University of Zurich[UZH]

Mixed QCD-EW corrections to the Drell–Yan process

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in collaboration with T. Armadillo, R. Bonciani, S. Devoto, M. Grazzini, S. Kallweit, N. Rana, C. Savoini, F. Tramontano, A. Vicini

Loops & Legs 2022
28th April 2022

Outline

- Motivations
- Methodology: infrared subtraction
- 2-loop virtual amplitude
- Phenomenological results
- Conclusions&Outlook

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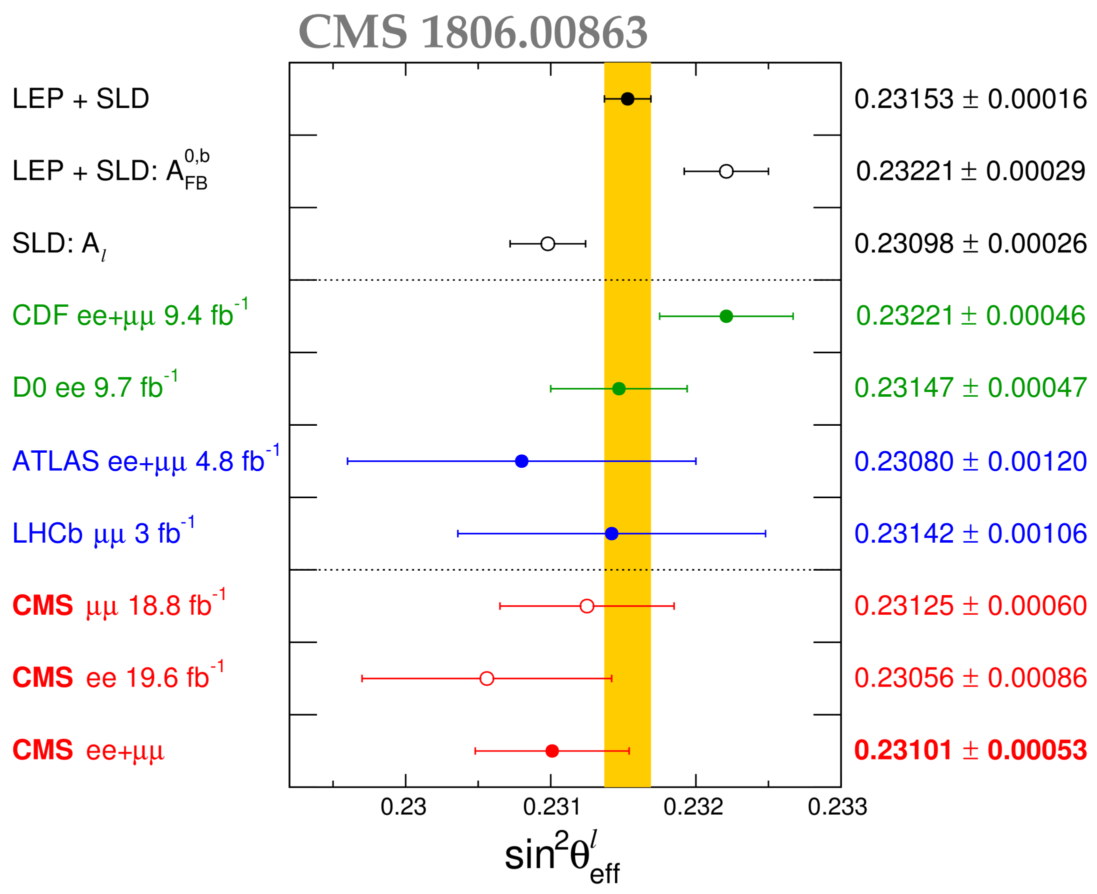
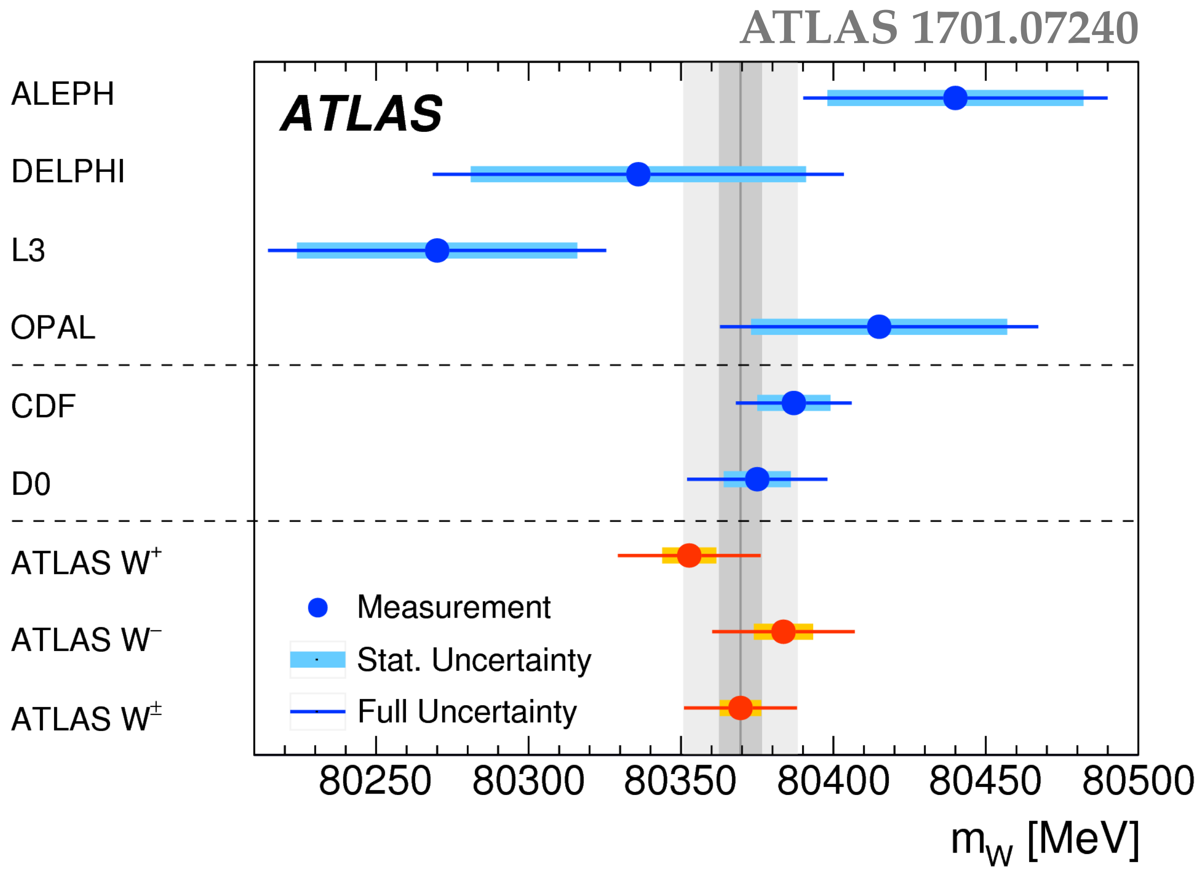
Introduction

Resonant Region

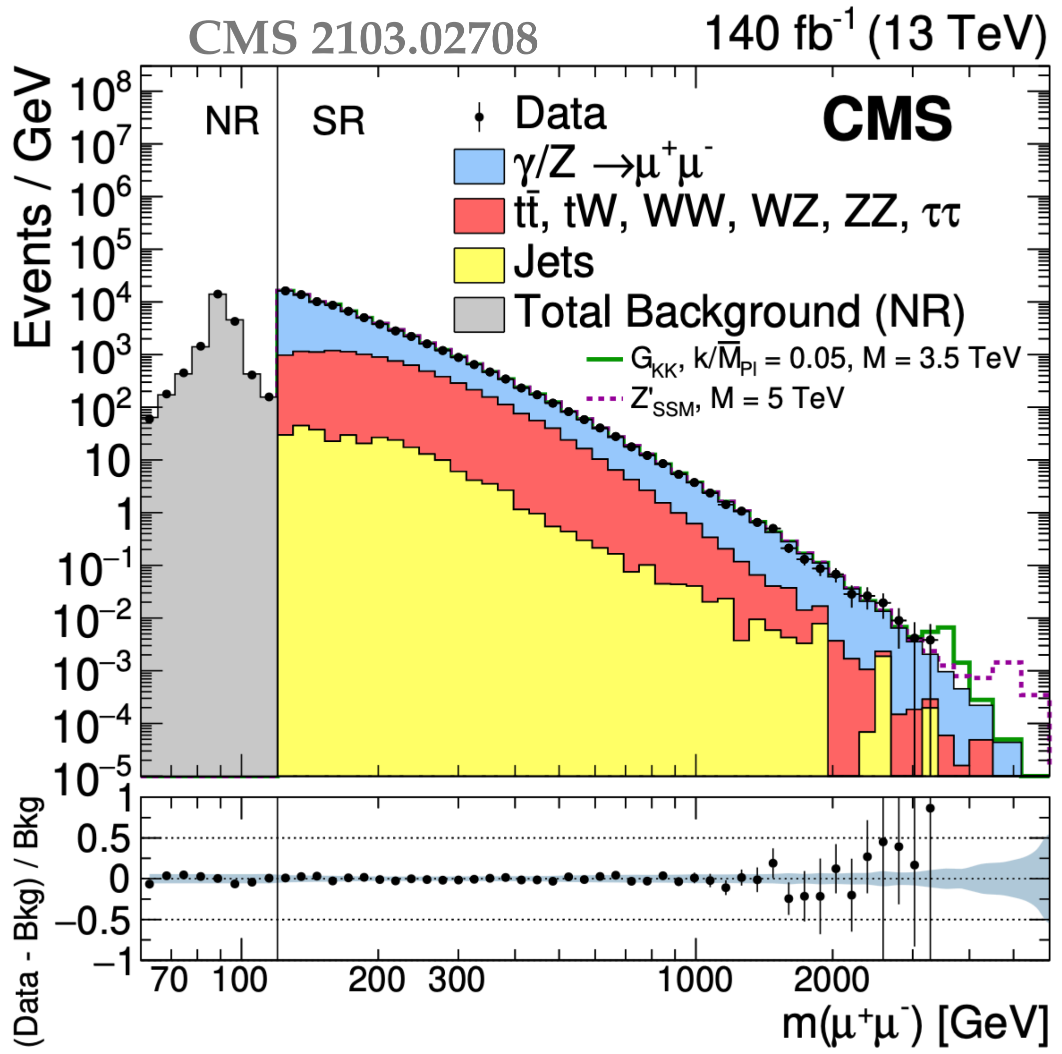
LHC Electro-Weak precision physics:

- extremely **precise** determination of **W mass** 80.354±0.007 GeV with expected **uncertainties** <u>at the level of $\mathcal{O}(10\,\text{MeV})$</u> at the end of HL-LHC
- measurement of the effective mixing angle starts to **compete with LEP**: $\sin^2\theta^{\ell}_{\text{eff}} = 0.23101 \pm 0.00053$





Off-Shell Region



mass window [GeV]	stat. unc. 140fb^{-1}	stat. unc. 3ab^{-1}
$600<m_{\mu\mu}<900$	1.4%	0.2%
$900<m_{\mu\mu}<1300$	3.2%	0.6%

- Modelling of the SM background **relevant** for new physics searches
- Measurement of the dilepton invariant mass spectrum **expected at $\mathcal{O}(1\%)$ at $m_{\ell\ell} \sim 1$ TeV**
- Requires control of the SM prediction at the $\mathcal{O}(0.5\%)$ level in the TeV

Introduction

→ very accurate SM predictions!

$$\sigma = \int dx_1 dx_2 f_{a/h_1}(x_1, \mu_F) f_{b/h_2}(x_2, \mu_F) \hat{\sigma}_{ab}(\hat{s}, \mu_R, \mu_F) + \mathcal{O}(\Lambda/Q)$$

Disclaimer: I will focus on fixed-order computations!

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0,0)} + \hat{\sigma}_{ab}^{(1,0)} + \hat{\sigma}_{ab}^{(2,0)} + \hat{\sigma}_{ab}^{(3,0)} + \dots \quad \text{QCD}$$

☑ QCD corrections dominant effects. They are known up to

- **NNLO differential cross sections**
  [Anastasiou, Dixon, Melnikov, Petriello (2003)], [Melnikov, Petriello (2006)] [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)] [Catani, Ferrera, Grazzini (2010)]
- **N^3LO inclusive cross sections and di-lepton rapidity distribution**
  [Duhr, Dulat, Mistlberger (2020)] [Chen, Gehrmann, Glover, Huss, Yang, and Zhu (2021)] [Duhr, Mistlberger (2021)]
- **N^3LO fiducial cross sections and distributions**
  [Camarda, Cieri, Ferrera (2021)], [Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli (2022)]

see talk by X. Chen

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$$+ \hat{\sigma}_{ab}^{(0,1)} + \dots \quad \text{EW}$$

☑ **NLO EW** corrections

- known since long
  [S. Dittmaier and M. Kramer (2002)], [Baur,Wackeroth (2004)], [Baur, Brein, Hollik, Schappacher, Wackeroth (2002)]
- nowadays **automatised** in different available generators
  [Les Houches 2017, 1803.07977]

Introduction

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$$+\hat{\sigma}_{ab}^{(0,1)} + \dots \quad \text{EW}$$

$$+\hat{\sigma}_{ab}^{(1,1)} + \dots \quad \text{QCD-EW}$$

Remark: N^3LO results displays a **slower convergence** of the perturbative series than expected from previous orders

Mixed QCD-EW corrections

- should **compete with N^3LO** according to the physical counting $\alpha \approx \alpha_S^2$ and represent the leading residual theoretical uncertainties due to truncation of the perturbative expansion
- is **highly desirable** in view of the expected precision target at HL-LCH, both in the resonant and and in the off-shell regions

→ from factorised ansatz, $\mathcal{O}(-2\%)$ at $m_{\ell\ell} = 1$ TeV

$$\frac{d\sigma}{dX} = \frac{d\sigma^{(1,0)}}{dX}\frac{d\sigma_{qq}^{(0,1)}}{dX}\left(\frac{d\sigma^{(0,0)}}{dX}\right)^{-1}$$

Mixed QCD-EW corrections: state of the art

The computation of fully differential mixed QCD-EW corrections to the production of an electro-weak boson is a **non trivial task**

Theoretical developments

- progress on two-loop master integrals
  [Bonciani, Di Vita, Mastrolia, Schubert (2016)], [Heller, von Manteuffel, Schabinger (2019)], [Hasan, Schubert (2020)]
- renormalization
  [Dittmaier, Schmidt, Schwarz (2020)]
- 2-loop amplitudes for $2 \to 2$ neutral current DY for massless leptons
  [Heller, von Manteuffel, Schabinger, Spiesberger (2020)]
- 2-loop amplitudes for $2 \to 2$ neutral current DY (retaining logarithms of the lepton mass)
  [Armadillo, Bonciani, Devoto, Rana, Vicini (2022)]

On-shell Z/W production ($2 \to 1$ process)

- analytical mixed QCD–QED corrections to the inclusive production of an on- shell Z
  [De Florian, Der, Fabre (2018)]
- fully differential mixed QCD–QED corrections to the production of an on-shell Z
  [Delto, Jaquier, Melnikov, Röntsch (2019)]
- total Z production cross section in fully analytical form including exact NNLO QCD-EW corrections
  [Bonciani, Buccioni, Rana, Vicini (2020)]
- fully differential on-shell Z and W production including exact NNLO QCD-EW corrections
  [F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch (2020)], [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)]

Mixed QCD-EW corrections: state of the art

The computation of fully differential mixed QCD-EW corrections to the production of an electro-weak boson is a **non trivial task**

Beyond on-shell computations

- dominant Mixed QCD-EW corrections in Pole Approximation for neutral- and charged- DY processes
  [Dittmaier, Huss, and Schwinn (2014,2015)]
- approximate corrections available in parton showers based on a factorised approach
  [Balossini et al (2010)], [Bernaciak,Wackeroth (2012)], [Barze' et al (2012,2013], [Calame et al (2017)]
- neutrino-pair production including NNLO QCD-QED corrections
  [Cieri, Der, De Florian, Mazzitelli (2020)]

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This talk

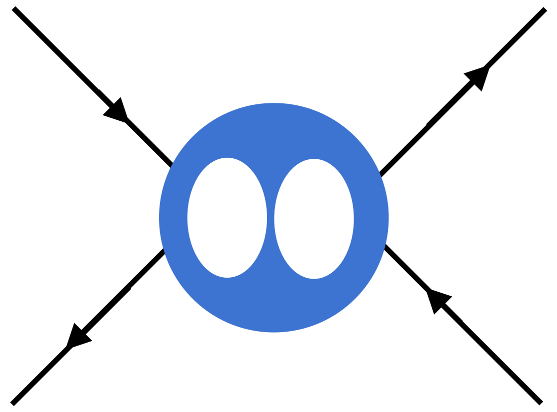
- mixed QCD-EW corrections to charged current Drell-Yan with approximate 2-loop amplitude
  [**LB**, Grazzini, Kallweit, Savoini, Tramontano (2021)]
- mixed QCD-EW corrections to neutral current Drell-Yan
  [Bonciani, **LB**, Grazzini, Kallweit, Rana, Tramontano, Vicini (2021)]
  [Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Rontsch, Signorile-Signorile (2022)]

see talk by Signorile-Signorile

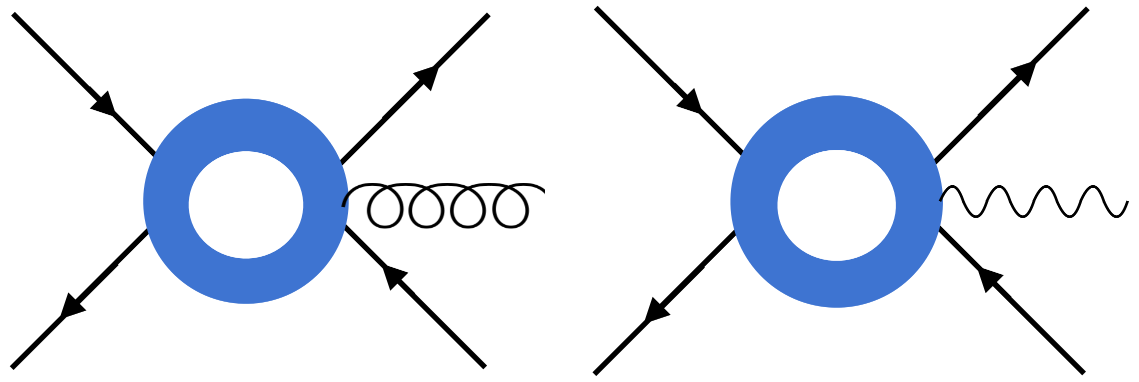
Important to have an independent calculation based on very different approaches for the infrared subtraction and for the calculation of the two-loop virtual amplitude

