

Axion

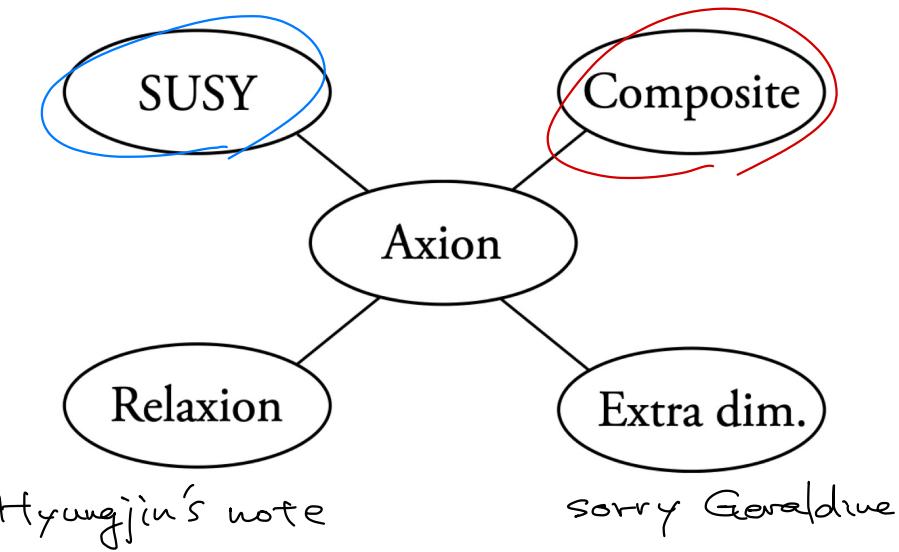
& other frameworks addressing the hierarchy problem

07.13.2021

Ryusuke Jinno



1. Overview



Axion + SUSY

- Brief recap of SUSY
Motivation, Wess-Zumino, Superpotential, F-term,
SUSY breaking, μ -problem, ...
- Saxon & Axino : general properties
- SUSY DFSZ (Kim-Nilles '84)
- SUSY KSVZ (Asaka-Yamaguchi 9805449)

Axion + composite

- Recap of minimal (+ α) composite axion
(Kim '85, Choi-Kim '85)
- Composite axion in composite Higgs
(Gherghetta & Nguyen 2007.10275)

2. Axion in SUSY

2-1. Recap of SUSY

- Motivation : hierarchy prob. $M_{EW} \ll M_{GUT}, M_P$

- More precisely : $\mathcal{L}_{toy} \supset -(y\Phi\bar{\Psi}\Psi + h.c.) - \lambda|\Phi|^2|S|^2.$
- \uparrow \uparrow \uparrow
 Dirac fermion Complex scalars

1-loop mass correction

$$-i\Delta m_{\tilde{\Phi}}^2 = \left\{ \begin{array}{l} \text{---} \circlearrowleft \tilde{\Phi} \text{---} \\ \text{---} \circlearrowright S \text{---} \end{array} \right. = i \frac{|y|^2}{8\pi^2} \Lambda_{UV}^2$$

$$= -i \frac{\lambda}{16\pi^2} \Lambda_{UV}^2$$

So, naively $m_{\tilde{\Phi}}^2 \sim \Lambda_{UV}^2$ but we observe $m_{\tilde{\Phi}}^2 \sim M_{EW}^2$

Hierarchy problem

Solution? : Weyl fermion \leftrightarrow Complex scalar
 (half dof of Dirac)

$$\text{with } |y|^2 = \lambda$$

- Simplest model : Wess-Zumino $\mathcal{L}_{WZ?} = -|\partial\phi|^2 + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$

$$\mathcal{L}_{WZ} = -|\partial\phi|^2 + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + |F|^2,$$

	ϕ	ψ	F
real dof (off shell)	2	4	2
real dof (on shell)	2	2	0

This model has a symmetry btwn. boson \leftrightarrow fermion.

$$\delta\phi = \epsilon\psi, \quad \delta\psi = -i\sigma^\mu \epsilon^\dagger \partial_\mu \phi + \epsilon F, \quad \delta F = -i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi.$$

SUSY transformation!

ϵ : infinitesimal parameter

complex, 2-component, anti-commuting, mass dim $-1/2$.

$$\epsilon \neq \epsilon^* \quad \epsilon_1, \epsilon_2 \quad \epsilon_1 \epsilon_2 = -\epsilon_2 \epsilon_1$$

$$\epsilon_1^*, \epsilon_2^*$$

- Superpotential, F-term

Suppose we have many copies of (ϕ, ψ, F)

$$\mathcal{L}_{\text{free}} = -(\partial^\mu \phi^{*i})(\partial_\mu \phi_i) + i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + \boxed{F^{*i} F_i}.$$

What about interactions?

$$\mathcal{L}_{\text{int}} = [((\dim 1)^{ij} \underbrace{\psi_i \psi_j}_3 + (\dim 2)^i \underbrace{F_i}_2 + (\dim 0)^{ij} \underbrace{F_i F_j}_4) + \text{c.c.}] - (\dim 4),$$

func. of $\{\phi_i\}$.

These functions cannot be arbitrary  because SUSY transf.

$$(\dim 1)^{ij} = -\frac{1}{2} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} (\equiv W^{ij}), \quad (\dim 2)^i = \frac{\partial W}{\partial \phi_i} (\equiv W^i).$$

W is called SUPERPOTENTIAL

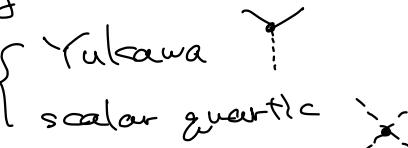
$$W = L^i \phi_i + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k.$$

neglected.

↑ ↑ ↑

gives gives gives

$\left\{ \begin{array}{l} \text{scalar} \\ \text{fermion} \end{array} \right\}$ mass $\left\{ \begin{array}{l} \text{Yukawa} \\ \text{scalar quartic} \end{array} \right\}$



What happens after integrating out $\{F_i\}$?

$$\text{solution : } F_i = -W_i^*$$

This gives rise to a scalar potential

$$\mathcal{L} \supset - \underbrace{\left| \frac{\partial W}{\partial \phi_i} \right|^2}_{(= -|F_i|^2)}.$$

F-term contribution to the potential.

