

Higgs pair production in SMEFT at NLO QCD: truncation uncertainties

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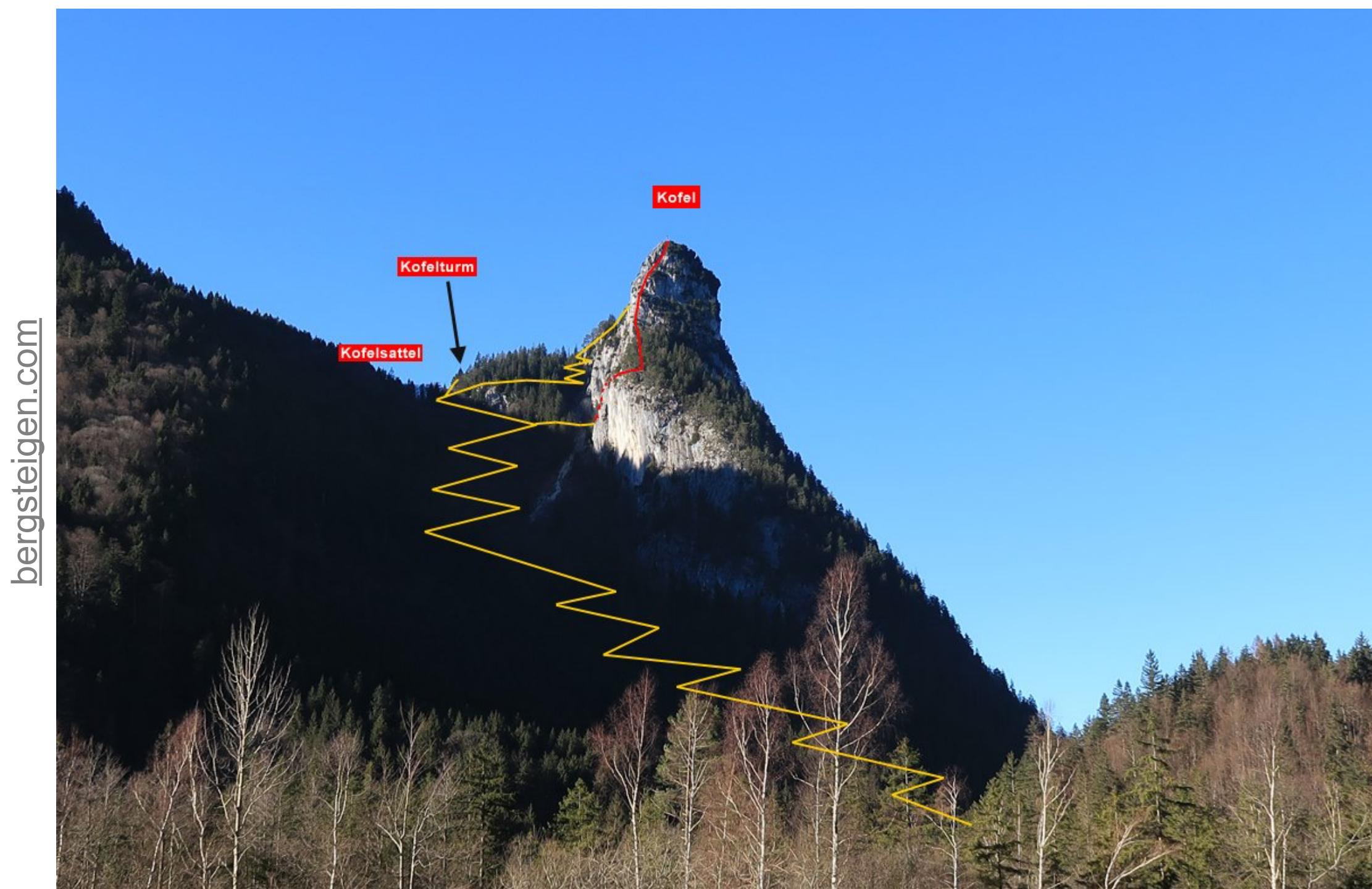
in collaboration with
Jannis Lang, Ludovic Scyboz
2204.13045

*builds on work with
Stephen Jones, Matthias Kerner et al.*

Loops & Legs 2022, Ettal

April 28, 2022

www.kit.edu



Motivation

How to identify imprints of New Physics?

need:

- precise SM predictions
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legs: up to 5 at 1-loop
scales: up to 4 at 2-loop (s , t , mt , mH)

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- investigate concrete BSM models, or
 - - parametrise effects of heavy New Physics by Effective Field Theory (EFT)

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Wilson coefficients: up to five!

