

# Electroweak corrections to $H \rightarrow WW/ZZ \rightarrow 4 \text{ leptons}$

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in collaboration with A. Bredenstein, A. Denner and M.M. Weber

based on hep-ph/0604011



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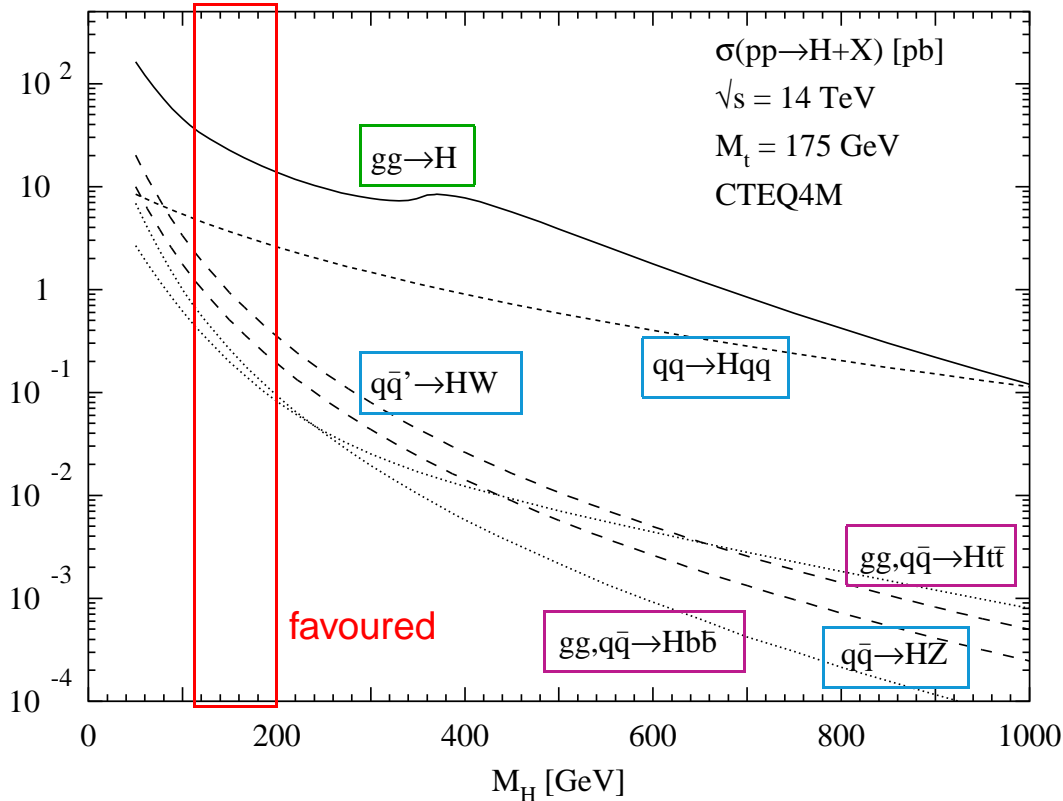
- 1 Introduction — the decays  $H \rightarrow WW/ZZ \rightarrow 4$  fermions
- 2 Calculation of electroweak corrections
- 3 Numerical results
- 4 Conclusions



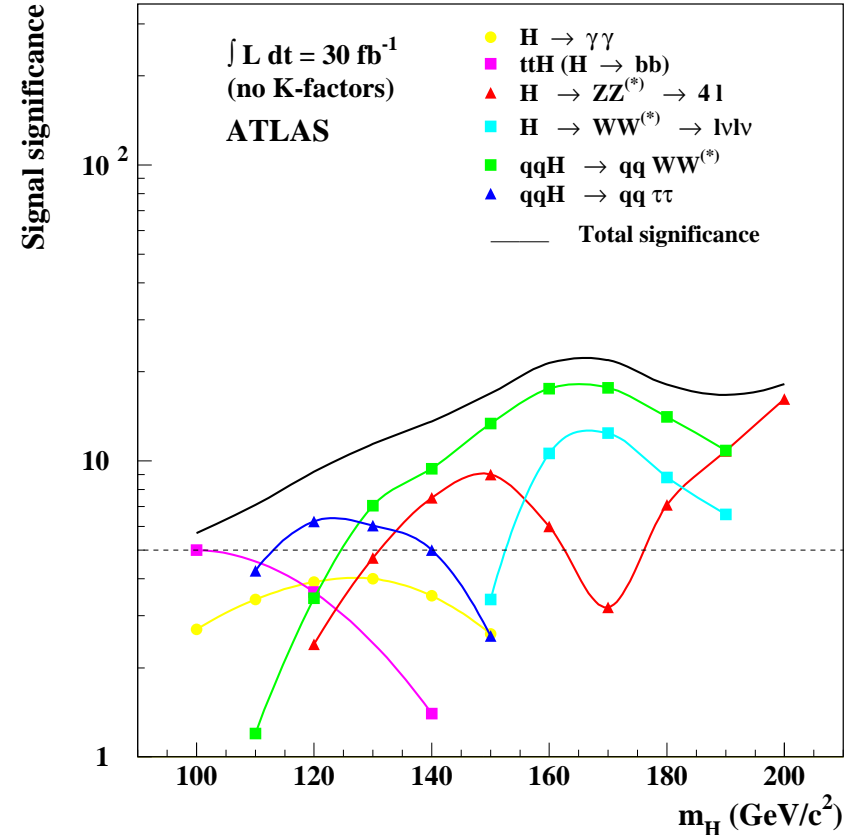
# 1 Introduction — the decays $H \rightarrow WW/ZZ \rightarrow 4$ fermions

## Cross sections and significance of the Higgs signal at the LHC

Spira et al. '98



ATLAS '03



Importance of decays  $H \rightarrow WW^{(*)}/ZZ^{(*)}$ :

- most important Higgs decay channels for  $M_H \gtrsim 125$  GeV
- most precise determination of  $M_H$  via  $H \rightarrow ZZ \rightarrow 4$  leptons for  $M_H \gtrsim 130$  GeV



## Theoretical description of $H \rightarrow WW^{(*)}/ZZ^{(*)}$ :

- existing work on partial decay widths:
    - ◇  $\mathcal{O}(\alpha)$  corrections to  $H \rightarrow WW/ZZ$  with stable  $W$ 's/ $Z$ 's  
Fleischer, Jegerlehner '81; Kniehl '91; Bardin, Vilenskii, Khristova '91
    - ◇ lowest-order predictions for  $H \rightarrow WW^{(*)}/ZZ^{(*)}$   
e.g. by Hdecay (Djouadi, Kalinowski, Spira '98)
  - however: proper description of distributions required
    - ◇ for the kinematical reconstruction of  $Z$ 's,  $W$ 's, and  $H$   
(including radiative corrections, in particular  $\gamma$  radiation)  
↪ invariant-mass distributions
    - ◇ for the verification of spin 0 and CP parity for the Higgs boson  
↪ angular and invariant-mass distributions  
Nelson '88; Soni, Xu '93; Chang et al.'93;  
Skjold, Osland '93; Barger et al.'93;  
Arens, Sehgal '94; Buszello et al.'02; Choi et al.'03
- ⇒ Monte Carlo generator for  $H \rightarrow WW/ZZ \rightarrow 4f$  with corrections needed

## Recent work and work in progress:

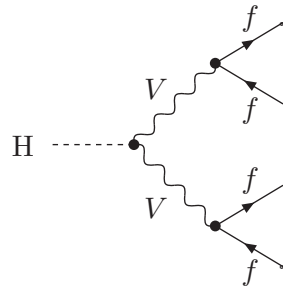
- PROPHECY4F: generator for  $H \rightarrow WW/ZZ \rightarrow 4f$  with electroweak corrections  
Bredenstein, Denner, S.D., Weber '06
- generator for  $H \rightarrow ZZ \rightarrow 4l$  with QED corrections  
Carloni-Calame et al.



## 2 Calculation of electroweak corrections

### Survey of Feynman diagrams

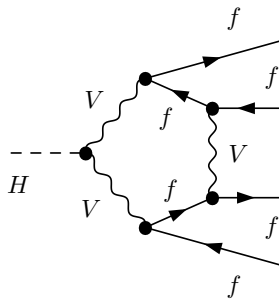
Lowest order:



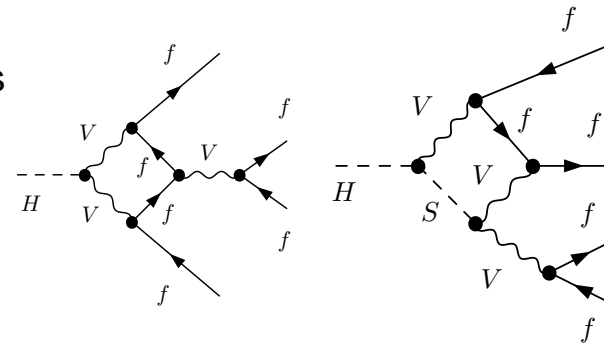
Electroweak  $\mathcal{O}(\alpha)$  corrections:

typical one-loop diagrams: # diagrams =  $\mathcal{O}(200-400)$

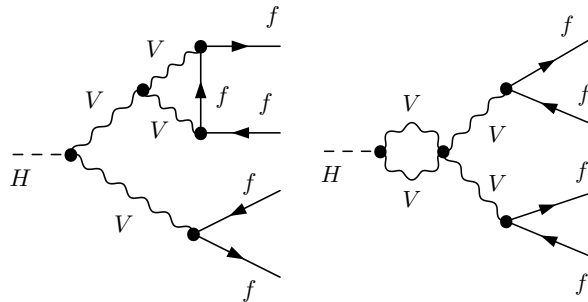
pentagons



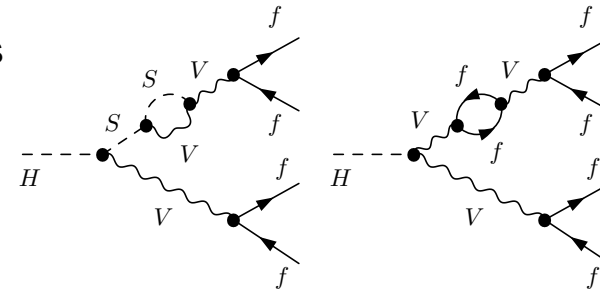
boxes



vertices



self-energies



+ photon bremsstrahlung (final-state radiation only)



## Comments on the $\mathcal{O}(\alpha)$ calculation

- Main complications in the loop calculation:

- ◇ gauge-invariant treatment of W and Z resonances

↪ “complex-mass scheme” Denner, S.D., Roth, Wieders '05

↪ talk of A. Denner

- ◇ numerical instabilities in Passarino–Veltman reduction of tensor integrals

↪ new reduction methods developed Denner, S.D. '05

New concepts already used in  $\mathcal{O}(\alpha)$  correction to  $e^+e^- \rightarrow 4f$

Denner, S.D., Roth, Wieders '05

- Features of PROPHECY4F:

- ◇  $\mathcal{O}(\alpha)$  calculation to all channels  $H \rightarrow WW/ZZ \rightarrow 4f$

- ◇ improved Born approximation for simplified evaluation

- ◇ final-state radiation beyond  $\mathcal{O}(\alpha)$  via structure functions

- ◇ multi-channel Monte Carlo integration (checked by VEGAS)

Berends, Kleiss, Pittau '94; Kleiss, Pittau '94

- ◇ in progress: unweighted events and QCD corrections for final-state quarks



## Numerical evaluation of one-loop integrals

### Passarino–Veltman reduction of tensor to scalar integrals

- ↪ inverse Gram determinants of external momenta
- ↪ **serious numerical instabilities where  $\det(G) \rightarrow 0$**   
(at phase-space boundary but not only !)

Our solutions: Denner, S.D., NPB734 (2006) 62 [hep-ph/0509141]

- **1- and 2-point integrals** → stable direct calculation
- **3- and 4-point integrals** → two hybrid methods
  - Passarino–Veltman  $\oplus$  seminumerical method  $\oplus$  analytical special cases
  - Passarino–Veltman  $\oplus$  expansions in small Gram and other kin. determinants
- **5- and 6-point integrals**
  - ↪ stable reduction to lower-point integrals without Gram determinants

⇒ **Techniques ready for further applications**

(dim. regularization for IR singularities possible; complex masses supported)

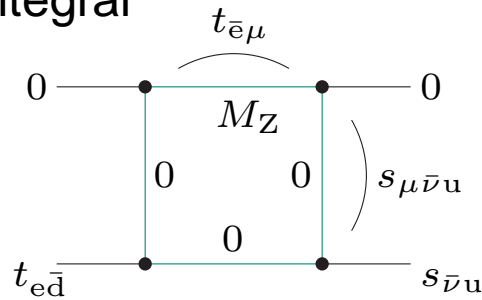
### Practical experience

↪ **Power + reliability of techniques can only be assessed via non-trivial applications !**

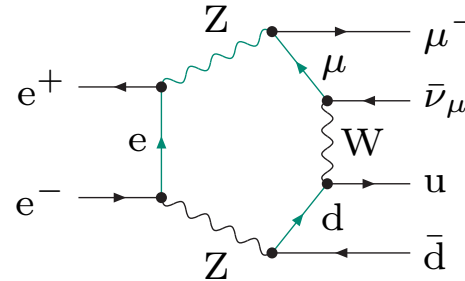


# A typical example with small Gram determinant:

Box integral

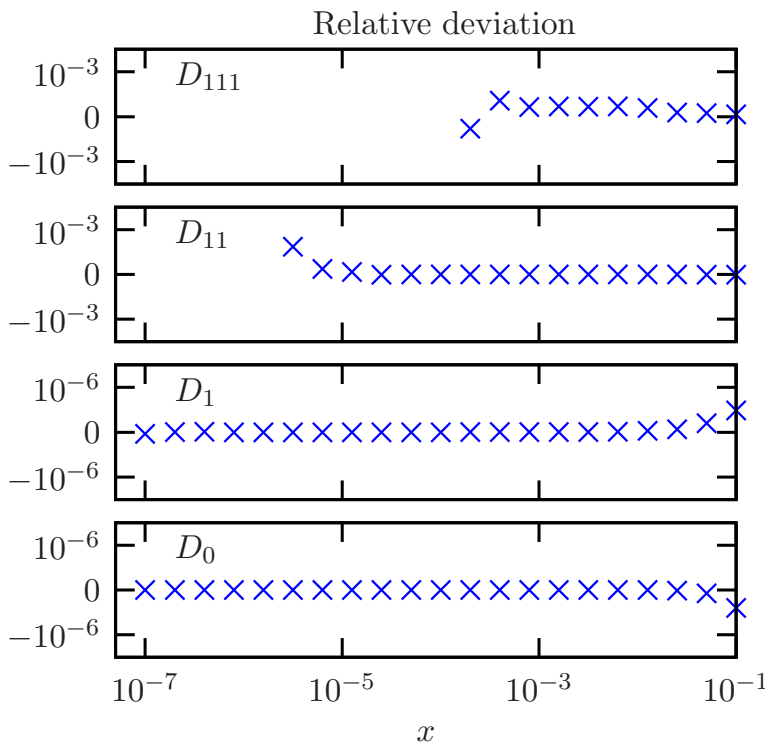
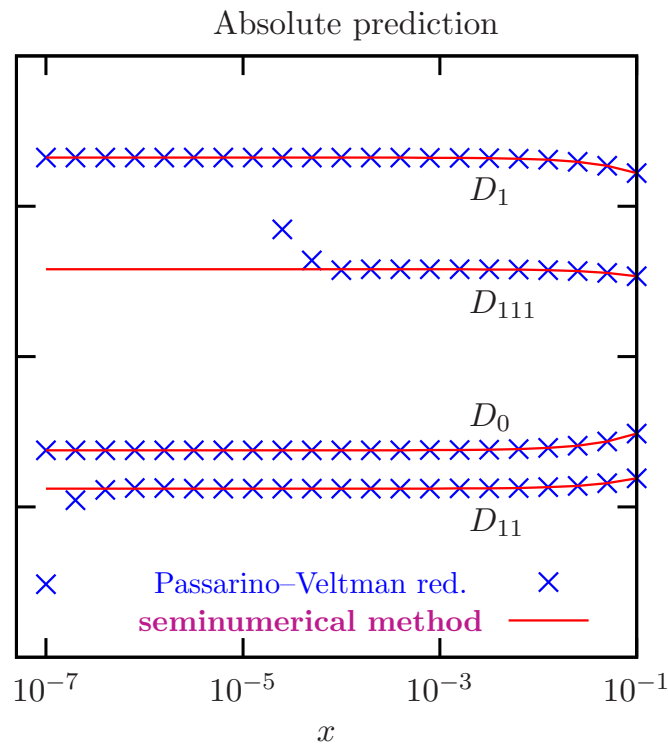


appears, e.g., in subgraph of diagram



Gram det.:  $\det(G) \rightarrow 0$  if  $t_{e\bar{d}} \rightarrow t_{\text{crit}} \equiv \frac{s_{\mu\bar{\nu}u}(s_{\mu\bar{\nu}u} - s_{\bar{\nu}u} + t_{\bar{e}\mu})}{s_{\mu\bar{\nu}u} - s_{\bar{\nu}u}}$

## Passarino–Veltman versus seminumerical method:



$$x \equiv \frac{t_{e\bar{d}}}{t_{\text{crit}}} - 1$$

$$\begin{aligned} s_{\mu\bar{\nu}u} &= +2 \times 10^4 \text{ GeV}^2 \\ s_{\bar{\nu}u} &= +1 \times 10^4 \text{ GeV}^2 \\ t_{\bar{e}\mu} &= -4 \times 10^4 \text{ GeV}^2 \\ t_{\text{crit}} &= -6 \times 10^4 \text{ GeV}^2 \end{aligned}$$

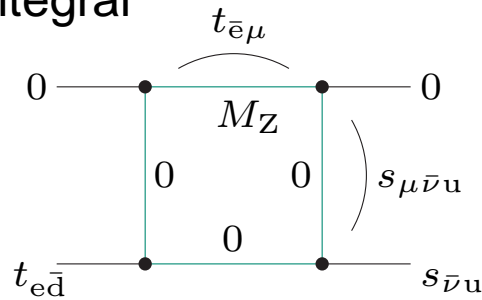
PV reduction breaks down,  
but seminum. method stable  
for  $\det(G) \rightarrow 0$  !



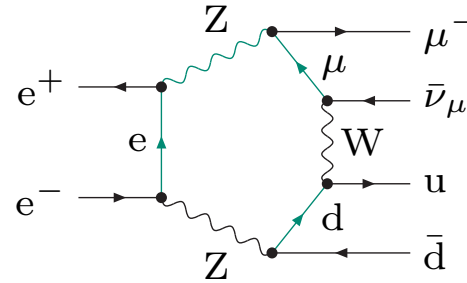


# A typical example with small Gram determinant:

Box integral

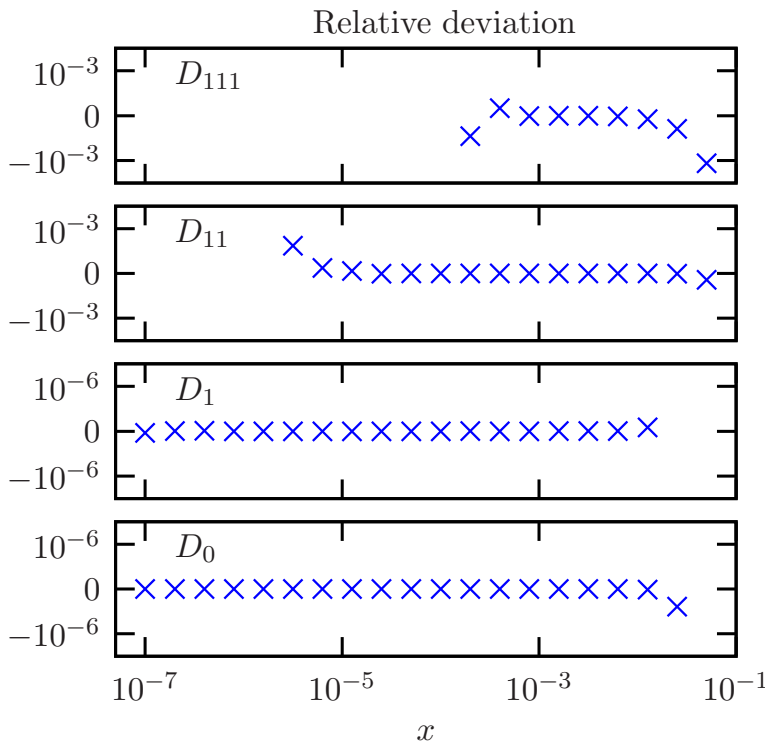
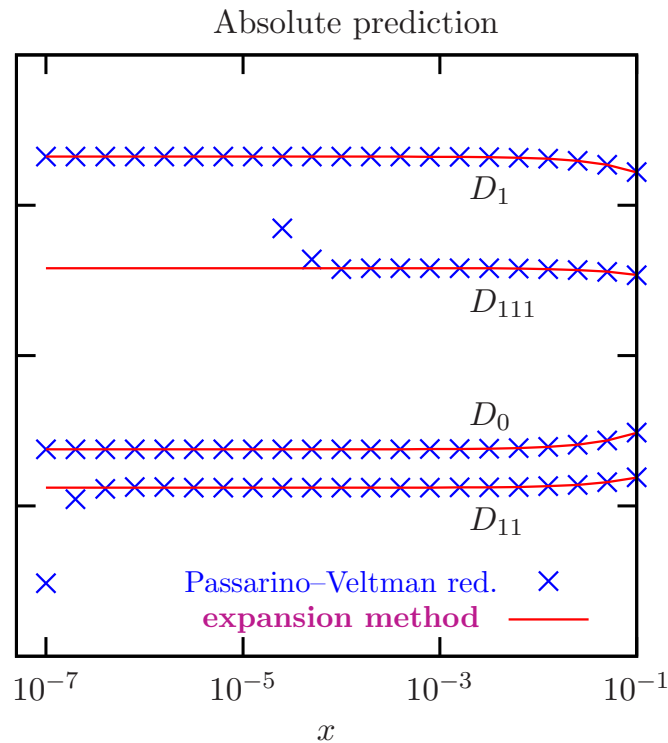


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Gram det.:  $\det(G) \rightarrow 0$  if  $t_{e\bar{d}} \rightarrow t_{\text{crit}} \equiv \frac{s_{\mu\bar{\nu}u}(s_{\mu\bar{\nu}u} - s_{\bar{\nu}u} + t_{\bar{e}\mu})}{s_{\mu\bar{\nu}u} - s_{\bar{\nu}u}}$

## Passarino–Veltman versus expansion method:



$$x \equiv \frac{t_{e\bar{d}}}{t_{\text{crit}}} - 1$$

$$s_{\mu\bar{\nu}u} = +2 \times 10^4 \text{ GeV}^2$$

$$s_{\bar{\nu}u} = +1 \times 10^4 \text{ GeV}^2$$

$$t_{\bar{e}\mu} = -4 \times 10^4 \text{ GeV}^2$$

$$t_{\text{crit}} = -6 \times 10^4 \text{ GeV}^2$$

PV reduction breaks down,  
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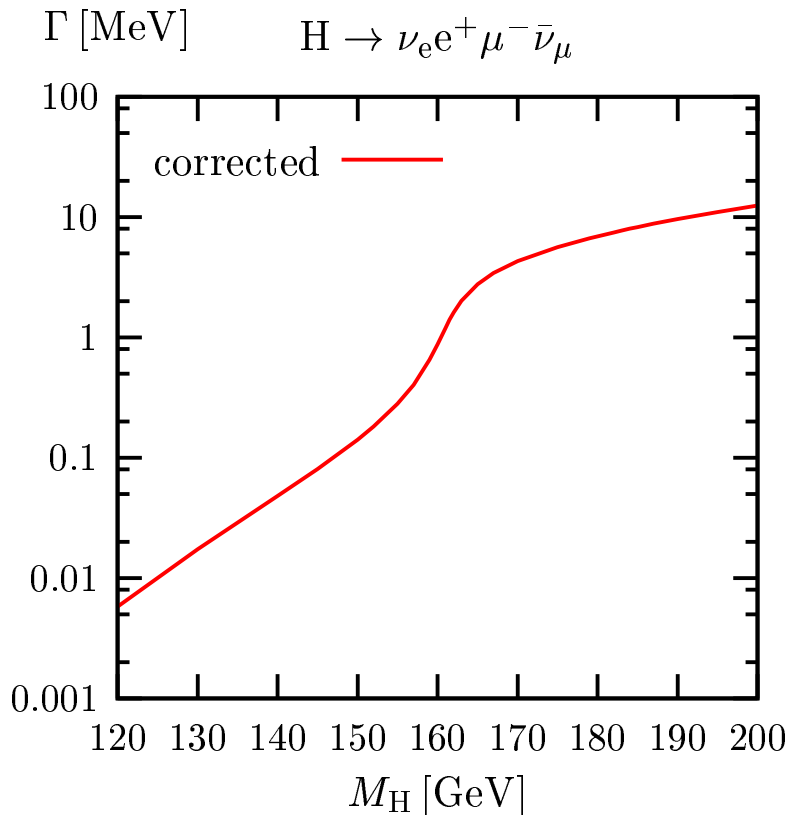
## Checks:

- **UV structure** of virtual corrections
  - ↪ independence of reference mass  $\mu$  of dimensional regularization
- **IR structure** of virtual + soft-photon corrections
  - ↪ independence of  $\ln m_\gamma$  ( $m_\gamma =$  infinitesimal photon mass)
- **mass singularities** of virtual + related collinear photonic corrections
  - ↪ independence of  $\ln m_{f_i}$  ( $m_{f_i} =$  small masses of external fermions)
- **gauge invariance** of amplitudes with  $\Gamma_W, \Gamma_Z \neq 0$ 
  - ↪ identical results in 't Hooft–Feynman and background-field gauge  
Denner, S.D., Weiglein '94
- **real corrections**
  - ↪ squared amplitudes compared with MADGRAPH  
Stelzer, Long '94
- **combination of virtual and real corrections**
  - ↪ identical results with two-cutoff slicing and dipole subtraction  
Catani, Seymour '96; S.D. '00
- **two completely independent calculations of all ingredients !**

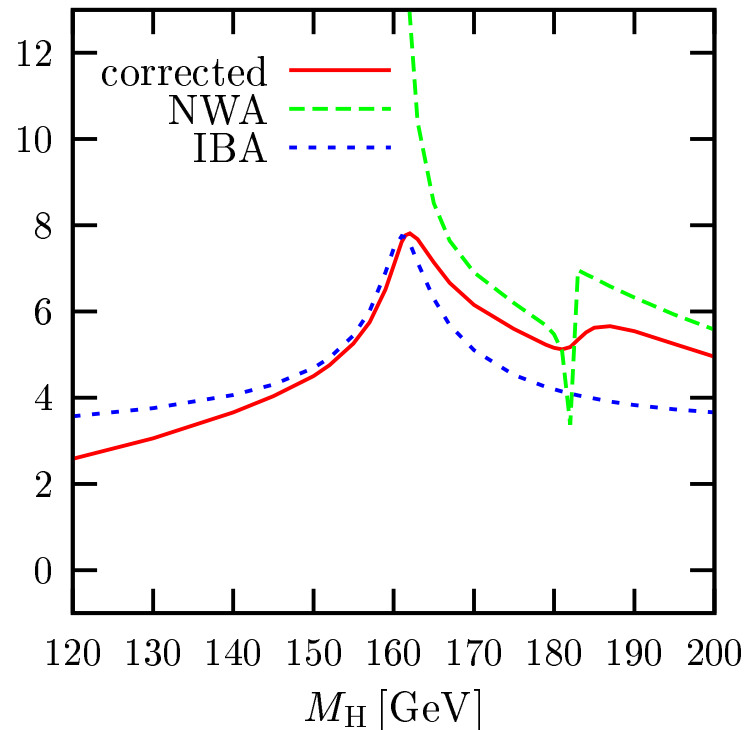


### 3 Numerical results

Partial decay width for  $H \rightarrow WW \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$   $G_\mu$ -scheme



$\delta$  [%]  $H \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$   $\delta = \frac{\Gamma}{\Gamma_{\text{Born}}} - 1$



Bredenstein, Denner,  
S.D., Weber '06

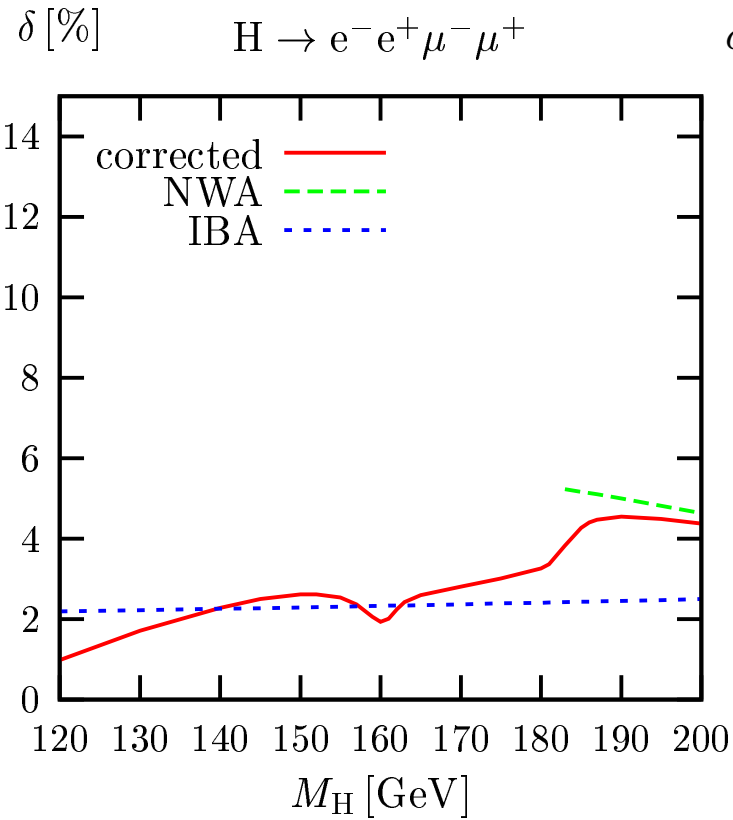
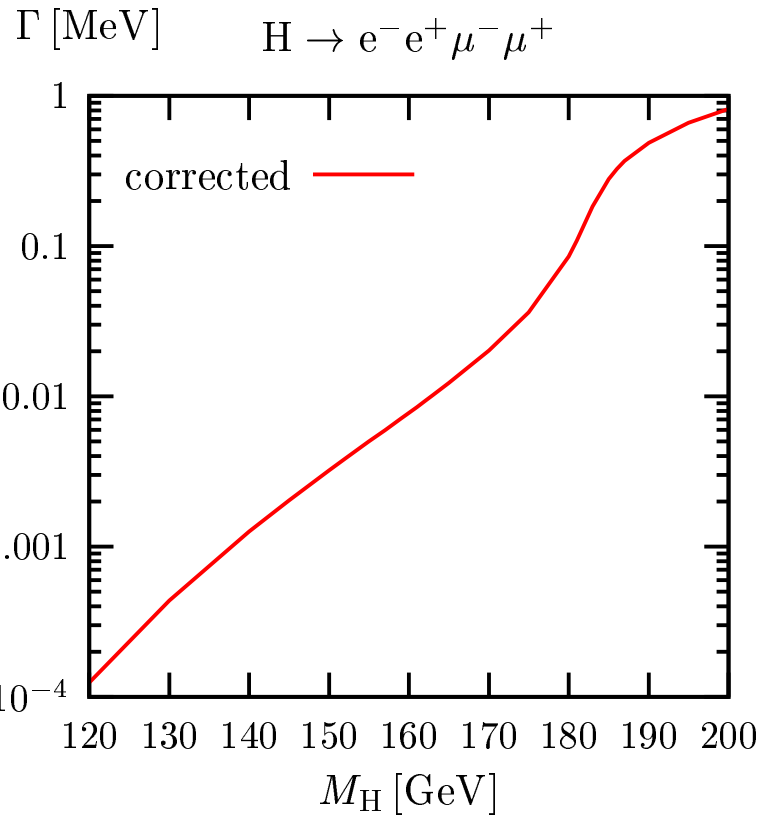
↑ Coulomb singularity for  $M_H \sim 2M_W$       ↑ threshold effect in loops for  $M_H \sim 2M_Z$

NWA = narrow-width approximation

IBA = improved Born approximation (Coulomb singularity, fitting constant, leading effects for  $M_H, m_t \gg M_W$ )



# Partial decay width for $H \rightarrow ZZ \rightarrow e^-e^+\mu^-\mu^+$ $G_\mu$ -scheme



$$\delta = \frac{\Gamma}{\Gamma_{\text{Born}}} - 1$$

Bredenstein, Denner,  
S.D., Weber '06

↑  
threshold effect in loops  
for  $M_H \sim 2M_W$

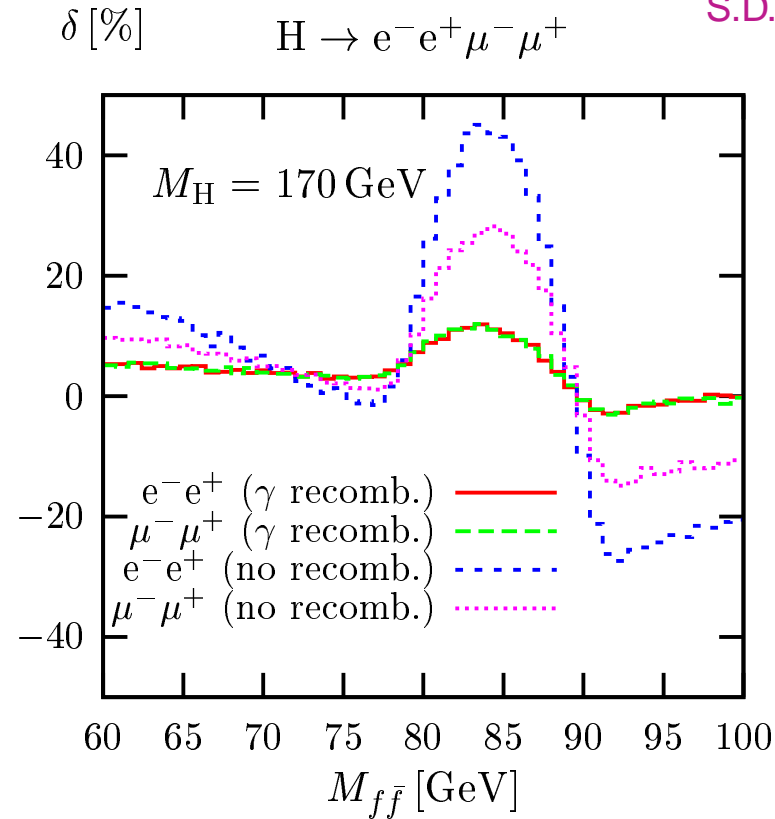
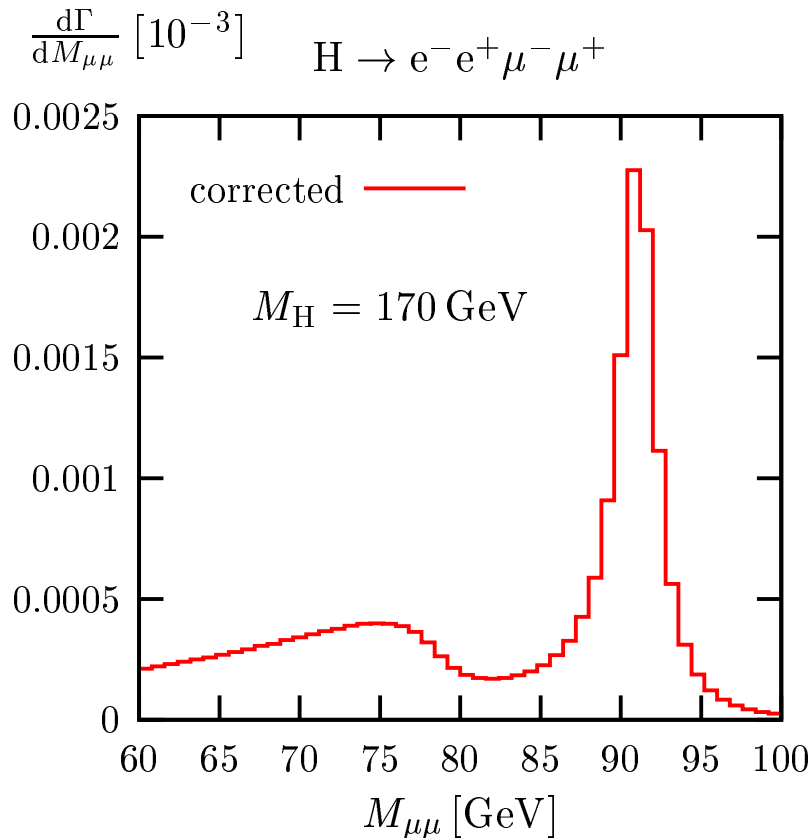
↑  
kinematic threshold  
at  $M_H \sim 2M_Z$



# Invariant-mass distribution for the Z boson in $H \rightarrow ZZ \rightarrow e^-e^+\mu^-\mu^+$

$G_\mu$ -scheme

Bredenstein, Denner,  
S.D., Weber '06



$\gamma$  recombination if  $M_{e\gamma/\mu\gamma} < 5 \text{ GeV}$

Large corrections from photon radiation in Z reconstruction

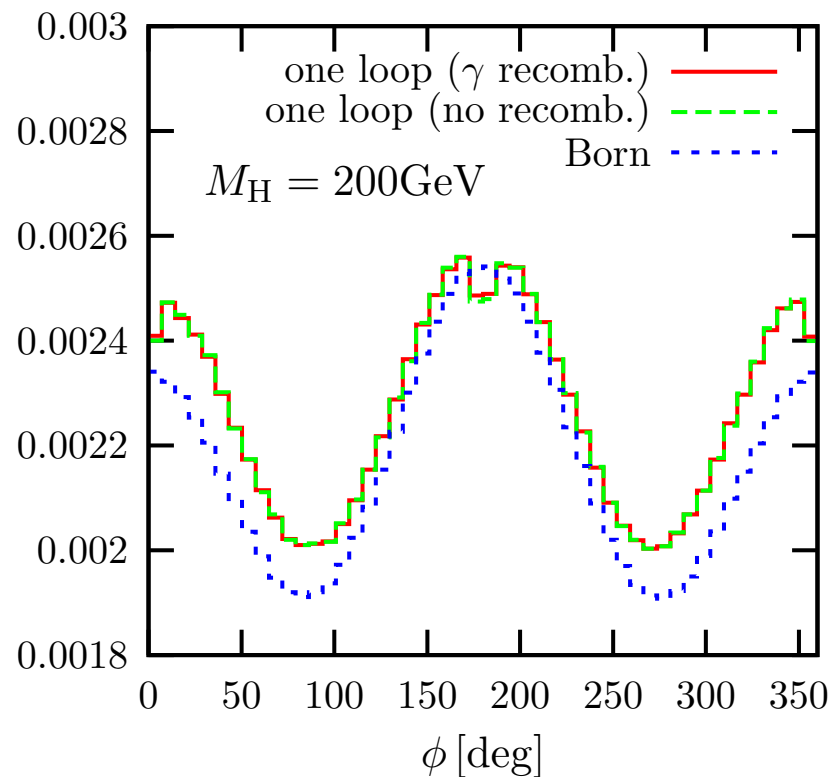


# Angle between decay planes for $H \rightarrow ZZ \rightarrow e^-e^+\mu^-\mu^+$

$G_\mu$ -scheme

$\frac{d\Gamma}{d\phi} \left[ \frac{\text{MeV}}{\text{deg}} \right]$

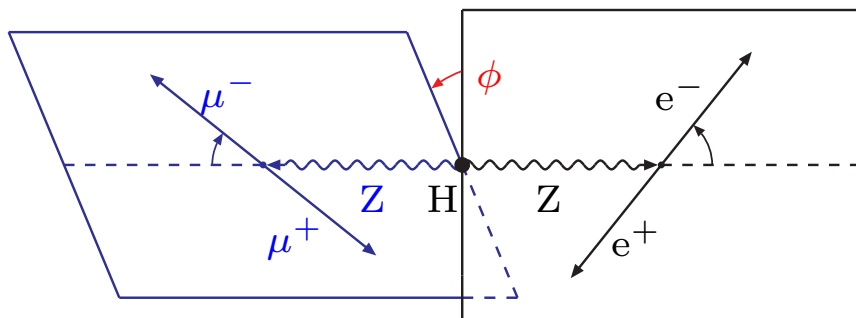
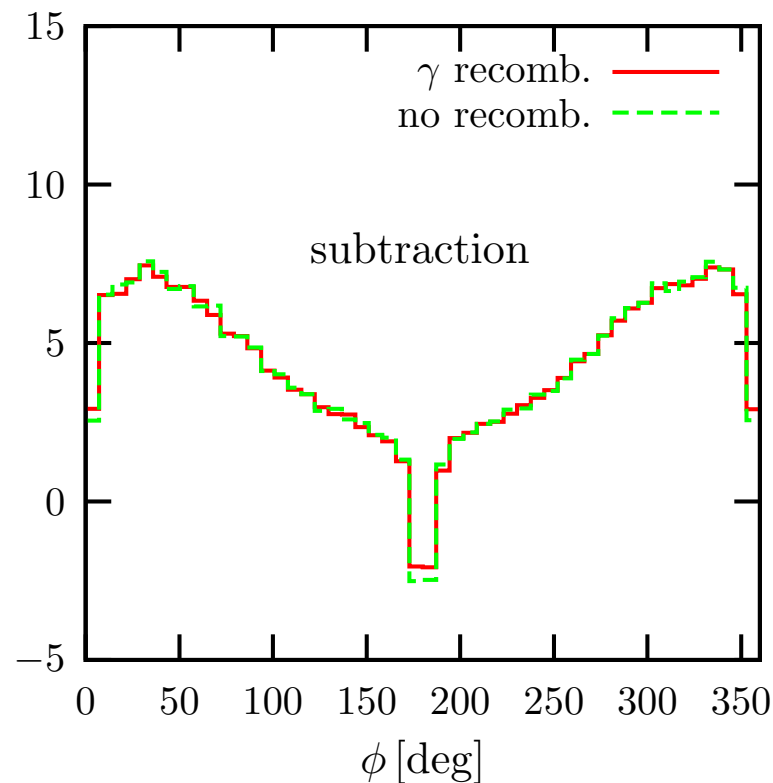
$H \rightarrow e^-e^+\mu^-\mu^+$



$\delta [\%]$

$H \rightarrow e^-e^+\mu^-\mu^+$

Bredenstein, Denner, S.D., Weber '06



$$\cos \phi = \frac{((\mathbf{p}_{e^-} + \mathbf{p}_{e^+}) \times \mathbf{p}_{e^-}) \cdot (-(\mathbf{p}_{\mu^-} + \mathbf{p}_{\mu^+}) \times \mathbf{p}_{\mu^-})}{|(\mathbf{p}_{e^-} + \mathbf{p}_{e^+}) \times \mathbf{p}_{e^-}| |-(\mathbf{p}_{\mu^-} + \mathbf{p}_{\mu^+}) \times \mathbf{p}_{\mu^-}|}$$

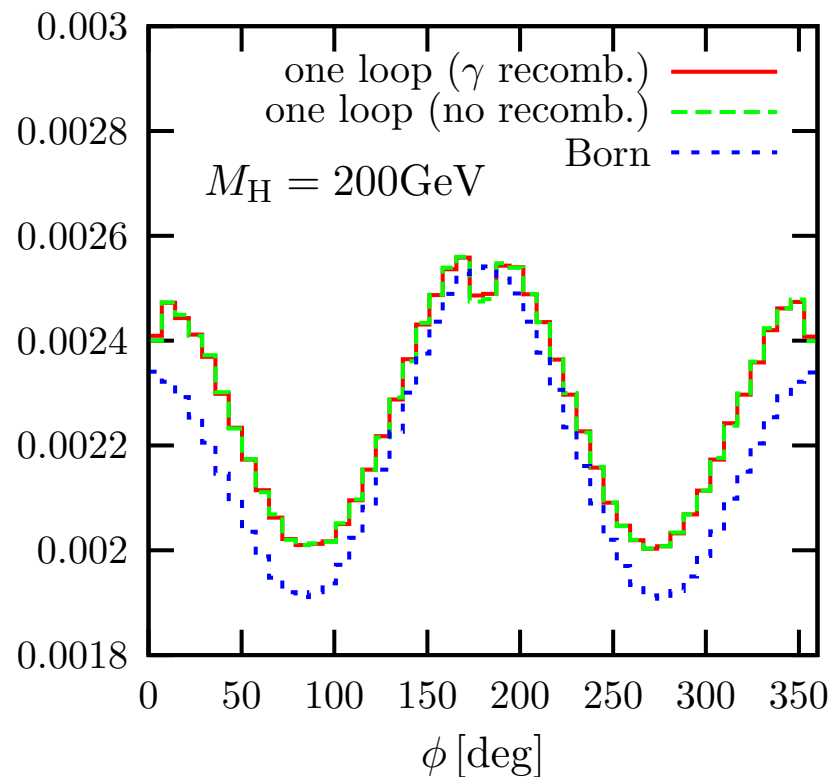


# Angle between decay planes for $H \rightarrow ZZ \rightarrow e^-e^+\mu^-\mu^+$

$G_\mu$ -scheme

$\frac{d\Gamma}{d\phi} \left[ \frac{\text{MeV}}{\text{deg}} \right]$

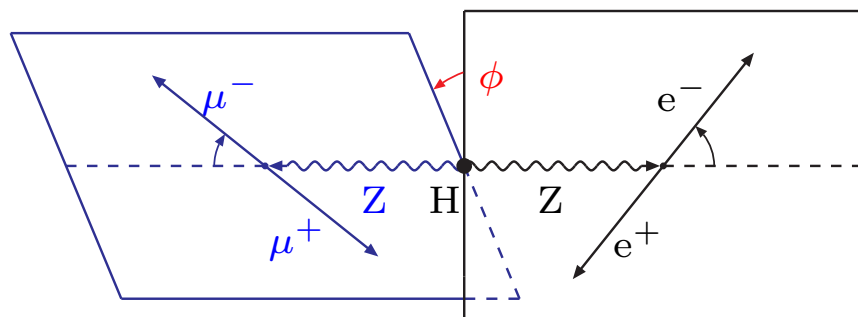
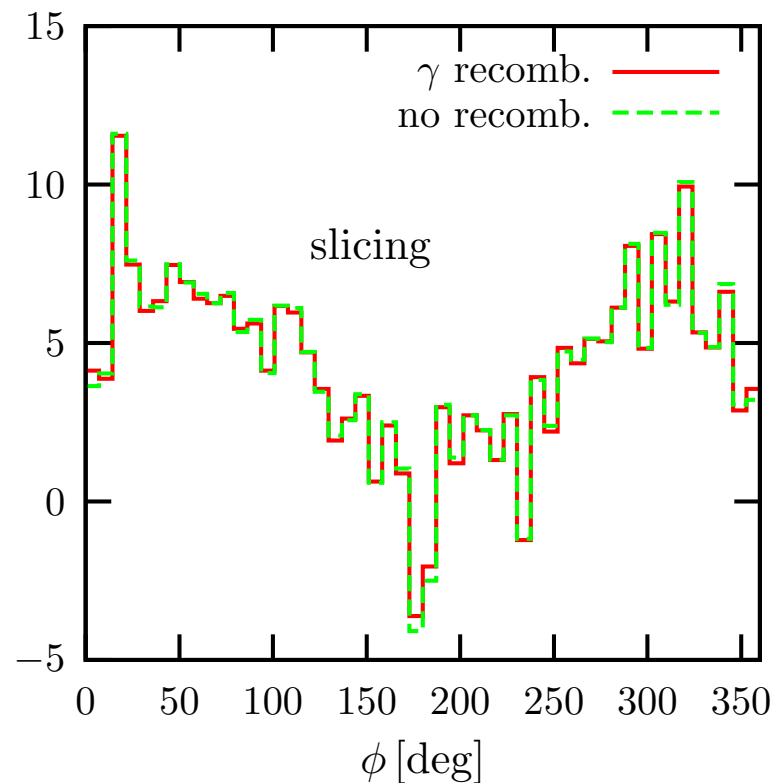
$H \rightarrow e^-e^+\mu^-\mu^+$



$\delta [\%]$

$H \rightarrow e^-e^+\mu^-\mu^+$

Bredenstein, Denner, S.D., Weber '06



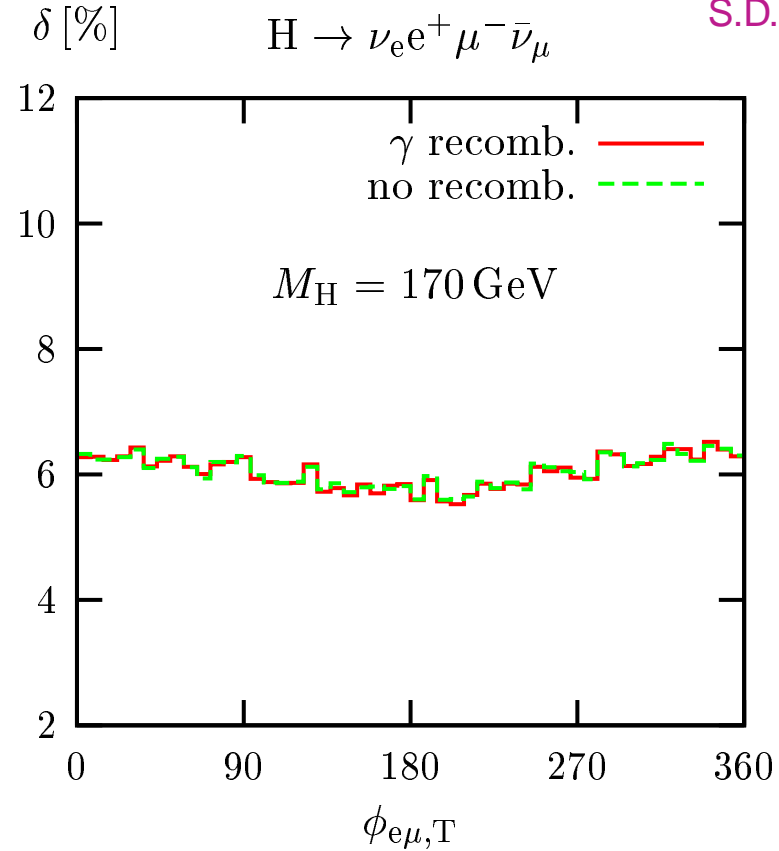
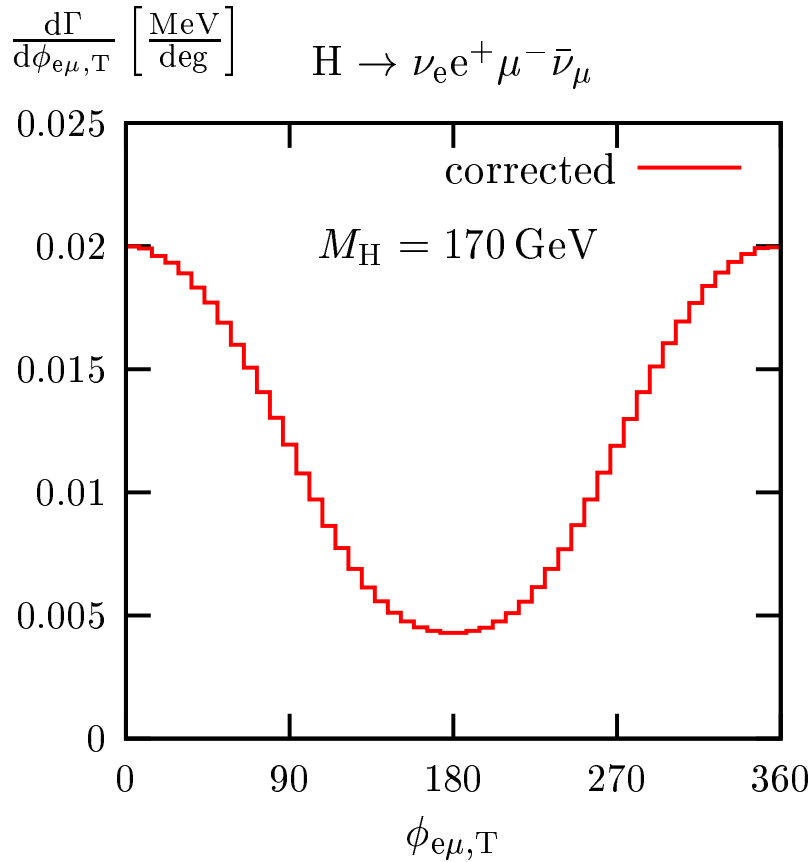
$$\cos \phi = \frac{\left( (\mathbf{p}_{e^-} + \mathbf{p}_{e^+}) \times \mathbf{p}_{e^-} \right) \left( -(\mathbf{p}_{\mu^-} + \mathbf{p}_{\mu^+}) \times \mathbf{p}_{\mu^-} \right)}{\left| (\mathbf{p}_{e^-} + \mathbf{p}_{e^+}) \times \mathbf{p}_{e^-} \right| \left| -(\mathbf{p}_{\mu^-} + \mathbf{p}_{\mu^+}) \times \mathbf{p}_{\mu^-} \right|}$$



# Distribution in the transverse angle between $e^+$ and $\mu^-$ in $H \rightarrow WW \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$

$G_\mu$ -scheme

Bredenstein, Denner,  
S.D., Weber '06



No significant distortion of shape by electroweak corrections





## 4 Conclusions

Higgs decays  $H \rightarrow WW/ZZ \rightarrow 4f$  are important for

- Higgs discovery at the LHC and Higgs mass measurement
- confirmation of Higgs quantum numbers (spin, CP) via differential distributions

**NEW:** PROPHECY4F – a generator for  $H \rightarrow WW/ZZ \rightarrow 4f$  including

- **full  $\mathcal{O}(\alpha)$  electroweak corrections**
  - ◊ W and Z resonances treated within the complex-mass scheme
  - ◊ tensor reduction numerically stabilized via seminumerical or expansion methods
- **universal corrections beyond  $\mathcal{O}(\alpha)$**  (FSR via structure functions, large- $M_H$  effects)
- QCD corrections to hadronic final states (in progress)

First results of PROPHECY4F on  $H \rightarrow WW/ZZ \rightarrow 4l$

- **partial decay widths:** corrections of  $\mathcal{O}(8\%)$  for  $M_H \lesssim 500$  GeV  
(reproduced by a simple improved Born approximation within  $\lesssim 2\%$  for  $M_H \lesssim 400$  GeV)
- **angular distributions:** corrections of  $\mathcal{O}(5-10\%)$  distort shapes
- **invariant-mass distributions** of W's and Z's:  
corrections of several 10% distort shapes (depend on inclusiveness of  $\gamma$  radiation)