• Superspace formalism

Lagrangian above can be derived in one stroke. How?

→ Superspace formalism.

$$\mathcal{L} = \int d^2\theta d^2\theta^\dagger \Phi^{*i} \Phi_i + \left[\int d^2\theta W(\{\Phi\}) + \text{c.c.} \right]$$

What is \emptyset ? What is \mathbb{I} ?

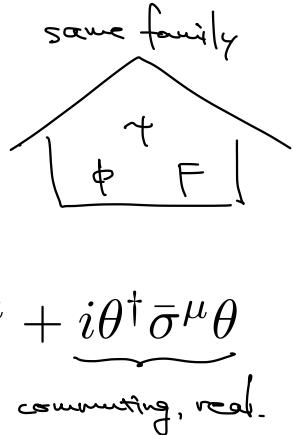
θ : Grassmann coordinate, (on top of
usual coordinate x^μ)

complex, 2-component, anti-commuting, dim $-1/2$.

(Left-chiral)

Real scalar	Weyl fermion	Auxiliary field	
----------------	-----------------	--------------------	--

$$\Phi(x, \theta, \theta^\dagger) = \underline{\phi}(y) + \sqrt{2}\underline{\theta}\underline{\psi}(y) + \theta\theta\underline{F}(y).$$



$$y^\mu \equiv x^\mu + \underbrace{i\theta^\dagger \bar{\sigma}^\mu \theta}_{\text{commuting, real.}}$$

trick.

$$\phi(y) = \phi(x) + \partial\phi \cdot i\theta^\dagger \bar{\sigma}\theta + \partial\partial\phi \cdot (i\theta^\dagger \bar{\sigma}\theta)^2 + \underbrace{\partial\partial\partial\phi \cdot (i\theta^\dagger \bar{\sigma}\theta)^3}_{\text{zero } \Theta \text{ too many } \Theta} + \dots$$

$$\theta\psi(y) = \theta\psi(x) + \theta\partial\psi \cdot i\theta^\dagger \bar{\sigma}\theta + \dots$$

zero Θ too many Θ

$$\theta\theta F(y) = \theta\theta F(x) + \dots$$

e.g. $\theta_1\theta_2\theta_3 = 0$

- To summarize, in order to have SUSY Lagrangian,

1. Fix your $W(\{\Phi\})$ (Don't use $\{\tilde{\Phi}^\dagger\}$)

2. Calculate $\mathcal{L} = \int d^2\theta d^2\theta^\dagger \Phi^{*i} \Phi_i + \left[\int d^2\theta W(\{\Phi\}) + \text{c.c.} \right]$

$$\left\{ \begin{array}{l} \text{Scalar potential } V = \sum_i \left| \frac{\partial W}{\partial \Phi_i} \right|^2 \\ \quad \quad \quad \Phi_i \rightarrow \phi_i \\ \quad \quad \quad \text{multiplet scalar.} \end{array} \right. \quad \left(\begin{array}{l} \text{contains} \\ \text{quartic } \sim y^2 \end{array} \right)$$

$$\text{Yukawa} \sim \left. \frac{\partial^2 W}{\partial \tilde{\Phi}_i \partial \tilde{\Phi}_j} \right|_{\tilde{\Phi} \rightarrow \Phi} \times \psi_i \psi_j$$

I will not talk about gauge & D-terms...

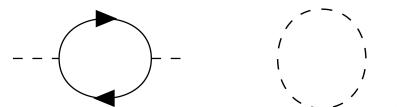
SUSY breaking.

In reality, SUSY must be broken.

(we don't observe
any scalar
at $m = 511 \text{ keV}$)

But we want to break it softly.

If we break it badly,



will not cancel.

→ Meaning
 $\frac{\partial W}{\partial \Phi_i} = 0$
 does not hold
 for all i
 at the same time

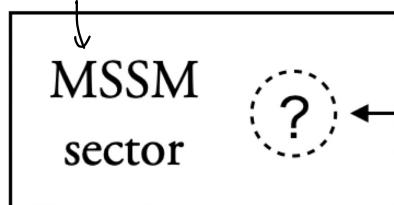
How is it possible?

SUSY is spontaneously broken if some sector has $\langle F_i \rangle \neq 0$.

So, assume "~~SUSY~~ sector" $\langle F \rangle \neq 0$

and communicate the effect to our sector.

Minimal
Supersymmetric
Standard Model



Messenger

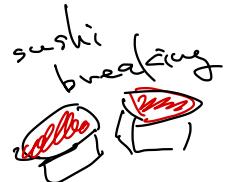
$m_{3/2} \sim \frac{\langle F \rangle}{M_P}$ Gravitino



mass

$\phi, \psi \rightarrow \psi \uparrow m_{\text{soft}} \sim \text{TeV}$

SUSY SUSY
conserving softly broken



- MSSM Lagrangian & μ -problem.

We need
2 Higgses
 $\downarrow \downarrow$

$$W_{\text{MSSM}} = \underline{\underline{y_u}} Q H_u \bar{u} - \underline{\underline{y_d}} Q H_d \bar{d} - \underline{\underline{y_e}} L H_d \bar{e} + \underline{\underline{\mu}} H_u H_d$$

dimless dimless dimless DIMENSION

$\downarrow \quad \downarrow \quad \swarrow$

gives { Yukawa } { scalar quartic }

ONE
Needs to be $\approx \text{TeV}$

We recognize μ - problem

It's not unnatural to have $m_{\text{soft}} \sim \text{TeV}$

(:() ~~SUSY~~ can be weakly communicated to our sector.

But why $\mu \sim \text{TeV}$ even though it's ^{conserving} SUSY part?

We wanted to explain that, right?

2-2. Saxon & Axino : general properties.