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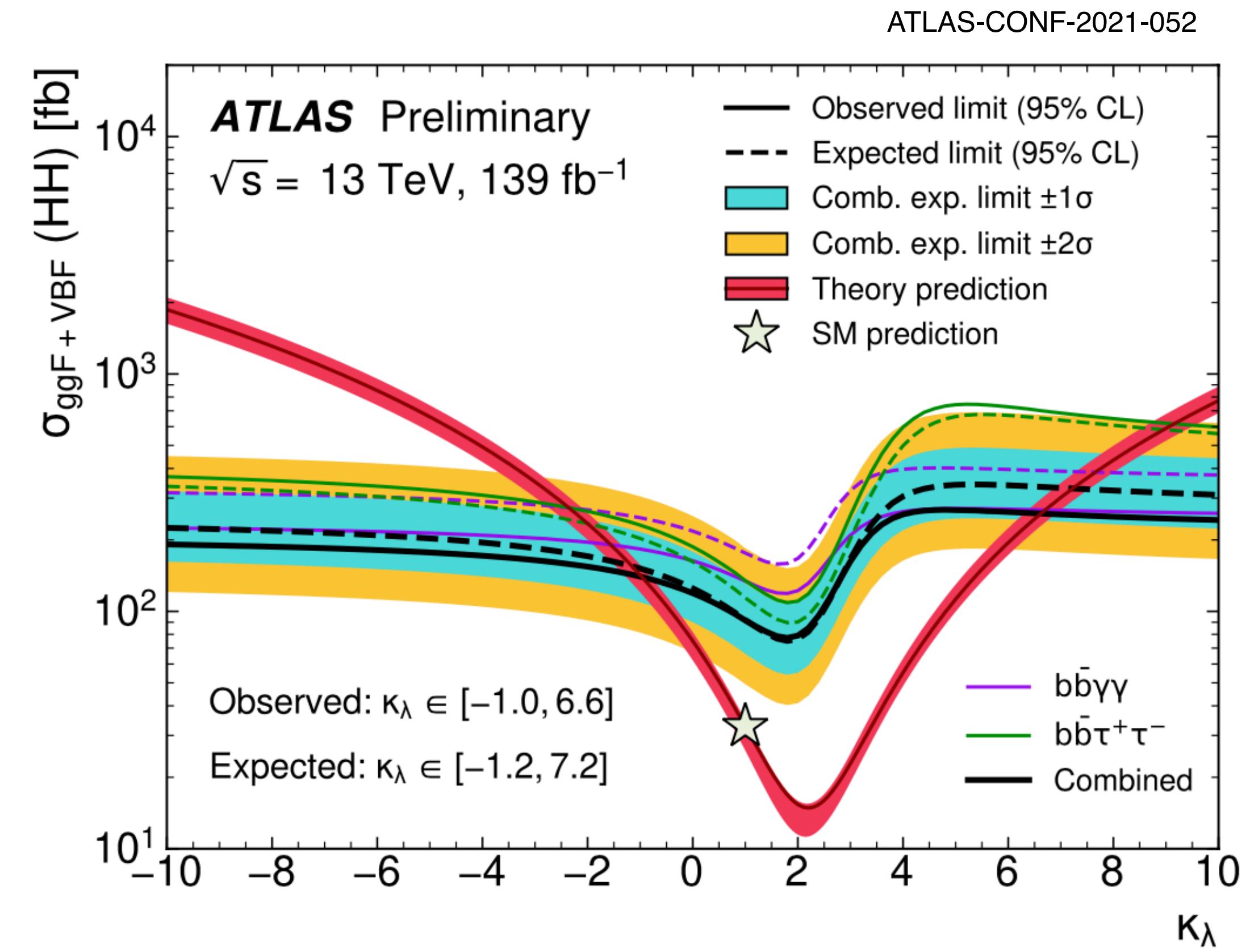
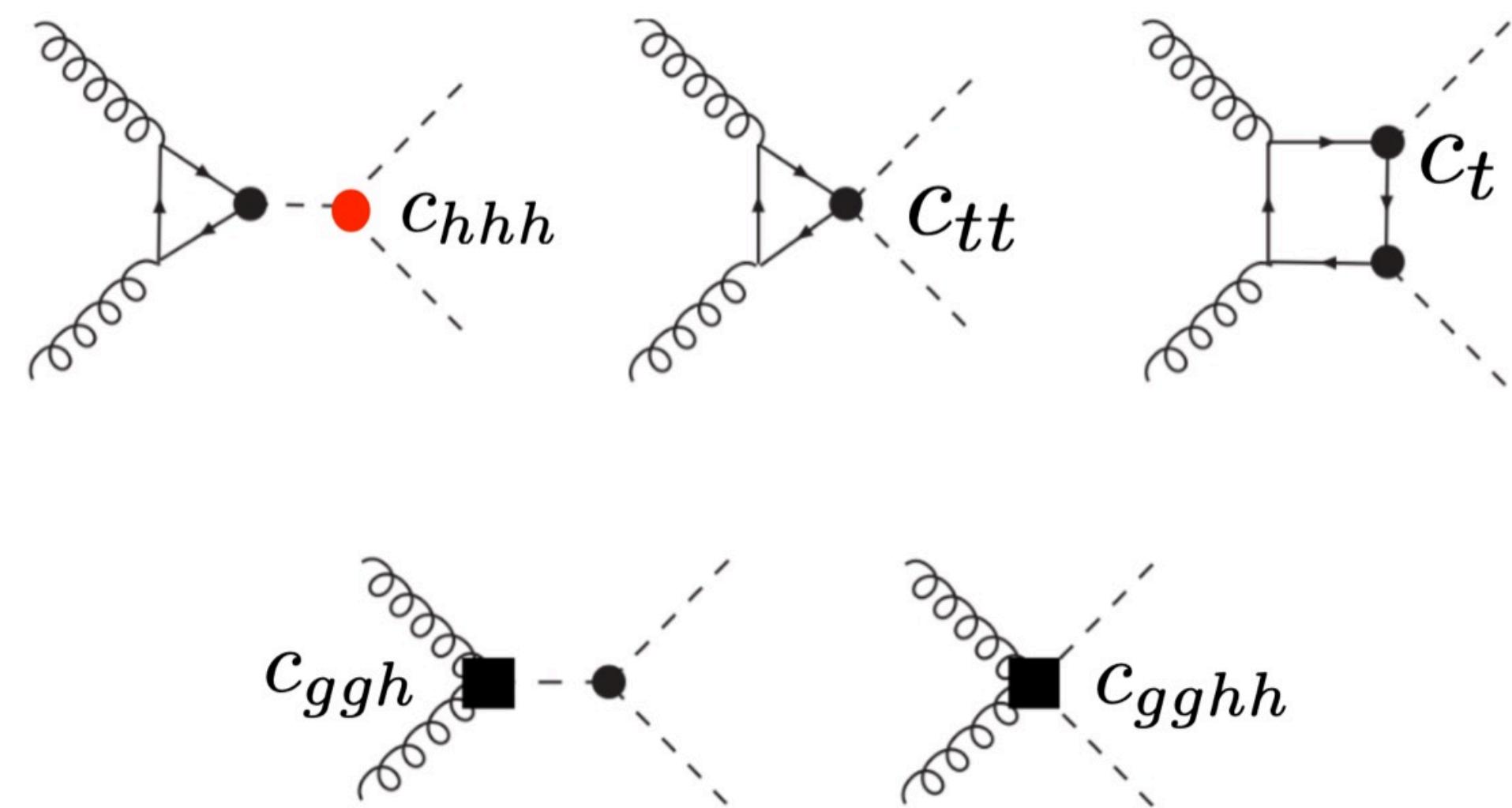
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Higgs boson pair production

- prime process to explore the Higgs potential
(Higgs boson trilinear coupling)
- if trilinear coupling different from SM, other couplings
are likely to be non-SM as well

→ EFT parametrisation by extra operators



Effective Field Theory expansion schemes

SMEFT (Standard Model Effective Field Theory):

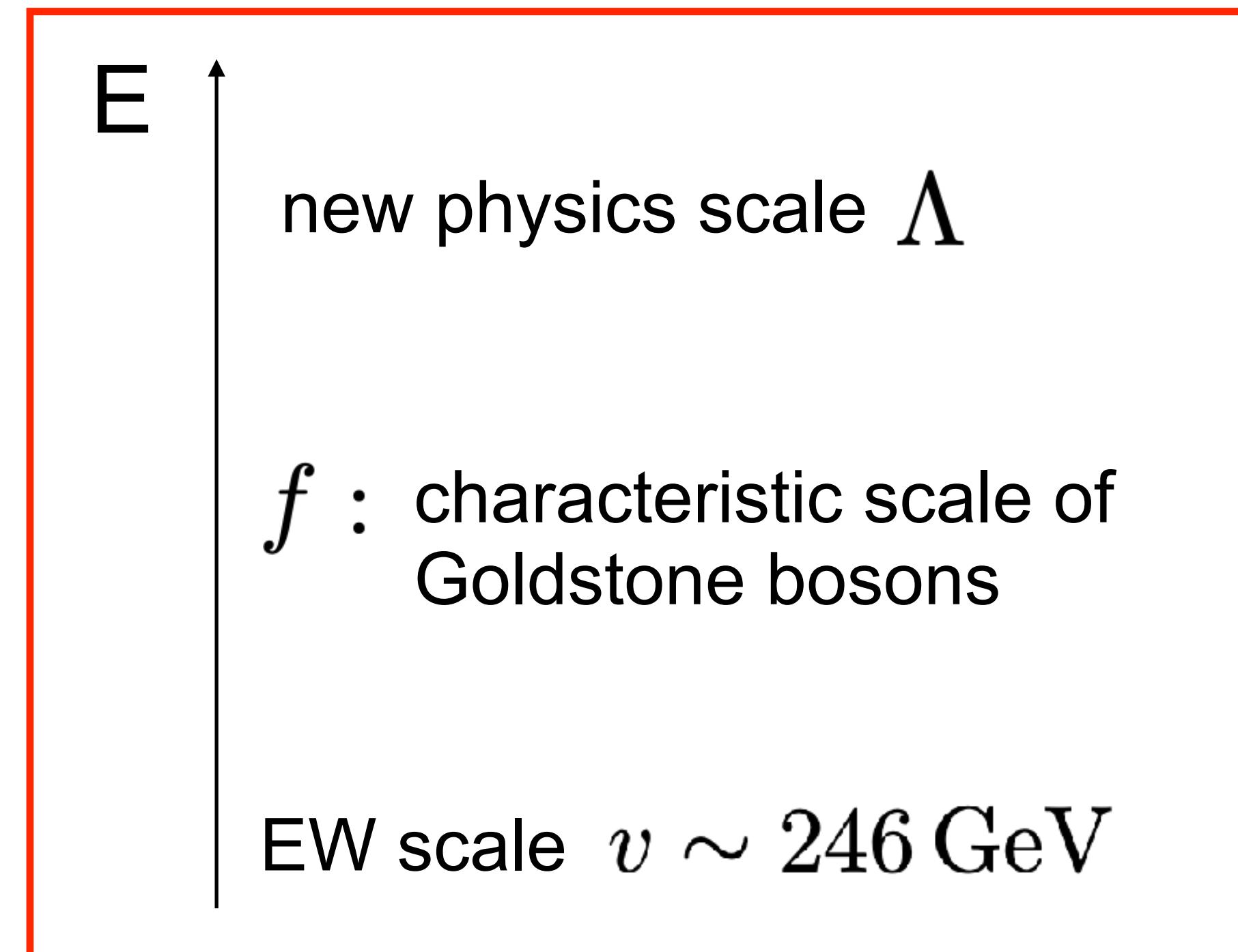
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{\text{dim6}} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

