Top Threshold Physics

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thanks to T. Teubner, A. Manohar, I. Stewart, P. Ruiz-Femenia

C. Reisser, C. Farrell, M. Stahlhofen

Outline

- Threshold Physics at the ILC
- Top Pair Threshold
  - measurements, experimental issues
  - theory issues
- Effective Theory (A): QCD $\longrightarrow$ stable top quarks
- Effective Theory (B): e.w. effects $\longrightarrow$ e.g. finite lifetime effects
- Other Application:
  - $e^+e^- \longrightarrow t\bar{t}H$
  - $e^+e^- \longrightarrow \tilde{t}\bar{\tilde{t}}$

Top Physics and the ILC

- e^+e^- collider: $E_{\rm cm} = 350$ GeV – 1 TeV
- Luminosity: $10^{34}\,{\rm cm}^{-2}{\rm s}^{-1} \to 100\,{\rm fb}^{-1}$/year

  LC $\sim 10^5$ $t\bar t$ pairs [$\sigma_{\rm tot} \sim 1\,{\rm pb}$] ($e^+e^- \to t\bar t$)

  LHC $\sim 10^8$ $t\bar t$ pairs [$\sigma_{\rm tot} \approx 850\,{\rm pb}$] ($gg \to t\bar t$)

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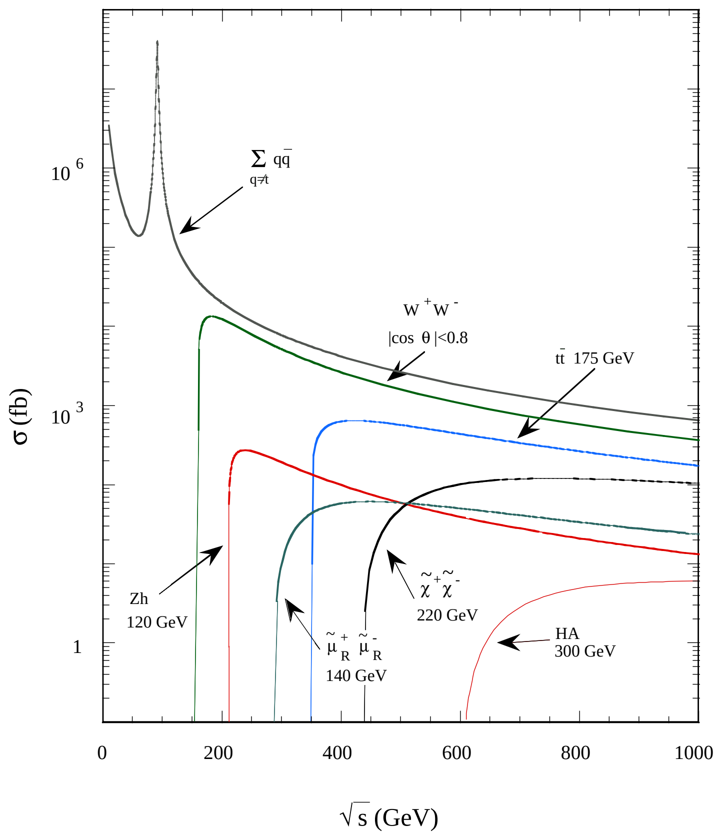
- Initial state tunable and very well known
  - Centre of mass energy variable

    $\rightarrow$ threshold & continuum



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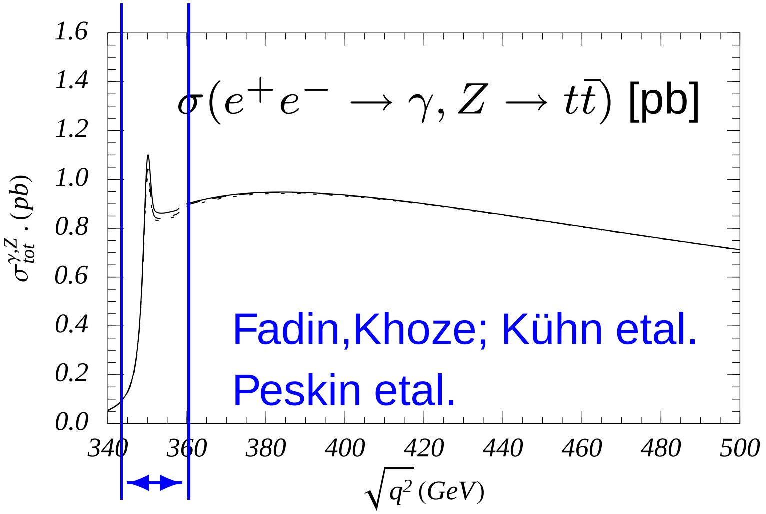
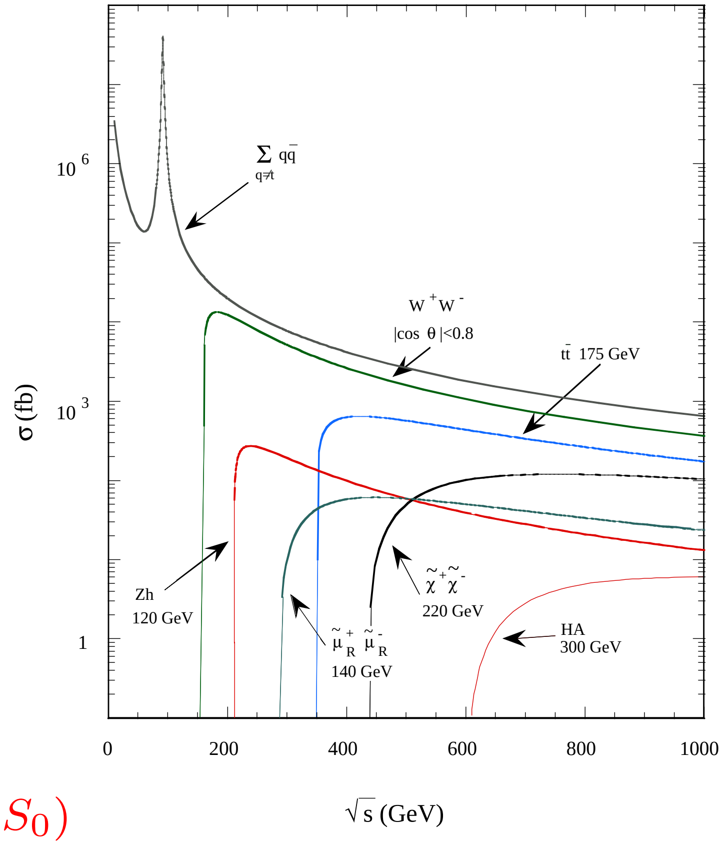


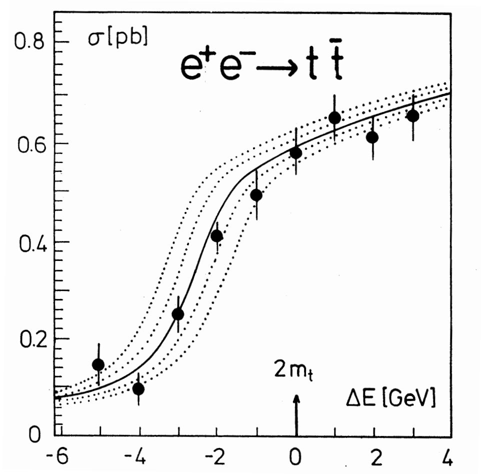


  - $e^\pm$ polarization: $P_- \sim 80\%,\ P_+ \sim 60\%$
  - $\gamma\gamma$ option: $e^+e^- \rightarrow t\bar{t}\,(^3S_1),\quad \gamma\gamma \rightarrow t\bar{t}\,(^1S_0)$

Threshold Measurements

Threshold Scan: $\sqrt{s} \simeq 350$ GeV (Phase I)

- ▷ count number of $t\bar{t}$ events
- ▷ color singlet state
- ▷ background is non-resonant
- ▷ QCD effects well understood (renormalons, NNLL QCD)



$\rightarrow \delta m_t^{\rm exp,stat} \simeq 20$ MeV

$\mathcal{L} = 300\,{\rm fb}^{-1}$
$9+1$ scan points
[Peralta, Martinez, Miquel]

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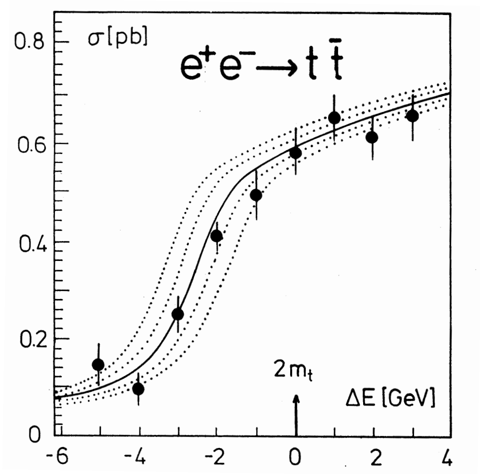

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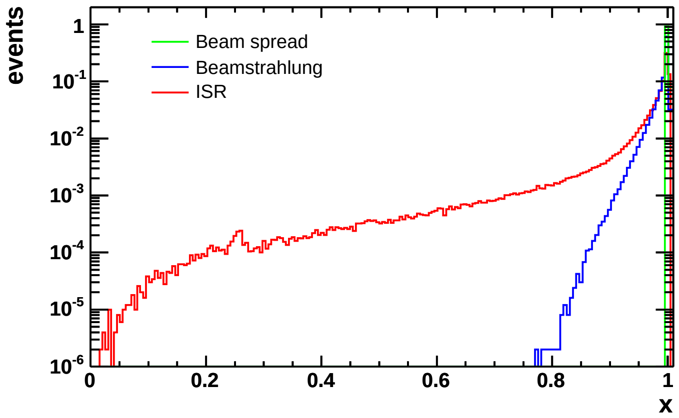
$\rightarrow \delta m_t^{\text{exp},\mathcal{L}\text{umi}} \lesssim 50$ MeV

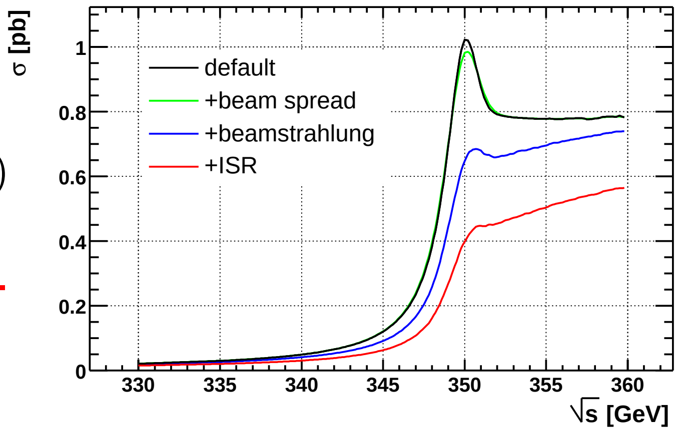
Simulations

Influence of Luminosity spectrum

Boogert 2004



$$\sigma(s) = \int_0^1 dx L(x) \sigma^0(x^2 s)$$



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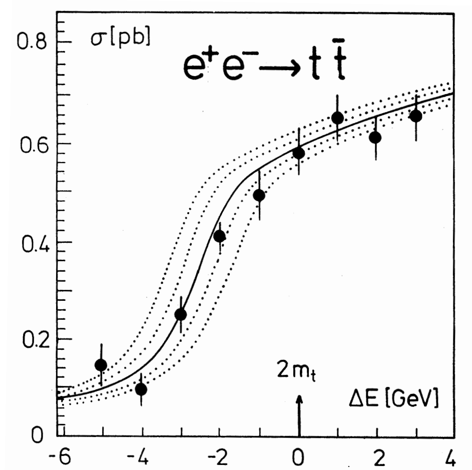

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$\rightarrow \delta m_t^{\rm theory} \simeq 100$ MeV

What mass?