- In SUSY theories, axion lives in a multiplet.

$$A = \frac{s + ia}{\sqrt{2}} + \sqrt{2}\theta \tilde{a} + \theta\bar{\theta} F_a$$

↑
axion
superfield

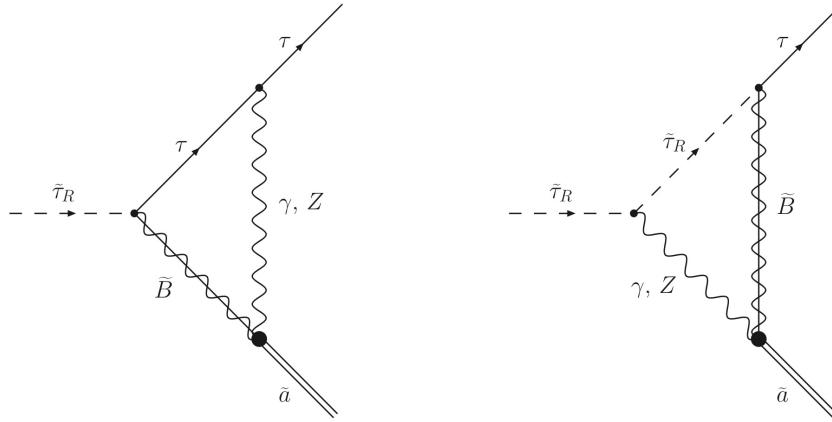
saxion s
 axion a
 auxiliary
 (not the decay const.)
 { saxion S } form a complex scalar field.
 { axion a }

- We expect S and \tilde{a} to be light because they are in the same multiplet as axion a
- Saxion S can be a cosmological problem
 - Why ? { Light scalar fields tend to oscillate at late times
→ Energy domination
 - It has couplings suppressed by $1/f_a$, mainly decaying into aa or $\tilde{a}\tilde{a} (\rightarrow aa)$
→ Dark radiation constraints.

• Axino \tilde{a}

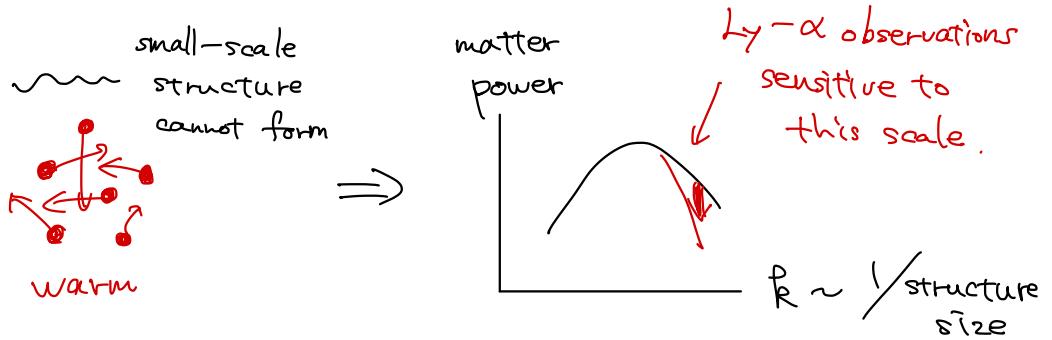
has couplings suppressed by $1/f_a$

→ behaves similarly to gravitino in colliders



can be (part of) dark matter.

→ if it's too light, it behaves as warm dark matter,
and can be observed/excluded by small-scale structure



can pop up from thermal plasma

$$\Omega_{\tilde{a}} h^2 \sim \mathcal{O}(1) g_s^6 \left(\frac{m_{\tilde{a}}}{100 \text{ MeV}} \right) \left(\frac{10^{11} \text{ GeV}}{f_a/N} \right)^2 \left(\frac{T_R}{10^4 \text{ GeV}} \right),$$

$$\Omega_{\tilde{G}} h^2 \sim \mathcal{O}(0.1) g_s^2 \left(1 + \frac{m_{\tilde{G}}^2}{3m_{\tilde{G}}^2} \right) \left(\frac{m_{\tilde{G}}}{100 \text{ GeV}} \right) \left(\frac{T_R}{10^{10} \text{ GeV}} \right).$$

- SUSY DFSZ (PQ - charged Higgs)

Kim-Nilles mechanism : solve $\left\{ \begin{array}{l} \text{strong CP} \\ \text{ee-problem} \end{array} \right\}$ at once.

$$W \supset \frac{\lambda S^2}{M_P} H_u H_d$$

↓ New field λS^2 (scalar component develops VEV $\sim 10^0 \text{ GeV}$)
 H_u H_d
 U(1)_{PQ} +1 -1 -1
 MSSM Higgs.

Strong CP : Same as (non-SUSY) DFSZ.

Recap. $\mathcal{L}_{\text{DFSZ:II}} \supset -y_u Q_L^\dagger \underline{H}_u \underline{U}_R - y_d Q_L^\dagger \underline{H}_d \underline{d}_R - y_e L_L^\dagger \underline{H}_d \underline{e}_R$

↓
 $- \lambda \underline{H}_u \underline{H}_d (\underline{\Phi}^\dagger)^2 - \text{(other terms)}$
 Integrate out Φ radial. $\underline{\Phi} \sim v_\Phi e^{i\alpha/v_\Phi}$ radial → bye bye
 phase → alive

↓
 Integrate out H_u, H_d Eliminate phase in $\lambda H_u H_d v_\Phi^2 e^{-i\alpha/v_\Phi}$
 \rightarrow Phases in quark mass
 (* otherwise non-decoupling effect appears)

(optional) Rotate quarks Path integral measure $\rightarrow \frac{a}{v_\Phi} G \tilde{G}$
 Axion!

μ -problem : Naturally explained if $\langle S \rangle \sim 10^{10} \text{ GeV}$.

$$\frac{\lambda \langle S \rangle^2}{\mu_p} H_u H_d \sim (\text{TeV}) \times H_u H_d.$$

* How can we explain $\langle S \rangle \sim 10^{10} \text{ GeV}$?

Logic in the original paper

"If we solve strong CP with axion, then $f_a \sim 10^{10} \text{ GeV}$.

