,c} &= -784.7 \text{ GeV} \; ; \; A_t = -535.4 \text{ GeV} \\
A_{d,s} &= -1025.7 \text{ GeV} \; ; \; A_b = -938.5 \text{ GeV} \\
A_{e,\mu} &= -449.0 \text{ GeV} \; ; \; A_\tau = -445.5 \text{ GeV}
\end{aligned}
$$

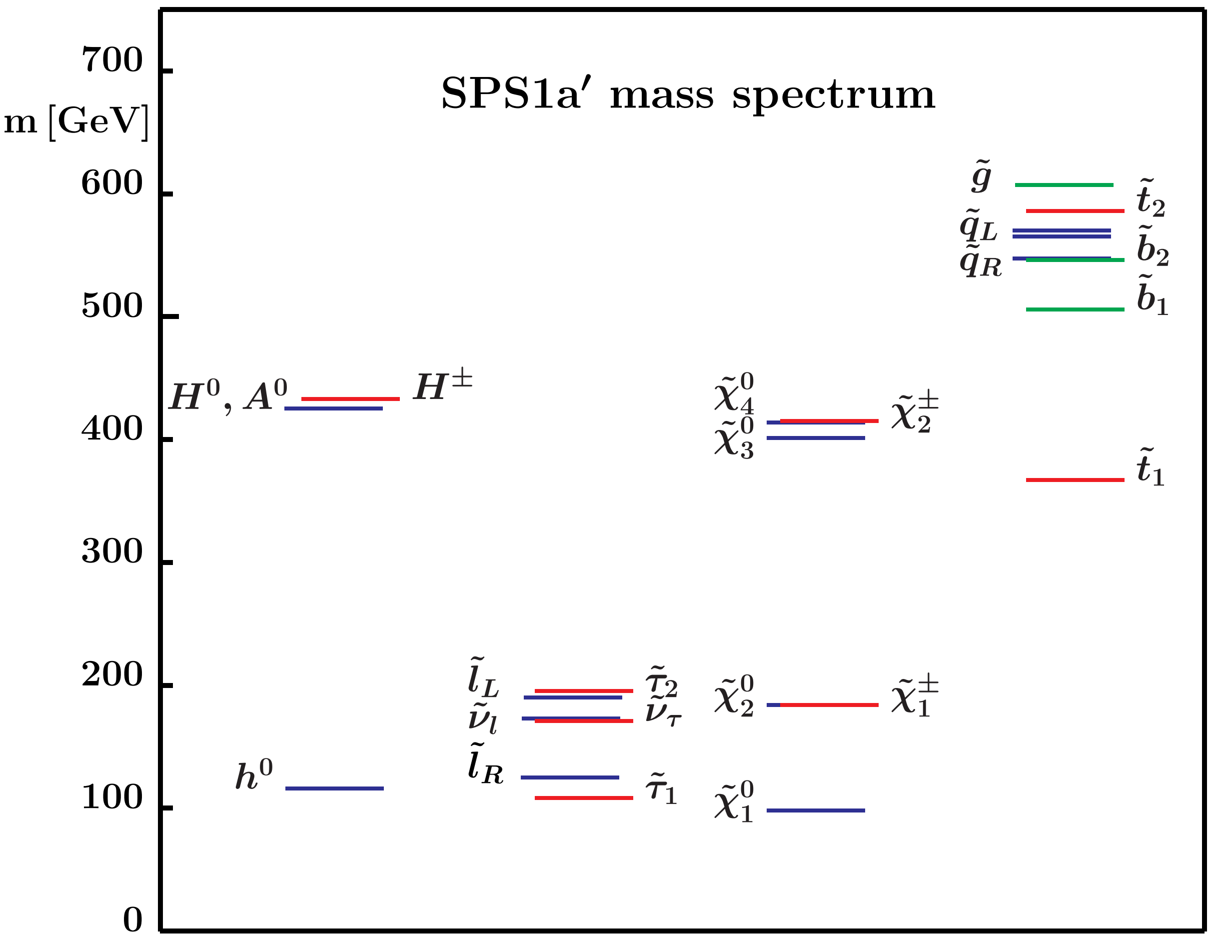
$$
\begin{aligned}
m_{\tilde{l}}^1 &= 184.12 \text{ GeV} \; ; \; m_{\tilde{l}}^2 = 184.11 \text{ GeV} \; ; \; m_{\tilde{l}}^3 = 182.18 \text{ GeV} \\
m_{\tilde{e}} &= 118.02 \text{ GeV} \; ; \; m_{\tilde{\mu}} = 117.99 \text{ GeV} \; ; \; m_{\tilde{\tau}} = 111.29 \text{ GeV} \\
m_{\tilde{q}}^1 &= 565.97 \text{ GeV} \; ; \; m_{\tilde{q}}^2 = 565.91 \text{ GeV} \; ; \; m_{\tilde{q}}^3 = 453.05 \text{ GeV} \\
m_{\tilde{u}} &= 546.78 \text{ GeV} \; ; \; m_{\tilde{c}} = 546.84 \text{ GeV} \; ; \; m_{\tilde{t}} = 460.52 \text{ GeV} \\
m_{\tilde{d}} &= 544.95 \text{ GeV} \; ; \; m_{\tilde{s}} = 544.97 \text{ GeV} \; ; \; m_{\tilde{t}} = 538.13 \text{ GeV}
\end{aligned}
$$

$\overline{\text{DR}}$ masses $\longrightarrow$ pole masses (SPS1a')

		m	δm	m_{phys}			m	δm	m_{phys}
•	$m_{\tilde{\chi}_1^+}$	$= 181.026 +$	3.178	$= 184.204$	•	$m_{\tilde{\chi}_1^0}$	$= 100.706 +$	(-2.958)	$= 97.748$
•	$m_{\tilde{\chi}_2^+}$	$= 423.420 +$	(-2.181)	$= 421.239$		$m_{\tilde{\chi}_2^0}$	$= 181.404 +$	3.022	$= 184.425$
						$m_{\tilde{\chi}_3^0}$	$= 408.579 +$	(-1.626)	$= 406.952$
•	$m_{\tilde{g}}$	$= 572.330 +$	40.524	$= 612.854$		$m_{\tilde{\chi}_4^0}$	$= 422.991 +$	(-3.310)	$= 419.681$
•	$m_{\tilde{\nu}_1}$	$= 169.890 +$	2.804	$= 172.695$	•	$m_{\tilde{u}_1^1}$	$= 506.424 +$	39.255	$= 545.680$
•	$m_{\tilde{e}_1^1}$	$= 123.574 +$	1.878	$= 125.452$	•	$m_{\tilde{u}_1^2}$	$= 524.275 +$	39.157	$= 563.433$
	$m_{\tilde{e}_1^2}$	$= 186.905 +$	3.082	$= 189.986$	•	$m_{\tilde{d}_1^1}$	$= 506.097 +$	39.409	$= 545.506$
						$m_{\tilde{d}_1^2}$	$= 530.033 +$	38.826	$= 568.859$
•	$m_{\tilde{\nu}_2}$	$= 169.884 +$	2.804	$= 172.688$	•	$m_{\tilde{u}_2^1}$	$= 506.410 +$	39.254	$= 545.664$
•	$m_{\tilde{e}_2^1}$	$= 123.510 +$	1.877	$= 125.387$	•	$m_{\tilde{u}_2^2}$	$= 524.285 +$	39.158	$= 563.444$
	$m_{\tilde{e}_2^2}$	$= 186.929 +$	3.080	$= 190.009$	•	$m_{\tilde{d}_2^1}$	$= 506.092 +$	39.408	$= 545.500$
						$m_{\tilde{d}_2^2}$	$= 530.034 +$	38.825	$= 568.859$
•	$m_{\tilde{\nu}_3}$	$= 168.001 +$	2.629	$= 170.630$	•	$m_{\tilde{u}_3^1}$	$= 333.171 +$	35.334	$= 368.504$
•	$m_{\tilde{e}_3^1}$	$= 106.080 +$	1.595	$= 107.674$	•	$m_{\tilde{u}_3^2}$	$= 549.649 +$	34.223	$= 583.872$
	$m_{\tilde{e}_3^2}$	$= 192.418 +$	2.786	$= 195.203$		$m_{\tilde{d}_3^1}$	$= 470.247 +$	34.711	$= 504.958$
					•	$m_{\tilde{d}_3^2}$	$= 506.244 +$	38.129	$= 544.374$

SPS1a' scenario

$M_{1/2}$	=	250 GeV	sign(μ)	=	+1
M_0	=	70 GeV	$\tan\beta(\tilde{M})$	=	10
A_0	=	−300 GeV			



Renormalization of $\tan\beta$

from the Higgs sector

$$\tan\beta = \frac{v_2}{v_1} \quad \rightarrow \quad \sqrt{\frac{Z_{H_2}}{Z_{H_1}}} \cdot \frac{v_2 + \delta v_2}{v_1 + \delta v_1}$$

$$= \quad \frac{v_2}{v_1}\left(1 + \delta Z_{H_2} - \delta Z_{H_1} + \frac{\delta v_2}{v_2} - \frac{\delta v_1}{v_1}\right)$$

$$\overline{DR} \qquad\qquad = 0$$

Separation of "QED corrections"

Full calculation inevitable

- separation of diagrams with virtual photons not UV-finite
- soft-photon bremsstrahlung necessary for IR-finite result
- hard bremsstrahlung for realistic treatments

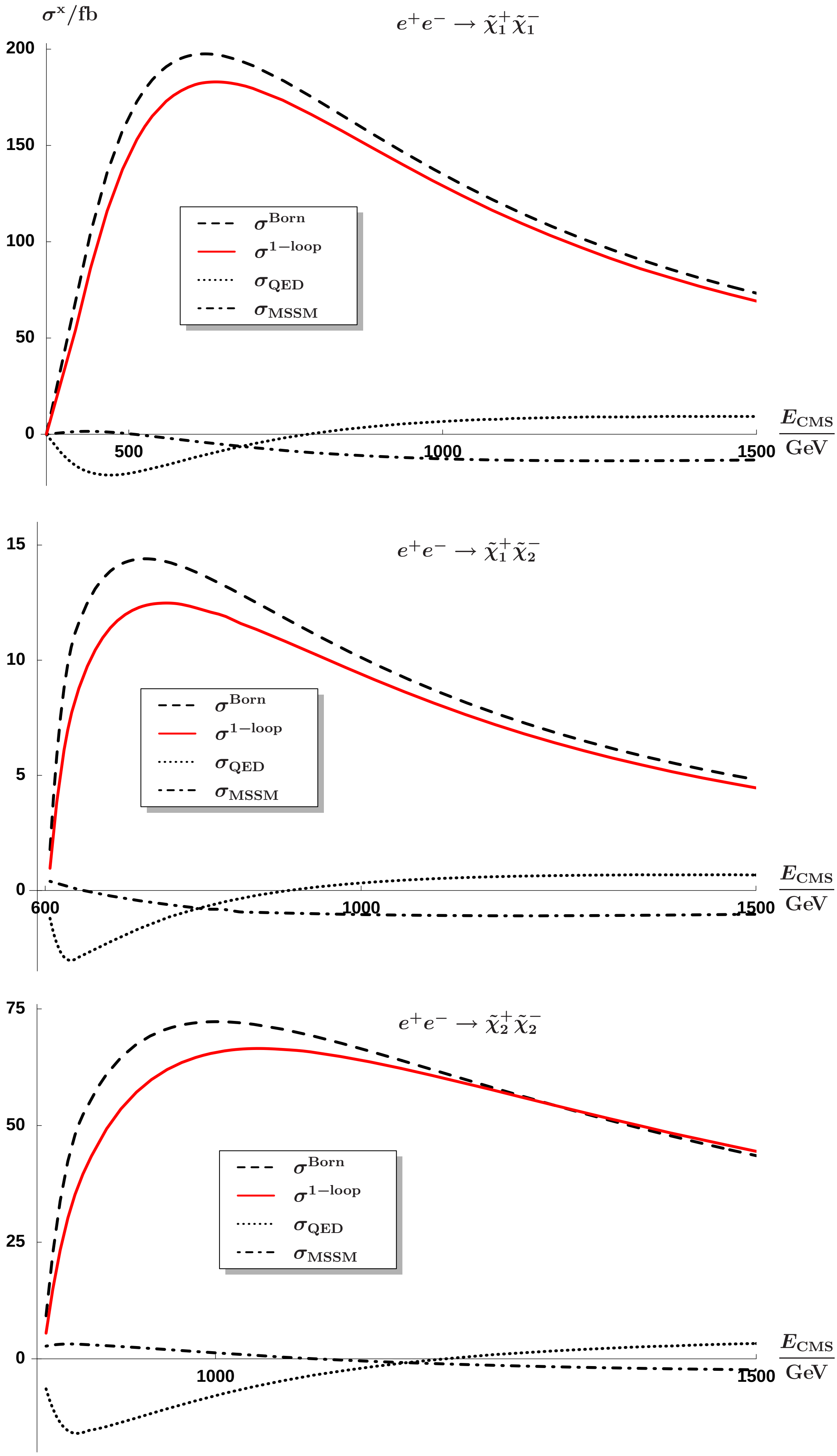
Reasonable separation ($L_e = \log \frac{s}{m_e^2}$, $\Delta E = E^{\max}_{\gamma\,\mathrm{soft}}$):

$$\sigma = \sigma_{\mathrm{QED}} + \sigma_{\mathrm{MSSM}}\,,$$

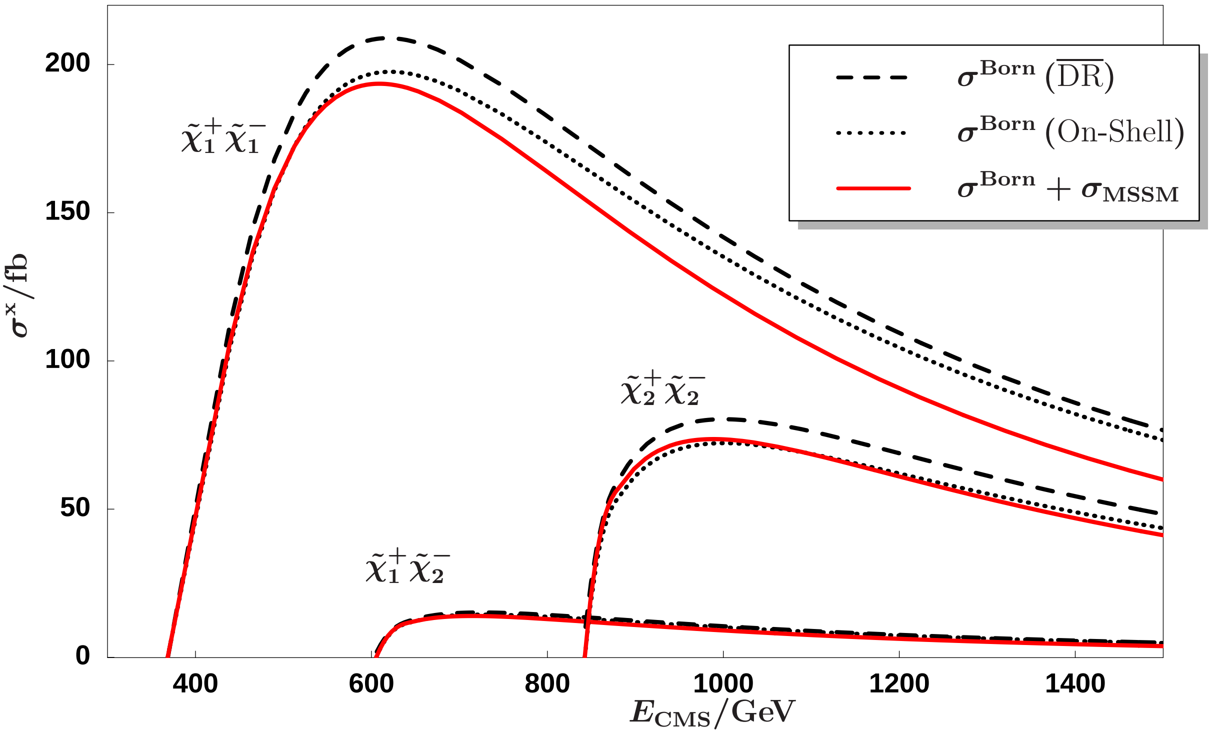
$$\sigma_{\mathrm{QED}} = \sigma^{\mathrm{hard}} + \frac{\alpha}{\pi}\left[(L_e - 1)\log\frac{4\,\Delta E^{\,2}}{s} + \frac{3}{2}L_e\right]\sigma_0$$

$$\sigma_{\mathrm{MSSM}} = \sigma^{\mathrm{v+s}} - \frac{\alpha}{\pi}\left[(L_e - 1)\log\frac{4\,\Delta E^{\,2}}{s} + \frac{3}{2}L_e\right]\sigma_0$$

- gauge invariant
- σ_{MSSM} free of large soft and collinear photon contributions

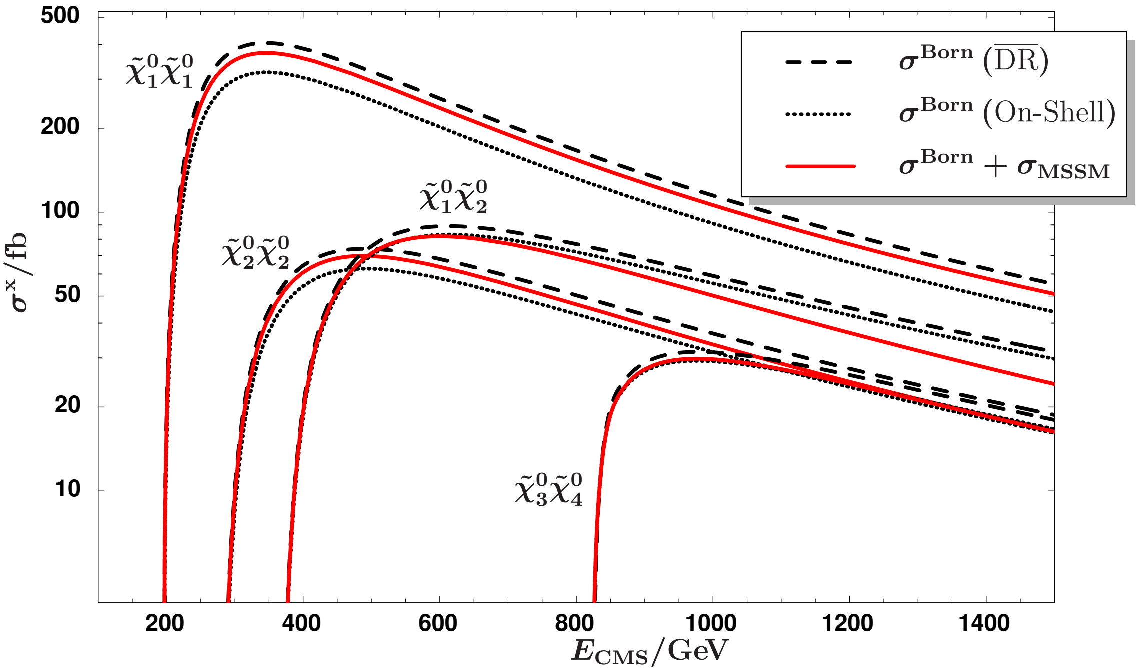


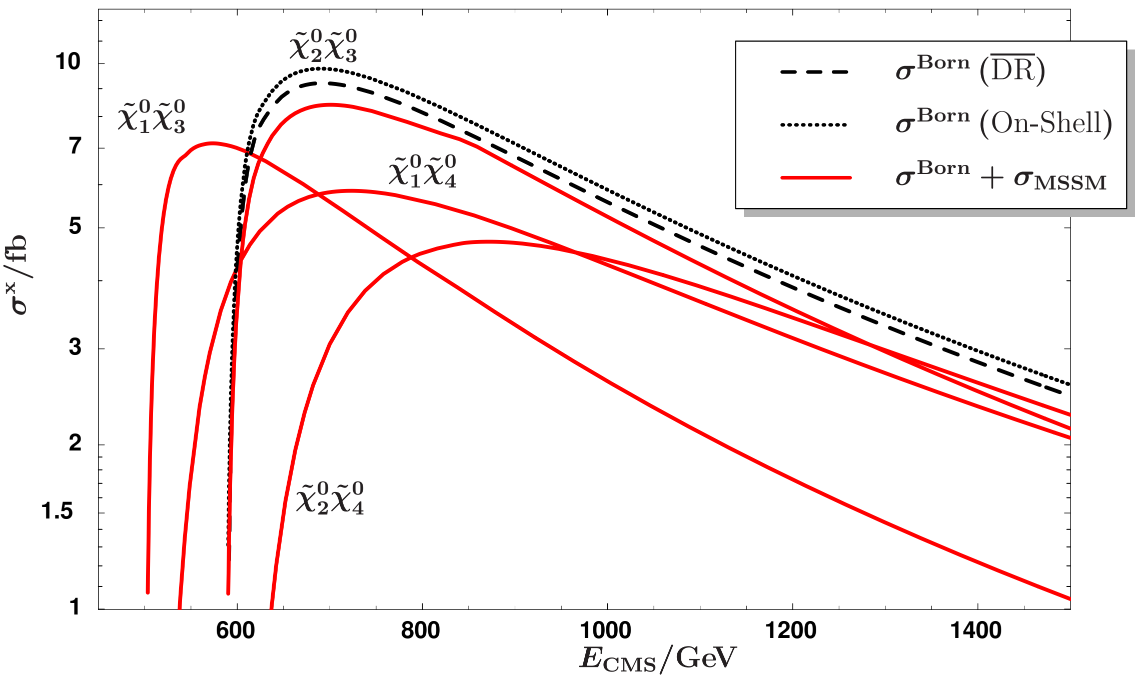
σ^{x}/fb
$e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$
200
150
100
50
0
500
1000
1500
$\frac{E_{\mathrm{CMS}}}{\mathrm{GeV}}$
σ^{Born}
$\sigma^{1-\mathrm{loop}}$
σ_{QED}
σ_{MSSM}
$e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$
15
10
5
0
600
1000
1500
$\frac{E_{\mathrm{CMS}}}{\mathrm{GeV}}$
σ^{Born}
$\sigma^{1-\mathrm{loop}}$
σ_{QED}
σ_{MSSM}
$e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_2^-$
75
50
25
0
1000
1500
$\frac{E_{\mathrm{CMS}}}{\mathrm{GeV}}$
σ^{Born}
$\sigma^{1-\mathrm{loop}}$
σ_{QED}
σ_{MSSM}



$\sigma^{\rm Born}$ ($\overline{\rm DR}$)
$\sigma^{\rm Born}$ (On-Shell)
$\sigma^{\rm Born} + \sigma_{\rm MSSM}$
$\tilde{\chi}_1^+\tilde{\chi}_1^-$
$\tilde{\chi}_2^+\tilde{\chi}_2^-$
$\tilde{\chi}_1^+\tilde{\chi}_2^-$
$\sigma^{\rm x}/{\rm fb}$
0
50
100
150
200
400
600
800
1000
1200
1400
$E_{\rm CMS}/{\rm GeV}$

$$e^+e^- \to \tilde{\chi}^0_i \tilde{\chi}^0_j$$





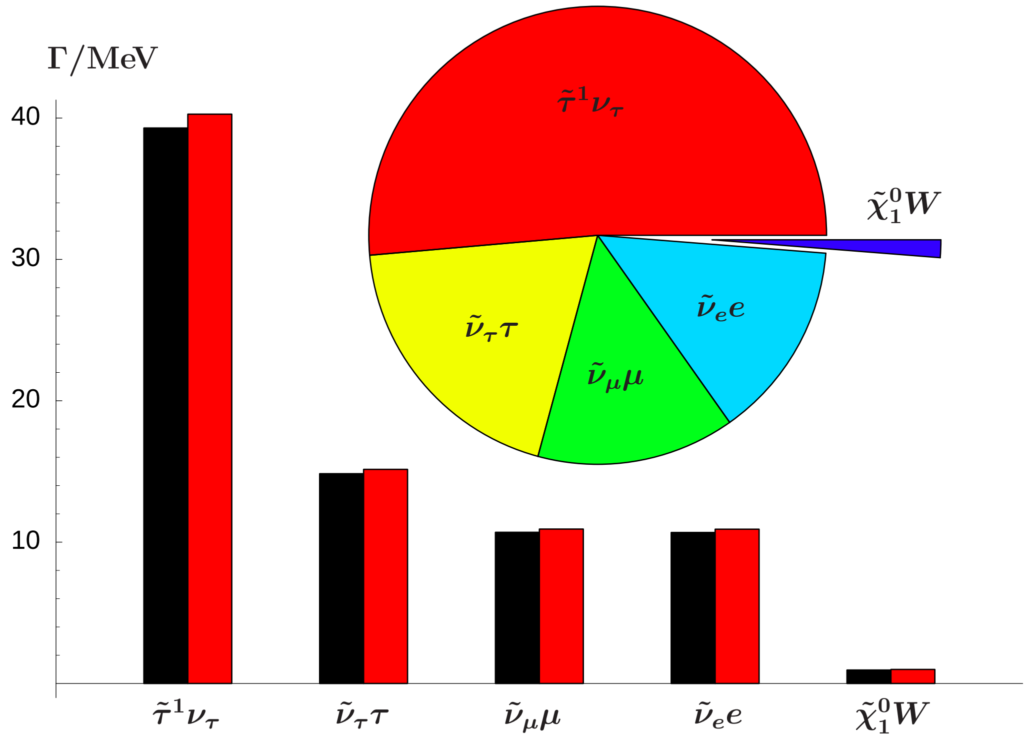
SUSY particle decay rates

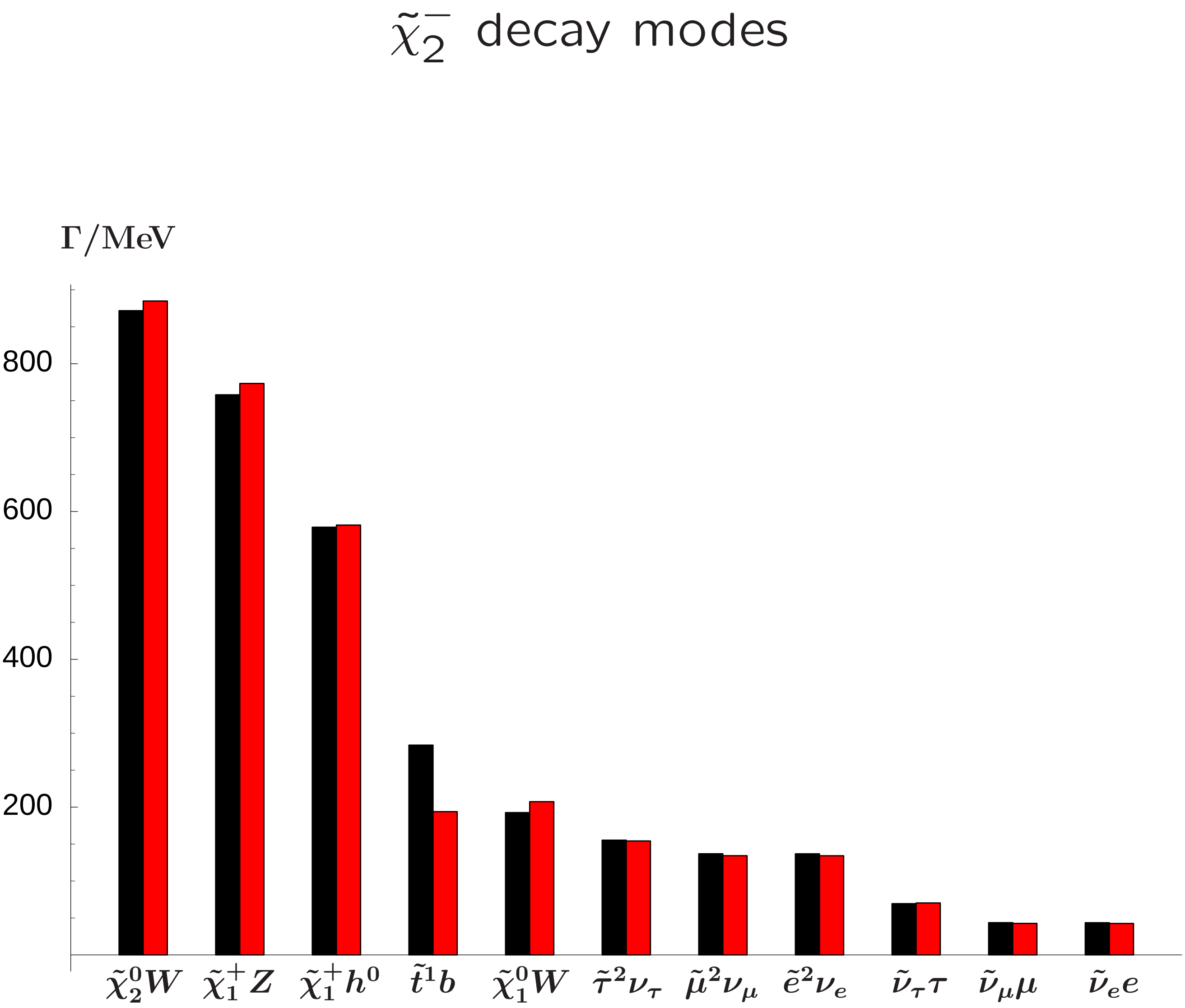
2-particle decays of $\tilde{\chi}^{\pm}_{1,2}$ and $\tilde{\chi}^{0}_{2,3,4}$

tree level (black) and 1-loop (red)

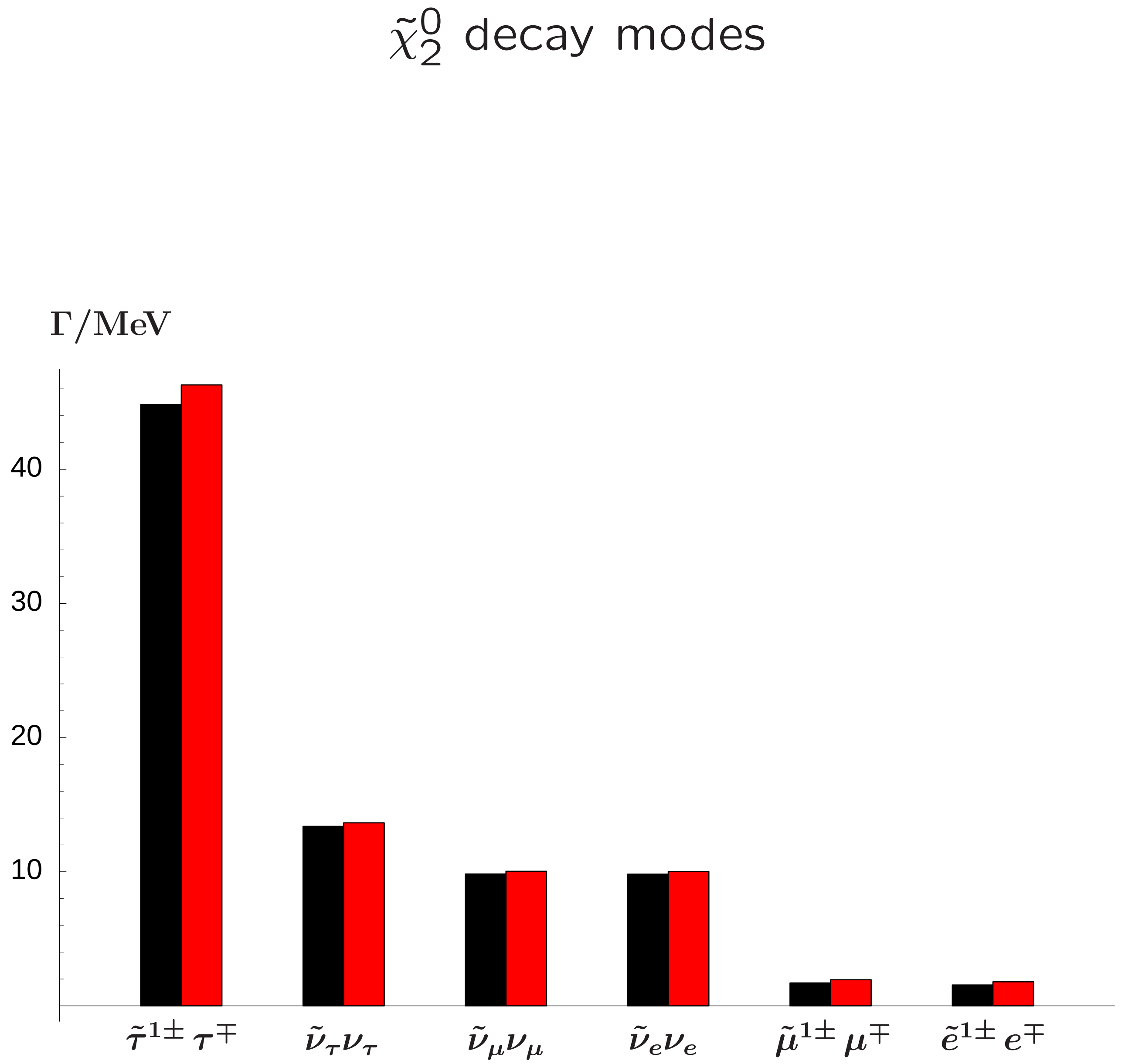
[Fritzsche, WH]

$\tilde{\chi}^{-}_{1}$ decay modes (SPS1a')





$\tilde{\chi}_2^-$ decay modes
Γ/MeV
800
600
400
200
$\tilde{\chi}_2^0 W$
$\tilde{\chi}_1^+ Z$
$\tilde{\chi}_1^+ h^0$
$\tilde{t}^1 b$
$\tilde{\chi}_1^0 W$
$\tilde{\tau}^2 \nu_\tau$
$\tilde{\mu}^2 \nu_\mu$
$\tilde{e}^2 \nu_e$
$\tilde{\nu}_\tau \tau$
$\tilde{\nu}_\mu \mu$
$\tilde{\nu}_e e$



$\tilde{\chi}_2^0$ decay modes
Γ/MeV
40
30
20
10
$\tilde{\tau}^{1\pm}\,\tau^{\mp}$
$\tilde{\nu}_\tau\nu_\tau$
$\tilde{\nu}_\mu\nu_\mu$
$\tilde{\nu}_e\nu_e$
$\tilde{\mu}^{1\pm}\,\mu^{\mp}$
$\tilde{e}^{1\pm}\,e^{\mp}$

Conclusions

- The MSSM is competitive to the SM
  - precision observables are well described
  - light Higgs boson natural

- precision-observable calculations advanced
  $\rightarrow$ 2-loop level

- m_{h^0} is another precision observable
  - dependent on all SUSY sectors
  - accurate theoretical evaluation ($\delta m_{h^0} \simeq 4$ GeV), to be further improved

- progress for one-loop studies for SUSY processes

  many results and tools already available