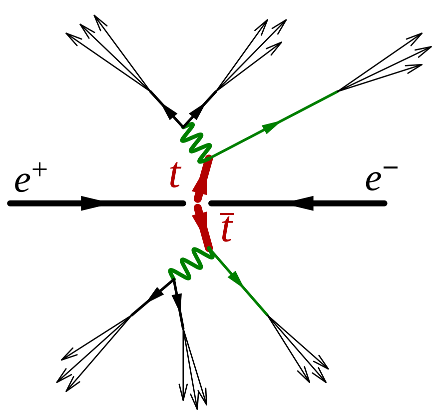
$\sqrt{s}_{\rm rise} \sim 2m_t^{\rm thr}$ + conv.pert.series

(short distance mass: 1S $\leftrightarrow$ $\overline{\rm MS}$)

Reconstruction: any $\sqrt{s}$ (Phase I + II)

Chekanov,Morgunov:

- ▷ $e^+e^- \rightarrow 6$ jets ($y_{\rm cut}^6$)
- ▷ b-tagging
- ▷ $\vec{P}_1 + \vec{P}_2 < \Delta_p$
- ▷ $M_1 - M_2 < \Delta_M$



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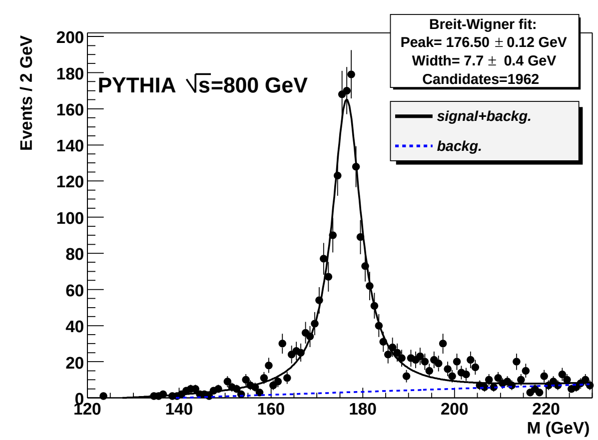

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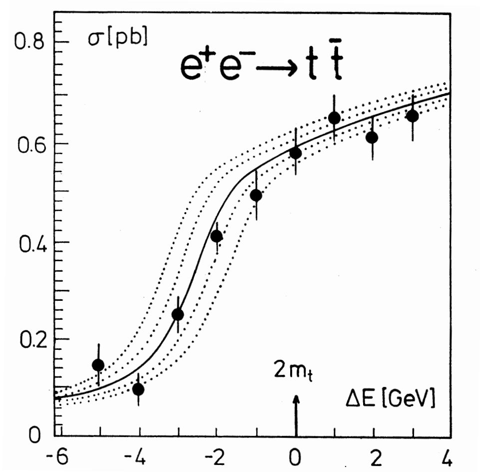

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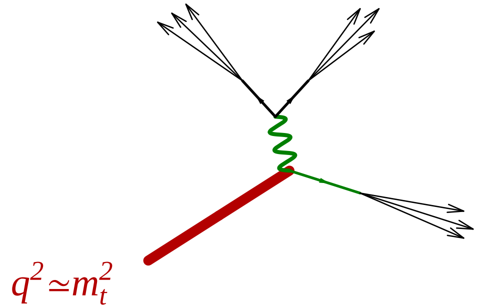

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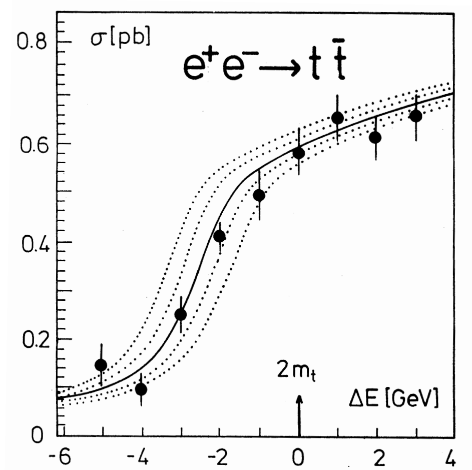

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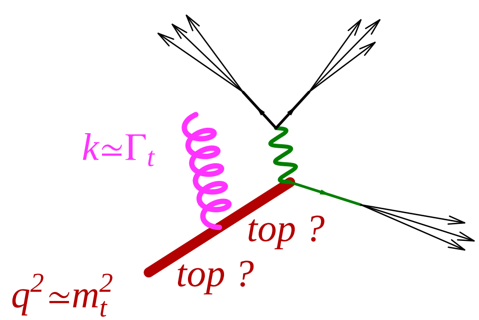

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($\mathcal{L} = 300\,{\rm fb}^{-1}$)

What mass?

Pole Mass ?

ambiguity: $\Delta m_t \sim \Lambda_{\rm QCD}$

$\Delta m_t \sim \pi\alpha_s(\Gamma_t)\,\Gamma_t$

There is s.th. to understand here !

Threshold Measurements

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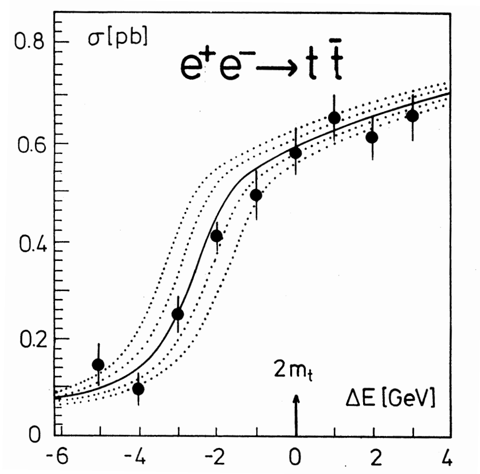

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(short distance mass: 1S $\leftrightarrow$ $\overline{\mathrm{MS}}$)

Simulations δ(line-shape form) $\leftrightarrow$ δ(Lumi spectrum)

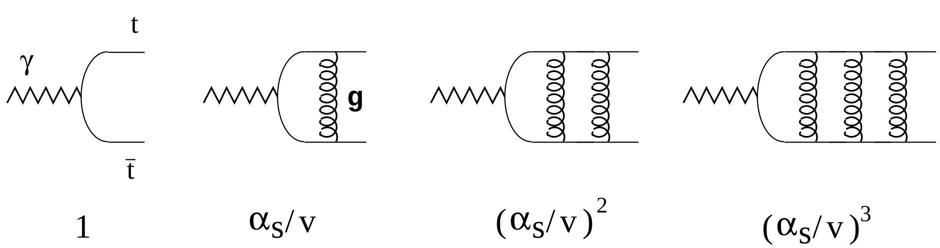
$(\delta\lambda_t/\lambda_t)^{\mathrm{stat}} = 15 - 50\%$	$(\delta\lambda_t/\lambda_t)^{\mathrm{syst}} = ?$	$(\delta\lambda_t/\lambda_t)^{\mathrm{theo}} \sim ?$
$(\delta\alpha_s(M_Z))^{\mathrm{stat}} = 0.001$	$(\delta\alpha_s(M_Z))^{\mathrm{syst}} = 0.002$	$(\delta\alpha_s(M_Z))^{\mathrm{theo}} \sim ?$
$(\delta\Gamma_t)^{\mathrm{stat}} = 50$ MeV	$(\delta\Gamma_t)^{\mathrm{syst}} = 15$ MeV	$(\delta\Gamma_t)^{\mathrm{theo}} \sim ?$

$\Rightarrow$ goal: $(\delta\sigma/\sigma)^{\mathrm{theo}} \leq 3\,\%$ $\Rightarrow$ NNLL

Theory Issues

m_t (hard) $\gg$ $\mathbf{p} \sim m_t v$ (soft) $\gg$ $E \sim m_t v^2$ (ultrasoft)

- perturbation theory in α_s breaks down $(\alpha_s/v)^n$



"Coulombsingularities"

$\longrightarrow$ Schrödinger Equation

$v \sim \alpha_s$

- perturbation theory in α_s breaks down $\longrightarrow$ large logs $(\alpha_s \ln v)^n$

$m_t = 175$ GeV, $\mathbf{p} \sim 25$ GeV, $E \sim 4$ GeV $\Rightarrow \ln\left(\frac{m_t^2}{E^2}\right) = 8$ $\longrightarrow$ RGE's

"multi-scale problem"

Theory Issues

- $\boxed{\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\rm QCD}}$ $\Rightarrow \quad v = \sqrt{\frac{E}{m}} \;\longrightarrow\; v_{\rm eff} = \sqrt{\frac{E+i\Gamma_t}{m}}$

  (Fadin,Khoze)

  $\Rightarrow \; m_t \gg p \gg E \gtrsim \Lambda_{\rm QCD}$ always true !

  $\Rightarrow$ top threshold entirely perturbative ! $\longrightarrow$ “Schrödinger theory”

- $\boxed{E \sim \Gamma_t:}$ top quarks are always produced off-shell !

