

N3LO CORRECTIONS TO NEUTRAL AND CHARGED CURRENT AT THE LHC



LOOPS AND LEGS IN QUANTUM FIELD THEORY 2022

Xuan Chen ITP, IAP, Karlsruhe Institute of Technology *Ettal, 26 April, 2022*



"THREE CLOUDS" IN PARTICLE PHYSICS SINCE 2021

Standard Mode

18.5

18.0

- ► Fermilab's Muon g-2 experiment
- ► Measure the precession difference of Muon in magnetic field
- \blacktriangleright Observe 4.2 σ deviation from SM
- Precision at 0.46 ppm

 $a_{\mu} = \frac{g - 2}{2} \begin{array}{c} a_{\mu}^{EXP} = 116592061(41) \times 10^{-11} \\ Phys.Rev.Lett. 126, 141801 \\ a_{\mu}^{SM} = 116591810(43) \times 10^{-11} \end{array}$ Phy. Reports 887 (2020) 1-116



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Science 376 (2022) 170-176 N3LO corrections to neutral and charged current at the LHC



► W boson mass measurements > CDFII full statistic result reports 7 σ

	C PERSONAL ST	SM	
1	D0 I	80478 ± 83	
1	CDF I	80432 ± 79	
	DELPHI	80336 ± 67	
	L3	80270 ± 55	
	OPAL	80415 ± 52	
	ALEPH	80440 ± 51	
	D0 II	80376 ± 23	
4	ATLAS	80370 ± 19	
	CDF II	80433 ± 9	
799	200 80000	80100 80200 80300 80400 80);
		W boson mass (MeV/c ²)	
Dist	ribution	W boson mass (MeV/c ²) W boson mass (MeV)	
Dist $n_{T}(e)$ $p_{T}^{\ell}(e)$ $n_{T}(\mu)$ $p_{T}^{\ell}(\mu)$ $p_{T}^{\ell}(\mu)$	ribution (\mathbf{r}, \mathbf{v}) (\mathbf{r}, \mathbf{v}) (\mathbf{r}, \mathbf{v}) (\mathbf{r}, \mathbf{v}) (\mathbf{r}, \mathbf{v}) (\mathbf{r}, \mathbf{v}) (\mathbf{r}, \mathbf{v}) (\mathbf{r}, \mathbf{v}) (\mathbf{r}, \mathbf{v})	W boson mass (MeV/c ²) W boson mass (MeV) $80,429.1 \pm 10.3_{stat} \pm 8.5_{syst}$ $80,411.4 \pm 10.7_{stat} \pm 11.8_{syst}$ $80,426.3 \pm 14.5_{stat} \pm 11.7_{syst}$ $80,446.1 \pm 9.2_{stat} \pm 7.3_{syst}$ $80,428.2 \pm 9.6_{stat} \pm 10.3_{syst}$ $80,428.9 \pm 13.1_{stat} \pm 10.9_{syst}$ $80,433.5 \pm 6.4_{stat} \pm 6.9_{syst}$	





"THREE CLOUDS" IN PARTICLE PHYSICS SINCE 2021

est of lepton universality in be	eauty-quark decays	#1		
LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Mar 22, 2021)				
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ATLAS, LHCb, CMS all have on-going analysis of W mass.

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N3LO corrections to neutral and charged current at the LHC

- ing the elephant with BSM free parameters
- ne "oblique corrections" S-T-U in vacuum polarisation:

$$= 4e^{2}[\Pi_{33}^{'}(0) - \Pi_{3Q}^{'}(0)]$$

= $\frac{e^{2}[\Pi_{11}(0) - \Pi_{33}(0)]}{\sin^{2}(\theta_{W})\cos^{2}(\theta_{W})m_{Z}^{2}}$
= $4e^{2}[\Pi_{11}^{'}(0) - \Pi_{33}^{'}(0)]$

