Two-parton contribution to the heavy-quark A_{FB} in NNLO QCD ($e^+e^- collisions$)

Roberto BONCIANI

IFIC, Instituto de Física Corpuscular

E-46071 València, Spain

In collaboration with: W. Bernreuther, T. Gehrmann, R. Heinesch, T. Leineweber, P. Mastrolia and E. Remiddi

Plan of the Talk

Plan of the Talk

- Introduction

Plan of the Talk

- Introduction
- The A_{FB} and the vertex form factors

arXiv:hep-ph/0604031

Plan of the Talk

- Introduction
- The A_{FB} and the vertex form factors

arXiv:hep-ph/0604031

  - The calculation of the Form Factors

Plan of the Talk

- Introduction
- The A_{FB} and the vertex form factors

arXiv:hep-ph/0604031

  - The calculation of the Form Factors

Nucl. Phys. B **706** (2005) 245 [arXiv:hep-ph/0406046]
Nucl. Phys. B **712** (2005) 229 [arXiv:hep-ph/0412259]
Nucl. Phys. B **723** (2005) 91 [arXiv:hep-ph/0504190]

Plan of the Talk

- Introduction
- The A_{FB} and the vertex form factors

  arXiv:hep-ph/0604031

  - The calculation of the Form Factors

    Nucl. Phys. B **706** (2005) 245 [arXiv:hep-ph/0406046]
    Nucl. Phys. B **712** (2005) 229 [arXiv:hep-ph/0412259]
    Nucl. Phys. B **723** (2005) 91 [arXiv:hep-ph/0504190]

  - Numerical results

Plan of the Talk

- Introduction
- The A_{FB} and the vertex form factors

  arXiv:hep-ph/0604031

  - The calculation of the Form Factors

    Nucl. Phys. B **706** (2005) 245 [arXiv:hep-ph/0406046]
    Nucl. Phys. B **712** (2005) 229 [arXiv:hep-ph/0412259]
    Nucl. Phys. B **723** (2005) 91 [arXiv:hep-ph/0504190]

  - Numerical results
  - The cross section near the threshold

Plan of the Talk

- Introduction
- The A_{FB} and the vertex form factors

  arXiv:hep-ph/0604031

  - The calculation of the Form Factors

    Nucl. Phys. B **706** (2005) 245 [arXiv:hep-ph/0406046]
    Nucl. Phys. B **712** (2005) 229 [arXiv:hep-ph/0412259]
    Nucl. Phys. B **723** (2005) 91 [arXiv:hep-ph/0504190]

  - Numerical results
  - The cross section near the threshold
- Summary

Introduction

- Forward-Backward Asymmetries in the production of fermions at high-energy e^+e^- collisions are important for the precise determination of the respective fermionic neutral current couplings. For instance at LEP a precision in the determination of A_{FB}^b on the Z peak of about 1.7% led to a determination of $\sin^2\theta_{W,eff}$ with an accuracy of 1 per mille

Introduction

- Forward-Backward Asymmetries in the production of fermions at high-energy e^+e^- collisions are important for the precise determination of the respective fermionic neutral current couplings. For instance at LEP a precision in the determination of A^b_{FB} on the Z peak of about 1.7% led to a determination of $\sin^2\theta_{W,eff}$ with an accuracy of 1 per mille
- Future linear colliders will be able to run at the Z peak with huge statistics and improve the accuracy in the measurement of A^b_{FB} to about 0.1%

Introduction

- Forward-Backward Asymmetries in the production of fermions at high-energy e^+e^- collisions are important for the precise determination of the respective fermionic neutral current couplings. For instance at LEP a precision in the determination of A^b_{FB} on the Z peak of about 1.7% led to a determination of $\sin^2\theta_{W,eff}$ with an accuracy of 1 per mille
- Future linear colliders will be able to run at the Z peak with huge statistics and improve the accuracy in the measurement of A^b_{FB} to about 0.1%
- Moreover, at a future linear collider (ILC) A^t_{FB} will be experimentally accessible

Introduction

- Forward-Backward Asymmetries in the production of fermions at high-energy e^+e^- collisions are important for the precise determination of the respective fermionic neutral current couplings. For instance at LEP a precision in the determination of A^b_{FB} on the Z peak of about 1.7% led to a determination of $\sin^2\theta_{W,eff}$ with an accuracy of 1 per mille
- Future linear colliders will be able to run at the Z peak with huge statistics and improve the accuracy in the measurement of A^b_{FB} to about 0.1%
- Moreover, at a future linear collider (ILC) A^t_{FB} will be experimentally accessible

$\Longrightarrow$ need for NNLO calculation retaining the dependence on the mass of the heavy quark

Introduction

- The EW corrections in the literature:

Introduction

- The EW corrections in the literature:
  - Massive NLO (Böhm et al. ’89, Bardin-Christova-Jack-Kalynovskaya-Olchevski-Riemann-Riemann ’01, Freitas-Mönig ’05)

Introduction

- The EW corrections in the literature:
  - Massive NLO (Böhm et al. '89, Bardin-Christova-Jack-Kalynovskaya-Olchevski-Riemann-Riemann '01, Freitas-Mönig '05)
- The QCD corrections in the literature:

Introduction

- The EW corrections in the literature:
  - Massive NLO (Böhm et al. '89, Bardin-Christova-Jack-Kalynovskaya-Olchevski-Riemann-Riemann '01, Freitas-Mönig '05)
- The QCD corrections in the literature:
  - Massive NLO (Arbuzov-Bardin-Leike '92, Djouadi-Lampe-Zerwas '95)

Introduction

- The EW corrections in the literature:
  - Massive NLO (Böhm et al. '89, Bardin-Christova-Jack-Kalynovskaya-Olchevski-Riemann-Riemann '01, Freitas-Mönig '05)
- The QCD corrections in the literature:
  - Massive NLO (Arbuzov-Bardin-Leike '92, Djouadi-Lampe-Zerwas '95)
  - Massless NNLO (Altarelli-Lampe '93, Ravindran-van Neerven '98)

Introduction

- The EW corrections in the literature:
  - Massive NLO (Böhm et al. '89, Bardin-Christova-Jack-Kalynovskaya-Olchevski-Riemann-Riemann '01, Freitas-Mönig '05)
- The QCD corrections in the literature:
  - Massive NLO (Arbuzov-Bardin-Leike '92, Djouadi-Lampe-Zerwas '95)
  - Massless NNLO (Altarelli-Lampe '93, Ravindran-van Neerven '98)
  - Massless NNLO with the leading logs ($\log(s/m_b^2)$) plus constant terms (Catani-Seymour '99)

Introduction

- The EW corrections in the literature:
  - Massive NLO (Böhm et al. ’89, Bardin-Christova-Jack-Kalynovskaya-Olchevski-Riemann-Riemann ’01, Freitas-Mönig ’05)
- The QCD corrections in the literature:
  - Massive NLO (Arbuzov-Bardin-Leike ’92, Djouadi-Lampe-Zerwas ’95)
  - Massless NNLO (Altarelli-Lampe ’93, Ravindran-van Neerven ’98)
  - Massless NNLO with the leading logs ($\log\left(s/m_b^2\right)$) plus constant terms (Catani-Seymour ’99)

Our goal is the determination of A_{FB} at $\mathcal{O}(\alpha^2\alpha_S^2)$ retaining the full dependence on the mass of the heavy quark $\Longrightarrow$ terms $(m_Q^2/s)^\alpha$ and $(m_Q^2/s)^\alpha \log^\beta\left(s/m_Q^2\right)$

Introduction

- The full NNLO QCD corrections to A_{FB}^{Q} involve:

Introduction

- The full NNLO QCD corrections to A_{FB}^{Q} involve:
  - Tree-Level $1 \longrightarrow 4$ (at least one pair of heavy quarks)

Introduction

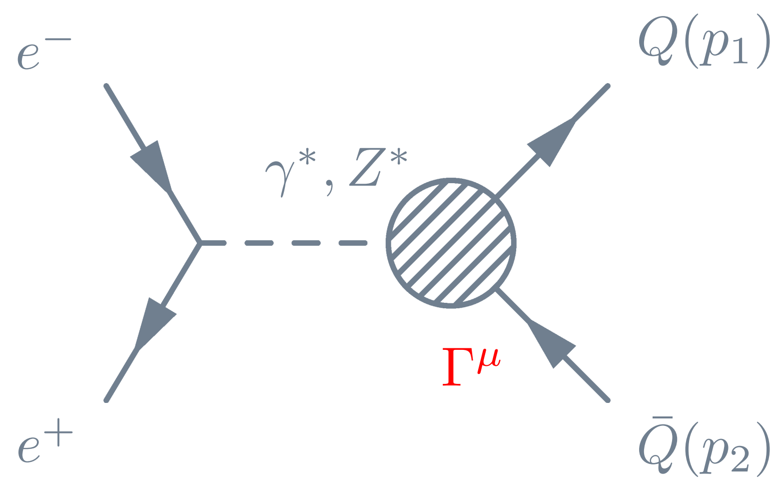
- The full NNLO QCD corrections to A_{FB}^{Q} involve:
  - Tree-Level $1 \longrightarrow 4$ (at least one pair of heavy quarks)
  - One-Loop $1 \longrightarrow 3$ (one pair of heavy quarks and a gluon)