  - non-factorizable corrections
  - final-state interferences (signal $\leftrightarrow$ non-signal/background)
  - gauge invariance

“theory for unstable particles”

Degrees of Freedom

- fields for degrees of freedom that can resonate for the quark-antiquark system

potential quarks	ψ_p, χ_p:	$(k_0, \mathbf{k}) \sim (mv^2, mv)$	$m_t \gg \mathbf{p} \gg E$
soft gluons	$A_q{}^\mu$:	$(k_0, \mathbf{k}) \sim (mv, mv)$	&
ultrasoft gluons	A^μ:	$(k_0, \mathbf{k}) \sim (mv^2, mv^2)$	$E = \mathbf{p}^2/m$

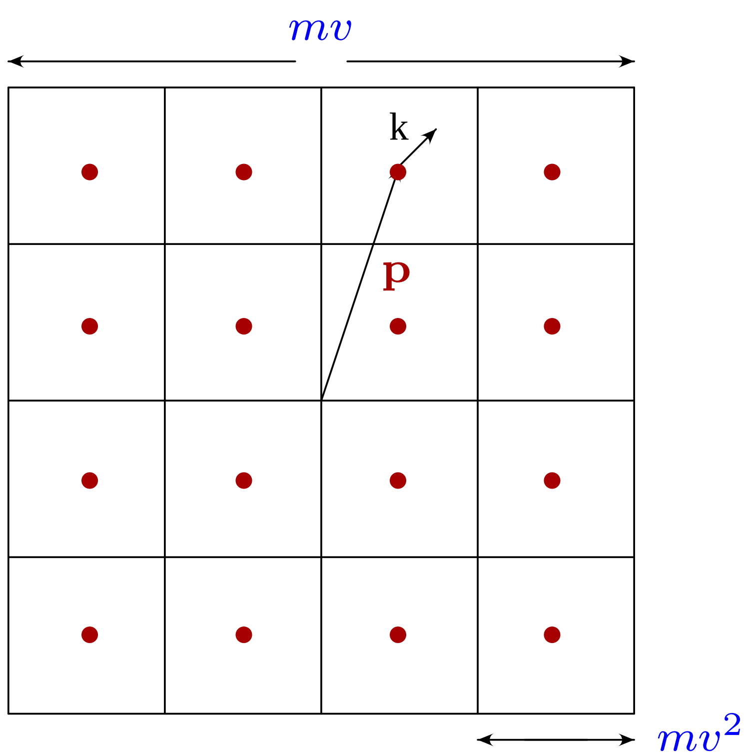
Labelle; Beneke, Smirnov

- vNRQCD label formalism:

$$(P^0, \mathbf{P}) = (0, \mathbf{p}) + (k^0, \mathbf{k})$$

soft component label — ultrasoft component dynamic variable

$$\psi_{\rm QCD}(x) \to \sum_{\mathbf{p}} e^{i\mathbf{p}.\mathbf{x}}\, \psi_{\mathbf{p}}(x)$$



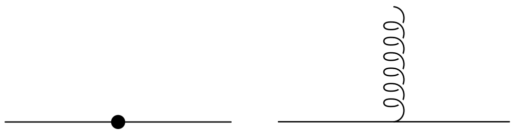
vNRQCD (stable quarks)

$\underline{\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm potential} + \mathcal{L}_{\rm soft}}$

Luke, Manohar, Rothstein; Stewart, AH

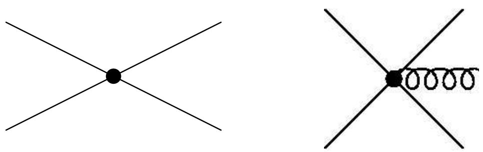
$$D^\mu = \partial^\mu + i\mu_U^\epsilon g_s(m\nu^2)A^\mu$$

$\mathcal{L}_{\rm usoft}$:



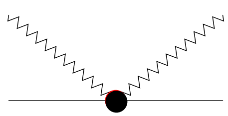
$$\psi_{\mathbf{p}}^\dagger(x)\left\{iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} - \delta m_t(\nu)\right\}\psi_{\mathbf{p}}(x)$$

$\mathcal{L}_{\rm potential}$:



$$\mu_S^{2\epsilon}\, V(\nu)\, \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}}$$

$\mathcal{L}_{\rm soft}$:



$$\mu_S^{2\epsilon}\, U_{\mu\nu}(\nu)\ \psi_{\mathbf{p}'}^\dagger A_q^\mu A_{q'}^\nu \psi_{\mathbf{p}}$$

$$\mu_U = \mu_S^2/m_t = m_t\nu^2$$

$$V = \left[\frac{\mathcal{V}_c(\nu)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(\nu)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(\nu)(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} + \frac{\mathcal{V}_s(\nu)}{m^2}\mathbf{S}^2 + \frac{\mathcal{V}_\Lambda(\nu)}{m^2}\Lambda + \frac{\mathcal{V}_t(\nu)}{m^2}\,T\right]$$

$$\mathbf{k} \equiv \mathbf{p} - \mathbf{p}'$$

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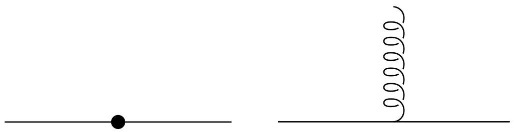

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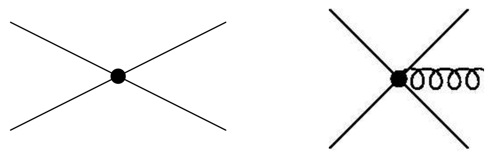

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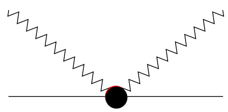
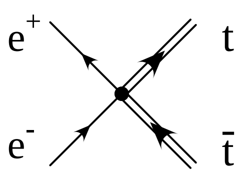


$$\mu_S^{2\epsilon}\, U_{\mu\nu}(\nu)\;\psi_{\mathbf{p}'}^\dagger A_q^\mu A_{q'}^\nu \psi_{\mathbf{p}}$$

<u>external currents:</u> (production & annihilation)



$$\mathbf{O}_{\mathbf{p}} = C_{V,A}(\nu)\cdot[\bar{e}\gamma^i(\gamma_5)e][\psi_{\mathbf{p}}^\dagger\,\boldsymbol{\sigma}^i\,\tilde{\chi}_{-\mathbf{p}}^*] + \cdots \qquad t\bar{t}\;(^3S_1)$$

Cross Section at NNLL Order

Schematic:

$$\sigma_{\rm tot} \quad \propto \quad {\rm Im}\left[\int {\rm d}^4{\rm x}\,{\rm e}^{-{\rm i}\hat{\rm q}{\rm x}}\left\langle\,0\,\right|{\rm T}\,\mathbf{O}_{\mathbf{p}}^{\dagger}(0)\,\mathbf{O}_{\mathbf{p}'}({\rm x})\left|\,0\,\right\rangle\right]$$

$$\propto \quad {\rm Im}\left[({\rm C}_{\rm A}(\nu)^2 + {\rm C}_{\rm V}(\nu)^2)\,{\rm G}(0,0,\sqrt{\rm s})\right]$$

$$\left(-\frac{\nabla^2}{m}-\frac{\nabla^4}{4m^3}+V(\mathbf{r})-(\sqrt{s}-2m-2\delta m)-i\Gamma_t\right)G(\mathbf{r},\mathbf{r}')=\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

Cross Section at NNLL Order

Schematic:

$$\sigma_{\rm tot} \;\propto\; {\rm Im}\left[\int d^4x\, e^{-i\hat{q}x} \left\langle 0 \left| T\, \mathbf{O}^\dagger_{\mathbf{p}}(0)\, \mathbf{O}_{\mathbf{p}'}(x) \right| 0 \right\rangle\right]$$

$$\propto\; {\rm Im}\left[(C_A(\nu)^2 + C_V(\nu)^2)\, G(0,0,\sqrt{s})\right]$$

$$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s} - 2m - 2\delta m) - i\Gamma_t\right) G(\mathbf{r},\mathbf{r}') = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

$V(\mathbf{r})$: fully known at NNLL order ✓

Manohar,Stewart; AH ’99-’03
Pineda,Soto ’00-’01
Peter ’94

C_A, C_V:

NLL ✓

NNLL (matching) ✓ Benke etal; Czarnecki etal ’99

NNLL (non-mixing) ✓ AH ’03

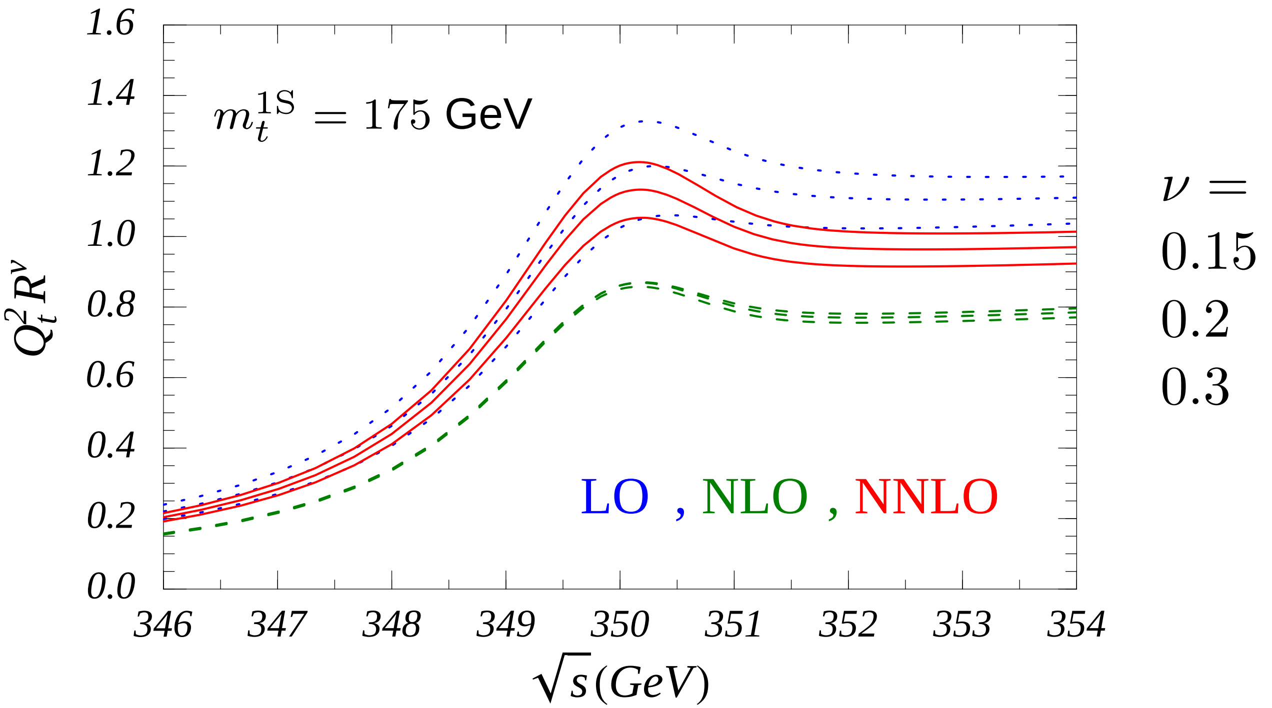
NNLL (mixing) ess. unknown

- spin-dependent (soft) Penin etal. ’04
- usoft n_f Stahlhofen, AH ’05

Cross Section at NNLL Order

1S mass - fixed order approach

Teubner,AH; Melnikov, Yelkovski;Yakovlev; Beneke,Signer,Smirnov; Sumino, Kiyo

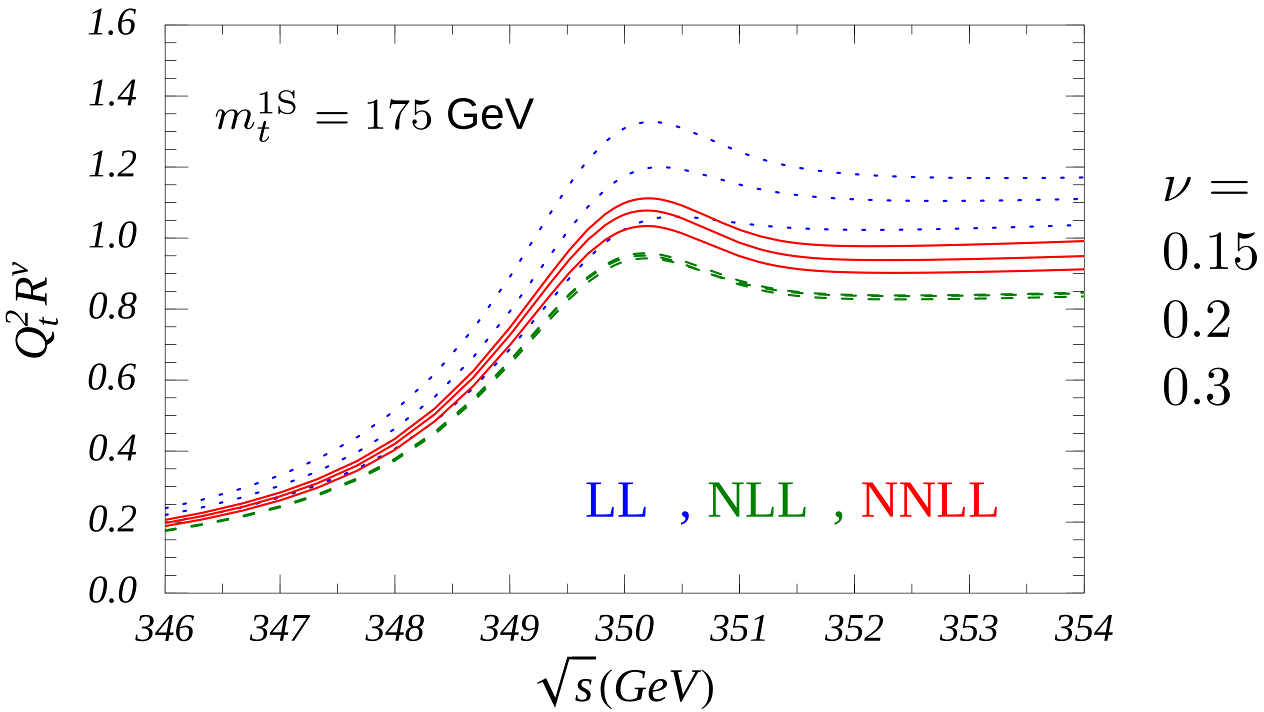


- peak position stable (threshold masses: 1S, PS, . . .)
- large sensitivity to factorization/renormalization scale setting
- NNNLO partial results: Penin etal. ’02 ’05, Beneke etal. ’05, Eiras etal. ’05

Cross Section at NNLL Order

1S mass - RG-improved, with NNLL non-mixing terms

Manohar,Stewart,Teubner,AH



- RGI expansion shows better convergence
- theory error: $\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim \pm 6\%$ goal: 3%

$\longrightarrow$ full NNLL (mixing) running of $C(\nu)$ required $\rightarrow$ w.i.p.

Electroweak Effects

3 classes:

- "Hard" electroweak — AHH, hep-ph/0604185
- Electromagnetic
- Finite lifetime

$\longrightarrow$ No general theory for all cases and observables !

$\longrightarrow$ EFT for certain observables and given powercounting.

$$\Gamma_t \sim m_t\alpha \sim m_t\alpha_s^2 \quad \Rightarrow \quad \boxed{v \sim \alpha_s \sim \alpha^{1/2}}$$

status for $\sigma_{\rm tot}$:

	LL	NLL	NNLL
"Hard" e.w.	✓	✓	✓
El. mag.	✓	(✓)	?
Fin. life.	✓	w.i.p.	w.i.p.

Finite Lifetime Effects

"inclusive treatment"

⇒ Optical Theory: effective complex indices of refraction for absorptive processes

⇒ vNRQCD: contributions from real Wb final states included in EFT matching conditions to QCD+ew. theory (=SM)

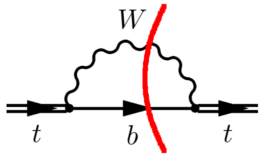
- complex matching conditions
- effective Lagrangian non-hermitian
- total rates through the optical theorem

Christoph Reisser, AH; Phys. Rev. D 71, 074022 (2005)

Finite Lifetime Effects

quark bilinears:

time dilatation correction



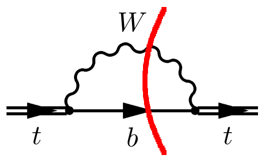
$$iD^0 - \frac{\mathbf{p}^2}{2m_t} + \delta m_t \quad \Longrightarrow \quad iD^0 - \frac{\mathbf{p}^2}{2m_t} + \delta m_t + i\frac{\Gamma_t}{2}\left(1 - \frac{\mathbf{p}^2}{2m_t^2}\right)$$

- power counting: $\Gamma_t \propto m_t g^2 \sim m_t v^2 \quad \Rightarrow \quad \boxed{g \sim g' \sim v \sim \alpha_s}$ $\quad \longrightarrow$ gauge invariance
- finite lifetime is LL effect, LO: $E \to E + i\Gamma_t$ Fadin,Khoze

Finite Lifetime Effects

time dilatation correction

quark bilinears:



$$iD^0 - \frac{\mathbf{p}^2}{2m_t} + \delta m_t \quad \Longrightarrow \quad iD^0 - \frac{\mathbf{p}^2}{2m_t} + \delta m_t + i\frac{\Gamma_t}{2}\left(1 - \frac{\mathbf{p}^2}{2m_t^2}\right)$$

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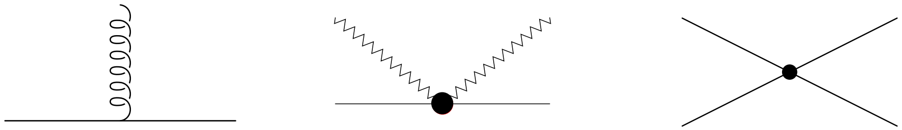
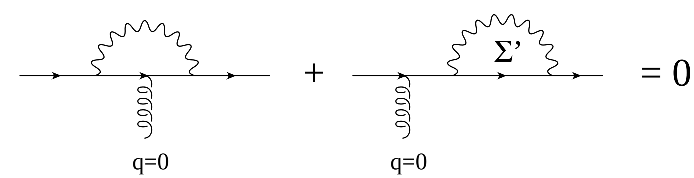
- finite lifetime is LL effect, LO: $E \longrightarrow E + i\Gamma_t$ Fadin,Khoze

gluon interactions & potentials:



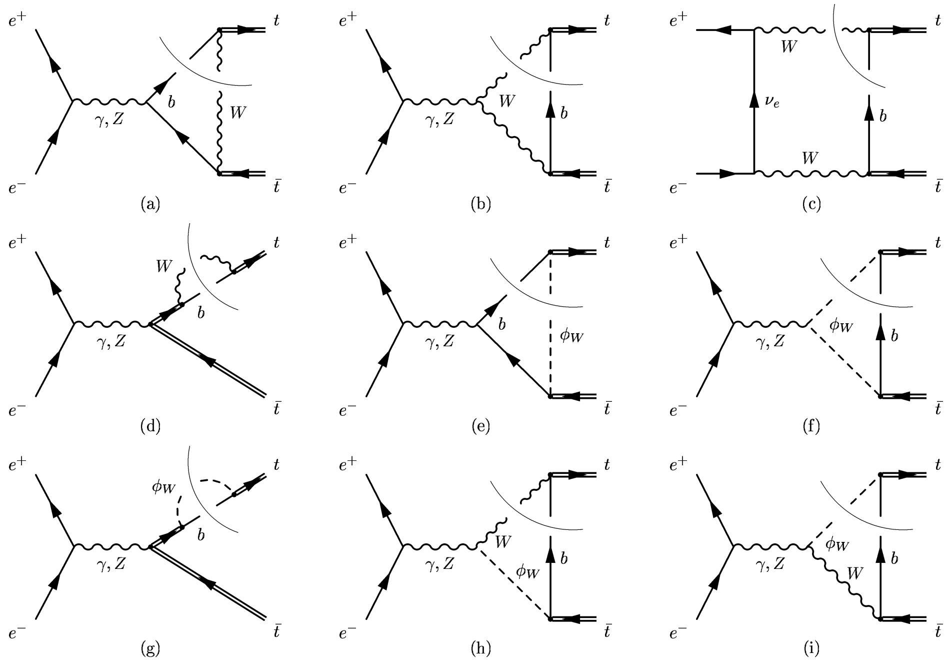


- electroweak corrections either beyond NNLL order or vanish due to gauge cancellations

$\longrightarrow$ ultrasoft gluon interference effects vanish at NLL and NNLL order (new !) [Khoze etal., Melnikov etal.]

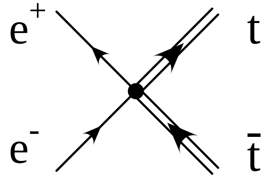
Finite Lifetime Effects

Currents:



Hard electroweak &
QCD matching corrections

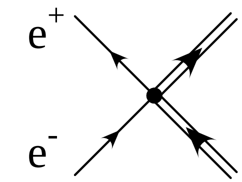
bW^+ and $\bar{b}W^-$ cuts

$$\mathbf{O_p} = \left[\, C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + iC^{\rm NNLL}_{\rm abs} \cdots \right] \cdot \Bigg($$  $$\Bigg) + \cdots$$

Finite Lifetime Effects

Currents:

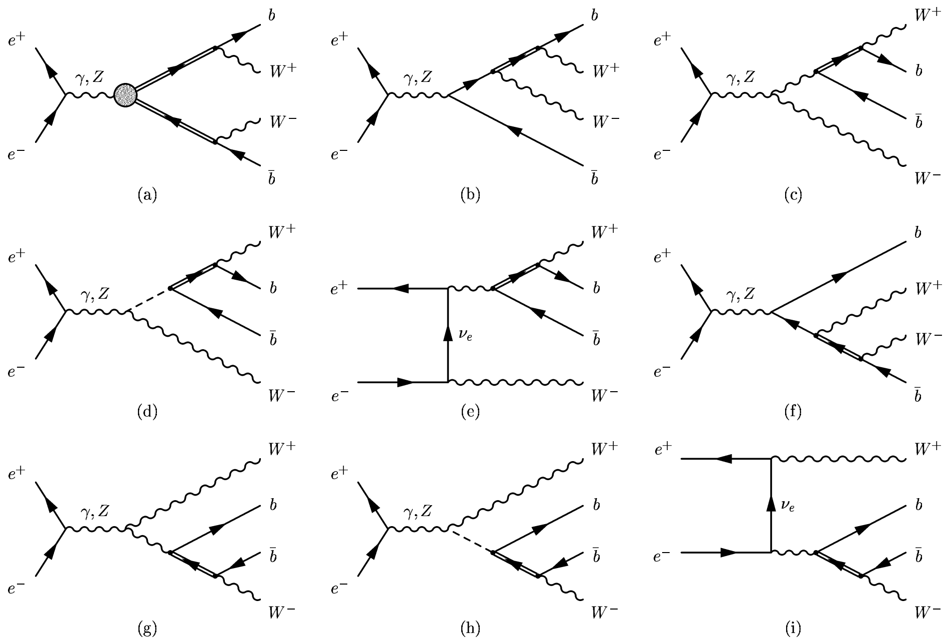
$$\mathbf{O_p} = \left[C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + iC^{\rm NNLL}_{\rm abs} \ldots \right] \cdot \left(\quad \right) + \cdots$$



$$\sigma_{\rm tot} \propto {\rm Im}\left[{\rm C}(\nu)^2\, {\rm G}(0,0,\sqrt{\rm s} + {\rm i}\Gamma_{\rm t})\right]$$