eskin and Takeuchi `92



3

"THREE CLOUDS" IN PARTICLE PHYSICS SINCE 2021

Test of lepton universality in beauty-quark decays		#1	- Fitt
LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Mar 22, 2021)			
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Measurement of the Positive Muon Anomalous Magnetic Mor	ment to 0.46	5 #1	$\alpha S =$
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High-precision measurement of the W boson mass with the CDF Collaboration • T. Aaltonen (Helsinki U. and Helsinki Inst. of Phys.) et a	CDF II detect	tor #1	αU :
Published in: Science 376 (2022) 6589, 170-176		8288	P
pdf ∂ links ∂ DOI	€ 49 c	itations	
Statistics from iNSPIRE-HEP by 19-0	04-2022	BMWc'20	0 -
Eurthen our evine entel confirm	ation	Mainz'19	9 -
Further experimental commis	ation	FHM'19	9 -
Earmilab Dun 2 - Dun 5 analyzi	0	ETM'19	9 -
Ferminal Run 2 ~ Run 5 analysi	5	RBC 18	7
► I HCh Upgrade I (2025) and II (2030)		
EITCO Opgrade I (2023) and II (2030)	KNT'1	9
► ATLAS, LHCb, CMS all have on-	-going	CHHKS'1	9
analysis of W mass.	00		660
uan Chen (KIT)	NSIO	correction	ns to nu

ing the elephant with BSM free parameters ne "oblique corrections" S-T-U in vacuum polarisation: Best-fit Peskin-Takeuchi param

22

Carpenter, Murphy, Smylie

$$= 4e^{2}[\Pi'_{33}(0) - \Pi'_{3Q}(0)]$$
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eutral and charged current at the LHC



- Challenge experiment with better/alternative predictions
 - ► Lattice prediction of HVP in g-2
 - Improve template fit in CDFII (ResBos@NLO+NNLL)



PRECISION PREDICTIONS AT THE LHC

QCD improved parton model

 P_A

± 3-5 % a LHC energy

 $\sigma_{AB} = \sum_{ab} \int_{0}^{1} dx_{0} \int_{0}^{1} dx_{b} f_{a|A}$ Parton distribution functions
Surgy evolution from all Hard scattering (*Perturbative quantum field theory*) ± 10 % level! N3LO corrections to neutral and charged current at the LHC

 $x_a P_A$

 $f_{a|A}(x_a)$

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ANATOMY OF HARD SCATTERING $\hat{\sigma}_{ab}$

► All matrix elements have Infrared (IR) divergency **Single soft** gluon with momentum $p_i \rightarrow 0$:

> $|\mathscr{M}^{0}_{m+1}(\cdots,i,j,k,\cdots)|^{2} \to s_{ijk} |\mathscr{M}^{0}_{m}(\cdots,i,k,\cdots)|^{2} \quad \text{with Eikonal factor} \quad s_{ijk} = \frac{2s_{ik}}{s_{ii}s_{ik}}$ $\frac{1}{s_{cd}} + \frac{1}{s_{cd}s_{abc}} - \frac{4}{s_{abc}s_{bcd}} + \frac{2(1-\epsilon)}{s_{bc}} \left(\frac{s_{ab}}{s_{abc}} + \frac{s_{cd}}{s_{bcd}} - 1\right)^2$

Double soft gluon with momentum $p_b + p_c \rightarrow 0$:

$$s_{abcd} = \frac{2s_{ad}^2}{s_{ab}s_{bcd}s_{abc}s_{cd}} + \frac{2s_{ad}}{s_{bc}}\left(\frac{1}{s_{ab}s_{cd}} + \frac{1}{s_{ab}s_{dcd}}\right)$$

Various double unresolved limits: Double soft, triple collinear, soft and collinear (Gehrmann-De Ridder, Gehrmann, Glover `05) (Braun-White, Glover `22)