Introduction

- The full NNLO QCD corrections to A_{FB}^{Q} involve:
  - Tree-Level $1 \longrightarrow 4$ (at least one pair of heavy quarks)
  - One-Loop $1 \longrightarrow 3$ (one pair of heavy quarks and a gluon)
  - Two-Loop virtual corrections $1 \longrightarrow 2$

Introduction

$$e^+e^- \to \gamma^*, Z^* \to Q\bar{Q}$$



$$V^\mu_{c_1c_2}(p_1,p_2) = \bar{u}_{c_1}(p_1)\Gamma^\mu_{c_1c_2}(q)v_{c_2}(p_2)$$

$$\Gamma^\mu_{c_1c_2}(q) = -i\delta_{c_1c_2}\Big[v_Q F_1(s)\gamma^\mu + v_Q \frac{1}{2m}F_2(s)i\sigma^{\mu\nu}q_\nu + a_Q G_1(s)\gamma^\mu\gamma_5 + a_Q\frac{1}{2m}G_2(s)\gamma_5 q^\mu\Big]$$

$$v_Q^\gamma = eQ_Q\,, \quad v_Q^Z = \frac{e}{s_w c_w}\left(\frac{T_3^Q}{2} - s_w^2 Q_Q\right), \quad a_Q = -\frac{e}{s_w c_w}\frac{T_3^Q}{2}, \quad s = \frac{S}{m^2}\,, \quad \sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu,\gamma^\nu\right]$$

$$A_{FB} = \frac{\sigma_A}{\sigma_S} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

where

$$\sigma_F = \int_0^1 d\cos\theta \frac{d\sigma}{d\cos\theta}\,, \quad \sigma_B = \int_{-1}^0 d\cos\theta \frac{d\sigma}{d\cos\theta}$$

Expanding in powers of α_S we have

$$\sigma_{A,S} = \sigma_{A,S}^{(2,0)} \quad + \sigma_{A,S}^{(2,1)} + \sigma_{A,S}^{(3,1)} \quad + \sigma_{A,S}^{(2,2)} + \sigma_{A,S}^{(3,2)} + \sigma_{A,S}^{(4,2)} + \mathcal{O}(\alpha_S^3)$$

the superscripts (i,j) give the "*number of partons in the final state*" and the "*order*" of α_S

Introduction

$$A_{FB} = A_{FB,0}\left(1 + A_{FB,1} + A_{FB,2}\right)$$

with

$$A_{FB,0} = \frac{\sigma_A^{(2,0)}}{\sigma_S^{(2,0)}}$$

$$A_{FB,1} = \frac{\sigma_A^{(2,1)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(2,1)}}{\sigma_S^{(2,0)}} + \frac{\sigma_A^{(3,1)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(3,1)}}{\sigma_S^{(2,0)}}$$

$$A_{FB,2} = \frac{\sigma_A^{(2,2)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(2,2)}}{\sigma_S^{(2,0)}} + \frac{\sigma_A^{(3,2)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(3,2)}}{\sigma_S^{(2,0)}} + \frac{\sigma_A^{(4,2)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(4,2)}}{\sigma_S^{(2,0)}} - \frac{\sigma_S^{(2,1)} + \sigma_S^{(3,1)}}{\sigma_S^{(2,0)}} \left[\frac{\sigma_A^{(2,1)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(2,1)}}{\sigma_S^{(2,0)}} + \frac{\sigma_A^{(3,1)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(3,1)}}{\sigma_S^{(2,0)}}\right]$$

$$\sigma_A^{(2p)} = \frac{N_c}{8\pi}\,\frac{s\beta^2}{D_Z}\,a_e^Z a_Q^Z \left[v_e^Z v_Q^Z + \frac{1}{2}\left(1-\frac{m_Z^2}{s}\right)v_e^\gamma v_Q^\gamma\right]\left(\tilde{F}_1^* G_1 + \tilde{F}_1 G_1^*\right)$$

$$\sigma_S^{(2p)} = \frac{N_c}{24\pi}\left\{\frac{\beta}{s}(v_e^\gamma v_Q^\gamma)^2 + \frac{1}{D_Z}\left[2(s-m_Z^2)\beta v_e^\gamma v_Q^\gamma v_e^Z v_Q^Z + s\beta\left(v_Q^Z\right)^2\left[\left(a_e^Z\right)^2+\left(v_e^Z\right)^2\right]\right]\right\}\times$$

$$\times\left[(3-\beta^2)\tilde{F}_1\tilde{F}_1^* + \beta^2(\tilde{F}_1 F_2^* + \tilde{F}_1^* F_2) + \frac{\beta^4}{1-\beta^2}F_2F_2^*\right]$$

$$+\frac{N_c}{12\pi}\,\frac{s\beta^3}{[(s-m_Z^2)^2+m_Z^2\Gamma_Z^2]}\left(a_Q^Z\right)^2\left[\left(a_e^Z\right)^2+\left(v_e^Z\right)^2\right]G_1G_1^*$$

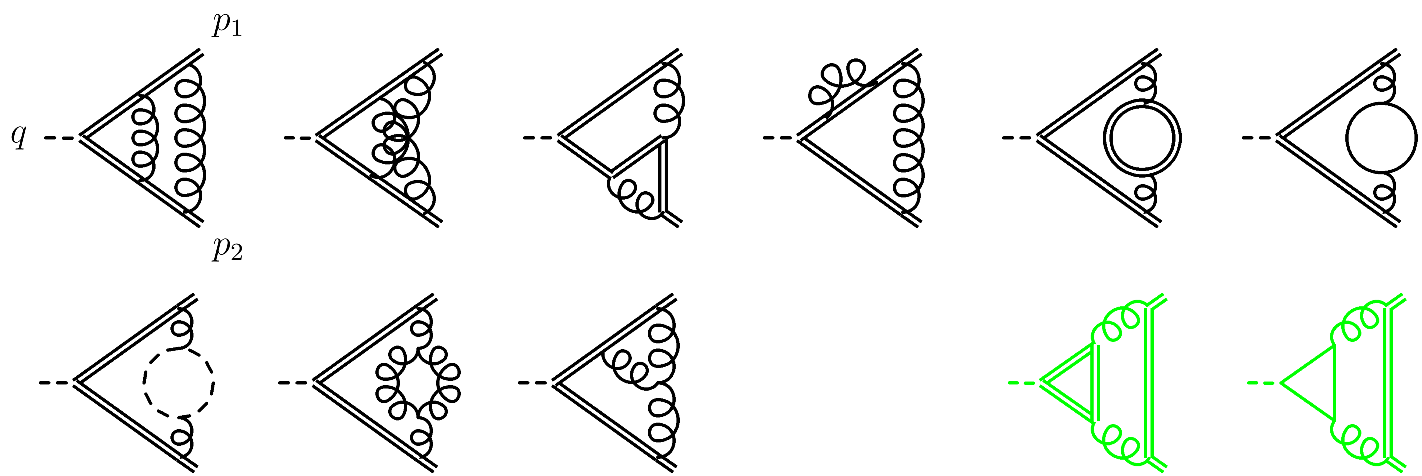
$$\tilde{F}_1 = F_1 + F_2, \qquad \beta = \sqrt{1-4m^2/s}, \qquad D_Z = \left[(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2\right], \qquad m_e = 0$$

The form factors can be expanded in powers of α_S as follows:

$$
\begin{aligned}
F_1(s) &= 1 + \left(\frac{\alpha_S}{2\pi}\right) F_1^{(1)}(s) + \left(\frac{\alpha_S}{2\pi}\right)^2 F_1^{(2)}(s) + \ldots.. \\
F_2(s) &= \left(\frac{\alpha_S}{2\pi}\right) F_2^{(1)}(s) + \left(\frac{\alpha_S}{2\pi}\right)^2 F_2^{(2)}(s) + \ldots.. \\
G_1(s) &= 1 + \left(\frac{\alpha_S}{2\pi}\right) G_1^{(1)}(s) + \left(\frac{\alpha_S}{2\pi}\right)^2 G_1^{(2)}(s) + \ldots.. \\
G_2(s) &= \left(\frac{\alpha_S}{2\pi}\right) G_2^{(1)}(s) + \left(\frac{\alpha_S}{2\pi}\right)^2 G_2^{(2)}(s) + \ldots..
\end{aligned}
$$

We are interested in the form factors at order $\alpha_S/(2\pi)$, that can be extracted by the following Feynman diagrams

Feynman Diagrams



Projectors

$$F_i(s) = tr_D\left(P_\mu^{(i)}\,\Gamma^\mu\right), \qquad G_i(s) = tr_D\left(R_\mu^{(i)}\,\Gamma^\mu\right)$$

$$F(s) \sim \int d^Dk_1\, d^Dk_2\, \frac{(k_i\cdot p_j, k_i\cdot k_j)}{D_1\cdots D_t}$$

$$P_\mu^{(i)} = (\not{p}_2 - m)\left[ic_1^{(i)}\gamma_\mu + \frac{i}{2m}c_2^{(i)}(p_1-p_2)_\mu\right](\not{p}_1 + m)$$