- accounts for irreducible interference contributions:

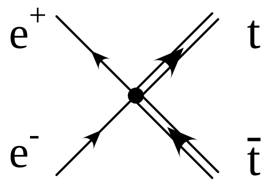
  resonant $\leftrightarrow$ non-resonant
  $W^+W^-b\bar{b}$ final states



Finite Lifetime Effects

Currents:

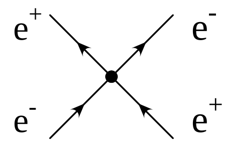
$$\mathbf{O_p} = \left[C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + i C^{\rm NNLL}_{\rm abs} \ldots \right] \cdot \Big(\quad \Big) + \cdots$$



$$\sigma_{\rm tot} \;\propto\; {\rm Im}\left[{\rm C}(\nu)^2\, {\rm G}(0,0,\sqrt{\rm s} + {\rm i}\Gamma_{\rm t}) \right]$$

- $(\Delta\sigma^{\Gamma}_{\rm tot}) \sim \alpha_s \Gamma_t \frac{1}{\epsilon}$ $\Rightarrow$ logarithmic phase space UV divergences

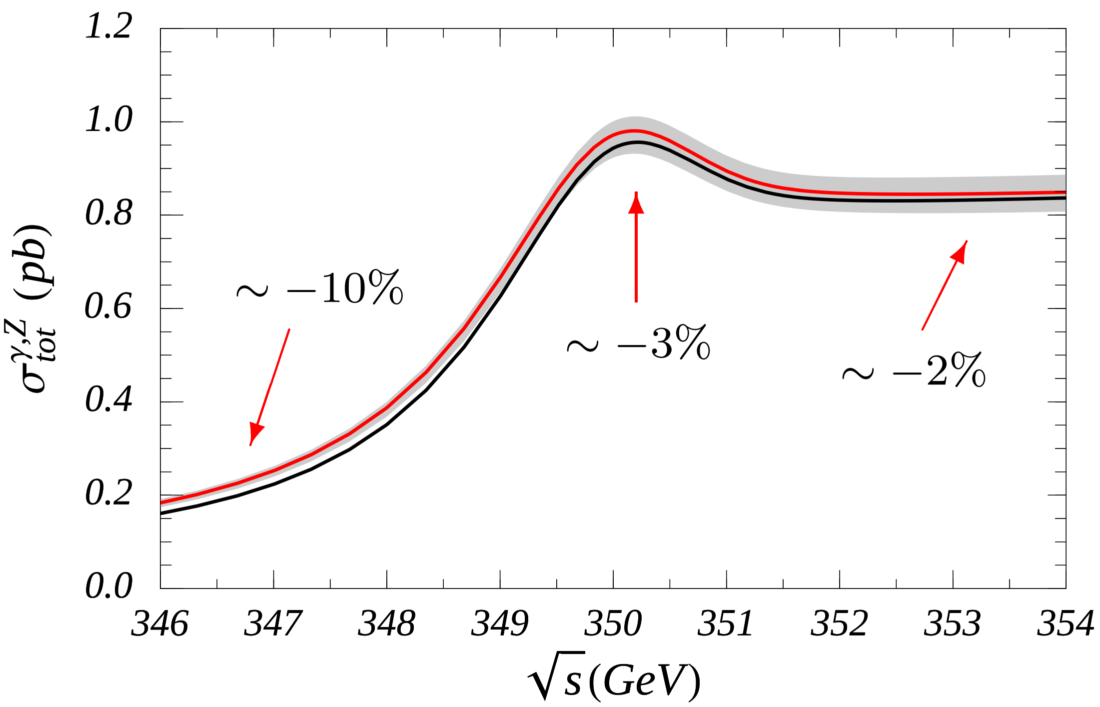
$\rightarrow$ anom. dim. for $(e^+e^-)(e^+e^-)$ operator $\rightarrow i\tilde{C}(\mu) \cdot \Big(\quad \Big)$ ✓



$\rightarrow$ matching for $i\tilde{C}(\mu)$:

- physical $W^+W^-b\bar{b}$ phase space $\rightarrow$ w.i.p.
- $W^+W^-b\bar{b}$ final state without tops input: Rieman,Kolodzej '05

Finite Lifetime Effects

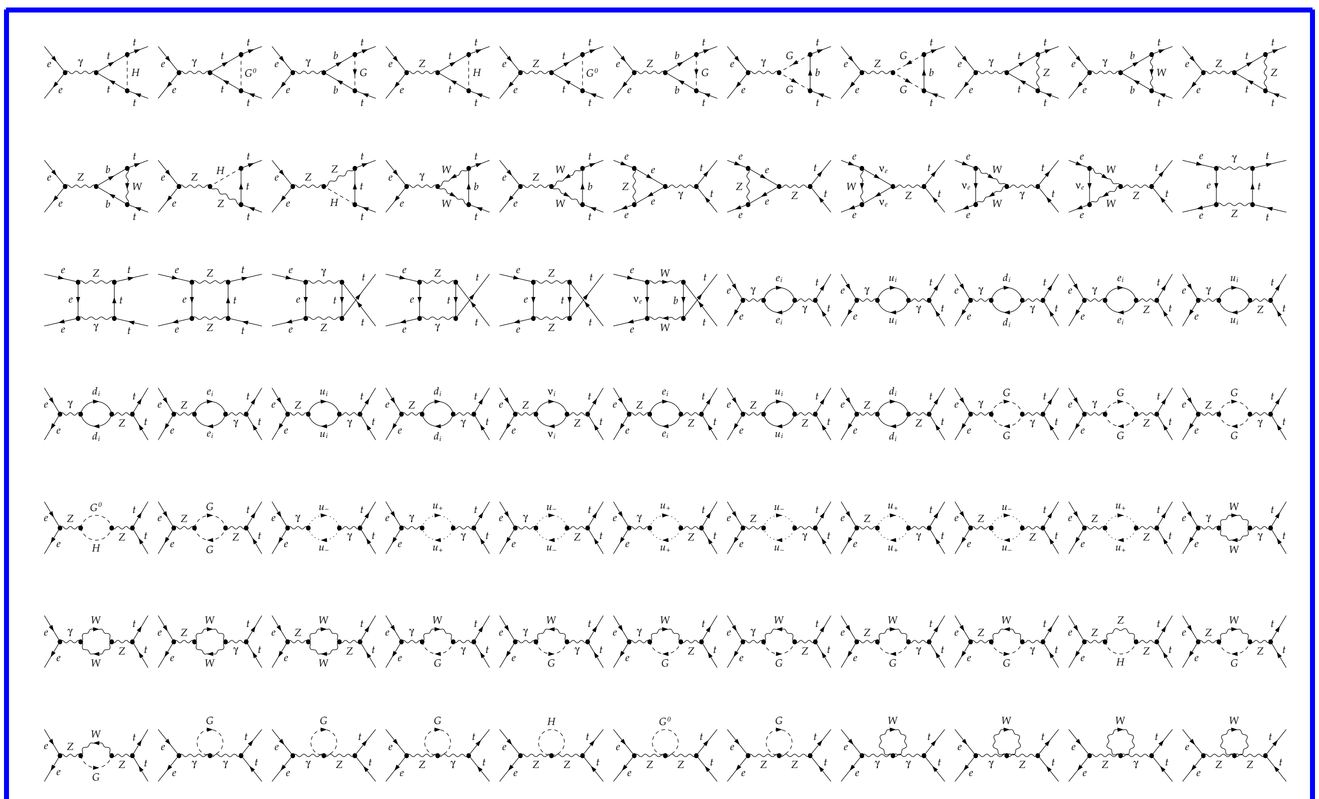


C. Reisser, AH

- corrections comparable to NNLL QCD corrections
- shift in the peak position: $30 - 50$ MeV $\qquad$ ($\delta m_t^{\text{ex}} \approx 50$ MeV)

Hard Electroweak Effects

- “real” short-distance electroweak corrections: $\mathcal{O}(\alpha_{\rm em}) \sim$ NNLL
- only matching conditions to $(e^+e^-)(t\bar{t})$ operators exist !

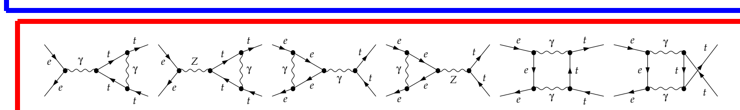


Grzadkowski, Kühn, etal. (1987)

Guth, Kühn (1992)

updated:

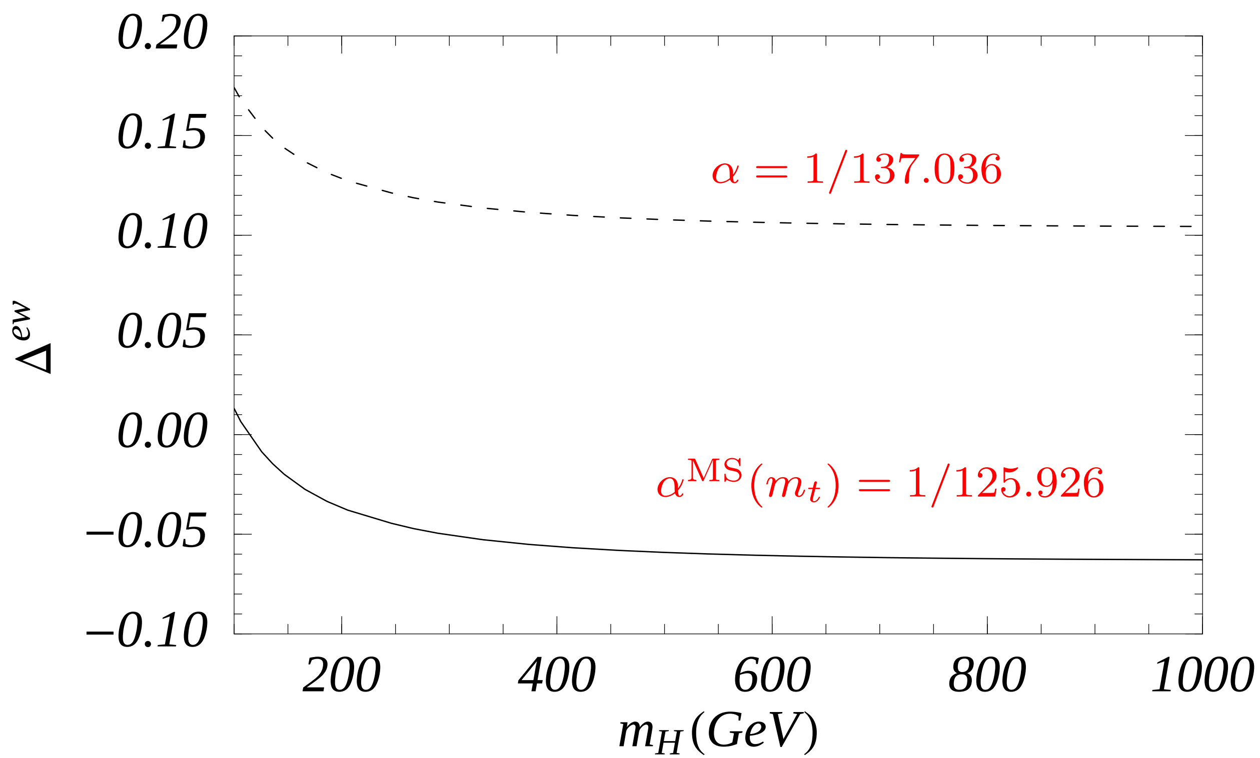
Reisser, AH hep-ph/0604104



QED corrections (+ real radiation, ISR) not yet determined $\longrightarrow$ w.i.p.

Hard Electroweak Effects

- “real” short-distance electroweak corrections: $\mathcal{O}(\alpha_{\rm em}) \sim$ NNLL
- global normalization correction: $\Delta^{\rm ew} \sim$ Re$[2c(1)^{\rm NNLL}_{\rm ew}]$

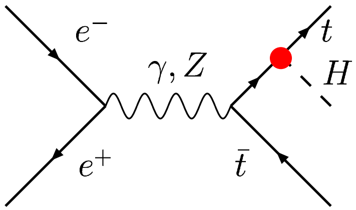


alt. scheme for $\alpha_{\rm QED}$:
$\alpha^{\overline{\rm MS},n_f=8}(\mu = m_t)$

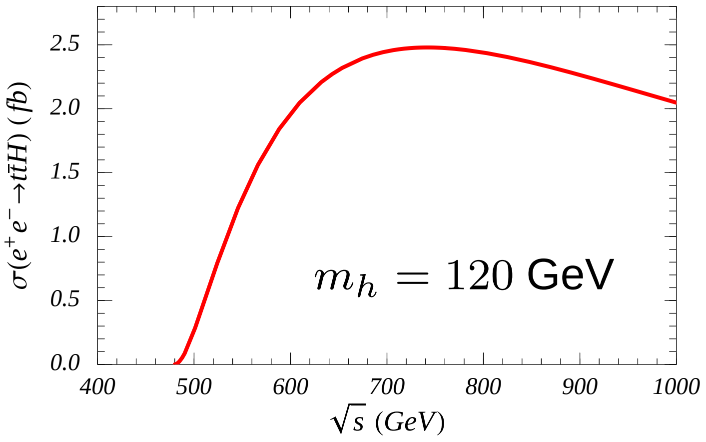
$m_t = 175$ GeV

Other Application

$e^+e^- \to t\bar{t}H$ $\longrightarrow$ top-Yukawa coupling



- Theory Status: $\sigma(e^+e^- \to t\bar{t}H)$
  Born ✓ [Gaemers et al., Djouadi et al.]
  1-loop ew. ✓ [Denner et al., Belanger et al., You et al.]
  $\mathcal{O}(\alpha_s)$ fixed-order ✓ [Dittmaier et al., Dawson et al.]
  NLL large-E_H QCD endpoint corrections ✓ [Cailin Farrell, AHH]



800 GeV: $\delta\lambda_t/\lambda_t = 5\%$ $(\mathcal{L} = 1000\text{fb}^{-1})$
[Gay;Besson;Winter]

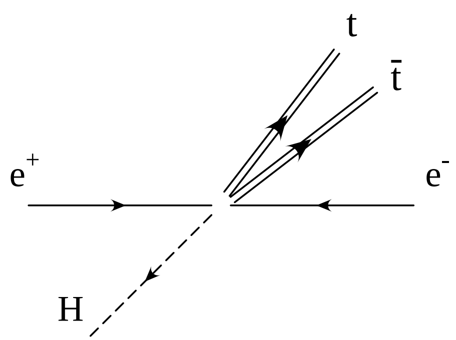
compare LHC: $\delta\lambda_t/\lambda_t = 12\%$ $(\mathcal{L} = 300\text{fb}^{-1})$

500 GeV: $(\delta\lambda_t/\lambda_t)^{\text{stat}} = 30\%$ $(\mathcal{L} = 1000\text{fb}^{-1})$
[Juste] $\to$ Born theory predictions

Other Application

$e^+e^- \longrightarrow t\bar{t}H$

$\longrightarrow$ region of large Higgs energy



$\longrightarrow$ $t\bar{t}$ collinear

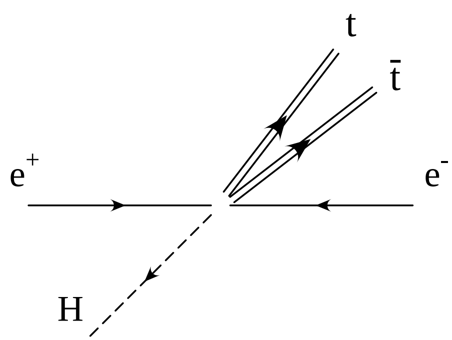
$\longrightarrow$ QCD effects localized in $t\bar{t}$ system

$\Rightarrow$ $t\bar{t}$ dynamics non-relativistic

Other Application

$e^+e^- \longrightarrow t\bar{t}H$

$\longrightarrow$ region of large Higgs energy



$\longrightarrow t\bar{t}$ collinear

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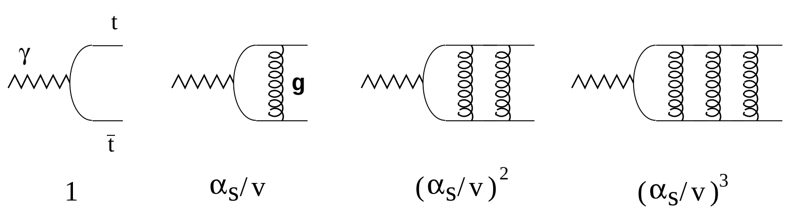
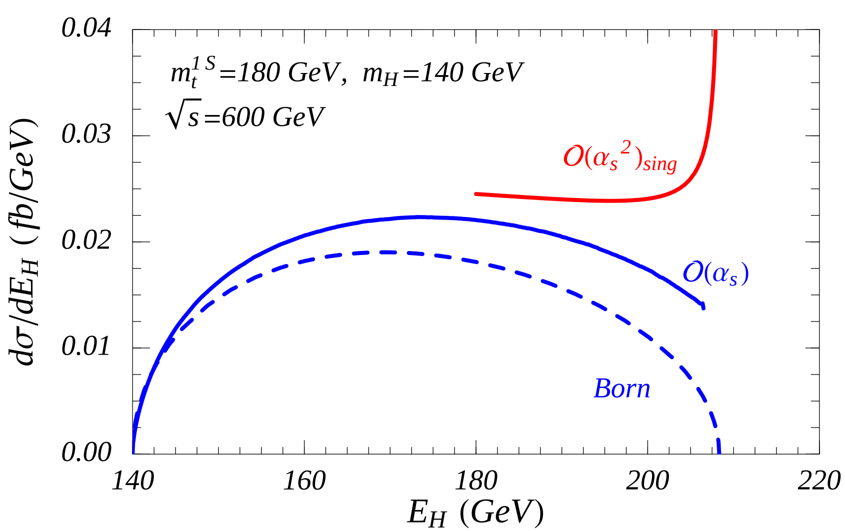




$\longrightarrow$ singularities: $\sim (\alpha_s/v)^n$,

$\sim (\alpha_s \ln v)^n$

$\longrightarrow$ fixed order expansion breaks down

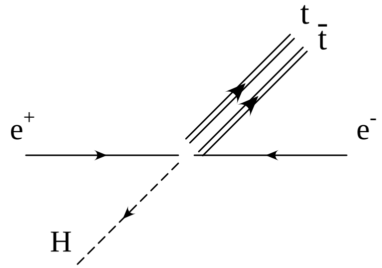
$\Rightarrow$ summation of singular terms

Other Application

$e^+e^- \to t\bar{t}H$ $\to$ factorization formula

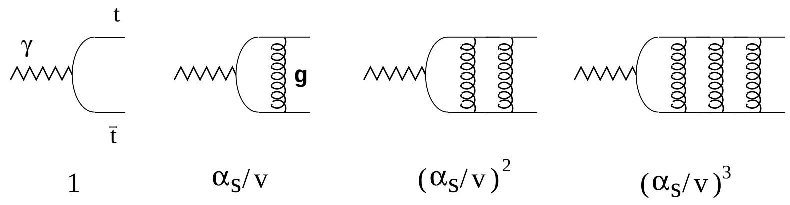
numerical input: Denner, Dittmaier,Roth,Weber ’04

$$\left(\frac{d\sigma}{dE_H}\right)_{\mathrm{E_H \approx E_H^{max}}} \sim C^2(\mu, \sqrt{s}, m_t, m_H) \times \mathrm{Im}[G(0,0,v,\mu)]$$