Explicit IR divergence for 1-loop matrix elements (Catani '98) $|\mathcal{M}_m^1\rangle = |\mathcal{M}_m^1(\mu^2; \{p\})\rangle = \mathbf{I}^{(1)}(\epsilon, \mu^2; \{p\}) |\mathcal{M}_m(\{p\})\rangle + |\mathcal{M}_m^{1, fin}(\mu^2; \{p\})\rangle$ **Explicit IR** divergence for 2-loop matrix elements $|\mathcal{M}_{m}^{2}\rangle = |\mathcal{M}_{m}^{2}(\mu^{2}; \{p\})\rangle = \mathbf{I}^{(1)}(\epsilon, \mu^{2}; \{p\}) |\mathcal{M}_{m}^{1}(\mu^{2}; \{p\})\rangle + \mathbf{I}^{(2)}(\epsilon, \mu^{2}; \{p\}) |\mathcal{M}_{m}^{0}(\mu^{2}; \{p\})\rangle + |\mathcal{M}_{m}^{2, fin}(\mu^{2}; \{p\})\rangle$ $\mathbf{I}^{(2)}(\epsilon,\mu^{2};\{p\}) = -\frac{1}{2}\mathbf{I}^{(1)}(\epsilon,\mu^{2};\{p\}) \left(\mathbf{I}^{(1)}(\epsilon,\mu^{2};\{p\}) + \frac{4\pi\beta_{0}}{\epsilon}\right) + \frac{e^{+\epsilon\psi(1)}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{2\pi\beta_{0}}{\epsilon} + K\right)\mathbf{I}^{(1)}(2\epsilon,\mu^{2};\{p\}) + \mathbf{H}^{(2)}(\epsilon,\mu^{2};\{p\}) + \mathbf{H}^{(2)}(\epsilon,\mu^{2$ Kinoshita-Lee-Nauenberg theorem (`64): all IR divergences cancel at each perturbation order of QFT N3LO corrections to neutral and charged current at the LHC

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Various N3LO unresolved limits: Triple soft (Catani, Colferai, Torrini `19) Quad collinear (Del Duca, Duhr et. al. `19), One-loop unresolved (Badger, Buciuni, Peraro `15, Catani, Cieri `21) Two-loop unresolved (Duhr, Gehrmann, Jaquier `15) etc.



- ► Assemble each $\hat{\sigma}_{ab}(x_a, x_b)$ at N3LO
 - ► Integration of QCD radiation with unitarity cuts
 - > Standard treatment of multi-loop calculations except elliptic integrals with $\tau = m^2/\hat{s}$ where $\hat{s} = x_a x_b s$

Application of ggF Higgs production ISTW08 68d $\mu = \mu_R = \mu_F \in [m_H/4, m_H]$

- ► Remarkable precision of the first N3LO XS (Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger `15 to `18)
- ► Available in public code iHixs 2 (Dulat, Lazopoulos, Mistlberger `18)
- ► Further application to bbF Higgs (Dulat, Lazopoulos, Mistlberger 19)

► VBF to Higgs and HH using DIS structure function (Dreyer, Karlberg 17 19)

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N3LO corrections to neutral and charged current at the LHC

-0.1

-0.2





> Use threshold expansion at different region of τ and truncate at sufficiently high orders. (Mistlberger `18) ► Use generalised power series ansatz to test the approximation and match coeff. of overlapping regions.

Not exact analytical solution of elliptic integrals but numerically precise enough for phenomenology













N3LO corrections to neutral and charged current at the LHC

 $pp \rightarrow W^{\pm}$

 $pp \rightarrow \gamma^*$

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► Method to control IR divergence?

Subtraction of IR divergence NNLO: Antenna subtraction (Gehrmann-De Ridder, Gehrmann et al. `05), STRIPPER (Czakon `10), ColourfullNNLO (Del Duca, Kardos et al. `16), Nested-soft-collinear subtraction (Caola, Melnikov, Rontsch `17), Analytic Sector Subtraction (Magnea, Maina, Pelliccioli, Signorile-Signorile, Torrielli, Uccirati 18), Geometric IR subtraction (Herzog `18)

► Slice IR sensitive phase space region (applicable to all orders)

qT slicing (Bozzi, Catani, De Florian, Grazzini et al. `06 `07), N-jettiness slicing (Boughezal, Liu, Petriello et al. `15)

> Projection to Born (applicable to all orders) (Cacciari, Dreyer, Karlberg, Salam, Zanderighi 15)

> Amplitude level removal et al. `20) Universal factorisation (Anastasiou, Haindl, Sterman, Yang, Zeng `18 to `20) *Xuan Chen (KIT)*

- NLO: di-pole (Catani, Seymour `96), FKS (Frixione et al. `95), Nagy-Soper (Nagy, Soper `07, Chung et al. '11, Bevilacqua et al. `13)

Loop-Tree Duality (Bierenbaum, Catani, Draggiotis et al. '08 to '10, Runkel, Szor et al. '19, Capatti, Hirschi et al. '19 '20, Aguilera-Verdugo N3LO corrections to neutral and charged current at the LHC



