

BERNHARD MISTLBERGER



COLLINEAR EXPANSION OF CROSS SECTIONS: TRICKS, RESULTS AND PHENOMENOLOGY

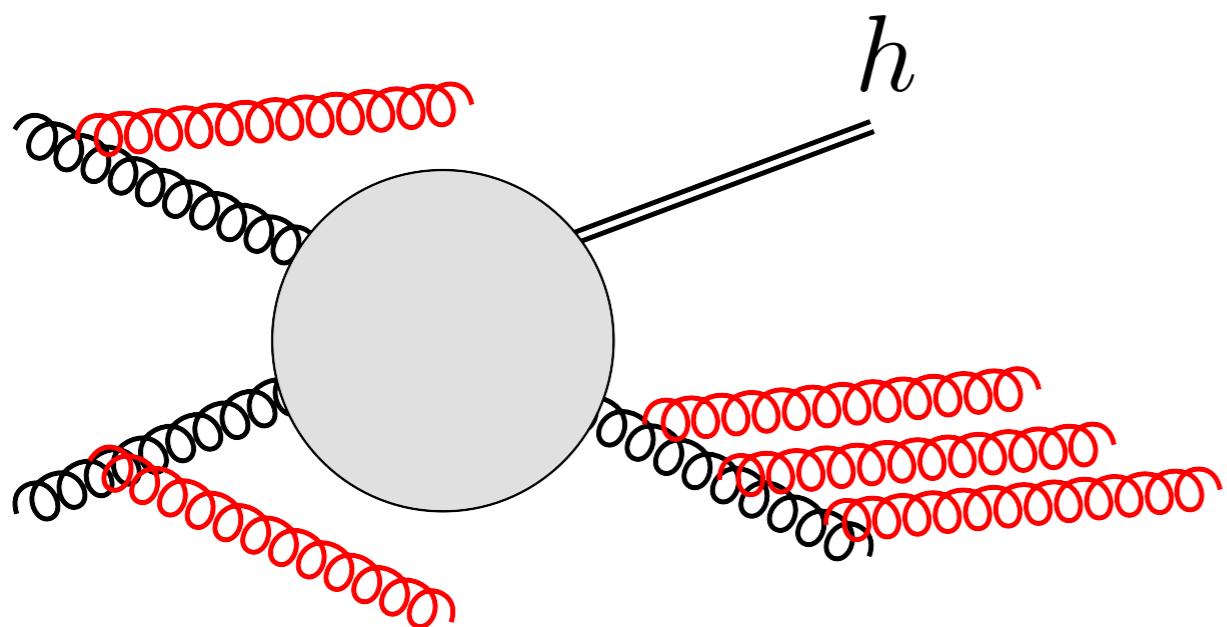
And with **Markus A. Ebert and Gherardo Vita**

[arXiv:2006.03055](https://arxiv.org/abs/2006.03055) [arXiv:2006.03056](https://arxiv.org/abs/2006.03056) [arXiv:2006.05329](https://arxiv.org/abs/2006.05329) [arXiv:2012.07853](https://arxiv.org/abs/2012.07853) [arXiv:2012.07859](https://arxiv.org/abs/2012.07859)

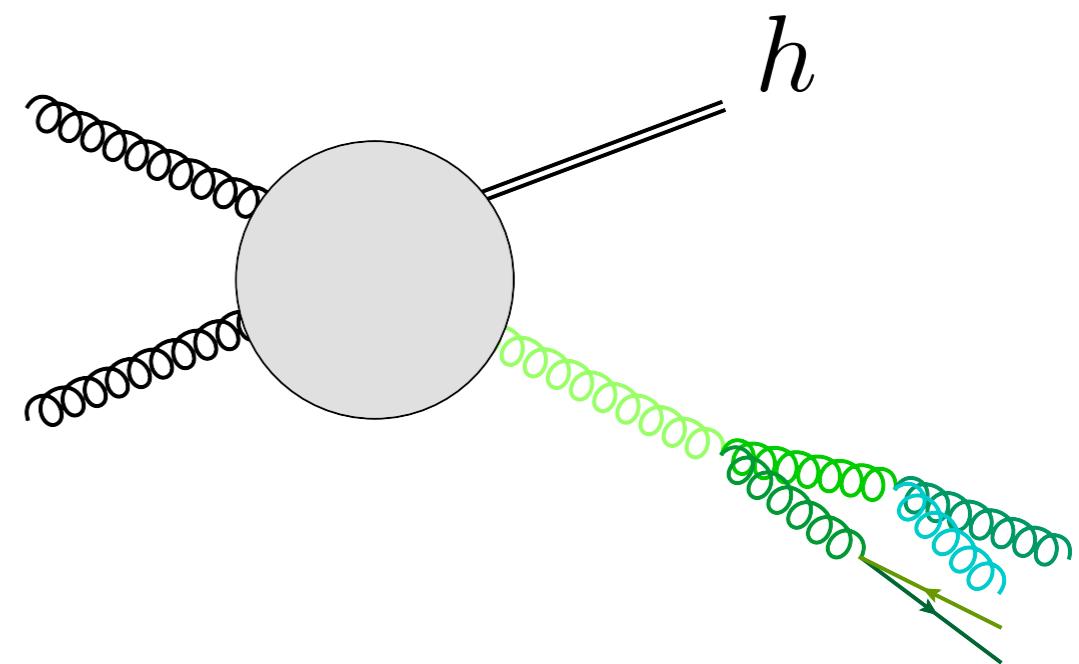
Introduction: Interesting Kinematic Limits

- ▶ Hard scattering processes are described in terms of the scattering of particles.

Soft Particles



Collinear Particles

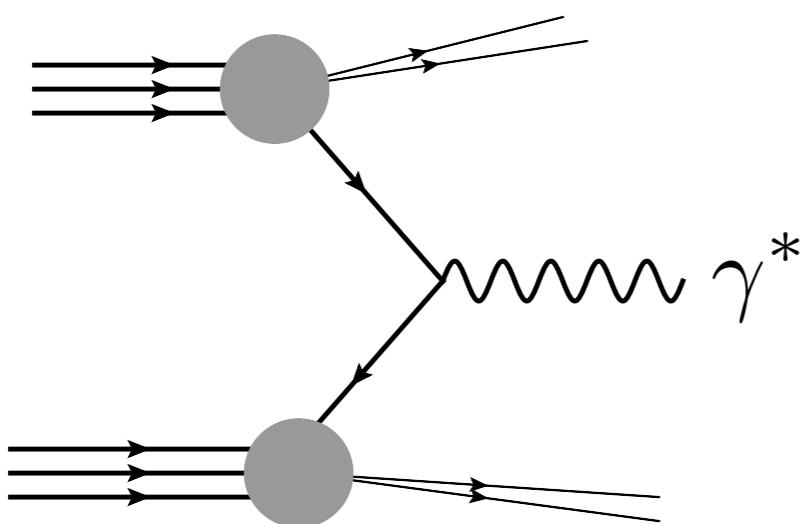


- ▶ Breakdown of perturbation theory in limit of small particle energies or angles.

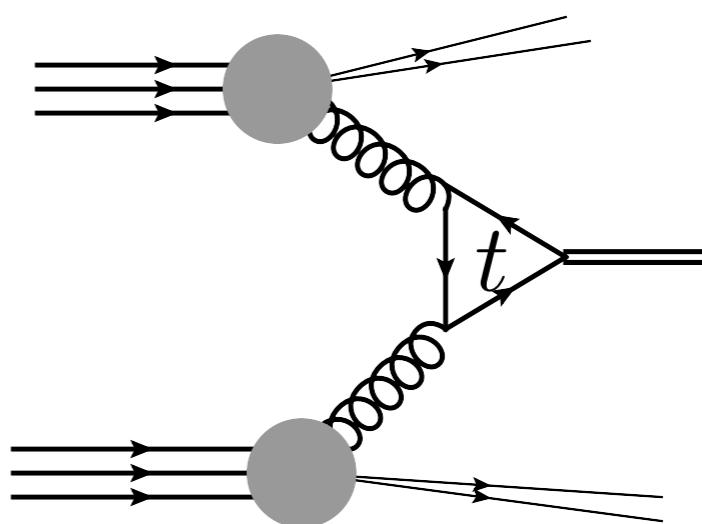
Motivation: Improve our Understanding and our Ability to make Predictions

- ▶ Improve fixed order precision predictions by obtaining ingredients and understanding for subtraction schemes.
- ▶ Learn about structure of kinematic limits.
- ▶ Obtain universal quantities for resummed cross sections.
- ▶ Find new ways to approximate perturbative cross sections.
- ▶ Study the universal structure of QCD beyond the leading term in kinematic expansions (beyond leading power)
- ▶ ...

Drell - Yan

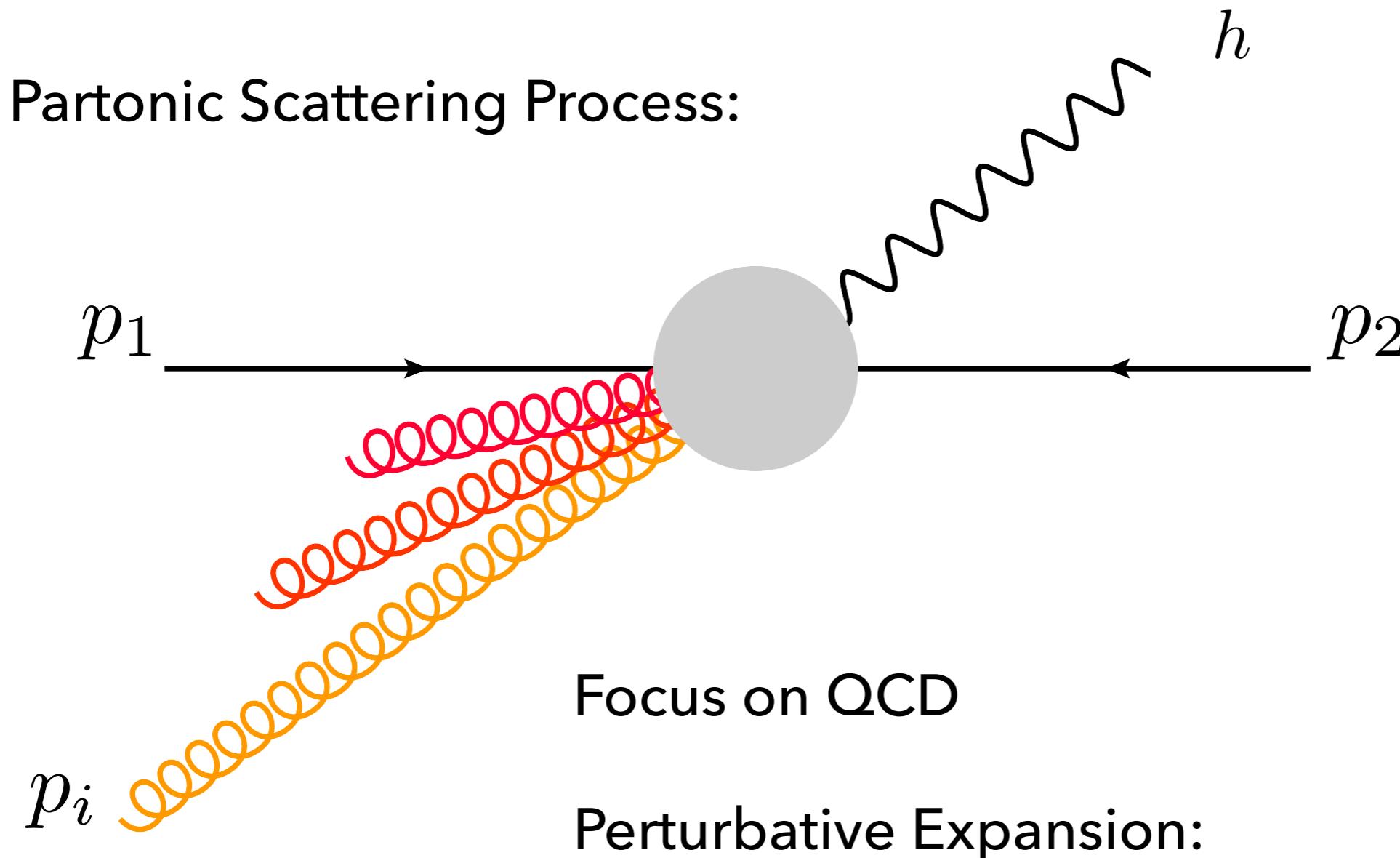


Higgs



- ▶ Having these specific processes in mind.
- ▶ Applicable to much larger class of processes.