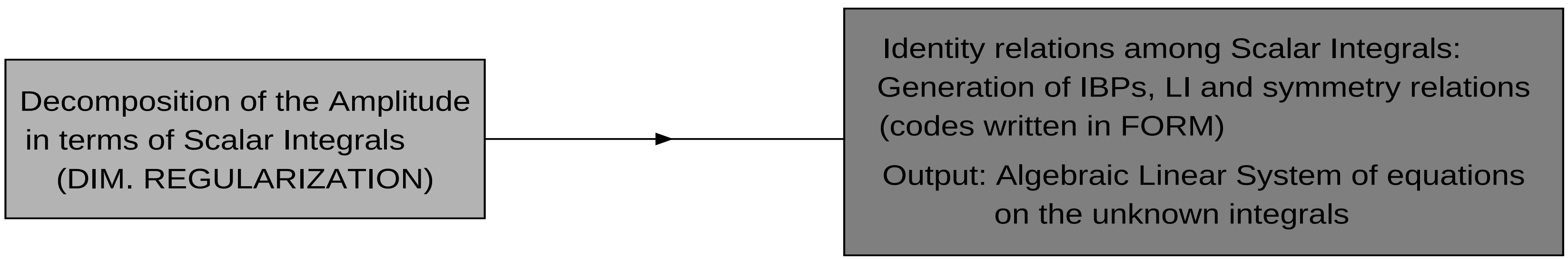
$$R_\mu^{(i)} = (\not{p}_2 - m)\left[ic_3^{(i)}\gamma_\mu\gamma_5 + \frac{i}{2m}c_4^{(i)}\gamma_5(p_1-p_2)_\mu\right](\not{p}_1 + m)$$

Laporta Algorithm and Diff. Equations

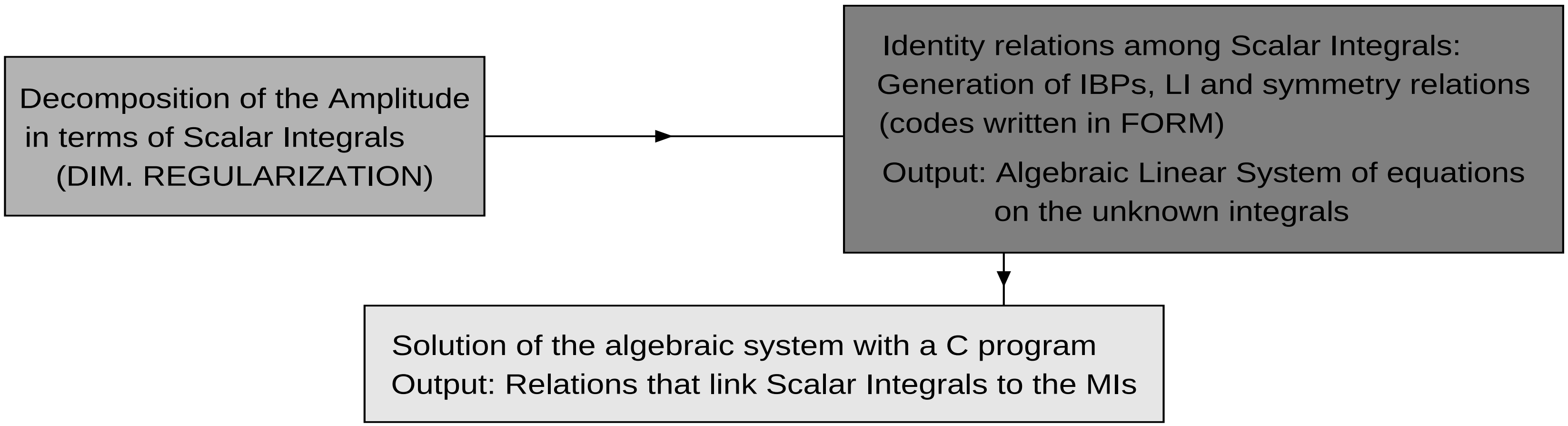
Laporta Algorithm and Diff. Equations

Decomposition of the Amplitude
in terms of Scalar Integrals
(DIM. REGULARIZATION)