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 $\int_{d\Phi_{N+2}} d\hat{\sigma}_{NNLO}^S + \int_{d\Phi_{N+1}} d\hat{\sigma}_{NNLO}^T + \int_{d\Phi_N} d\hat{\sigma}_{NNLO}^U = 0$ $d\hat{\sigma}_{NNLO} = \int_{d\Phi_{N+2}} \left(d\hat{\sigma}_{NNLO}^{RR} - d\hat{\sigma}_{NNLO}^{S} \right)$ $+ \int_{d\Phi_{N+1}} (d\hat{\sigma}_{NNLO}^{RV} - d\hat{\sigma}_{NNLO}^{T})$ $+ \int_{d\Phi_N} (d\hat{\sigma}_{NNLO}^{VV} - d\hat{\sigma}_{NNLO}^{U})$

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$$d\sigma_{N^{k}LO}^{F} = \mathcal{H}_{N^{k}LO}^{F} \otimes d\sigma_{LO}^{F}\Big|_{\delta(\tau)} + \left[d\sigma_{N^{k-1}LO}^{F+jet}\right]_{\delta(\tau)}$$

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► Differential N3LO accuracy

- ► Projection to Born
- ► Jet production in DIS (NNLOJET) Currie, Gehrmann, Glover, Huss, Niehues `18 ► Higgs decay to bb (MCFM) Mondini, Schiavi, Williams `19 ► Higgs production via ggF (RapidiX+NNLOJET) XC, Gehrmann, Glover, Huss, Mistlberger, Pelloni `21 ►qT slicing
- ► Higgs production via ggF (HN3LO+NNLOJET) Cieri, XC, Gehrmann, Glover, Huss `18 ► Higgs pair production via ggF (with modified iHixs2) Chen, Li, Shuo, Wang `19 ► Drell-Yan production (NNLOJET) XC, Gehrmann, Glover, Huss, Yang, Zhu `21 ► Combined with resummation (N3LL at small qT)

Drell-Yan production (DYTurbo) Camarda, Cieri, Ferrera `21 (RadISH+NNLOJET) XC, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli 22

► Higgs production via ggF (SCETlib) Billis, Dehnadi, Ebert, Michel, Tackmann `21 *Xuan Chen (KIT)*





STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$ Differential N3LO predictions for neutral current production > Fully differential N3LO Drell-Yan production (via γ^*) (XC, Gehrmann, Glover, Huss, Yang, Zhu 21) > Apply qt-slicing at N3LO with SCET factorisation and expand to N3LO:

$$egin{split} rac{d^3\sigma}{dQ^2d^2ec{q_T}dy} &= \int rac{d^2b_{\perp}}{(2\pi)^2} e^{-iq_{\perp}\cdot b_{\perp}} \sum_q \sigma_{
m LO}^{\gamma^*} H_{qar{q}} iggl[\sum_k \int_{x_1}^1 rac{dz_1}{z_1} \mathcal{I}_{qk}\left(z_1, b_T^2, \mu
ight) f_{k/h_1}(x_1/z_1, \mu) \ & imes \sum_j \int_{x_2}^1 rac{dz_2}{x_2} \mathcal{I}_{ar{q}j}\left(z_2, b_T^2, \mu
ight) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}\left(b_{\perp}, \mu
ight) + (q \leftrightarrow ar{q}) iggr] + \mathcal{O}\left(rac{q_T^2}{Q^2}
ight) \end{split}$$

> All factorised functions are recently known up to N3LO: 1) 3-loop hard function $H_{a\bar{a}}^{(3)}$ (Gehrmann, Glover, Huber, Ikizlerli, Studerus `10) 2) Transverse-momentum-dependent (TMD) soft function $S(b_1, \mu)$ at α_s^3 (Li, Zhu `16) > Apply qt cut to factorise N3LO contribution into two parts:

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3) Matching kernel of TMD beam function I_{qk} at α_s^3 (Luo, Yang, Zhu, Zhu `19, Ebert, Mistlberger, Vita `20)

 $\mathrm{d}\sigma_{N^{3}LO}^{\gamma^{*}} = \left[\mathcal{H}^{\gamma^{*}} \otimes \mathrm{d}\sigma^{\gamma^{*}}\right]_{N^{3}LO} \Big|_{\delta(p_{T,\gamma^{*}})} + \left[\mathrm{d}\sigma_{NNLO}^{\gamma^{*}+jet} - \mathrm{d}\sigma_{N^{3}LO}^{\gamma^{*}}\right]_{p_{T,\gamma^{*}} > qt_{cut}} + \mathcal{O}(qt_{cut}^{2}/Q^{2})$



>Differential N3LO predictions for neutral current production $\mathcal{O}(\alpha \alpha_s^3)$

- > Computational setup for $pp \rightarrow \gamma^*$
 - > Fix Q value for γ^* at 100 GeV (NNLO and N3LO scale variations deviate)
 - Use central value of PDF4LHC15_nnlo_mc as benchmark input
 - $\sim \alpha_s(m_7) = 0.118$ with scale variation values calculated from LHAPDF
 - ► G_{μ} EW-scheme with fixed α value
 - $\mu_R = \mu_F = 100 \text{ GeV}$ for central QCD scale and use 7-point variations for uncertainty estimation
 - > Consider LO decay with $m_e = m_\mu = 0$
- > Apply p_{T,γ^*} > 0.25 GeV constrain for NNLO γ^* + *Jet* without jet definition
- ➤ Use SCET factorisation to integrate contributions below qt cut.
- Dedicated MC adaption to four sub-channels:

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gg, qqb = (qqb+qbq), qg = (qg+gq+qbg+gqb), qq = (qq+qbqb)N3LO corrections to neutral and charged current at the LHC



(identical setup in the inclusive calculation by Durh, Dulat and Mistlberger in Phys. Rev. Lett. 125 (2020) 17, 172001)





Xuan Chen (KIT)

XC, Gehrmann, Glover, Huss, Yang, Zhu Phys.Rev.Lett. 128 (2022) 5 N3LO corrections to neutral and charged current at the LHC





Xuan Chen (KIT)

XC, Gehrmann, Glover, Huss, Yang, Zhu Phys.Rev.Lett. 128 (2022) 5 N3LO corrections to neutral and charged current at the LHC



- Differential N3LO predictions for charged current production
 - > Fully differential N3LO W^{\pm} production (XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation)
 - Computational setup for LHC observables:
 - ► No constrain on W boson mass, $m_{l\nu} \in [0, +\infty]$
 - ► Use NNPDF31 nnlo PDFs with α_s values from LHAPDF.
 - ► G_{μ} EW-scheme with fixed α value.
 - > Dynamic QCD scale $\mu_R = \mu_F = m_{l\nu}$ with 7 variations.
 - > Consider LO decay with $m_e = m_\mu = 0$
 - > Apply $p_{T,l\nu}$ > 0.5 GeV constrain for NNLO W+jet
 - > Use SCET factorisation for $p_{T,l\nu} < 0.5$ GeV and to check asymptotic agreement of pT distribution.

> Dedicated MC adaption to six sub-channels (symmetric in rapidity distribution): gg, qqb = (qqb+qbq), qg = (qg+gq), qbg = (qbg+gqb), qq, qbqb*Xuan Chen (KIT)*



SCET+NNLOJET





Differential N3LO predictions for charged current production

$d\sigma_{FO}^{W^-}/d|y_{W^-}|$



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XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation N3LO corrections to neutral and charged current at the LHC





pp to W[±] SCET+NNLOJET

 $\sqrt{s} = 13 \text{ TeV}$





Differential N3LO predictions for charged current production $m_T = (E_T^l + E_T^{\nu})^2 - (\overrightarrow{p}_T^l + E_T^$

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$$+\overrightarrow{p}_{T}^{\nu})^{2} = \sqrt{2E_{T}^{l}E_{T}^{\nu}(1-\cos\phi)}$$

Breit-Wigner form (running decay width):

$$s^2 - m_W^2 + is^2 \Gamma_W / m_W$$

	(MeV)	Wmass	Wwidth
	PDG (2020)	80379 ± 12	2085 ± 42
1	CDFII	80433 ± 9	2089.5 ± 0.6
	L3	80270 ± 55	2180 ± 14



Differential N3LO predictions for charged current production

$$m_T = (E_T^l + E_T^{\nu})^2 - (\overrightarrow{p}_T^l + \overrightarrow{p}_T^{\nu})^2 = \sqrt{2E_T^l E_T^{\nu} (1 - \cos\phi)}$$