$\gamma^* \ WZ \ H \ bbH \ H^* \ G \ WH \ ZH$

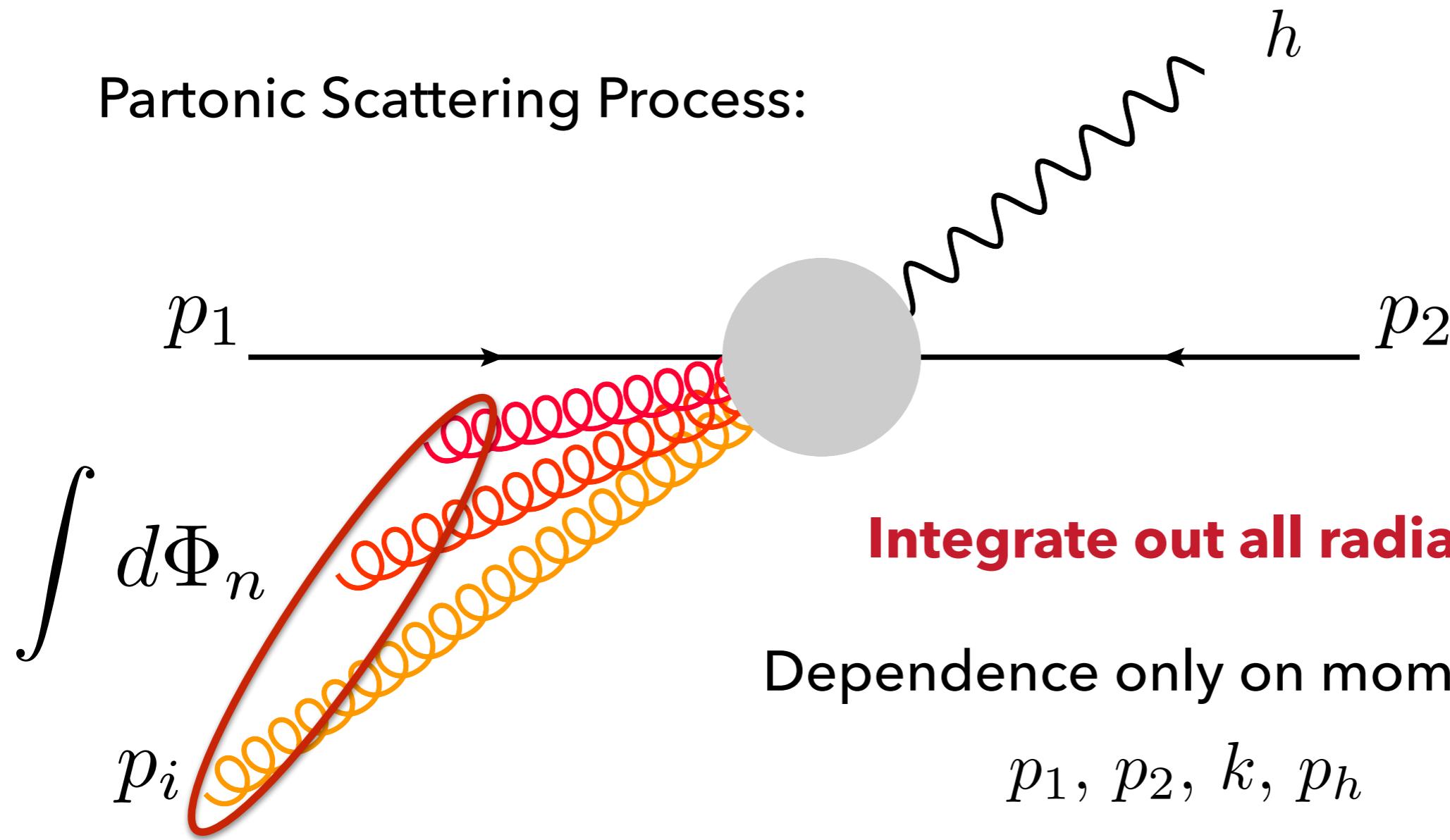


$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \dots$$

INTEGRATE OUT RADIATION

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Partonic Scattering Process:



$$k = \sum_i p_i$$

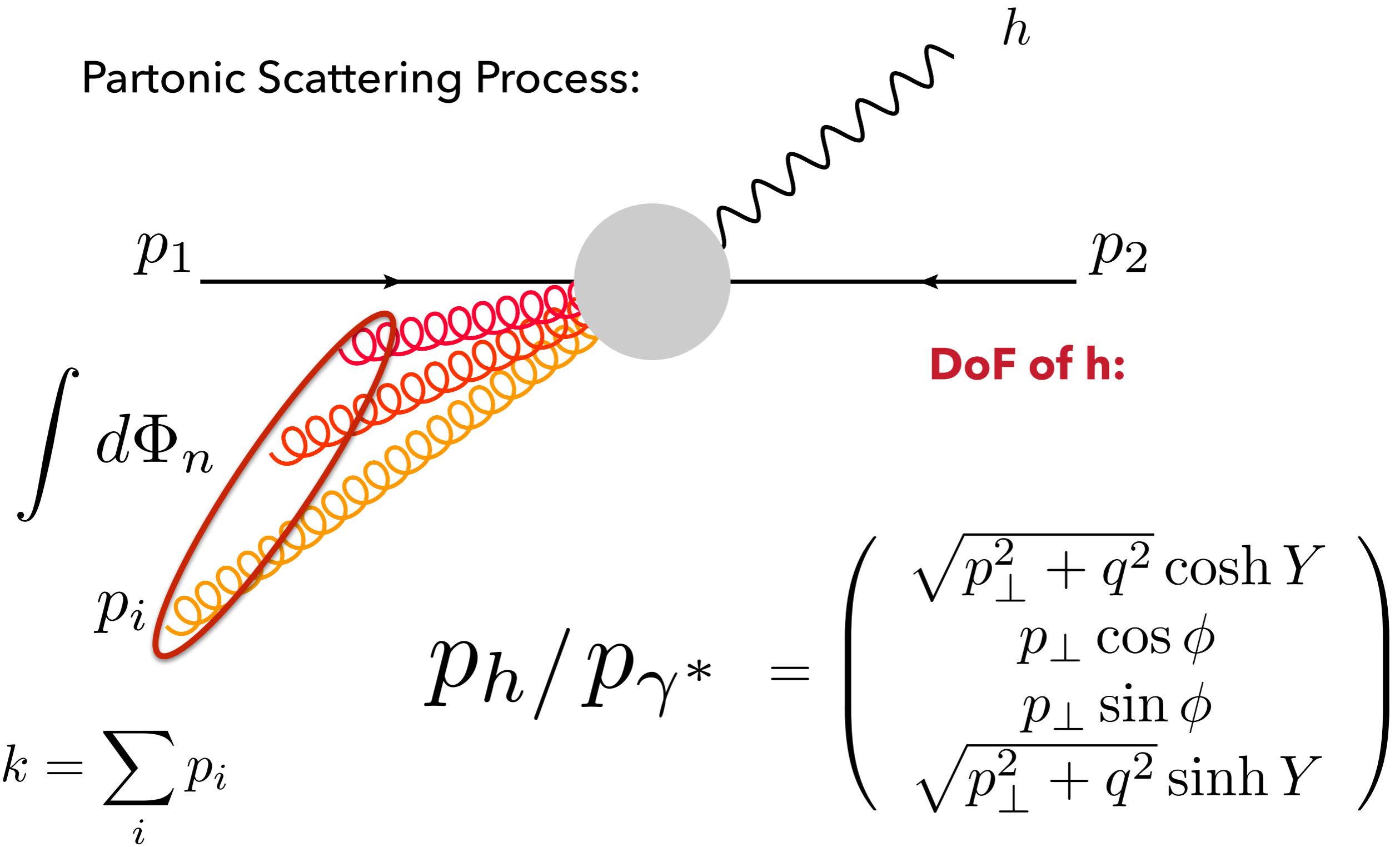
Momentum conservation:

$$p_1 + p_2 + p_h + k = 0$$

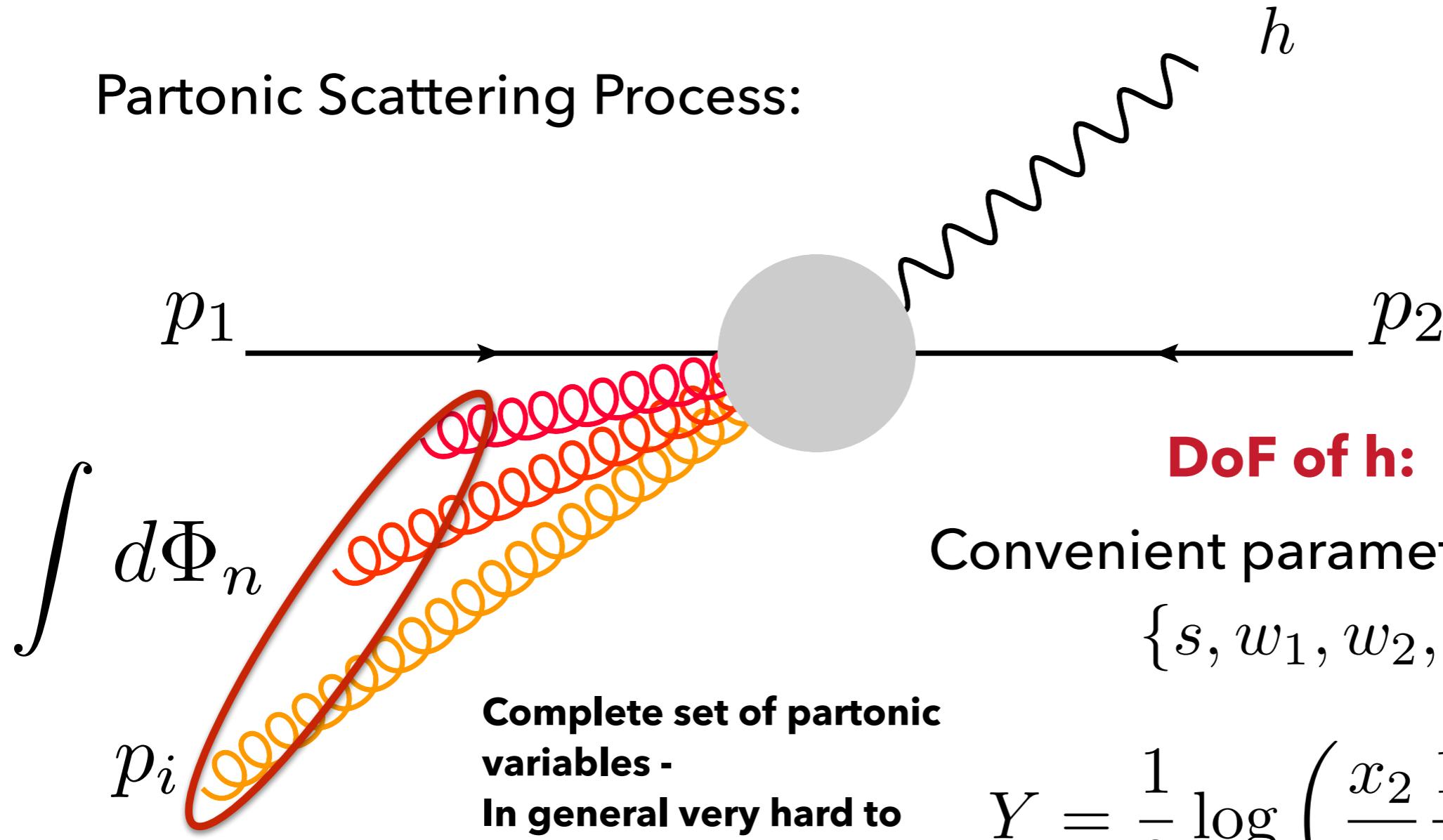
INTEGRATE OUT RADIATION

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Partonic Scattering Process:



Partonic Scattering Process:



DoF of h :