We can solve μ -problem with $\frac{\lambda S^2}{\mu_p} H_u H_d$

without any additional theoretical cost."

* Domain walls?

$N_{DW} = \text{same as non-SUSY DFSZ} = 6$

Why? Higgs rotation affects all generations of quarks.

- SUSY KSVZ (PQ charged heavy quarks)

$$W \supset \lambda X Q \bar{Q}$$

heavy quarks.

"bar" does not mean
 any operation.
 \bar{Q} is one object.

	X	Q	\bar{Q}
$U(1)_{PQ}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$
$\rightarrow SU(3)_c$	1	□	□

$=$ $=$

Strong CP

$$X \text{ gets VEV } \langle X \rangle \sim f_a e^{i\alpha/f_a}$$

↓

radial \rightarrow bye
phase \rightarrow alive

$$Q, \bar{Q} \text{ have a mass } \sim f_a e^{i\alpha/f_a}$$

Rotate Q, \bar{Q} to eliminate the phase

(* otherwise
non-decoupling effect)

↓

Path integral measure gives $\frac{a}{f_a} \tilde{Q} \tilde{\bar{G}}$

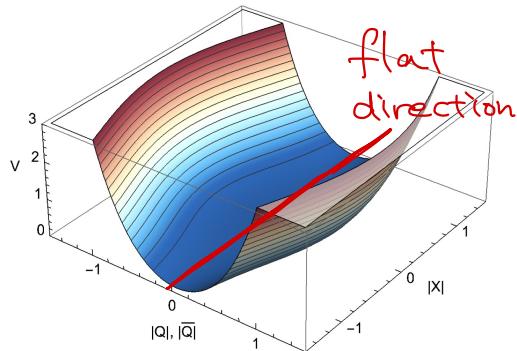
Axion!

Saxion dynamics

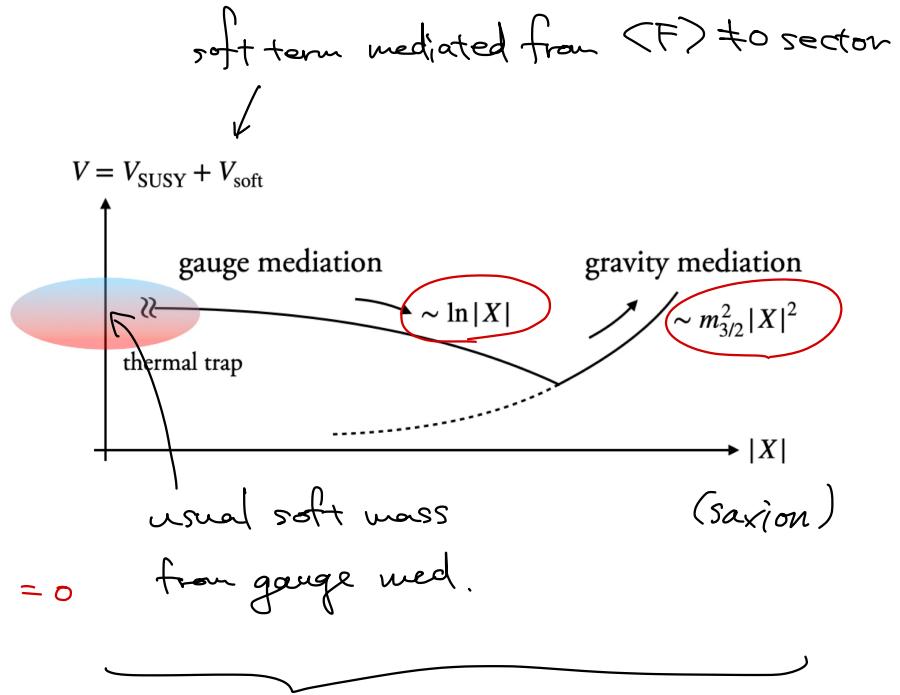
$$V_{\text{SUSY}} \supset \left| \frac{\partial W}{\partial X} \right|^2 + \left| \frac{\partial W}{\partial Q} \right|^2 + \left| \frac{\partial W}{\partial \bar{Q}} \right|^2 = |Q|^2 |\bar{Q}|^2 + |X|^2 (|Q|^2 + |\bar{Q}|^2)$$

↑
SUSY-conserving level

Where's saxion S ?



saxion = $|X|$ direction
along $|Q| = |\bar{Q}| = 0$



SUSY-conserving (eve)

After including ~~SUSY~~

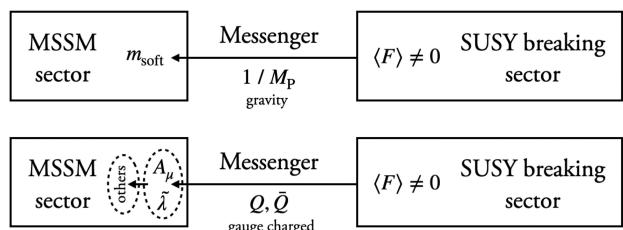
* Assume gauge mediation, $m_{\text{soft}} \sim \alpha \frac{\langle F_{\text{soft}} \rangle}{\mu_{\text{mess}}}$