+ hard QCD corrections

soft & ultrasoft QCD corrections

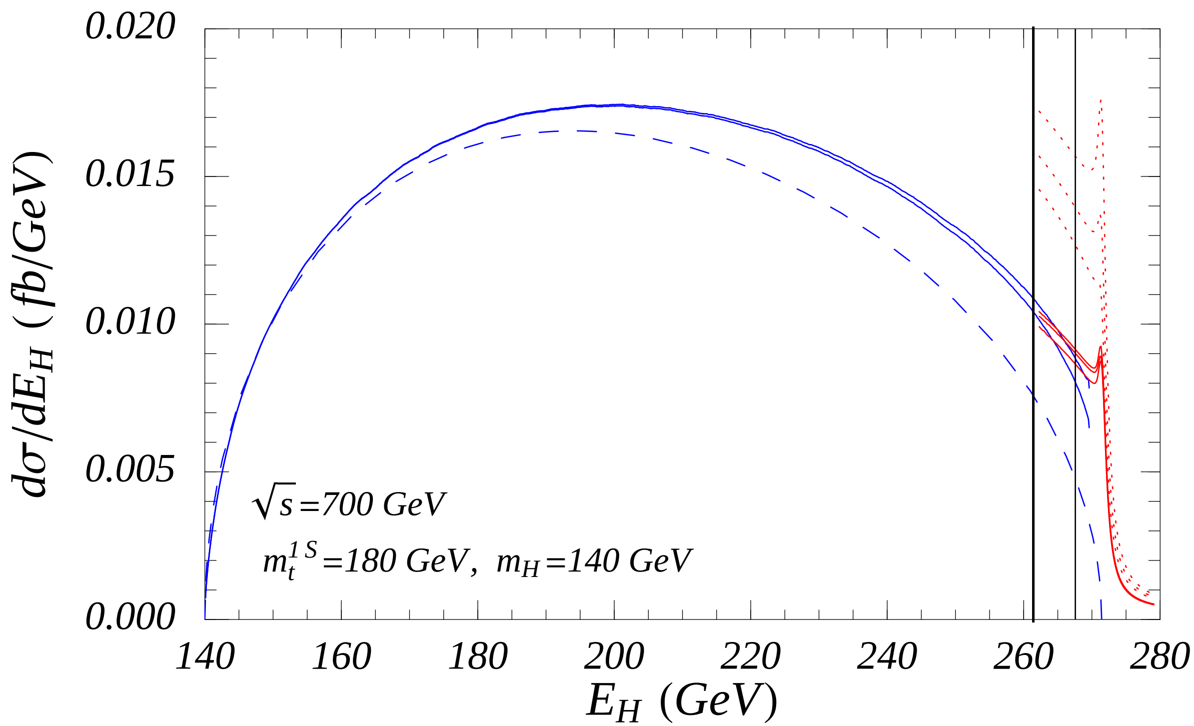


NLL formalism: Cailin Farrell, AHH; Phys.Rev.D72,014007 (2005)

Cailin Farrell, AHH; hep-ph/0604166

Other Application

$e^+e^- \to t\bar{t}H$ $\to$ NLL Higgs energy spectrum

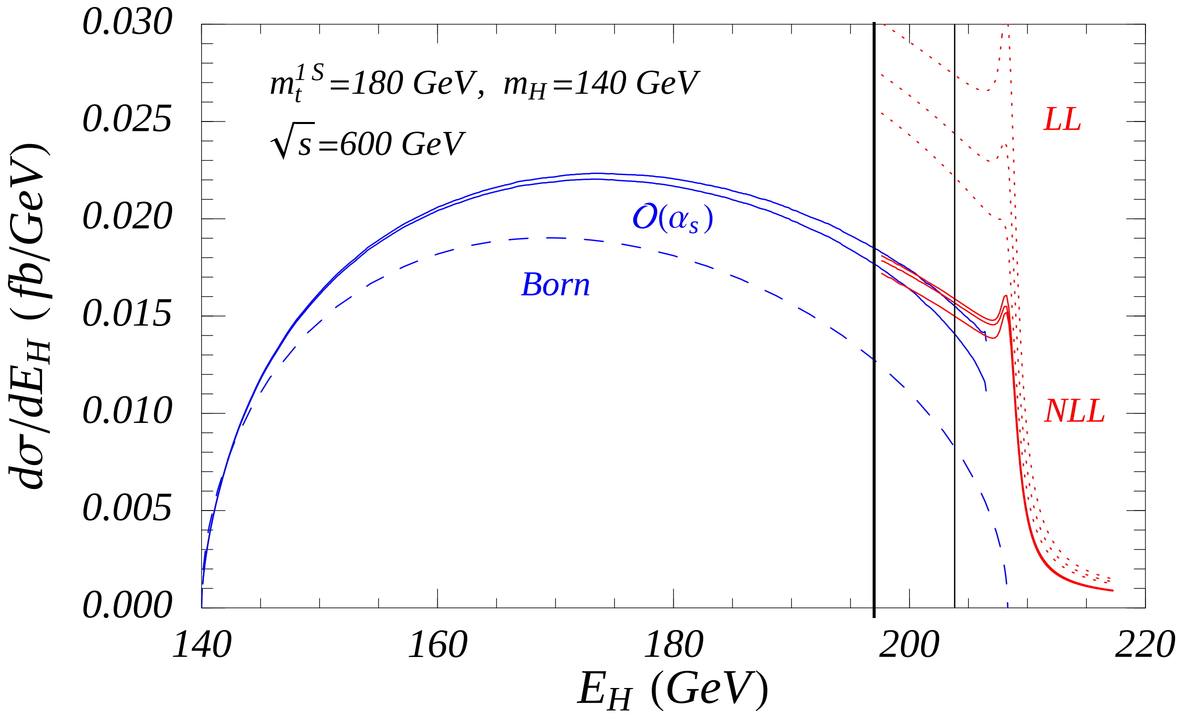


Farrell, AHH

- large E_H endpoint regions increases for smaller $\sqrt{s}$ / larger m_H

Other Application

$e^+e^- \to t\bar{t}H$ $\to$ NLL Higgs energy spectrum

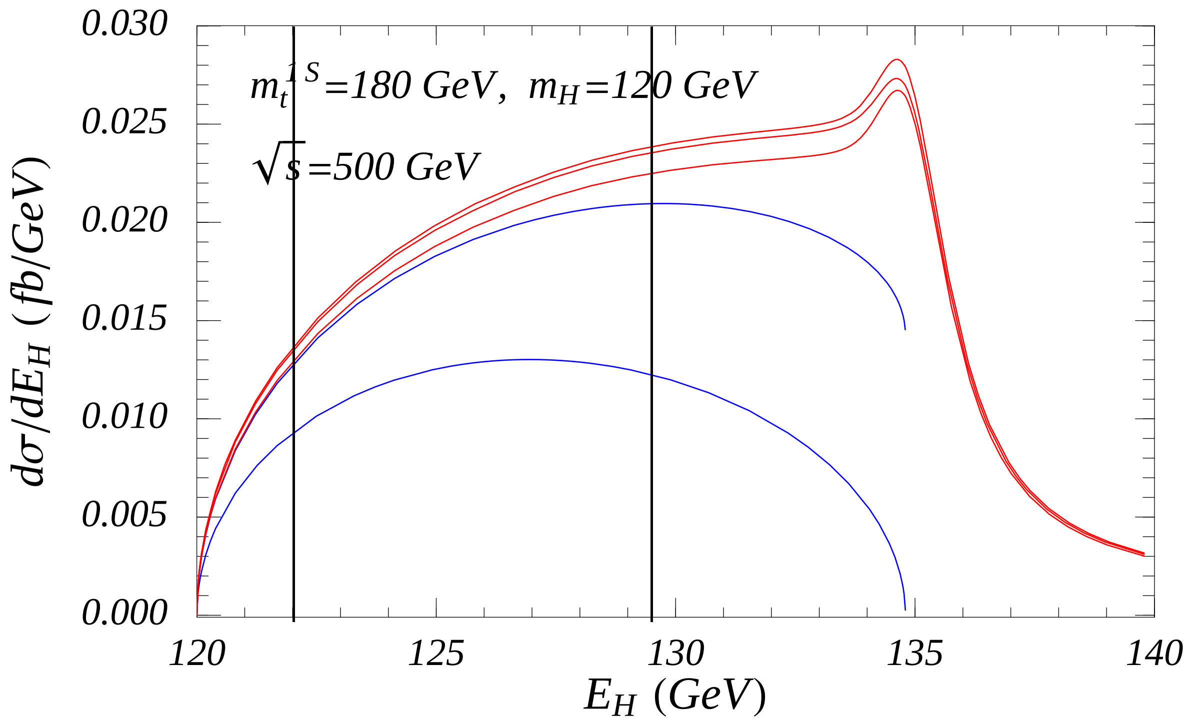


Farrell, AHH

- large E_H endpoint regions increases for smaller $\sqrt{s}$ / larger m_H

Other Application

$e^+e^- \to t\bar{t}H$ $\to$ NLL Higgs energy spectrum

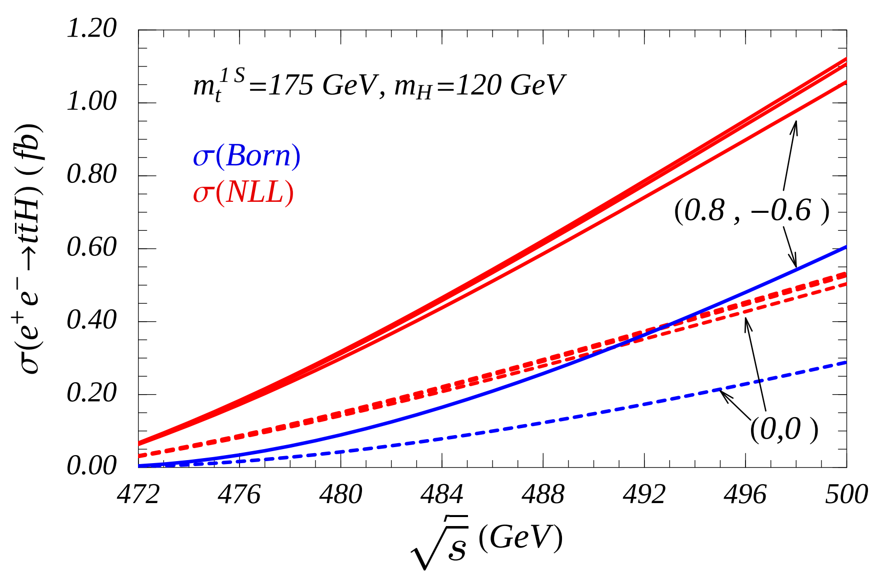


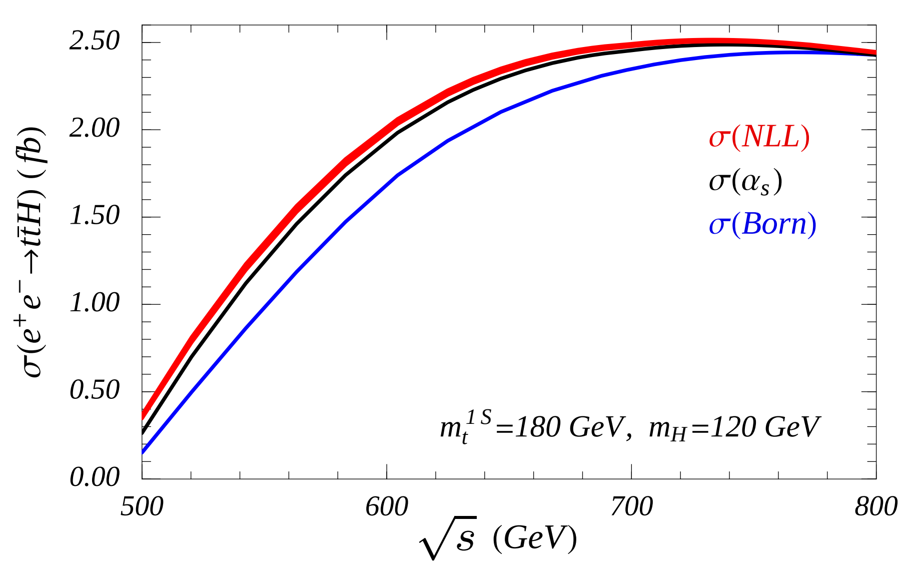
Farrell, AHH

- large E_H endpoint regions increases for smaller $\sqrt{s}$ / larger m_H

Other Application

$e^+e^- \to t\bar{t}H$ $\to$ total cross section





500 GeV:

- factor 2 enhancement over tree level from summation of $(\alpha_s/v)^n$, $(\alpha_s \ln v)^n$ terms
- another factor of 2 enhancement for $P_- = -80\%$, $P_+ = +60\%$
- essential for realistic studies for ILC (phase I) Juste '02, '06