Convenient parametrisation

$$\{s, w_1, w_2, x\}$$

$$Y = \frac{1}{2} \log \left(\frac{x_2}{x_1} \frac{1-w_1}{1-w_2} \right)$$

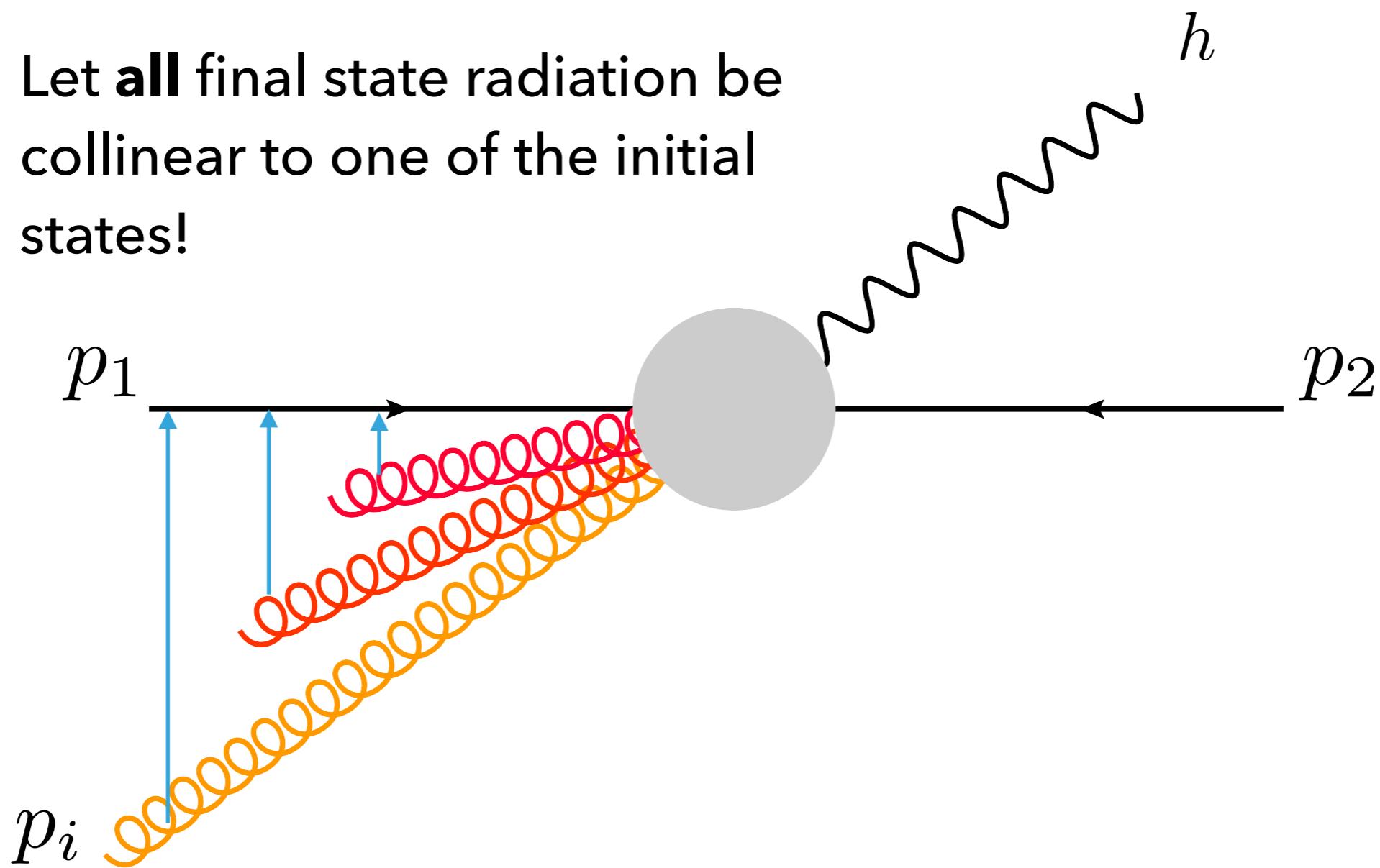
$$k = \sum_i p_i$$

$$s = (p_1 + p_2)^2$$

$$p_{\perp}^2 = sw_1w_2(1-x)$$

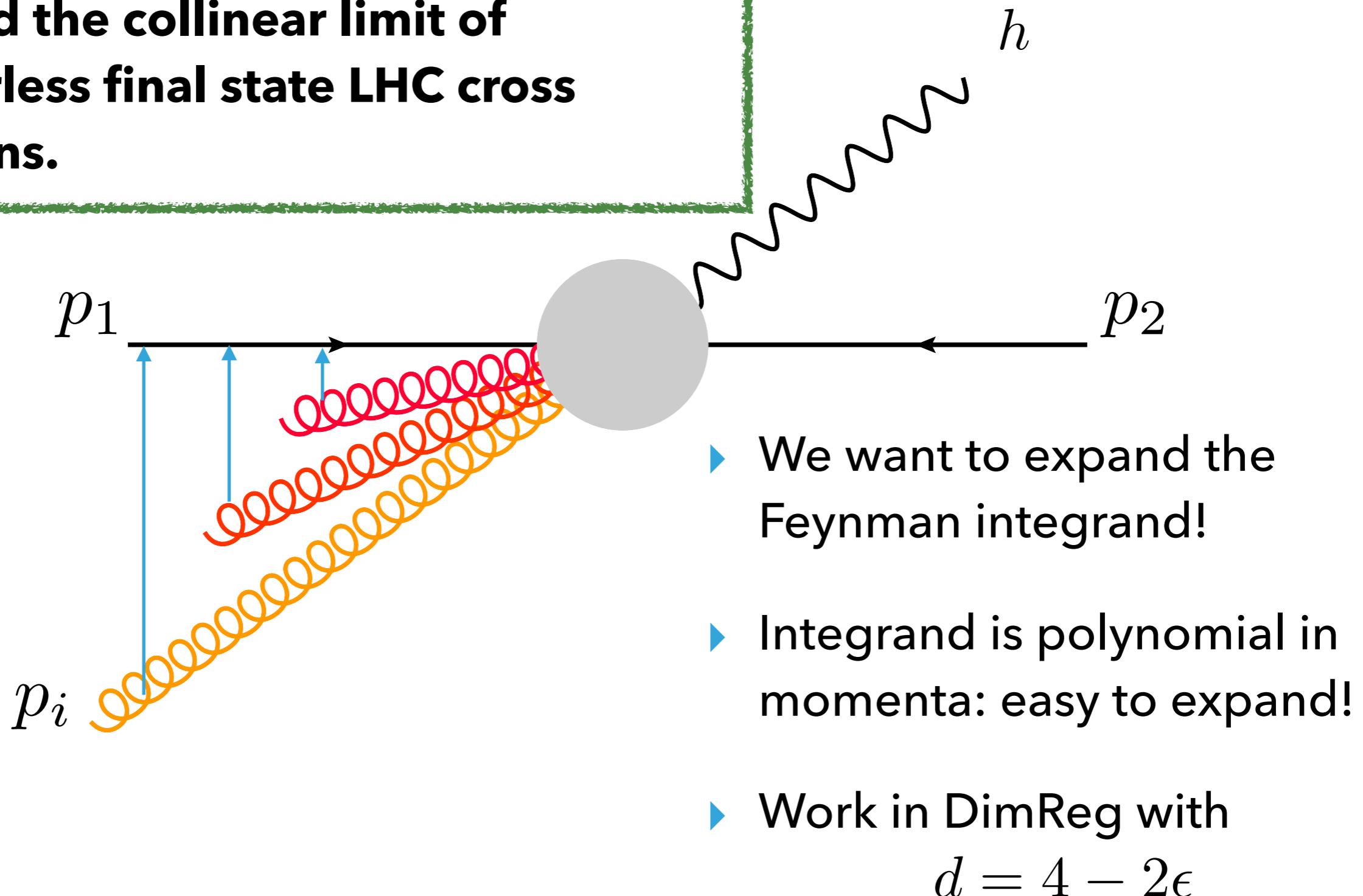
EXPANDING AROUND COLLINEAR LIMITS

Let **all** final state radiation be collinear to one of the initial states!



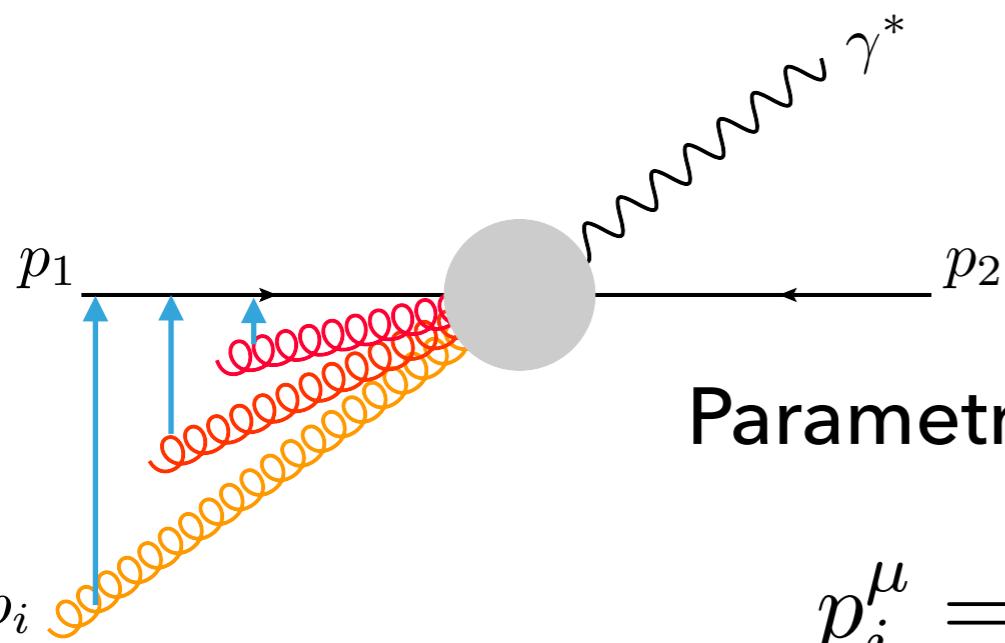
Goal:

**Formulate a systematic expansion
around the collinear limit of
colourless final state LHC cross
sections.**



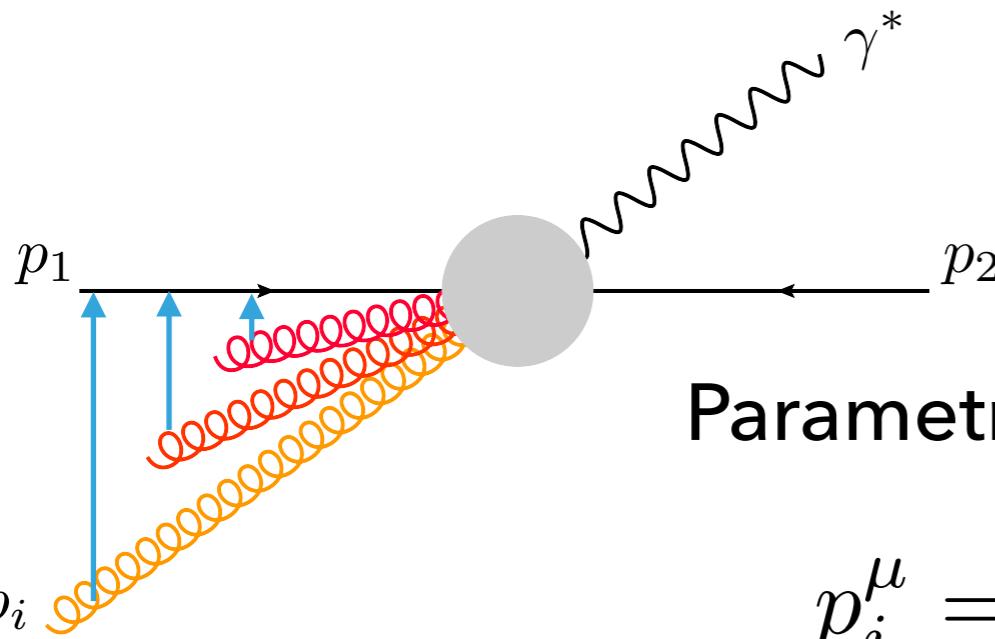
RESCALING TRANSFORMATION

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Parametrise all final state momenta.

$$p_i^\mu = \frac{(2p_2 p_i)}{s} p_1^\mu + \frac{(2p_1 p_i)}{s} p_2^\mu + p_{i,\perp}^\mu.$$



Parametrise all final state momenta.

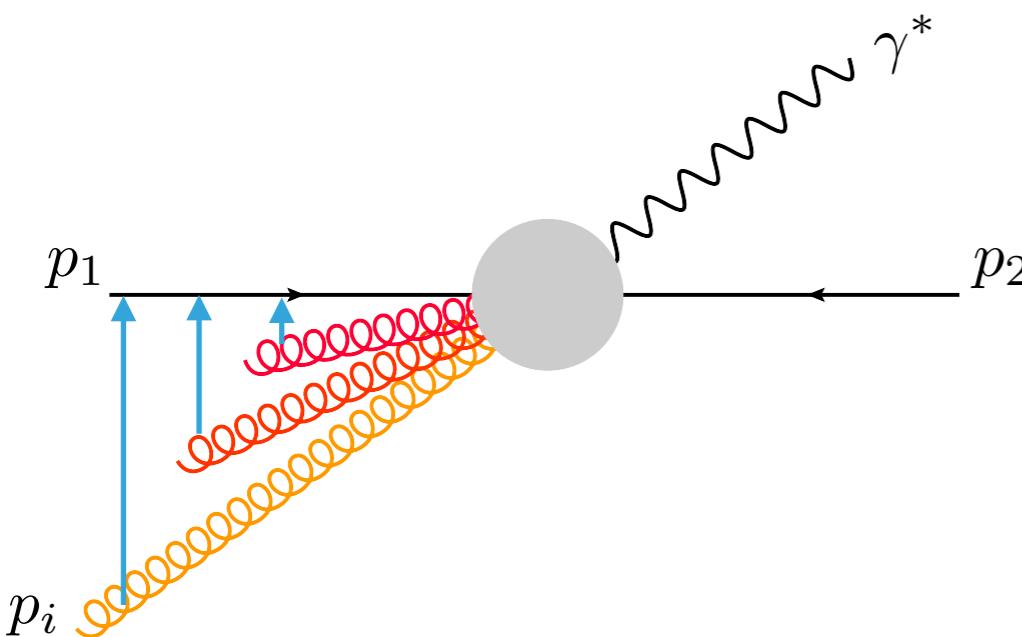
$$p_i^\mu = \frac{(2p_2 p_i)}{s} p_1^\mu + \frac{(2p_1 p_i)}{s} p_2^\mu + p_{i,\perp}^\mu.$$

Define a rescaling transformation.

$$p_i \rightarrow \lambda \frac{(2p_2 p_i)}{s} p_1^\mu + \frac{(2p_1 p_i)}{s} p_2^\mu + \sqrt{\lambda} p_{i,\perp}^\mu.$$

Artificial, small expansion parameter: λ

Same as for SCET / CSS / Splitting functions.



Transformation on variables:

$$s \rightarrow s$$

$$x \rightarrow x$$

$$w_1 \rightarrow w_1$$

$$w_2 \rightarrow \lambda w_2$$

Rescaling transformation.

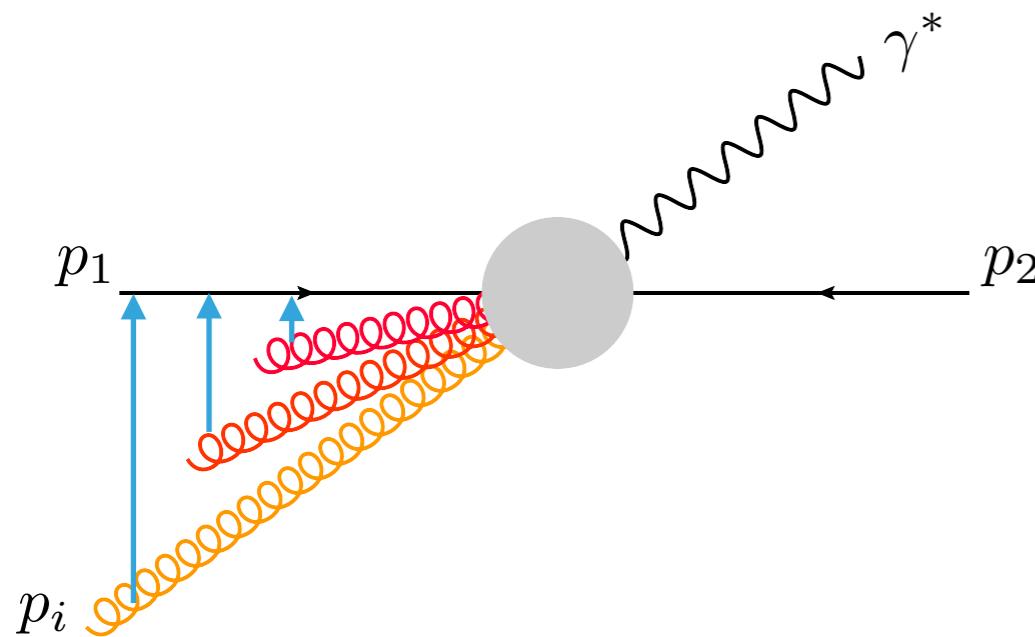
$$p_i \rightarrow \lambda \frac{(2p_2 p_i)}{s} p_1^\mu + \frac{(2p_1 p_i)}{s} p_2^\mu + \sqrt{\lambda} p_{i,\perp}^\mu.$$

Variables:

$$w_i = -\frac{2p_i k}{2p_1 p_2} \quad x = \frac{k^2}{sw_1 w_2}$$

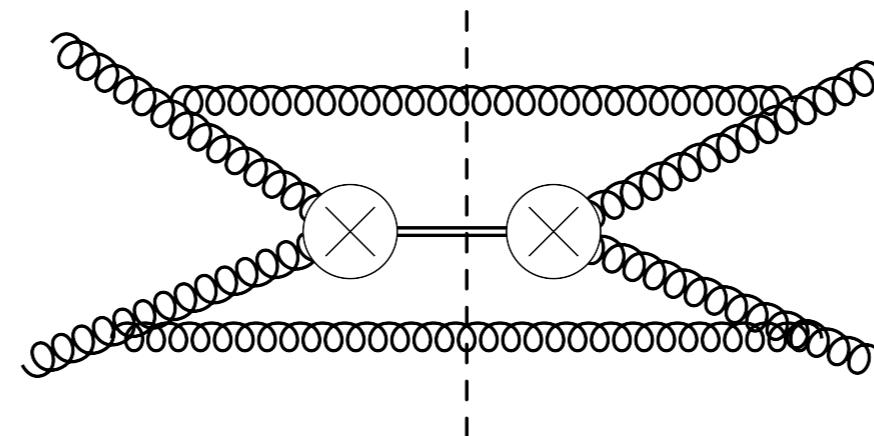
At the integrated level the expansion will automatically yield an expansion in

w_2



- ▶ We want to expand the Feynman integrand!
- ▶ Integrand is polynomial in momenta: easy to expand!

- ▶ Example: Phase Space Integrals:



PHASE SPACE INTEGRALS

- ▶ A simple double-real phase space integral:

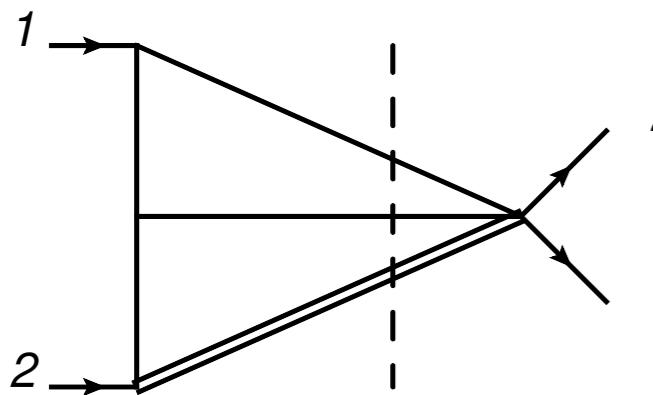
$$\int d\Phi_2^{\text{diff}} \frac{1}{(p_1 + p_3)^2(p_1 + p_3 + p_4)^2}$$

- ▶ One propagator rescales non-trivially

$$\begin{aligned} \frac{1}{(p_1 + p_3 + p_4)^2} &= \frac{1}{2p_1p_3 + 2p_1p_4 + 2p_3p_4} \rightarrow \frac{1}{2p_1p_3 + 2p_1p_4 + 2\lambda p_3p_4} \\ &= \sum_{i=0}^{\infty} (-\lambda)^i \frac{(2p_3p_4)^i}{(2p_1p_3 + 2p_1p_4)^{1+i}}. \end{aligned}$$

PHASE SPACE INTEGRALS

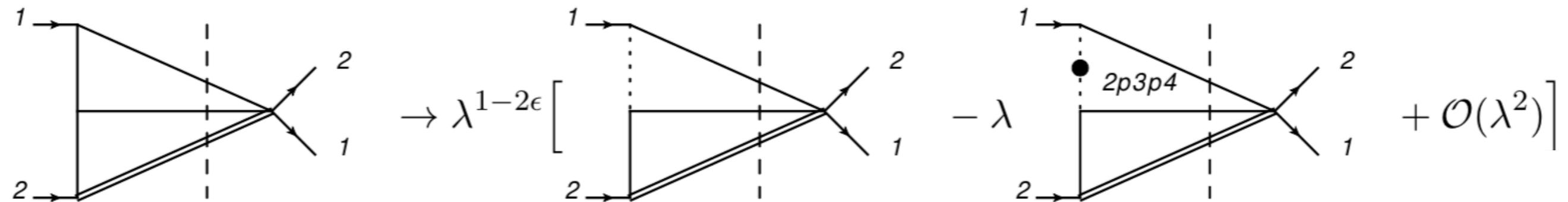
- ▶ Expanding our integral



$$\rightarrow \sum_{i=0}^{\infty} (-\lambda)^{1+i-2\epsilon} \int d\Phi_2^{\text{diff}} \frac{(2p_3 p_4)^i}{(p_1 + p_3)^2 (2p_1 p_3 + 2p_1 p_4)^{1+i}}.$$

~Collinear Wilson line!

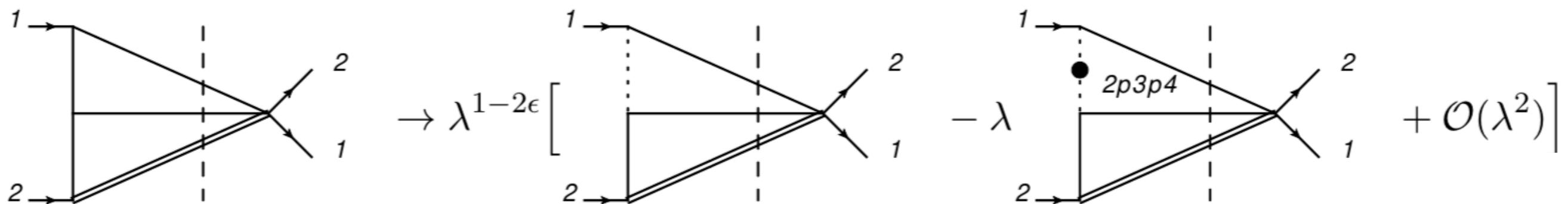
- ▶ Diagrammatic representation for terms in our expansion



$$\rightarrow \lambda^{1-2\epsilon} \left[\dots \right] + \mathcal{O}(\lambda^2)$$

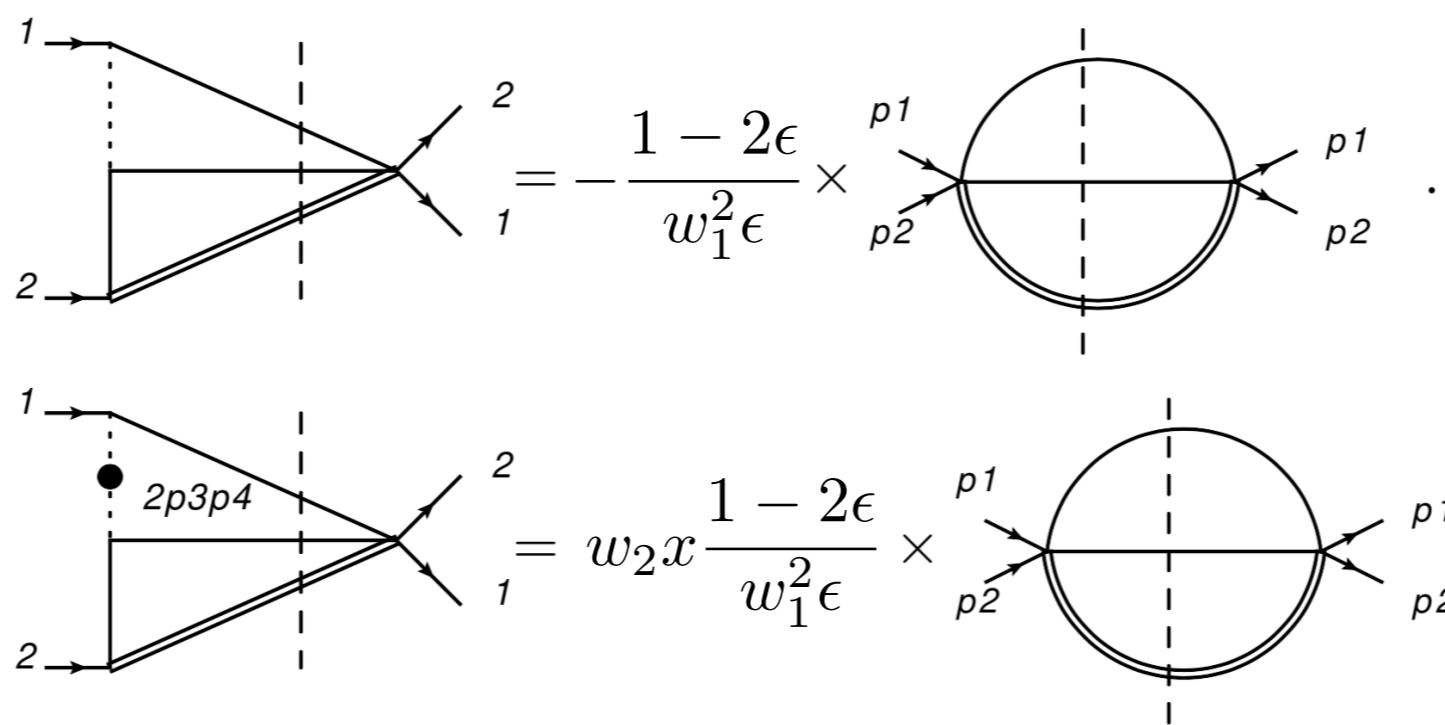
PHASE SPACE INTEGRALS

- Diagrammatic representation for terms in our expansion



$$\lambda^{1-2\epsilon} \left[\text{dashed rectangle} - \lambda \text{ (solid rectangle with dot)} \right] + \mathcal{O}(\lambda^2)$$

- IBP identities!



$$\text{rectangle} = - \frac{1-2\epsilon}{w_1^2 \epsilon} \times \text{circle}$$

$$\text{rectangle with dot} = w_2 x \frac{1-2\epsilon}{w_1^2 \epsilon} \times \text{circle}$$

PHASE SPACE INTEGRALS

- ▶ Series expansion for our example

$$\rightarrow -\frac{1-2\epsilon}{w_1^2 \epsilon} \lambda^{1-2\epsilon} \times [1 + \lambda w_2 x + \mathcal{O}(\lambda^2)]$$

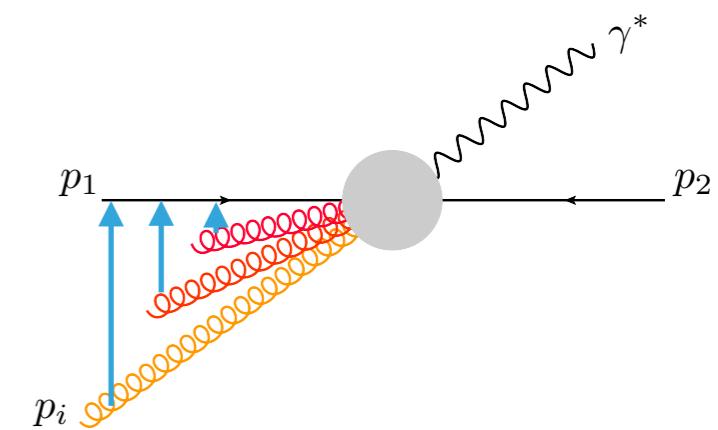
- ▶ Indeed matches the exact result!

$$= -\frac{1-2\epsilon}{w_1^2 \epsilon} \frac{1}{1-w_2 x}$$

- ▶ Loops: Method of Regions

SUMMARY

- ▶ Method for the expansion of cross sections around the collinear limit.
- ▶ Parametric expansion in terms of W_2
- ▶ Systematically improvable.
- ▶ Expansion at the integrand level.
- ▶ Simplification of individual terms in the expansion.
(IBPs, Differential Eqs., etc.)
- ▶ Higher order terms contain the same functions as first order terms and are easily relatable to those.

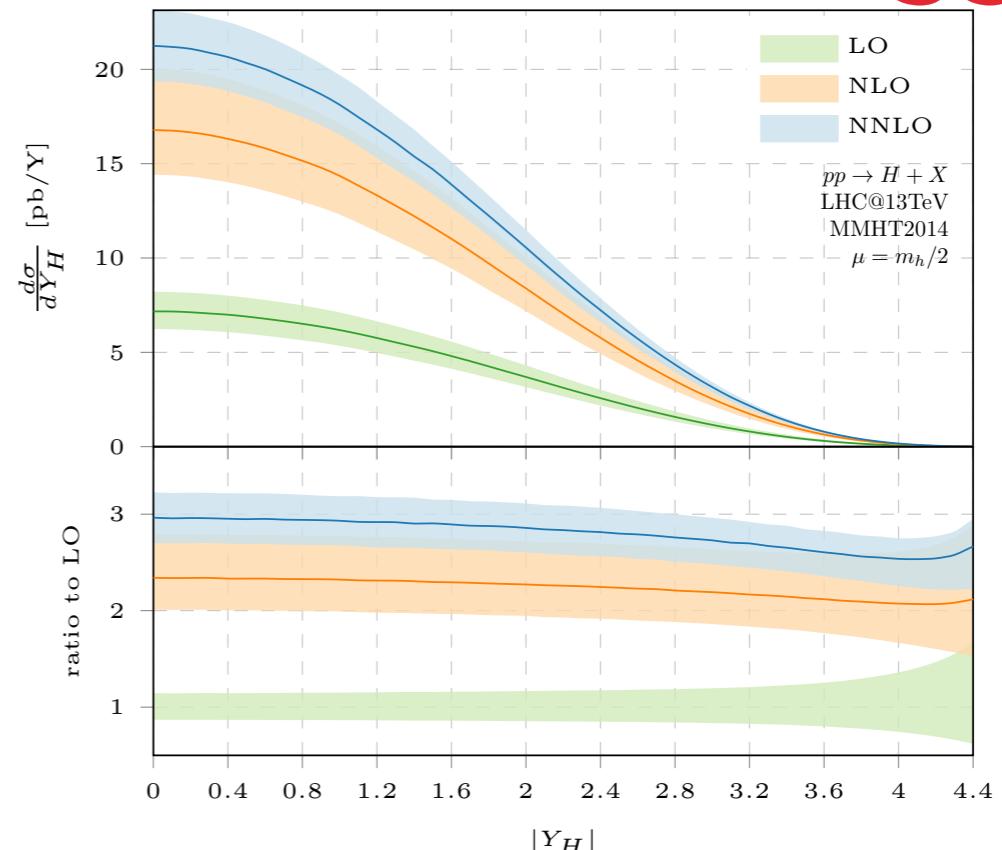


RAPIDITY DISTRIBUTION

$$\frac{\partial \sigma}{\partial Y}$$

- ▶ The differential observable!