Mixed QCD-EW corrections: state of the art

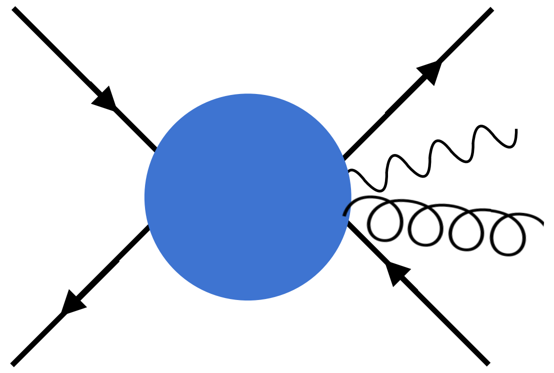
The complexity is similar to the calculation of NNLO QCD corrections for a $2 \to 2$ **multi-scale** process including **emission from final state legs**



Two-loop virtual diagrams (plus one-loop squared)



One-loop diagrams with one gluon or one photon emission



Tree-level diagrams with one gluon and one photon emission

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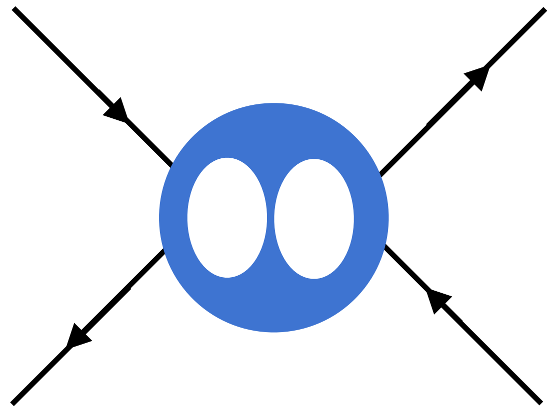

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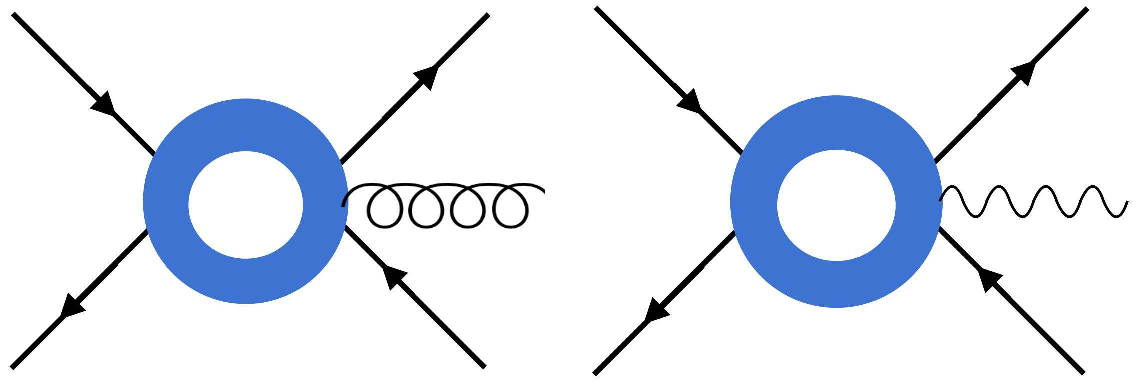

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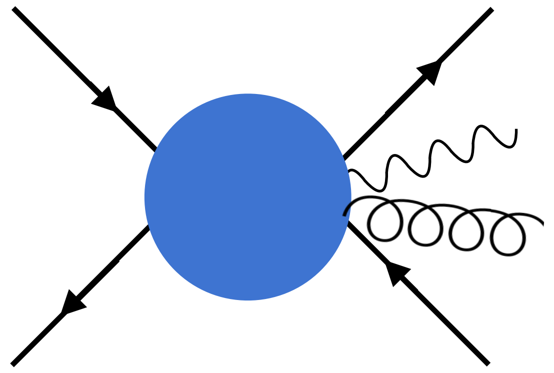

Tree-level diagrams with one gluon and one photon emission

- The computation of **tree-level** and **one-loop** amplitude is nowadays fully automatised, using tools like **OpenLoops** and **Recola**
- The double-real and real-virtual corrections known from studies of the **large transverse momentum lepton pair final state**
  [A.Denner, S.Dittmaier,T.Kasprzik,A.Muck (2011), A.Denner, S.Dittmaier, M.Hecht, C.Pasold (2015)] [J.Lindert et al., 1705.04664]
- **Complications**: numerical stabilities in the deep infrared regions

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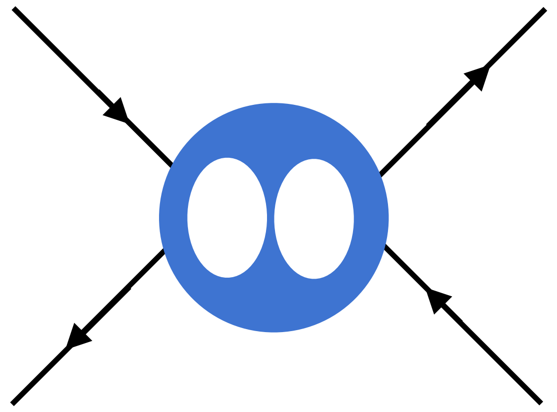

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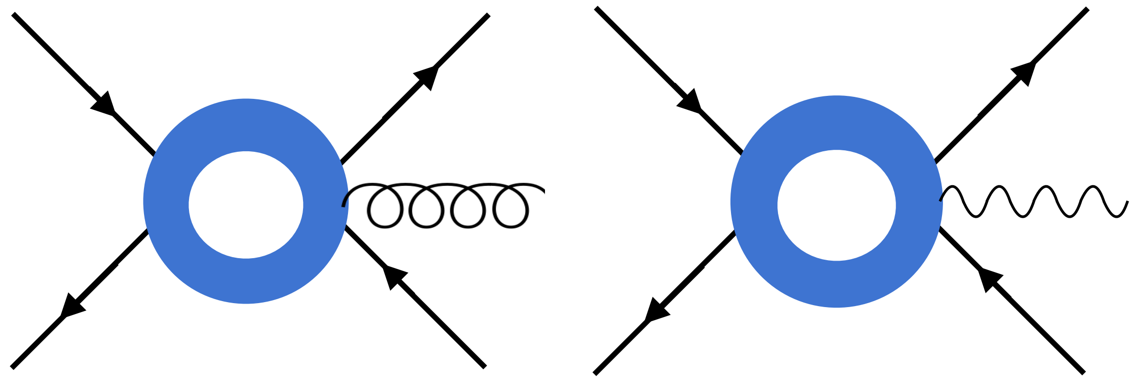

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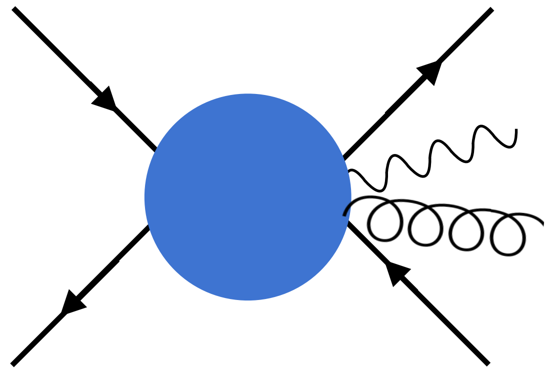

Tree-level diagrams with one gluon and one photon emission

ISSUES

- computation of two-loop virtual amplitudes (generation of the amplitudes, γ_5 treatment, 2-loop UV renormalization, subtraction of IR divergences, IBP reduction, **evaluation of Master Integrals**)
- combining all contributions to obtain the prediction for physical cross section and differential observable is a non-trivial task due to **the presence of IR singularities** (from intermediate virtual particles and real emission phase space integrals)

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The q_T subtraction formalism for NNLO QCD-EW corrections

GENERAL IDEAS

- **Do not reinvent the wheel:** the structure of IR singularities is associated to only the QCD-QED subpart and can be worked out by a dedicated **abelianisation** of the NNLO QCD results [de Florian, Rodrigo, Sborlini (2016)], [de Florian, Der, Fabre (2018)]
- We rely on the q_T **subtraction formalism and** its extension to the case of **massive final-state emitters** (heavy quarks in QCD and leptons in EW) [Catani, Grazzini (2007)], [Catani, Torre, Grazzini (2014)], [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan (2018)], [LB, Grazzini, Tramontano, 2019]

Resolution variable

q_T := **transverse momentum** of the dilepton final state
Q := invariant mass of the dilepton final state

One emission is always resolved for $q_T/Q > 0$

The q_T subtraction formalism for NNLO QCD-EW corrections

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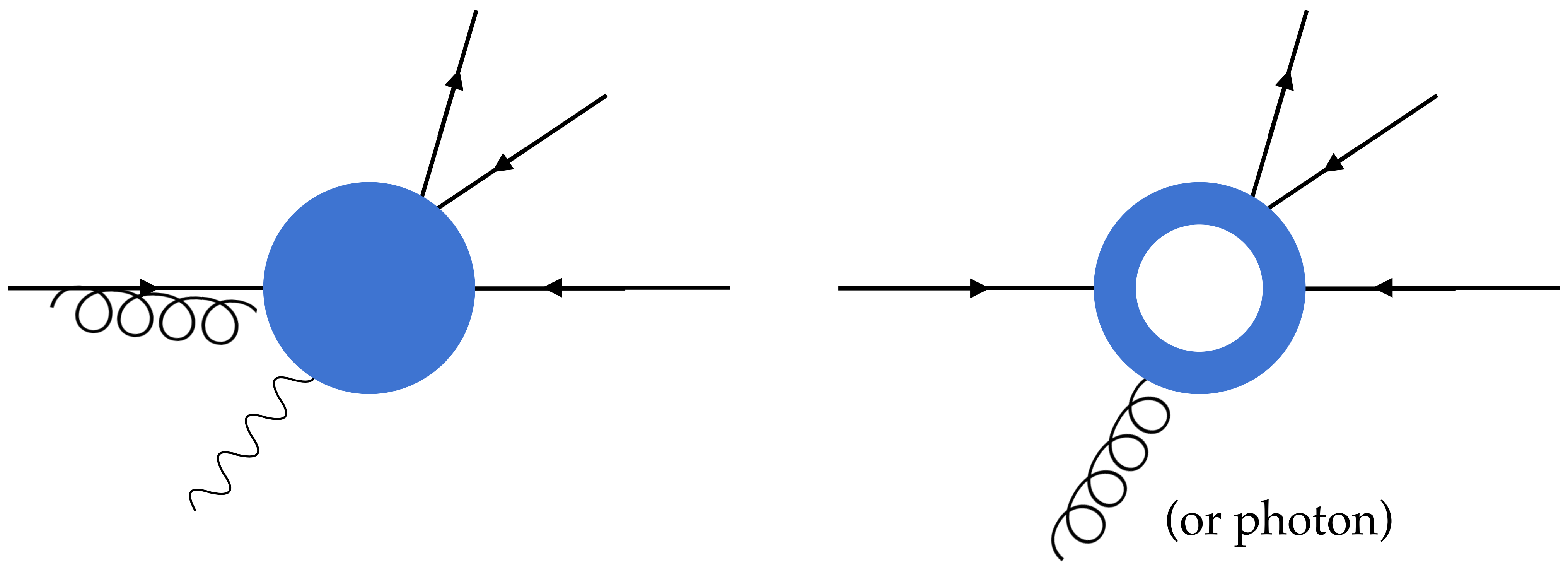
Lepton must be massive!

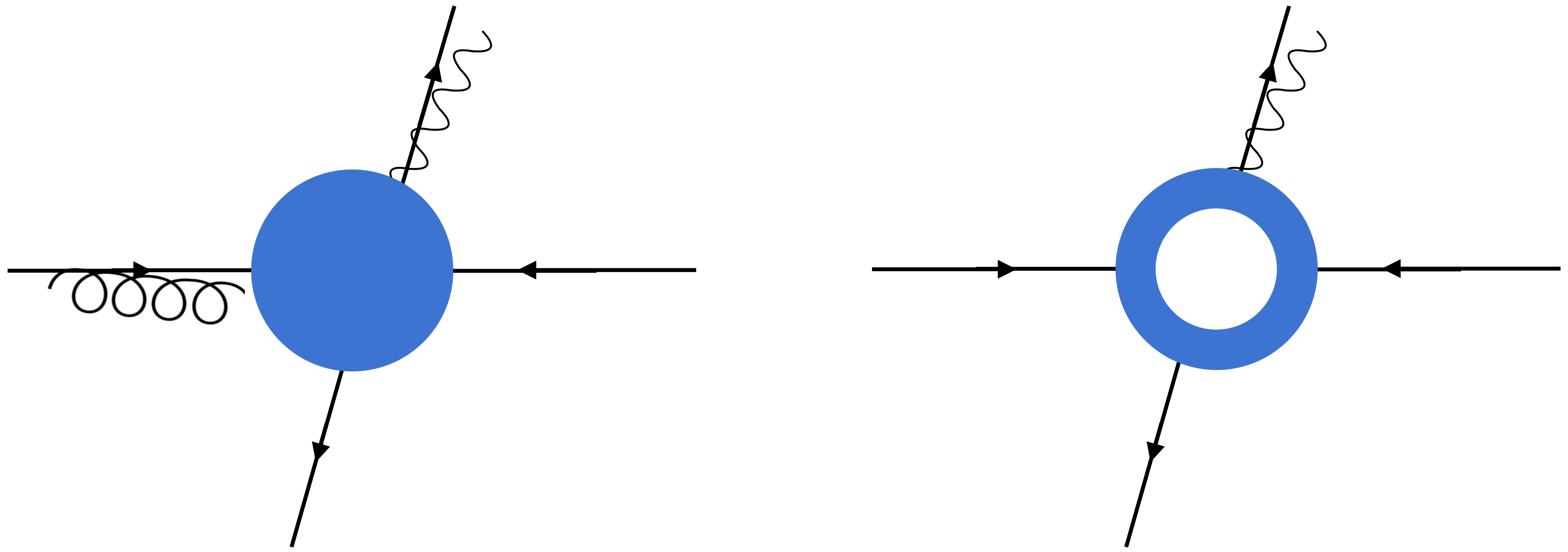
Initial-state radiation

For $q_T/Q > 0$ **one emission is always resolved**



Final-state (collinear) radiation

There are configurations with $q_T/Q > 0$ and **two unresolved emission if leptons are massless**



The q_T subtraction formalism in a nutshell

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)}$$

$\mathcal{O}(\alpha_s^m \alpha^n)$ term

$$d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{\mathrm{cut}}}$$

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- Double real + real-virt have only **NLO-type singularities** above the cut
- Apply a suitable **NLO subtraction scheme** (CS dipole in our case**)**
- Their contribution is **finite**
- **Double unresolved singularities** display themselves as **large logarithms** of the cutoff parameter r_{cut}

$$\int_{r_{\mathrm{cut}}} d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} C_i \ln^i r_{\mathrm{cut}} + C_0 + \mathcal{O}(r_{\mathrm{cut}}^m)$$

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- The **counterterm** is obtained by expanding at fixed order the q_T **resummation formula**
- The counterterm **removes the IR sensitivity** associated to the cutoff variable

through abelianisation of available QCD results

$$\int_{r_{\rm cut}} d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} c_i \ln^i r_{\rm cut} + C_0 + \mathcal{O}(r_{\rm cut}^m) \quad \longrightarrow \quad \int_{r_{\rm cut}} \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right] \sim C_0 + \mathcal{O}(r_{\rm cut}^m)$$

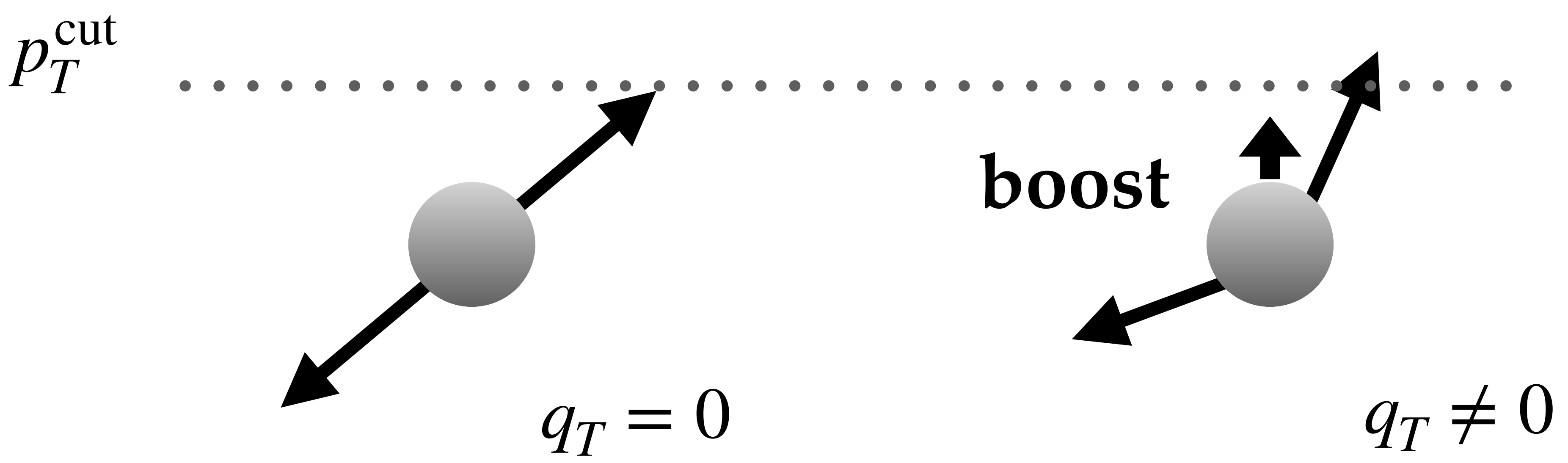
Size of **power corrections** affects the **performance** of the method ; trade off in the choice of $r_{\rm cut}$

- sufficiently small to render power corrections negligible
- sufficiently large to reduce numerical instabilities due to (global) cancellation of large quantities

Power Corrections (PCs) in the Drell-Yan process

- QCD corrections: **quadratic** ($m = 2$) for inclusive setups, **linear** power corrections may arise for **fiducial cuts** [Ebert, Tackmann, 2019], [Salam, Slade, 2021]

[Ebert, Michel, Stewart, Tackmann, 2020], [Catani, de Florian, Ferrera, Grazzini, 2015]

