

Recent developments within the MATRIX framework

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Precision calculations — the key to fully exploit LHC measurements

Sample case: diboson production

- important SM test → trilinear couplings
- background for Higgs analyses and BSM searches
- very clean signatures in leptonic decay channels
- good statistics already with available data

All diboson processes available at NNLO QCD accuracy in the public **MATRIX** framework

[Grazzini, SK, Wiesemann (2018)]

- inevitable for data–theory agreement

Mandatory steps to match experimental precision also in the future

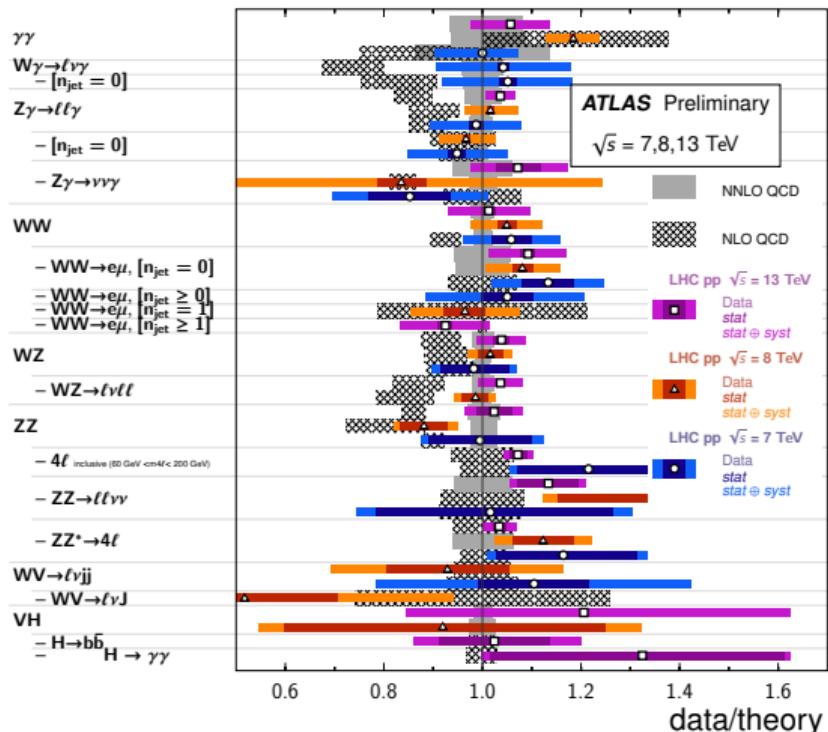
- leading QCD corrections beyond NNLO
- EW corrections and combination with QCD

→ **MATRIX v2** [Grazzini, SK, Wiesemann (2021)]

[ATLAS collaboration (2022)]

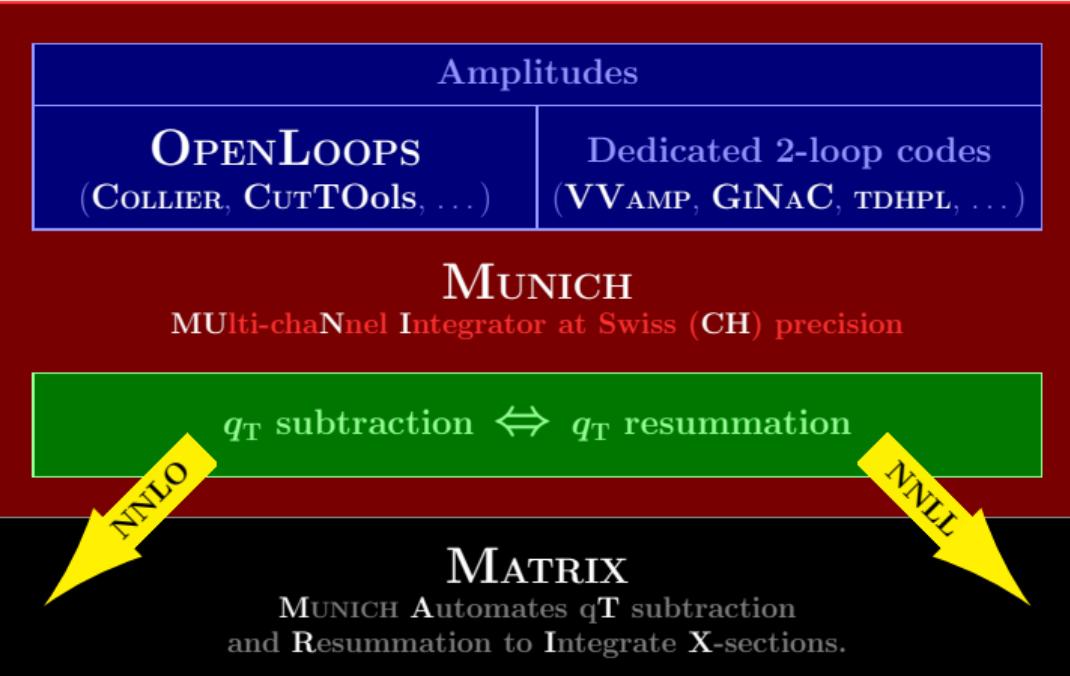
Diboson Cross Section Measurements

Status: February 2022



The MATRIX framework for automated NNLO QCD calculations (and beyond)

[Grazzini, SK, Wiesemann (2018) + Rathlev; Buonocore, Devoto, Mazzitelli, Rottoli, Sargsyan, Savoini, Yook, ...]



available under <https://matrix.hepforge.org/>

MATRIX v1 (fall 2017)

- H, V, $\gamma\gamma$, $V\gamma$, VV at NNLO QCD for all leptonic decay channels

MATRIX v2 (summer 2021)

- combination with NLO EW for all leptonic V and VV processes
- loop-induced gg channel at NLO QCD for neutral VV processes

MATRIX v2.1 (spring 2022)

- bin-wise $q_{T,\text{cut}} \rightarrow 0$ extrapolation also for all distributions
- recoil-driven linear power corrections (relevant for Drell-Yan)
- $\gamma\gamma\gamma$ at NNLO QCD ($2 \rightarrow 3$)
- $t\bar{t}$ at NNLO QCD (heavy-quark FS)

Outline

1 The MATRIX framework for precision calculations

2 Single boson production

- Recoil-driven linear power corrections in Drell–Yan processes

3 Diboson production

- Combination of NNLO QCD and NLO EW predictions
- NLO QCD corrections to loop-induced gluon fusion

4 Triboson production

- Triphoton production at NNLO QCD accuracy
- Feasibility studies for other triboson processes at NNLO QCD accuracy

5 Heavy-quark production

- Production of top-quark pairs at NNLO QCD accuracy
- Feasibility studies for associated heavy-quark pair production at NNLO QCD accuracy

6 Conclusions & Outlook

The MUNICH/MATRIX framework for automated NNLO calculations

MATRIX — MUNICH Automates qT-subtraction and Resummation to Integrate X-sections

[Grazzini, SK, Wiesemann (2018)]

- first public tool that performs NNLO QCD calculations for a large class of processes
- core of the framework: the C++ parton-level Monte Carlo generator

MUNICH — MUlti-chaNnel Integrator at swiss (CH) precision [SK]

- bookkeeping of partonic subprocesses for all contributions
 - fully automated dipole subtraction for NLO calculations (massive, QCD and EW)
[Catani, Seymour (1997), Catani, Dittmaier, Seymour, Trocsanyi (2002), Dittmaier (2000), SK, Lindert, Maierhöfer, Pozzorini, Schönherr (2015)]
 - general amplitude interface
 - 1-loop amplitudes
 - 2-loop amplitudes
 - highly efficient multi-channel Monte Carlo integration with several optimization features
 - simultaneous monitoring of slicing parameter and automated extrapolation
- tail enhancement
- PYTHON script to simplify the use of MATRIX
 - installation of MUNICH and all supplementary software
 - interactive shell steering all run phases without human intervention (grid-, pre-, main-run, summary)
 - organization of parallelized running on multicore machines and commonly used clusters:
SLURM, HTCONDOR, LSF, etc.

Idea of the q_T subtraction method for (N)NLO cross sections

Consider the production of a **colourless final state F** via $q\bar{q} \rightarrow F$ or $gg \rightarrow F$: $d\sigma_F^{(N)\text{NLO}} \Big|_{q_T \neq 0} = d\sigma_{F+\text{jet}}^{(N)\text{LO}}$
where q_T refers to the transverse momentum of the colourless system F [Catani, Grazzini (2007)]

- $d\sigma_F^{(N)\text{NLO}} \Big|_{q_T \neq 0}$ is singular for $q_T \rightarrow 0$
 - limiting behaviour known from transverse-momentum resummation [Bozzi, Catani, de Florian, Grazzini (2006)]
- Define a universal counterterm Σ with the complementary $q_T \rightarrow 0$ behaviour [Bozzi, Catani, de Florian, Grazzini (2006)]

$$d\sigma^{\text{CT}} = \Sigma(q_T/q) \otimes d\sigma^{\text{LO}}$$
 where q is the invariant mass of the colourless system F
- Add the $q_T = 0$ piece with the hard-virtual coefficient \mathcal{H}_F , which contains the 1-(2-)loop amplitudes at (N)NLO and compensates for the subtraction of Σ [Catani, Cieri, de Florian, Ferrera, Grazzini (2013)]
 - Master formula for (N)NLO cross section in q_T subtraction method

$$d\sigma_F^{(N)\text{NLO}} = \mathcal{H}_F^{(N)\text{NLO}} \otimes d\sigma^{\text{LO}} + \left[d\sigma_{F+\text{jet}}^{(N)\text{LO}} - \Sigma^{(N)\text{NLO}} \otimes d\sigma^{\text{LO}} \right]_{\text{cut } q_T \rightarrow 0}$$

- all ingredients known for extension to $N^3\text{LO}$ [Luo, Yang, Zhu, Zhu (2019; 2020), Ebert, Mistlberger, Vita (2020), Cieri, Chen, Gehrmann, Glover, Huss (2019), Camarda, Cieri, Ferrera (2021), Chen, Gehrmann, Glover, Huss, Yang, Zhu (2021)]

Investigation of $r_{\text{cut}} = \text{cut}_{q_T/q}$ dependence — sample case pp $\rightarrow \gamma\gamma + X$

Result for $r_{\text{cut}} \rightarrow 0$ via extrapolation

- automated and simultaneous scan over reasonable range of r_{cut} values
- quadratic least- χ^2 fit with variable range

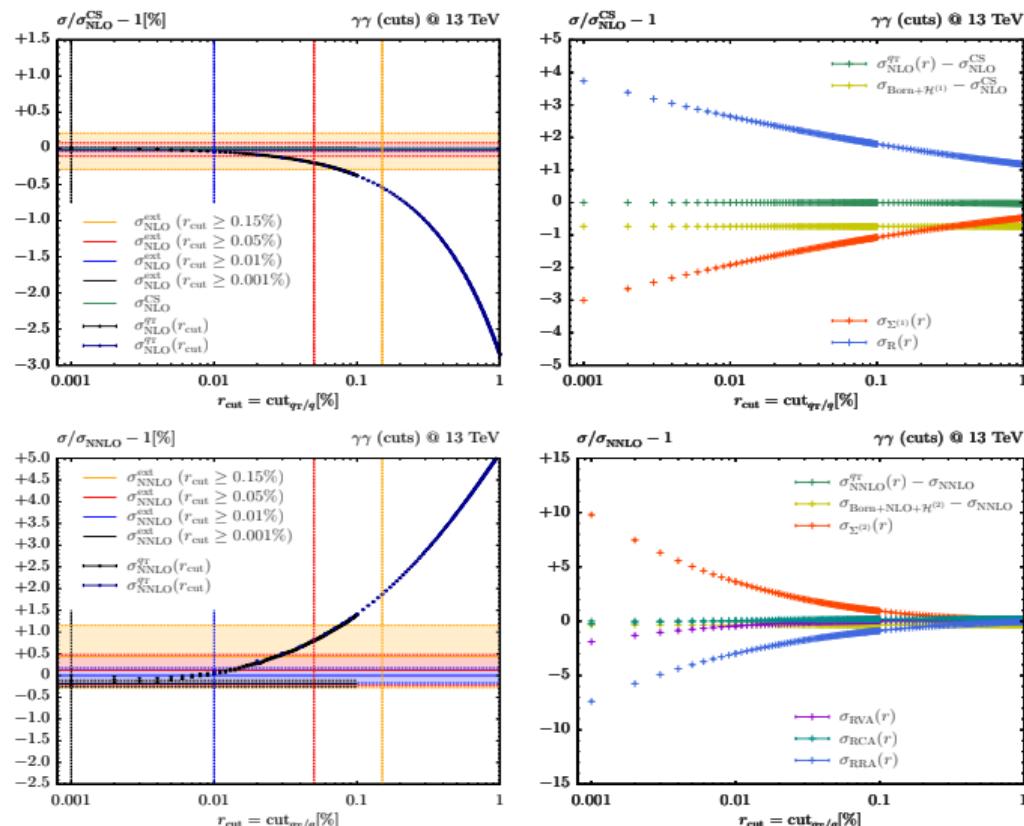
$$\sigma_{(N)\text{NLO}}(r_{\text{cut}}) = Ar_{\text{cut}}^2 + Br_{\text{cut}} + \sigma_{(N)\text{NLO}}$$

- error estimate based on combination of statistical error and variation of r_{cut} range

→ Significant r_{cut} dependence for processes involving isolated photons (similar between NLO and NNLO QCD)

→ good agreement of extrapolated results within errors for different start values

- $r_{\text{cut}} \geq 0.15\%$
- $r_{\text{cut}} \geq 0.05\%$
- $r_{\text{cut}} \geq 0.01\%$
- $r_{\text{cut}} \geq 0.001\%$



Investigation of $r_{\text{cut}} = \text{cut}_{q_T/q}$ dependence — sample case $\text{pp} \rightarrow \ell^-\ell^+\ell^-\ell^+ + X$

Result for $r_{\text{cut}} \rightarrow 0$ via extrapolation

- same procedure for all processes

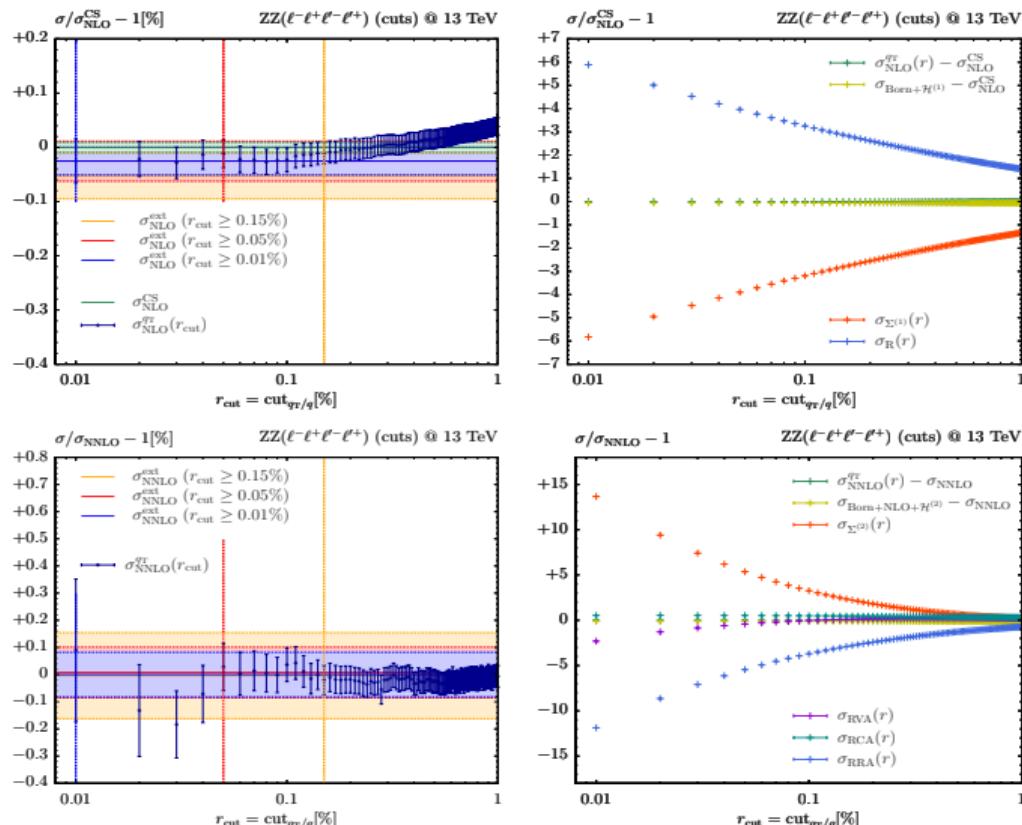
$$\sigma_{(N)\text{NLO}}(r_{\text{cut}}) = Ar_{\text{cut}}^2 + Br_{\text{cut}} + \sigma_{(N)\text{NLO}}$$

- No significant r_{cut} dependence for processes **without** isolated photons (similar between NLO and NNLO QCD)
- good agreement of extrapolated results within errors for different start values

- $r_{\text{cut}} \geq 0.15\%$
- $r_{\text{cut}} \geq 0.05\%$
- $r_{\text{cut}} \geq 0.01\%$

- larger cancellation between contributions (factor of ≈ 15 at $r_{\text{cut}} = 0.01\%$)

- Important exception:** linear power corrections induced by particular fiducial cut configurations



Recoil-driven linear power corrections in neutral-current Drell–Yan process

Transverse-momentum cuts on undistinguished particles in two-body final states introduce enhanced sensitivity to low momentum scales [Salam, Slade (2021)]

→ **Linear power corrections (linPCs) in context of q_T subtraction**

- have been resummed to all orders for s -channel (DY, Higgs) production
[Ebert, Michel, Stewart, Tackmann (2021); Billis, Dehnadi, Ebert, Michel, Tackmann (2021)]

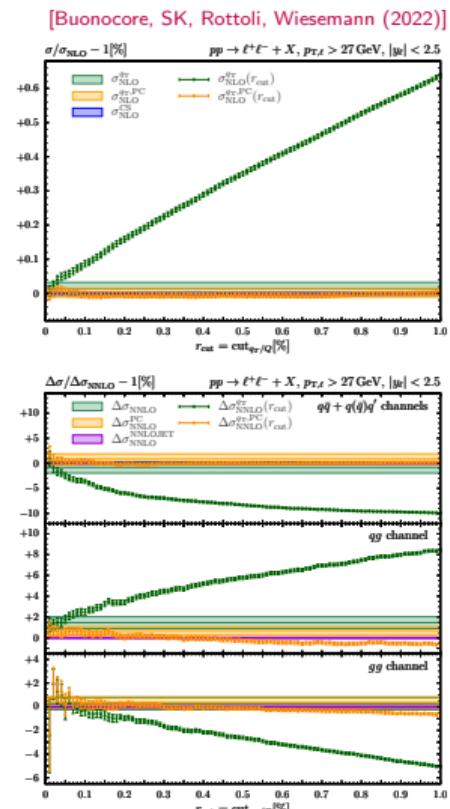
→ **Recoil prescription** can be used to predict linPCs also in fixed-order calculations [Buonocore, SK, Rottoli, Wiesemann (2022), Camarda, Cieri, Ferrera ('21)] :

$$\Delta\sigma^{\text{linPCs}}(r_{\text{cut}}) = \int d\Phi_F \int_{\epsilon}^{r_{\text{cut}}} dr' \left(\frac{d\sigma^{\text{CT}}}{d\Phi_F dr'} \Theta_{\text{cuts}}(\Phi_F^{\text{rec}}) - \frac{d\sigma^{\text{CT}}}{d\Phi_F dr'} \Theta_{\text{cuts}}(\Phi_F) \right)$$

- Φ_F^{rec} describes frame where system F is assigned a recoil q_T (boost from Collins–Soper frame, but precise prescription irrelevant)

→ **Adding the contribution $\Delta\sigma^{\text{linPCs}}(r_{\text{cut}})$ reduces leading (recoil-driven) r_{cut} dependence from linear (without linPCs) to (at most) quadratic**

- Illustration for **symmetric cuts** at NLO (upper plot) and NNLO (lower plot; reference result from NNLOjet [Bizon et al. (2021)])



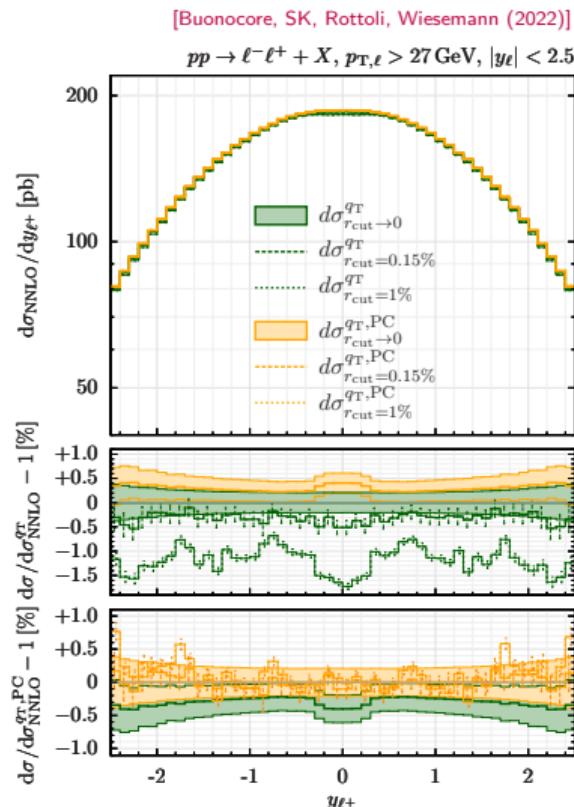
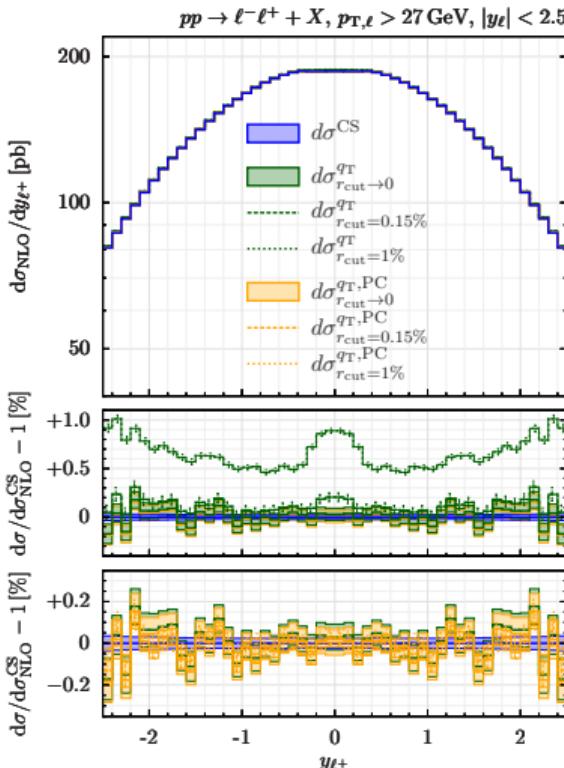
Distributions with linPCs for neutral-current DY process with symmetric cuts

Sample distribution: ℓ^+ rapidity at NLO (left) and NNLO (right)

- up to $\sim 2\%$ deviations for highest considered value $r_{\text{cut}} = 1\%$ without linPCs
- good agreement between considered r_{cut} values with linPCs (within errors)

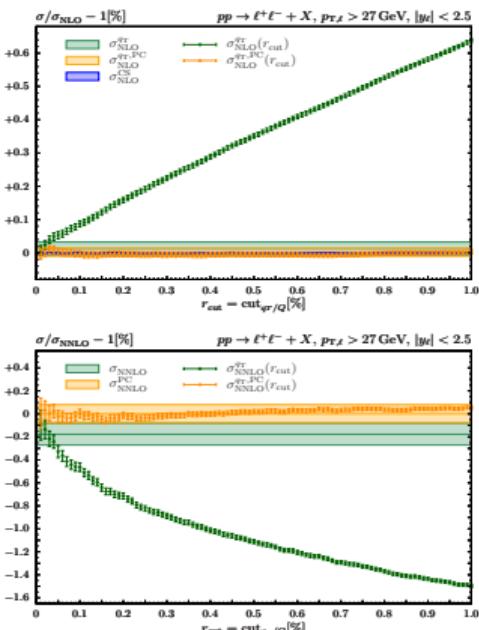
Note: The extrapolated results with linPCs and without linPCs agree well within errors!

- higher efficiency with linPCs (larger r_{cut} values sufficient)
- accurate results also from binwise extrapolation without including linPCs

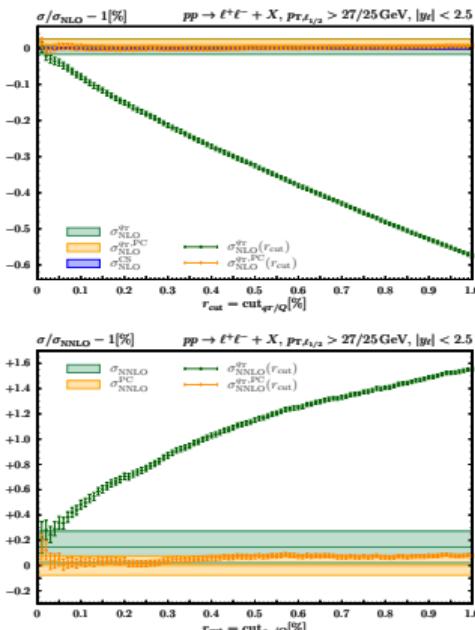


Recoil-driven linear power corrections for different fiducial cut configurations

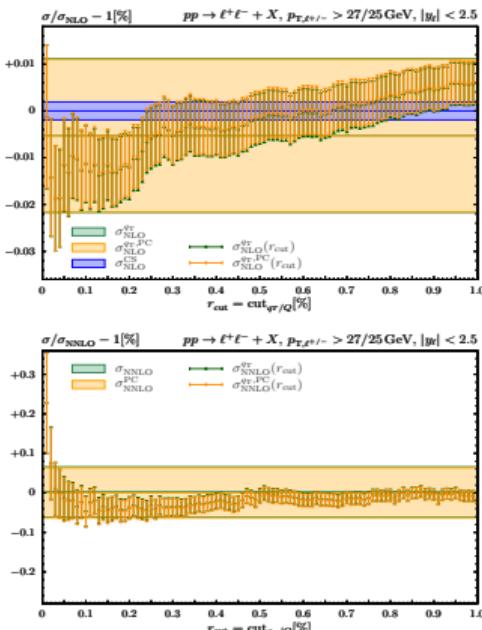
symmetric ($p_{T,\ell} > 27 \text{ GeV}$)



asymmetric ($p_{T,\ell_{1/2}} > 27/25 \text{ GeV}$)



staggered ($p_{T,\ell^\pm} > 27/25 \text{ GeV}$)



NLO

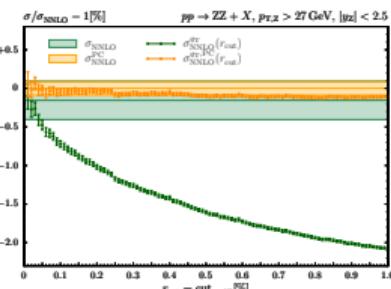
NNLO

- similarly sizable linPCs (with opposite sign) for **symmetric** and **asymmetric** cuts
- **linPCs absent for staggered cuts** (as long as $q_T < \delta p_T$) ➔ also for other alternative cuts [Salam, Slade (2021)]

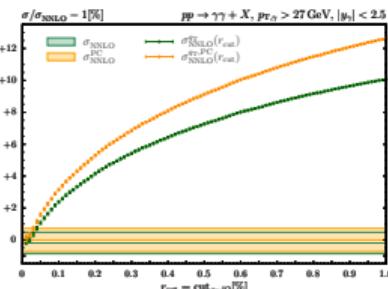
Recoil-driven linear power corrections for diboson processes

Investigation of linPCs for diboson processes (two-body kinematics and symmetric cuts)

ZZ



$\gamma\gamma$

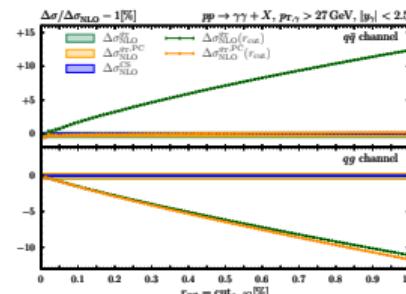


- For ZZ production, linPCs from recoil prescription reduce r_{cut} dependence to (at most) quadratic
 - formally proven only for s -channel production [Ebert, Michel, Stewart, Tackmann (2021)]
- For $\gamma\gamma$ production, r_{cut} dependence from photon isolation dominates over recoil-driven linPCs

Note: Recoil-driven linPCs absent for staggered cuts (e.g. on $|y|$ -ordered bosons), like in Drell–Yan case

Breakdown into partonic channels for $\gamma\gamma$ case

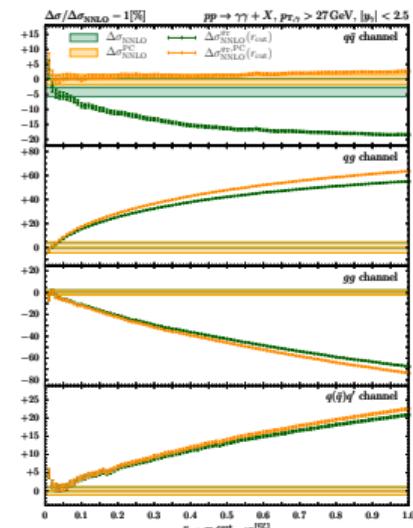
[Buonocore, SK, Rottoli, Wiesemann (2022)]



\uparrow NLO

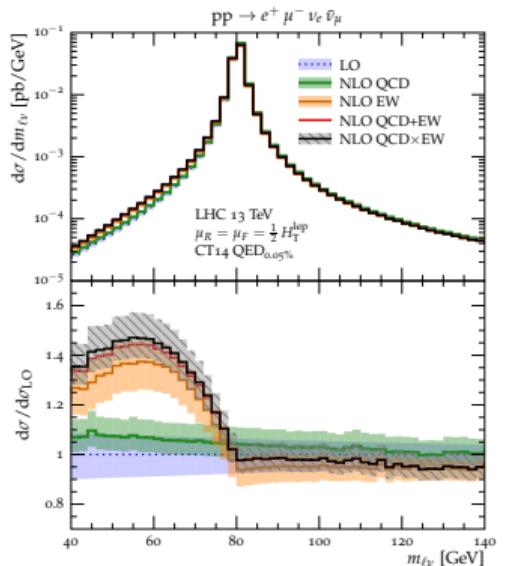
NNLO \rightarrow

- linear r_{cut} dependence in $q\bar{q}$ channel only due to kinematic effects
- cured by recoil-driven linPCs for $q\bar{q}$ channel



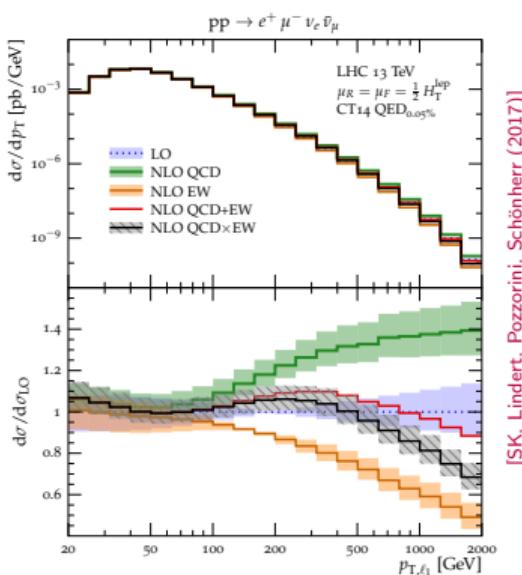
Main features of EW corrections

- Shape corrections in invariant-mass distributions



Pure QED effect:
photon bremsstrahlung off decay
leptons (migration effect)

- Negative corrections in high-energy observables



Genuine EW effect:
enhancement due to large universal
Sudakov logarithms

- Photon-induced processes

inclusion via **LUXQED** PDFs
[Manohar, Nason, Salam, Zanderighi (2016; 2017)]

- as Born processes,
e.g. $\gamma\gamma \rightarrow WW$
- as EW corrections,
from IS $\gamma \rightarrow q\bar{q}^*$ splittings

- Subdominant production modes (not maximal in α_s)

- e.g. $q\bar{q} \rightarrow Z^*/\gamma^* \rightarrow t\bar{t}$
- interferences between QCD and EW production modes
- corresponding tower of NLO contributions that cannot be uniquely qualified as QCD or EW corrections (in parts)

Combination of QCD and EW corrections for diboson production – p_{T,V_2}

Both corrections sizable, particularly in high-energy tails of distributions

- approximation of leading $\mathcal{O}(\alpha_s \alpha)$ effects desirable

Different combination approaches

- additive:

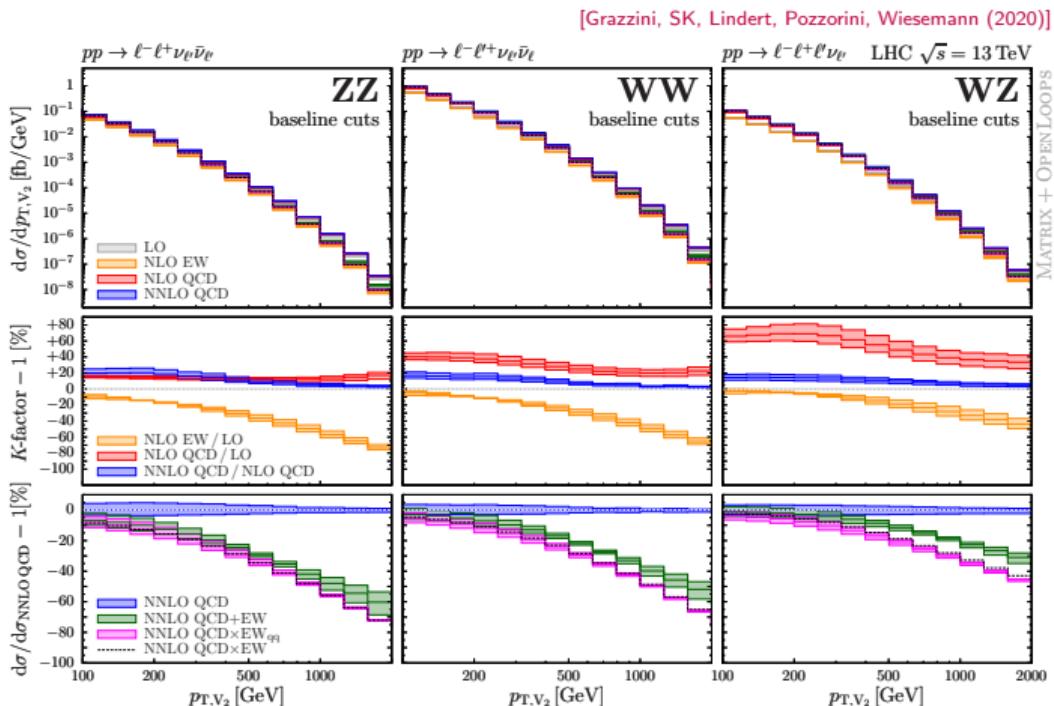
$$d\sigma^{\text{LO}}(1 + \delta_{\text{QCD}}^{(\text{N})\text{NLO}} + \delta_{\text{EW}}^{\text{NLO}})$$

- multiplicative:

$$d\sigma^{\text{LO}}(1 + \delta_{\text{QCD}}^{(\text{N})\text{NLO}})(1 + \delta_{\text{EW}}^{\text{NLO}})$$

- multiplicative (only $q\bar{q}$):

$$d\sigma_{q\bar{q}}^{\text{LO}}(1 + \delta_{q\bar{q}, \text{QCD}}^{(\text{N})\text{NLO}})(1 + \delta_{q\bar{q}, \text{EW}}^{\text{NLO}}) + \sigma_{\gamma\text{-ind., EW}}^{\text{NLO}}$$



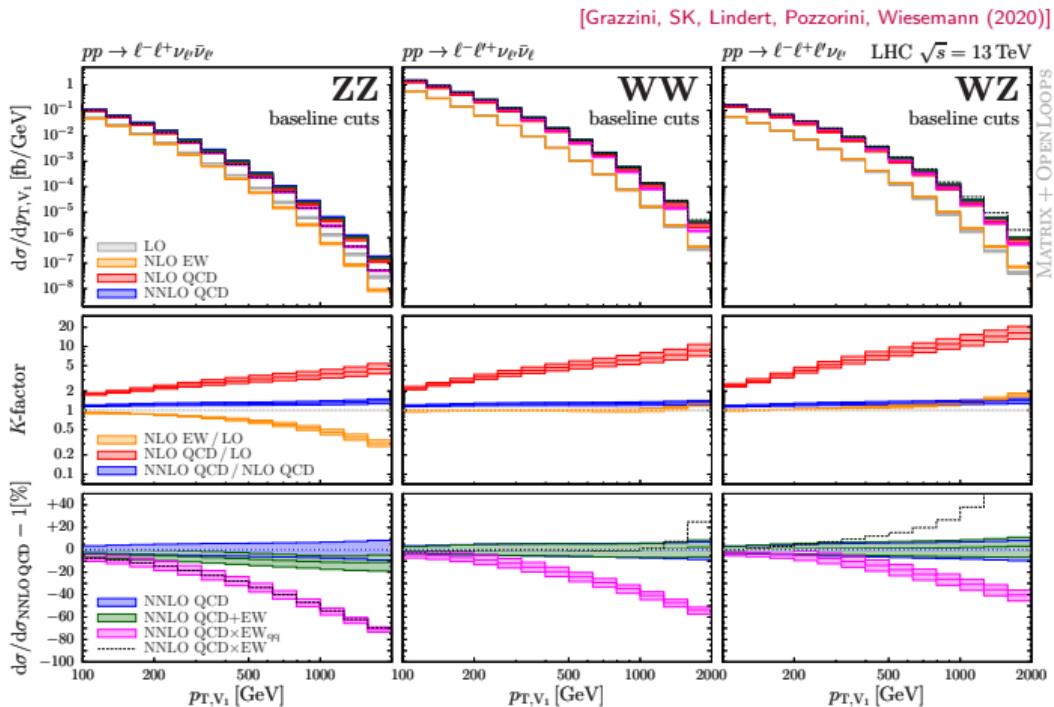
Factorized approaches well motivated for **genuine VV observables** (dominated by hard-VV topologies)

- catch leading mixed QCD–EW effects and may thus be considered preferable

Combination of QCD and EW corrections for diboson production – p_{T,V_1}

Situation more involved in presence of so-called **giant K-factors**

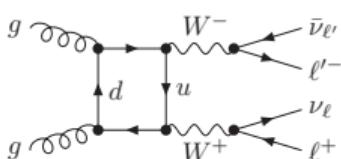
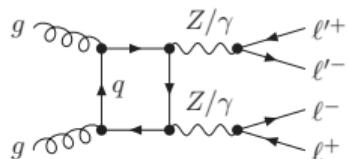
- QCD corrections in tails dominated by **hard-Vj topologies**
- also large positive EW corrections (photon-induced hard-Vj topologies)
- **additive** underestimates EW effects
- **multiplicative** combination multiplies large QCD and EW K -factors
↪ discarded
- **multiplicative (only $q\bar{q}$)** shows expected Sudakov behaviour, but overestimates the EW effects (VV K -factor applied in hard-Vj region)



None of the approaches works perfectly well for **observables dominated by hard-Vj topologies**

- merged prediction, full mixed QCD–EW calculation, or phase space restriction to hard-VV topologies

NLO QCD corrections to loop-induced gluon channel in ZZ/WW production



- only LO-accurate at $\mathcal{O}(\alpha_s^2)$
- enhanced by large gluon luminosity
- presumably dominant $\mathcal{O}(\alpha_s^3)$ contribution

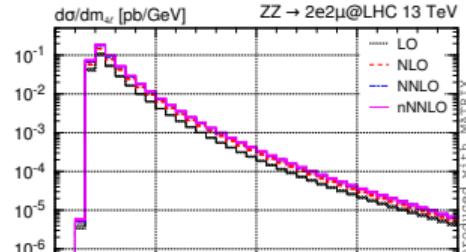
Approximate nNNLO QCD prediction

- full m_t dependence in 1-loop terms
 - massless 2-loop amplitudes from ggVVAMP
[von Manteuffel, Tancredi (2015)]
- ➡ reweighted by full- m_t Born amplitude

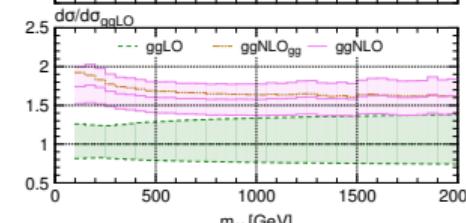
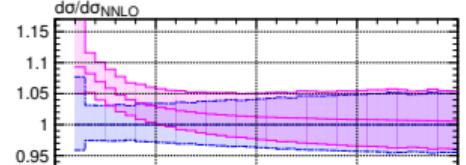
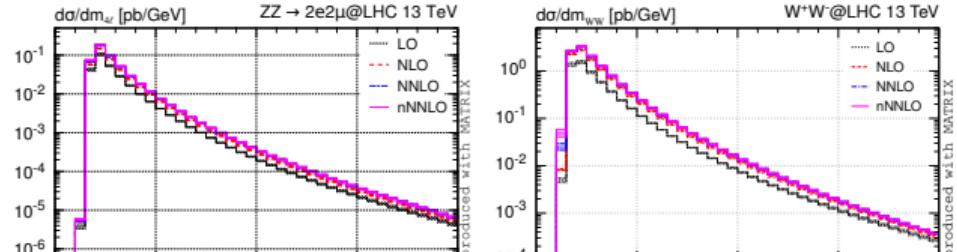
Effect of NLO QCD in gg channel

- highest impact in low-energy regions
- enhancement beyond NNLO QCD scale band

[Grazzini, SK, Wiesemann, Yook (2019)]



[Grazzini, SK, Wiesemann, Yook (2020)]



Best available fixed-order predictions for ZZ/WW production

- NNLO QCD for $q\bar{q}$ channel
- NLO EW combination for $q\bar{q}$ channel
- NLO QCD corrections for gg channel

→ available for all VV processes in
MATRIX v2 [Grazzini, SK, Wiesemann (2021)]

(refinements on m_t effects in gg amplitudes
 still to be added [Grazzini, SK, Wiesemann, Yook (2021)])

Diboson production beyond fixed order

- resummation ($p_{T,VV}^{\text{veto}}, p_{T,\text{jet}}, \dots$)
MATRIX+RADISH [SK, Re, Rottoli, Wiesemann (2020)]

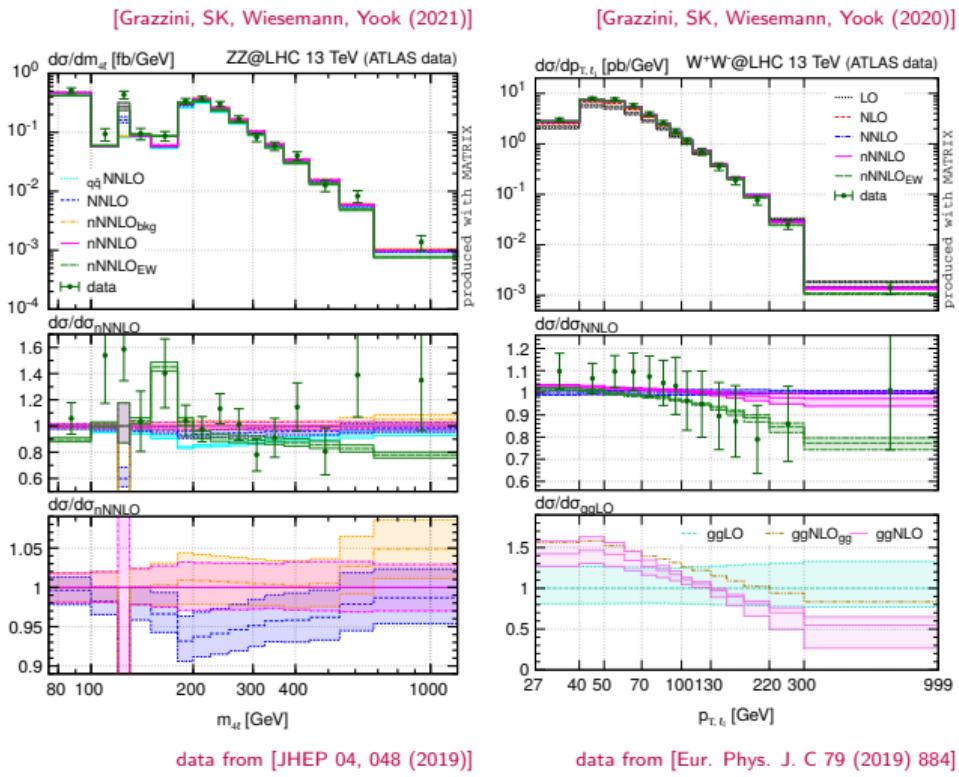
- event generation at NNLO QCD

NNLOPS [WW: Re, Wiesemann, Zanderighi (2018)]

MINNLO_{PS} [Z γ : Lombardi, Wiesemann, Zanderighi (2021);
 WW: Lombardi, Wiesemann, Zanderighi (2021);
 ZZ: Buonocore, Koole, Lombardi, Rottoli, Wiesemann, Zanderighi (2022)]

GENEVA

[$\gamma\gamma$: Alioli, Broggio, Gavardi, SK, Lim, Nagar, Napoletano, Rottoli (2021)
 ZZ: Alioli, Broggio, Gavardi, SK, Lim, Nagar, Napoletano (2021);
 W γ : Cridge, Lim, Nagar (2022)]



Triboson production at NNLO QCD accuracy

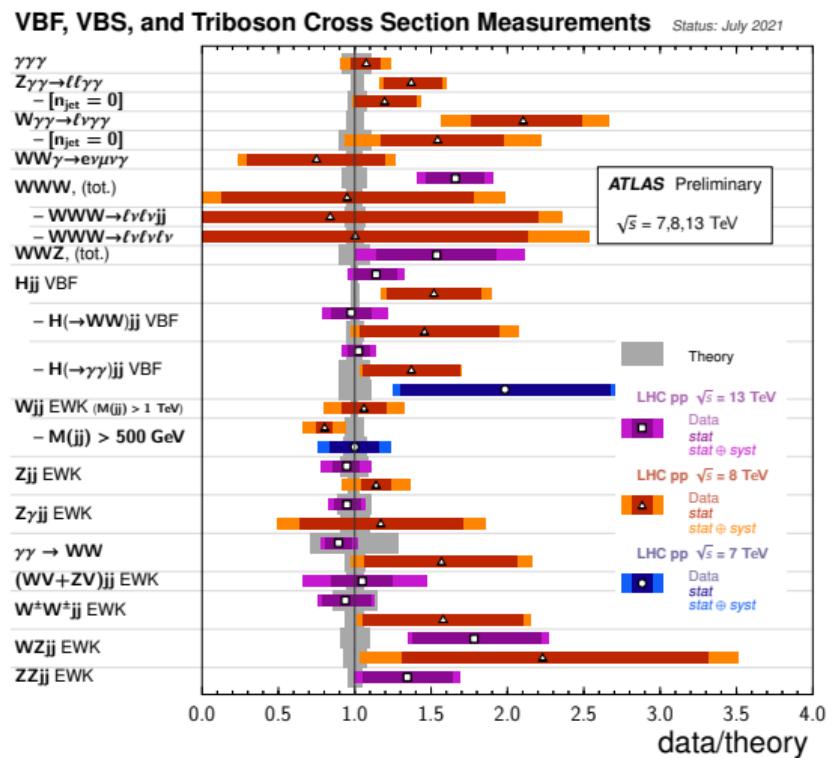
Relevance of triboson production

- important SM test ➔ quartic gauge couplings
- background for BSM searches
- very clean signatures in leptonic decay channels
- at least $\gamma\gamma\gamma$ and $V\gamma\gamma$ processes measured already with statistics collected during LHC Run I
 - ➔ much smaller cross sections for massive VVV
 - ➔ only WWW and WWZ observed recently

Prospects for triboson predictions at NNLO

- **two-loop amplitudes as the bottleneck**
(only available for triphoton production by now)
- NNLO subtraction methods applicable in principle
- complicated calculation (amplitudes, phase spaces)
 - ➔ **challenge for technical performance, but feasible in the **MATRIX** framework**

[ATLAS collaboration (2021)]



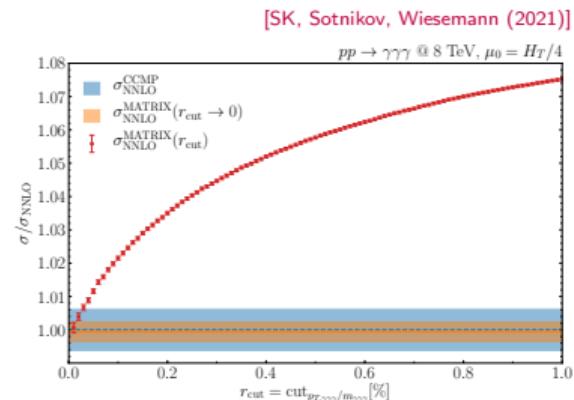
Triphoton production at NNLO QCD accuracy

First MATRIX calculation for genuine $2 \rightarrow 3$ process at NNLO QCD

- q_T subtraction method for colourless final states directly applicable:

$$d\sigma_{\text{NNLO}}^{\gamma\gamma\gamma} = \mathcal{H}_{\text{NNLO}}^{\gamma\gamma\gamma} \otimes d\sigma_{\text{LO}} + \left[d\sigma_{\text{NLO}}^{\gamma\gamma\gamma+\text{jet}} - d\sigma_{\text{NNLO}}^{\gamma\gamma\gamma, \text{CT}} \right]_{r_{\text{cut}} \rightarrow 0}$$

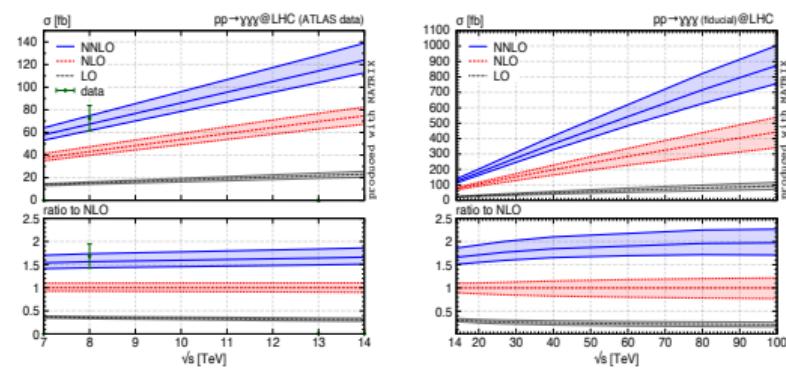
- remarkable numerical control over slicing parameter dependence
- full agreement with independent calculation [Chawdhry, Czakon, Mitov, Poncelet (2020)]



Further important ingredients of the calculation

- highly efficient phase space integration in MUNICH [SK]
- fast and stable 1-loop amplitudes from OPENLOOPS
[Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)]
- fast and stable 2-loop amplitudes [Abreu, Page, Pascual, Sotnikov (2021)]
generated with CARAVEL, using PENTAGONFUNCTIONS++
[Abreu et al. (2020)] [Chicherin, Sotnikov (2020)]

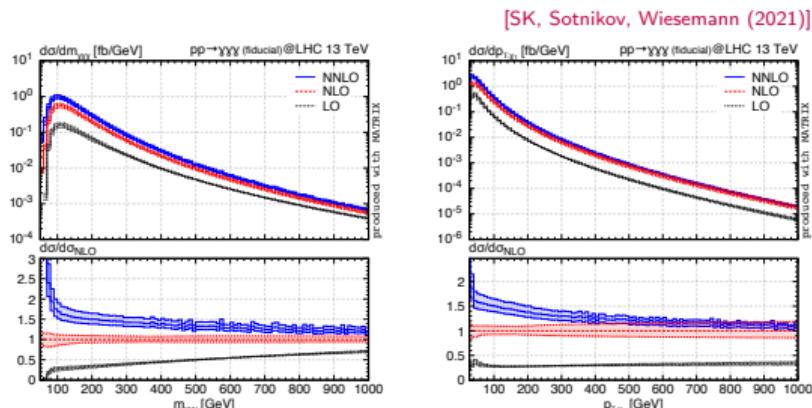
→ talks by Vasily Sotnikov and Ben Page



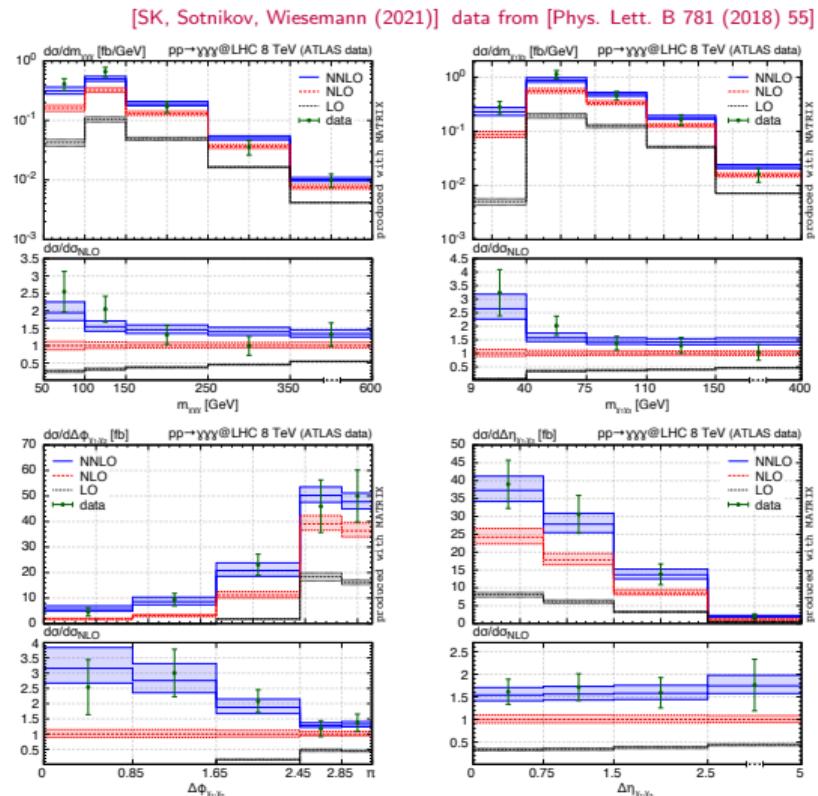
Triphoton production at NNLO QCD accuracy

Comparison with ATLAS data at 8 TeV

- perfect agreement with NNLO QCD predictions, due to both normalization and shape corrections
- significant discrepancies at lower orders



- great numerical performance also with refined resolution and in suppressed phase space regions
- **MATRIX fully suitable for triboson processes**

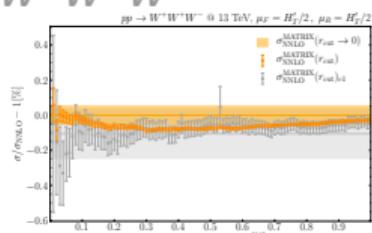


Feasibility studies on massive triboson production at NNLO QCD accuracy

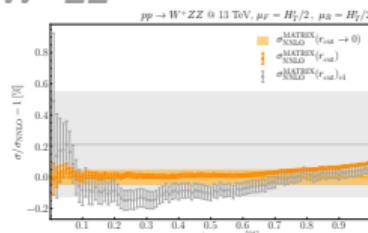
Studies on r_{cut} dependence for inclusive massive VVV production processes

- very good numerical control ➔ permille-level precision achievable within reasonable runtime

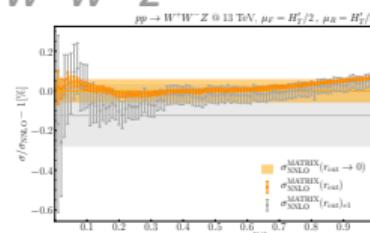
$W^+ W^+ W^-$



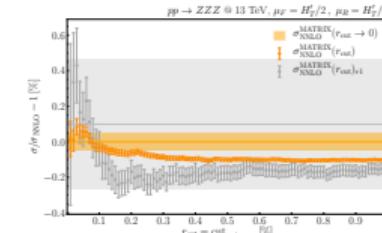
$W^+ ZZ$



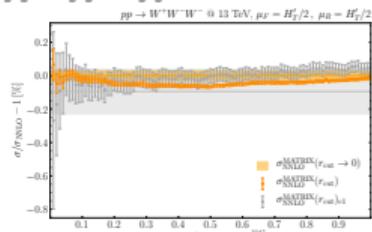
$W^+ W^- Z$



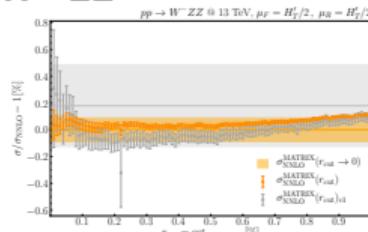
ZZZ



$W^+ W^- W^-$



$W^- ZZ$



Obviously, no robust conclusion on the size of NNLO QCD corrections can be drawn without knowledge of the two-loop amplitudes

➔ Finite remainder of two-loop amplitudes set to zero for these technical feasibility studies.

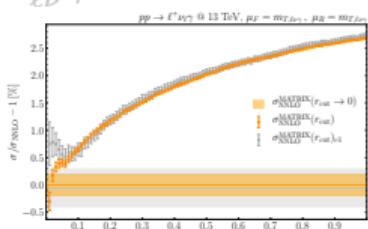
- impact of finite remainder of two-loop amplitudes $H^{(2)}$ typically small, only $\mathcal{O}(2\%)$ for VV processes
- NNLO K-factors without $H^{(2)}$: $\mathcal{O}(1 - 2\%)$ ($\approx +6\%$ from gg channel for WWZ)
 - ➔ no huge NNLO QCD corrections expected for massive VVV production processes

Feasibility studies on $V\gamma\gamma$ production processes at NNLO QCD accuracy

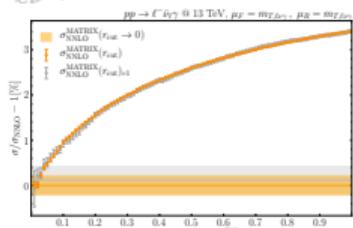
Studies on r_{cut} dependence for $V\gamma\gamma$ processes (standard cuts and photon isolation)

- large power corrections due to photon isolation (as for $\gamma\gamma\gamma$), but numerically well under control

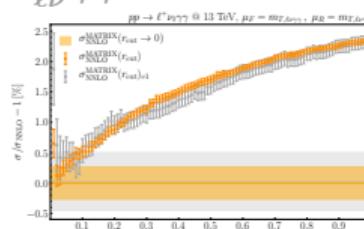
$W_{e\nu}^+ \gamma\gamma$



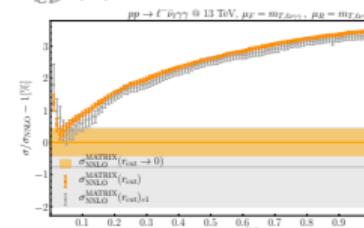
$W_{e\nu}^- \gamma\gamma$



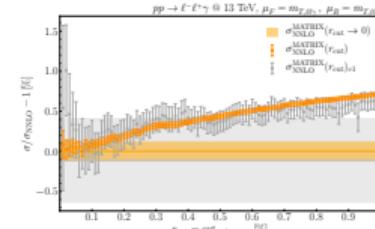
$W_{e\nu}^+ \gamma\gamma$



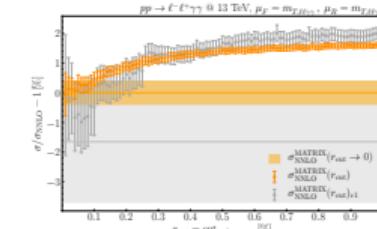
$W_{e\nu}^- \gamma\gamma$



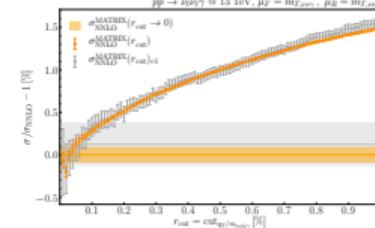
$Zee\gamma\gamma$



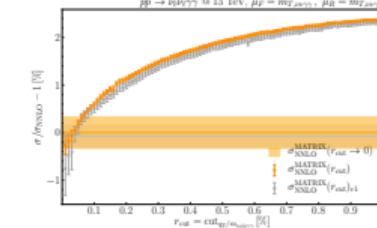
$Zee\gamma\gamma$



$Z\nu\nu\gamma\gamma$



$Z\nu\nu\gamma\gamma$



- impact of finite remainder of two-loop amplitudes $H^{(2)}$ small, only $\mathcal{O}(2 - 3\%)$ for $V\gamma$ processes
- NNLO K -factors without $H^{(2)}$: $\mathcal{O}(10 - 30\%)$ (depending on final state and fiducial cuts)
 - comparable size of NNLO QCD corrections as for $V\gamma$ expected for $V\gamma\gamma$ processes

Production of heavy coloured particles at NNLO QCD accuracy

Extension of q_T subtraction method to production of heavy coloured particles (e.g. top-quark pairs)

$$d\sigma_{\text{NNLO}}^{t\bar{t}} = \mathcal{H}_{\text{NNLO}}^{t\bar{t}} \otimes d\sigma_{\text{LO}} + \left[d\sigma_{\text{NLO}}^{t\bar{t}+\text{jet}} - d\sigma_{\text{NNLO}}^{t\bar{t}, \text{CT}} \right]_{r_{\text{cut}} \rightarrow 0}$$

- counterterm accounts for IR behaviour of real contribution, including soft singularities related to emissions from final-state quarks [Catani, Grazzini, Torre (2014), Ferroglio, Neubert, Pecjak, Yang (2009), Li, Li, Shao, Yang, Zu (2013)]
- $\mathcal{H}_{\text{NNLO}}^{t\bar{t}}$ contains remainder of integrated final-state soft singularities
[Catani, Devoto, Grazzini, Mazzitelli (to appear), Angeles-Martinez, Czakon, Sapeta (2018)]
- massive NLO subtraction required for real-emission part, e.g. massive dipole subtraction
[Catani, Seymour (1997), Catani, Dittmaier, Seymour, Trocsanyi (2002)]

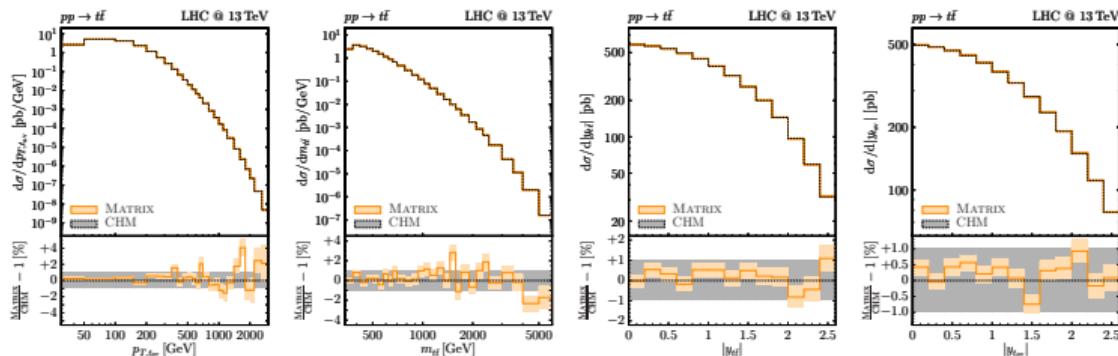
Associated heavy-quark pair production ($t\bar{t}\text{H}$, $t\bar{t}\text{V}$, ...) with identical singularity structure

- numerical solutions required for evaluation of the soft function due to more involved kinematics
 - ➡ no back-to-back configuration of heavy quarks
- proof-of-principle calculation for non-diagonal channels in $t\bar{t}\text{H}$ [Catani, Fabre, SK, Grazzini (2021)]
- two-loop amplitudes as the bottleneck for any beyond $2 \rightarrow 2$ NNLO calculations

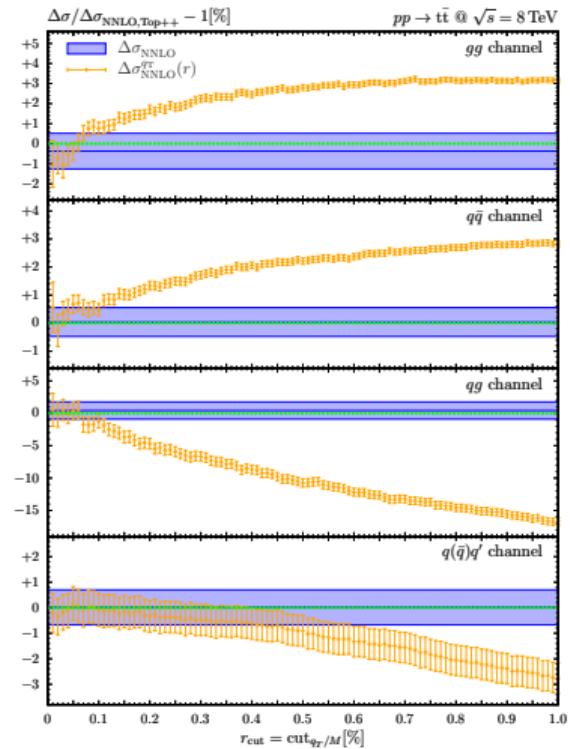
Top-quark pair production at NNLO QCD accuracy

First MATRIX calculation for colourful final states at NNLO QCD

- 2-loop amplitudes from numerical result [Bärnreuther, Czakon, Fiedler (2014)]
- slicing parameter dependence under good numerical control; investigation after splitting into partonic channels
 - full agreement with TOP++ [Czakon, Mitov (2014)]
- successful validation also on the level of differential distributions
 [Catani, Devoto, Grazzini, SK, Mazzitelli (2019)]
 - (comparison against results from [Czakon, Heymes, Mitov (2017)])



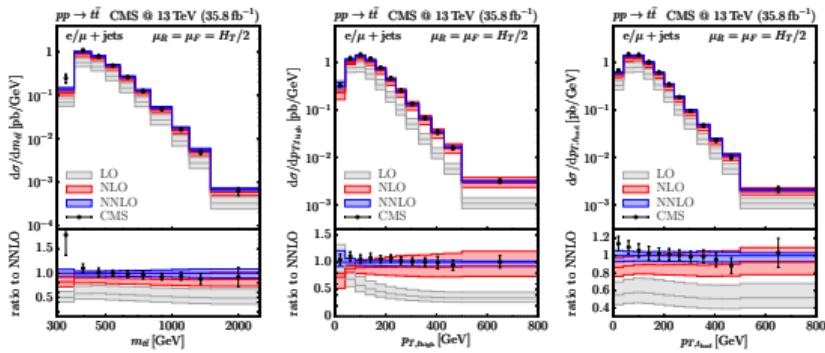
[Catani, Devoto, Grazzini, SK, Mazzitelli, Sargsyan (2019)]



Top-quark pair production at NNLO QCD accuracy

Good agreement with (multi)differential CMS data

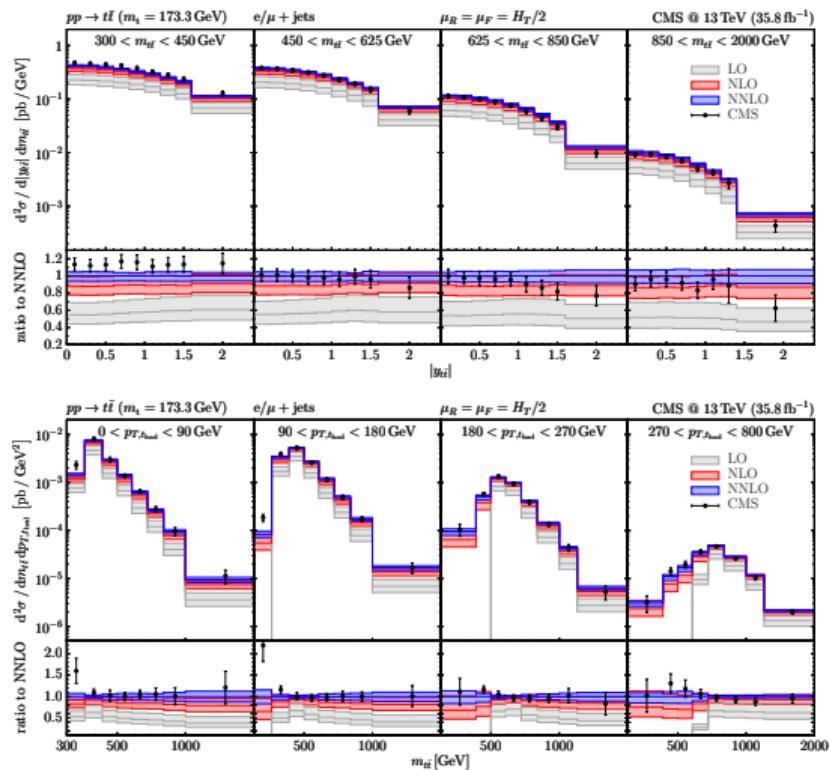
- lowest $m_{t\bar{t}}$ bin problematic: sensitivity to m_t value, threshold effects, extrapolation to stable tops, ...
- instabilities related to $p_{T,t\bar{t}} \rightarrow 0$ region
 - would require resummation/shower matching



Indications for perturbative convergence

- widely overlapping bands from NLO to NNLO with reduced scale variation uncertainties

[Catani, Devoto, Grazzini, SK, Mazzitelli (2019)] data from [Phys. Rev. D 97 (2018) 112003]

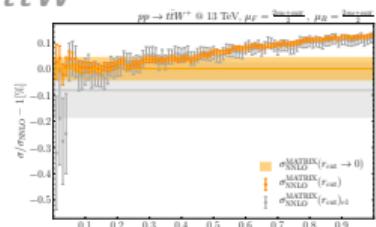


Feasibility studies on associated heavy-quark pair production at NNLO QCD accuracy

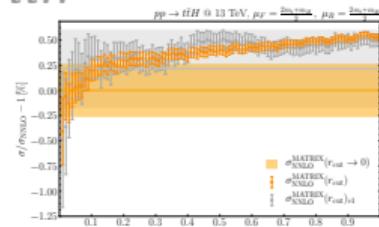
Studies on r_{cut} dependence for inclusive $Q\bar{Q} + X$ processes

- proof-of-principle for non-diagonal channels in $t\bar{t}H$ [Catani, Fabre, SK, Grazzini (2021)]
- finite remainders of two-loop amplitudes and of the soft function neglected in these studies
 - no reliable estimate of NNLO QCD result, but cancellation of IR divergences well under control
 - permille-level precision achievable within reasonable runtime

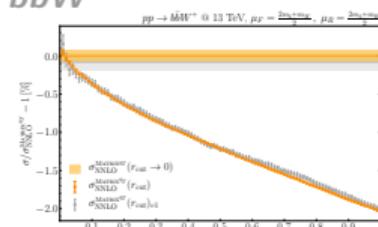
$t\bar{t}W^+$



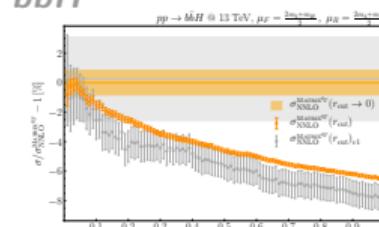
$t\bar{t}H$



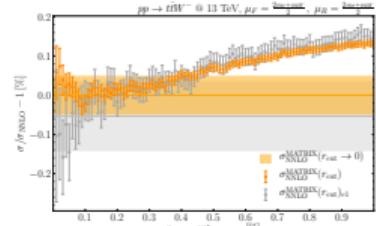
$b\bar{b}W^+$



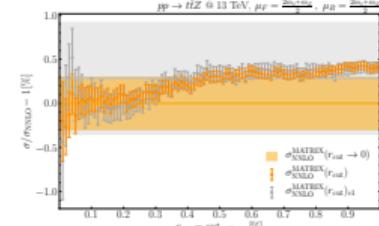
$b\bar{b}H$



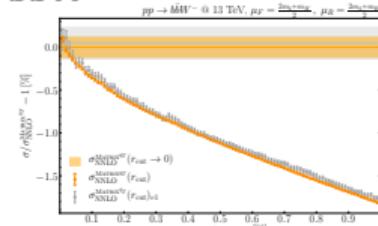
$t\bar{t}W^-$



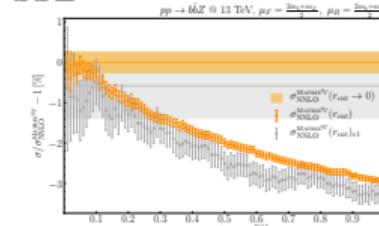
$t\bar{t}Z$



$b\bar{b}W^-$



$b\bar{b}Z$



Conclusions & Outlook

The **MATRIX** framework for NNLO QCD calculations (fall 2017)

- based on the **MUNICH** integrator, amplitudes from **OPENLOOPS** and dedicated 2-loop amplitudes (e.g. **VWAMP**)
- available processes: H, V, $\gamma\gamma$, $V\gamma$, VV at NNLO QCD for all leptonic decay channels

New features in the release **MATRIX v2** (summer 2021)

- combination of NNLO QCD and NLO EW corrections for massive diboson processes
- leading N^3LO contributions in terms of NLO QCD corrections to loop-induced gluon fusion channels

Additional features in the upcoming release **MATRIX v2.1** (spring 2022)

- recoil-driven linear power corrections and bin-wise r_{cut} extrapolation for distributions
- new processes: $\gamma\gamma\gamma$ (first $2 \rightarrow 3$ application), $t\bar{t}$ (heavy-quark final state)

Recent achievements and future applications of the **MATRIX** framework

- application to $2 \rightarrow 3$ processes at NNLO QCD ➔ feasibility studies shown beyond purely massless case
- mixed NNLO QCD–EW corrections for massive Drell–Yan production ➔ talk by Luca Buonocore
- towards a treatment of light jets at the Born level using the k_T^{ness} variable ➔ talk by Chiara Savoini

Overview

Recent developments within the Macro framework

01 Precision evaluations – the key to fully regular MC@NLO calculations

02 The Macro framework for automated NNLO/QCD calculations [and beyond]

03 Gluon

04 The Macro framework for automated NNLO calculations

05 The Macro framework for automated NNLO calculations

06 Panel shows linear gauge corrections for gluon-gluon fusion

07 Description of $\pi^+ \pi^- \rightarrow \text{diquark-antidiquark}$ – results over $m = 1.5$ GeV

08 Description of $\pi^+ \pi^- \rightarrow \text{diquark-antidiquark}$ – results over $m = 1.5$ GeV

09 Description of linear gauge corrections in neutral current Drell-Yan process

10 Descriptions with NNLO for central energy Drell-Yan process with symmetric cuts

11 Descriptions linear gauge corrections for different initial and final configurations

12 Descriptions linear gauge corrections for different initial and final configurations

13 Main features of EW corrections

14 Combination of QCD and EW corrections for dijet production – $\mu_{\text{EW}} = 0$

15 Combination of QCD and EW corrections for dijet production – $\mu_{\text{EW}} = 1$

16 NNLO QCD corrections to loop-induced gluon channel in ZZ-WW production

17 One additional order predictions for ZZ-WW production

18 Descriptions of NNLO QCD accuracy

19 Top/bottom production at NNLO QCD accuracy

20 Top/bottom production at NNLO QCD accuracy

21 Possibility studies on massive dilepton production at NNLO QCD accuracy

22 Possibility studies on VBF production processes at NNLO QCD accuracy

23 Production of heavy colored particles at NNLO QCD accuracy

24 Top/bottom production at NNLO QCD accuracy

25 Descriptions of cross sections with NNLO QCD accuracy

26 Descriptions of cross sections with NNLO QCD accuracy

Backup

This figure displays 12 panels of ATLAS physics results, organized into four columns and three rows.

- Column 1:**
 - Top Panel:** Summary of ATLAS Monte-Carlo (MC) and Experimental results for $\gamma\gamma \rightarrow \text{hadrons}$.
 - Middle Panel:** Comparison of NLO QCD predictions for $\gamma\gamma \rightarrow \text{hadrons}$ with experimental data.
 - Bottom Panel:** Results for mixed NLO QCD-EW corrections for Drell-Yan processes.
- Column 2:**
 - Top Panel:** Interfering dedicated 2-loop amplitudes to $\text{Muons}/\text{Muons}$.
 - Middle Panel:** Performance features of the Muon phase space integrator.
 - Bottom Panel:** Summary of ATLAS diboson measurements.
- Column 3:**
 - Top Panel:** Illustration of effect of off-shellness in $\gamma\gamma \rightarrow \text{hadrons}$ at NLO.
 - Middle Panel:** Mixed NLO QCD-EW calculation for production of massive charged particles.
 - Bottom Panel:** Results for mixed NLO QCD-EW corrections in off-shell Z production.
- Column 4:**
 - Top Panel:** Combination of QCD and EW corrections for diboson production – $\mu_{\text{F}} = -H_{\text{vis}}$, $\mu_{\text{R}} = 0$.
 - Middle Panel:** NLO QCD corrections to loss-induced gluon channel in ZZ-WW production.
 - Bottom Panel:** Bottom-quark pair production at NNLO QCD accuracy.

Supplying MUNICH/MATRIX with 1-loop amplitudes

Process-independent interfaces to general automated amplitude generators

- **OPENLOOP_S** [Cascioli, Maierhöfer, Pozzorini (2012); SK, Lindert, Maierhöfer, Pozzorini, Schönherr (2015)], written in **FORTRAN**
 - general code and process libraries
 - on-the-fly tensor reduction [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)] with hybrid-precision stability system
 - scalar integrals from **COLLIER** [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2011)] or **ONELOOP** [van Hameren (2011)]
- **RECOLA** [Actis, Denner, Hofer, Lang, Scharf, Uccirati (2017)], written in **FORTRAN**
 - on-the-fly generation of amplitudes
 - tensor reduction and scalar integrals via **COLLIER** [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2003, 2006, 2011)]
 - different model files available, also for SMEFT and BSM applications
- modular structure of **MUNICH** allows other generators to be interfaced as well

Several dedicated interfaces developed in context of **MATRIX** applications

- loop×tree and loop×loop colour (and spin) correlators
- helicity amplitudes, colour-stripped amplitudes to construct 4-colour correlators
- imaginary parts of loop×tree amplitudes and correlators, helicity-flip amplitudes

Interfacing dedicated 2-loop amplitudes to MUNICH/MATRIX

- Higgs, Drell–Yan, **VH**, $\gamma\gamma$, $V\gamma$ production
 - direct implementation of public analytic results, e.g. for $V\gamma$ [Gehrmann, Tandredi (2012)]
- **VV** production — **qqVVAMP** [Gehrmann, von Manteuffel, Tancredi (2015)] and **ggVVAMP** [von Manteuffel, Tancredi (2015)] libraries
 - C++ libraries using **GINAC** [Bauer, Frink, Kreckel (2002); Vollinga, Weinzierl (2005)] and **CLN** for arbitrary precision arithmetics
 - IBP approach, generated using **MATHEMATICA**, **FORM** [Vermaseren et al.], **REDUCE2** [von Manteuffel, Studerus ('12)]
 - independent calculation of amplitudes in [Caola, Henn, Melnikov, Smirnov, Smirnov (2015; 2016)]
 - Higgs-mediated helicity amplitudes with full m_t dependence from [Harlander, Prausa, Usovitsch (2019; 2020)]
- $\gamma\gamma\gamma$ production — amplitudes from [Abreu, Page, Pascual, Sotnikov ('20)]
 - C++ library, generated by **CARAVEL** [Abreu et al. (2020)], applying **PENTAGONFUNCTIONS++** [Chicherin, Sotnikov (2020)]
 - numerical unitarity and analytic reconstruction techniques [Ita (2015); Abreu et al. (2018; 2018; 2019; 2019)]
- **HH** production (full m_t dependence) — **HHGRID** library [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]
 - PYTHON based numerical interpolation of amplitude grid
 - generated by 2-loop extension of **GoSAM** [Jones (2016)], **REDUCE2** [von Manteuffel, Studerus ('12)], **SECDEC3** [Borowka et al. (2015)]
- **Q \bar{Q}** production — amplitude grids from [Bärnreuther, Czakon, Fiedler (2014)]
 - FORTRAN routine for numerical interpolation of 2-dimensional grid, improved by expansions

Performance features of the MUNICH phase space integrator

Issue of poorly populated regions

- sample case: high-energy tails
- standard phase space optimization samples points in bulk region

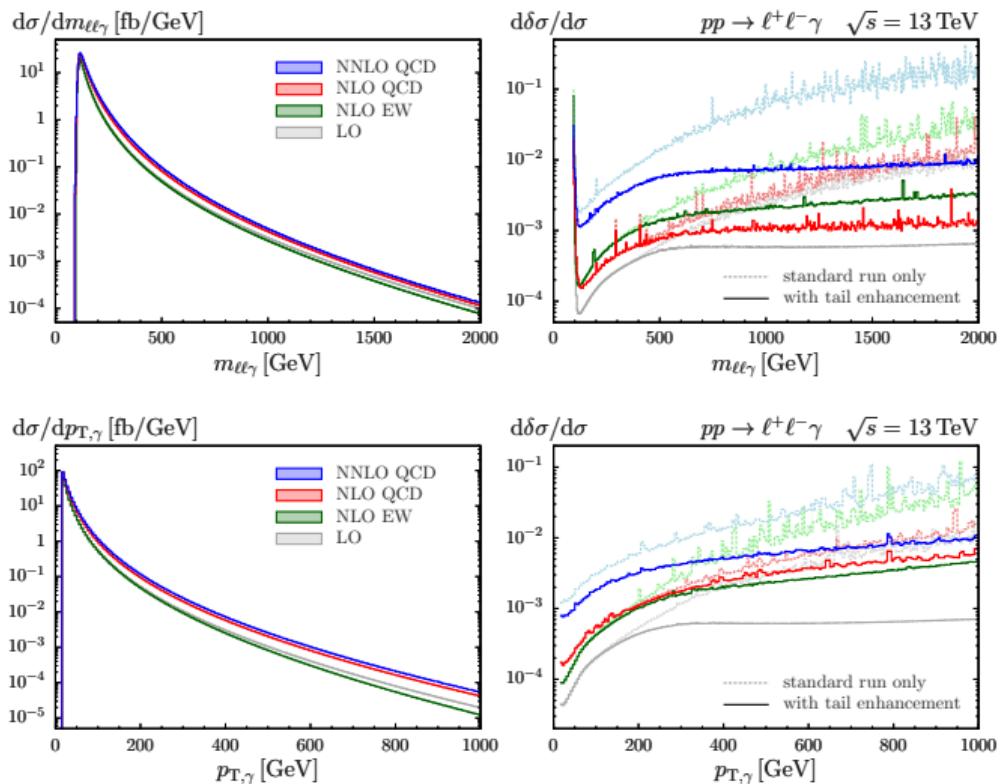
Solution in MUNICH integrator

- additional runs with optimization including a general bias factor
- sophisticated automated combination with results from standard runs

Significantly improved errors

- $\mathcal{O}(10)$ and better with doubled runtime
- simultaneous enhancement of observables

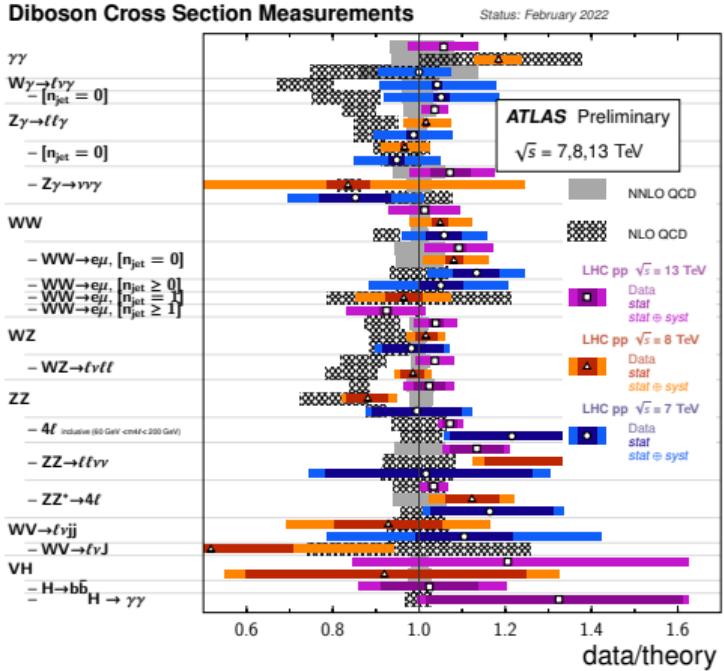
Good performance also for off-shell regions of intermediate resonances



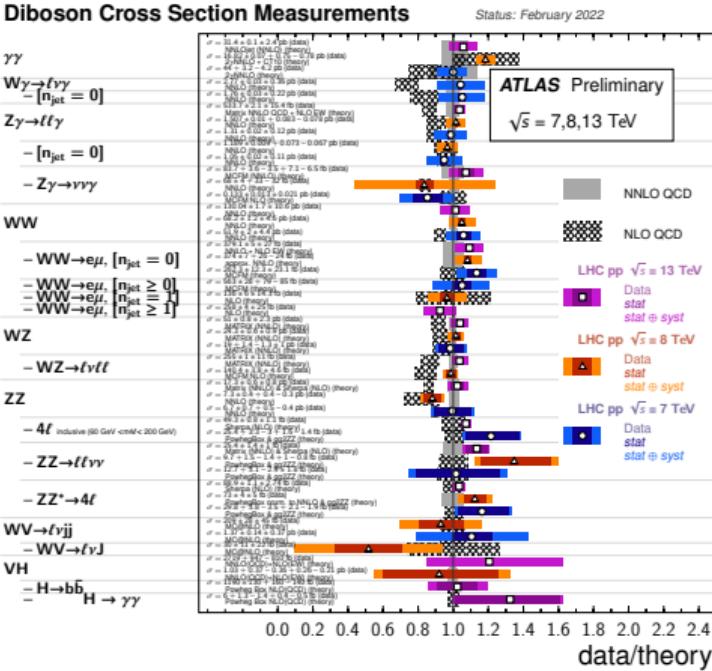
Summary of ATLAS diboson measurements

[ATLAS collaboration (2022)]

Diboson Cross Section Measurements



Diboson Cross Section Measurements



$\mathcal{L} dt$ [fb \cdot s]	Reference
130	JHEP 11 (2021) 168
30.2	PRD 95 (2021) 116005
30.2	PRD 95 (2021) 116006
4.6	PRD 97 (2022) 112003 [2020]
4.6	arXiv:1807.16116
4.6	PRD 97 (2022) 112004 [2020]
36.1	JHEP 09 (2020) 054
20.3	PRD 93 (2022) 076006
4.6	PRD 95 (2022) 076005 [2020]
20.3	PRD 93 (2022) 076007 [2020]
4.6	PRD 97 (2022) 112003 [2020]
36.1	JHEP 12 (2021) 016
20.3	PRD 93 (2022) 076008 [2020]
4.6	PRD 97 (2022) 112004 [2020]
36.1	EPJC 79 (2019) 884
20.3	PLB 763, 111 (2014)
4.6	Phys. Rev. D 87 (2013) 112001
36.1	PRD 93 (2022) 112002 [2020]
20.3	JHEP 09 (2016) 039
4.6	PRD 93 (2021) 050003
20.3	PLB 763, 111 (2016)
130	ATLAS-COM-PHYS-2020-074
36.1	EPJC 79 (2019) 535
20.3	PRD 93 (2022) 060006 [2020]
4.6	EPJC 79 (2019) 2173
36.1	EPJC 79 (2019) 335
20.3	PRD 93 (2022) 060004 [2020]
36.1	PRD 97 (2018) 033004
20.3	JHEP 01, 098 (2017)
4.6	EPJC 79 (2019) 172
130	JHEP 07 (2021) 065
4.6	ATLAS-CONF-2021-055
36.1	JHEP 10 (2021) 127
20.3	JHEP 07 (2021) 127
4.6	JHEP 07 (2021) 056
130	JHEP 07 (2021) 055
20.3	PLB 753, 033-073 (2018)
4.6	JHEP 02, 138 (2013)
20.3	EPJC 77 (2017) 563
4.6	JHEP 01, 049 (2015)
20.2	EPJC 77 (2017) 563
36.1	JHEP 12 (2017) 004
20.3	JHEP 12 (2017) 024
130	ATLAS-COM-2020-027
130	ATLAS-COM-2021-053

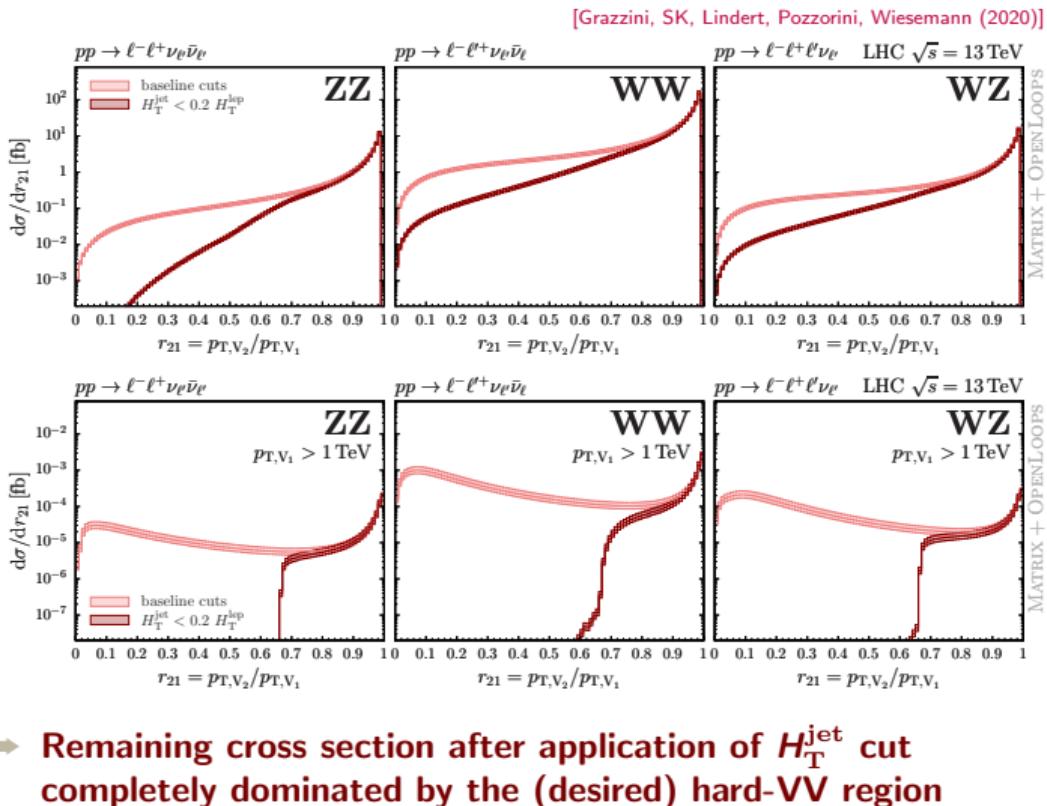
Combination of QCD and EW corrections for diboson production – $p_{\text{T},V_2}/p_{\text{T},V_1}$

Restriction to hard-VV topologies

- direct requirement on the hardness of the two vector bosons
- moderate jet veto to suppress hard-Vj topologies: $H_{\text{T}}^{\text{jet}} < 0.2 H_{\text{T}}^{\text{lep}}$

Illustration of effect on distribution in $r_{21} = p_{\text{T},V_2}/p_{\text{T},V_1}$ at NLO QCD
($r_{21} = 1$ for LO kinematics)

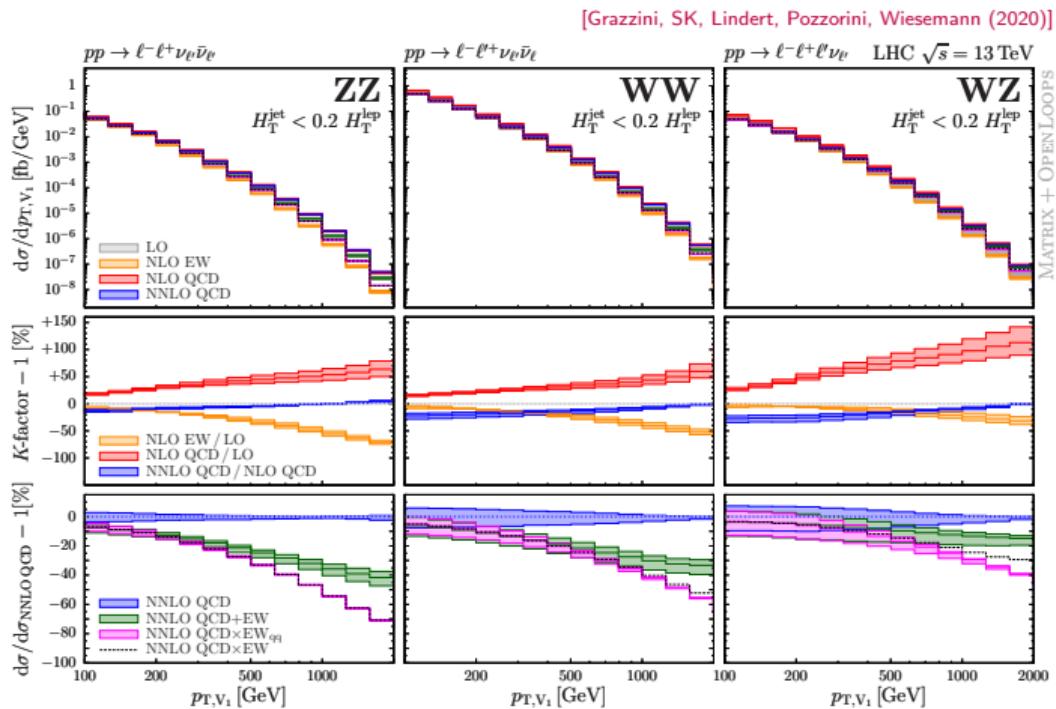
- only baseline cuts (upper panels)
 - cross section peaked at $r_{21} \gtrsim 0.8$; widely unaffected by $H_{\text{T}}^{\text{jet}}$ cut
- region $p_{\text{T},V_1} > 1 \text{ TeV}$ (lower panels)
 - second peak at $r_{21} \lesssim 0.3$ where hard-Vj topologies dominate; completely removed by $H_{\text{T}}^{\text{jet}}$ cut



Combination of QCD and EW corrections for diboson production – $p_{\text{T},V_1} - H_{\text{T,jet}}$ cut

With $H_{\text{T}}^{\text{jet}} < 0.2 H_{\text{T}}^{\text{lep}}$, cross section dominated by hard-VV region

- justification to apply NLO EW correction factor on (N)NLO QCD cross sections
- **additive** underestimates EW effects; difference to multiplicative would overestimate uncertainties
- **both multiplicative** approaches well motivated to catch leading effects from mixed QCD–EW corrections
 - difference between the two as a rough estimate of $\mathcal{O}(\alpha_s \alpha)$ uncertainties
- Restriction to hard-VV topologies **reduces uncertainties from mixed QCD–EW higher-order effects**, but should also **increase the sensitivity of experimental searches for BSM effects (like AGCs)**



NLO QCD corrections to loop-induced gluon channel in ZZ/WW production

Integrated cross sections for $pp \rightarrow e^- e^+ \mu^- \mu^+$ (setup of ATLAS 8 TeV ZZ analysis [JHEP 01, 099 (2017)])

[Grazzini, SK, Wiesemann, Yook (2019)]

\sqrt{s}	8 TeV		13 TeV		$\sigma/\sigma_{\text{NLO}} - 1$
	σ [fb]		σ [fb]		
LO	8.1881(8) ^{+2.4%} _{-3.2%}	13.933(1) ^{+5.5%} _{-6.4%}	-27.5%	-29.8%	
NLO	11.2958(4) ^{+2.5%} _{-2.0%}	19.8454(7) ^{+2.5%} _{-2.1%}	0%	0%	
$q\bar{q}\text{NNLO}$	12.09(2) ^{+1.1%} _{-1.1%}	21.54(2) ^{+1.1%} _{-1.2%}	+7.0%	+8.6%	
σ [fb]			$\sigma/\sigma_{\text{ggLO}} - 1$		$\sigma/\sigma_{\text{ggLO}} - 1$
	σ [fb]		σ [fb]		
ggLO	0.79355(6) ^{+28.2%} _{-20.9%}	2.0052(1) ^{+23.5%} _{-17.9%}	0%	0%	
ggNLO _{gg}	1.4787(4) ^{+15.9%} _{-13.1%}	3.626(1) ^{+15.2%} _{-12.7%}	+86.3%	+80.8%	
ggNLO	1.3892(4) ^{+15.4%} _{-13.6%}	3.425(1) ^{+13.9%} _{-12.0%}	+75.1%	+70.8%	
σ [fb]			$\sigma/\sigma_{\text{NLO}} - 1$		$\sigma/\sigma_{\text{NLO}} - 1$
	σ [fb]		σ [fb]		
NNLO	12.88(2) ^{+2.8%} _{-2.2%}	23.55(2) ^{+3.0%} _{-2.6%}	+14.0%	+18.7%	
nNNLO	13.48(2) ^{+2.6%} _{-2.3%}	24.97(2) ^{+2.9%} _{-2.7%}	+19.3%	+25.8%	

- relatively large corrections of 5% – 7% for ZZ, exceeding the NNLO QCD uncertainty bands (slightly smaller effect of 2% – 3% for WW)

Integrated cross sections for $pp \rightarrow e^- \mu^+ \nu_\mu \bar{\nu}_e$ (setup of ATLAS 13 TeV WW analysis [EPJC 79, 884 (2019)])

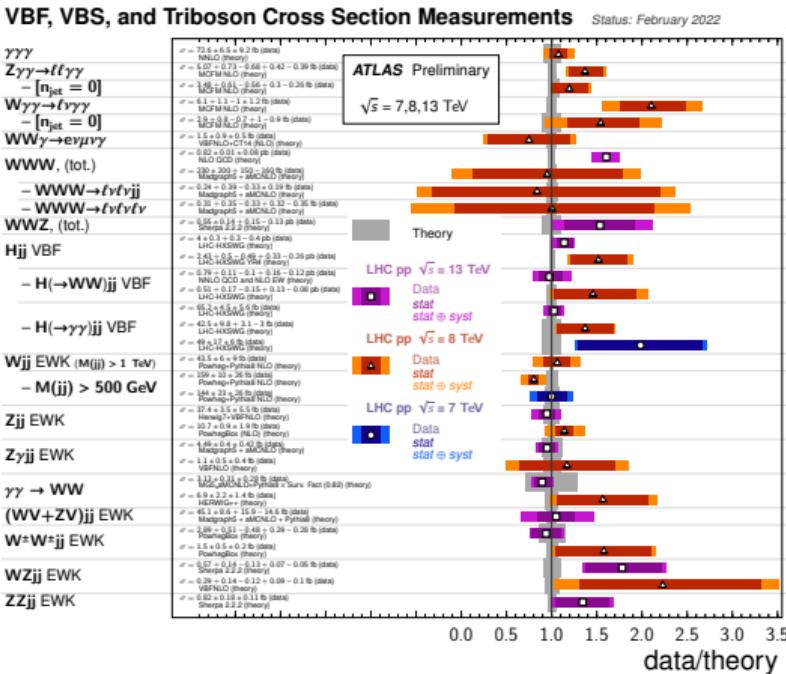
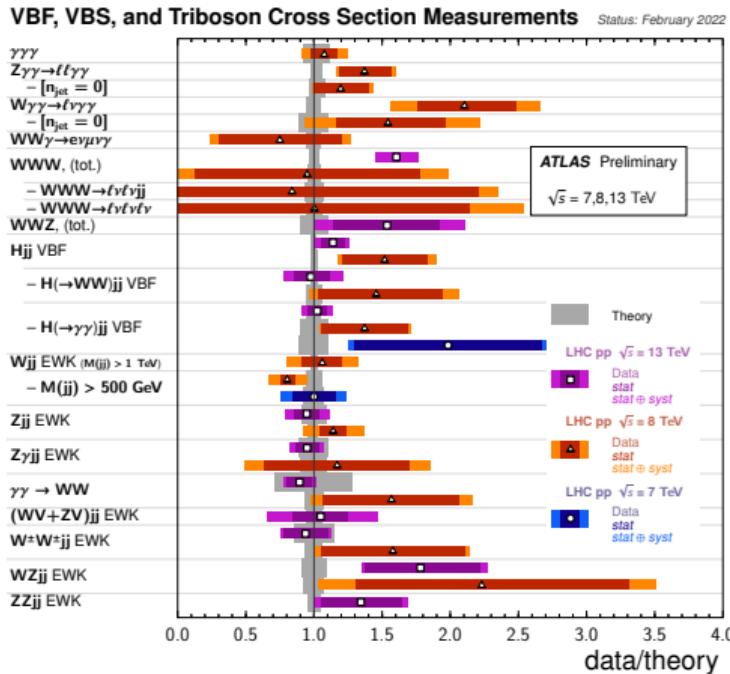
[Grazzini, SK, Wiesemann, Yook (2020)]

\sqrt{s}	jet veto		no veto		$\sigma/\sigma_{\text{NLO}} - 1$
	σ [fb]		σ [fb]		
13 TeV					
LO	284.11(1) ^{+5.5%} _{-6.5%}		284.11(1) ^{+5.5%} _{-6.5%}		-15.5% -43.7%
NLO	336.42(3) ^{+1.6%} _{-2.0%}		504.36(3) ^{+4.1%} _{-3.3%}		+0.% +0.%
$q\bar{q}\text{NNLO}$	336.8(2) ^{+0.7%} _{-0.5%}		558.5(2) ^{+2.1%} _{-1.9%}		+0.1% +10.7%
σ [fb]			$\sigma/\sigma_{\text{ggLO}} - 1$		$\sigma/\sigma_{\text{ggLO}} - 1$
	σ [fb]		σ [fb]		
ggLO	21.965(4) ^{+25.7%} _{-18.4%}		21.965(4) ^{+25.7%} _{-18.4%}		+0.% +0.%
ggNLO _{gg}	31.68(6) ^{+10.8%} _{-10.6%}		38.49(6) ^{+15.9%} _{-13.3%}		+44.2% +75.2%
ggNLO	28.79(6) ^{+7.8%} _{-9.1%}		37.57(6) ^{+15.3%} _{-13.0%}		+31.1% +71.0%
σ [fb]			$\sigma/\sigma_{\text{NLO}} - 1$		$\sigma/\sigma_{\text{NLO}} - 1$
	σ [fb]		σ [fb]		
NNLO	358.7(2) ^{+1.2%} _{-0.9%}		580.5(2) ^{+2.9%} _{-2.6%}		+6.6% +15.1%
nNNLO	365.6(2) ^{+0.4%} _{-0.6%}		596.1(2) ^{+2.8%} _{-2.6%}		+8.7% +18.2%
σ [fb]			$\sigma/\sigma_{\text{nNNLO}}$		$\sigma/\sigma_{\text{nNNLO}}$
	σ [fb]		σ [fb]		
nNNLO _{EW}	354.3(2) ^{+0.5%} _{-0.8%}		580.2(2) ^{+2.7%} _{-2.6%}		-3.1% -2.7%

- similar size as (negative) EW corrections

Summary of ATLAS triboson, VBF and VBS measurements

[ATLAS collaboration (2022)]

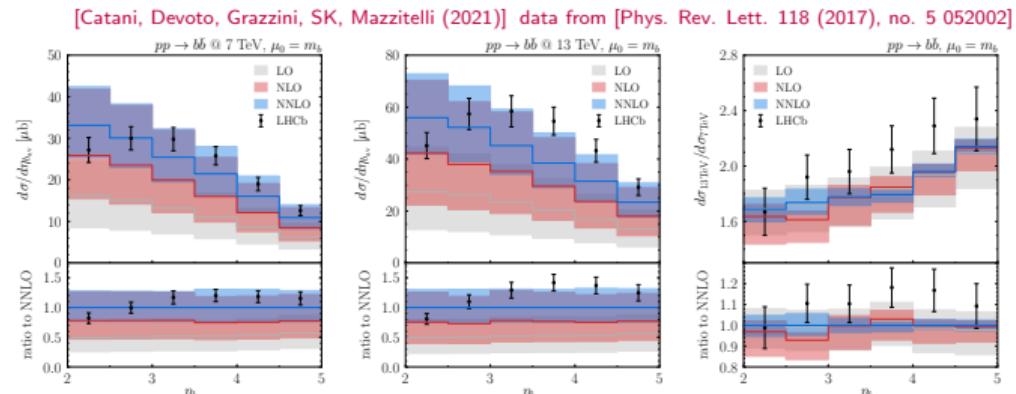
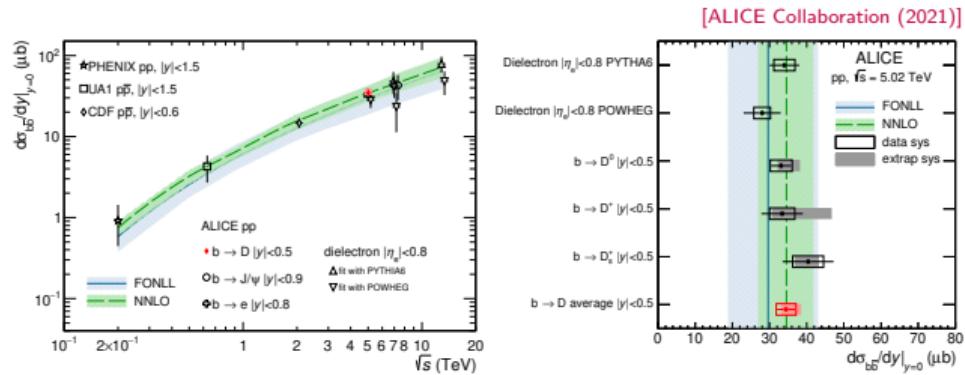
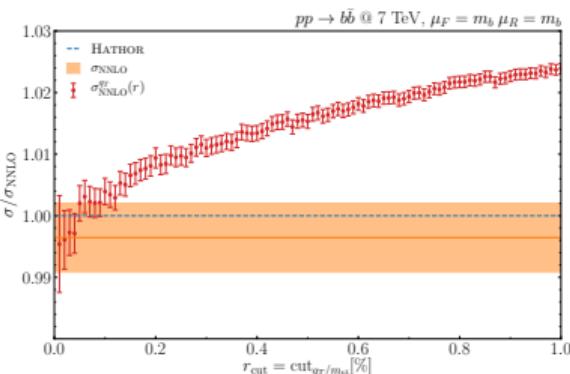


$\int L dt$ [fb $^{-1}$]	Reference
20.2	PLB 781 (2018) 55
20.3	PRD 93, 112002 (2016)
20.3	PRD 93, 112002 (2016)
20.3	PRD 93, 031802 (2016)
20.3	PRD 93, 031802 (2016)
20.2	EPJC 77 (2017) 646
139	arXiv:2201.13045
20.3	EPJC 77 (2017) 141
20.3	EPJC 77 (2017) 141
79.8	PLB 798 (2019) 134913
139	ATLAS-CONF-2021-053
20.3	EPJC 76 (2016) 6
139	ATLAS-CONF-2021-014
20.3	PRD 92, 012006 (2015)
139	ATLAS-CONF-2019-029
20.3	ATLAS-CONF-2015-060
4.5	ATLAS-CONF-2015-060
20.2	EPJC 77 (2017) 474
20.2	EPJC 77 (2017) 474
4.7	EPJC 77 (2017) 474
139	EPJC 81 (2021) 163
20.3	JHEP 04, 031 (2014)
139	ATLAS-CONF-2021-038
20.3	JHEP 07 (2017) 107
139	PLB 816 (2021) 136190
35.5	PRD 94, 032007 (2019)
36.1	PRL 123, 161801 (2019)
20.3	PRD 96, 012007 (2017)
36.1	PLB 793 (2019) 469
20.3	PRD 93, 092004 (2016)
139	arXiv:2004.10612

Bottom-quark pair production at NNLO QCD accuracy

Application to bottom quarks

- conventionally similar to $t\bar{t}$ production
- applications in all LHC experiments
- larger uncertainties due to lower scales
 - reduction through ratios with partial cancellation of uncertainties
- numerically more challenging ($m_b \ll m_t$)
 - calculation still remarkably stable



Mixed NNLO QCD-EW calculation for production of massive charged particles

Extension of q_T subtraction method to mixed QCD-EW corrections of $\mathcal{O}(\alpha_s^m \alpha^n)$

$$d\sigma_{(m,n)}^{\ell\ell/\ell\nu} = \mathcal{H}_{(m,n)}^{\ell\ell/\ell\nu} \otimes d\sigma_{\text{LO}} + \left[d\sigma_{(m,n)}^{\ell\ell/\ell\nu, \text{R}} - d\sigma_{(m,n)}^{\ell\ell/\ell\nu, \text{CT}} \right]_{r_{\text{cut}} \rightarrow 0}$$

- $m = 1(2)$ and $n = 0$: (N)NLO QCD corrections
- $m = 0$ and $n = 1$: NLO EW corrections
- $m = 1$ and $n = 1$: mixed NNLO QCD-EW corrections

(limitation: no massless jets (for $m \geq 1$) and no massless charged particles (for $n \geq 1$) allowed at LO)

Strategy to cancel IR singularities in mixed QCD-EW corrections

- abelianisation procedure, starting from heavy-quark pair production at NNLO QCD
 - ➡ for neutral final states, abelianisation of standard q_T subtraction method is sufficient
(mixed QCD-QED corrections on $pp \rightarrow Z$ [De Florian, Der, Fabre (2018)], $pp \rightarrow \nu\bar{\nu}$ [Cieri, De Florian, Der, Mazzitelli (2020)])
- colourless final state ($\ell\ell/\ell\nu$) results in soft final-state singularities of pure QED origin
 - ➡ much simpler IR structure than in heavy-quark pair production at NNLO QCD
- finite charged-lepton mass required to regularize collinear final-state singularities

Towards mixed NNLO QCD-EW corrections for Drell-Yan processes

Mixed NNLO QCD-QED corrections

- on-shell Z production [De Florian, Der, Fabre (2018)]
- off-shell $Z \rightarrow \nu\bar{\nu}$ production
[Cieri, De Florian, Der, Mazzitelli (2020)]
- on-shell Z production with decay, including NLO QCD (production) \times NLO QED (decay)
[Delto, Jaquier, Melnikov, Röntsch (2019)]

Mixed NNLO QCD-EW corrections for on-shell DY production

- on-shell Z production [Bonciani, Buccioni, Rana, Vicini (2020)]
- on-shell W production
[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2021)]
- estimate of the impact on W mass extraction
[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2021)]

Mixed NNLO QCD-EW corrections beyond the on-shell approximation

- pole approximation (PA) [Dittmaier, Huss, Schwinn (2014 & 2015)]

Recent progress towards full mixed NNLO QCD-EW corrections

- two-loop master integrals
[Bonciani, Di Vita, Mastrolia, Schubert (2016),
Heller, von Manteuffel, Schabinger (2019), Hasan, Schubert (2020)]
- two-loop amplitude for neutral current DY
[Heller, von Manteuffel, Schabinger, Spiesberger (2020)]
- complete result for $\mathcal{O}(n_F \alpha_s \alpha)$ effects
[Dittmaier, Schmidt, Schwarz (2020)]

Mixed NNLO QCD-EW calculation within the MATRIX framework with massive leptons (muons)

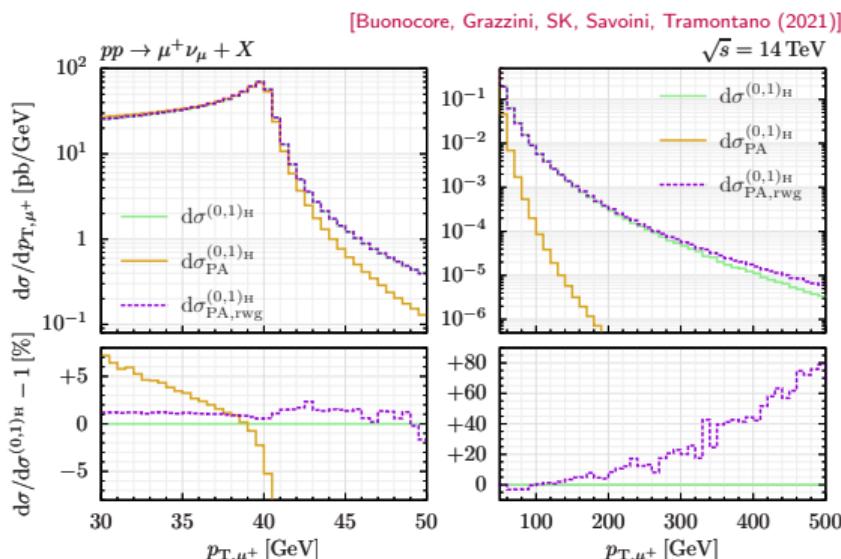
- off-shell $pp \rightarrow W \rightarrow \mu^+ \nu_\mu$ calculation with two-loop amplitudes in a reweighted PA approach
[Buonocore, Grazzini, SK, Savoini, Tramontano (2021)]
- off-shell $pp \rightarrow Z/\gamma \rightarrow \mu^+ \mu^-$ calculation with full two-loop amplitudes [Bonciani, Rana, Vicini (to appear)]
[Bonciani, Buonocore, Grazzini, SK, Tramontano, Rana, Vicini ('21)]

Justification of (reweighted) pole approximation approaches

PA vs. reweighted PA at NLO EW

$$H_{\text{PA}}^{(0,1)} = 2\text{Re} \left(\mathcal{M}_{\text{fin}}^{(0,1)} \mathcal{M}^{(0,0)*} \right)_{\text{PA}} / \left| \mathcal{M}^{(0,0)} \right|^2$$

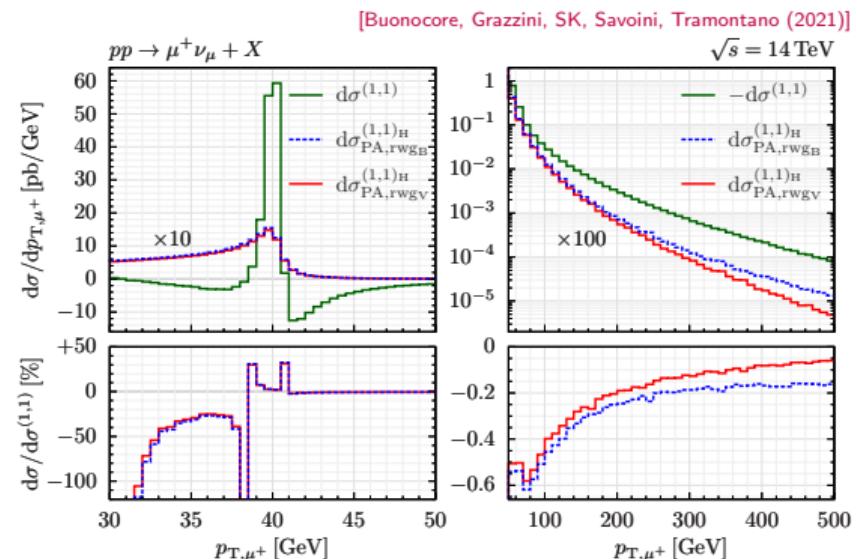
$$H_{\text{PA,rwg}}^{(0,1)} = 2\text{Re} \left(\mathcal{M}_{\text{fin}}^{(0,1)} \mathcal{M}^{(0,0)*} \right)_{\text{PA}} / \left| \mathcal{M}_{\text{PA}}^{(0,0)} \right|^2$$



Reweighted PA versions at NNLO QCD-EW

$$H_{\text{PA,rwgB}}^{(1,1)} = H_{\text{PA}}^{(1,1)} \times \left| \mathcal{M}^{(0,0)} \right|^2 / \left| \mathcal{M}_{\text{PA}}^{(0,0)} \right|^2$$

$$H_{\text{PA,rwgV}}^{(1,1)} = H_{\text{PA}}^{(1,1)} \times H^{(0,1)} / H_{\text{PA}}^{(0,1)}$$



Results for mixed NNLO QCD-EW corrections in off-shell W production

Setup and integrated results

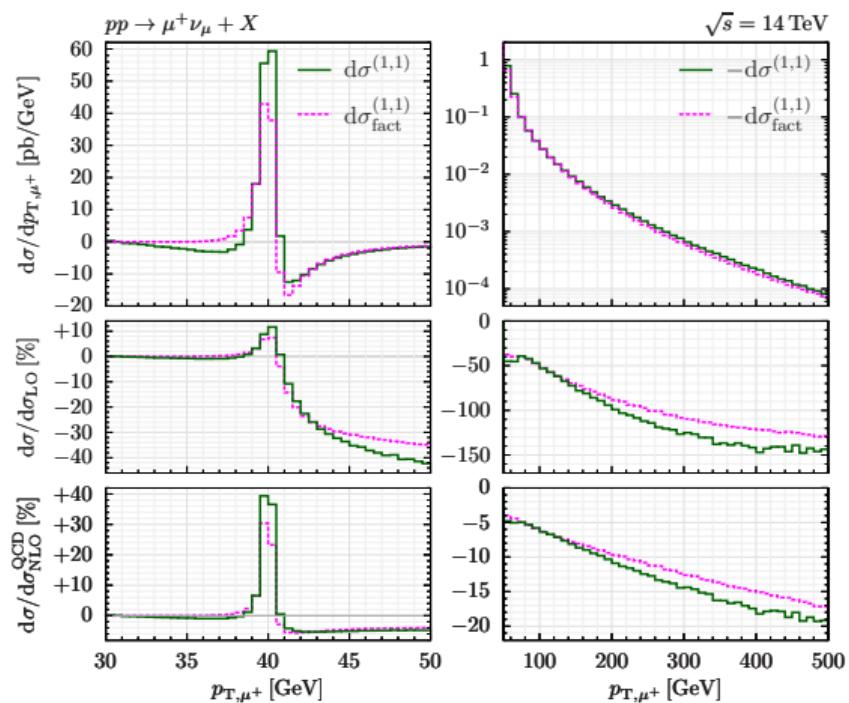
- NNPDF31_nnlo_as_0118_luxqed $\mu_R = \mu_F = m_W$
- $p_{T,\mu} > 25 \text{ GeV}$, $|y_\mu| < 2.5$, $p_{T,\text{miss}} > 25 \text{ GeV}$

		NLO QCD	NLO EW	NNLO QCD	NNLO QCD-EW
σ [pb]	σ_{LO}	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	5029.2	970.5(3)	-143.61(15)	251(4)	-7.0(1.2)
qg	—	-1079.86(12)	—	-377(3)	39.0(4)
$q(g)\gamma$	—	—	2.823(1)	—	0.055(5)
$q(\bar{q})q'$	—	—	—	44.2(7)	1.2382(3)
gg	—	—	—	100.8(8)	—
tot	5029.2	-109.4(4)	-140.8(2)	19(5)	33.3(1.3)
		-2.2%	-2.8%	+0.4%	+0.6%

- large cancellations between partonic channels in NLO and NNLO QCD corrections
- similar size of NLO QCD and EW corrections
- no dedicated scale dependence studies performed

Differential distribution in p_{T,μ^+}

[Buonocore, Grazzini, SK, Savoini, Tramontano (2021)]



Results for mixed NNLO QCD-EW corrections in off-shell Z production

Setup and integrated results

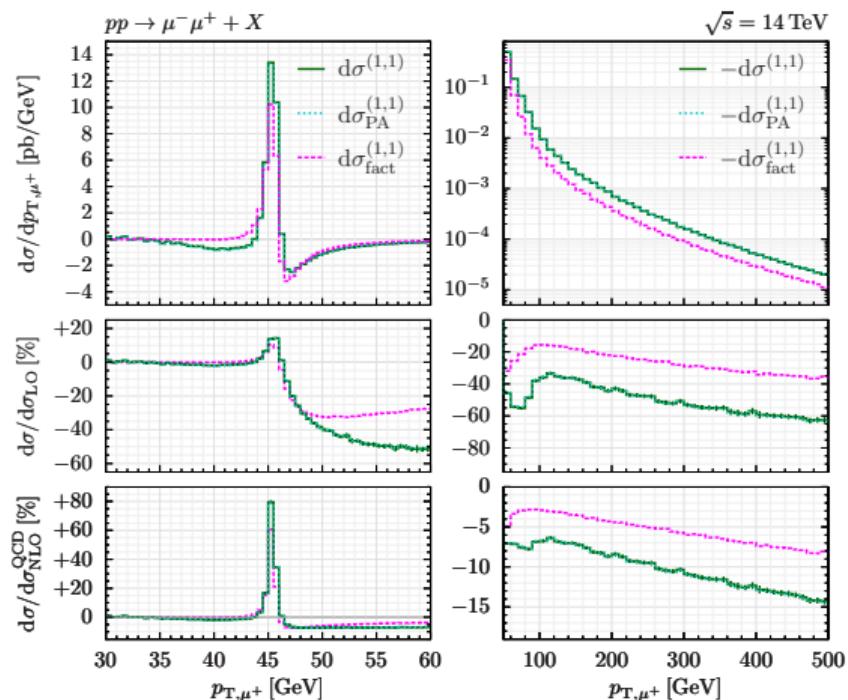
- NNPDF31_nnlo_as_0118_luxqed $\mu_R = \mu_F = m_Z$
- $p_{T,\mu} > 25 \text{ GeV}$, $|y_\mu| < 2.5$, $m_{\mu\mu} > 50 \text{ GeV}$

	NLO QCD	NLO EW	NNLO QCD	NNLO QCD-EW	
σ [pb]	σ_{LO}	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	
$q\bar{q}$	809.56(1)	191.85(1)	-33.76(1)	49.9(7)	-4.8(3)
qg	—	-158.08(2)	—	-74.8(5)	8.6(1)
$q(g)\gamma$	—	—	-0.839(2)	—	0.084(3)
$q(\bar{q})q'$	—	—	—	6.3(1)	0.19(0)
gg	—	—	—	18.1(2)	—
$\gamma\gamma$	1.42(0)	—	-0.0117(4)	—	—
tot	810.98(1)	33.77(2)	-34.61(1)	-0.5(9)	4.0(3)
		+4.2%	-4.3%	-0.1%	+0.5%

- large cancellations between partonic channels in NLO and NNLO QCD corrections
- almost complete (accidental) compensation between NLO QCD and EW corrections

Differential distribution in p_{T,μ^+}

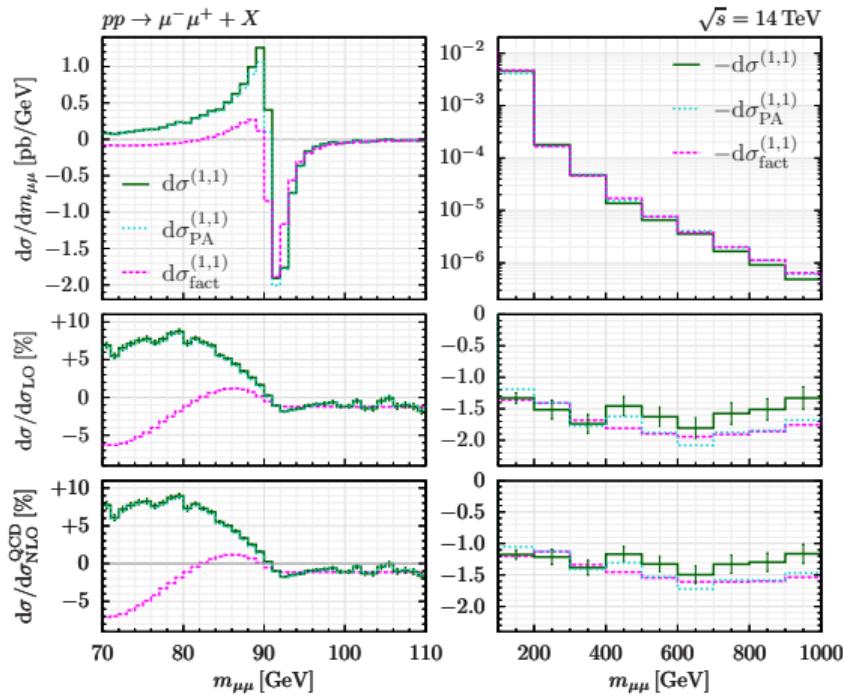
[Bonciani, Buonocore, Grazzini, SK, Tramontano, Rana, Vicini ('21)]



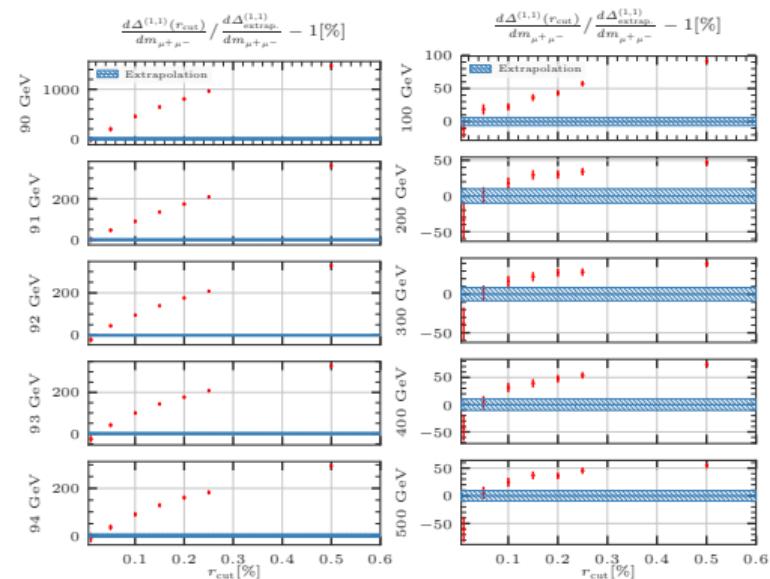
Results for mixed NNLO QCD-EW corrections in off-shell Z production

Differential distribution in $m_{\mu\mu}$

[Bonciani, Buonocore, Grazzini, SK, Tramontano, Rana, Vicini ('21)]



Numerical control over power corrections by binwise $r_{cut} \rightarrow 0$ extrapolation



→ inclusion of power corrections could significantly improve numerical performance

NNLO QCD subtraction/slicing methods and implementations (status Oct 2021)

Subtraction/slicing methods

- **q_T subtraction** [Catani, Grazzini (2007)]
- **N -jettiness subtraction**
[Boughezal, Focke, Liu, Petriello (2015); Gaunt, Stahlhofen, Tackmann, Walsh (2015)]
- **Antenna subtraction** [Gehrmann, Gehrmann-De Ridder, Glover (2005)]
- **Sector-improved residue subtraction**
[Czakon (2010); Boughezal, Melnikov, Petriello (2012)]
- **ColorFul subtraction** [Somogyi, Trocsanyi, Del Duca (2005)]
- **Nested soft–collinear subtraction**
[Caola, Melnikov, Röntsch (2017)]
- **Analytic local sector subtraction**
[Magnea, Maina, Pelliccioli, Signorile-Signorile, Torrielli, Uccirati (2018)]
- **Projection to Born** [Cacciari, Dreyer, Karlberg, Salam, Zanderighi (2015)]
- **Geometric subtraction** [Herzog (2018)]
- ...

→ Extension beyond $2 \rightarrow 2$ conceptionally straightforward if amplitudes become available!

General (public) frameworks

- **MATRIX** (q_T slicing) [Grazzini, SK, Wiesemann, ...]
 - $Z, W, H, \gamma\gamma, Z\gamma, W\gamma, WW, ZZ, WZ$
 - $ZH, WH, HH, t\bar{t}, b\bar{b}, \gamma\gamma\gamma, \dots$
- **MCFM** (N -jettiness slicing)
[Campbell, K. Ellis, Giele, Neumann, Williams, ...]
 - $Z, W, H, ZH, WH, \gamma\gamma, Z\gamma$
 - $W\gamma, \gamma j, Zj, Wj, Hj, \dots$
- **NNLOJET** (antennna subtraction)
[Gehrmann, Gehrmann-de Ridder, Glover, Huss, Chen, Gauld, ...]
 - $jj, \gamma j, Zj, Wj, Hj, Zb, \dots$
- **STRIPPER** (sector-improved residue subtraction)
[Czakon, Mitov, Poncelet, Chawdhry, ...]
 - $t\bar{t}, jj, WW, Wc, \gamma\gamma\gamma, \gamma\gamma j, jjj, \dots$
- ...

Recent achievements in (N)NNLO QCD calculations (status Oct 2021)

First $2 \rightarrow 3$ calculations at NNLO QCD

- $\gamma\gamma\gamma$ [Chawdhry, Czakon, Mitov, Poncelet (2020), SK, Sotnikov, Wiesemann (2021)]
- $\gamma\gamma j$ [Chawdhry, Czakon, Mitov, Poncelet ('21)]
- jjj [Czakon, Mitov, Poncelet ('21)]

Recent achievements in 2-loop $2 \rightarrow 3$ amplitudes

- leading-colour jjj [Abreu, Page, Pascual, Sotnikov (2021),
Abreu, Febres Cordero, Ita, Page, Sotnikov (2021)]
- $q\bar{q} \rightarrow \gamma\gamma j$ [Agarwal, Buccioni, von Manteuffel, Tancredi ('21)]
- $gg \rightarrow \gamma\gamma g$ [Badger, Brønnum-Hansen, Chicherin, Gehrmann, Hartanto,
Henn, Marcoli, Moodie, Peraro, Zoia ('21)]
- leading-colour $Wb\bar{b}$ [Badger, Hartanto, Zoia (2021)]
- leading-colour $Hb\bar{b}$ [Badger, Hartanto, Kryš, Zoia ('21)]

Heavy-quark loops for $gg \rightarrow$ diboson processes

- HH [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016),
Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann (2019)]
- $\gamma\gamma$ [Maltoni, Mandal, Zhao (2019), Chen, Heinrich, Jahn, Jones, Kerner (2020)]
- ZZ [Agarwal, Jones, von Manteuffel (2021), Brønnum-Hansen, Wang (2021)]
- WW [Brønnum-Hansen, Wang (2021)]
- ZH [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk (2021)]

Inclusive $2 \rightarrow 1$ calculations at N^3LO QCD

- H [Anastasiou, Duhr, Dulat, Herzog, Mistlberger (2015),
+ Furlan, Gehrmann, Lazopoulos (2016), Mistlberger (2018)]
- $b\bar{b} \rightarrow H$ [Duhr, Dulat, Mistlberger (2020), + Hirschi (2020)]
- W [Duhr, Dulat, Mistlberger (2020 & 2020)]
- γ^* [Duhr, Dulat, Mistlberger (2020)]

Fully differential $2 \rightarrow 1$ calculations at N^3LO QCD

- H [Dulat, Mistlberger, Pelloni (2019), Cieri, Chen, Gehrmann, Glover, Huss (2019)]
- $H (\rightarrow \gamma\gamma)$ [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni ('21)]
- Z/γ^* [Camarda, Cieri, Ferrera ('21), Chen, Gehrmann, Glover, Huss, Yang ('21)]
- ➡ combination of local NNLO subtraction with
slicing/projection methods promoted to N^3LO

First 3-loop amplitudes beyond $2 \rightarrow 1$

- leading-colour $\gamma\gamma$ [Caola, von Manteuffel, Tancredi (2021)]
- four-quark scattering [Caola, Chakraborty, Gambuti,
von Manteuffel, Tancredi ('21)]

Status of NLO EW calculations (status Oct 2021)

Dedicated comparison in Les Houches 2017 proceedings

- BBMC + RECOLA
- MUNICH/MATRIX + OPENLOOPs
- MoCANLO + RECOLA
- SHERPA + GoSAM/ OPENLOOPs/ RECOLA
- MADGRAPH5_AMC@NLO + MADLOOP
- conceptionally solved, as for NLO QCD calculations

Recent highlights: high-multiplicity processes

- off-shell $t\bar{t}W$ production ($2 \rightarrow 8$) [Denner, Pelliccioli (2021)]
- off-shell $t\bar{t}H$ production ($2 \rightarrow 7$) [Denner, Lang, Pellen, Uccirati (2017)]
- off-shell WWW production ($2 \rightarrow 6$) [Dittmaier, Knippen, Schwan (2020)]
- vector boson scattering ($2 \rightarrow 6$)
 - $W^\pm W^\pm$ [Biedermann, Denner, Pellen (2017), Denner, Lang, Pellen, Uccirati (2017)]
 - WZ [Denner, Dittmaier, Maierhöfer, Pellen, Schwan (2019)]
 - ZZ [Denner, Franken, Pellen, Schmidt (2020)]
 - $W^+ W^-$ [Denner, Franken, Pellen, Schmidt (preliminary results at RADCOR 2021)]