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$$m_T = (E_T^l + E_T^{\nu})^2 - (\overrightarrow{p}_T^l)^2$$



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- Differential N3LO predictions for neutral current production with fiducial cuts
 - Apply ATLAS fiducial cuts at 13 TeV
 - > Dynamical scale $\mu_F = \mu_R = \sqrt{m_{ll}^2 + p_T^{ll^2}}$

► $m_{ll} \in [66, 116] \text{ GeV}, |\eta^{l^{\pm}}| < 2.5$

- Symmetric cuts: $|p_T^{l^{\pm}}| > 27 \text{ GeV}$ Introduce power correction at $O(q_T^{cut}/m_{ll})$
- ► Solution:

> Apply Lorentz Boost below q_T^{cut} Buonocore, Rottoli, Kallweit, Wiesemann 21 Camarda, Cieri, Ferrera `21 ➤ Product cuts: $\sqrt{p_T^{l^+} p_T^{l^-}} > 27 \text{ GeV}$

 $\min\{p_T^{l^+}, p_T^{l^-}\} > 20 \text{ GeV}$ Salam, Slade `21 > Typical fiducial cuts for m_T^V , p_T^V in DY production > Large log terms appear in $p_T^l \sim m_V/2, m_T^V \sim 2 \times \min[p_T^l], p_T^V \sim 0$ *Xuan Chen (KIT)*





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[dd]

D0

NNPDF4.0 NNLO, 13 TeV, $pp \rightarrow Z/\gamma^* (\rightarrow \ell^+ \ell^-) + X$



 $p_T^{\rm cut} \, [{
m GeV}]$

XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli 22





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[qd]

6 1

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 $p_T^{\rm cut} \, [{\rm GeV}]$ XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli 22

$$\times \min[p_T^l], p_T^V \sim 0$$

sutral and charged current at the LHC

[qd]

6 1





[qd]

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 $p_T^{\rm cut} \, [{\rm GeV}]$ XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli 22

Require resummation at small p_T^V N3LO corrections to neutral and charged current at the LHC



► Differential N3LO predictions for neutral current production with fiducial cuts



Xuan Chen (KIT)

CONCLUSION AND OUTLOOK

- > Precision phenomenology could be the key to reveal new physics principles.
- > For theory predictions of LHC observables, there has been rapid progress in perturbative QCD calculations at NNLO and N3LO accuracy.
- ► Differential N3LO precision is now available for neutral and charged current production at the LHC.
- challenged at N3LO.
- stabilise the convergence of scale variations.
- key to make accurate interpretation of experiment data.

Xuan Chen (KIT)

N3LO corrections to neutral and charged current at the LHC

> Our standard methodology to estimate theoretical uncertainties via scale variation is

Resumed N3LO+N3LL predictions are essential to compare with data and help to

> EW, QCD-EW corrections are not included in this talk but equivalent important at the level of accuracy. How to combine different source of corrections/uncertainties is the



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Thank You for Your Attention



BACKUP SLIDES

- production with fiducial cuts
 - and matched to N3LO



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BACKUP SLIDES

Differential N3LL +NNLO predictions for charged current production with fiducial cuts

- ► Precise W measurement with calibration against Z.
- ► Improved QCD uncertainties through out pT.
- ► Different EW and QCD-EW correction between Z and W are not yet considered.

Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Walker `19

Xuan Chen (KIT)



N3LO corrections to neutral and charged current at the LHC

BACKUP SLIDES

- ► Budget of theoretical uncertainties ► Dominant theory error of $\sigma_{N^3LO}^{pp \to H}$ from PDFs.
 - ► EW corrections especially $\alpha_s \alpha$ corrections become relevant and even dominant.
 - ► Time to reflect all approximations being involved: 5-flavour, Heavy Top limit, running of m_t, m_W etc. ATLAS (7 TeV, 4.6 fb⁻¹)

F. Dulat, A. Lazopoulos, B. Mistlberger 2018
 Fiducial cuts introduce linear power correction due to illy defined region for QCD factorisation
 The effect can be dominant at N3LO precision
 New ideas emerge recently to rescue
 Mathematical and charged current at the LHC

- Ideal for legacy experimental analysis.

Xuan Chen (KIT)

► Defiducialization (A. Glazov 2020) N3LO corrections to neutral and charged current at the LHC