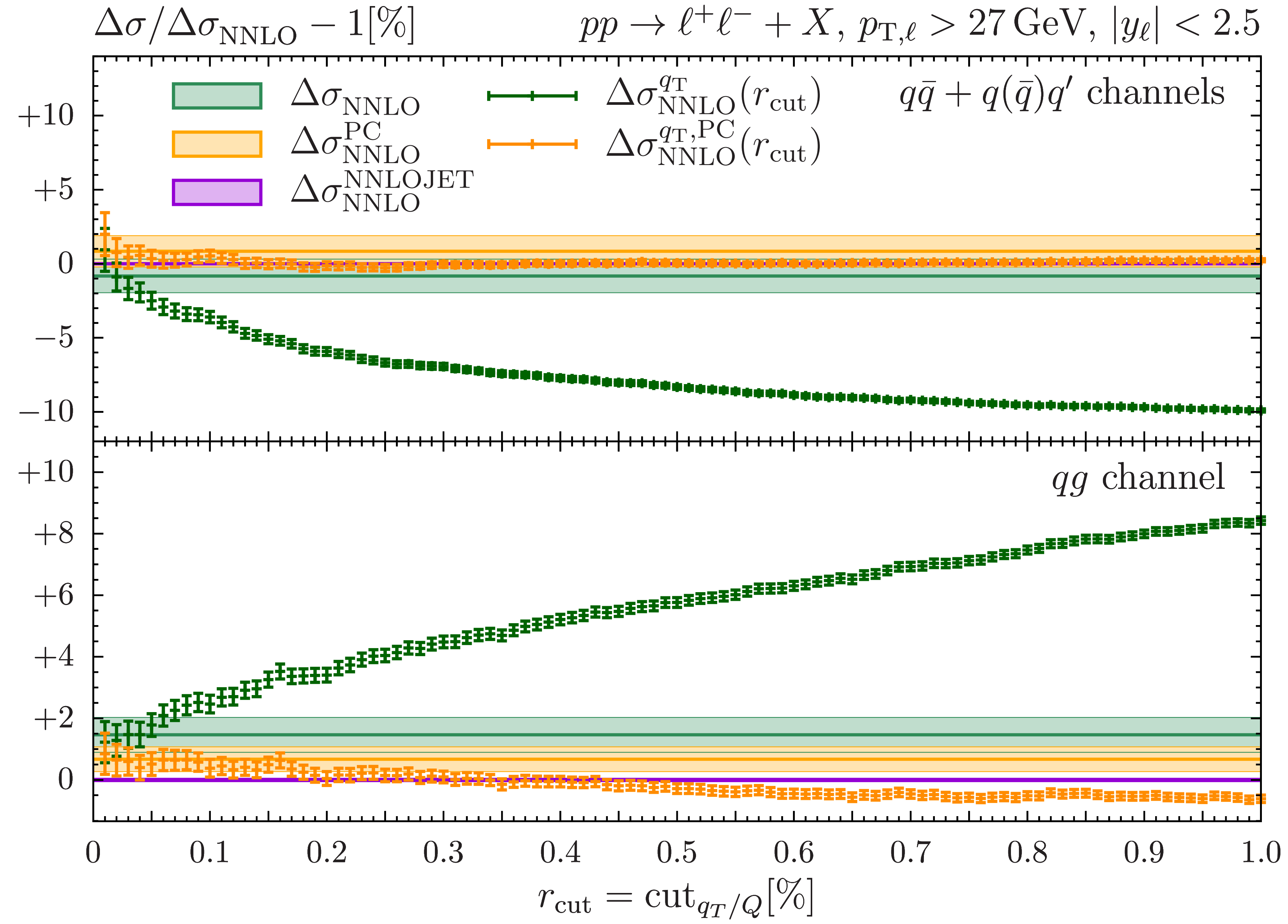


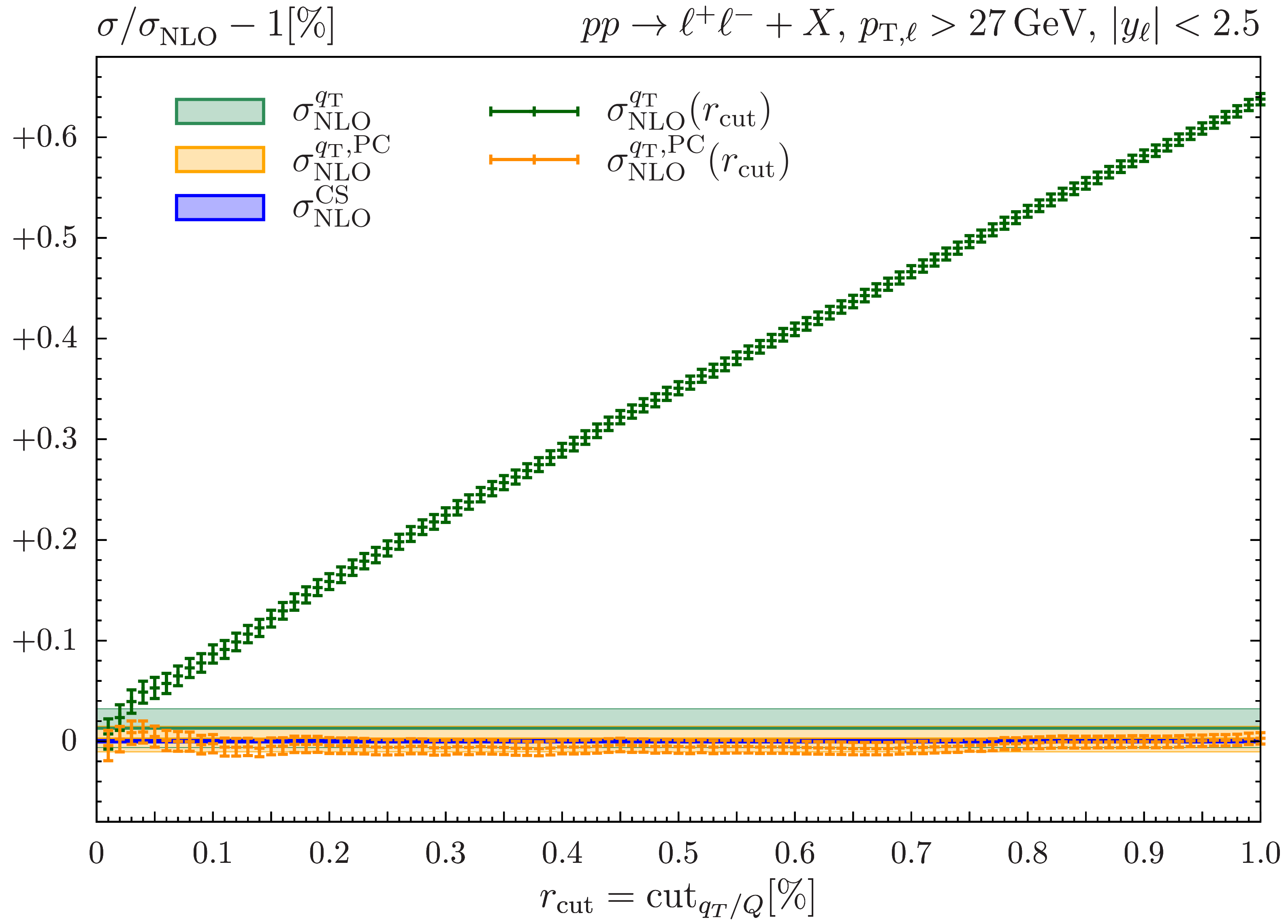
general solution for fiducial power corrections through a modification of the subtraction formula

$$\sigma = \sigma_{q_T} + \Delta\sigma^{\rm linPCs}(r_{\rm cut}) + \mathcal{O}\left(r_{\rm cut}^{k'}\right)$$

$$\Delta\sigma^{\rm linPCs}(r_{\rm cut}) = \int d\Phi_{\rm F} \int_0^{r_{\rm cut}} dr' \left[\frac{d\sigma^{\rm F}_{\rm CT}}{d\Phi_{\rm F}} \Theta_{\rm cuts}\left(\Phi_{\rm F}^{\rm rec}(\Phi_{\rm F}, r')\right) - \frac{d\sigma^{\rm F}_{\rm CT}}{d\Phi_{\rm F}} \Theta_{\rm cuts}(\Phi_{\rm F})\right]$$

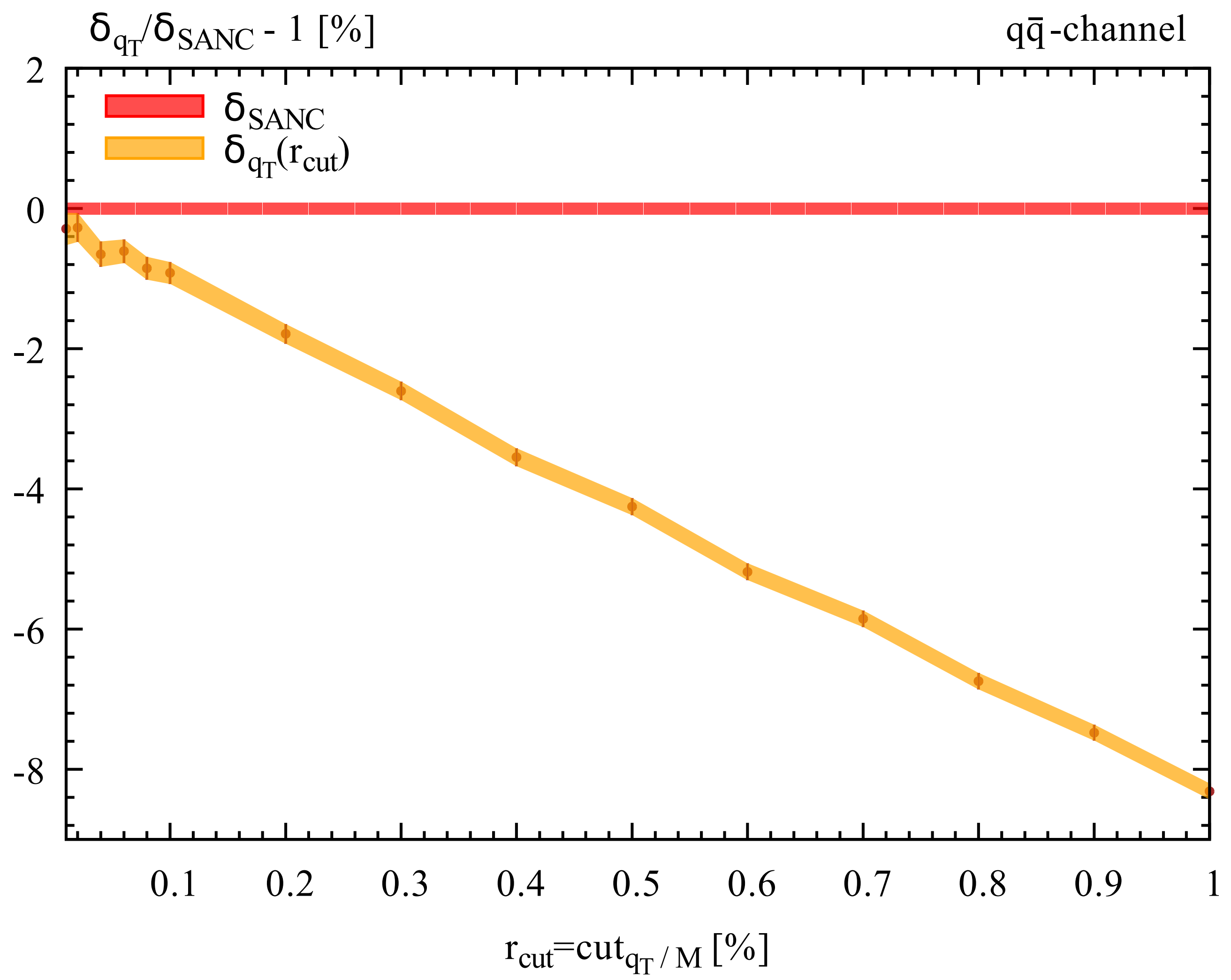
[**LB**, Kallweit, Rottoli, Wiesemann, 2021], [Camarda, Cieri, Ferrera, 2021]





Power Corrections (PCs) in the Drell-Yan process

- EW corrections: **linear** ($m = 1$) power corrections due to final-state emission



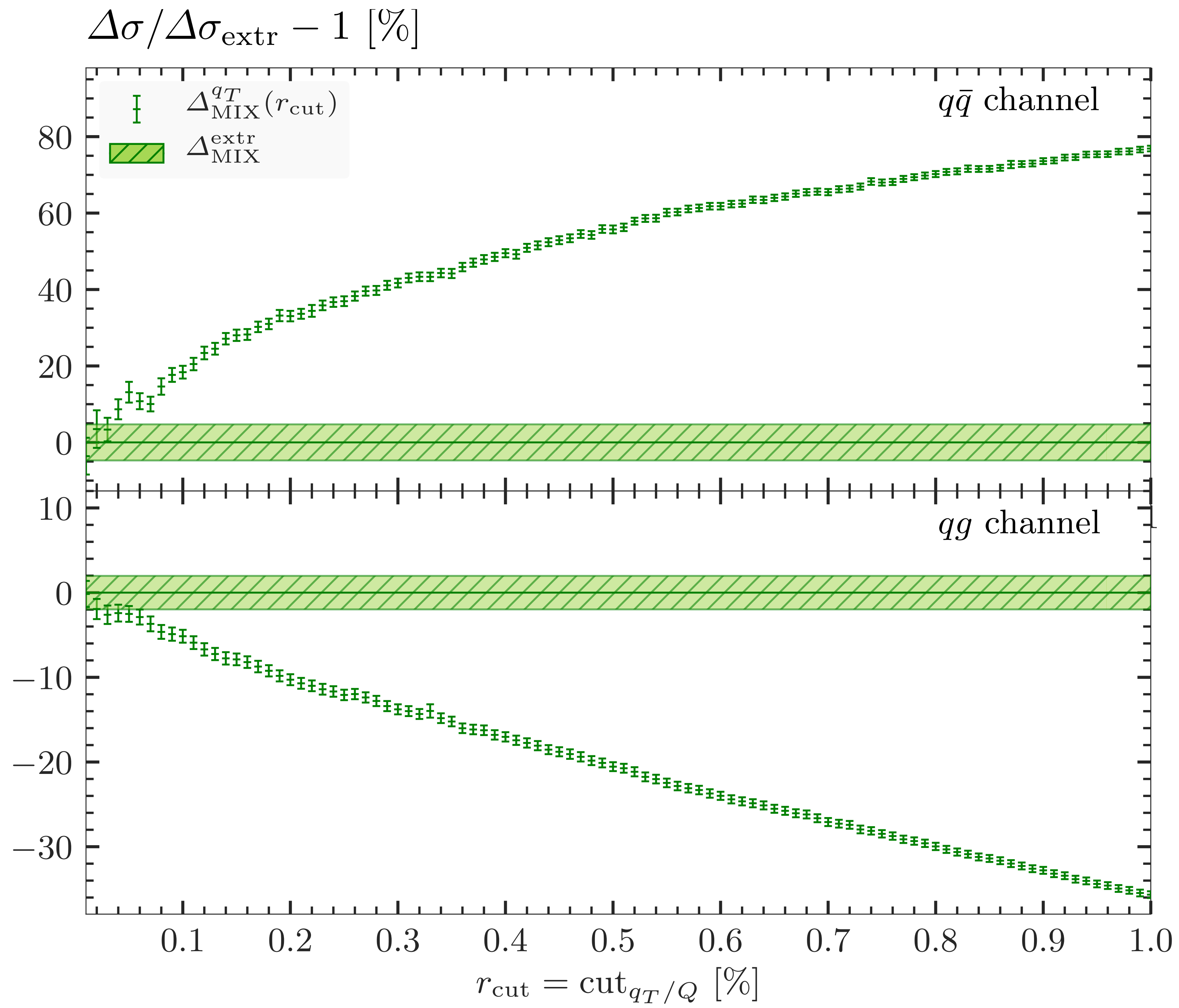
analytical insight for **inclusive cross section** in pure QED

$$\sigma^{\rm NLP}(s; r_{\rm cut}) = -\frac{3\pi}{8}\frac{\alpha}{2\pi} r_{\rm cut}\left[\frac{6(5-\beta^2)}{3-\beta^2} + \frac{-47+8\beta^2+3\beta^4}{\beta(3-\beta^2)}\log\frac{1+\beta}{1-\beta}\right]\sigma_B(s)$$

$$\beta = \sqrt{1-\frac{4m^2}{s}}$$

[**LB**, Grazzini, Tramontano, 2019]

in general we have to rely on an extrapolation procedure!



- Mixed QCD-NLO EW: linear (m=1) + log enhancement
  - Rather **large** $r_{\rm cut}$ dependence on the mixed corrections
  - Control of the mixed corrections at $\mathcal{O}$(5-10%) which translates into **per mille level accuracy on the total cross section**
  - **Bin-wise extrapolation** for distributions

The q_T subtraction formalism in a nutshell: Hard-Virtual coefficient

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)}$$

$\mathcal{O}(\alpha_s^m \alpha^n)$ term

$$d\sigma^{(1,1)} = \mathcal{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{\rm cut}}$$

- The **hard-collinear** coefficient brings in the **virtual corrections** and finite remainder that lives at $q_T = 0$, restoring the correct **normalisation**

$$\mathcal{H}^F = [\ H^F C_1 C_2]$$

Process dependent hard-virtual functions: universal relation with the all-order virtual amplitude
[Catani, Cieri, de Florian, Ferrera Grazzini (2013)]

Process independent (universal) **collinear functions** known up N³LO in QCD
[Catani,Grazzini (2011)],
[Catani, Cieri, de Florian, Ferrera, Grazzini (2012)
[Luo, Yang, Zhu, Zhu (2019)]
[Ebert, Mistlberger, Vita (2020)]

$$|\tilde{\mathcal{M}} > = (1 - \tilde{I})|\mathcal{M} >$$

$$H^F \sim < \tilde{\mathcal{M}} | \tilde{\mathcal{M}} >$$

computed with abelianisation

$$\mathcal{H}^{(m,n)} = H^{(m,n)} \delta(1 - z_1)\delta(1 - z_2) + \delta\mathcal{H}^{(m,n)}$$

IR subtracted amplitude

$$H^{(1,1)} \equiv \frac{2\mathrm{Re}\left(\mathcal{M}_{\rm fin}^{(1,1)} \mathcal{M}^{(0,0)*}\right)}{|\mathcal{M}^{(0,0)}|^2}$$

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2-loop virtual NC-DY: exact calculation

WORKFLOW

1. Feynman diagrams: QGRAF
2. Dirac Algebra and interference terms: FORM
3. IBPs: KIRA, LITERED, REDUZE2
4. Evaluation of MIs
5. UV renormalisation, subtraction of IR poles
6. Numerical grid

Treatment of γ_5: Naive anti-commuting γ_5 with reading point prescription

MIs with **up to one massive** boson exchange are evaluated analytically
[Bonciani, Di Vita, Matrolia, Schubert, 2016], [Heller, von Manteuffel, and Schabinger, 2020] [Hasan, Schubert, 2020], [R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, 2008], [R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, 2008], [P. Mastrolia, M. Passera, A. Primo, and U. Schubert, 2017]

5 MIs with **two massive** bosons cannot be easily expressed in terms of GPls
Require an **alternative strategy** (see also [Heller, von Manteuffel, Schabinger (2019)])