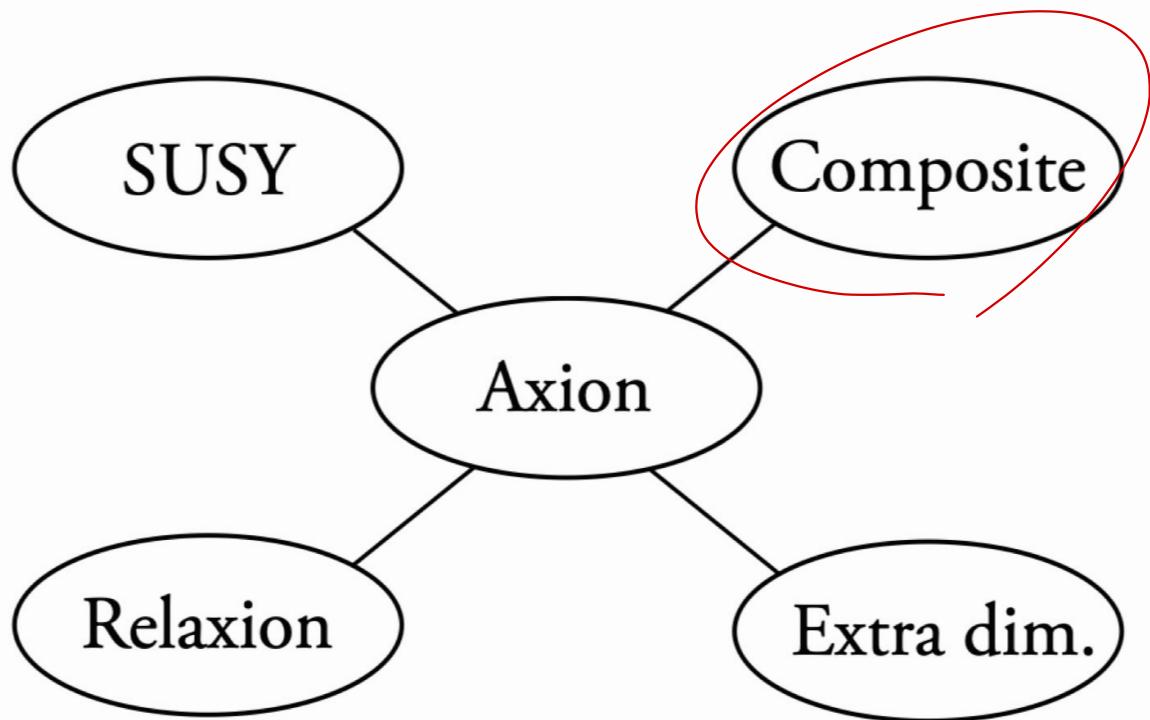
① At large $|X|$, $\sim \ln |X|$ term appears.

② We still have effect from gravity med. $m_{3/2}^2 |X|^2$

⇒ Balance b/wn. ① & ②
can create $f_a \sim 10^{10} \text{ GeV}$

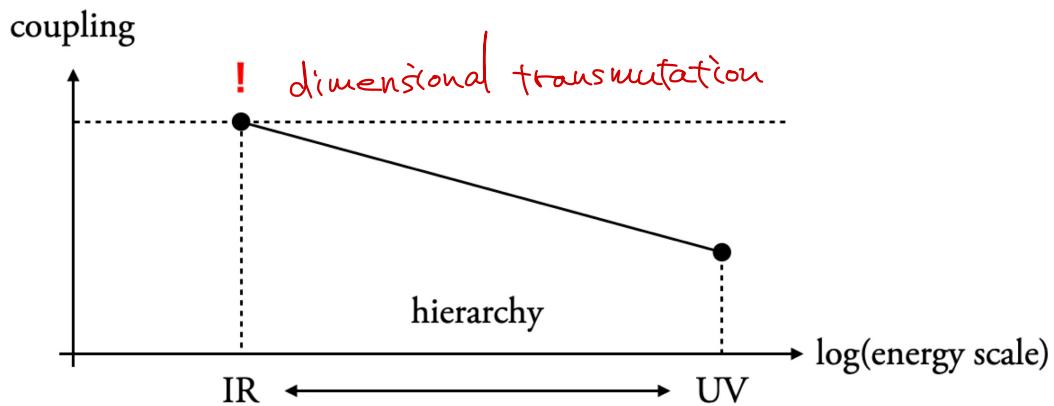
Asaka-Yamaguchi 9805779





3. Axion in composite Higgs

Motivation

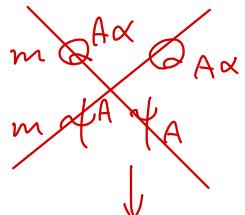


3-1. Composite axion (not composite Higgs)

Minimal (+ α) model

Philip's talk

minimal model



$$\mathcal{L} = i(Q^\dagger)_{A\alpha} \bar{\sigma}^\mu D_\mu Q^{A\alpha} + i(Q^\dagger)^{A\alpha} \bar{\sigma}^\mu D_\mu Q_{A\alpha} + i(\psi^\dagger)_\alpha \bar{\sigma}^\mu D_\mu \psi^\alpha + i(\psi^\dagger)^\alpha \bar{\sigma}^\mu D_\mu \psi_\alpha + \dots$$

	$Q^{A\alpha}$	$Q_{A\alpha}$	ψ^A	ψ_A
axicolor \rightarrow	SU(N) _a	\square	$\bar{\square}$	\square
our color \rightarrow	SU(3) _c	\square	$\bar{\square}$	1
axial symmetries	A_1	1	1	0
	A_2	0	0	1

\square : fundamental

$\bar{\square}$: anti-fund.

no $m Q^{A\alpha} Q_{A\alpha}$

no $m \psi^A \psi_A$

- Remember pions: $U(2)_L \times U(2)_R = SU(2)_V \times SU(2)_A \times U(1)_V \times U(1)_A$
- spontaneously broken \rightarrow pions
broken from the beginning