Gluon-Fusion: Higgs



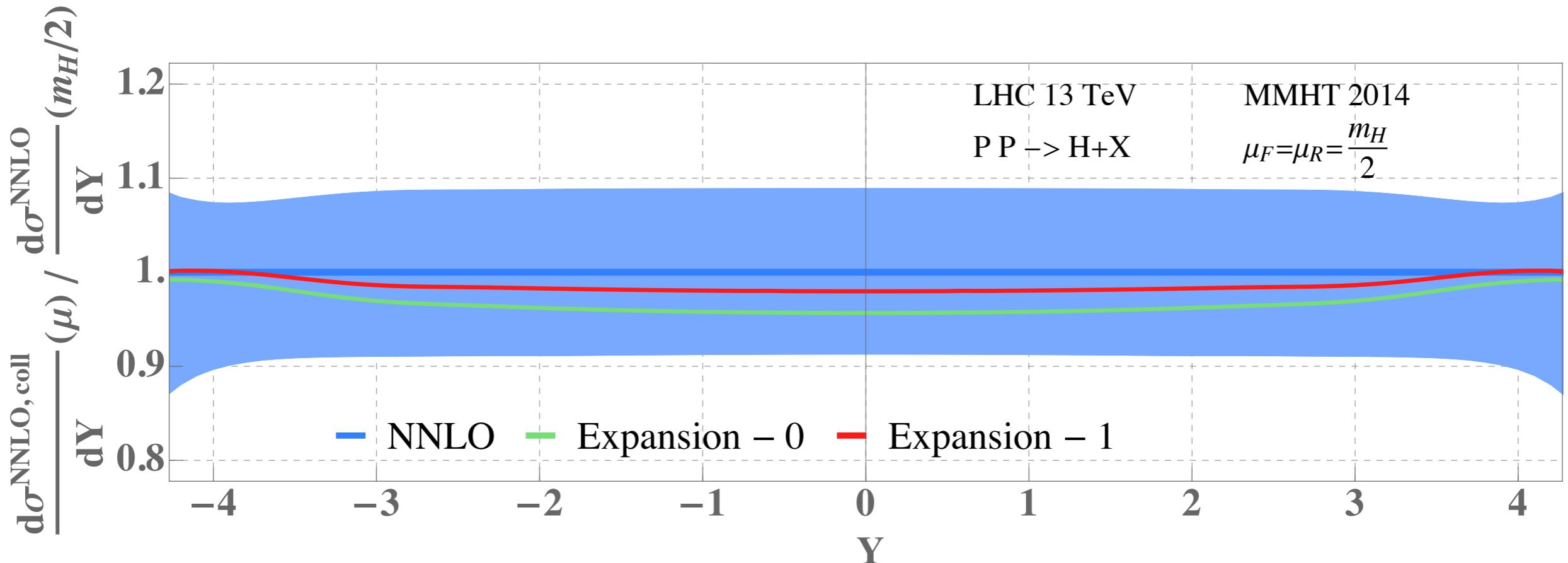
Expanded the NNLO partonic cross section to second power in collinear limit.

$$\frac{d\eta_{ij}^{R, \text{approx.}}(z_1, z_2)}{dQ^2 dY} = \left. \frac{d\eta_{ij}^R}{dQ^2 dY} \right|_{\bar{z}_2 \sim \lambda^2} + \left. \frac{d\eta_{ij}^R}{dQ^2 dY} \right|_{\bar{z}_1 \sim \lambda^2} - \left. \frac{d\eta_{ij}^R}{dQ^2 dY} \right|_{\bar{z}_{1,2} \sim \lambda^2} + \mathcal{O}(\lambda^2).$$

neglect:

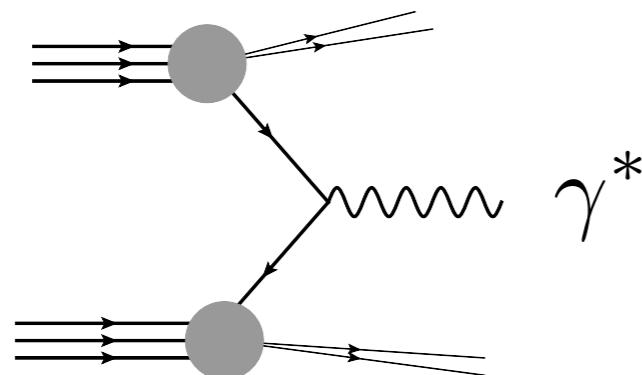
$$\sim \mathcal{O}(w_2) \quad \sim \mathcal{O}(w_1) \quad \sim \mathcal{O}(w_1, w_2)$$

RAPIDITY DISTRIBUTION

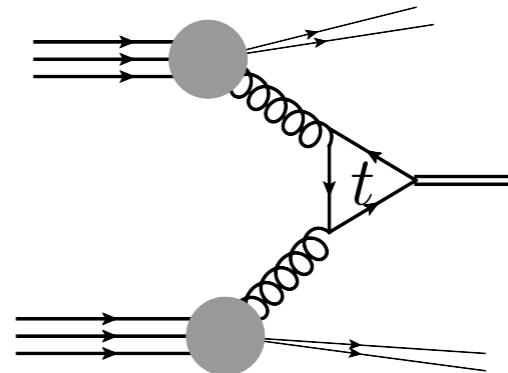


- ▶ Approximation works well!
- ▶ Systematically improvable.
- ▶ Currently, an example; **N3LO** in progress.

Drell - Yan



Higgs



- ▶ Expanded partonic cross section up to **N3LO to first order in collinear limit.**

- ▶ Feynman diagrams: $\mathcal{O}(10^6)$
- ▶ 492 new master integrals using differential equations
- ▶ Fully analytic results for the collinear limit of the partonic cross section

$$\frac{d\eta_{ij}}{dQ^2 dw_1 dw_2 dx}$$

- ▶ Cross sections for infrared sensitive observables can be written in general as:

$$\frac{d\sigma}{dQ^2 dY d\mathcal{T}} = \sigma_0 \sum_{i,j} H_{ij}(Q^2) [B_i(x_1^B, \mathcal{T}) \otimes B_j(x_2^B, \mathcal{T}) \otimes S(\mathcal{T})] \times [1 + \mathcal{O}(\mathcal{T}/Q)].$$

The diagram consists of three labels with arrows pointing to specific parts of the equation. A blue arrow points from the word "Hard" to the term $H_{ij}(Q^2)$. A green double-headed arrow points between the terms $B_i(x_1^B, \mathcal{T})$ and $B_j(x_2^B, \mathcal{T})$. A red arrow points from the word "Soft" to the term $S(\mathcal{T})$.

The only part that depends on the specific process.

B_i & S_i **Universal, process independent.**

Beam Functions: B_i

Since they encode collinear physics, they can naturally be extracted from our computation of the collinear limit of cross sections.

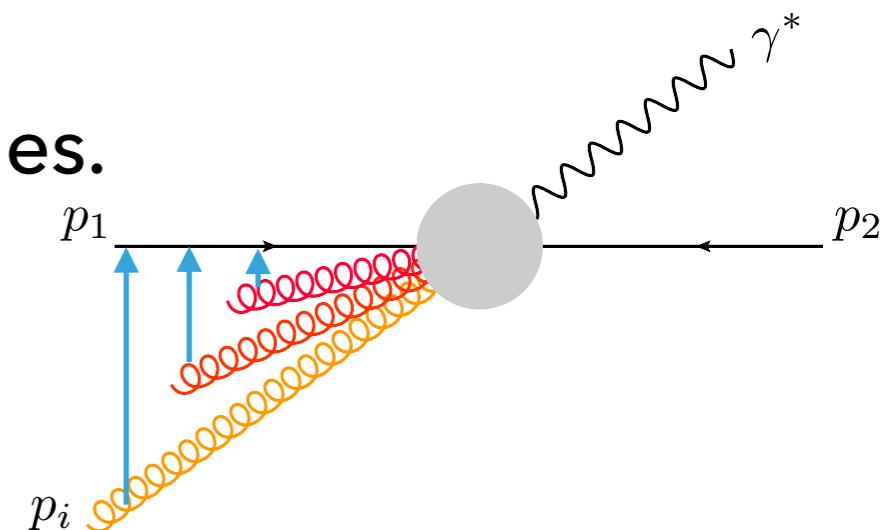
- ▶ Explicit relation of beam functions to the collinear limit of colour singlet cross sections:

$$B_i(x_1^B, \mathcal{T}) = \sum_j \int_{x_1^B}^1 \frac{dz_1}{z_1} f_j\left(\frac{x_1^B}{z_1}\right) \times \int_0^1 dx \int_0^\infty dw_1 dw_2 \delta[z_1 - (1 - w_1)] \\ \times \lim_{\text{strict } n-\text{coll.}} \left\{ \delta[\mathcal{T} - \mathcal{T}(Q, Y, w_1, w_2, x)] \frac{d\eta_{j\bar{i}}}{dQ^2 dw_1 dw_2 dx} \right\}.$$

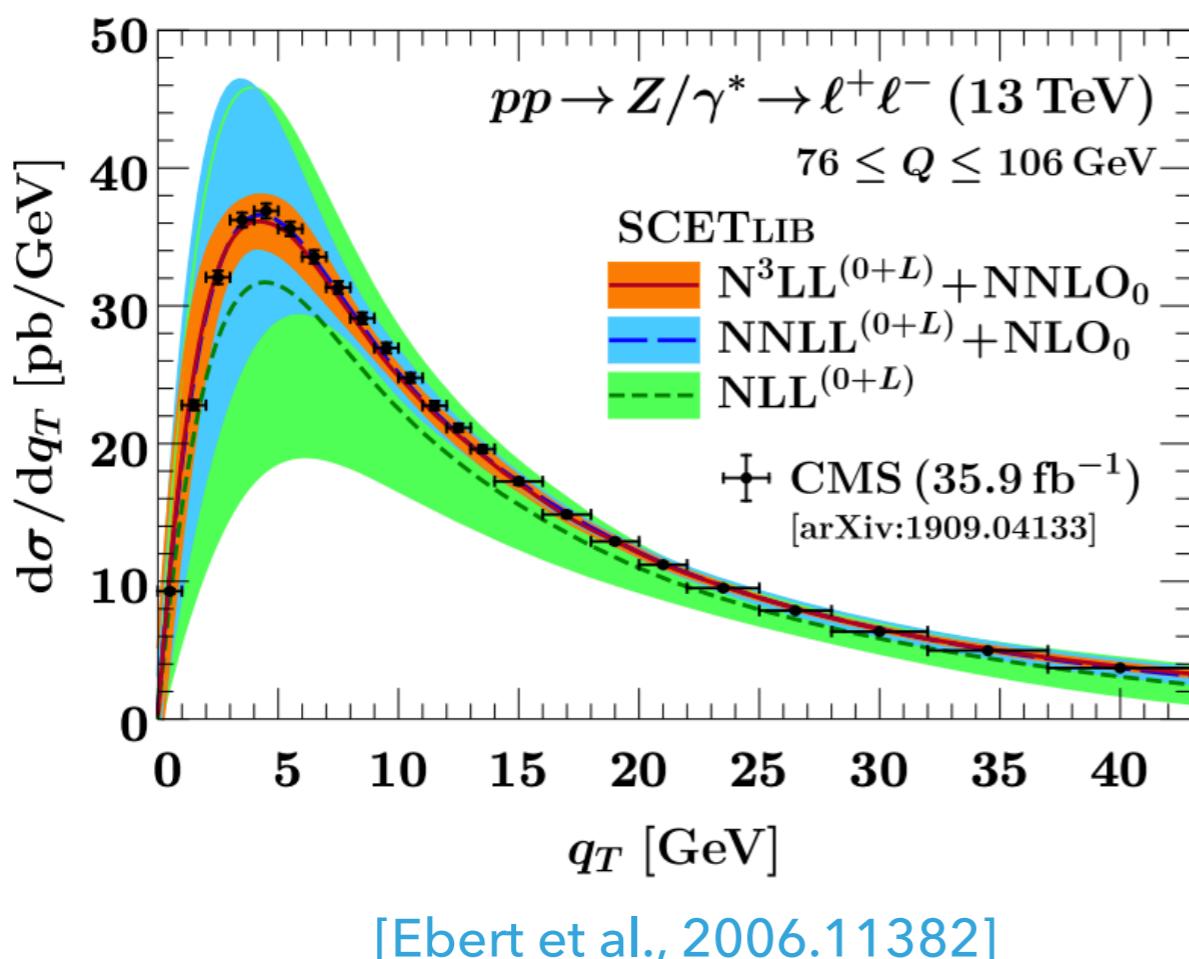
- ▶ Beam functions: Probability to find a parton in a proton with momentum fraction x_1^B and observable value \mathcal{T} .
- ▶ Related to standard PDFs via

$$B_i(x, \mathcal{T}, \mu) = \sum_j \mathcal{I}_{ij}(x, \mathcal{T}, \mu) \otimes_x f_j^R(x, \mu) \times [1 + \mathcal{O}(\Lambda_{\text{QCD}}/\mathcal{T})].$$

- ▶ Collinear Limit: Transverse momentum vanishes.
- ▶ Transverse momentum dependent beam functions play a role in many applications:



Resummation:



Slicing:

Approximate!

$$\frac{d\sigma}{d\mathcal{O}} = \int d\mathcal{T} \frac{d\tilde{\sigma}}{d\mathcal{T} d\mathcal{O}}$$

$$+ \int d\mathcal{T} \left[\frac{d\sigma}{d\mathcal{T} d\mathcal{O}} - \frac{d\tilde{\sigma}}{d\mathcal{T} d\mathcal{O}} \right]$$

Kallweit's talk

Chen's talk

Lower Order!

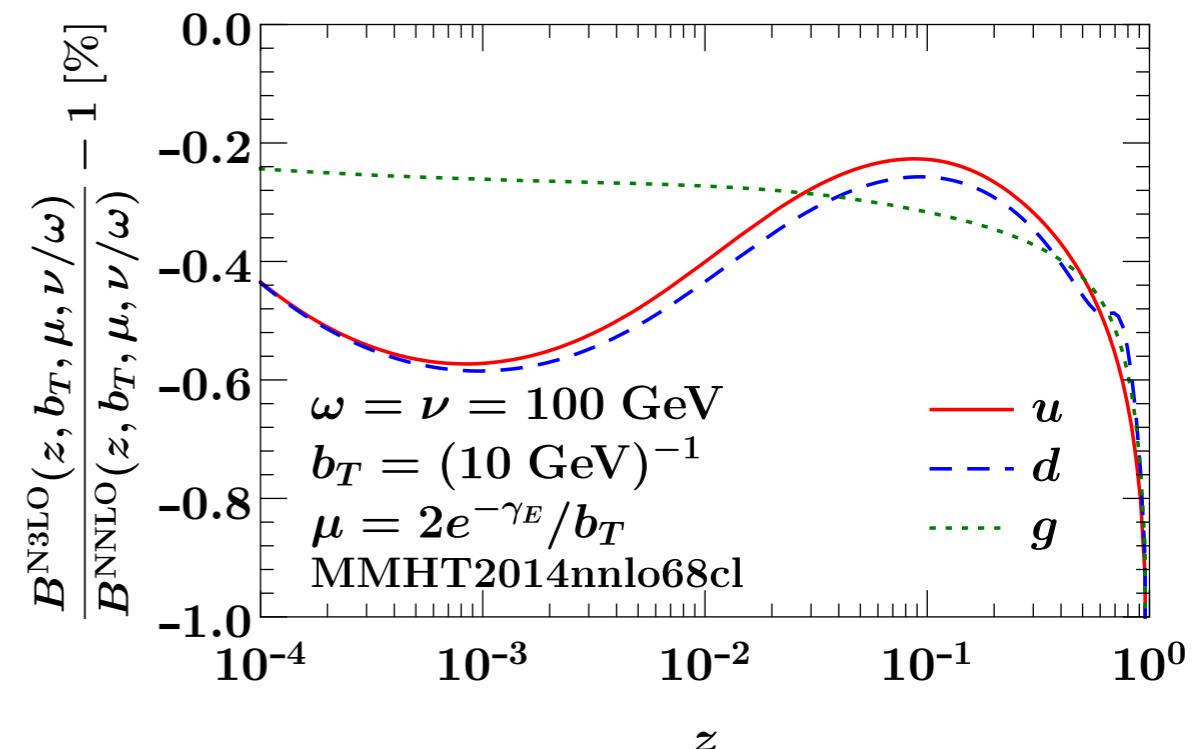
- ▶ Computed the N3LO corrections to the transverse momentum dependent beam functions.

- ▶ All channels, fully analytic.

See also [\[Luo et al., 1912.05778\]](https://arxiv.org/abs/1912.05778)

- ▶ Last missing universal ingredient for q_\perp subtraction at N3LO.

$\gamma^* W Z H \ bbH \ H^* G \ WH \ ZH \ ZZ \ WW$

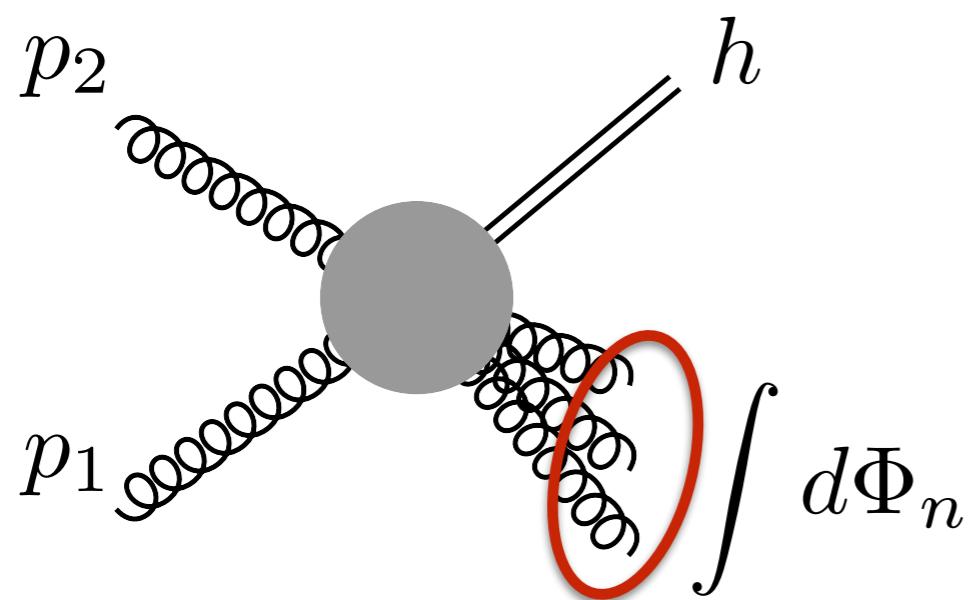


In other news: We also computed the
N-Jettiness Beam Functions at N3LO.
[arXiv:2006.03056](https://arxiv.org/abs/2006.03056)

Behring's talk

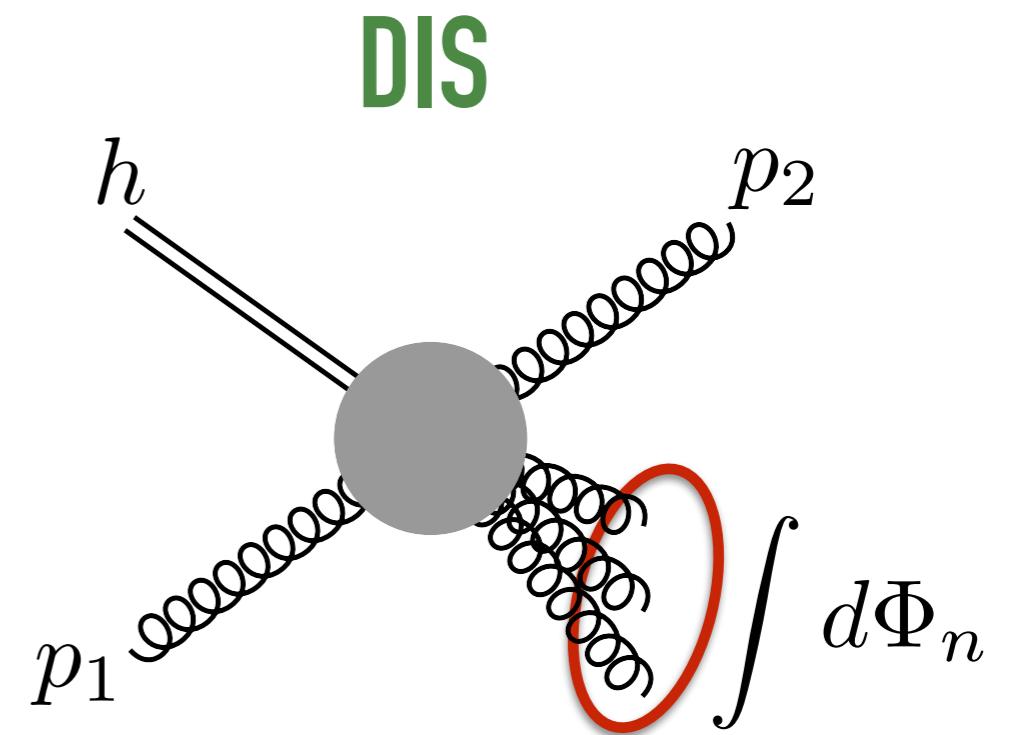
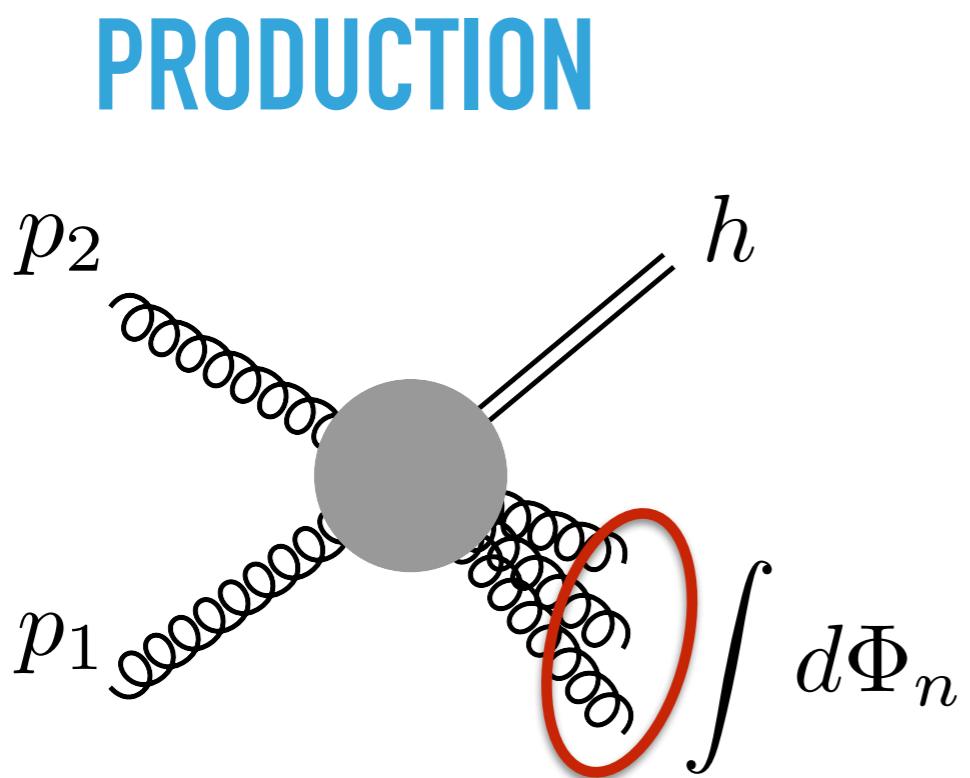
- ▶ We started with a production process:

PRODUCTION

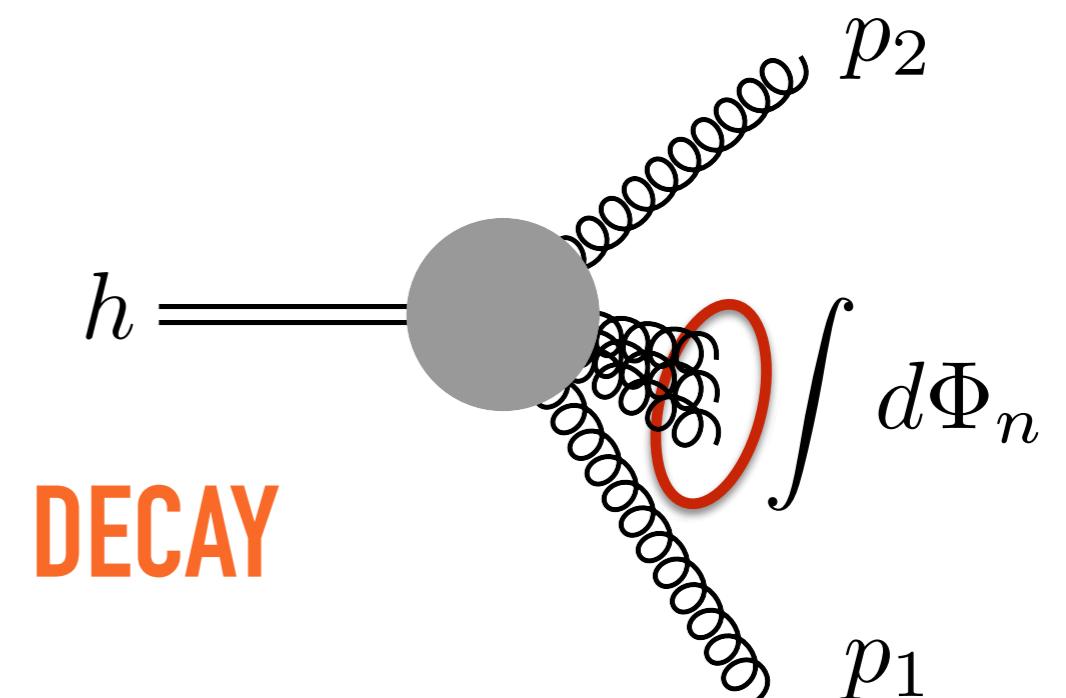
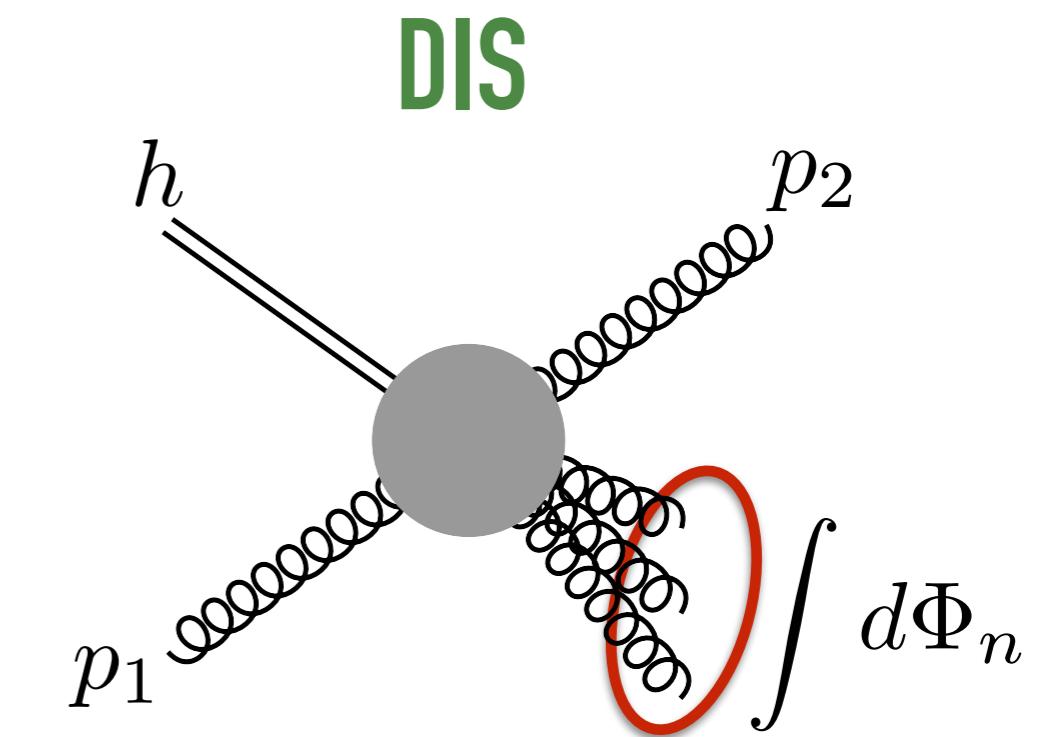
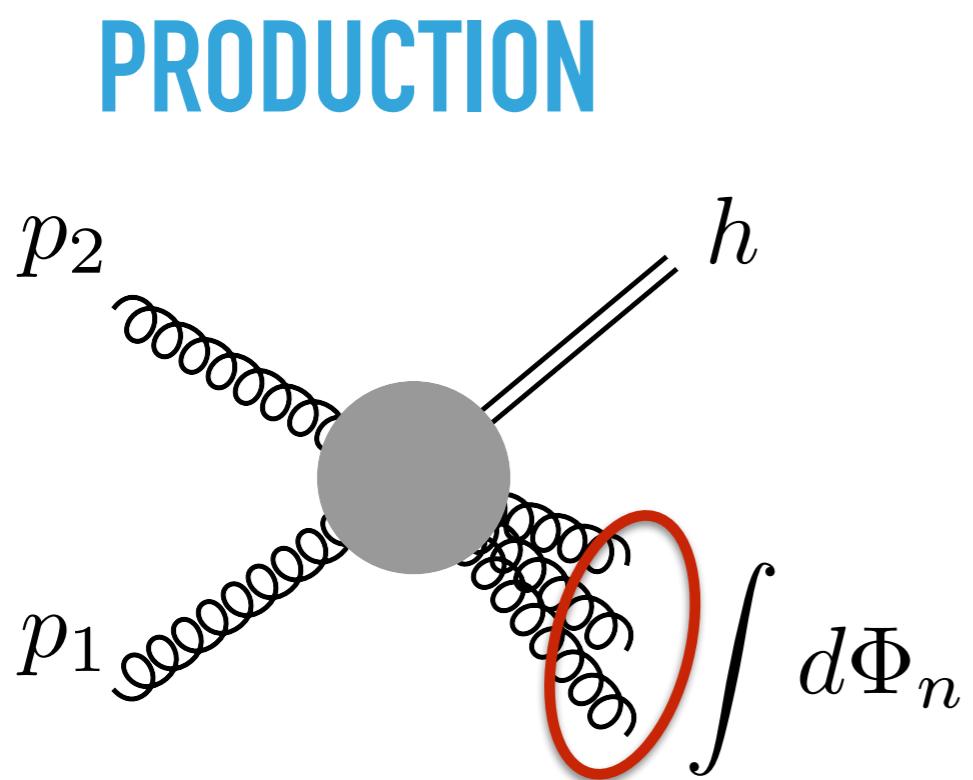


- ▶ Radiation in final state.
- ▶ Differential in the other momenta.

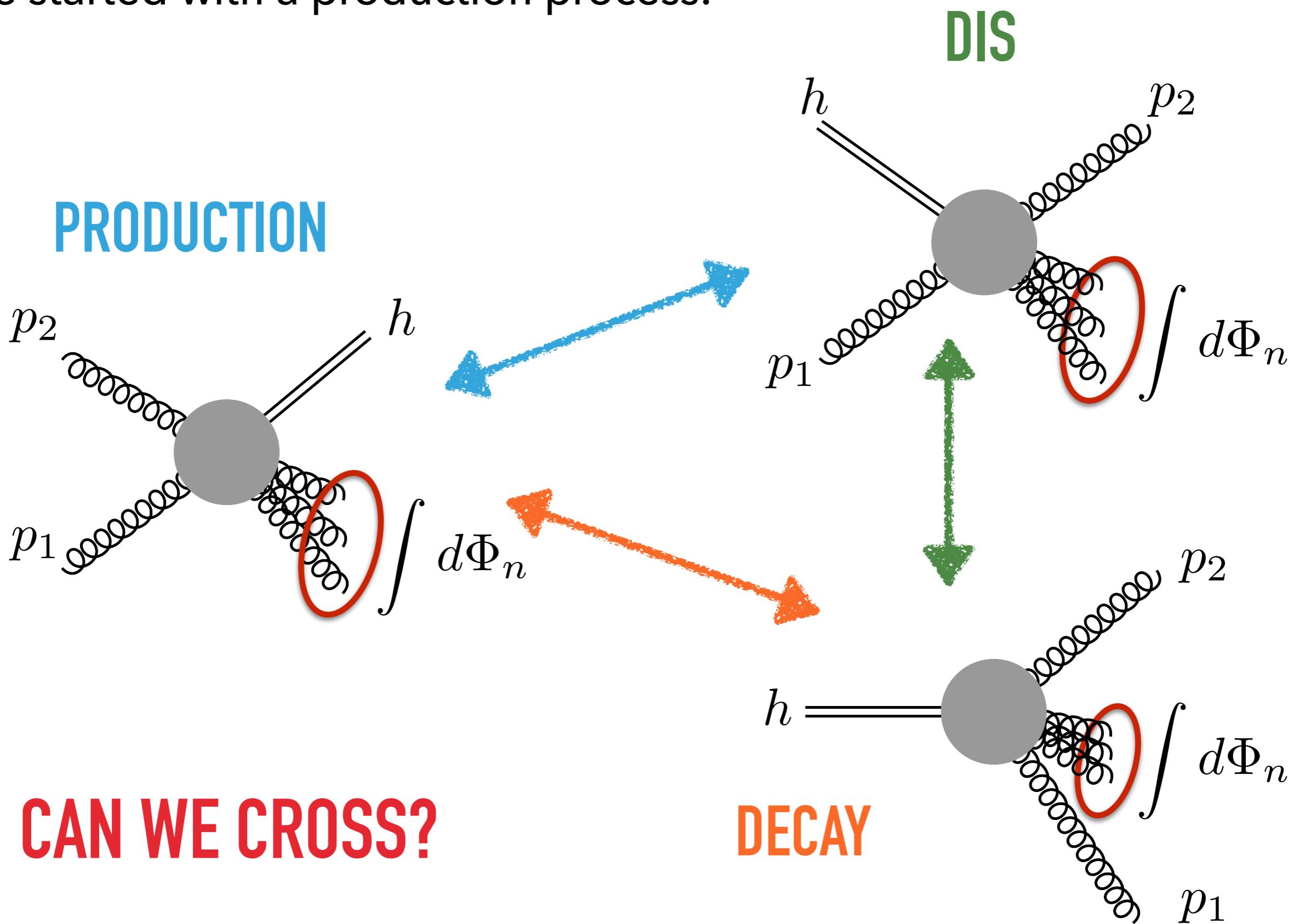
- ▶ We started with a production process:



- ▶ We started with a production process:



- We started with a production process:



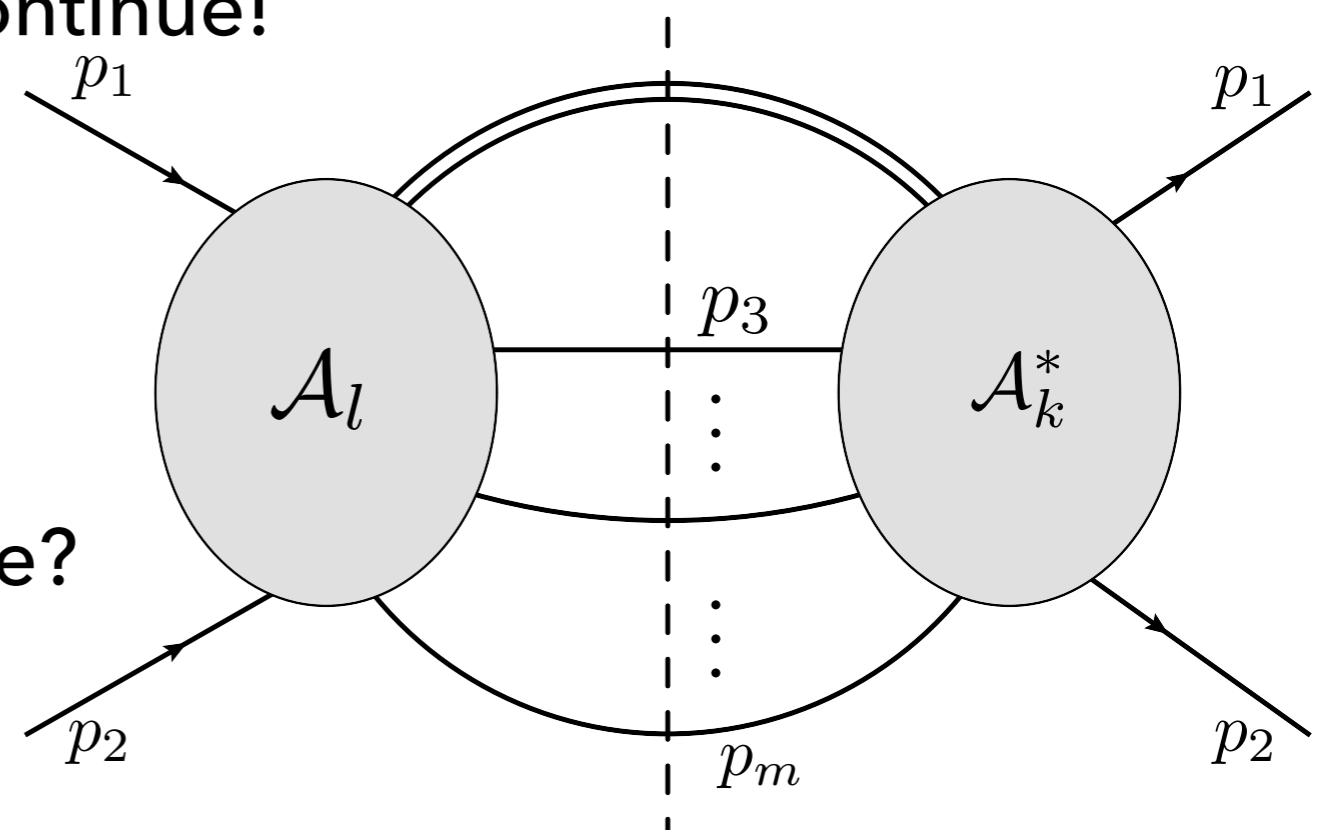
- ▶ To cross means to analytically continue!

- ▶ Our variables change sign:

$$\{s, w_1, w_2\}$$

- ▶ What is their branch cut structure?

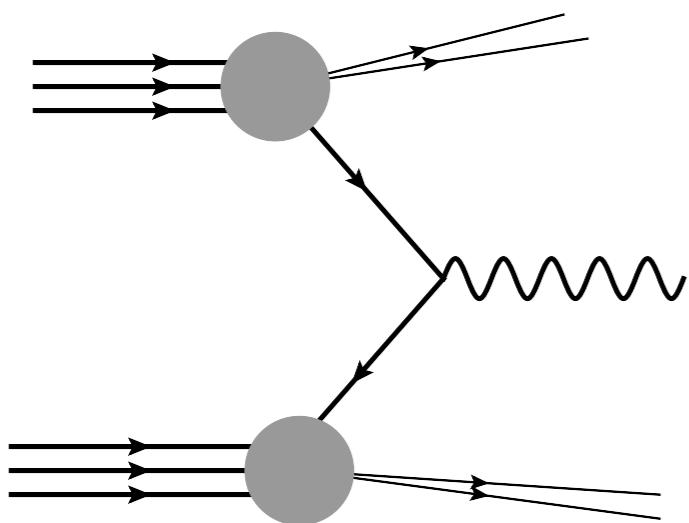
[cross section]



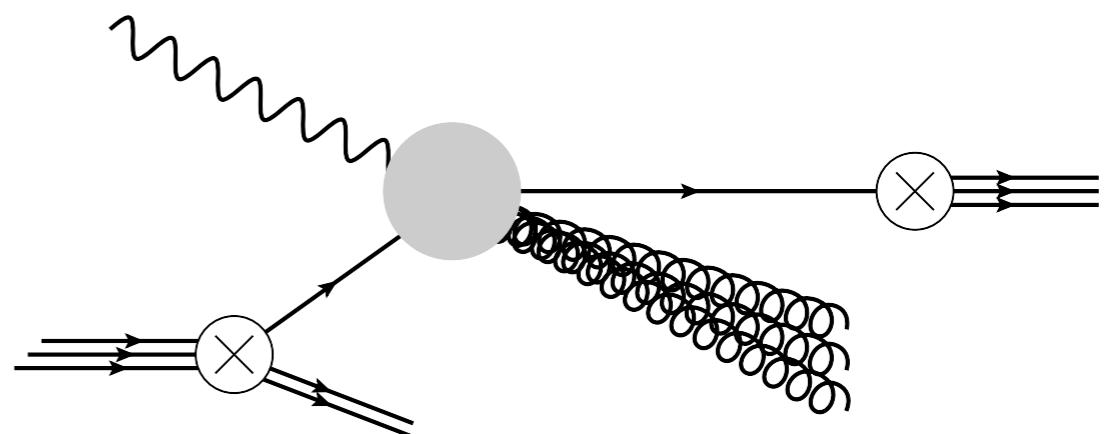
$$\frac{d\eta_{ij}^{(m+l+k)}}{dQ^2 dw_1 dw_2 dx} = (sw_1 w_2)^{-m\epsilon} \times \left[\sum_{i_1, i_2=0}^l \sum_{j_1, j_2=0}^k \frac{d\eta_{ij}^{(m+l+k, i_1, i_2, j_1, j_2)}}{dQ^2 dw_1 dw_2 dx} \right. \\ \left. \times \Re \left(\left[(-s)^{(i_1+i_2-l)\epsilon} (sw_1)^{-i_1\epsilon} (sw_2)^{-i_2\epsilon} \right] \left[(-s)^{(j_1+j_2-k)\epsilon} (sw_1)^{-j_1\epsilon} (sw_2)^{-j_2\epsilon} \right]^* \right) \right].$$

[Mandelstam Invariants! - Continue!]