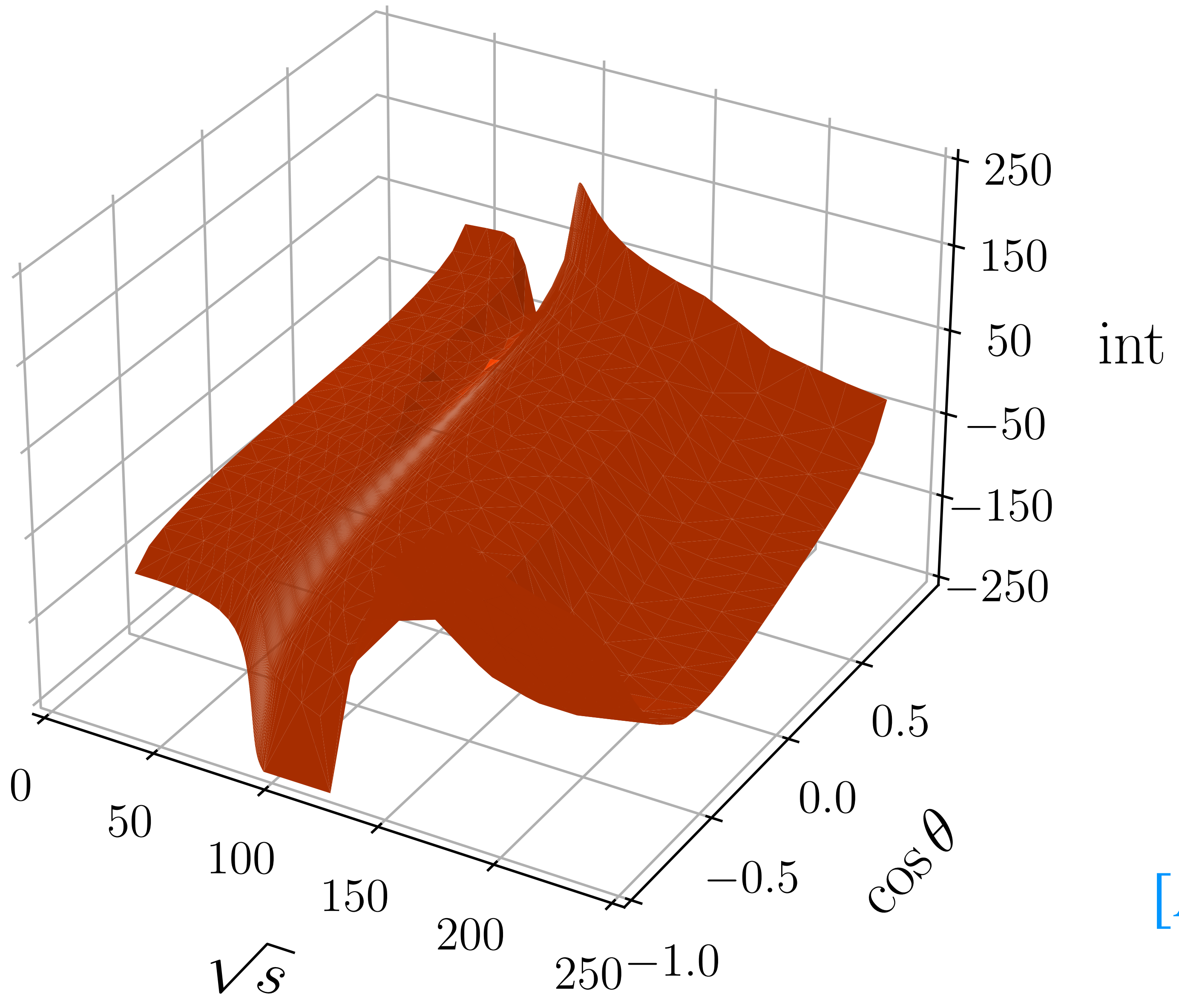
Semi-analytical evaluation of tree-loop interference
[Armadillo, Bonciani, Devoto, Rana, Vicini 2022] see talk by S. Devoto

- Numerical resolution of differential equations for MIs via **series expansions,** inspired by DiffExp [Hidding (2006)] but extended for **complex masses**
- **Arbitrary number of significant digits** (with analytic boundary condition)
- The method is **general** (applicable to other processes)
- Numerical evaluation of amplitudes takes $\mathcal{O}$(10 min/point) per core

for **fast** numerical integration

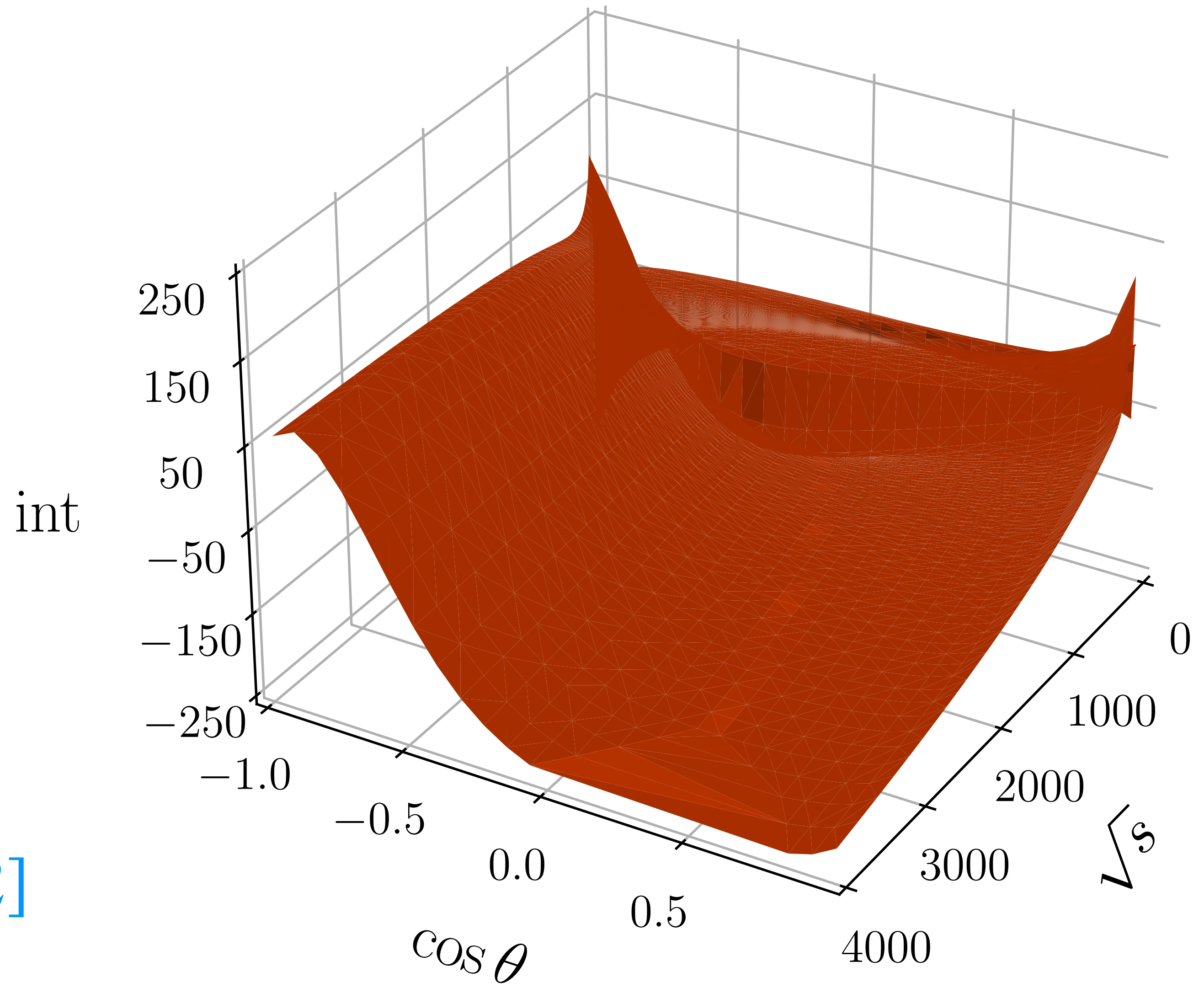
2-loop virtual NC-DY: numerical evaluation

- **Validation**: several checks of the MIs performed with Fiesta and PySecDec, comparison with the PA in the resonant region
- **Evaluation:** preparation of **an optimised numerical grid** covering the physical $2 \to 2$ phase space relevant the LHC in $(s, \cos\theta)$ with GiNaC and DiffExp/SeaFire [T. Armadillo et al in preparation] and **interpolation with cubic splines** for numerical integration
  - $\mathcal{O}(9\text{ h})$ on a 32-cores machines for 3000 grid points
  - the **lepton mass is kept finite** wherever needed to regularise the final state collinear divergence; the logarithms of the lepton mass are subtracted from the numerical grid and added back analytically
  - the resulting UV- and IR-subtracted Hard-Virtual coefficient is a **smooth, slowly varying** function



$$H^{(1,1)} \equiv \frac{2\mathrm{Re}\left(\mathcal{M}^{(1,1)}_{\mathrm{fin}} \mathcal{M}^{(0,0)*}\right)}{|\mathcal{M}^{(0,0)}|^2}$$

in units $\frac{\alpha_s}{\pi}\frac{\alpha}{\pi}\sigma_{\mathrm{LO}}$



[Armadillo, Bonciani, Devoto, Rana, Vicini 2022]

Outline

- Motivations
- Methodology: infrared subtraction
- 2-loop virtual amplitude
- Phenomenological results
- Conclusions&Outlook

MATRIX framework

MATRIX *[Grazzini, Kallweit, Wiesemann, 2018]*

MUNICH (by S. Kallweit)

- **efficient multichannel** phase space generation
- **bookkeeping** all subprocesses
- automatic implementation of dipole subtraction

AMPLITUDES

- 1-loop amplitudes: **OpenLoops, Recola** (Collier, CutTOols,…)
- 2-loop: dedicated 2-loop codes (VVamp, GiNac, TDHPL,…)

SUBTRACTION SCHEME

- @NLO: dipole and q_T subtraction
- @NNLO: q_T subtraction

MATRIX v2.0

- NNLO QCD differential predictions for many color singlet processes: H, V, $\gamma\gamma$, $V\gamma$, VV for all leptonic decays
- combination with NLO EW for all leptonic V and VV processes
- loop-induced gluon fusion channel at NLO QCD for neutral VV processes

NEW MATRIX v2.1 (beta version) matrix.hepforge.org

see talk by Stefan Kallweit

- NNLO QCD for $t\bar{t}$ and $\gamma\gamma\gamma$ production
- **bin-wise extrapolation** and inclusion of QCD **fiducial power corrections** in 2-body kinematics

MATRIX framework

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Mixed QCD-EW corrections for Drell-Yan available in a future release

Mixed QCD-EW corrections for NC-DY

First calculation of complete mixed QCD-EW correction to Drell-Yan [LB, Bonciani, Grazzini, Kallweit, Rana, Tramontano, Vicini 2021]

SETUP (LHC @ $\sqrt{s}$ = 14 TeV)

- NNPDF31_nnlo_as_0118_luxqed
- $p_{T,\mu} > 25$ GeV, $|y_\mu| < 2.5$, $m_{\mu^+\mu^-} > 50$ GeV
- massive muons (no photon lepton recombination)
- G_μ scheme, complex mass scheme
- fixed scale $\mu_F = \mu_R = m_Z$

σ [pb]	$\sigma_{\rm LO}$	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	809.56(1)	191.85(1)	−33.76(1)	49.9(7)	−4.8(3)
qg	—	−158.08(2)	—	−74.8(5)	8.6(1)
$q(g)\gamma$	—	—	−0.839(2)	—	0.084(3)
$q(\bar{q})q'$	—	—	—	6.3(1)	0.19(0)
gg	—	—	—	18.1(2)	—
$\gamma\gamma$	1.42(0)	—	−0.0117(4)	—	—
tot	810.98(1)	33.77(2)	−34.61(1)	−0.5(9)	4.0(3)
$\sigma^{(m,n)}/\sigma_{\rm LO}$		+4.2 %	−4.3 %	~ 0 %	+0.5 %

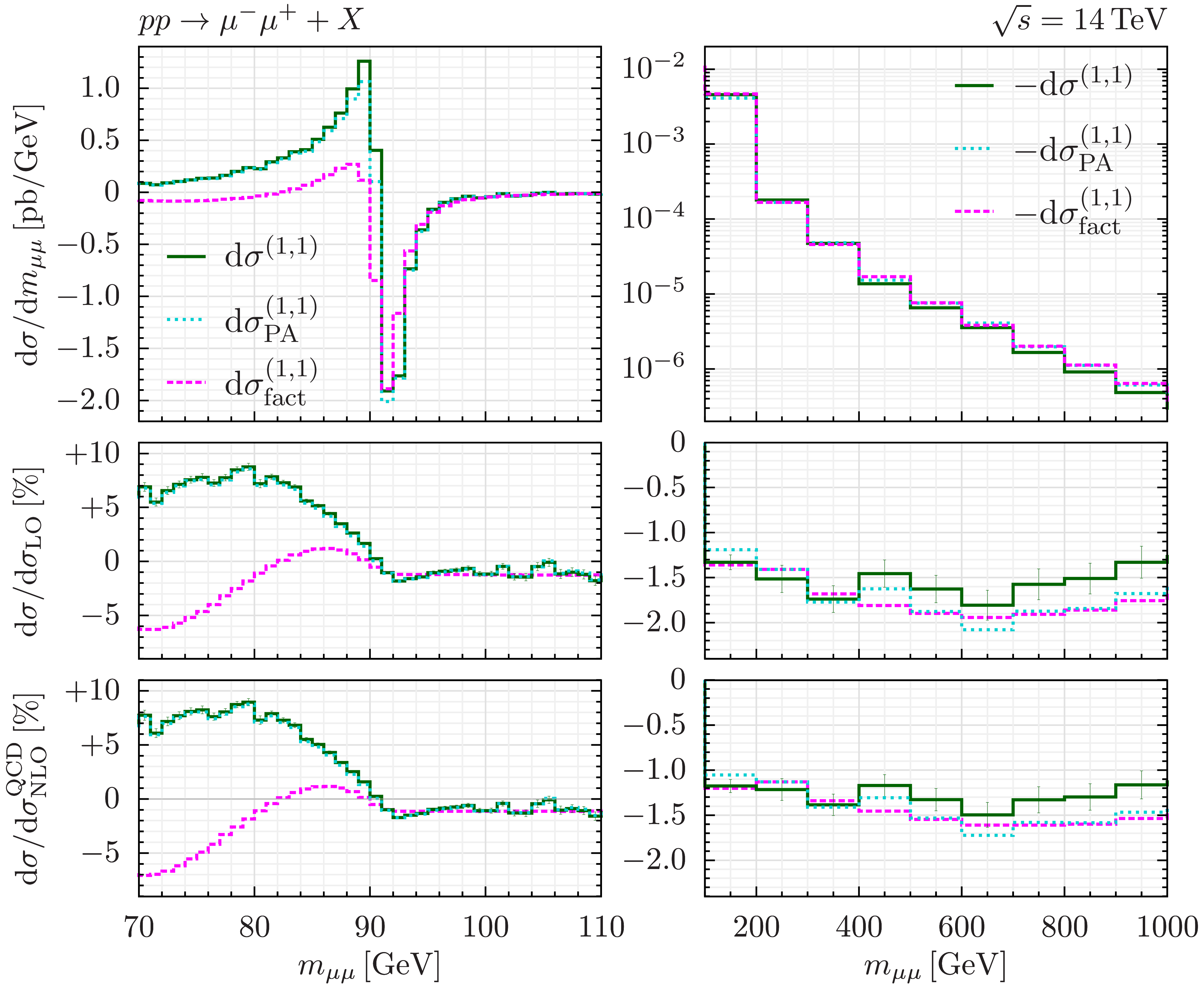
- NLO and NNLO QCD corrections show large cancellations among the partonic channels (especially between $q\bar{q}$ and qg)
- NLO QCD and NLO EW corrections are of the same order and opposite sign (accidental cancellation)
- Mixed QCD-EW corrections are dominated by the qg channel and are larger than NNLO QCD (for the particular chosen setup)
- Photon induced processes rather suppressed

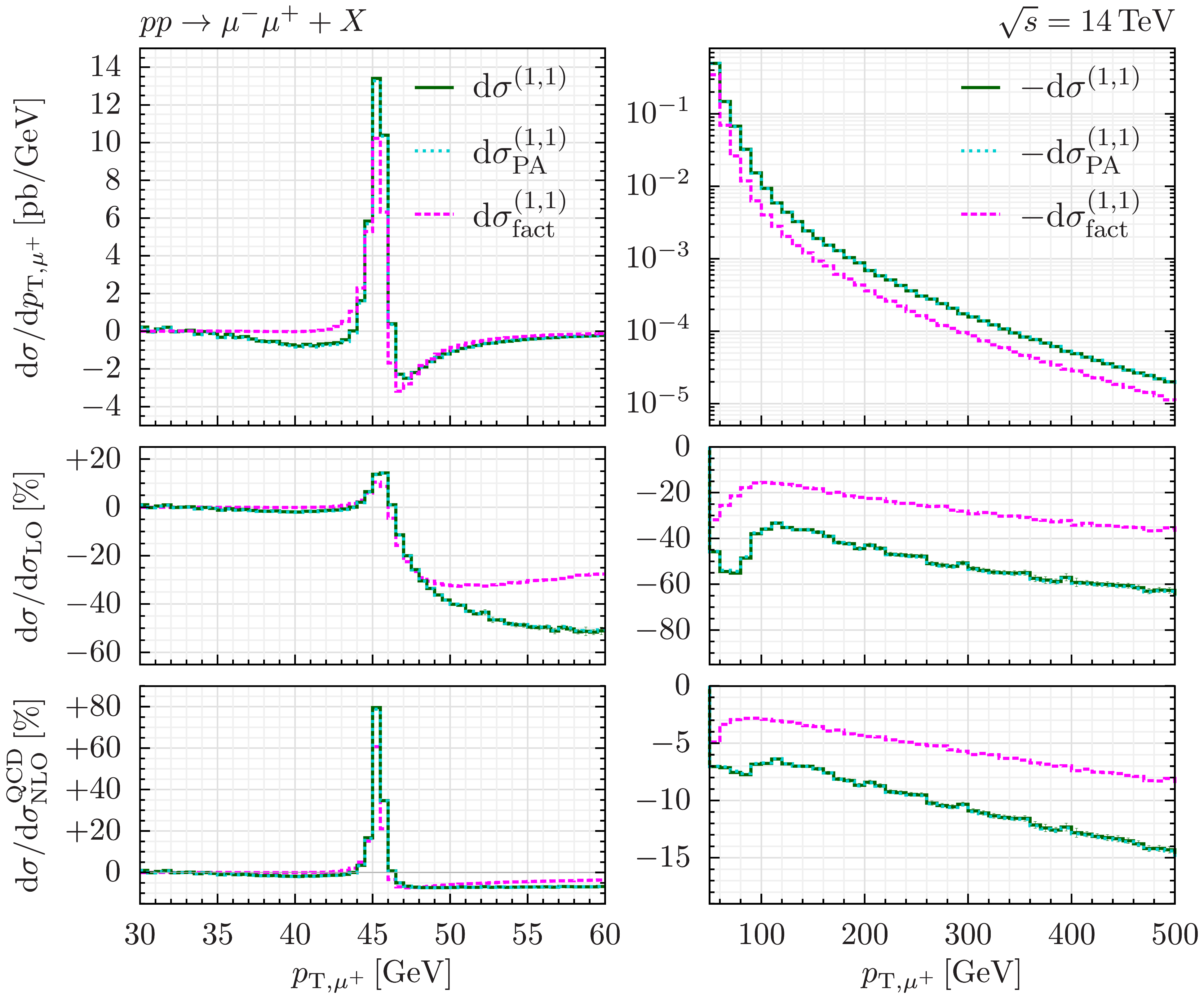
Computational resources

- $\mathcal{O}$(120k) core hours for NNLO QCD (reduced by a factor 2-3 by including fiducial PCs)
- $\mathcal{O}$(180k) core hours for mixed QCD-EW