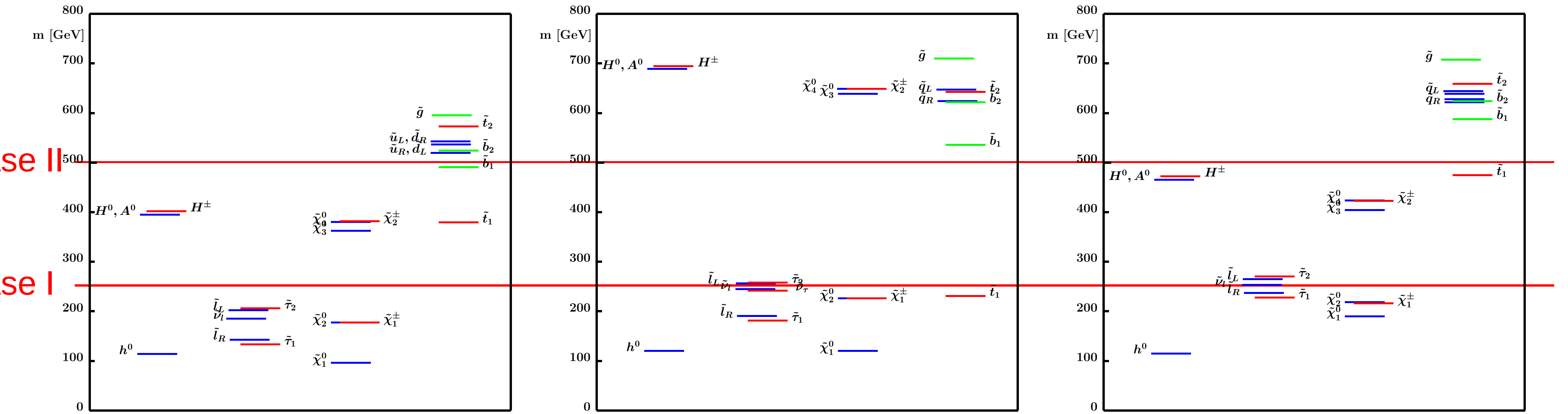
$\Rightarrow$ $(\delta\lambda_t/\lambda_t)^{\rm ILC}_{500\ {\rm GeV}} \sim 30\%$ $\stackrel{\sim}{\longrightarrow}$ $10 - 15\%$

Other Applications

$$e^+e^- \to \tilde{q}\bar{\tilde{q}}$$

- in many models for SUSY breaking squark pair production is possible at the ILC

SPS 1a SPS 5 SPS 6

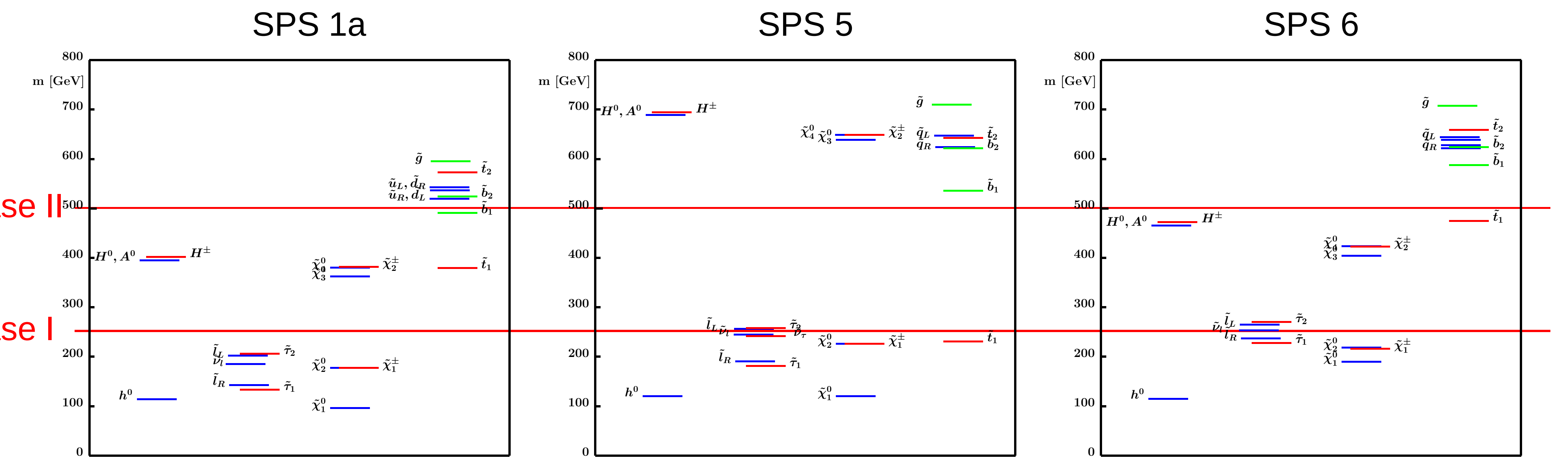


SPS 1a	SPS 5	SPS 6
$m_{\tilde{t}_1} = 396$ GeV	$m_{\tilde{t}_1} = 240$ GeV	$m_{\tilde{t}_1} \simeq 490$ GeV
$\tilde{t}_1 \to b\chi_1^+, \ldots$	$\tilde{t}_1 \to b\chi_1^+, c\chi_1^0$	$\tilde{t}_1 \to b\chi_1^+, \ldots$
$\Gamma_{\tilde{t}_1} = 1.92$ GeV	$\Gamma_{\tilde{t}_1} = 0.04$ GeV	$\Gamma_{\tilde{t}_1} \simeq 3.2$ GeV

Other Applications

$e^+e^- \to \tilde{q}\bar{\tilde{q}}$

- in many models for SUSY breaking squark pair production is possible at the ILC



- low-energy QCD dynamics very similar to $t\bar{t}$ threshold physics
- e.w. weak effects & phenomenology can differ significantly: $m_{\tilde{q}}, \Gamma, \ldots$
- no coherent theoretical analysis of QCD & electroweak effects exists

Ruiz-Femenia, Teubner, AHH

Other Applications

$e^+e^- \to \tilde{q}\bar{\tilde{q}}$ $\to$ vNRQCD

quarks: $\Lambda = -i\dfrac{\mathbf{S}\cdot(\mathbf{p}'\times\mathbf{p})}{\mathbf{k}}$, $T = \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 - \dfrac{3(\mathbf{k}\boldsymbol{\sigma}_1)(\mathbf{k}\boldsymbol{\sigma}_2)}{\mathbf{k}^2}$

$$V = \left[\frac{\mathcal{V}_c(\nu)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(\nu)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(\nu)(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} + \frac{\mathcal{V}_s(\nu)}{m^2}\mathbf{S}^2 + \frac{\mathcal{V}_\Lambda(\nu)}{m^2}\Lambda + \frac{\mathcal{V}_t(\nu)}{m^2}T\right]$$

$$\frac{d}{d\ln\nu}\ln C_{^3S_1}(\nu) = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left(\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\mathcal{V}_s(\nu)\right) + \frac{\mathcal{V}_k(\nu)}{2}$$

$$C_{^3S_1}(\nu = 1) = 1 - \frac{4}{3}\frac{\alpha_s(m_t)}{\pi}$$

Other Applications

$e^+e^- \to \tilde{q}\bar{\tilde{q}}$ $\to$ scalar vNRQCD

squarks:

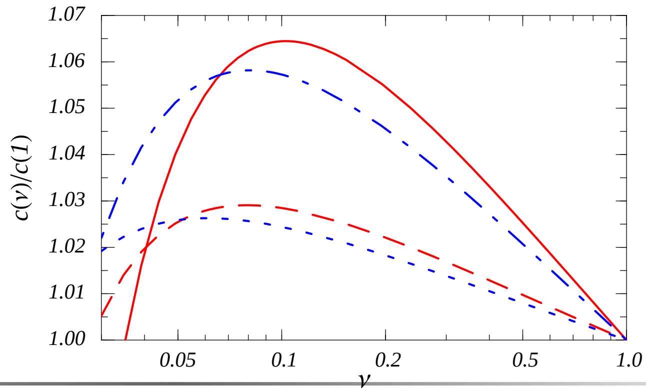
- no spin-dependent interactions
- P-wave production
- full NLL QCD running completed

Ruiz-Femenia, AH [hep-ph/0511102]

$$V = \left[\frac{\mathcal{V}_c(\nu)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(\nu)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(\nu)(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} \right]$$

$$\frac{d}{d\ln\nu} \ln C_{1_{P_1}}(\nu) = -\frac{\mathcal{V}_c(\nu)}{48\pi^2}\left(\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu)\right) + \frac{\mathcal{V}_k(\nu)}{6}$$

$$C_{1_{P_1}}(\nu = 1) = 1 - \frac{4}{3}\frac{\alpha_s(m_t)}{\pi}$$



Conclusion

- Top threshold physics ($m_{t\bar{t}} \approx 2m_t$) unique QCD $\leftrightarrow$ electroweak laboratory
  - hard & nonrelativistic & soft-collinear dynamics
  - unstable particles ($\Gamma_t \sim E_{\rm kinetic}$)
- Master application: $e^+e^- \to t\bar{t}$ @ ILC
  - QCD dynamics understood (NNLL RGE-improved, N^{3}LO fixed-order)
  - effects of ew. corr.'s and finite top lifetime $\to$ w.i.p.
  - top mass: $\delta m_t^{1S} \simeq (\pm 100$ GeV$)^{\rm theo} + (\pm 50$ GeV$)^{\rm exp}$ ✓
  - λ_t, Γ_t: $\to d\sigma/\sigma \approx \pm 6\%$ goal: 3%
- top threshold essential for $\sigma(e^+e^- \to t\bar{t}H)$ @ 500 GeV $\to \lambda_t$
- Other applications:
  - ⋆ $t\bar{t}$ threshold at Tevtraon/LHC (presumably small, but maybe important)
  - ⋆ squark pair production ILC+LHC (particularly for heavy squarks)
  - ⋆ particle decay rates: $H \to gg$ ($m_h \approx 2m_t$)

$$X \to t\bar{t} + Y$$

Colors

This is blue

This is red

This is brown

This is magenta

This is Dark Green

This is Dark Blue

This is Green

This is Cyan

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks