

Dimensional regularization and γ_5 — no-compromise* approach to the BMHV scheme

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28th April, Loops&Legs 2022

Collaborators: Bélusca-Maïto, Ilakovac, Kühler, Mađor-Božinović

* or: traditional/old-fashioned/stubborn. . .

γ_5 and DReg: well-known problem

Three properties in 4-dimensions:

$$\{\gamma_5, \gamma^\mu\} = 0, \quad (1)$$

$$\text{Tr}(\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4i \epsilon^{\mu\nu\rho\sigma}, \quad (2)$$

$$\text{Tr}(\Gamma_1 \Gamma_2) = \text{Tr}(\Gamma_2 \Gamma_1). \quad (3)$$

Inconsistent in $D \neq 4$ (can prove that trace=0).

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Give up at least one \Rightarrow many proposals!

- “Naive” anticommuting? Reading point? Larin? ... Many alternatives!
- Often limited range of applicability
- **BMHV** (non-anticommuting, very complicated, breaks gauge inv. But unitary, consistent)

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Our idea: No-compromise approach to BMHV — apply it and accept/deal with its difficulties! Goal: EWSM!

(So far 1-loop YM and 2-loop abelian)

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Study motivated by need for precision and general effort on regularization schemes

(\rightsquigarrow reports “To d or not to d” and “May the four be with you”)

BMHV scheme — non-anticommuting γ_5

QFT consistent, unitary; breaks symmetries, complicated

- “ D -dim space” split into pure 4-dim space $\oplus (-2\epsilon)$ -dim space

$$X^\mu = \bar{X}^\mu + \hat{X}^\mu$$

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$$

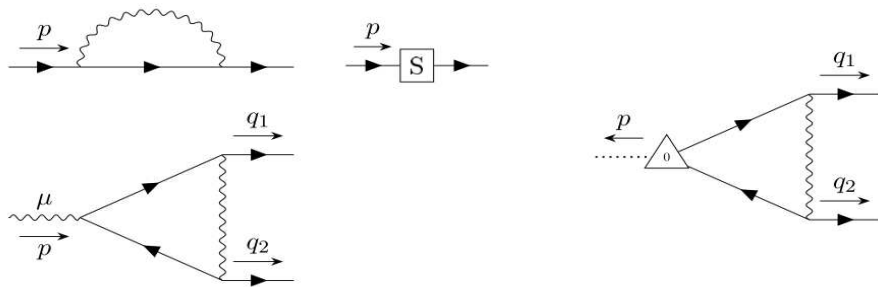
$$\{\gamma_5, \bar{\gamma}^\mu\} = 0$$

$$[\gamma_5, \hat{\gamma}^\mu] = 0$$

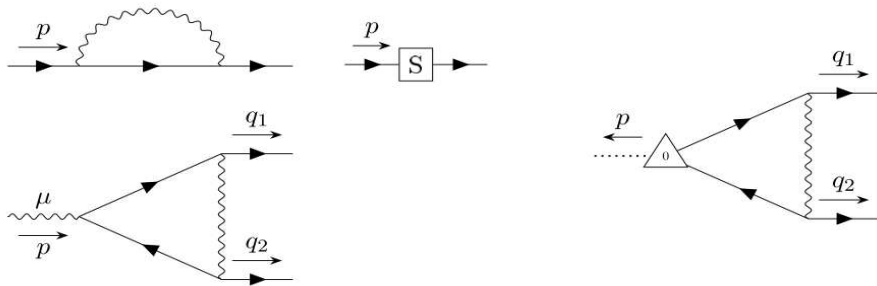
Goals:

- Take seriously, apply to 1-loop, 2-loop ... EW calculations
- Technical questions: restore gauge invariance (here)
- Conceptual: relation to other schemes, RGEs (future)
- **Progress will feed back to other schemes**

Preview: Problem in a nutshell

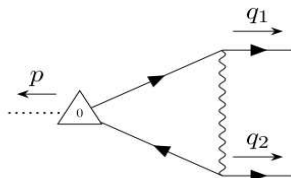
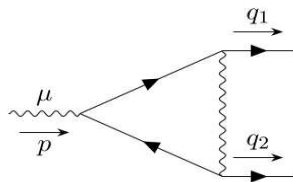
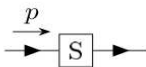
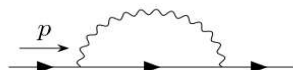


Preview: Problem in a nutshell



Ward identity
violated

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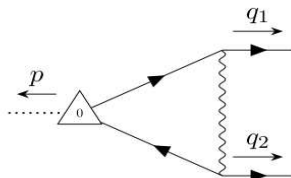
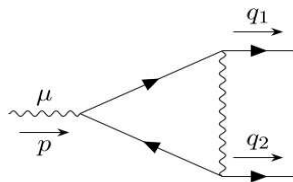
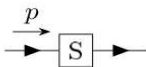


Ward identity
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compensated by
special c.t.

(our main task)

Preview: Problem in a nutshell



Ward identity
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Alternative:
breaking via $\frac{\epsilon}{\epsilon}$ term

(our main task)

(tool)

Preview: Example of main technical result

symmetry-restoring counterterm for this case:

$$S_{\text{fct}}^1 = \frac{e^2}{16\pi^2} \int d^4 x \left\{ \dots + \left(\frac{5 + \xi}{6} \right) (\mathcal{Y}_R^j)^2 (\bar{\psi}_j i \not{\partial} P_R \psi_j) \right\}.$$

Finite, NON-evanescent counterterms. Not gauge invariant!

Modify all self-energies and some interactions!

But rather compact, universal, could be implemented e.g. in FeynArts

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Q: How do we derive this? At 2-loop order?

see also: SUSY in DRed (3-loop) [DS'05, Hollik, DS'05, DS, Unger'18]

Plan here: chiral “QED” (only $P_R\psi$) at 1-/2-loop

[Bélusca-Maïto, Ilakovac, Kühler Mador-Božinović, DS, 2021]

Abelian theory like $U(1)_Y$ -part of SM, only ψ_{Ri} interact

$$\mathcal{L} = i\overline{\psi_{Ri}}\not{D}\psi_{Ri} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \text{g.fix, ghost, source terms}$$
$$D^\mu = \partial^\mu - ieA^\mu\gamma_R,$$

Description of symmetry: gauge invariance \rightarrow BRST invariance \rightarrow
Slavnov-Taylor identity is requirement on renormalized theory:

$$S(\Gamma_{\text{ren}}) = 0$$

combines well-known STI/WIs between 1PI Green functions e.g.

- Ward identity between fermion self energy and vertex function
- photon self energy = transverse, etc

Plan here: chiral “QED” (only $P_R\psi$) at 1-/2-loop

[Bélusca-Maïto, Ilakovac, Kühler Mador-Božinović, DS, 2021]

1. Define D -dimensional Lagrangian compute symmetry breaking
2. Determine 1-loop UV divs $\rightsquigarrow \mathcal{L}_{\text{sct}}$
3. Determine 1-loop violation of Slavnov-Taylor identity
4. Determine 1-loop symmetry-restoring counterterms $\rightsquigarrow \mathcal{L}_{\text{fct}}$
5. Repeat at 2-loop new features?

1. Define D -dimensional Lagrangian

What should we take: \mathcal{L} in D -dim?

Our choice has D -dim kinetic, 4-dim interaction term

$$\mathcal{L}_{\text{fermions}} = i\bar{\psi}_i \not{\partial} \psi_i + e\gamma_{Ri} \bar{\psi}_{Ri} \mathbf{A} \psi_{Ri}.$$

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- Interaction term: choose $P_L \gamma^\mu P_R$.
- future: can study different options, $\gamma^\mu P_R$ or $P_L \gamma^\mu$ or DRED, FDH

1. Define D -dimensional Lagrangian

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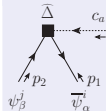
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$$\mathcal{L}_{\text{fermions}} = i\bar{\psi}_i \not{\partial} \psi_i + e\mathcal{Y}_{Ri} \bar{\psi}_{Ri} \mathbf{A} \psi_{Ri}.$$

It breaks D -dim gauge/BRST invariance

⇒ and leads to breaking of tree-level Slavnov-Taylor identity

$$\mathcal{S}_d(\mathcal{S}_0) = \hat{\Delta} \equiv \int d^d x (e\mathcal{Y}_{Ri}) c \left\{ \bar{\psi}_i \left(\overleftarrow{\not{\partial}} P_R + \overrightarrow{\not{\partial}} P_L \right) \psi_i \right\}.$$



$$= (e\mathcal{Y}_{Ri}) \left(\hat{p}_1 P_R + \hat{p}_2 P_L \right)_{\alpha\beta}$$

This is the core of the difficulties.
Can be written as a
local Feynman rule

2. Compute Green functions to determine UV divs

Many 1-loop diagrams (not shown) \rightsquigarrow divergent counterterms:

$$S_{\text{sct}}^1 = S_{\text{sct,inv}}^1 + S_{\text{sct,break}}^1 ,$$

2. Compute Green functions to determine UV divs

Many 1-loop diagrams (not shown) \rightsquigarrow divergent counterterms:

$$S_{\text{sct}}^1 = S_{\text{sct,inv}}^1 + S_{\text{sct,break}}^1,$$

First part as usual

$$S_{\text{ct,inv}}^1 = \frac{\delta Z_A^1}{2} L_A + \frac{\delta Z_C^1}{2} L_C + \frac{\delta Z_{\psi_R}^1}{2} L_{\psi_R} + \frac{\delta e_A^1}{e_A} L_{e_A},$$

second part is special for BMHV, sym-breaking and “evanescent”

$$S_{\text{sct,break}}^1 = \frac{-\hbar e_A^2}{16\pi^2 \epsilon} \frac{\text{Tr}(\mathcal{Y}_R^2)}{3} \left(2(\bar{S}_{AA} - S_{AA}) + \int d^d x \frac{1}{2} \bar{A}^\mu \hat{\partial}^2 \bar{A}_\mu \right).$$

Divergences for evanescent operators with independent coefficients, beyond the usual field/parameter renormalization

well-known in DRed: needed for unitarity/finiteness at higher orders

3. Determine 1-loop violation of Slavnov-Taylor id.

Ultimate structure at 1-loop (finite ct to be determined)

$$\Gamma_{\text{DReg}}^{(1)} = \Gamma^{(1)} + \mathcal{S}_{\text{sct}}^1 + \mathcal{S}_{\text{fct}}^1,$$

Evaluate STI at 1-loop order, div-parts cancel, fin-parts t.b.d.

$$\mathcal{S}_d(\Gamma_{\text{DReg}}^{(1)}) = \underbrace{\mathcal{S}_d(\Gamma^{(1)})}_{\text{finite}} + \mathcal{S}_d \mathcal{S}_{\text{fct}}^1$$

Left term means: breaking of regularized STI; must be computed.

In principle this corresponds to checking all STIs/WIs, e.g. Fermion 2-point/3-point function etc.

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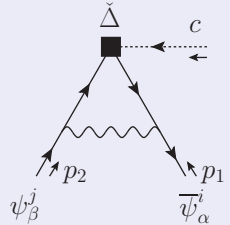
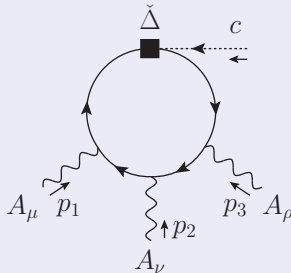
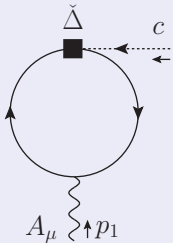
In principle this corresponds to checking all STIs/WIs, e.g. Fermion 2-point/3-point function etc.

But can be simplified by using quantum action principle (BM)

$$\mathcal{S}_d(\Gamma^{(1)}) = \hat{\Delta} \cdot \Gamma^{(1)},$$

Bonneau (1980): only power-counting divergent diagrams matter!

The complete set of power-counting divergent 1-loop diagrams with insertion of $\widehat{\Delta}$:



Results mean: breaking of three concrete WI/STIs.

They have the form $\frac{\epsilon/\text{evanescent}}{\epsilon} \times (\text{local})$

\rightsquigarrow local counterterms can repair the symmetry!

(There is an additional diagram corresponding to the fermion triangle loop and the true anomaly (assumed absent))

4. Determine symmetry-restoring counterterms

$$\mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}} + \mathcal{S}_d \mathcal{S}_{\text{fct}}^1 \stackrel{!}{=} 0$$

Requiring this renormalized STI to hold leads to the result

$$\mathcal{S}_{\text{fct}}^1 = \frac{e^2}{16\pi^2} \int d^4 x \left\{ \frac{-\text{Tr}(\mathcal{Y}_R^2)}{6} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + \frac{e^2 \text{Tr}(\mathcal{Y}_R^4)}{12} (\bar{A}^2)^2 + \left(\frac{5 + \xi}{6} \right) (\mathcal{Y}_R^j)^2 (\bar{\psi}_j i \bar{\not{\partial}} P_R \psi_j) \right\}.$$

This is the full 1-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.

Finite, NON-evanescent counterterms. Not gauge invariant!

Modify both self-energies and A^4 interaction

2. Determine UV divs at 2-loop

Many 2-loop diagrams (not shown) \rightsquigarrow divergent counterterms:

$$S_{\text{sct}}^2 = S_{\text{sct,inv}}^2 + S_{\text{sct,break}}^2,$$

First part as usual \sim field and parameter renormalization
second part is special for BMHV, “sym-breaking” (partially non-evan.)

$$S_{\text{sct,break}}^2 = -\frac{e^4}{256\pi^4\epsilon} \frac{\text{Tr}(\mathcal{Y}_R^4)}{3} \left(2(\bar{S}_{AA} - S_{AA}) + \left(\frac{1}{2\epsilon} - \frac{17}{24} \right) \int d^d x \frac{1}{2} \bar{A}^\mu \widehat{\partial}^2 \right) \\ - \frac{e^4}{256\pi^4} \frac{(\mathcal{Y}_R^j)^2}{3\epsilon} \left(\frac{5}{2} (\mathcal{Y}_R^j)^2 - \frac{2}{3} \text{Tr}(\mathcal{Y}_R^2) \right) \overline{S_{\bar{\psi}\psi_R}^j}$$

3. Determine 2-loop violation of Slavnov-Taylor id.

Ultimate structure at 2-loop (fct to be determined)

$$\Gamma_{\text{DReg}}^{(2)} = \Gamma^{(2)} + \mathcal{S}_{\text{sct}}^2 + \mathcal{S}_{\text{fct}}^2,$$

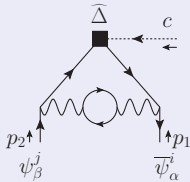
Evaluate STI at 2-loop order, div-parts cancel, fin-parts t.b.d.

$$\mathcal{S}_d(\Gamma_{\text{DReg}}^{(2)}) = \mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d \mathcal{S}_{\text{fct}}^2$$

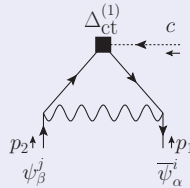
Left term (breaking of regularized STI) must be computed, use q.a.p.

$$\mathcal{S}_d(\Gamma^{(2)}) = \hat{\Delta} \cdot \Gamma^{(2)} + \Delta_{\text{ct}}^1 \cdot \Gamma^{(1)}$$

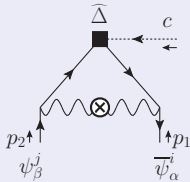
2-loop Slavnov-Taylor breaking — many diagrams of four types:



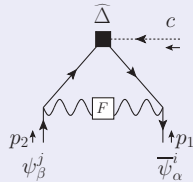
2-loop insertion of $\widehat{\Delta}$



1-loop insertion of Δ_{ct}^1



insertion of $\widehat{\Delta}$ into 1-loop diagram with 1-loop ct insertion



Sum gives $\mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} = \text{local}$. Can cancel by local counterterms

4. Determine sym-restoring counterterms at 2-loop

$$\mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d \mathcal{S}_{\text{fct}}^2 \stackrel{!}{=} 0$$

Requiring this renormalized STI to hold leads to the result

$$\mathcal{S}_{\text{fct}}^2 = \frac{e^4}{(16\pi^2)^2} \int d^4 x \left\{ \text{Tr}(\mathcal{Y}_R^4) \frac{11}{48} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + e^2 \frac{\text{Tr}(\mathcal{Y}_R^6)}{8} (\bar{A}^2)^2 \right. \\ \left. - (\mathcal{Y}_R^j)^2 \left(\frac{127}{36} (\mathcal{Y}_R^j)^2 - \frac{1}{27} \text{Tr}(\mathcal{Y}_R^2) \right) (\bar{\psi}_j i \not{\partial} P_R \psi_j) \right\}$$

This is the full 2-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.

Finite, NON-evanescent counterterms. Not gauge invariant!

Same structure as at 1-loop

Generalization to YM

[Bélusca-Maïto, Ilakovac, Mađor-Božinović, DS, 2020]

symmetry-restoring counterterm for YM+fermions+scalars (1-loop)

$$\begin{aligned} S_{\text{fct,restore}}^1 = & \frac{\hbar}{16\pi^2} \left\{ g^2 \frac{S_2(R)}{6} \left(5S_{GG} + S_{GGG} - \int d^4x G^{a\mu} \partial^2 G_\mu^a \right) + \frac{Y_2(S)}{3} S_{\Phi\Phi} \right. \\ & + g^2 \frac{(T_R)^{abcd}}{3} \int d^4x \frac{g^2}{4} G_\mu^a G^{b\mu} G_\nu^c G^{d\nu} - \frac{(C_R)^{abmn}}{3} \int d^4x \frac{g^2}{2} G_\mu^a G^{b\mu} \Phi^m \Phi^n \\ & + g^2 \left(1 + \frac{\xi - 1}{6} \right) C_2(R) S_{\bar{\psi}\psi} - \frac{((Y_R^m)^* T_R^a Y_R^m)_{ij}}{2} \int d^4x g \bar{\psi}_i G^a P_R \psi_j \\ & \left. - g^2 \frac{\xi C_2(G)}{4} (S_{\bar{R}C\psi_R} + S_{RC\bar{\psi}_R}) \right\}, \end{aligned}$$

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Modify all self-energies and some interactions!

But rather compact, universal, could be implemented e.g. in FeynArts

Summary and outlook

- BMHV breaks gauge invariance already in \mathcal{L}_D
- Symmetry-restoring counterterms: 1-loop YM, 2-loop abelian
- Method established, result has compact simple structure
- Conceptual: non- $\overline{\text{MS}}$ cts, evanescent cts
- **Outlook:** 2-loop YM, 2-loop EWSM
- **Outlook:** 3-loop
- **Outlook:** automatize, implement in FeynRules, FeynArts

...,Kühler,Weisswange

sample operators = $c\bar{\psi}\psi$

ghost #,fermionic

$\hat{A}^\mu \hat{\partial}^2 \hat{A}_\mu$

evanescent fields, momenta

$K_\psi c\psi$

ext. bosonic spinor field, ghost #

- **Outlook:** alternative \mathcal{L}_D , schemes (Larin, FDH, DRed, etc)