Mixed QCD-EW corrections for NC-DY

First calculation of complete mixed QCD-EW correction to Drell-Yan [LB, Bonciani, Grazzini, Kallweit, Rana, Tramontano, Vicini 2021]





- Breakdown of naive QCD-QED factorisation below Z peak
- PA provides an excellent description **near the resonance**
- $\mathcal{O}(1\%)$ corrections at 1 TeV, PA slightly off

- Around resonance, breakdown of fixed-order
- Naive QCD-QED factorisation fails to describe the high-tail
- The high-tail is dominated by Z+1jet configurations, with Z almost on shell; qg channel by far dominant

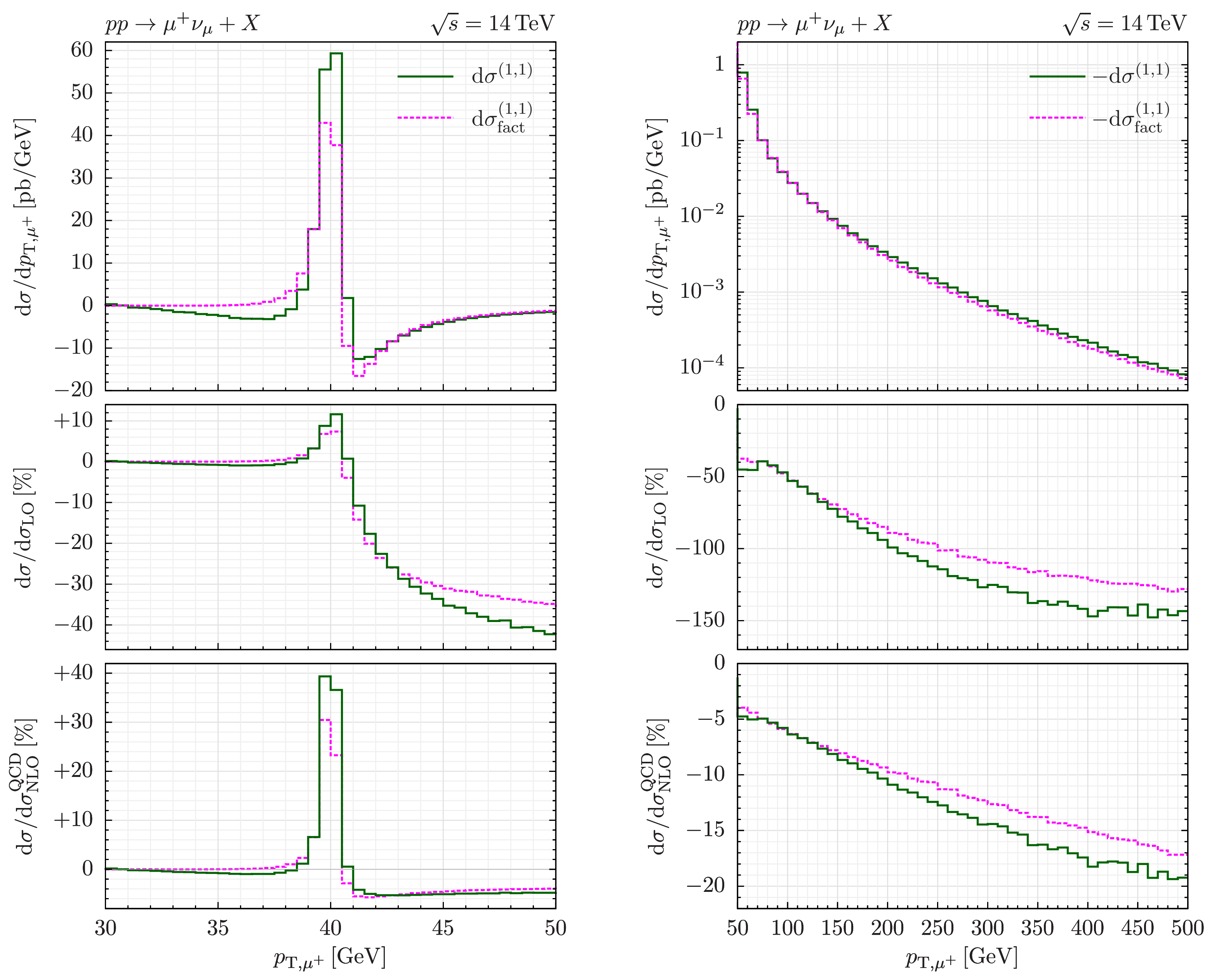
Mixed QCD-EW corrections for CC-DY

Mixed QCD-EW correction to Drell-Yan with **two-loop virtual approximated in PA** [LB, Grazzini, Kallweit, Savoini, Tramontano, 2021]

SETUP (LHC @ $\sqrt{s} = 14$ TeV)

- `NNPDF31_nnlo_as_0118_luxqed`
- $p_{T,\mu} > 25$ GeV, $|y_\mu| < 2.5$, $p_{T,\nu_\mu} > 25$ GeV
- massive muons (no photon lepton recombination)
- G_μ scheme, complex mass scheme
- fixed scale $\mu_F = \mu_R = m_W$

σ [pb]	$\sigma_{\rm LO}$	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	5029.2	970.5(3)	−143.61(15)	251(4)	− 7.0(1.2)
qg	—	−1079.86(12)	—	−377(3)	39.0(4)
$q(g)\gamma$	—	—	2.823(1)	—	0.055(5)
$q(\bar{q})q'$	—	—	—	44.2(7)	1.2382(3)
gg	—	—	—	100.8(8)	—
tot	5029.2	−109.4(4)	−140.8(2)	19(5)	33.3(1.3)
$\sigma^{(m,n)}/\sigma_{\rm LO}$		−2.2 %	−2.8 %	+0.4 %	+0.6 %



- Same pattern of corrections as in NC-DY
- Mixed QCD-EW corrections **dominated by qg channel (exact)**
- Focus on lepton transverse momentum, but **transverse mass** around the W peak should well described by PA

Possibility to estimate the impact on W mass as in [Dittmaier, Huss, Schwinn (2015)]

Ongoing work

- Computation of the exact 2-loop amplitude

Mixed QCD-EW corrections for CC-DY

Mixed QCD-EW correction to Drell-Yan with **two-loop virtual approximated in PA** [LB, Grazzini, Kallweit, Savoini, Tramontano, 2021]

SETUP (LHC @ $\sqrt{s} = 14$ TeV)

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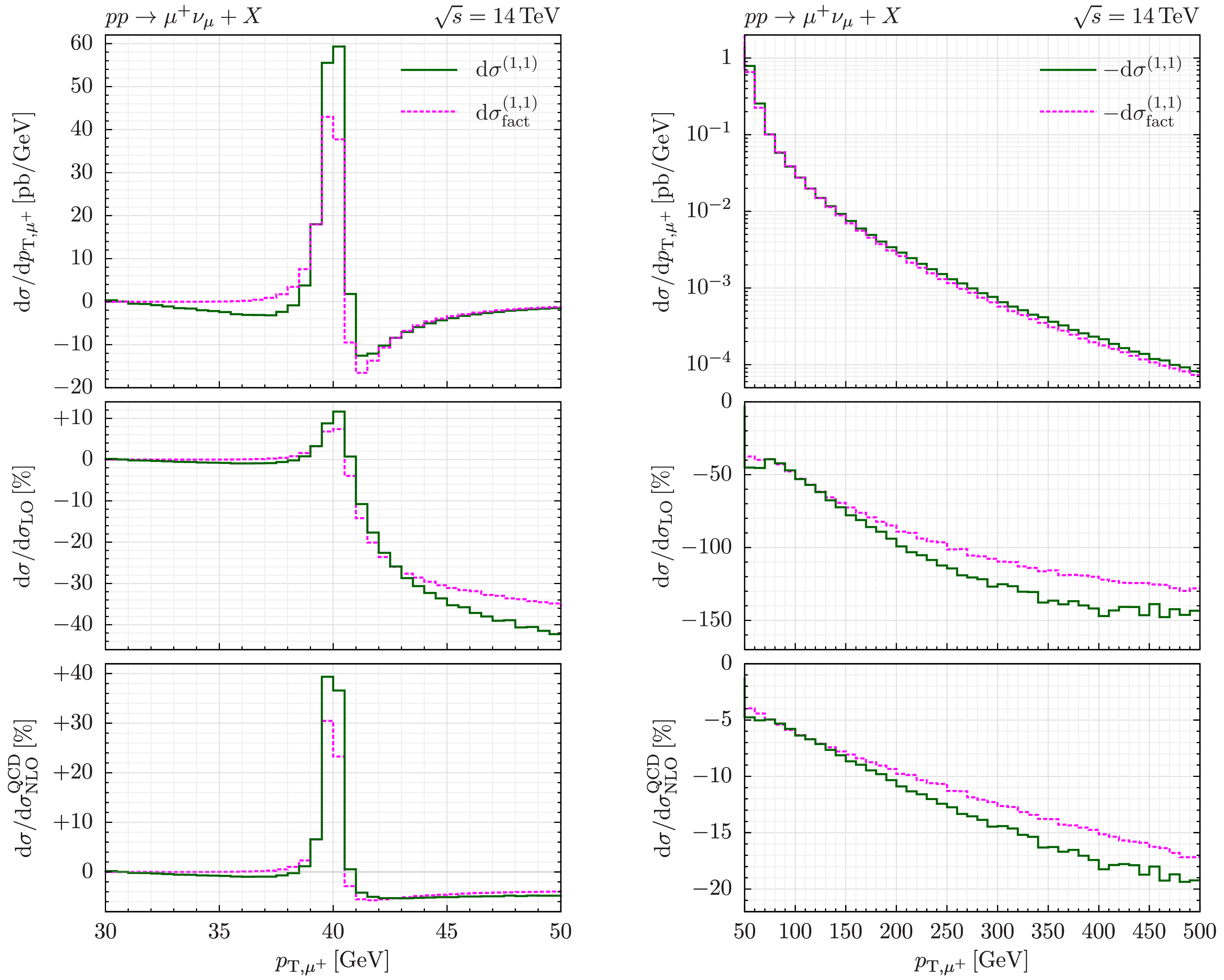
tot	5029.2	−109.4(4)	−140.8(2)	19(5)	33.3(1.3)
$\sigma^{(m,n)}/\sigma_{\rm LO}$		−2.2 %	−2.8 %	+0.4 %	+0.6 %
$\sigma^{(m,n)}/\sigma_{\rm LO}$		+10 %	−2.9 %	+4.2 %	+0.8 %



Remark: the pattern of QCD correction is sensitive to the scale choice

$\mu_F = \mu_R = m_W/2$

Phenomenology of mixed QCD-EW corrections for NC-DY

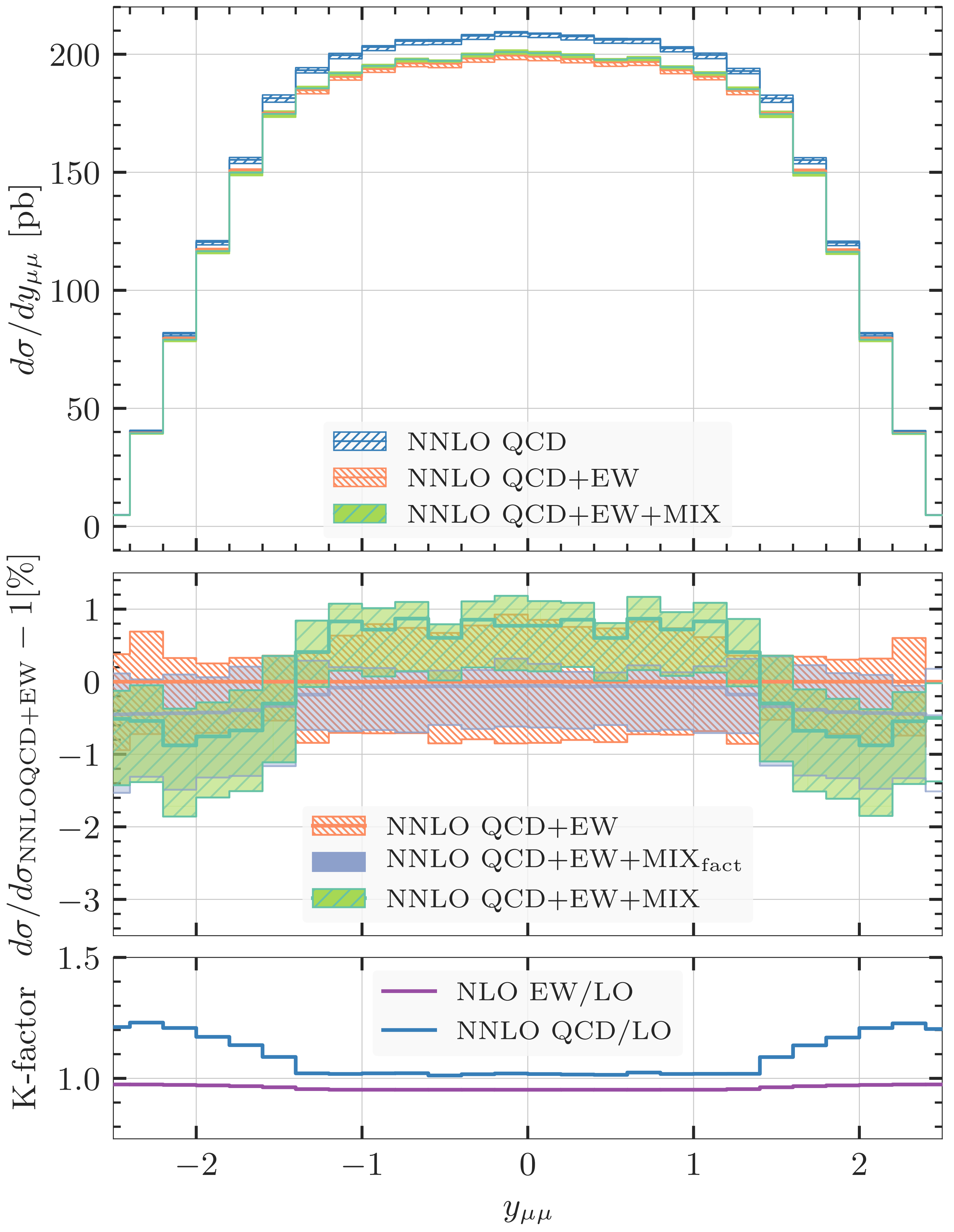
PRELIMINARY

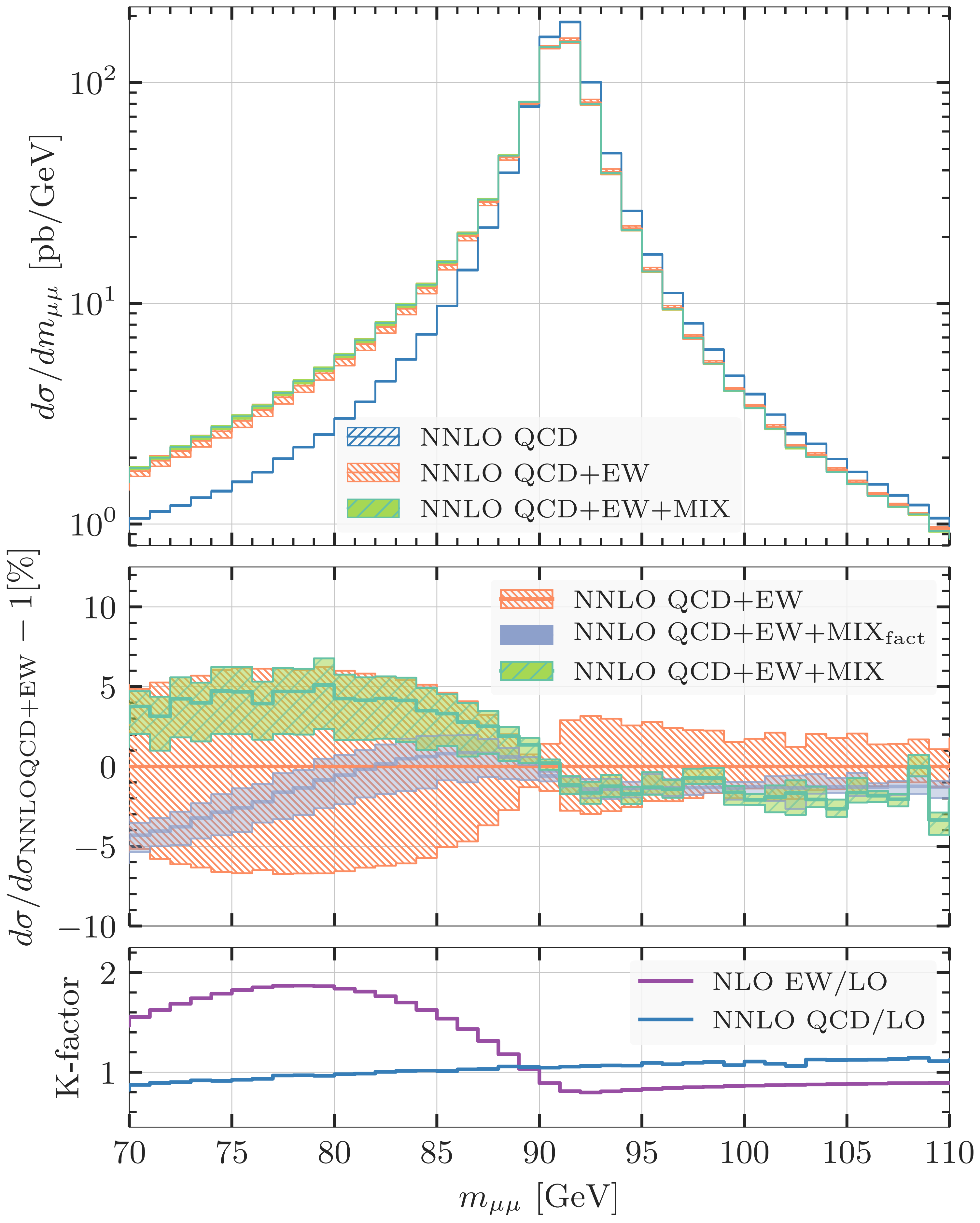
[LB, Bonciani, Devoto, Grazzini, Kallweit, Rana, Tramontano, Vicini in preparation]

SETUP (LHC @ $\sqrt{s}$ = 13.6 TeV)

- `NNPDF31_nnlo_as_0118_luxqed`
- $p_{T,\mu} > 25$ GeV, $|y_\mu| < 2.5$, 66 GeV $< m_{\mu^+\mu^-} <$ 116 GeV
- massive muons (no photon lepton recombination)
- G_μ scheme, complex mass scheme
- fixed scale $\mu_F = \mu_R = m_Z$

G_μ-scheme	σ [pb]	$\sigma^{(i,j)}$ [pb]	$\sigma^{(i,j)}/\sigma_{LO}$
LO	$763.40(2)^{+12.7\%}_{-13.6\%}$	–	–
NLO QCD	$802.26(6)^{+2.7}_{-4.2\%}$	38.86(6)	5.1%
NNLO QCD	$802.5(7)^{+0.4}_{-0.8\%}$	0.2(7)	0.0%
NLO EW	$730.76(2)^{+12.7}_{-13.6\%}$	−32.65(3)	−4.3%
NNLO QCD+EW	$769.8(7)^{+0.5}_{-0.6\%}$	–	–
NNLO QCD+EW+MIX$_{fact}$	$768.2(7)^{+0.3}_{-0.7\%}$	−2.0(1)	−0.2%
NNLO QCD+EW+MIX	$772.4(8)^{+0.3}_{-0.7\%}$	2.6(2)	0.3%