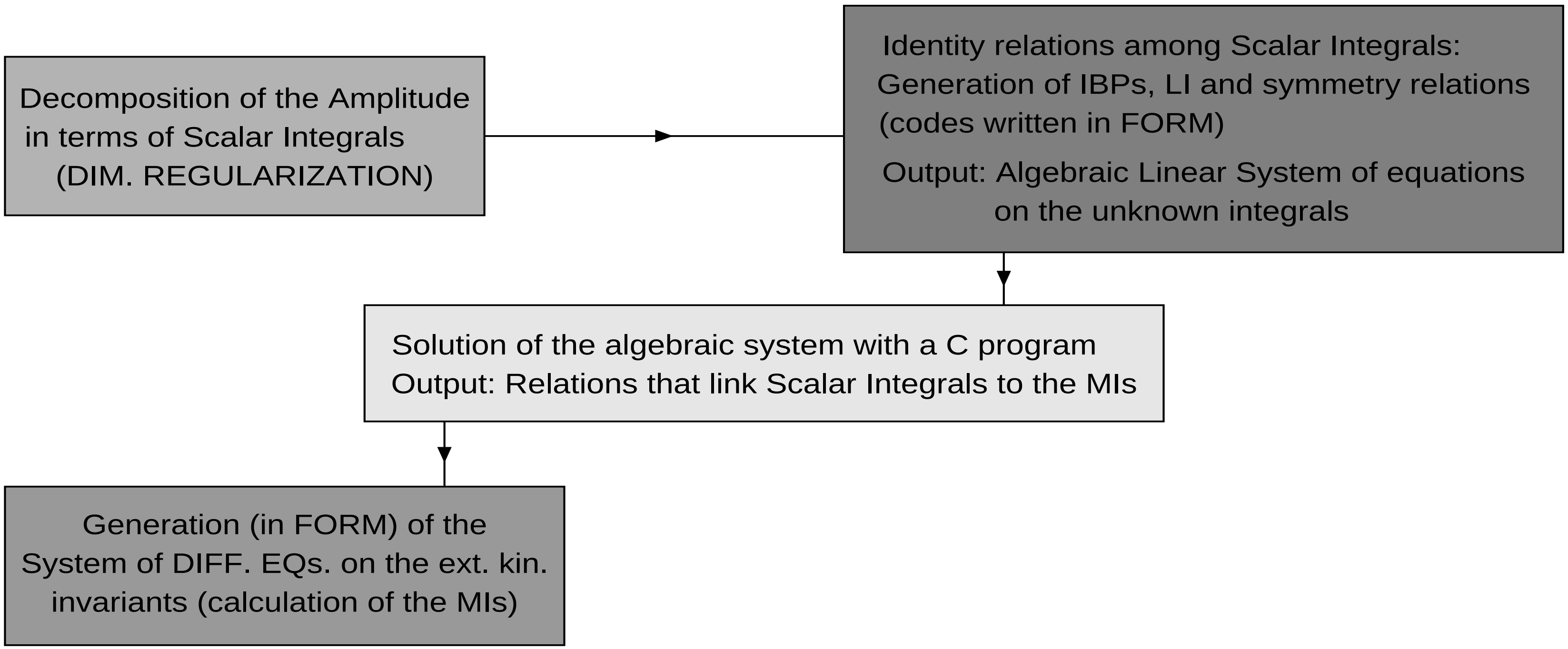
Laporta Algorithm and Diff. Equations



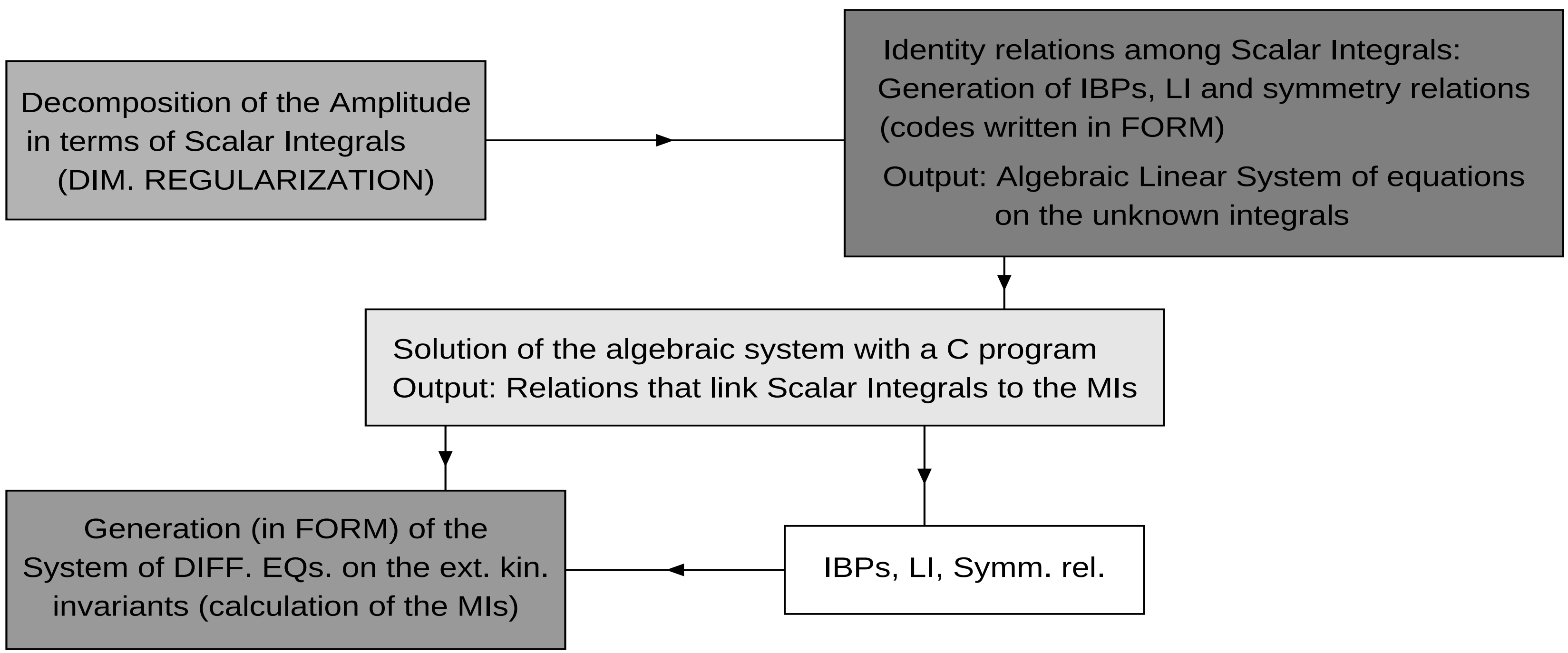
Laporta Algorithm and Diff. Equations



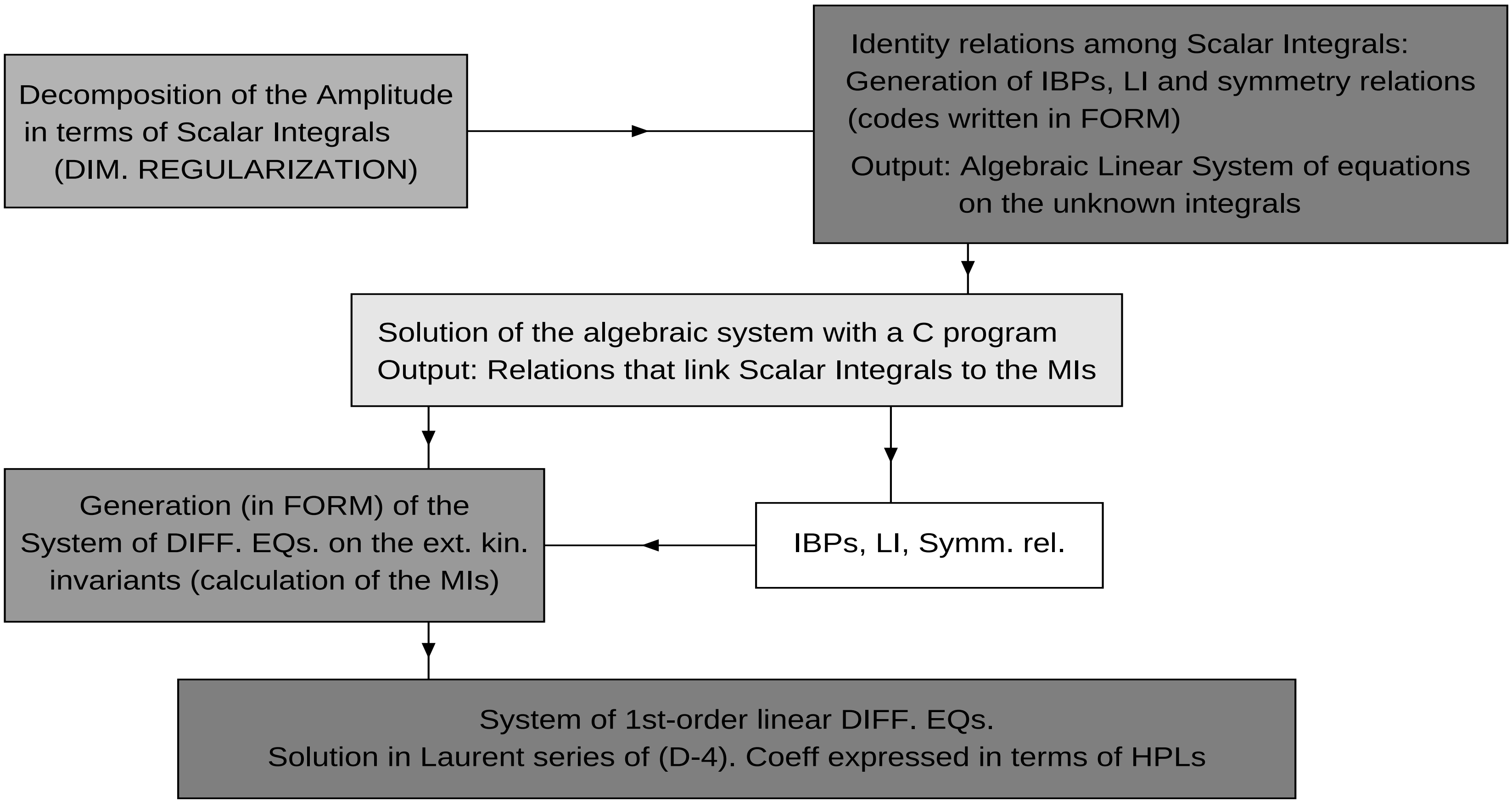
Laporta Algorithm and Diff. Equations



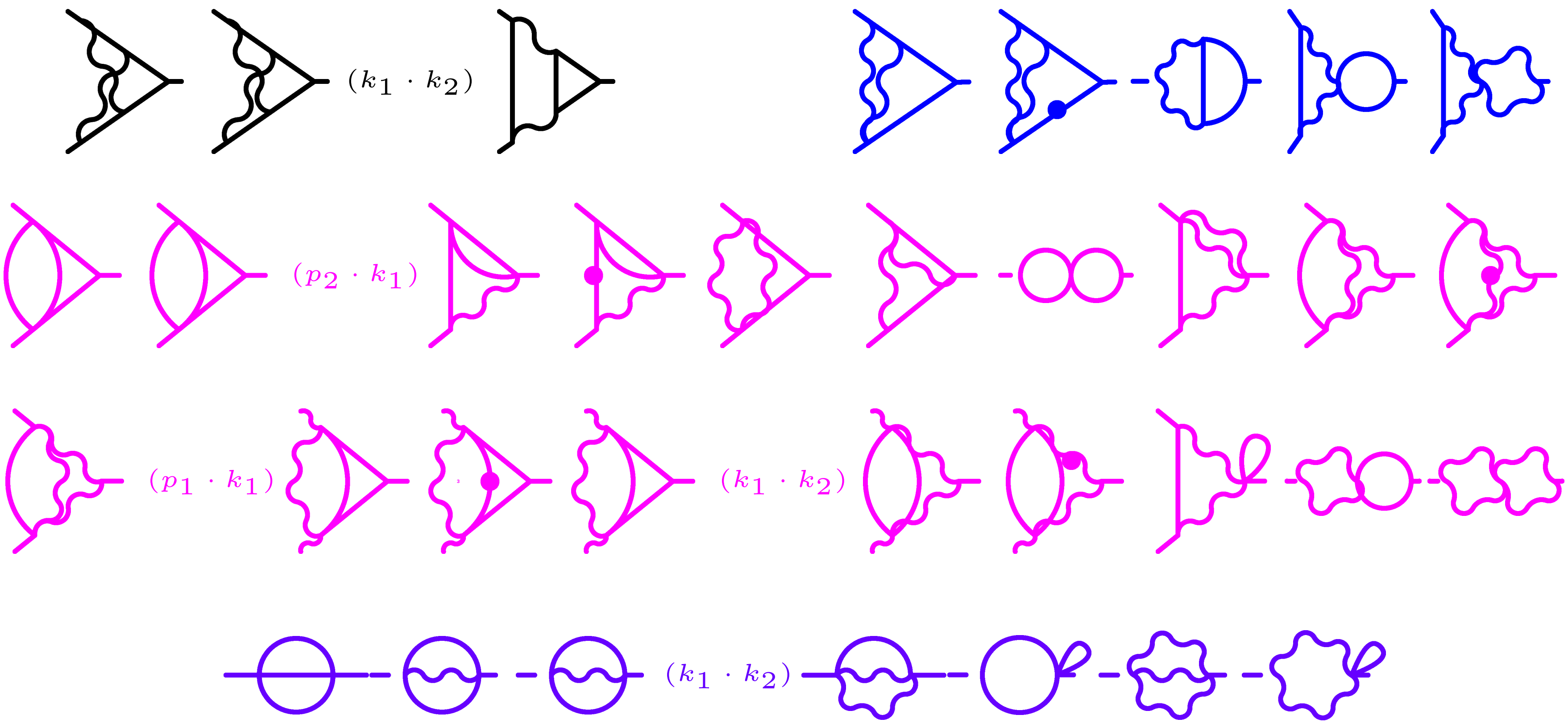
Laporta Algorithm and Diff. Equations



Laporta Algorithm and Diff. Equations



The 34 Master Integrals



R. B., P. Mastrolia and E. Remiddi, *Nucl. Phys.* **B661 (2003) 289.**
R. B., P. Mastrolia and E. Remiddi, *Nucl. Phys.* **B690 (2004) 138.**
M. Czakon, J. Gluza and T. Riemann, *Nucl. Phys. Proc. Suppl.* **135 (2004) 83.**

- Renormalization scheme:
  - mass and wave function of the heavy quark $\rightarrow$ OS
  - coupling constant and gluon wave function $\rightarrow$ $\overline{\text{MS}}$
- γ_5 prescription in D dimensions:
  - diagrams without fermionic triangle $\rightarrow$ naive anticommuting γ_5
  - diagrams with fermionic triangle $\rightarrow$ 't Hooft-Veltman γ_5
- For the first kind of diagrams the Ward Identities are directly satisfied.
- For the second kind of diagrams, the use of 't Hooft-Veltman γ_5 in D dimensions brakes the Ward Identities that have to be restored with a finite renormalization.

Tree-Level

$$A_{FB,0} = \frac{\sigma_A^{(2,0)}}{\sigma_S^{(2,0)}}$$

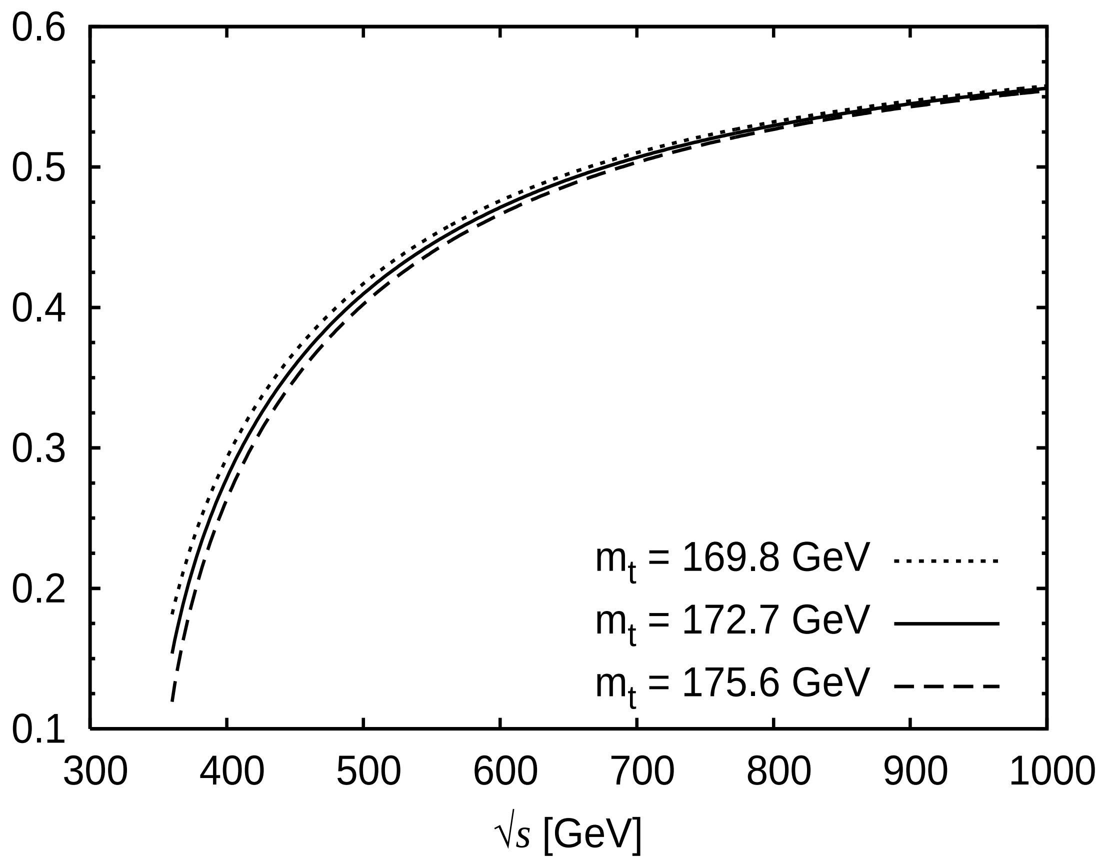
$$\sigma_A^{(2,0)} = \frac{N_c}{4\pi}\,\frac{s}{D_Z}\,\beta^2\,a_e^Z a_Q^Z\left[v_e^Z v_Q^Z + \frac{1}{2}\left(1-\frac{m_Z^2}{s}\right)\,v_e^\gamma v_Q^\gamma\right]$$

$$\sigma_S^{(2,0)} = \sigma_S^{(2,0,\gamma)} + \sigma_S^{(2,0,Z)} + \sigma_S^{(2,0,\gamma Z)},$$

where:

$$\begin{aligned}
\sigma_S^{(2,0,\gamma)} &= \frac{N_c}{24\pi}\,\frac{1}{s}\,\beta\,\left(v_e^\gamma v_Q^\gamma\right)^2(3-\beta^2) \\
\sigma_S^{(2,0,Z)} &= \frac{N_c}{24\pi}\,\frac{s}{D_Z}\,\beta\left[\left(a_e^Z\right)^2+\left(v_e^Z\right)^2\right]\left[2\left(a_Q^Z\right)^2\beta^2+\left(v_Q^Z\right)^2(3-\beta^2)\right] \\
\sigma_S^{(2,0,\gamma Z)} &= \frac{N_c}{12\pi}\,\frac{s}{D_Z}\,\beta\left(1-\frac{m_Z^2}{s}\right)\,v_e^\gamma v_Q^\gamma v_e^Z v_Q^Z\,(3-\beta^2)
\end{aligned}$$

Tree-Level



Leading order asymmetry $A_{\mathrm{FB},0}^{(t\bar{t})}$ for three values of the top quark mass.

One-Loop Level

$$A_{FB,1}^{(2p)} = \frac{\sigma_A^{(2,1)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(2,1)}}{\sigma_S^{(2,0)}}$$

$$\sigma_A^{(2,1)} = \sigma_A^{(2,0)} \left(\frac{\alpha_s}{2\pi}\right) \left[\mathrm{Re}\,\tilde{F}_1^{(1\ell)} + \mathrm{Re}\,G_1^{(1\ell)}\right]$$

$$\sigma_S^{(2,1)} = \sigma_S^{(2,0,\gamma)}\sigma_S^{(2,1,\gamma)} + \sigma_S^{(2,0,Z)}\sigma_S^{(2,1,Z)} + \sigma_S^{(2,0,\gamma Z)}\sigma_S^{(2,1,\gamma Z)}$$

where

$$\sigma_S^{(2,1,\gamma)} = \left(\frac{\alpha_s}{2\pi}\right)\left\{2\mathrm{Re}\,\tilde{F}_1^{(1\ell)} + \frac{2\beta^2}{3-\beta^2}\mathrm{Re}\,F_2^{(1\ell)}\right\}$$

$$\sigma_S^{(2,1,Z)} = \frac{\left(v_Q^Z\right)^2 (3-\beta^2)\,\sigma_S^{(2,1,\gamma)} + 4\left(\frac{\alpha_s}{2\pi}\right)\left(a_Q^Z\right)^2 \beta^2 \mathrm{Re}\,G_1^{(1\ell)}}{2\left(a_Q^Z\right)^2\beta^2 + (3-\beta^2)\left(v_Q^Z\right)^2}$$

$$\sigma_S^{(2,1,\gamma Z)} = \sigma_S^{(2,1,\gamma)}$$

One-Loop Level

Because

$$\sigma_S^{(2,1)} = \sigma_S^{(2,0)} \sigma_S^{(2,1,\gamma)} + \sigma_S^{(2,0,Z)} \left[\sigma_S^{(2,1,Z)} - \sigma_S^{(2,1,\gamma)}\right]$$

we can write

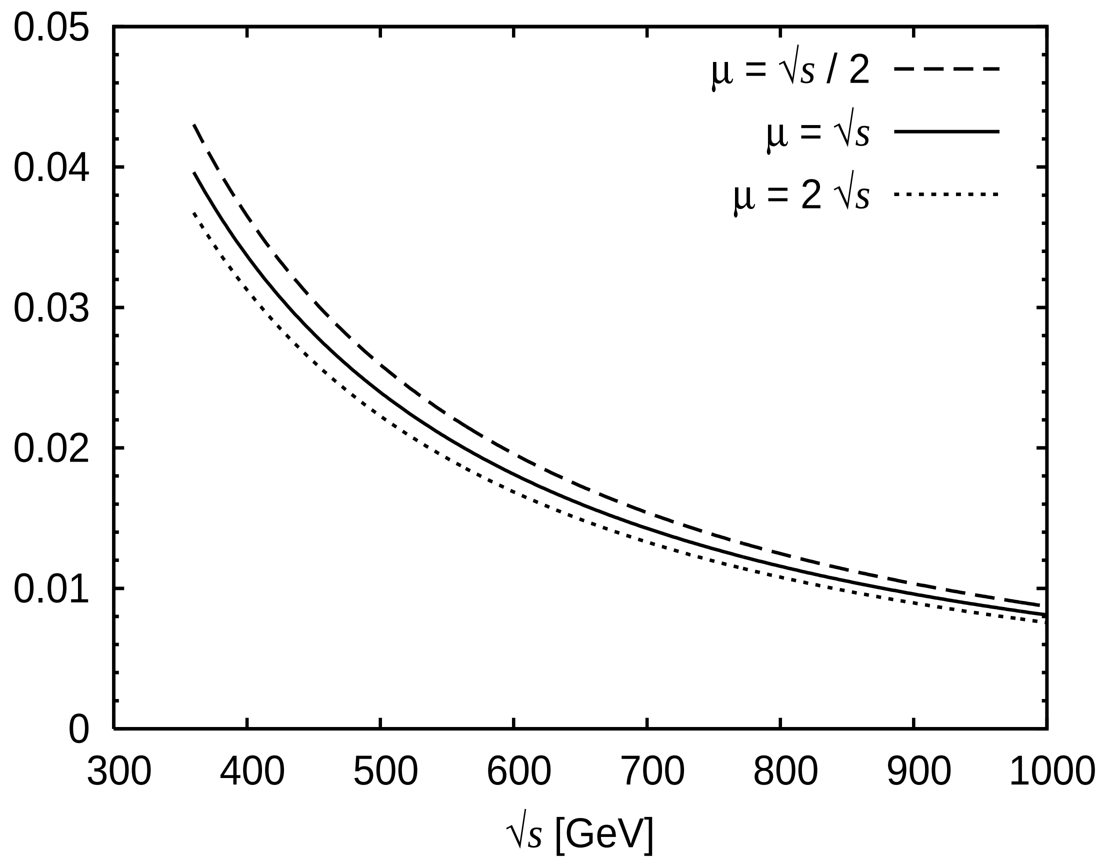
$$\begin{aligned} A_{FB,1}^{(2p)} &= \frac{\sigma_A^{(2,1)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(2,1)}}{\sigma_S^{(2,0)}} \\ &= \frac{\sigma_A^{(2,1)}}{\sigma_A^{(2,0)}} - \sigma_S^{(2,1,\gamma)} - \frac{1}{\sigma_S^{(2,0)}} \left\{ \sigma_S^{(2,0,Z)} \left[\sigma_S^{(2,1,Z)} - \sigma_S^{(2,1,\gamma)}\right] \right\} \mathcal{A}_{1l} - \frac{1}{\sigma_S^{(2,0)}} \mathcal{B}_{1l} \end{aligned}$$

with

$$\begin{aligned} \mathcal{A}_{1l} &= -ReF_1^{(1l)} + ReG_1(1l) + ReF_2^{(1l)} \left(1 - \frac{6}{3-\beta^2}\right) \\ \mathcal{B}_{1l} &= \frac{s\beta^3 \, a_Q^2 (v_e^2 + a_e^2)}{s_W^4 c_W^4 D_Z} \left[\frac{8}{3} (-ReF_1^{(1l)} + ReG_1^{(1l)}) + 8 \left(1 - \frac{3}{3-\beta^2}\right) ReF_2^{(1l)} \right] \end{aligned}$$

We have $Pole(ReF_1^{(1l)}) = Pole(ReG_1^{(1l)}) \Longrightarrow A_{FB,1}^{(2p)}$ is IR finite.

One-Loop Level



Order α_s correction $A_1^{(t\bar{t})}$ for three values of the renormalization scale μ, using $m_t = 172.7$ GeV

Two-Loop Level

$$A_{FB,2}^{(2p)} = \frac{\sigma_A^{(2,2)}}{\sigma_A^{(2,0)}} - \frac{\sigma_S^{(2,2)}}{\sigma_S^{(2,0)}} - \frac{\sigma_S^{(2,1)}}{\sigma_S^{(2,0)}} A_1^{(2p)}$$

$$\sigma_A^{(2,2)} = \sigma_A^{(2,0)} \left(\frac{\alpha_s}{2\pi}\right)^2 \left[\mathrm{Re}\,\tilde{F}_1^{(2\ell)} + \mathrm{Re}\,G_1^{(2\ell)} + \mathrm{Re}\,\tilde{F}_1^{(1\ell)}\mathrm{Re}\,G_1^{(1\ell)} + \pi^2\mathrm{Im}\,\tilde{F}_1^{(1\ell)}\mathrm{Im}\,G_1^{(1\ell)}\right]$$

$$\sigma_S^{(2,2)} = \sigma_S^{(2,0,\gamma)}\sigma_S^{(2,2,\gamma)} + \sigma_S^{(2,0,Z)}\sigma_S^{(2,2,Z)} + \sigma_S^{(2,0,\gamma Z)}\sigma_S^{(2,2,\gamma Z)}$$

where

$$\sigma_S^{(2,2,\gamma)} = \left(\frac{\alpha_s}{2\pi}\right)^2 \left\{\frac{2\beta^2}{3-\beta^2}\left[\mathrm{Re}\,F_2^{(2\ell)} + \mathrm{Re}\,\tilde{F}_1^{(1\ell)}\mathrm{Re}\,F_2^{(1\ell)} + \pi^2\,\mathrm{Im}\,\tilde{F}_1^{(1\ell)}\mathrm{Im}\,F_2^{(1\ell)}\right]\right.$$

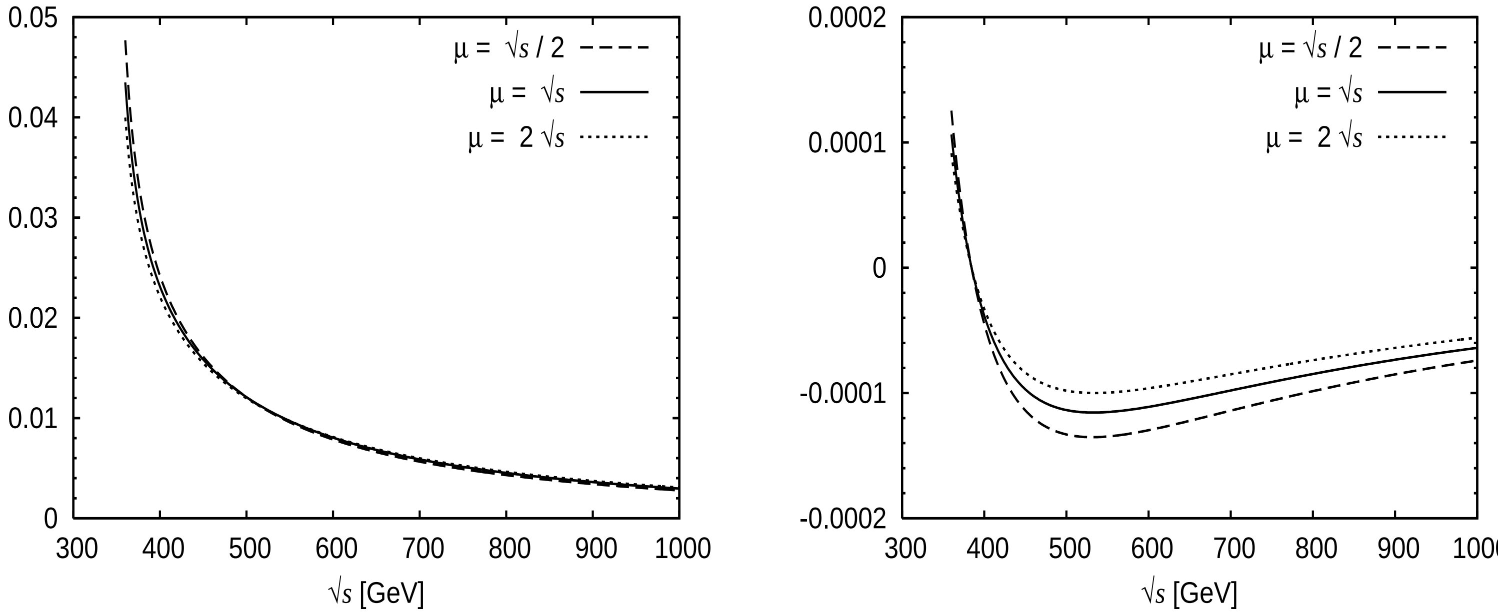
$$\left. + \frac{\beta^4}{(3-\beta^2)(1-\beta^2)}\left[(\mathrm{Re}\,F_2^{(1\ell)})^2 + \pi^2\,(\mathrm{Im}\,F_2^{(1\ell)})^2\right] + (\mathrm{Re}\,\tilde{F}_1^{(1\ell)})^2 + \pi^2\,(\mathrm{Im}\,\tilde{F}_1^{(1\ell)})^2 + 2\mathrm{Re}\,\tilde{F}_1^{(2\ell)}\right\}$$

$$\sigma_S^{(2,2,Z)} = \frac{1}{2\left(a_Q^Z\right)^2\beta^2 + (3-\beta^2)\left(v_Q^Z\right)^2}\left\{\left(v_Q^Z\right)^2(3-\beta^2)\,\sigma_S^{(2,2,\gamma)} + 4\left(\frac{\alpha_s}{2\pi}\right)^2\left(a_Q^Z\right)^2\beta^2\mathrm{Re}\,G_1^{(2}\right.$$

$$\left. +2\left(\frac{\alpha_s}{2\pi}\right)^2\left(a_Q^Z\right)^2\beta^2\left[(\mathrm{Re}\,G_1^{(1\ell)})^2 + \pi^2\,(\mathrm{Im}\,G_1^{(1\ell)})^2\right]\right\}$$

