Determining fundamental parameters of QCD on the lattice

SFB/TR 9

Rainer Sommer

DESY, Zeuthen

April 2006, “loops & legs”, Eisenach

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$\Lambda_{\rm QCD}$
$M_{\rm strange}$
$M_{\rm charm}$
$M_{\rm beauty}$

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Λ_{QCD}
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not a review

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What can be done by lattice QCD?

- applicable for all scales
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  and high energy perturbative regimes

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- no questions about "non-perturbative effects",

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- potentially very good precision

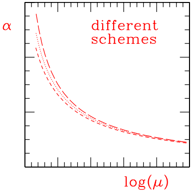
Renormalization group and Λ-parameter (mass–independent scheme)

RGE

$$\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \qquad \bar{g}(\mu)^2 = 4\pi\alpha(\mu)$$

$$\beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3 \left\{ b_0 + b_1 \bar{g}^2 + b_2 \bar{g}^4 + \ldots \right\}$$

$$b_0 = \frac{1}{(4\pi)^2} \left(11 - \frac{2}{3} N_\mathrm{f}\right)$$



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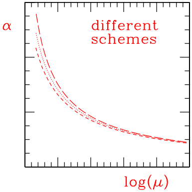

- *exact equation* for Λ in a mass–independent scheme $(\bar g \equiv \bar g(\mu))$

$$\Lambda = \mu\,(b_0\bar g^2)^{-b_1/2b_0^2}\mathrm{e}^{-1/2b_0\bar g^2}\,\exp\left\{-\int_0^{\bar g}\mathrm{d}g\left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

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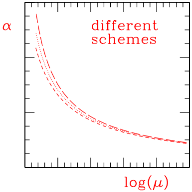

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- trivial scheme dependence:

$$\Lambda_a/\Lambda_b = \exp\{c_{a,b}/(4\pi b_0)\}\,, \qquad \alpha_a = \alpha_b + c_{a,b}\,\alpha_b^2 + \mathrm{O}(\alpha_b^3)$$

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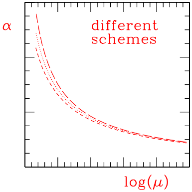

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- use a suitable physical coupling (scheme) and compute Λ

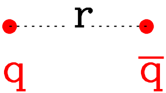
  Requires non-perturbative computation of $\beta(\bar{g})$

Physical couplings

– what is a non-perturbatively defined coupling?

Example

$m_q \to \infty$



$$F(r) = \frac{4}{3}\frac{1}{r^2}\left\{\alpha_{\overline{\rm MS}}(\mu) + c_1[\alpha_{\overline{\rm MS}}(\mu)]^2 + \ldots\right\}, \quad \mu = 1/r$$

$$\equiv \frac{4}{3}\frac{1}{r^2}\,\alpha_{\rm qq}(\mu)$$

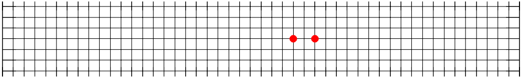
Physical couplings: their properties

- defined for all energies μ
- independent of the regularization procedure
  $\longrightarrow$ i.e. (on the lattice) the continuum limit can be taken
- any one of them defines a renormalization scheme
- the usual perturbative properties when α is small, e.g.

$$\alpha_a(\mu) = \alpha_b(\mu) + c_{a,b}\,[\alpha_b(\mu)]^2 + \ldots$$

(also for $a = \overline{\mathrm{MS}}$).

Problem in a lattice computation ($\alpha_{\rm qq}$ as an example)



$$L \gg \frac{1}{0.2\mathrm{GeV}} \gg \frac{1}{\mu} \sim \frac{1}{10\mathrm{GeV}} \gg a$$

box size ↑ confinement scale, m_π ↑ spacing ↑

$$\Downarrow$$

$$L/a \gg 50$$

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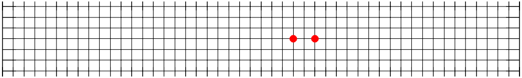


$$L \gg \frac{1}{0.2\mathrm{GeV}} \gg \frac{1}{\mu} \sim \frac{1}{10\mathrm{GeV}} \gg a$$

$\uparrow$ box size (L) — $\uparrow$ confinement scale, m_π — $\uparrow$ spacing (a)

$$\Downarrow$$

$$L/a \gg 50$$

Solution: $\boxed{L = 1/\mu}$ $\longrightarrow$ left with $L/a \gg 1$ [Wilson, ... , Lüscher, Weisz, Wolff]

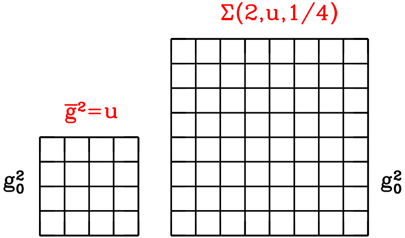
Finite size effect as a physical observable; finite size scaling!