Drell - Yan



Deep In-Elastic Scattering

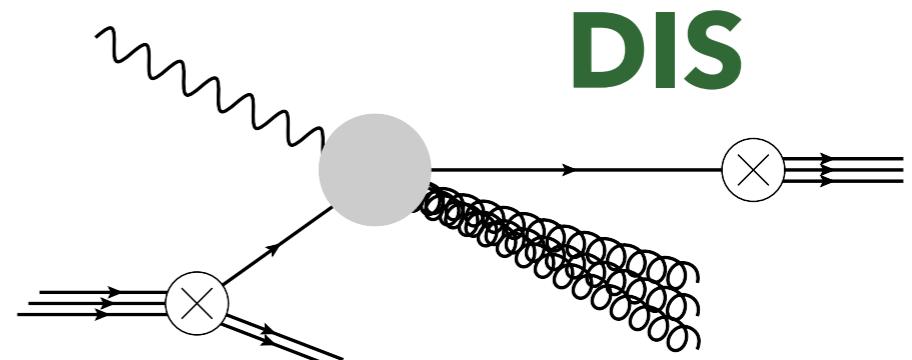


- ▶ Semi-Inclusive DIS in the small $P_{2,\perp}$ limit:

$$\frac{d\sigma_{P+h \rightarrow H+X}}{d\vec{P}_{2,\perp}^2 d\xi} = \hat{\sigma}_0 \sum_{i,j} H_{ij} \times (Q^2, \mu) \tilde{f}_i(P_{2,\perp}) \otimes d_j(P_{2,\perp}, \xi) \left[1 + \mathcal{O}\left(\vec{P}_{2,\perp}^2/Q^2\right) \right]$$

- ▶ Semi-Inclusive DIS in the small $P_{2,\perp}$ limit:

$$\frac{d\sigma_{P+h \rightarrow H+X}}{d\vec{P}_{2,\perp}^2 d\xi} = \hat{\sigma}_0 \sum_{i,j} H_{ij} \times (Q^2, \mu) \tilde{f}_i(P_{2,\perp}) \otimes d_j(P_{2,\perp}, \xi) \left[1 + \mathcal{O}\left(\vec{P}_{2,\perp}^2/Q^2\right) \right]$$



- ▶ Transverse Momentum Dependent Fragmentation Function (TMDFF)

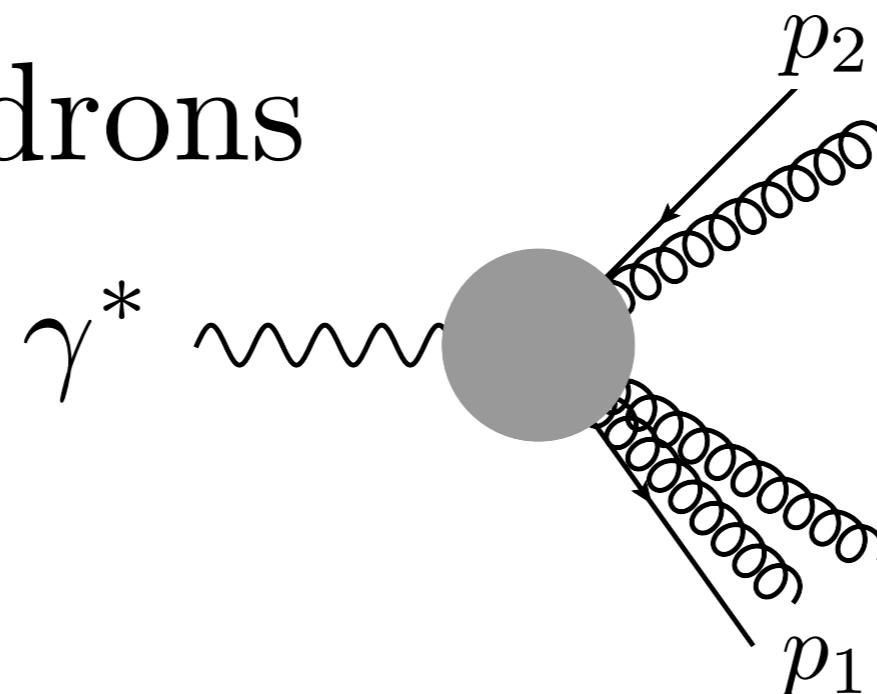
The probability to find a Hadron inside a parton with a given longitudinal momentum fraction ξ and transverse momentum $P_{2,\perp}$

- ▶ Related to the longitudinal fragmentation function via

$$\tilde{d}_j(\xi, b_T, \mu, \tau, \omega_b) = \sqrt{\tilde{S}(b_T, \mu, \tau)} \sum_j \int_\xi^1 \frac{d\zeta}{\zeta} \tilde{\mathcal{I}}_{ij}^{\text{FF}}(\zeta, b_T, \mu, \tau, \omega_b) D_j\left(\frac{\xi}{\zeta}, \mu\right).$$

TMDFF
 Soft Function
 Matching Kernel
 FF

$e^+e^- \rightarrow \text{hadrons}$



[2012.07859]

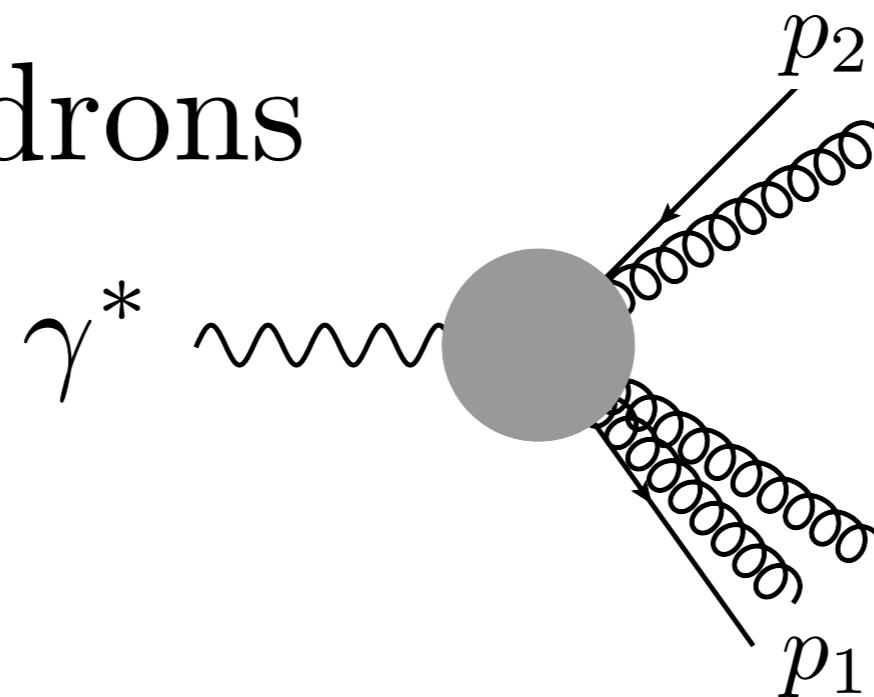
$$z \equiv \frac{1}{2}(1 - \cos \chi)$$

- ▶ Production of two hadrons with small transverse momentum.
- ▶ Related to Energy-Energy Correlation function:

$$\text{EEC}(\chi) = \frac{d\sigma}{d\chi} = \sum_{i,j} \int d\sigma_{e^+e^- \rightarrow ij + X} \frac{E_i E_j}{Q^2} \delta(\cos \theta_{ij} - \cos \chi)$$

$$\lim_{z \rightarrow 1} \frac{d\sigma}{dz} = \int_0^1 dz_1 dz_2 \frac{z_1 z_2}{2} \int d^2 \vec{q}_T \delta\left(1 - z - \frac{q_T^2}{Q^2}\right) \lim_{q_T \rightarrow 0} \sum_{h_1, h_2} \frac{d\sigma_{e^+e^- \rightarrow h_1 h_2}}{dz_1 dz_2 d^2 \vec{q}_T}$$

$e^+e^- \rightarrow \text{hadrons}$



[2012.07859]

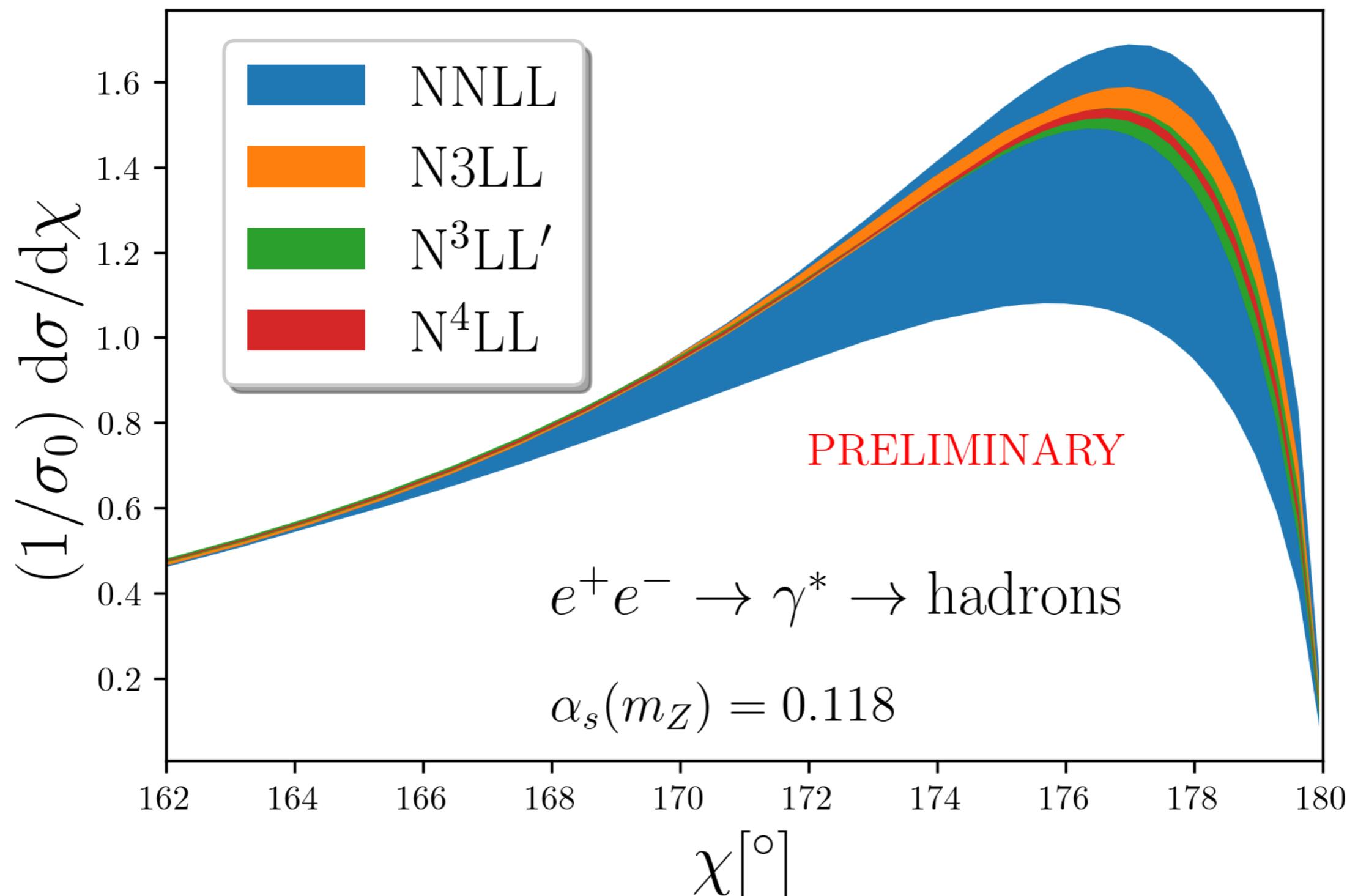
$$z \equiv \frac{1}{2}(1 - \cos \chi)$$

- ▶ Factorisation Theorem for the EEC at $z=1$:

$$\begin{aligned} \frac{d\sigma_{e^+e^- \rightarrow h_1 h_2}}{dz_1 dz_2 d^2 \vec{q}_T} &= \hat{\sigma}_0 H_{q\bar{q}}(Q, \mu) \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \tilde{D}_{h_1/q}\left(z_1, b_T, \mu, \frac{\nu}{Q}\right) \tilde{D}_{h_2/\bar{q}}\left(z_2, b_T, \mu, \frac{\nu}{Q}\right) \\ &\times \tilde{S}_q(b_T, \mu, \nu) \times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]. \end{aligned} \quad ($$

- ▶ Recently, collected anomalous dimensions for N4LL resummation.

$e^+e^- \rightarrow \text{hadrons}$ ▶ Energy-Energy Correlator:

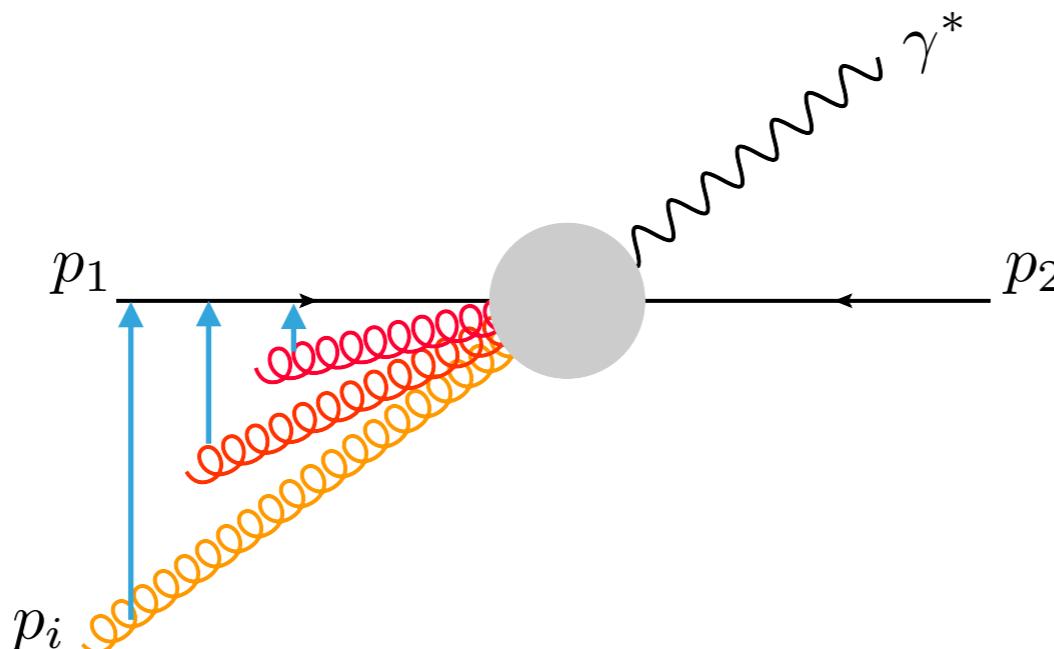


- ▶ The beginning of **N3LO** phenomenology!
- ▶ Collinear expansions for colour singlet processes.

- ▶ Efficient way of performing approximations
for collider cross sections.

Higgs Boson Rapidity at NNLO

- ▶ First results:
 - Beam Functions at **N3LO** for q_\perp & \mathcal{T}
 - Fragmentation Functions at **N3LO** for q_\perp
 - EEC at $z=1$ at **N4LL**



Thank you!