The one-loop amplitude for six gluon scattering

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In collaboration with Walter Giele and Keith Ellis

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- $2 \to 2$: well established in the SM (QCD & EW) and beyond (MSSM)
- $2 \to 3$: a variety of calculations, but some still missing
- $2 \to 4$: only very few full calculations exist
  - EW: $e^+e^- \to 4$ fermions [full] — Denner, Dittmaier '05
  - EW: $e^+e^- \to HH\nu\bar{\nu}$ [full] — GRACE '05
  - QCD: $gg \to 4\,g$ [virtual only] — Ellis, Giele, GZ '06

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What is the bottleneck?

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- the calculation of the tree graph rates with $N+1$ partons
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A similar automation of one-loop calculations would constitute a major progress $\Rightarrow$ this was our aim

Seminumerical method: sketch and remarks

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Ellis,Giele,GZ '05

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A: use qgraf/form/mathematica to generate the amplitude

N1: reduce tensor integrals to scalar integrals in higher D Davydychev '91

N2: use a complete set of recursions relations to reduce all integrals to a set of analytically known basis integrals Giele & Glover '04

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Note:

- ☛ we compute ϵ expansions $\Rightarrow$ no issue about IR divergeces
- ☛ analytical expressions for the basis integrals $\Rightarrow$no loss of accuracy
- ☛ use expanded relations for exceptional configurations $\Rightarrow$ results numerical stable in the whole phase space

☛ details about the method in W. Giele's talk

Exceptional momentum configurations

- standard recursions assume $\det(S_{ij}) \neq 0$ and $\det(G_{ij}) \neq 0$
- not true for exceptional configurations (degeneracies or thresholds)
- exceptional configurations have zero measure, but recursions are unstable close to those points

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Note:

- general method, no analytical understanding of "singularities" needed
- method should hold unaltered in the presence of internal masses

Warmup: calculation of virtual corrections to

✓ four photon amplitudes

✓ four gluon amplitudes

✓ five gluon amplitudes

New results: calculation of virtual corrections to

✗ Higgs + 4 partons in the large m_t limit

Ellis, Giele, GZ '05

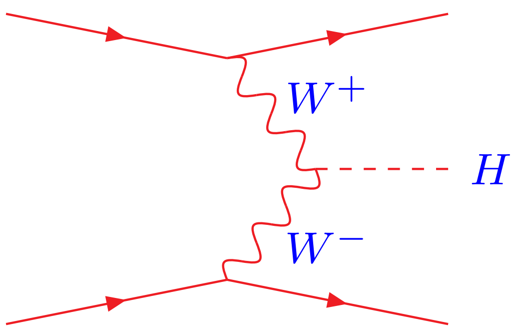
✗ six gluons amplitudes — Ellis, Giele, GZ '06

➥ *first full virtual six point calculation in QCD*

Higgs plus dijets production via VBF is one of the most promising channels to measure the Higgs couplings at the LHC

Two mechanisms:

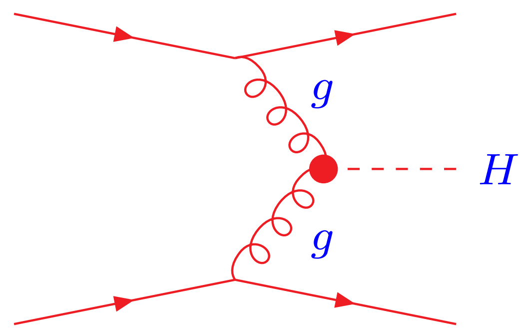
Vector Boson Fusion



$\mathcal{O}\left(\alpha_{ew}^2\right)$

- ✗ small QCD uncertainties
- ✗ suitable for coupling measurements

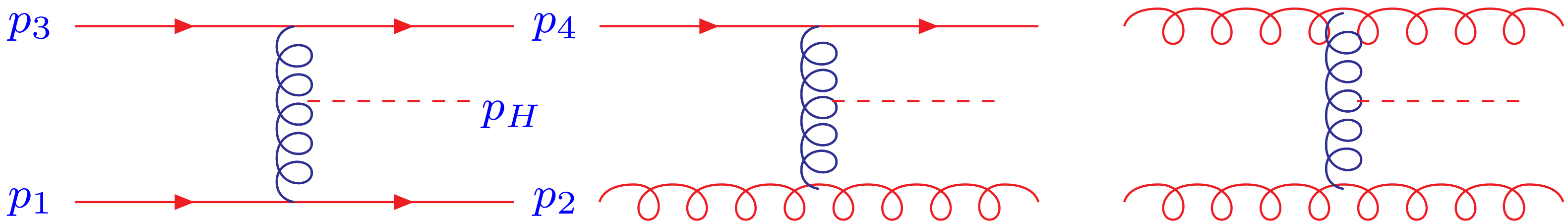
Gluon Gluon Fusion



$\mathcal{O}\left(\alpha_s^4\right)$

- ✗ large QCD uncertainties
- ✗ despite kinematical cuts remains dominant background

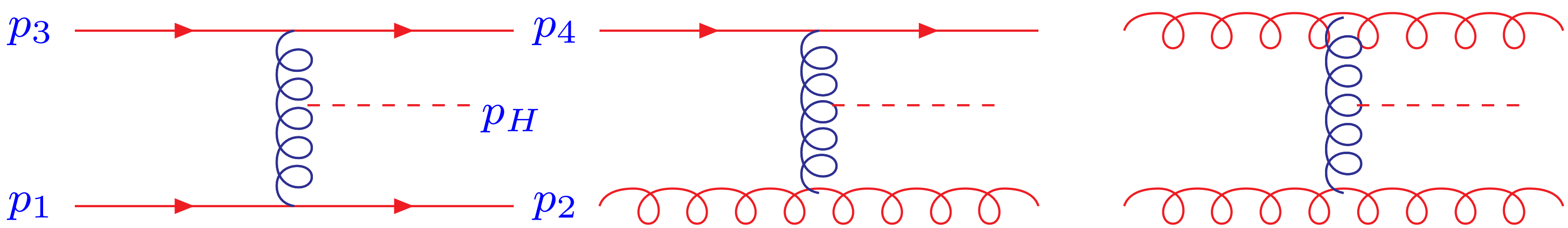
Gluon-gluon fusion processes



@NLO

First application: Higgs plus four partons

Gluon-gluon fusion processes



@NLO

Processes

$A:$	H	$\longrightarrow q\bar{q}q'\bar{q}'$	30 diagrams
$B:$	H	$\longrightarrow q\bar{q}q\bar{q}$	60 diagrams
$C:$	H	$\longrightarrow q\bar{q}gg$	191 diagrams
$D:$	H	$\longrightarrow gggg$	739 diagrams

Sample results for one random phase space point

	$1/\epsilon^2$	$1/\epsilon$	1
A_B	0	0	12.9
$A_{V,A}$	-68.9	-114.6	120.0
$\lvert A_{V,N} - A_{V,A}\rvert$	10^{-13}	$4 \cdot 10^{-12}$	$3 \cdot 10^{-11}$
B_B	0	0	858.9
$B_{V,A}$	-4580.6	-436.1	26471.0
$\lvert B_{V,N} - B_{V,A}\rvert$	$5 \cdot 10^{-11}$	$4 \cdot 10^{-9}$	$4 \cdot 10^{-10}$
C_B	0	0	968.6
$C_{V,A}$	-8394.4	-19808.0	not known
$\lvert C_{V,N} - C_{V,A}\rvert$ or $C_{V,N}$	$8 \cdot 10^{-10}$	$9 \cdot 10^{-9}$	-1287.9
D_B	0	0	$3.6 \cdot 10^6$
$D_{V,A}$	$-4.3 \cdot 10^7$	$-1.0 \cdot 10^8$	not known
$\lvert D_{V,N} - D_{V,A}\rvert$or$D_{V,N}$	$8 \cdot 10^{-8}$	$9 \cdot 10^{-7}$	$-6.8 \cdot 10^7$

Notation:

A, B, C, D four processes

X_B : Born

$X_{V,A}$: Virtual Analytical

$X_{V,N}$: Virtual Numerical

$\Rightarrow$ full analytical calculation done for $H + 4q$

$\Rightarrow$ relative accuracy $\mathcal{O}\left(10^{-13}\right)$

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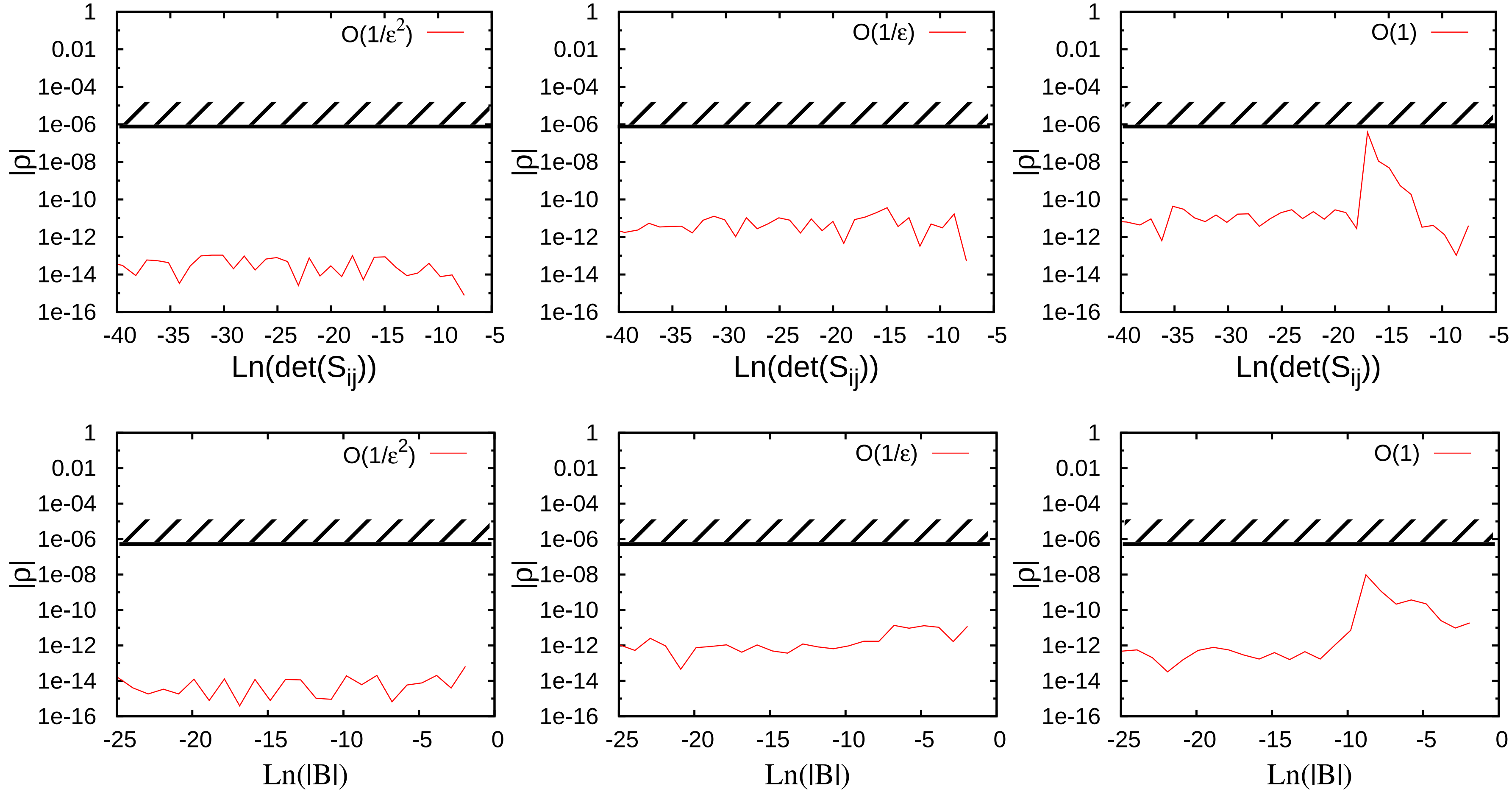
First full NLO (real+virtual) predictions for dijet Higgs production cross sections at the LHC implemented in MCFM $\Rightarrow$ see K. Ellis's talk

Results for exceptional phase space points



➥ reach predefined relative accuracy $\rho \equiv (A_{V,N} - A_{V,A})/A_{V,A}$ of 10^{-6}

Elements of the calculation

- color decomposition
- supersymmetric decomposition (comparison with from literature)
- helicity amplitudes
- modified tensor reduction $\Rightarrow$ see W. Giele's talk
- number of diagrams involved ~ 12000 (compare with H+4g $\sim$ 700)

