Real corrections to Higgs boson pair production at NNLO in the large top quark mass limit

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Loops and Legs in Quantum Field Theory, 2022

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Higgs self coupling

Standard Model Higgs potential:

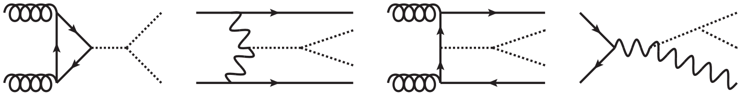
$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4,$$

where $\lambda = m_H^2/(2v^2) \approx 0.13$.

Want to measure λ, to determine if $V(H)$ is consistent with nature.

- Challenging! Cross-section $\approx 10^{-3} \times H$ prod.
- $-3.3 < \lambda/\lambda_{SM} < 8.5$ [CMS '21]

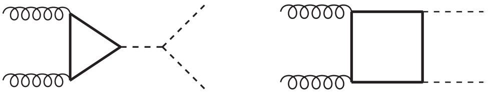
λ appears in various production channels:



- Gluon fusion – dominant, 10x
- VBF
- $t\bar{t}$ associated production
- H-strahlung

Gluon Fusion

Leading order (1 loop) partonic amplitude:



$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

- $\mathcal{F}_{tri}$ contains the dependence on λ

Form factors:

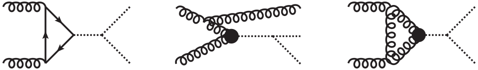
- LO: known exactly [Glover, van der Bij '88]
- Beyond LO... no fully-exact (analytic) results to date
  - numerical evaluation, expansion in various kinematic limits

$gg \to HH$ Beyond LO

$m_t \to \infty$ limit ("HEFT") used in many approximations:

- NLO "Born-improved" HTL: [Dawson, Dittmaier, Spira '98]

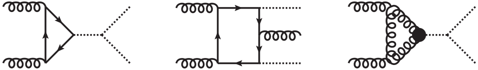
exact LO | B-i NLO real | B-i NLO virt



B-i: $d\sigma_{NLO}(m_t) \approx \frac{d\sigma_{LO}(m_t)}{d\sigma_{LO}(\infty)} d\sigma_{NLO}(\infty)$

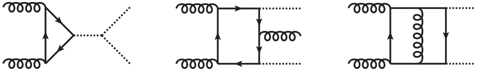
- NLO "FTapprox": [Maltoni, Vryonidou, Zaro '14]

exact LO | exact NLO real | B-i NLO virt



- NLO Full: [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16] [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]

exact LO | exact NLO real | exact NLO virt



$gg \to HH$ Beyond LO

NLO:

- large-m_t [Grigo, Hoff, Melnikov, Steinhauser '13]
- large-m_t + threshold exp. Padé [Gröber, Maier, Rauh '17]
- high-energy expansion [Davies, Mishima, Steinhauser, Wellmann '18,'19]
- small-p_T expansion [Bonciani, Degrassi, Giardino, Gröber '18]

NNLO:

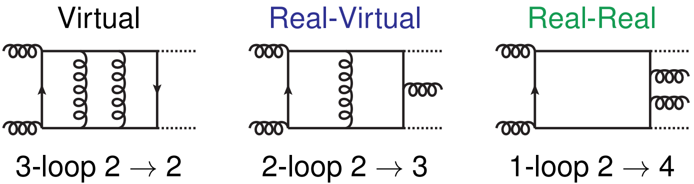
- large-m_t virtuals [Grigo, Hoff, Steinhauser '15][Davies, Steinhauser '19]
- HEFT + numeric reals [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
- large-m_t reals [Davies, Herren, Mishima, Steinhauser '21]

N3LO:

- Wilson coefficient C_{HH} [Spira '16][Gerlach, Herren, Steinhauser '18]
- HEFT [Chen, Li, Shao, Wang '19]

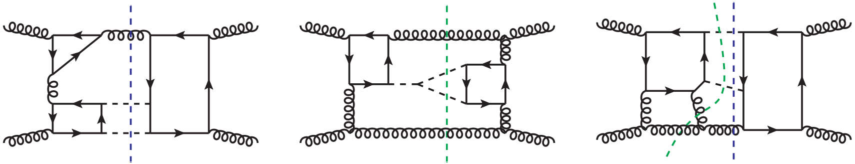
NNLO Total Cross Section

Need to compute: $\sigma_{ij}^{(2)} = \sigma_{ij,\mathrm{virt}}^{(2)} + \sigma_{ij,\mathrm{real}}^{(2)} + \sigma_{ij,\mathrm{coll}}^{(2)}$ $\quad (ij = gg, gq, q\bar{q}, qq')$



Each part is divergent, sum is finite (including also Collinear CTs).

Total XS: proceed via *Optical Theorem*. Phase-space → loop integrals:



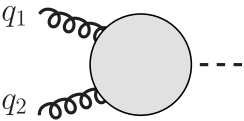
Three- and four-particle cuts of 5-loop 2 → 2 forward diagrams.

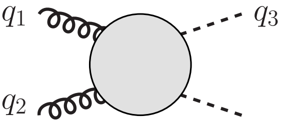
Large-m_t expansion

Expand integrals in the region "$m_t \gg$ other scales".

Result: series in powers of $\{q_i \cdot q_i,\ q_i \cdot q_j,\ q_j \cdot q_j,\ \ldots\}/m_t^2$

- $gg \to H(\to HH)$: $q_1 \cdot q_2/m_t^2$
- $gg \to HH$: $\{q_3 \cdot q_3,\ q_1 \cdot q_2,\ q_1 \cdot q_3,\ q_2 \cdot q_3\}/m_t^2$





Large-m_t expansion

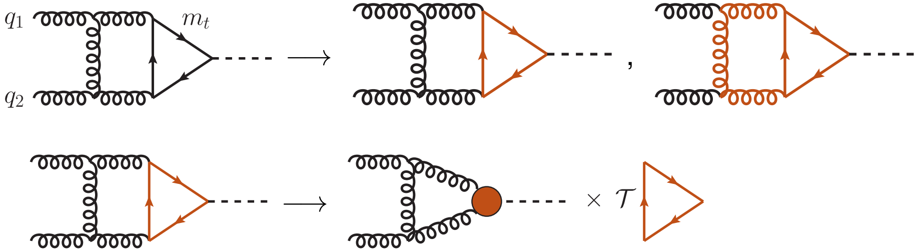
Expansion by sub-graph:

- sum over sub-graphs which contain m_t
- expand hard-scaling propagators in their small parameters

Diagrams factorize into:

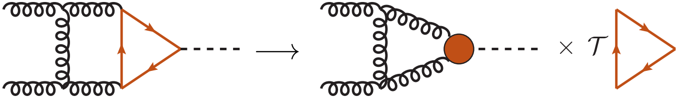
- massless integrals
- expanded hard sub-graphs $\to$ massive tadpole integrals

Example: 2-loop $gg \to H$ diagram:



Large-m_t expansion

More explicitly:



$$\iint \mathrm{d}^d l_1\, \mathrm{d}^d l_2\, \frac{1}{l_2^2} \frac{1}{(l_2+q_1)^2} \frac{1}{(l_2-q_2)^2} \frac{1}{(l_1+q_1)^2-m_t^2} \frac{1}{(l_1-q_2)^2-m_t^2} \frac{1}{(l_1-l_2)^2-m_t^2} \longrightarrow$$

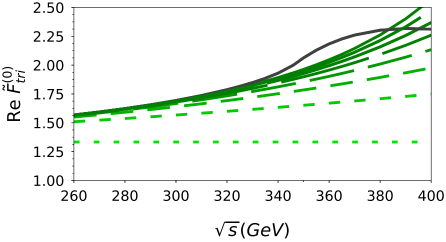
$$\int \mathrm{d}^d l_2\, \frac{1}{l_2^2} \frac{1}{(l_2+q_1)^2} \frac{1}{(l_2-q_2)^2} \int \mathrm{d}^d l_1 \left[\frac{1}{(l_1^2-m_t^2)^3} + \frac{2(q_1\cdot l_1 - q_2\cdot l_1 - l_2\cdot l_1) + l_2\cdot l_2}{(l_1^2-m_t^2)^4} + \cdots \right]$$

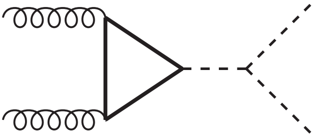
What remains?

- massless integral over l_2
- massive tadpole integrals: $l_1^{\mu_1} l_1^{\mu_2} \cdots l_1^{\mu_N} / (l_1^2 - m_t^2)^m$

Large-m_t expansion

Eg, LO $\mathcal{F}_{tri}$: expansion to $1/m_t^{14}$

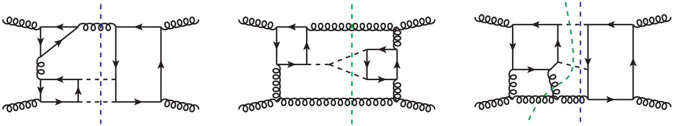




Software:

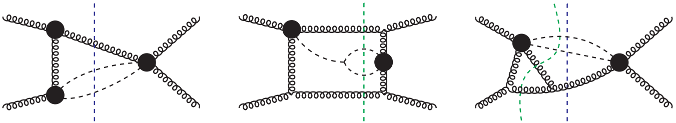
diagram generation	`qgraf`	[Nogueira '93]
large-m_t expansion	`q2e/exp`	[Harlander, Seidelsticker, Steinhauser '97]
	`FORM 4.2`	[Ruijl, Ueda, Vermaseren '17]
tadpoles (1-3 loop)	`MATAD`	[Steinhauser '00]
massless integrals	`FIRE 6`	[Smirnov '19]
	`LiteRed`	[Lee '12]

NNLO $gg \to HH$



`qgraf` generates a large number of 5-loop $2 \to 2$ diagrams:

- filter for valid cuts using `gen`
  - gg channel: 16.6M $\to$ 160.1K
  - gq channel: 1.7M $\to$ 5.4K

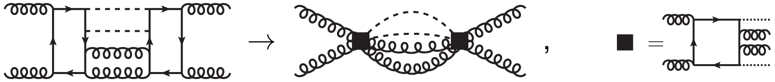


Expansion of such graphs is difficult, computationally.

- compute only $1/m_t^0$ (gg), $1/m_t^2$ (gq, $q\bar{q}$) in this style
- "careful `FORM` programming"

"Building blocks"

Approach: construct "building blocks", pre-expanded "effective vertices", legs off shell, open Lorentz+colour indices:



3600 diagrams → 1 diagram , ■ = 120 diagrams

Need to compute and expand various building blocks:

- *ggH*
- *ggHH*
- *gggH*
- *gggHH*
- *ggggH*
- *ggggHH*

Generate "building block" diagrams directly:

- *gg* channel: 16.6M → 160.1K → 4612
- *gq* channel: 1.7M → 5.4K → 336

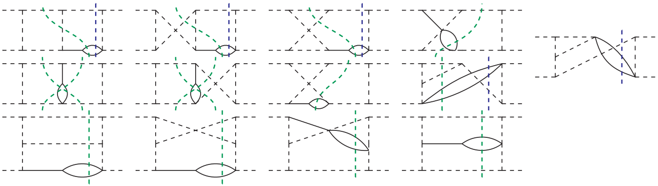
We still directly expand two-loop sub-graphs.

Phase-space Integrals

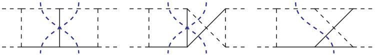
After large-m_t expansion, 2- and 3-loop “phase-space integrals” remain.

Partial fraction and map to a minimal set of families: `LIMIT` [Herren ‘20]

IBP reduce (`LiteRed`) to obtain three and four particle cuts of 74 3-loop MIs belonging to 13 topologies (RV+RR),



and 16 2-loop MIs belonging to 3 topologies (RV),



Phase-space Integrals

Compute MIs via differential equations w.r.t. $x = m_H^2/s$:

- $\partial_x \vec{I} = \epsilon \sum_i M_i/(x - x_i)\, \vec{I}$ (Libra) [Lee '20]
- exact solns (GPL), and via series expansion: $\delta = 1 - 4x \to 0$

$$I_1^{(3)} \approx \delta^{5/2} \left[\frac{s}{7680\pi^5 \epsilon} - \frac{s(15 \log \delta - 56 + 20 \log 2)}{38400\pi^5} + \mathcal{O}(\epsilon^1) \right] + \mathcal{O}(\delta^{7/2}) \,,$$

$$\vdots$$

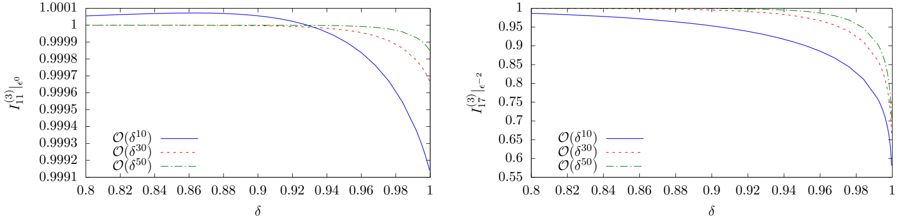
$$I_{17}^{(3)} \approx \sqrt{\delta} \left[-\frac{1}{3072\pi^5 s^2 \epsilon^4} + \frac{5 \log \delta - 10 + 8 \log 2}{3072\pi^5 s^2 \epsilon^3} + \mathcal{O}(\epsilon^{-2}) \right] + \mathcal{O}(\delta^{3/2}) \,,$$

$$\vdots$$

Easier to combine virtuals and compute collinear CTs in δ expansion.

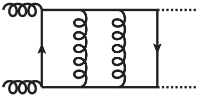
Phase-space Integrals

Two examples, ratio of $\delta = 1 - 4x \to 0$ expansion to exact (GPL) result:



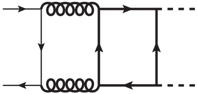
- δ^{30} reproduces exact result very well up to $\delta \approx 0.9$
- $2m_t$ threshold corresponds to $\delta = 1 - m_h^2/m_t^2 = 0.48$

Virtual Corrections



NNLO virtual corrections for *gg* channel are known [Davies, Steinhauser '19]

- expand in δ, square and integrate over t



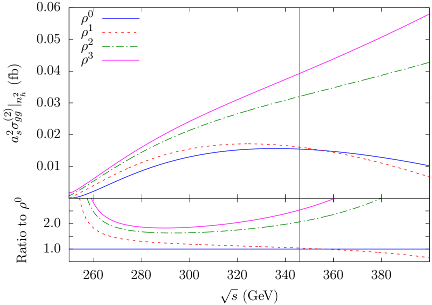
At NNLO there are two-loop $q\bar{q} \to HH$ virtuals

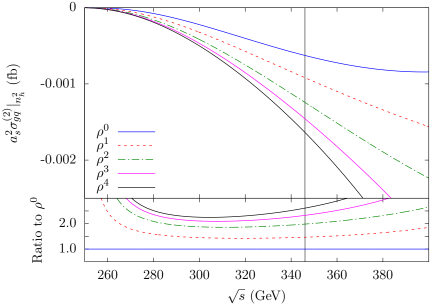
- compute large-m_t expansion, expand in δ, square, integrate over t...
- vanish in HEFT but contribute starting from $1/m_t^2$

NNLO $gg \to HH$, Partonic Cross Sections

Resulting expansions for channels $(\rho = m_h^2/m_t^2)$

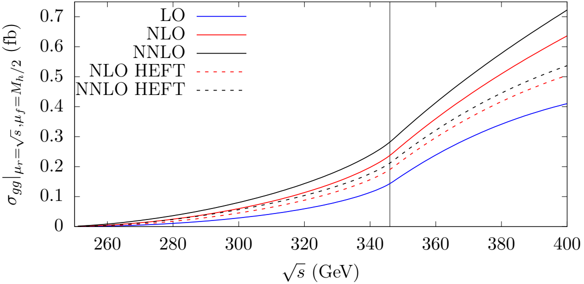
- gg, to $1/m_t^6$
- gq, to $1/m_t^8$





- m_t dependence is a 100% correction to HEFT at NNLO

NNLO $gg \to HH$, Partonic Cross Sections



gg, $1/m_t^6$ vs HEFT

- LO→NLO: +100%
- NLO→NNLO: +30%

Summary

Multi-scale multi-loop amplitudes are hard:

- study them via expansions in certain kinematic limits
- direct numerical evaluation

Large-m_t expansions:

- good description of amplitudes below top quark threshold
- can be combined with other expansions for a better approximation
  - large-m_t + threshold Padé at NNLO – work in progress
- differential XS? Requires large-m_t exp. of 2-loop $2 \to 3$.