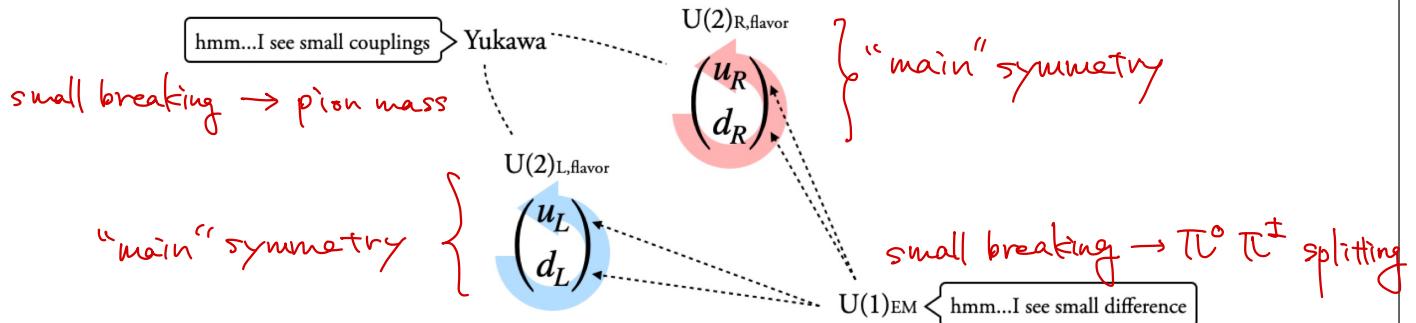


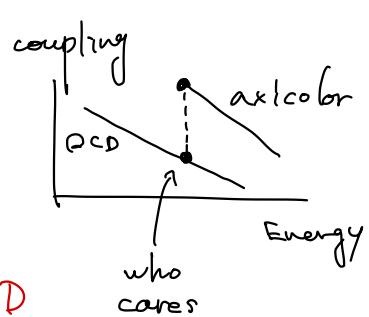
Figure 8: Approximate $U(2)_L \times U(2)_R$ and its small breaking seen by $U(1)_{EM}$ and Yukawas in QCD. On top of this, $U(1)_A$ is anomalous in reality.

- In the present model

$$q^A = \begin{pmatrix} Q^{A1} \\ Q^{A2} \\ Q^{A3} \\ \psi^A \end{pmatrix}, \quad q_A = \begin{pmatrix} Q_{A1} \\ Q_{A2} \\ Q_{A3} \\ \psi_A \end{pmatrix}$$

} feels QCD } does not feel QCD

slight difference
seen from axicolor.



$$U(4)_L \times U(4)_R = SU(4)_V \times SU(4)_A \times U(1)_V \times U(1)_A$$

direction of

$$4^2 - 1 = 15 \text{ NG bosons.}$$

$$15 = 8 + 3 + \bar{3} + 1$$

decomposition
under QCD

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \end{pmatrix}$$

left axial left

$$q^A \rightarrow e^{i\varepsilon} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} q^A$$

$$q_A \rightarrow e^{i\varepsilon} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} q_A$$

- After all, we have 2 rotations in the beginning $\begin{pmatrix} Q \\ \chi \end{pmatrix}$, and one combination does not feel axicolor.

$$\delta\mathcal{L} = -\frac{2N\epsilon_1}{32\pi^2} G_{\mu\nu}^{\alpha'} \tilde{G}^{\alpha'\mu\nu} - \frac{2(3\epsilon_1 + \epsilon_2)}{32\pi^2} F_{\mu\nu}^{A'} \tilde{F}^{A'\mu\nu}$$

$$J_{A_1-3A_2}^{5\mu} = (Q^\dagger)_{A\alpha} \bar{\sigma}^\mu Q^{A\alpha} + (Q^\dagger)^{A\alpha} \bar{\sigma}^\mu Q_{A\alpha} - 3(\psi^\dagger)_\alpha \bar{\sigma}^\mu \psi^\alpha - 3(\psi^\dagger)^\alpha \bar{\sigma}^\mu \psi_\alpha$$

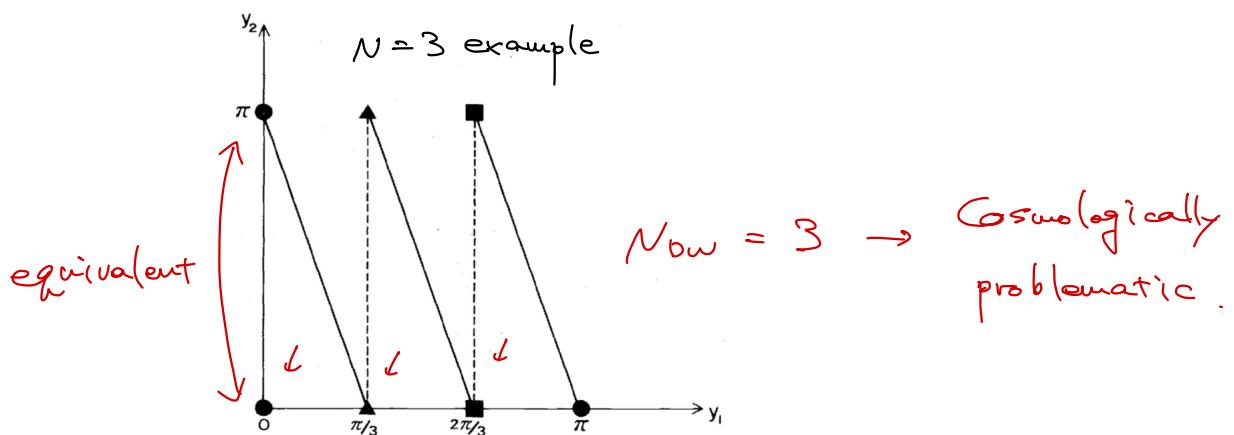
Strong CP is solved by this guy.

The other guy gets mass $\sim \mathcal{O}(\Lambda_{\text{axi}})$ from axicolor instanton.

- Low-energy Lagrangian = KSVE axion
- Problems?

Problems We learned that several problems exist in the minimal model.

- Asymptotic freedom: Is the axicolor $SU(N)_a$ really asymptotically free? In other words, does it really get strong at low energy?
- Domain walls: How can we design the model so that $N_{\text{DW}} = 1$?
- Unwanted relics: How can we remove the unwanted bound states of the axicolor?



Minimal + α model

Choi & Kim '85

	$Q^{A\alpha}$	$Q_{A\alpha}$	q^α	q_α	ψ^A	ψ_A	ν	ν'
$SU(2)_a$	$\square = \mathbf{2}$	$\square = \mathbf{2}$	1	1	$\square = \mathbf{2}$	$\square = \mathbf{2}$	1	1
$SU(3)_c$	$\square = \mathbf{3}$	$\bar{\square} = \bar{\mathbf{3}}$	$\square = \mathbf{3}$	$\bar{\square} = \bar{\mathbf{3}}$	1	1	1	1
A_1	1	1	-1	-1	0	0	0	0
A_2	0	0	0	0	1	1	-1	-1

$$\delta \mathcal{L} = -\frac{2\epsilon_1}{32\pi^2} G_{\mu\nu}^{\alpha'} \tilde{G}^{\alpha'\mu\nu} \stackrel{\text{QCD}}{=} -\frac{6\epsilon_1 + 2\epsilon_2}{32\pi^2} G_{\mu\nu}^{A'} \tilde{G}^{A'\mu\nu} \stackrel{\text{axicolor}}{=}$$

Cf. previously ...