The step scaling function: $\sigma(s,u) = \bar{g}^2(sL)$ with $u = \bar{g}^2(L)$

On the lattice:
additional dependence on the resolution a/L

g_0 fixed, L/a fixed:

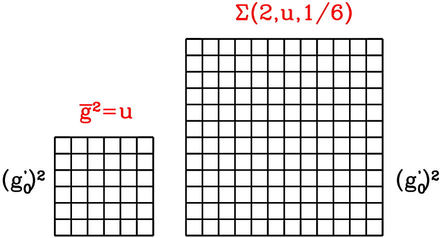
$$\bar{g}^2(L) = u, \qquad \bar{g}^2(sL) = u'\,,$$
$$\Sigma(s,u,a/L) \quad = \quad u'$$



continuum limit:

$$\Sigma(s,u,a/L) = \sigma(s,u) + \mathrm{O}(a/L)$$

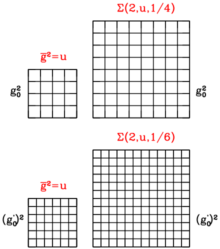
in the following always $s = 2$



The step scaling function: the continuum limit

$N_\mathrm{f} = 2$:

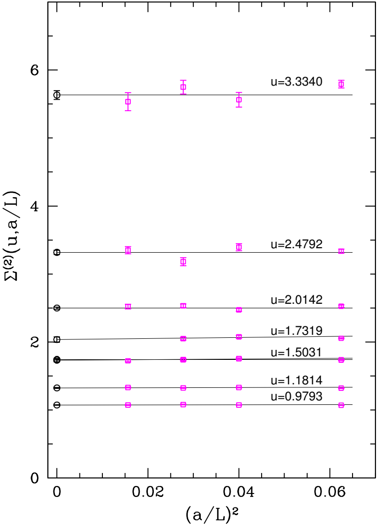
continuum limit



[ALPHA Collaboration

Bode, Frezzotti, Gehrmann, Hasenbusch, Heitger, Jansen, S. Kurth, Rolf, Simma, Sint, S., Weisz, Wittig, Wolff, 2001;

Della Morte, Frezzotti, Heitger, Rolf, S., Wolff, 2005]



Step scaling function as function of $u = \bar{g}^2$

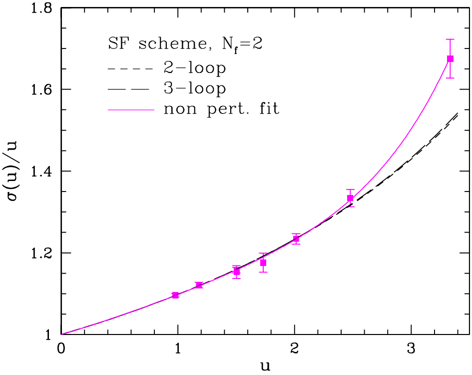
SSF $\sigma(u)$

comparison to PT

... and NP fit

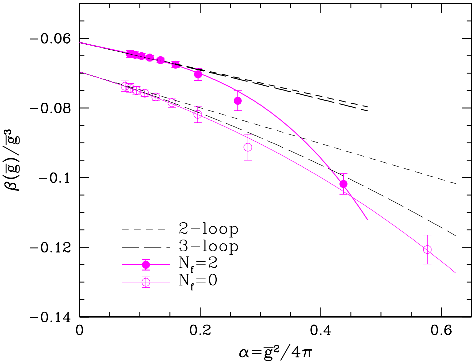
$$\sigma(u) = u + s_0 u^2 + s_1 u^3 + s_2 u^4 + s_3 u^5 + s_4 u^6$$

s_0, s_1 fixed from PT



β-function in SF scheme, $N_\mathrm{f} = 2$

comparison to PT and $N_\mathrm{f} = 0$



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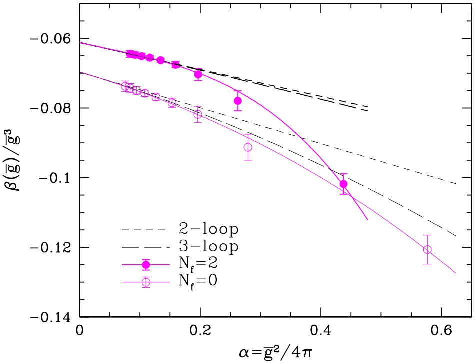

- non-perturbative deviations from 3-loop β for $\alpha_{\mathrm{SF}} > 0.25$

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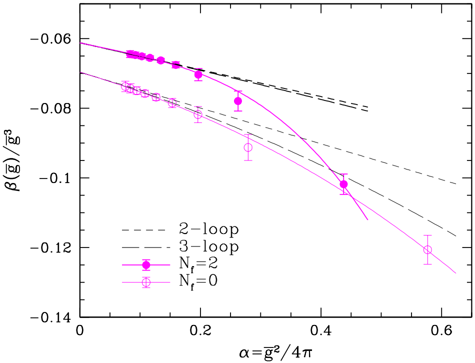
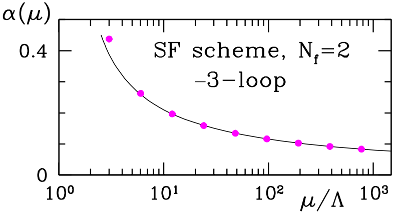


- non-perturbative deviations from 3-loop β for $\alpha_{SF} > 0.25$
- not clear from within perturbation theory

Non-perturbative running of α



[ALPHA Collaboration, 2005]

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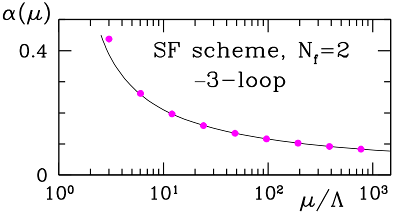

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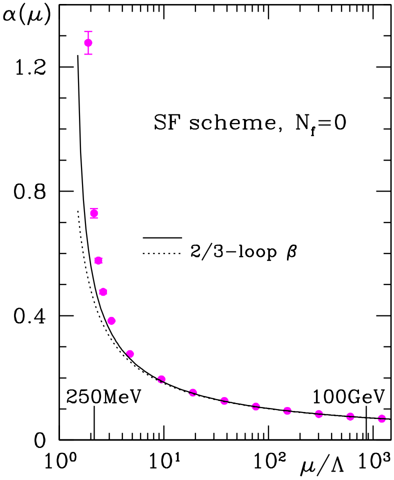

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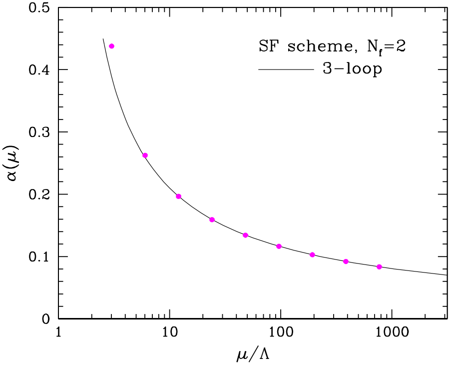
Non-perturbative running of α, $N_f = 2$

SF-scheme, NP, $N_f = 2$

error bars are smaller than symbol size



Experiment + PT

$\overline{\mathrm{MS}}$-scheme [S. Bethke 2000]

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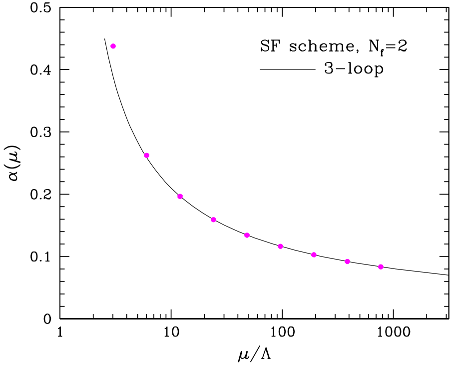

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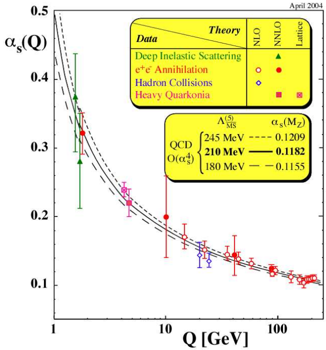

The Lambda parameter

Define $L_{\rm max}$ by $\boxed{\bar{g}^2(L_{\rm max}) = 5.5}$

k steps with NP σ:

$\bar{g}^2(L_{\rm max}/2^k) = \sigma(\bar{g}^2(L_{\rm max}/2^{k+1}))$

$\Downarrow$

k	u_k	$-\ln(\Lambda L_{\rm max})$
0	5.5	0.957
1	3.306(40)	1.071(25)
2	2.482(31)	1.093(37)
3	2.010(27)	1.093(48)
4	1.695(22)	1.089(57)
5	1.468(18)	1.087(65)
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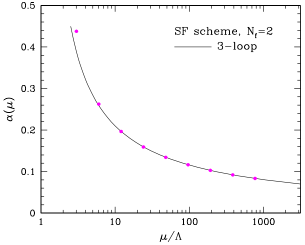

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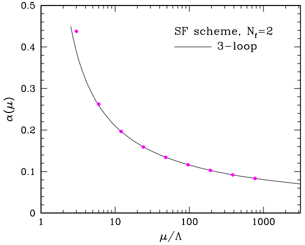

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Result: $\boxed{-\ln(\Lambda L_{\rm max}) = 1.09(7)}$

Λ in MeV

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  e.g. $F_{\rm K}$ (large volume computation)

  ... at present use $r_0 = 0.5\,\rm fm$ instead

  r_0 defined from QQ-Force $F(r_0)r_0^2 = 1.65$
  [R.S., 1994]

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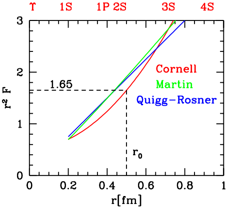

$$N_{\mathrm{f}} = 0 : \qquad r_0 \times F_{\mathrm{K}} = 0.5\,\mathrm{fm} \times F_{\mathrm{K}}^{\mathrm{experimental}} \pm 3\% \;\surd$$

several groups say $r_0 = 0.5\,\mathrm{fm} \pm 5\%$ holds also for $N_{\mathrm{f}} = 2, N_{\mathrm{f}} = 3$

Λ in MeV ...

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Λ in MeV ...

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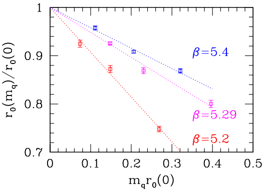

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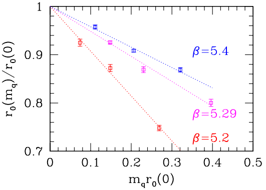

and determine L/a at fixed $\bar{g}^2$ for these β-values

$\beta = 6/g_0^2$	κ	L/a	c_t=1-loop $\bar{g}^2(L)$	c_t=2-loop $\bar{g}^2(L)$
5.20	0.13600	4	3.32(2)	3.65(3)
5.20	0.13600	6	4.31(4)	4.61(4)
5.29	0.13641	4	3.184(16)	3.394(17)
5.29	0.13641	6	4.059(32)	4.279(37)
5.29	0.13641	8	5.34(8)	5.65(9)
5.40	0.13669	4	3.016(20)	3.188(24)
5.40	0.13669	6	3.708(31)	3.861(34)
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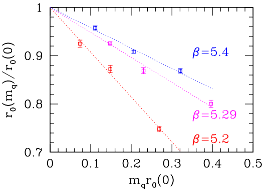

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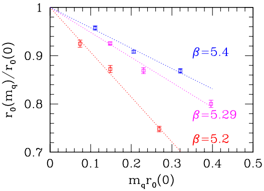
		$u_{\max} = 3.65$		$u_{\max} = 4.61$	
β	r_0/a	$L_{\max}/a$	$\Lambda_{\overline{MS}}\, r_0$	$L_{\max}/a$	$\Lambda_{\overline{MS}}\, r_0$
5.20	5.45(5)(20)	4.00(6)	0.655(27)	6.00(8)	0.610(25)
5.29	6.01(4)(22)	4.67(6)	0.619(25)	6.57(6)	0.614(24)
5.40	7.01(5)(15)	5.43(9)	0.621(17)	7.73(10)	0.609(16)

Λ in MeV ...