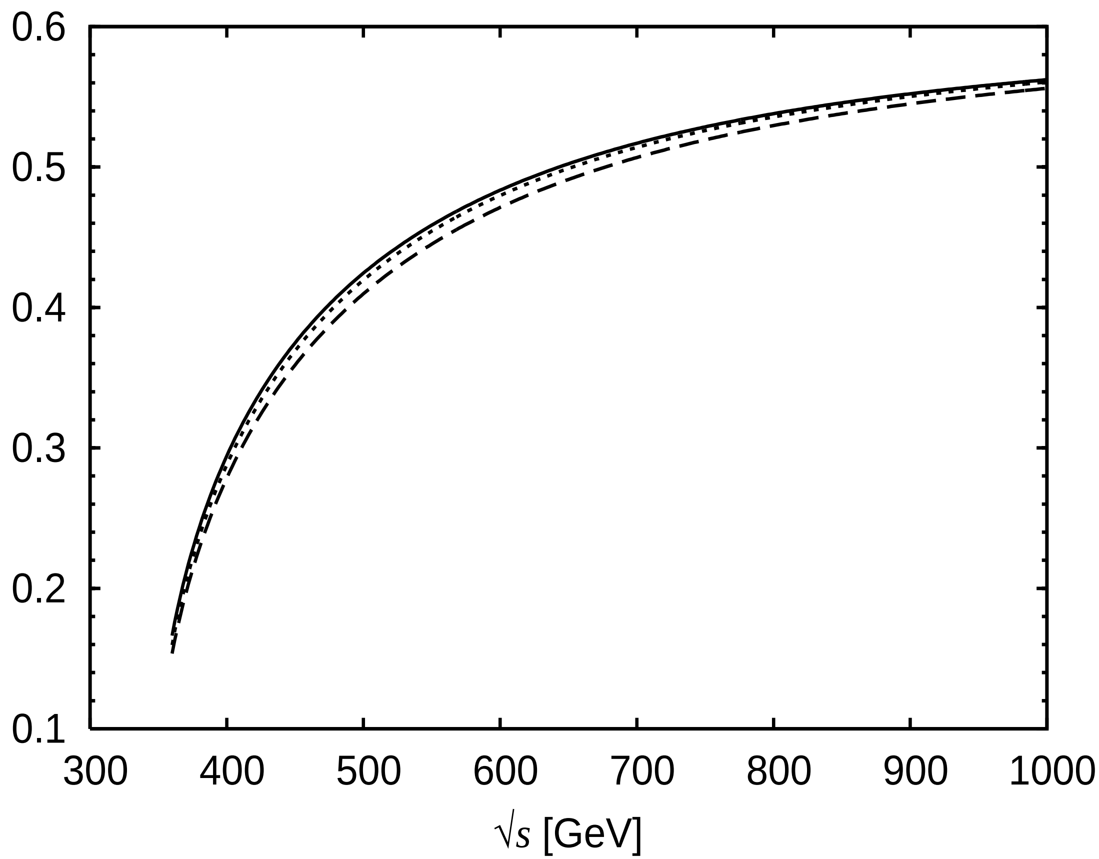
$$\sigma_S^{(2,2,\gamma Z)} = \sigma_S^{(2,2,\gamma)}$$

Two-Loop Level



Order α_s^2 correction $A_2^{(t\bar{t},A)}$ (left) and $A_2^{(t\bar{t},B)}$ (right) for three values of the renormalization scale μ, using $m_t = 172.7$ GeV.

Two-Parton Contribution at $\mathcal{O}(\alpha^2\alpha_S^2)$



Forward-backward asymmetry to lowest, first and second order in α_s using $m_t = 172.7$ GeV and $\mu = \sqrt{s}$. $A_{\text{FB},0}^{(t\bar{t})}$ (dashed), $A_{\text{FB}}^{(t\bar{t})}(\alpha_s)$ (dotted), $A_{\text{FB}}^{(t\bar{t})}(\alpha_s^2)$ (solid)

c and b quarks on the Z peak

The $b\bar{b}$ contributions to A_{FB} for bottom quarks at $\sqrt{s} = m_Z$

	$A_{FB,0}^{(bb)}$	$A_1^{(bb)}$	$A_2^{(bb,A)}$	$A_2^{(bb,B)}$	$A_{FB}^{(bb)}(\alpha_s)$	$A_{FB}^{(bb)}(\alpha_s^2)$
$\mu = \frac{m_Z}{2}$	0.103128	-0.000365	-0.000084	0.001147	0.103090	0.103200
$\mu = m_Z$	0.103128	-0.000326	-0.000100	0.000919	0.103094	0.103179
$\mu = 2m_Z$	0.103128	-0.000295	-0.000109	0.000753	0.103097	0.103164

The $c\bar{c}$ contributions to A_{FB} for charm quarks at $\sqrt{s} = m_Z$

	$A_{FB,0}^{(c\bar{c})}$	$A_1^{(c\bar{c})}$	$A_2^{(c\bar{c},A)}$	$A_2^{(c\bar{c},B)}$	$A_{FB}^{(c\bar{c})}(\alpha_s)$	$A_{FB}^{(c\bar{c})}(\alpha_s^2)$
$\mu = \frac{m_Z}{2}$	0.073592	-0.000170	-0.000060	-0.002418	0.073580	0.073397
$\mu = m_Z$	0.073592	-0.000152	-0.000063	-0.001938	0.073581	0.073434
$\mu = 2m_Z$	0.073592	-0.000138	-0.000064	-0.001586	0.073582	0.073460

$$m_c = 1.5\,\text{GeV}, \quad m_b = 5\,\text{GeV}, \quad m_t = 172.7 \pm 2.9\,\text{GeV},$$
$$m_Z = 91.1875\,\text{GeV}, \quad \Gamma_Z = 2.4952\,\text{GeV},$$
$$\sin^2\theta_W = 0.23153, \quad \alpha_s^{N_f=5}(m_Z) = 0.1187$$

Threshold expansions

In order to check our results we performed the expansion near the threshold in powers of $\beta = \sqrt{1 - 4m^2/s}$.
The cross section can be written as follows:

$$\sigma_{NNLO} = \sigma_S^{(2,0,\gamma)} \Big\{ 1 + \Delta^{(0,Ax)} + C_F \left(\frac{\alpha_s}{2\pi}\right) \Delta^{(1,Ve)} \left(1 + \Delta^{(0,Ax)}\right) + C_F \left(\frac{\alpha_s}{2\pi}\right) \Delta^{(1,Ax)} + C_F \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta^{(2,Ve)} \left(1 + \Delta^{(0,Ax)}\right) + C_F \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta^{(2,Ax)} \Big\}$$

where $$\sigma_S^{(2,0,\gamma)} = \frac{N_c}{24\pi} \frac{1}{s} \beta \left(v_e^\gamma v_Q^\gamma\right)^2 (3 - \beta^2)$$

$$\Delta^{(0,Ax)} = \frac{\sigma_S^{(2,0,Z)} + \sigma_S^{(2,0,\gamma Z)}}{\sigma_S^{(2,0,\gamma)}}$$

$$C_F \left(\frac{\alpha_s}{2\pi}\right) \Delta^{(1,Ax)} = \frac{\sigma_S^{(2,0,Z)}}{\sigma_S^{(2,0,\gamma)}} \left(\sigma_S^{(2,1,Z)} - \sigma_S^{(2,1,\gamma)}\right)$$

$$C_F \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta^{(2,Ax)} = \frac{\sigma_S^{(2,0,Z)}}{\sigma_S^{(2,0,\gamma)}} \left(\sigma_S^{(2,2,Z)} - \sigma_S^{(2,2,\gamma)}\right)$$

Top Threshold expansions

Up to order β^0 the cross section is IR-finite:

$$\Delta^{(1,Ve)} = \frac{6\zeta(2)}{\beta} - 8 + \mathcal{O}(\beta)$$

$$\Delta^{(2,Ve)} = C_F \Delta_A^{(2,Ve)} + C_A \Delta_{NA}^{(2,Ve)} + N_f T_R \Delta_L^{(2,Ve)} + T_R \Delta_H^{(2,Ve)}$$

with

$$\Delta_A^{(2,Ve)} = \frac{12\zeta^2(2)}{\beta^2} - \frac{48\zeta(2)}{\beta} + 24\zeta^2(2) - 4\zeta(2)\Big(\frac{35}{3} - 8\ln 2 + 4\ln\beta\Big) - 4\zeta(3) + 39$$

$$\Delta_{NA}^{(2,Ve)} = \frac{4\zeta(2)}{\beta}\Big(\frac{31}{12} - \frac{11}{2}\ln(2\beta)\Big) + 4\zeta(2)\Big(\frac{179}{12} - 16\ln 2 - 6\ln\beta\Big) - 26\zeta(3) - \frac{151}{9}$$

$$\Delta_L^{(2,Ve)} = \frac{4\zeta(2)}{\beta}\Big(2\ln(2\beta) - \frac{5}{3}\Big) + \frac{44}{9}$$

$$\Delta_H^{(2,Ve)} = -\frac{32}{3}\zeta(2) + \frac{176}{9}$$

A. H. Hoang, *Phys. Rev.* **D56 (1997) 7276.**
A. Czarnecki and K. Melnikov, *Phys. Rev. Lett.* **80 (1998) 2531.**
M. Beneke, A. Signer and V.A. Smirnov, *Phys. Rev. Lett.* **80 (1998) 2535.**

Threshold expansions

The cross section is IR finite at this order of β, but Coulomb divergencies $1/\beta$ are present

- The Sommerfeld-Sakharov factor (modulus square of the fermion wave function at the origin in NRQCD)

$$|\psi(0)|^2 = \frac{z}{1-e^{-z}}\,, \qquad z = C_F \frac{6\zeta(2)}{\beta}\frac{\alpha_S}{\pi}$$

  resums the Coulomb terms α_S^n/β^n of the C_F^2 part and improves, therefore, the convergence of the series when β approaches α_S

- If we multiply also by the so-called hard correction term $(1-4C_F\alpha/\pi)$, the subleading terms $\alpha_S^{(n+1)}/\beta^n$ can be taken into account

IR divergences for $\Delta^{(1,Ve)}$ and $\Delta^{(2,Ve)}$ IR divergences appear at $\mathcal{O}(\beta)$ and $\mathcal{O}(\beta^2)$, respectively.

Top Threshold expansions

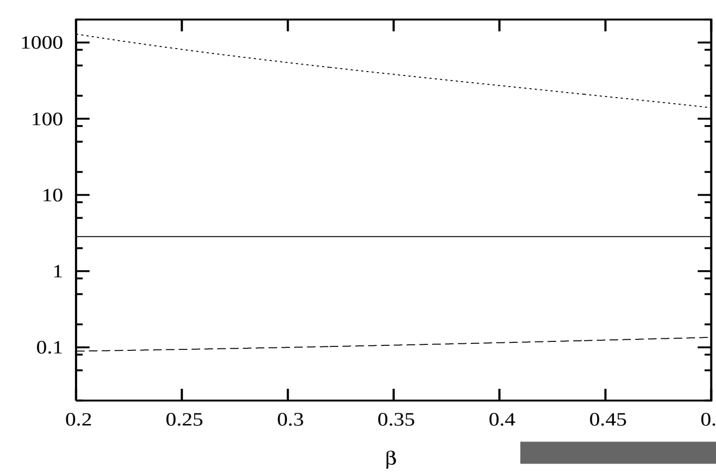
For the terms involving Z-boson exchange we have:

$$\Delta^{(0,Ax)} = \frac{s^2}{D_Z}\left\{\frac{\left(a_e^Z\right)^2+\left(v_e^Z\right)^2}{\left(v_e^\gamma v_Q^\gamma\right)^2}\left[2\left(a_Q^Z\right)^2\frac{\beta^2}{3-\beta^2}+\left(v_Q^Z\right)^2\right]+2\left(1-\frac{m_Z^2}{s}\right)\frac{v_e^Z v_Q^Z}{v_e^\gamma v_Q^\gamma}\right\}$$

$$\Delta^{(1,Ax)} = \mathcal{O}\left(\beta^2\right)$$

$$\Delta^{(2,Ax)} = \frac{64\zeta(2)m_Q^4(a_Q^Z)^2\left[(v_e^Z)^2+(a_e^Z)^2\right]}{(v_Q^\gamma v_e^\gamma)^2(4m_Q^2-m_Z^2)^2}C_F+\mathcal{O}(\beta)$$

- $\Delta^{(1,Ax)}$ and $\Delta^{(2,Ax)}$ are of order β^2 and β^0, respectively
- In the expansions of $\Delta^{(1,Ax)}$ and $\Delta^{(2,Ax)}$ IR divergences appear at order β^4 and β^3, respectively



Top Threshold expansions

The same happens for the asymmetric cross section

$$\sigma^{(A)}_{NNLO} = \sigma^{(2,0)}_{A} \left\{ 1 + C_F \left(\frac{\alpha_s}{2\pi} \right) \Delta^{(A,1)} + C_F \left(\frac{\alpha_s}{2\pi} \right)^2 \Delta^{(A,2)} \right\}$$

$$\Delta^{(A,1)} = \frac{6\zeta(2)}{\beta} - 6 + \mathcal{O}(\beta)$$

$$\Delta^{(A,2)} = C_F \Delta^{(A,2)}_{A} + C_A \Delta^{(A,2)}_{NA} + N_f T_R \Delta^{(A,2)}_{L} + T_R \left(\Delta^{(A,2)}_{H} + \Delta^{(A,2)}_{tr} \right)$$

with

$$\Delta^{(A,2)}_{A} = \frac{12\zeta^2(2)}{\beta^2} - \frac{36\zeta(2)}{\beta} + 24\zeta^2(2) - 4\zeta(2)\left(\frac{25}{6} - \frac{25}{4}\ln 2 + \frac{9}{2}\ln\beta \right) - \frac{35}{4}\zeta(3) + \frac{70}{3}$$

$$\Delta^{(A,2)}_{NA} = \frac{4\zeta(2)}{\beta}\left(\frac{16}{3} - \frac{11}{2}\ln(2\beta) \right) + 4\zeta(2)\left(\frac{67}{6} - \frac{25}{2}\ln 2 - 4\ln\beta \right) - \frac{35}{2}\zeta(3) - 14$$

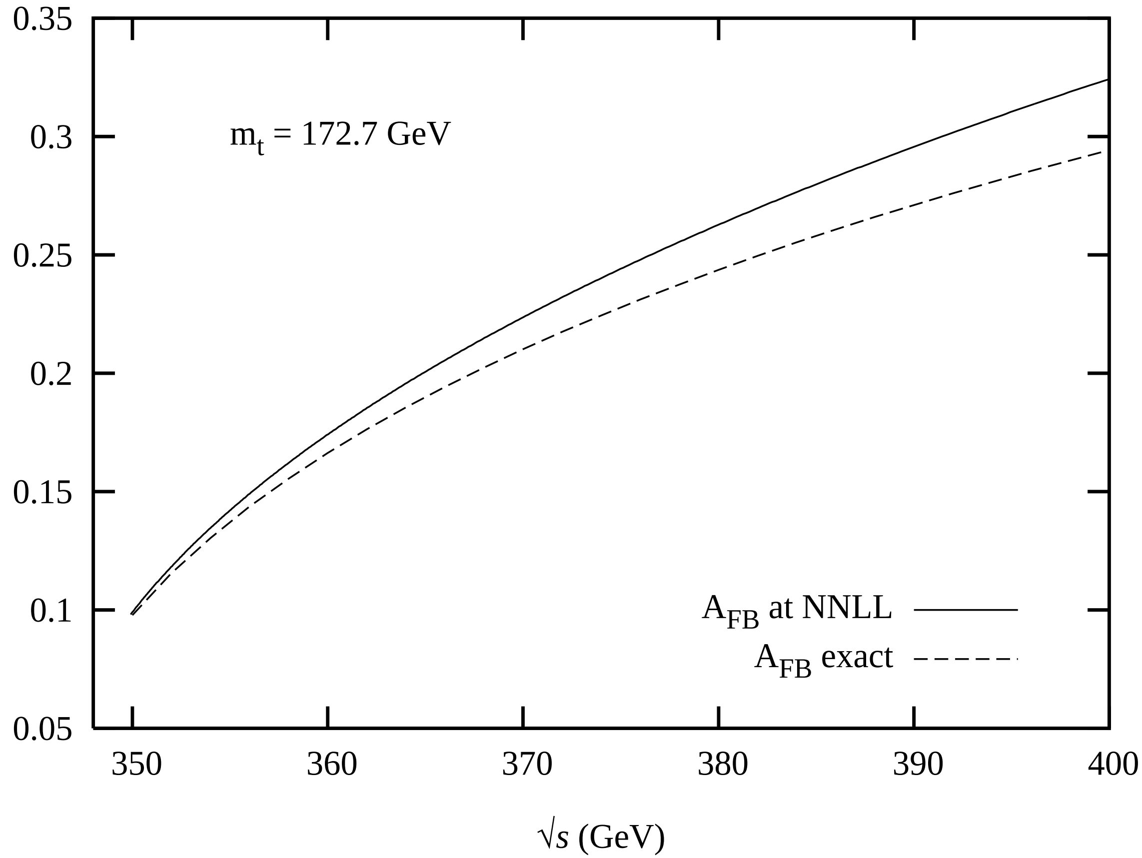
$$\Delta^{(A,2)}_{L} = \frac{4\zeta(2)}{\beta}\left(2\ln(2\beta) - \frac{8}{3} \right) + 4$$

$$\Delta^{(A,2)}_{H} = -\frac{32}{3}\zeta(2) + \frac{56}{3}$$

$$\Delta^{(A,2)}_{tr} = \zeta(2)\left(16\ln 2 - \frac{23}{3} \right) - 8\ln 2 + \frac{8}{3}\ln^2 2 + \mathcal{O}\left(\beta^2\right)$$

Top Threshold expansions

$$A_{FB}^{(Q\bar{Q})} = A_{FB,0}\, C_{FB}$$



The second order forward-backward asymmetry $A_{FB}^{(t\bar{t})}(\alpha_s^2)$ near threshold: exact values (dashed) and the values obtained from the near-threshold formula (solid), using $\mu = m_t = 172.7$ GeV

Checks

- For what concerns the vector form factors, results for the massive and massless fermion loops were calculated by Hoang-Jezabek-Kühn-Teubner. We are in full agreement for the fi rst ones and in agreement with the leading logarithmic terms for the massless fermion loop.
- In the threshold region, we compared our form factors at one loop with the ones by Hoang (replacing C_F with 1) and we found complete agreement once Hoang's results are translated in dimensional regularization. At the two-loop level we found agreement between the poles of our C_F^2 part and the poles of Hoang's results.
- Always in the threshold region we found agreement with the two-loop corrections to the cross section $e^+e^- \to Q\bar{Q}$ calculated by Czarnecki-Melnikov and Beneke-Signer-Smirnov.
- For the anomalous diagrams, we found full agreement with the triangle contribution to the $b\bar{b}$ vertex function of the non-singlet current $\bar{t}\gamma_\mu\gamma_5 t - \bar{b}\gamma_\mu\gamma_5 b$ computed for massless b quarks with the method of dispersion relations by Kniehl-Kühn.

Summary

- We presented the calculation of the two-parton contribution to the heavy-quark Forward-Backward Asymmetry in e^+e^- collisions, at the NNLO in QCD, retaining the complete dependence on the mass of the heavy quark
- Analytical expressions were obtained using the Laporta algorithm for the reduction to the Master Integrals and the Differential Equations Method for their calculation