+ for $N=2$.

$$\delta \mathcal{L} = -\frac{2N\epsilon_1}{32\pi^2} G_{\mu\nu}^{\alpha'} \tilde{G}^{\alpha'\mu\nu} - \frac{2(3\epsilon_1 + \epsilon_2)}{32\pi^2} F_{\mu\nu}^{A'} \tilde{F}^{A'\mu\nu}$$

$$\left\{ \begin{array}{l} 2\epsilon_1 = 2\pi \times \text{integer} \\ 6\epsilon_1 + 2\epsilon_2 = 2\pi \times \text{integer} \end{array} \right. \quad \begin{array}{l} \text{for } 0 \leq \epsilon_1 < \pi \\ \text{for } 0 \leq \epsilon_2 < \pi \end{array}.$$

$$\Rightarrow \text{Only } \epsilon_1 = 0, \epsilon_2 = 0 \Rightarrow N_{D\omega} = 1.$$

3 - 2. Axion in composite Higgs

Gheorghetta & Nguyen '20

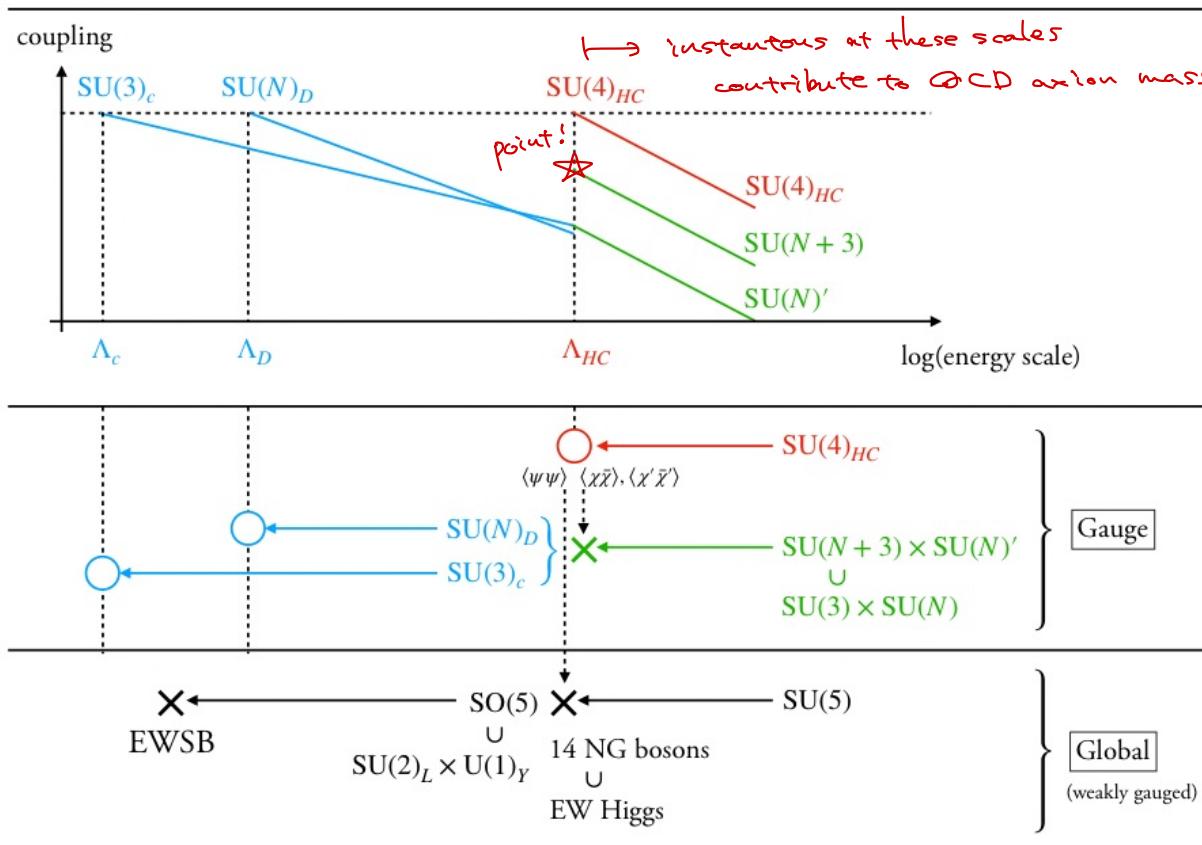


Figure 12: Symmetry breaking induced by the hypercolor confinement.

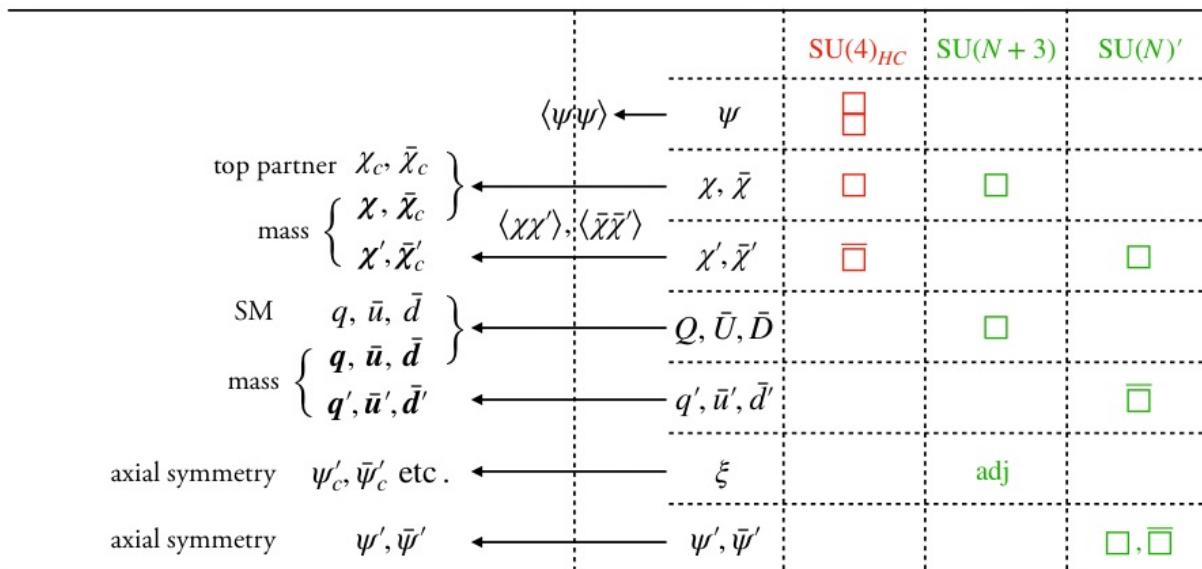


Figure 13: Fate of the particles.

	SU(4) _{HC}	SU($N + 3$)	SU(N)'	SU(5)
ψ	$\boxed{} = \mathbf{6}$	$\mathbf{1}$	$\mathbf{1}$	$\boxed{} = \mathbf{5}$
χ	$\boxed{} = \mathbf{4}$	$\boxed{} = \mathbf{N+3}$	$\mathbf{1}$	$\mathbf{1}$
χ'	$\boxed{} = \mathbf{4}$	$\mathbf{1}$	$\boxed{} = \mathbf{N}$	$\mathbf{1}$
$\bar{\chi}$	$\overline{\boxed{}} = \overline{\mathbf{4}}$	$\overline{\boxed{}} = \overline{\mathbf{N+3}}$	$\mathbf{1}$	$\mathbf{1}$
$\bar{\chi}'$	$\overline{\boxed{}} = \overline{\mathbf{4}}$	$\mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$	$\mathbf{1}$

Table 7: Charge assignments for particles relevant to the hypercolor confinement. They are all left-chiral. The first three groups are gauged, while the last is almost global, part of which is gauged. On top of this, χ and $\bar{\chi}$, and χ' and $\bar{\chi}'$, are assumed to have much smaller masses than Λ_{HC} .

	SU($N + 3$)	SU(N)'		SU(3) _c	SU(N) _D
Q	$\boxed{} = \mathbf{N+3}$	$\mathbf{1}$		$\boxed{} = \mathbf{3}$	$\mathbf{1}$
\bar{U}	$\overline{\boxed{}} = \overline{\mathbf{N+3}}$	$\mathbf{1}$		$\boxed{} = \overline{\mathbf{3}}$	$\mathbf{1}$
\bar{D}	$\overline{\boxed{}} = \overline{\mathbf{N+3}}$	$\mathbf{1}$		$\boxed{} = \mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$
\bar{q}'	$\mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$		$\overline{\boxed{}} = \overline{\mathbf{3}}$	$\mathbf{1}$
u'	$\mathbf{1}$	$\boxed{} = \mathbf{N}$		$\boxed{} = \mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$
d'	$\mathbf{1}$	$\boxed{} = \mathbf{N}$		$\overline{\boxed{}} = \overline{\mathbf{3}}$	$\mathbf{1}$
ξ	adj	$\mathbf{1}$		$\mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$
ψ'	$\mathbf{1}$	$\boxed{} = \mathbf{N}$		$\mathbf{1}$	$\mathbf{1}$
$\bar{\psi}'$	$\mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$		$\mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$

→

	SU(3) _c	SU(N) _D
q	$\boxed{} = \mathbf{3}$	$\mathbf{1}$
\mathbf{q}	$\mathbf{1}$	$\boxed{} = \mathbf{N}$
\bar{u}	$\overline{\boxed{}} = \overline{\mathbf{3}}$	$\mathbf{1}$
$\bar{\mathbf{u}}$	$\mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$
\bar{d}	$\overline{\boxed{}} = \overline{\mathbf{3}}$	$\mathbf{1}$
$\bar{\mathbf{d}}$	$\mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$
\bar{q}'	$\mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$
$\bar{\mathbf{u}}'$	$\mathbf{1}$	$\boxed{} = \mathbf{N}$
\bar{d}'	$\mathbf{1}$	$\boxed{} = \mathbf{N}$
ψ'_c	$\boxed{} = \mathbf{3}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$
$\bar{\psi}'_c$	$\overline{\boxed{}} = \overline{\mathbf{3}}$	$\boxed{} = \mathbf{N}$
λ_c	adj	$\mathbf{1}$
λ_D	$\mathbf{1}$	adj
ν'	$\mathbf{1}$	$\mathbf{1}$
ψ'	$\mathbf{1}$	$\boxed{} = \mathbf{N}$
$\bar{\psi}'$	$\mathbf{1}$	$\overline{\boxed{}} = \overline{\mathbf{N}}$

Table 8: Charge assignments for particles irrelevant to the hypercolor confinement. They are all left-chiral. The left and right tables are before and after the gauge breaking $\text{SU}(N + 3) \times \text{SU}(N)' \rightarrow \text{SU}(3)_c \times \text{SU}(N)_D$ caused by the hypercolor confinement in the ψ and χ sector.

Interesting point!

QCD axion does not follow $m_a \sim m_\pi f_\pi$
(solves strong CP)

$$\delta\mathcal{L} = -\frac{\Lambda_D^4}{2} \left((N+3) \frac{a_1}{f_D} + \frac{a_2}{f_D} \right)^2 - \frac{\Lambda_c^4}{2} \left((N+3) \frac{a_1}{f_D} \right)^2 - \boxed{\frac{\Lambda_I^4}{2} \left(\frac{a_2}{f_D} \right)^2}$$

reasonable reasonable ??

From small-scale instantons
above Λ_{HC}

* The original idea is from Patil's paper (805, 06465)

* Then what's new about this paper?

They combined the idea with composite Higgs framework.