- r_0/a from UKQCD/QCDSF + CP-PACS/JLQCD
- extrapolate (ourselves) to the chiral limit



and determine L/a at fixed $\bar{g}^2$ for these β-values

$\beta = 6/g_0^2$	κ	L/a	c_t=1-loop $\bar{g}^2(L)$	c_t=2-loop $\bar{g}^2(L)$
5.20	0.13600	4	3.32(2)	3.65(3)
5.20	0.13600	6	4.31(4)	4.61(4)
5.29	0.13641	4	3.184(16)	3.394(17)
5.29	0.13641	6	4.059(32)	4.279(37)
5.29	0.13641	8	5.34(8)	5.65(9)
5.40	0.13669	4	3.016(20)	3.188(24)
5.40	0.13669	6	3.708(31)	3.861(34)
5.40	0.13669	8	4.704(59)	4.747(63)

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$$\Lambda_{\overline{\rm MS}}\, r_0 = 0.62(4)(4)$$

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$N_{\rm f}$:	0	2	4	5
[ALPHA Collaboration]	0.60(5)	0.62(4)(4)		
[Bethke, L&L 2004] "experiment"			0.74(10)	0.54(8)
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- looks like an irregular $N_{\rm f}$-dependence ... but relatively large errors
- PT: $\Lambda_{\overline{\rm MS}}^{N_{\rm f}=4} \approx 1.4\,\Lambda_{\overline{\rm MS}}^{N_{\rm f}=5}$ [Bernreuther & Wetzel]

  How accurate is this? Need $\mu \ll m_{\rm beauty}$ where pert. theory is accurate.

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  Replace $L_{\rm max}/r_0$ by $F_{\rm K} \times L_{\rm max}$ with small quark masses, small a (will be done soon, god willing)

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  I essentially described **all** the sources of systematic errors.

  The assumptions made are minimal.

Quark masses

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- mass independent renormalization scheme, (even for m_{b})

$$\overline{m}_i(\mu) = \underbrace{Z_m(\mu a, g_0)}_{\text{flavor independent}} \quad m_i^{\mathrm{bare}}(g_0)$$

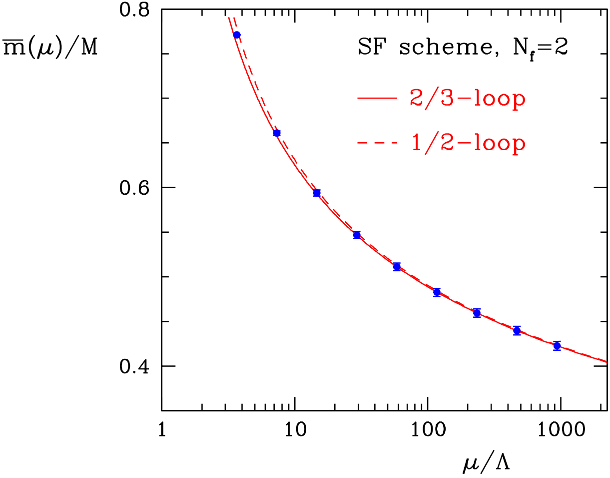
$\longrightarrow$ solve problem of running once and for all
RGI masses (scale and scheme inependent):

$$M_i = \lim_{\mu\to\infty} (2b_0\bar{g}^2(\mu))^{-d_0/2b_0}\,\overline{m}(\mu)_i$$

$$M_i/M_j = \overline{m}(\mu)_i/\overline{m}(\mu)_j = m_i^{\mathrm{bare}}/m_j^{\mathrm{bare}}$$

still due to convention, in the end, convert to $\overline{\mathrm{MS}}$-scheme

Non-perturbative running of $\overline{m}$, $N_{\mathrm{f}} = 2$



Results for m_{strange}

$\overline{\mathrm{MS}}$ scheme, $\mu = 2\mathrm{GeV}$, exp. input: $r_0 = 0.5\,\mathrm{fm}$, $m_{\mathrm{K}} = 495\,\mathrm{MeV}$

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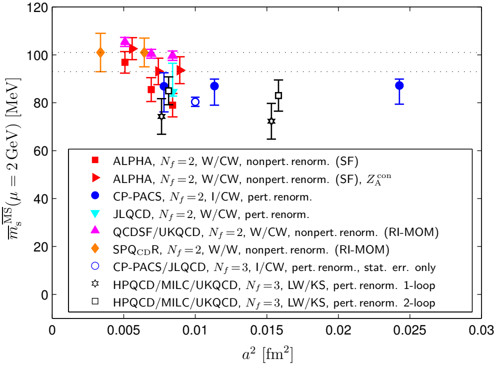