- Mixed QCD-EW corrections are smaller in this setup, but **non-trivial $\mathcal{O}(1\%)$ shape distortion** in the distributions
- Stabilisation of theory uncertainties

Uncertainities: 7-point scale variation
NNLO QCD+EW+MIX$_{fact}$: NNLO QCD+EW+ **factorised** approximation of mixed corrections

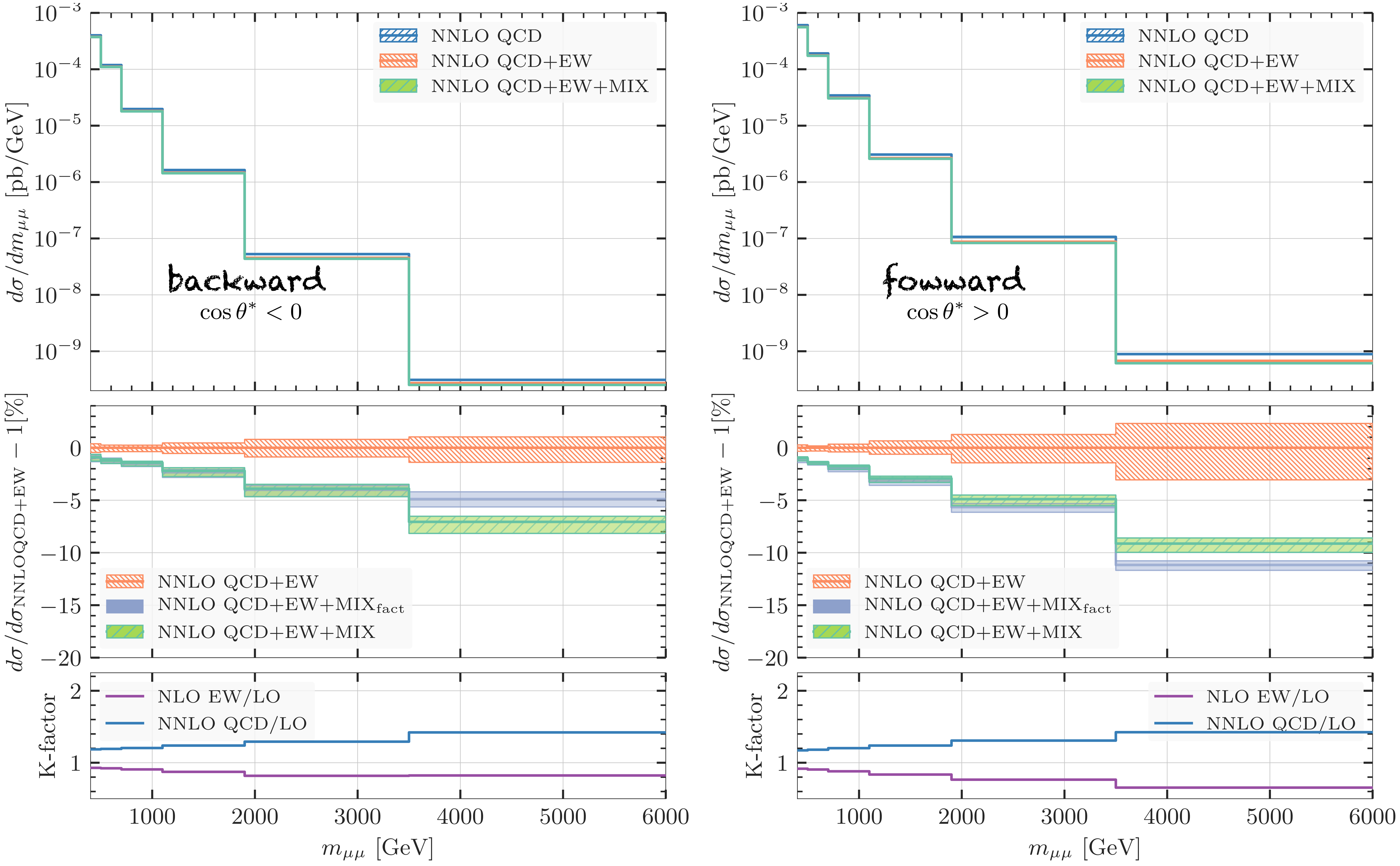
Phenomenology of mixed QCD-EW corrections for NC-DY

PRELIMINARY

[LB, Bonciani, Devoto, Grazzini, Kallweit, Rana, Tramontano, Vicini in preparation]

SETUP (LHC @ $\sqrt{s}$ = 13 TeV) **CMS 2103.02708**

- `NNPDF31_nnlo_as_0118_luxqed`
- $p_{T,\mu} > 53$ GeV, $|y_\mu| < 2.4$, $m_{\mu^+\mu^-} > 150$ GeV
- massive muons (no photon lepton recombination)
- G_μ scheme, complex mass scheme
- dynamic scale $\mu_F = \mu_R = m_{\mu^+\mu^-}$



- Negative corrections of several percents in the tails with respect to NNLO QCD+EW
- The **factorised approximation** catches the bulk of QCD-EW corrections pointing towards a factorisation of NLO QCD corrections and EW Sudakov logarithms

  as observed in [Buccioni et al (2022)]
- Small residual non-factorisable effects at (sub) percent level

Conclusions & Outlook

- The Drell-Yan process is a cornerstone of the LHC precision physics program: a lot of progress from **experimental** and **theory** pointing towards an astonishing target of (**sub) per mille accuracy**
- We have presented a **new computation** of the mixed QCD-EW corrections to the neutral and charged Drell-Yan processes with massive lepton
- For the first time, **all real and virtual contributions** are consistently included for the **neutral current process**. For the **charged current**, **only the finite part of the two-loop amplitude** is computed in the **pole approximation**
- The cancellation of the IR singularities is achieved with the q_T subtraction formalism while the two-loop virtual amplitude is computed by applying a semi-numerical approach
- Mixed QCD-EW corrections are small but usually larger than what expected by naive counting of couplings. They **improves the theoretical accuracy** and may lead to **non-trivial distortions** of the shape of differential observables
- Impact on the **high-energy tail** of the invariant dilepton mass: **~1-5% at** $m_{\ell\ell}$**~1-3TeV,** relevant for New Physics searches. It is described reasonably well by a **factorisation** of NLO QCD and NLO EW (Sudakov logs) corrections

Conclusions & Outlook

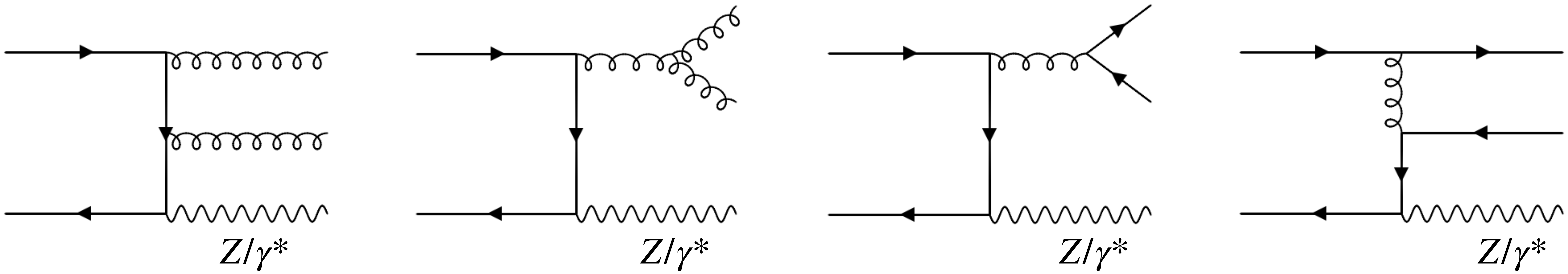
Prospects

- **Pheno studies** ongoing (electrons and comparison with Buccioni et al, A_{FB}, electroweak input scheme, implications on W mass determination)
- Computation of the **exact** 2 loop mixed QCD-EW amplitude for the **charged current process**
- Inclusion of mixed corrections in **resummed predictions** for the transverse momentum of the dilepton system and of the charged leptons

Back up

Abelianisation procedure

$q\bar{q}$ channel in NNLO QCD



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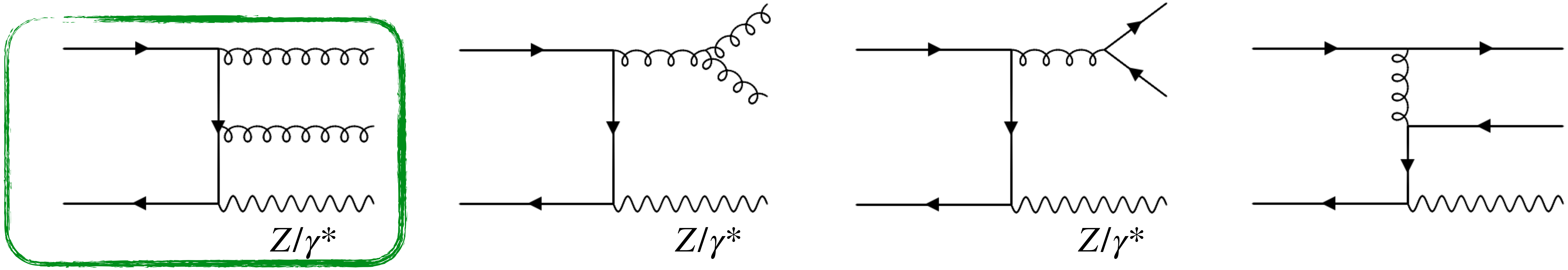
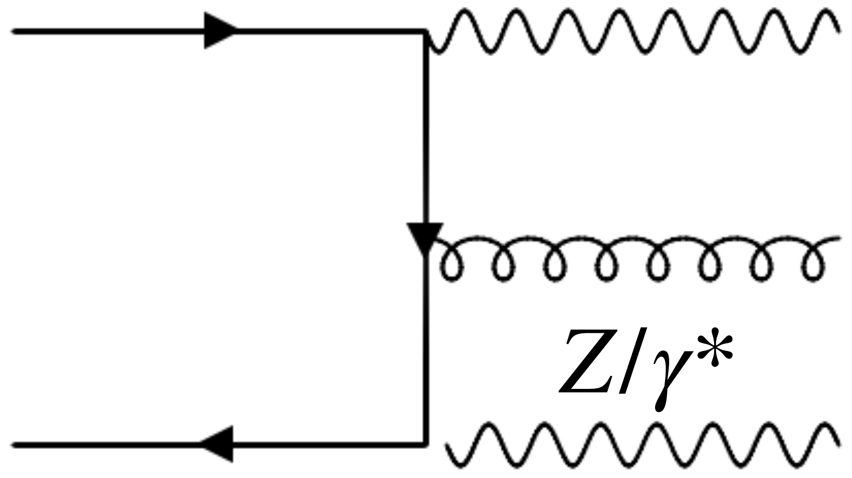
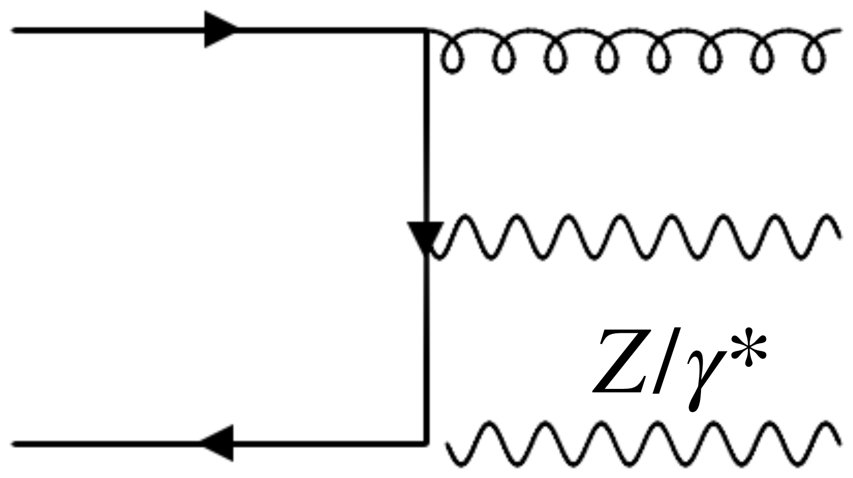


Color structure + symmetry factor (**identical gluons**)

$$\frac{1}{2N_C^2}\mathrm{Tr}[T^aT^aT^bT^b] = \frac{C_F^2}{2N_C} \qquad \frac{1}{2N_C^2}\mathrm{Tr}[T^aT^bT^aT^b] = \frac{1}{2N_C}C_F\left(C_F - \frac{C_A}{2}\right)$$

Photon-gluon replacement. Two **distinguishable** processes



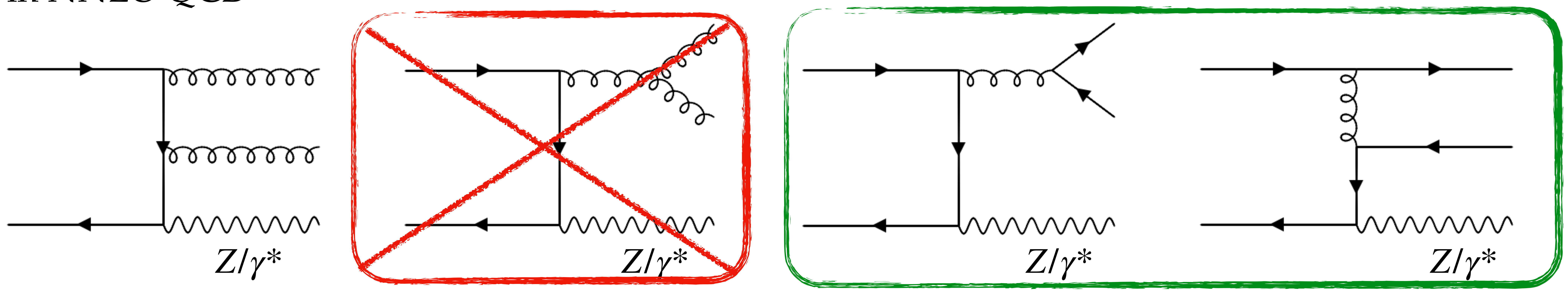


$$\frac{1}{N_C^2}\mathrm{Tr}[T^aT^a]e_f^2 = \frac{C_Fe_f^2}{N_C}$$

Replacement list: $C_A \to 0, \quad C_F^2 \to 2C_Fe_f^2$

Abelianisation procedure

$q\bar{q}$ channel in NNLO QCD



$C_A \to 0$

$C_A \to 0, \quad T_R \to 0, \quad C_F^2 \to 2C_F e_f^2$

Replacement rules

$q\bar{q}$ channel in NNLO QCD

$C_A \to 0, \quad T_R \to 0, \quad C_F^2 \to 2C_F e_f^2$

qg channel in NNLO QCD

$C_A \to 0, \quad C_F \to e_f^2$ — qg channel in QCD-QED

$C_A \to 0, \quad T_R \to N_C e_{q_f}^2$ — $q\gamma$ channel in QCD-QED

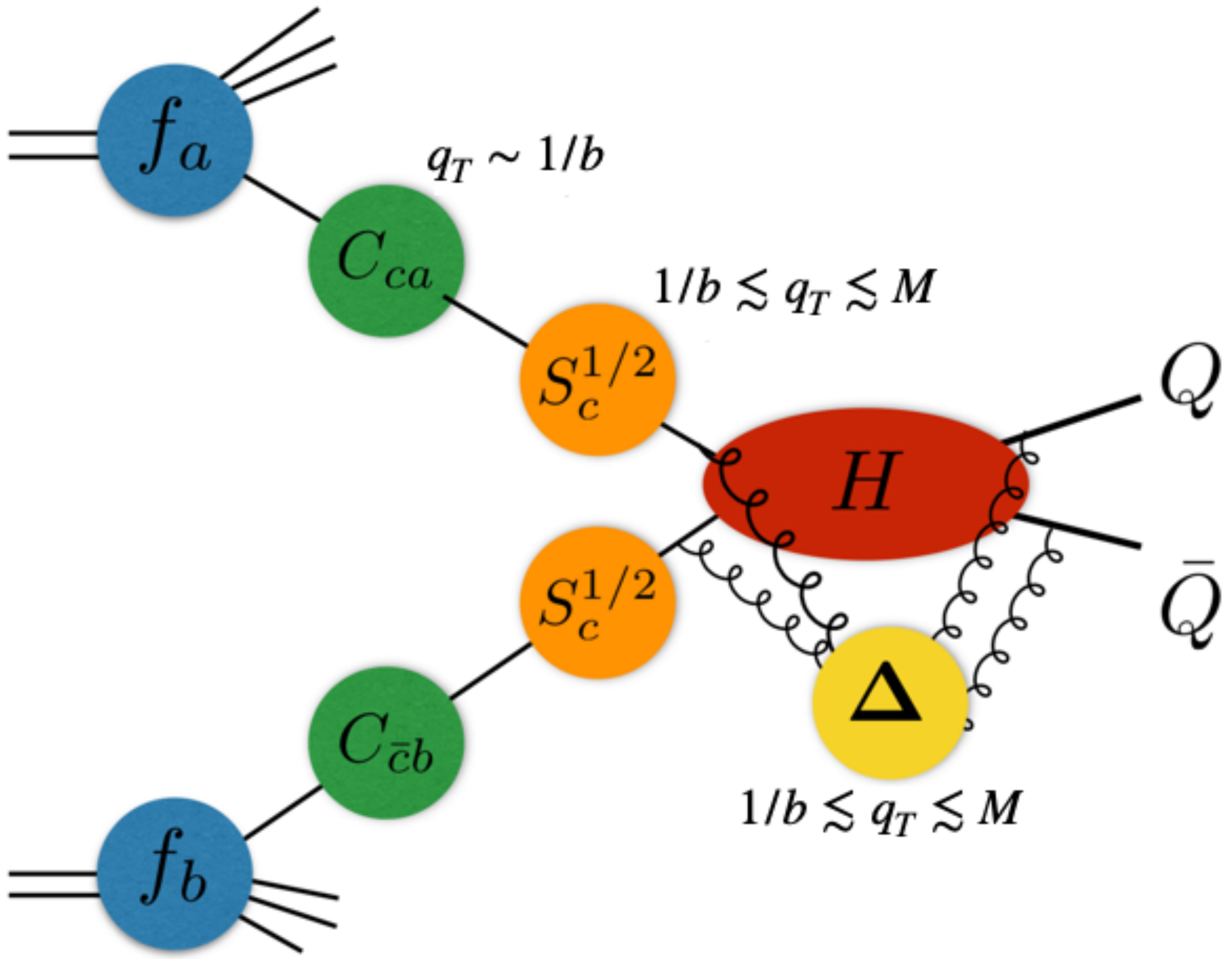
The q_T subtraction formalism for NNLO QCD-EW corrections

q_T subtraction formalism extended to the case of heavy quarks production [Catani, Grazzini, Torre (2014)]

Successful employed for computation of NNLO QCD corrections to the production of

- a top pair [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan (2019)]
- a bottom pair production [Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2021)]
- a top pair and a Higgs (off-diagonal channels) [Catani, Fabre, Grazzini, Kallweit, (2021)]

The resummation formula shows a **richer structure** because of additional soft singularities (four coloured patrons at LO)



- Soft logarithms controlled by the **transverse momentum anomalous dimension $\mathbf{\Gamma}_t$** known up to NNLO [Mitov, Sterman, Sung(2009)], [Neubert et al (2009)]
- Hard coefficient gets a **non-trivial** colour structure (matrix in colour-space)
- Non trivial azimuthal correlations
- Notice that is crucial that the final state is **massive:** the mass is the physical regulator of the final state collinear singularities

The q_T subtraction formalism for NNLO QCD-EW corrections

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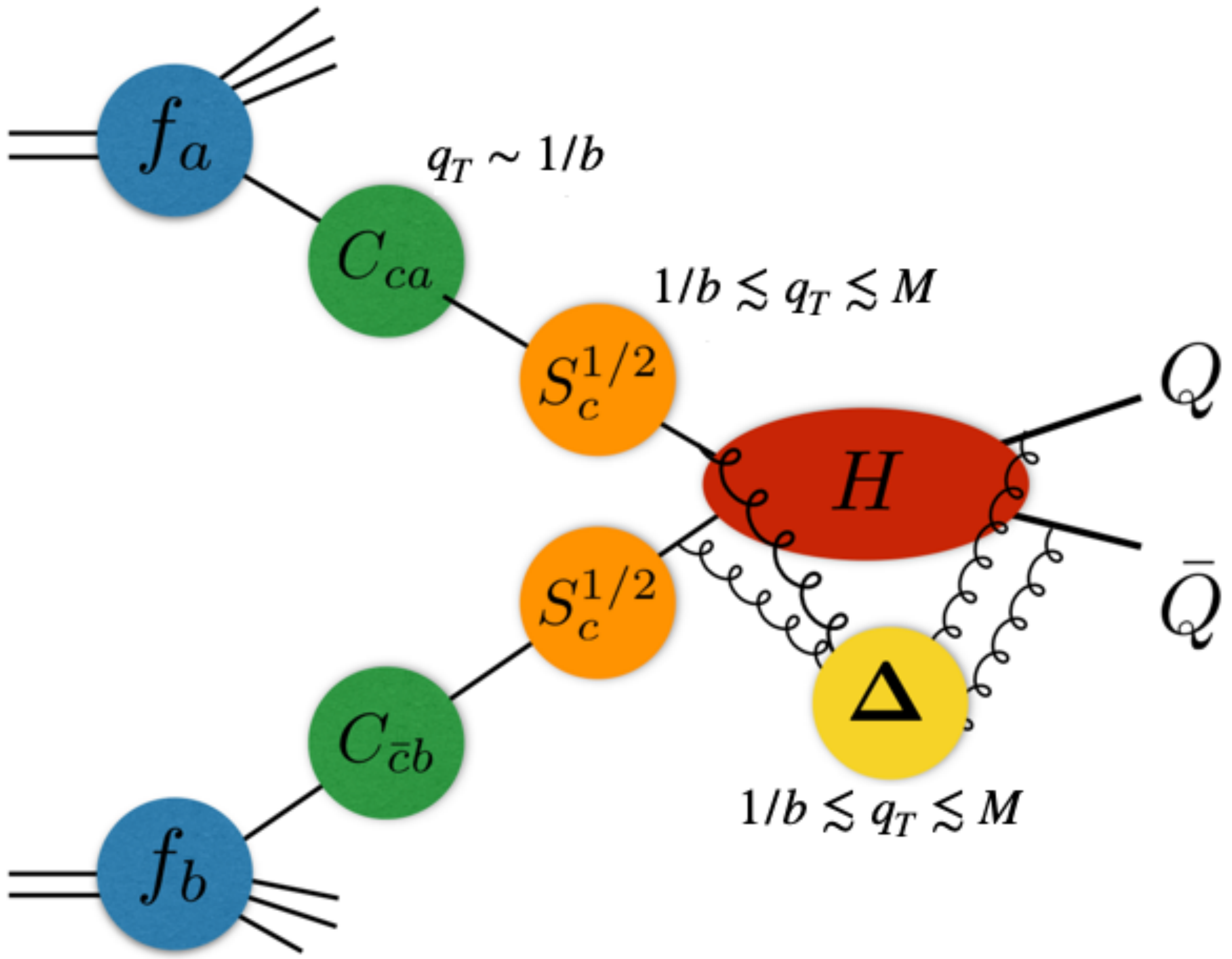


MIXED QCD-EW case

Final state is colour neutral

purely soft contributions exhibits a **much simpler structure**

the corresponding **soft logarithms** are **entirely controlled by the $\mathcal{O}(\alpha)$ soft anomalous dimension**

$$\Gamma_t = -\frac{1}{4}\left\{(e_3^2 + e_4^2)(1 - i\pi) + \sum_{i=1,2;j=3,4} e_i e_j \ln \frac{(2p_i \cdot p_j)^2}{Q^2 m_\ell^2}\right\}$$

$$q(p_1) + \bar{q}'(p_2) \to \ell(p_3) + \bar{\ell}(\bar{\nu}_\ell)(p_4)$$

the same is valid for the finite **soft function** (contact term)

Hard-Virtual coefficient: IR structure and finite amplitudes

$$\mathscr{M}^{(1,0)}_{\text{fin}} = \mathscr{M}^{(1,0)} + \frac{1}{2}\left(\frac{\alpha_s}{\pi}\right) C_F \left[\frac{1}{\epsilon^2} + \left(\frac{3}{2} + i\pi\right)\frac{1}{\epsilon} - \frac{\pi^2}{12}\right] \mathscr{M}^{(0)}$$

$$\mathscr{M}^{(0,1)}_{\text{fin}} = \mathscr{M}^{(0,1)} + \frac{1}{2}\left(\frac{\alpha}{\pi}\right) \left\{ \left[\frac{1}{\epsilon^2} + \left(\frac{3}{2} + i\pi\right)\frac{1}{\epsilon} - \frac{\pi^2}{12}\right] \frac{e_c^2 + e_{\bar{c}}^2}{2} - \frac{2\Gamma_t}{\epsilon} \right\} \mathscr{M}^{(0)}$$

$$\begin{aligned}
\mathscr{M}^{(1,1)}_{\text{fin}} = \; & \mathscr{M}^{(1,1)} - \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right) \left\{ \frac{1}{8\epsilon^4}(e_c^2 + e_{\bar{c}}^2) C_F + \frac{1}{2\epsilon^3} C_F \left[\left(\frac{3}{2} + i\pi\right)\frac{e_c^2 + e_{\bar{c}}^2}{2} - \Gamma_t\right] \right\} \mathscr{M}^{(0)} \\
& + \frac{1}{2\epsilon^2} \left\{ \left(\frac{\alpha}{\pi}\right) \frac{e_c^2 + e_{\bar{c}}^2}{2} \mathscr{M}^{(1,0)}_{\text{fin}} + C_F \left(\frac{\alpha_s}{\pi}\right) \mathscr{M}^{(0,1)}_{\text{fin}} \right. \\
& \qquad \left. + C_F \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right) \left[\left(\frac{7}{12}\pi^2 - \frac{9}{8} - \frac{3}{2} i\pi\right) \frac{e_c^2 + e_{\bar{c}}^2}{2} + \left(\frac{3}{2} + i\pi\right)\Gamma_t\right] \mathscr{M}^{(0)} \right\} \\
& + \frac{1}{2\epsilon} \left\{ \left(\frac{\alpha}{\pi}\right) \left[\left(\frac{3}{2} + i\pi\right)\frac{e_c^2 + e_{\bar{c}}^2}{2} - 2\Gamma_t\right] \mathscr{M}^{(1,0)}_{\text{fin}} + \left(\frac{\alpha_s}{\pi}\right) C_F \left[\frac{3}{2} + i\pi\right] \mathscr{M}^{(0,1)}_{\text{fin}} \right. \\
& \qquad \left. + \frac{1}{8} C_F \left(\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha}{\pi}\right) \left[\left(\frac{3}{2} - \pi^2 + 24\zeta(3) + \frac{2}{3} i\pi^3\right) \frac{e_c^2 + e_{\bar{c}}^2}{2} - \frac{2}{3}\pi^2\Gamma_t\right] \mathscr{M}^{(0)} \right\}
\end{aligned}$$

2-loop virtual CC-DY: Pole Approximation

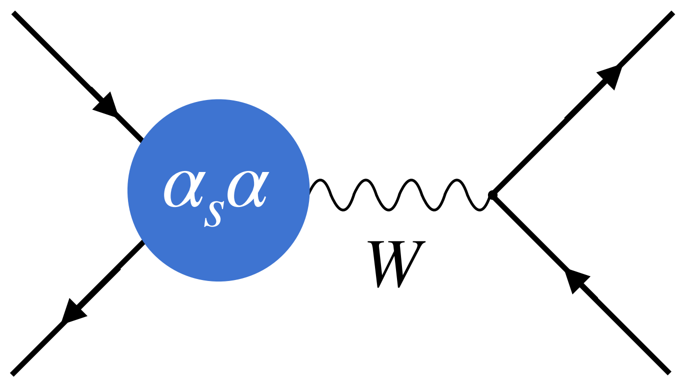
The **Pole Approximation** (PA) is a systematic expansion around the resonance pole with respect to the parameter Γ_W/M_W.

Beyond the narrow width approximation, the PA:

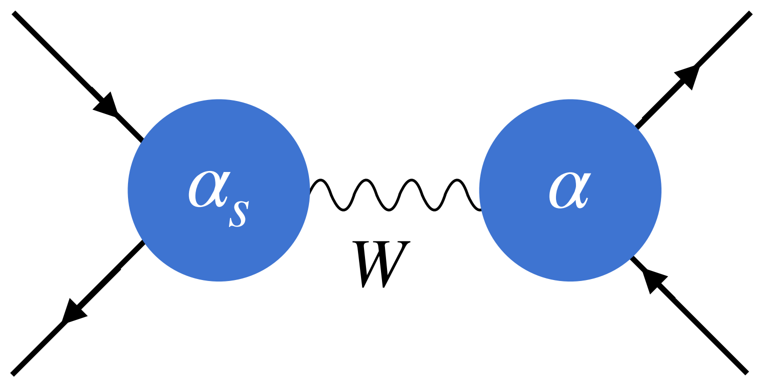
- keeps dominant (logarithmic) terms in Γ_W/M_W
- the structure of the IR singularities resembles that of the full computation

Factorisable corrections

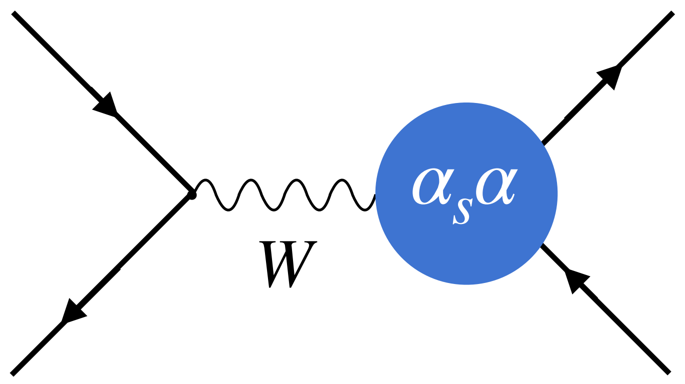
Corrections to the production and/or decay vertex



Initial-Initial: extracted from mixed QCD-EW form factors
- [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)] W boson
- Bonciani, Buccioni, Rana, Vicini (2020)] Z boson



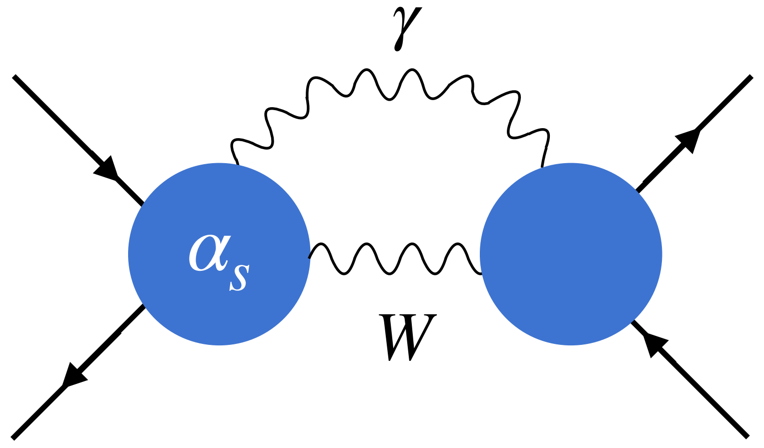
Initial-Final: computed using RECOLA



Final-Final: finite renormalisation constant [Dittmaier, Huss, and Schwinn (2015)]

Non-Factorisable corrections

Box topologies containing a soft photon linking production and decay



$$= \mathcal{F}^{(1,1)}_{\rm nf}\mathcal{M}^{(0)}_{\rm PA} = \delta^{(0,1)}_{\rm nf}\delta^{(1,0)}\mathcal{M}^{(0)}_{\rm PA}$$

[Dittmaier, Huss, and Schwinn (2014)]

We apply the PA (improved by a re-weighting procedure) **only** for the computation of the interference of **the two-loop virtual** with the tree-level amplitude for

- **charged current** Drell-Yan process
- **cross checks** and **validation** for the **neutral current** process

Hard-Virtual coefficient in PA: re-weighting

$$H_{\rm PA}^{(m,n)} = \frac{2{\rm Re}\left(\mathscr{M}_{\rm fin}^{(m,n)}\mathscr{M}^{(0,0)*}\right)_{\rm PA}}{|\mathscr{M}^{(0,0)}|^2}, \quad \text{for } m=0,1,\ n=1$$

Remark: since the Hard-Virtual term is eventually multiplied by $d\sigma_{\rm LO}$, the above definition corresponds to **compute the virtual-tree interference in PA**

We consider **alternative definitions** which differ for terms beyond the accuracy of the PA

- at NLO-EW ($m=0, n=1$)

$$H_{\rm PA,rwg}^{(0,1)} = \frac{2{\rm Re}\left(\mathscr{M}_{\rm fin}^{(0,1)}\mathscr{M}^{(0,0)*}\right)_{\rm PA}}{|\mathscr{M}_{\rm PA}^{(0,0)}|^2}$$

Cancellation of IR poles is exact

Effectively re-weights the virtual in PA with the exact Born amplitude

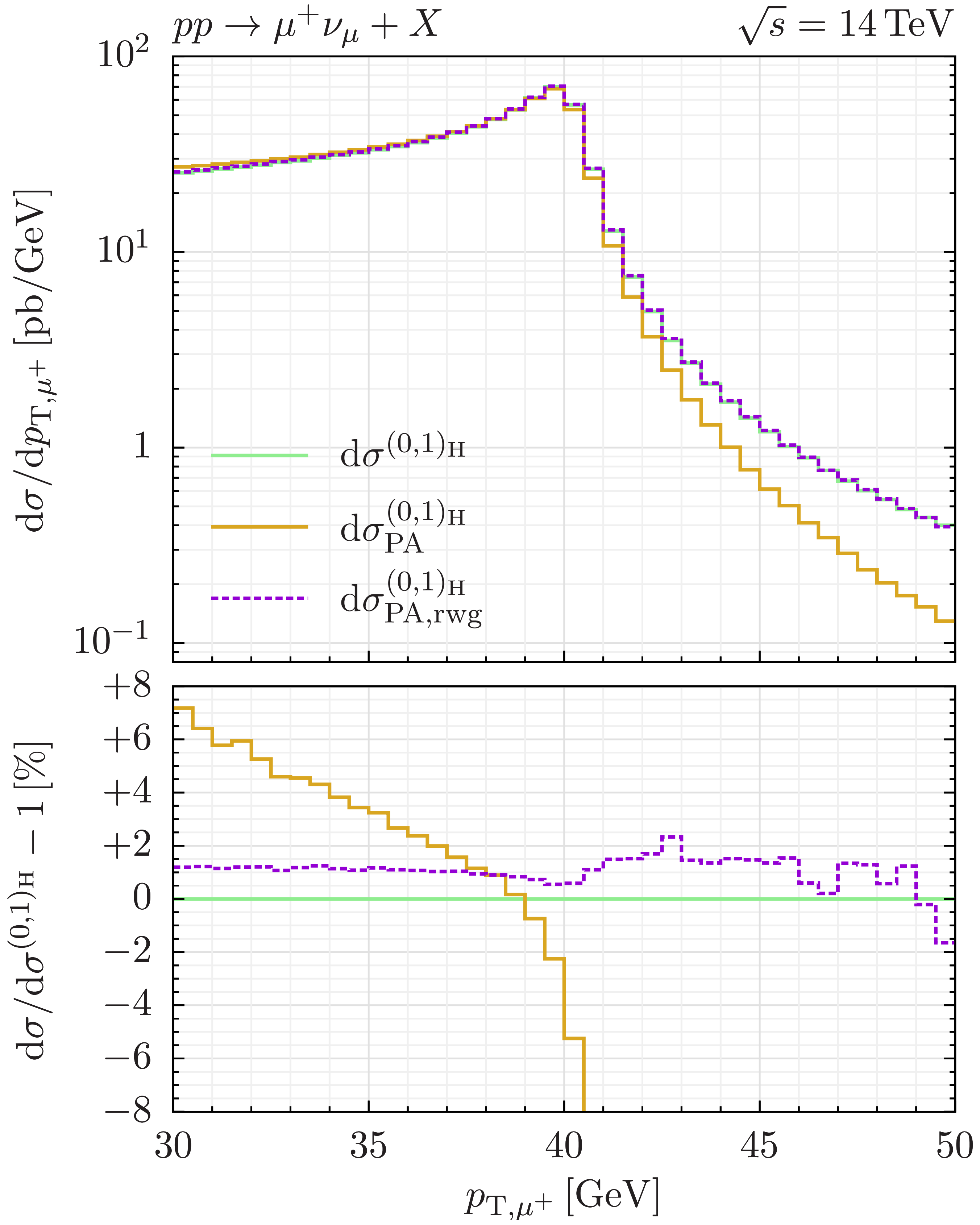
- at NNLO QCD-EW ($m=1, n=1$)

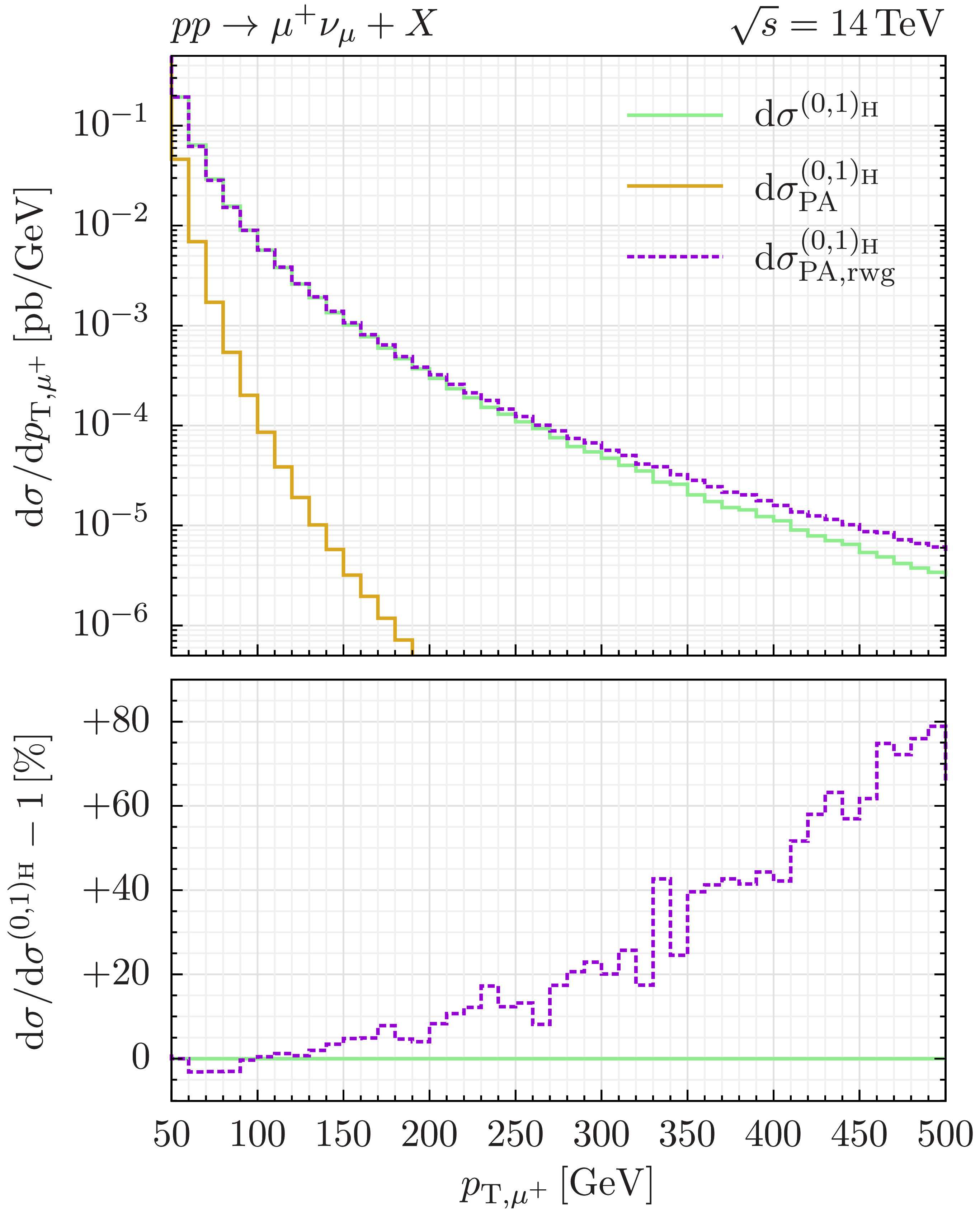
$$H_{\rm PA,rwg_B}^{(1,1)} = H_{\rm PA}^{(1,1)} \times \frac{|\mathscr{M}^{(0,0)}|^2}{|\mathscr{M}_{\rm PA}^{(0,0)}|^2} = \frac{2{\rm Re}\left(\mathscr{M}_{\rm fin}^{(1,1)}\mathscr{M}^{(0,0)*}\right)_{\rm PA}}{|\mathscr{M}_{\rm PA}^{(0,0)}|^2}$$

$$H_{\rm PA,rwg_V}^{(1,1)} = H_{\rm PA}^{(1,1)} \times \frac{H^{(0,1)}}{H_{\rm PA}^{(0,1)}} = \frac{2{\rm Re}\left(\mathscr{M}_{\rm fin}^{(1,1)}\mathscr{M}^{(0,0)*}\right)_{\rm PA}}{|\mathscr{M}^{(0,0)}|^2} \times \frac{2{\rm Re}\left(\mathscr{M}_{\rm fin}^{(0,1)}\mathscr{M}^{(0,0)*}\right)}{2{\rm Re}\left(\mathscr{M}_{\rm fin}^{(0,1)}\mathscr{M}^{(0,0)*}\right)_{\rm PA}}$$

Effectively re-weights with the exact one-loop EW virtual amplitude

Hard-Virtual coefficient in PA: validation @NLO EW





- The Pole Approximation **supplemented** with the **re-weighting**
  - agrees with the exact result at the **percent level** both below and above the **peak**
  - good modelling (**correct order of magnitude**) of the hard-virtual at high pT
  - difference with exact coefficient: $\mathcal{O}(20\%)$ at 300 GeV, $\mathcal{O}(80\%)$ at 500 GeV with PA systematically overshooting the exact result (**Sudakov Logs**)

Factorise ansatz

We present our prediction for the $\mathcal{O}(\alpha_s \alpha)$ correction as

- absolute correction
- normalised correction with respect to the LO cross section
- normalised correction with respect to the NLO QCD cross section

We compare our results with the naive factorised ansantz given by the formula

$$\frac{d\sigma^{(1,1)}_{\text{fact}}}{dX} = \left(\frac{d\sigma^{(1,0)}}{dX}\right) \times \left(\frac{d\sigma^{(0,1)}_{q\bar{q}}}{dX}\right) \times \left(\frac{d\sigma_{\text{LO}}}{dX}\right)^{-1}$$

Remark (especially for the transverse momentum distribution)

A factorised approach is justified if the dominant sources of QCD and EW corrections factorise with respect to the hard W production subprocesses.

At NLO, gluon/photon initiated channels open up populating the tail of the p_T spectrum, thus leading to large corrections (***giant K-factors***)

We <u>do not include</u> the **photon-induced** channels in the NLO-EW differential K-factor to avoid the multiplication of two giant K-factors of QCD and EW origin, which is not expected to work

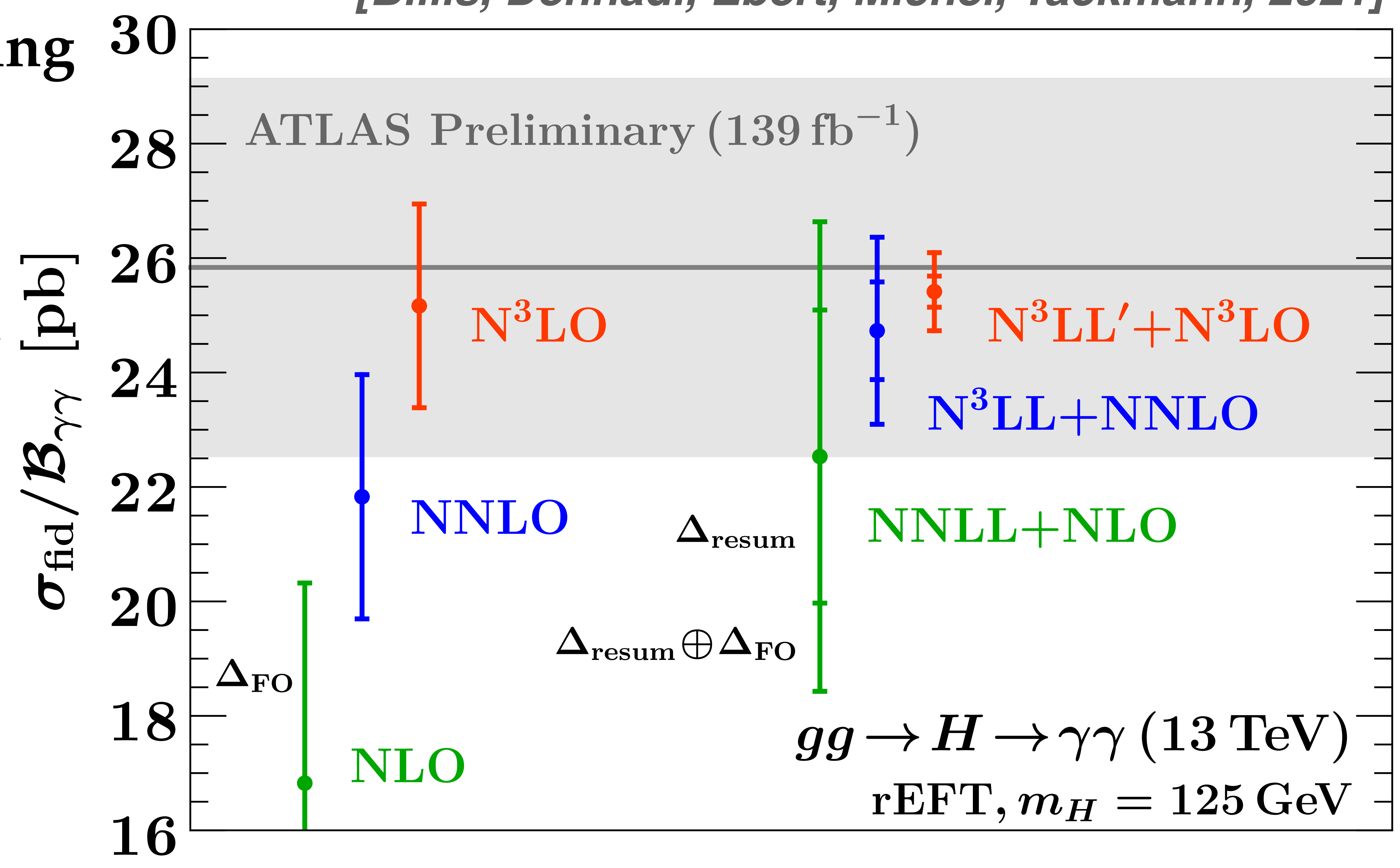
[Lindert, Grazzini, Kallweit, Pozzorini, Wiesemann (2019)]

Convergence of the perturbative expansion in the presence of fiducial cuts

Fiducial cuts may challenge the convergence of the perturbative fixed-order series

- **Symmetric cuts** on the transverse momentum of the two-body decay products lead to an **enhanced sensitivity to soft radiation** when the two particles are back-to-back in the transverse plane
  [Klaser, Kramer, 1996], [Harris, Owens, 1997], [Frixione, Ridolfi, 1997]
- They lead to **linear power corrections** in the transverse momentum spectrum of the color singlet q_T
- The linear dependence in q_T is related to a **factorial growth** of the coefficients in the perturbative series (with **alternating-sign** coefficient, hence Borel-summable) *[Salam, Slade, 2021]*
- The effect is larger for the case of the Higgs due to its **Casimir scaling**
- **Asymmetric cuts** on the transverse momentum of the hardest and the of the softest particle do not improve the situation.
- Symmetric cuts and asymmetric cuts are commonly used for Drell-Yan and Higgs analysis, respectively.

[Billis, Dehnadi, Ebert, Michel, Tackmann, 2021]



Viable resolution strategies

- improve the convergence by **resumming** the linear power corrections
- **alternative** choices of cuts *[Salam, Slade, 2021]*

Restoring the quadratic dependence on q_T

In the standard q_T subtraction master formula *[Catani, Grazzini, 2007]*

$$\sigma^{\rm F}_{\rm (N)NLO} = \int d\sigma^{\rm F}_{\rm LO} \otimes \mathscr{H} + \int \left[d\sigma^{\rm F+jet}_{\rm (N)LO} - d\sigma^{\rm F}_{\rm CT} \right] \theta(q_T/Q - r_{\rm cut}) + \mathcal{O}\left(r_{\rm cut}^k\right)$$

the counterterm is given by a **pure LP expansion** of the q_T spectrum

- above $r_{\rm cut}$: all power corrections are exactly provided by the real matrix element (avoiding any double counting)
- below $r_{\rm cut}$: all power corrections are missing

Formally, the residual dependence on the slicing parameter $r_{\rm cut}$ is given by the integral of the non-singular component of the real spectrum below the cut.

$$\int d\sigma^{\rm F+jet,reg}_{\rm (N)LO} \theta(r_{\rm cut} - q_T/Q)$$

For the case of **fiducial cuts**, the leading power correction is linear ($k = 1$). It can be **predicted by factorisation** and is **equivalent** to the q_T recoil prescription

$$\int d\Phi_{\rm F+jet} \frac{d\sigma^{\rm F+jet,reg}_{\rm (N)LO}}{d\Phi_{\rm F+jet}} \theta(r_{\rm cut} - q_T/Q)\Theta_{\rm cuts}(\Phi_{\rm F+jet}) = \int d\Phi_{\rm F} \int_0^{r_{\rm cut}} dr' \left[\frac{d\sigma^{\rm F}_{\rm CT}}{d\Phi_{\rm F}} \Theta_{\rm cuts}\left(\Phi^{\rm rec}_{\rm F}(\Phi_{\rm F}, r')\right) - \frac{d\sigma^{\rm F}_{\rm CT}}{d\Phi_{\rm F}} \Theta_{\rm cuts}(\Phi_{\rm F}) \right] + \mathcal{O}(r^2_{\rm cut})$$

where $\Theta_{\rm cuts}$ implements the fiducial cuts and $\Phi^{\rm rec}_{\rm F} = \Phi^{\rm rec}_{\rm F}(\Phi_{\rm F}, r')$ is the recoiled kinematics

Restoring the quadratic dependence on q_T

Improved q_T subtraction master formula

$$\sigma^{\mathrm{F}}_{\mathrm{(N)NLO}} = \int d\sigma^{\mathrm{F}}_{\mathrm{LO}} \otimes \mathscr{H} + \int \left[d\sigma^{\mathrm{F+jet}}_{\mathrm{(N)LO}} - d\sigma^{\mathrm{F}}_{\mathrm{CT}} \right] \theta(q_T/Q - r_{\mathrm{cut}}) + \Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) + \mathcal{O}\left(r_{\mathrm{cut}}^{k'}\right)$$

with the **linPC** term, $\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}})$, given by

$$\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) = \int d\Phi_{\mathrm{F}} \int_0^{r_{\mathrm{cut}}} dr' \left[\frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}\left(\Phi^{\mathrm{rec}}_{\mathrm{F}}(\Phi_{\mathrm{F}}, r')\right) - \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}(\Phi_{\mathrm{F}}) \right]$$

Remarks on the linPC term

- it affects the q_T subtraction formula at the **power corrections** level only
- its formulation is **fully differential** with respect to the Born phase space
- it is **integrable** in 4 dimensions (local cancellation of infrared singularities)
- it is completely determined by the **knowledge of the counterterm** (can be easily **implemented** in any code implementing the q_T subtraction method)

Restoring the quadratic dependence on q_T

Improved q_T subtraction master formula

$$\sigma^{\mathrm{F}}_{\mathrm{(N)NLO}} = \int d\sigma^{\mathrm{F}}_{\mathrm{LO}} \otimes \mathcal{H} + \int \left[d\sigma^{\mathrm{F+jet}}_{\mathrm{(N)LO}} - d\sigma^{\mathrm{F}}_{\mathrm{CT}} \right] \theta(q_T/Q - r_{\mathrm{cut}}) + \Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) + \mathcal{O}\left(r_{\mathrm{cut}}^{k'}\right)$$

with the **linPC** term, $\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}})$, given by

$$\Delta\sigma^{\mathrm{linPCs}}(r_{\mathrm{cut}}) = \int d\Phi_{\mathrm{F}} \int_0^{r_{\mathrm{cut}}} dr' \left[\frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}\left(\Phi^{\mathrm{rec}}_{\mathrm{F}}(\Phi_{\mathrm{F}}, r')\right) - \frac{d\sigma^{\mathrm{F}}_{\mathrm{CT}}}{d\Phi_{\mathrm{F}}} \Theta_{\mathrm{cuts}}(\Phi_{\mathrm{F}}) \right]$$

Remarks on the linPC term

- for the case of fiducial cuts, we expect that its inclusion will **change the power correction scaling from linear to quadratic** ($k = 1$ to $k' = 2$)
- for other cases, we expect that its inclusion will not make the power correction scaling worse
- in principle, given its formulation, it can be applied to any process

Origin of linear power corrections

Kinematics of the two-body decay

[Ebert, Michel, Stewart, Tackmann, 2020], [Alekin, Kardos, Moch, Trocsanyi, 2021], [Salam, Slade, 2021]

$$q^\mu = (m_T \cosh Y, q_T, 0, m_T \sinh Y)$$

$$p_1^\mu = p_{T,1}\left(\cosh(Y+\Delta y), \cos\phi, \sin\phi, m_T \sinh(Y+\Delta y)\right)$$

$$p_1^\mu = q^\mu - p_1^\mu$$

in the small q_T limit →

$$p_{T,1} = \frac{Q}{2\cosh\Delta y}\left[1 + \frac{q_T}{Q}\frac{\cos\phi}{\cosh\Delta Y} + \mathcal{O}\left(q_T^2/Q^2\right)\right]$$

$$p_{T,2} = p_{T,1} - q_T\cos\phi + \mathcal{O}\left(q_T^2/Q^2\right)$$

$$\eta_1 = Y + \Delta y$$

$$\eta_2 = Y - \Delta y - 2\frac{q_T}{Q}\cos\phi \sinh\Delta y + \mathcal{O}\left(q_T^2/Q^2\right)$$

The two-body decay **phase space** with cuts is given by

$$\Phi_{q\to p_1+p_2}(q_T) = \frac{1}{8\pi^2}\int_0^{2\pi} d\phi \int d\Delta y \frac{p_{T,1}^2}{Q^2}\Theta_{\text{cuts}}(q_T, \phi, \Delta y; \text{cuts})$$

The integrand has a dependence on q_T through the combinations q_T^2 and $q_T\cos\phi$. It follows that

presence of **linear fiducial power corrections** ⟷ use of cuts **breaking the azimuthal symmetry**

Origin of linear power corrections

Symmetric cuts: $p_{T,i} > p_T^{\rm cut}, \quad i = 1,2 \implies \min(p_{T,1}, p_{T,2}) > p_T^{\rm cut}$

two different integrands: **breaking of azimuthal symmetry**

$$\min(p_{T,1}, p_{T,2}) = \begin{cases} p_{T,1} & \cos\phi < 0 \\ p_{T,1} - q_T\cos\phi & \cos\phi > 0 \end{cases}$$

$$\Phi(q_T) - \Phi(0) = -\frac{1}{2\pi^2}\frac{q_T}{Q}\frac{p_T^{\rm cut}/Q}{\sqrt{1-(2p_T^{\rm cut})^2/Q^2}}\int_0^{\pi/2} d\phi\cos\phi$$

Asymmetric cuts: $p_T^{\rm hard} > p_T^{\rm cut,h}$ and $p_T^{\rm soft} > p_T^{\rm cut,s} \implies \min(p_{T,1}, p_{T,2}) > p_T^{\rm cut,s}$

Equivalent to the symmetric cuts case: **it does not solve the issue of the appearance of linear power corrections**!

Staggered cuts: $p_{T,1} > p_T^{\rm cut} + \delta p_T$ and $p_{T,2} > p_T^{\rm cut} \implies \min(p_{T,1} - \delta p_T, p_{T,2}) > p_T^{\rm cut}$

$$\min(p_{T,1} - \delta p_T, p_{T,2}) = \begin{cases} p_{T,1} - \delta p_T & \cos\phi < \delta p_T/q_T \\ p_{T,1} - q_T\cos\phi & \cos\phi > \delta p_T/q_T \end{cases}$$

In the region $q_T < \delta p_T$, the **quadratic dependence on q_T is restored,** as numerically observed in *[Grazzini,Kallweit,Wiesemann, 2017]*

Fiducial PCs and differential distributions