- tree level decomposition

$$\mathcal{A}_n^{\rm tree}(\{p_i,\lambda_i,a_i\}) = g^{n-2} \sum_{\sigma\in S_n/Z_n} {\rm Tr}(T^{a_{\sigma(1)}}\cdots T^{a_{\sigma(n)}})\; A_n^{\rm tree}(p_{\sigma(1)}^{\lambda_{\sigma(1)}},\ldots,p_{\sigma(n)}^{\lambda_{\sigma(n)}})$$

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- one-loop decomposition $\mathcal{A}_n(\{p_i,h_i,a_i\}) = g^n \sum_{c=1}^{\lfloor n/2\rfloor+1} \sum_{\sigma\in S_n/S_{n;c}} \text{Gr}_{n;c}(\sigma)\, A_{n;c}(\sigma)$
  - leading in color: $\text{Gr}_{n;1}(1) = N_c\ \text{Tr}(T^{a_1}\cdots T^{a_n})$
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- subleading amplitudes $A_{n;c}(c > 1)$ fully determined by the leading ones $A_{n;1}$

Bern, Kosower Nucl.Phys. B 362 (1991) 389

$\Rightarrow$ *need only leading color stripped amplitudes*

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$$A^{\mathrm{QCD}} = A^{[1]} + \frac{n_f}{N} A^{[1/2]}$$

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In the literature first contributions computed are

$$\begin{aligned} A^{\mathcal{N}=4} &= A^{[1]} + 4A^{[1/2]} + 3A^{[0]} \\ A^{\mathcal{N}=1} &= A^{[1/2]} + A^{[0]} \end{aligned}$$

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☛ *to construct full QCD amplitudes one needs also* $A^{[0]}$

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All the others obtained by parity operation or cyclic permutations

Comparison with the literature: bibliography

- $\mathcal{N} = 4$

Finite	*Bern, Dixon, Kosower, PRL 70, 2677 (1993)*
MHV	*Bern, Dixon, Dunbar, Kosower, NPB 425 (1994) 217*
NMHV	*Bern, Dixon, Dunbar, Kosower, NPB 435 (1995) 59*

- $\mathcal{N} = 1$

Finite	*Bern, Dixon, Kosower, PRL 70, 2677 (1993)*
MHV	*Bern, Dixon, Dunbar, Kosower, NPB 435 (1995) 59*
NMHV	*Bidder, Bjerrum-Bohr, Dixon, Dunbar, PLB 606, 189 (2005)*
	Britto, Buchbinder, Cachazo, Feng, PRD 72 (2005) 065012

- $\mathcal{N} = 0$

Finite	*Bern, Dixon, Kosower, PRD 72, 125003 (2005)*
MHV	*Bern, Dixon, Kosower, hep-ph/0507005* [only $(--++++)$]
NMHV	*Britto, Feng, Mastrolia, hep-ph/0602178* [no rational terms]
	BDK+Berger,Forde, hep-ph/0604195 [only $(---+++)$]

$\Rightarrow$ talks of Z. Bern, D. Kosower, P. Mastrolia's

Comparison with the literature: results

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> Six gluon amplitudes fully known numerically.
> Now further work needed to compute full cross sections.

In progress and outlook

In progress:

- calculation of Higgs + dijet cross section via gluon fusion at LHC and phenomenological studies

Next:

- optimization of the method
- extension to include internal masses
- work on phase space integration
- automate subtraction
- general six-point NLO calculations $\Rightarrow$ tackle Les Houches priority list
  NLO QCD corrections to six legs processes important and doable
  [of all six-point amplitudes the six-gluon one is numerically the most challenging one]

Experimental NLO priorities [from Les Houches 2005]

- $2 \to 3$
  - $pp \to WW +$ jet
    [general background to NP]
  - $pp \to VVV$
    [background to SUSY trilepton]
  - $pp \to H + 2$jets
    [background to VBF H]
- $2 \to 4$
  - $pp \to 4$jets
    [general background to NP]
  - $pp \to tt + 2$jets
    [background to $t\bar{t}H$]
  - $pp \to tt + b\bar{b}$
    [background to $t\bar{t}H$]
  - $pp \to V + 3$jets
    [general background to NP]
  - $pp \to VV + 2$jets
    [background to WBF $H \to WW$]
  - $pp \to VVV +$ jet
    [background to SUSY trilepton]
  - $pp \to WW + b\bar{b}$
    [background to $t\bar{t}$]