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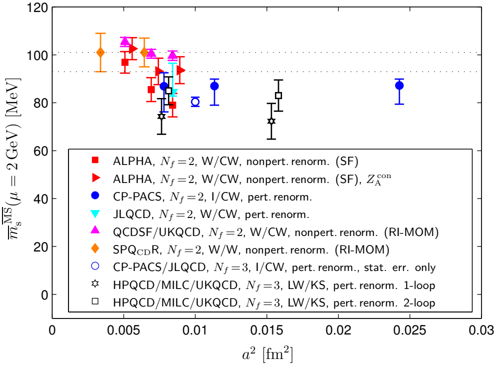

$\leftarrow$ $97(4)\,\mathrm{MeV}$

[ALPHA & UKQCD, 2001]

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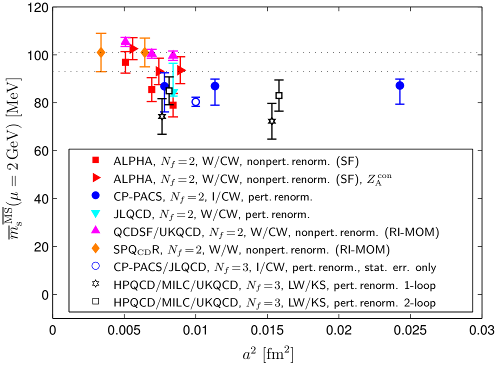

$\leftarrow$ 97(4) MeV

[ALPHA Collaboration & UKQCD, 2001]

- several results still with perturbative renormalization tend to be smaller than non-perturbative ones

Results for $m_{\rm charm}$

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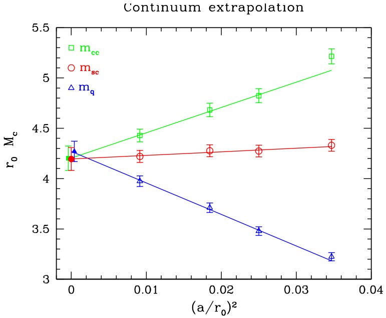

large mass causes $\mathrm{O}((am_{\rm charm})^2)$ errors
difficult (but successfull cont. extrapol.) [ALPHA Collaboration, J. Rolf & S. Sint, 2003]
$\overline{m}_{\rm charm}(\overline{m}) = 1.30(3)\,{\rm GeV}$ quenched!

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$L_2 = 2L_1$

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in the $\overline{\rm MS}$ scheme:

$$
\begin{aligned}
m_{\rm b}(m_{\rm b}) &= m_{\rm b}^{\rm stat} + m_{\rm b}^{(1)} \\
m_{\rm b}^{\rm stat} &= 4.35(6)\,{\rm GeV}\,(= {\rm O}(m_{\rm b}) + {\rm O}(\Lambda)\,) \\
m_{\rm b}^{(1)} &= -0.02(2)\,{\rm GeV}\,(= {\rm O}(\Lambda^2/m_{\rm b})\,)\,.
\end{aligned}
$$

in the quenched approximation!

[Della Morte,Garron,Papinutto & S, 2005]

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- non-perturbative running is significant, in case of $Z_{\rm A}^{\rm stat}$, $Z_{\Delta \rm S=2}$ very much so
- the perturbative series itselve does not "predict" where this happens

Shortcuts are popular: (improved) bare couplings and quark masses

[Creutz, 1980; Parisi; ...; HPQCD]

basic formula ("asymptotic scaling") a=lattice spacing:

$$\frac{\Lambda_{\mathrm{lat}}}{F_\pi} = \frac{1}{a\,F_\pi}\,(b_0 {g_0}^2)^{-b_1/2b_0^2}\mathrm{e}^{-1/2b_0{g_0}^2}\left\{1+\underbrace{c_1\,{g_0}^2+\ldots}_{\text{corrections}}\right\}, \quad \frac{\Lambda_{\mathrm{lat}}}{\Lambda_{\overline{\mathrm{MS}}}} = \#$$

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Massless schemes: a remark

Consider an observable Φ with any energy scale μ, even

$$\mu = \mathrm{O}(\overline{m}_\mathrm{t})$$

Non-perturbatively

$$\Phi = \Phi(\mu, \overline{m}_u(\mu), \ldots, \overline{m}_t(\mu), \alpha^{(6)}(\mu))$$

with $\overline{m}_i$, $\alpha^{(6)}$ in six-flavour massless renormalization scheme

$$\Phi = F(\mu/\Lambda, M_u/\Lambda, \ldots, M_t/\Lambda,) \text{ for dimensionless } \Phi$$

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- In principle, non-perturbatively, thresholds do not need to be resolved in practise: u,d,s, maybe also c