canonical dimension counting

HEFT (Higgs Effective Field Theory):

$$\mathcal{L}_{\text{HEFT}} = \mathcal{L}_2 + \sum_{L=1}^{\infty} \sum_i \left(\frac{1}{16\pi^2}\right)^L c_i^{(L)} O_i^{(L)}$$

counting of loop orders, expansion parameter: $f^2/\Lambda^2 \approx 1/(16\pi^2)$
(similar to chiral perturbation theory)



HEFT and SMEFT

- **HEFT:** Goldstone sector has a symmetry $SU(2)_L \times SU(2)_R$ (chiral)
which is broken to $SU(2)_{L+R}$ (“custodial symmetry”, protects the rho-parameter)
- physical Higgs field $h(x)$ is $SU(2)_L \times U(1)_Y$ **singlet** (cf. non-linear sigma-model)
 - Lagrangian can contain polynomials
$$\sum_n c_n \left(\frac{h}{v}\right)^n$$
 with no a priori relation among the c_n
- UV completion can be strongly coupled
model examples: composite H, H-dilaton, conformal H, induced EWSB, ...
- **SMEFT:** Higgs field $\Phi(x)$ is complex doublet, transforms linearly under $SU(2) \times U(1)$

Lagrangians relevant for HH production

SMEFT:

$$\begin{aligned}\Delta\mathcal{L}_{\text{Warsaw}} = & \frac{C_{H,\square}}{\Lambda^2}(\phi^\dagger\phi)\square(\phi^\dagger\phi) + \frac{C_{HD}}{\Lambda^2}(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi) + \frac{C_H}{\Lambda^2}(\phi^\dagger\phi)^3 \\ & + \left(\frac{C_{uH}}{\Lambda^2}\phi^\dagger\phi\bar{q}_L\phi^c t_R + h.c. \right) + \frac{C_{HG}}{\Lambda^2}\phi^\dagger\phi G_{\mu\nu}^a G^{\mu\nu,a} \quad (\text{Warsaw basis})\end{aligned}$$

Grzadkowski et al. 1008.4884

HEFT:

$$\mathcal{L} \supset -m_t \left(\textcolor{red}{c_t} \frac{h}{v} + \textcolor{red}{c_{tt}} \frac{h^2}{v^2} \right) \bar{t}t - \textcolor{red}{c_{hhh}} \frac{m_h^2}{2v} h^3 + \frac{\alpha_s}{8\pi} \left(\textcolor{red}{c_{ggh}} \frac{h}{v} + \textcolor{red}{c_{gghh}} \frac{h^2}{v^2} \right) G_{\mu\nu}^a G^{a,\mu\nu}$$

Feruglio '93, Buchalla et al. '13, '18

NLO with full top quark mass dependence implemented in

<http://powhegbox.mib.infn.it/User-Process-V2/ggHH>

GH, Jones, Kerner, Luisoni, Scyboz
2006.16877

NNLO': De Florian, Fabre, GH, Mazzitelli,
Scyboz 2106.14050

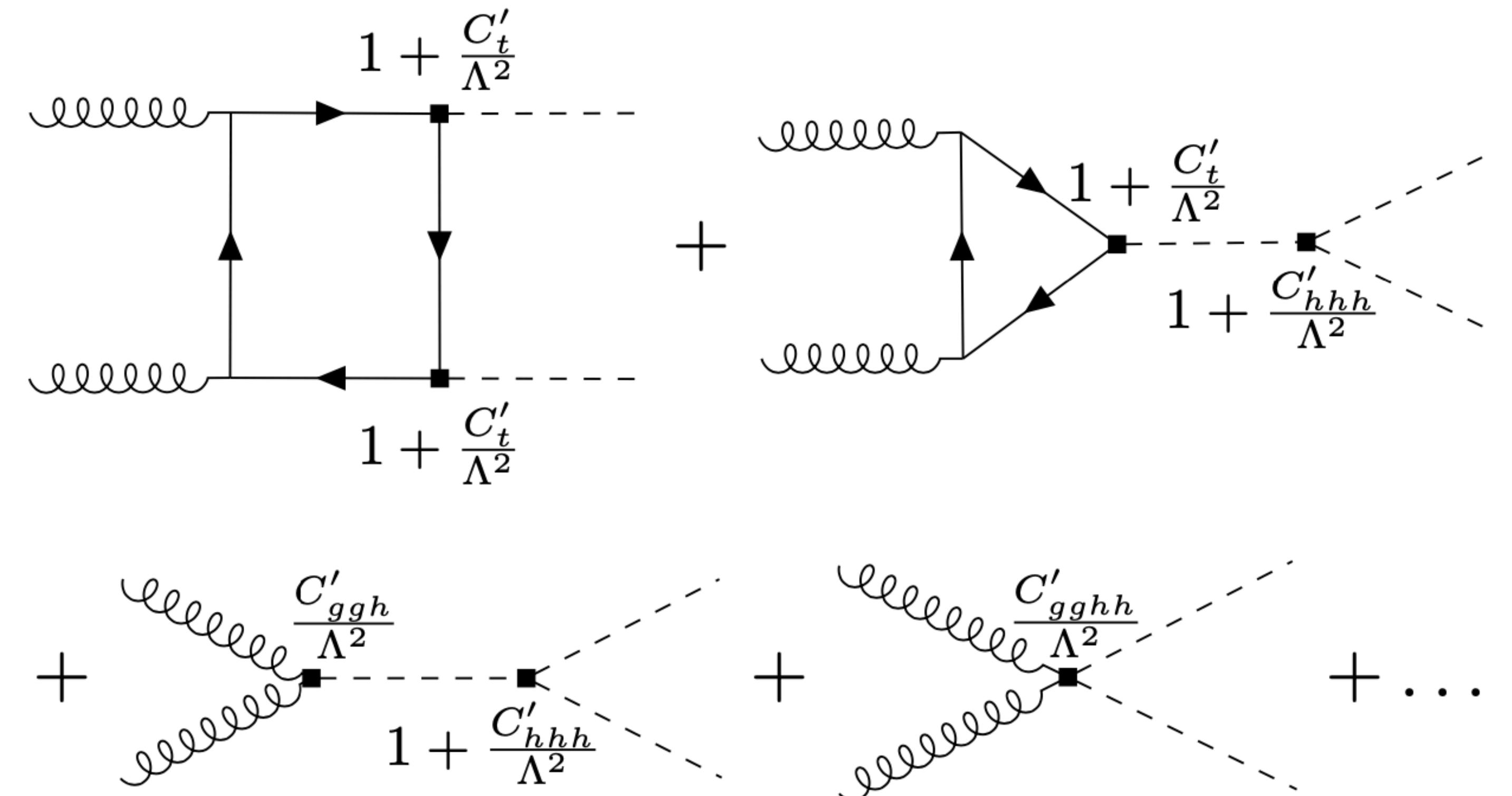
Lagrangians relevant for HH production

naive translation (comparing coefficients at Lagrangian level):

HEFT	Warsaw
c_{hhh}	$1 - 2 \frac{v^2}{\Lambda^2} \frac{v^2}{m_h^2} C_H + 3 \frac{v^2}{\Lambda^2} C_{H,\text{kin}}$
c_t	$1 + \frac{v^2}{\Lambda^2} C_{H,\text{kin}} - \frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2}m_t} C_{uH}$
c_{tt}	$-\frac{v^2}{\Lambda^2} \frac{3v}{2\sqrt{2}m_t} C_{uH} + \frac{v^2}{\Lambda^2} C_{H,\text{kin}}$
c_{ggh}	$\frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha_s} C_{HG}$
c_{gghh}	$\frac{v^2}{\Lambda^2} \frac{4\pi}{\alpha_s} C_{HG}$

$$C_{H,\text{kin}} = C_{H,\square} - \frac{1}{4} C_{HD}$$

SMEFT truncation

$$\mathcal{M} =$$

$$= \mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{single ins.}} + \mathcal{M}_{\text{double ins.}}$$

terms $\sim 1/\Lambda^4$ same order as dim 8 operators (which are not included)

SMEFT at amplitude squared level

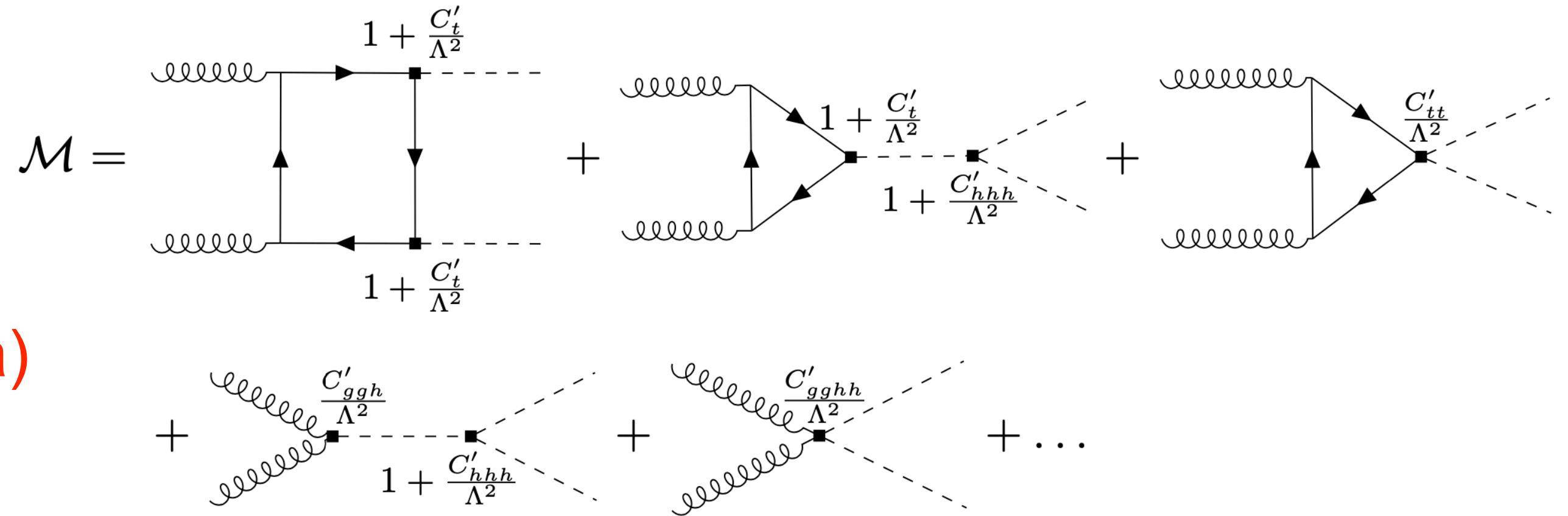
truncation options:

$$\sigma = \sigma_{\text{SM}} + \frac{1}{\Lambda^2} \sigma_{\text{SM} \times \text{single ins.}} \quad (\text{a})$$

$$+ \frac{1}{\Lambda^4} \sigma_{\text{single ins.} \times \text{single ins.}} \quad (\text{b})$$

$$+ \frac{1}{\Lambda^4} \sigma_{\text{SM} \times \text{double ins.}} \quad (\text{c})$$

$$+ \frac{1}{\Lambda^6} \sigma_{\text{single ins.} \times \text{double ins.}} + \frac{1}{\Lambda^8} \sigma_{\text{double ins.} \times \text{double ins.}} \quad (\text{d})$$



SMEFT at amplitude squared level

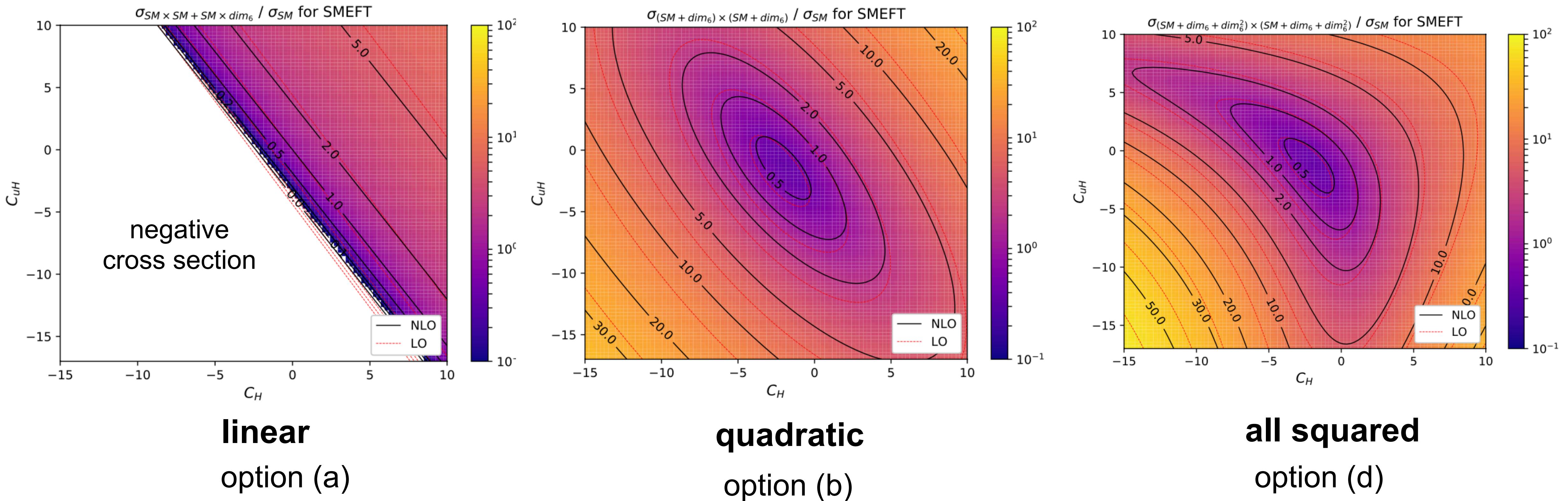
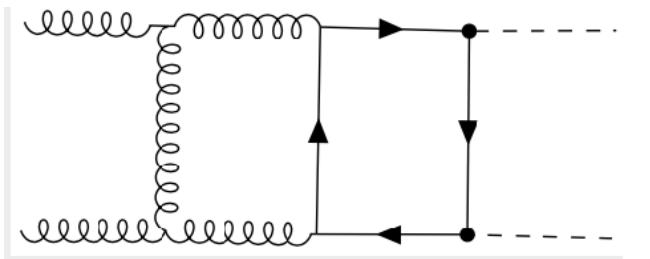
4 options:

$$\sigma \simeq \begin{cases} \sigma_{\text{SM}} + \sigma_{\text{SM} \times \text{dim6}} & (\text{a}) \\ \sigma_{(\text{SM}+\text{dim6}) \times (\text{SM}+\text{dim6})} & (\text{b}) \\ \sigma_{(\text{SM}+\text{dim6}) \times (\text{SM}+\text{dim6})} + \sigma_{\text{SM} \times \text{dim6}^2} & (\text{c}) \\ \sigma_{(\text{SM}+\text{dim6}+\text{dim6}^2) \times (\text{SM}+\text{dim6}+\text{dim6}^2)} & (\text{d}) \end{cases}$$

- (a): “linearised dim 6” (first order of expansion in $1/\Lambda^2$ at cross section level)
- (b): “quadratic dim 6” (first order of expansion in $1/\Lambda^2$ at amplitude level, then squared)
- (c): include all terms $\mathcal{O}(1/\Lambda^4)$ coming from dim6^2 and double operator insertions
- (d): would correspond to HEFT except for treatment of α_s

Results: total HH cross section

note: full NLO QCD corrections building on Borowka, Greiner, GH, Jones, Kerner, et al. '16

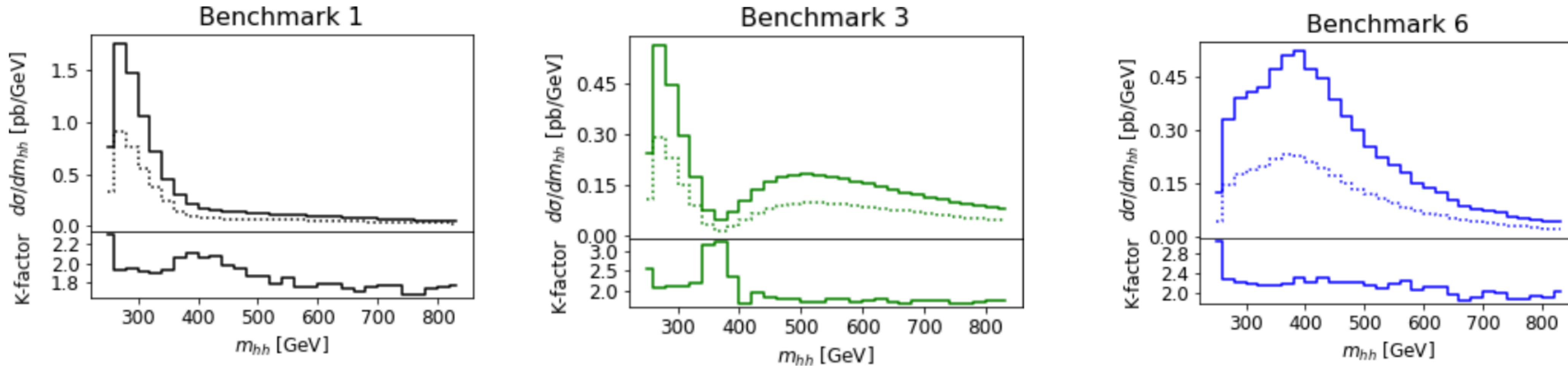


flat directions very different for different truncation options

figures: Jannis Lang

Results at benchmark points

consider benchmark points characteristic for a certain mHH shape



Capozi, GH,
1908.08923

benchmark (* = modified)	c_{hhh}	c_t	c_{tt}	c_{ggh}	c_{gghh}
SM	1	1	0	0	0
1*	5.105	1.1	0	0	0
3*	2.21	1.05	$-\frac{1}{3}$	0.5	0.25*
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25

modified: to fulfil SMEFT relation $c_{ggh} = 2c_{gghh}$

- benchmark 1: enhanced low mHH
- benchmark 3: dip
- benchmark 6: SM-like except for shoulder left of peak

new benchmarks fulfilling current constraints:
Ludovic Scyboz

Results: total HH cross sections

quadratic dim-6

benchmark	$\sigma_{\text{NLO}}[\text{fb}]$ option (b)	K-factor option (b)	ratio to SM option (b)	$\sigma_{\text{NLO}}[\text{fb}]$ option (a)	$\sigma_{\text{NLO}}[\text{fb}]$ HEFT
SM	$27.94^{+13.7\%}_{-12.8\%}$	1.67	1	-	-
$\Lambda = 1 \text{ TeV}$					
1	$74.29^{+19.8\%}_{-15.6\%}$	2.13	2.66	-61.17	94.32
3	$69.20^{+11.7\%}_{-10.3\%}$	1.82	2.47	29.64	72.43
6	$72.51^{+20.6\%}_{-16.4\%}$	1.90	2.60	52.89	91.40
$\Lambda = 2 \text{ TeV}$					
1	$14.03^{+12.0\%}_{-11.9\%}$	1.56	0.502	5.58	-
3	$30.81^{+16.0\%}_{-14.4\%}$	1.71	1.10	28.35	-
6	$35.39^{+17.5\%}_{-15.2\%}$	1.76	1.27	34.18	-

Naive translation at Lagrangian level:

benchmark (* = modified)	c_{hhh}	c_t	c_{tt}	c_{ggh}	c_{gghh}	$C_{H,\text{kin}}$	C_H	C_{uH}	C_{HG}	Λ
SM	1	1	0	0	0	0	0	0	0	1 TeV
1*	5.105	1.1	0	0	0	4.95	-6.81	3.28	0	1 TeV
3*	2.21	1.05	$-\frac{1}{3}$	0.5	0.25*	13.5	2.64	12.6	0.0387	1 TeV
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV

HEFT	Warsaw
c_{hhh}	$1 - 2\frac{v^2}{\Lambda^2} \frac{v^2}{m_h^2} C_H + 3\frac{v^2}{\Lambda^2} C_{H,\text{kin}}$
c_t	$1 + \frac{v^2}{\Lambda^2} C_{H,\text{kin}} - \frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2}m_t} C_{uH}$
c_{tt}	$-\frac{v^2}{\Lambda^2} \frac{3v}{2\sqrt{2}m_t} C_{uH} + \frac{v^2}{\Lambda^2} C_{H,\text{kin}}$
c_{ggh}	$\frac{v^2}{\Lambda^2} \frac{8\pi}{\alpha_s} C_{HG}$
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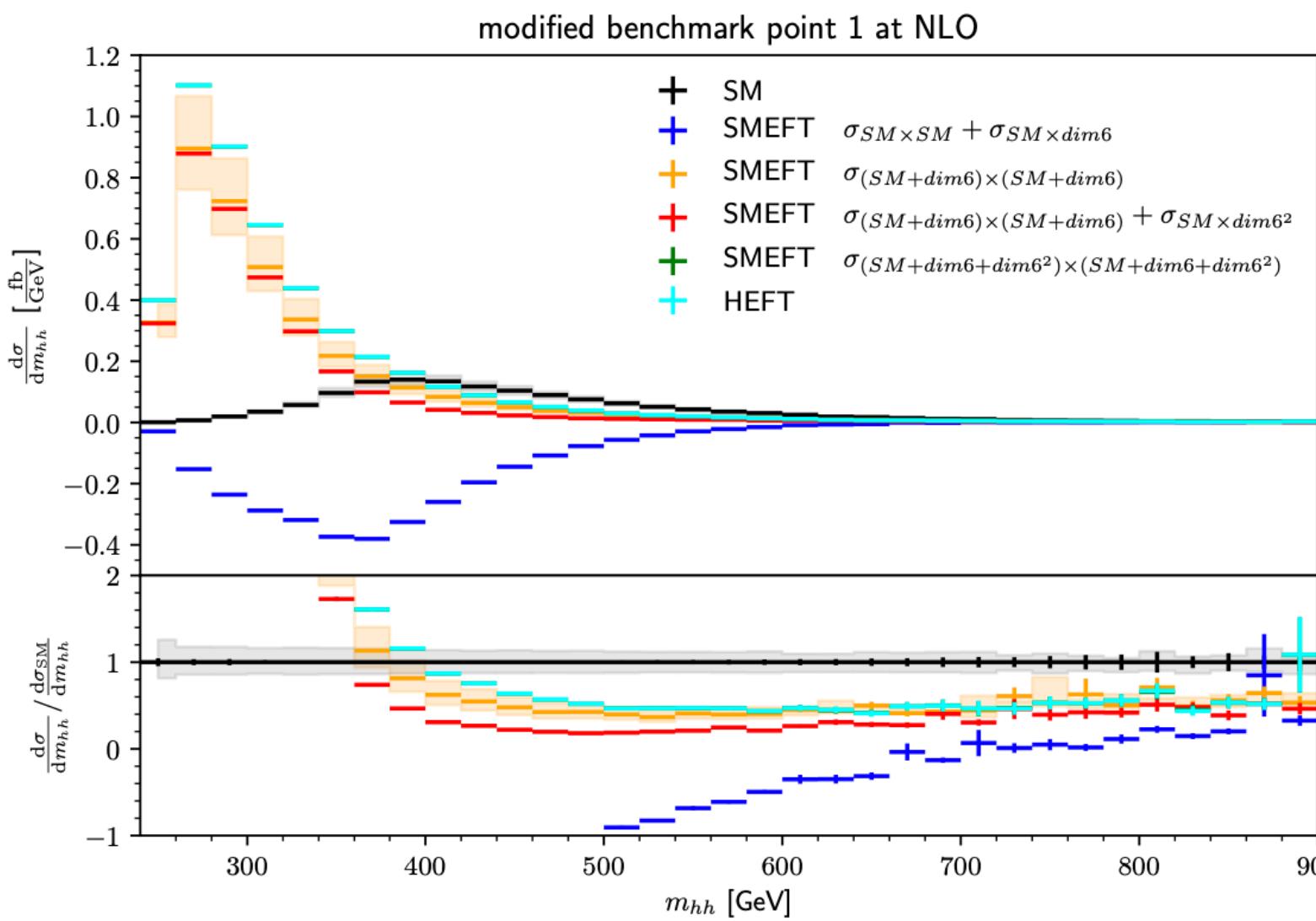
$$E^2 \frac{|C_i|}{\Lambda^2} \ll 1 \quad \text{not fulfilled for } \Lambda \simeq 1 \text{ TeV}$$

and $E \simeq m_{hh}$ up to ~ 1 TeV

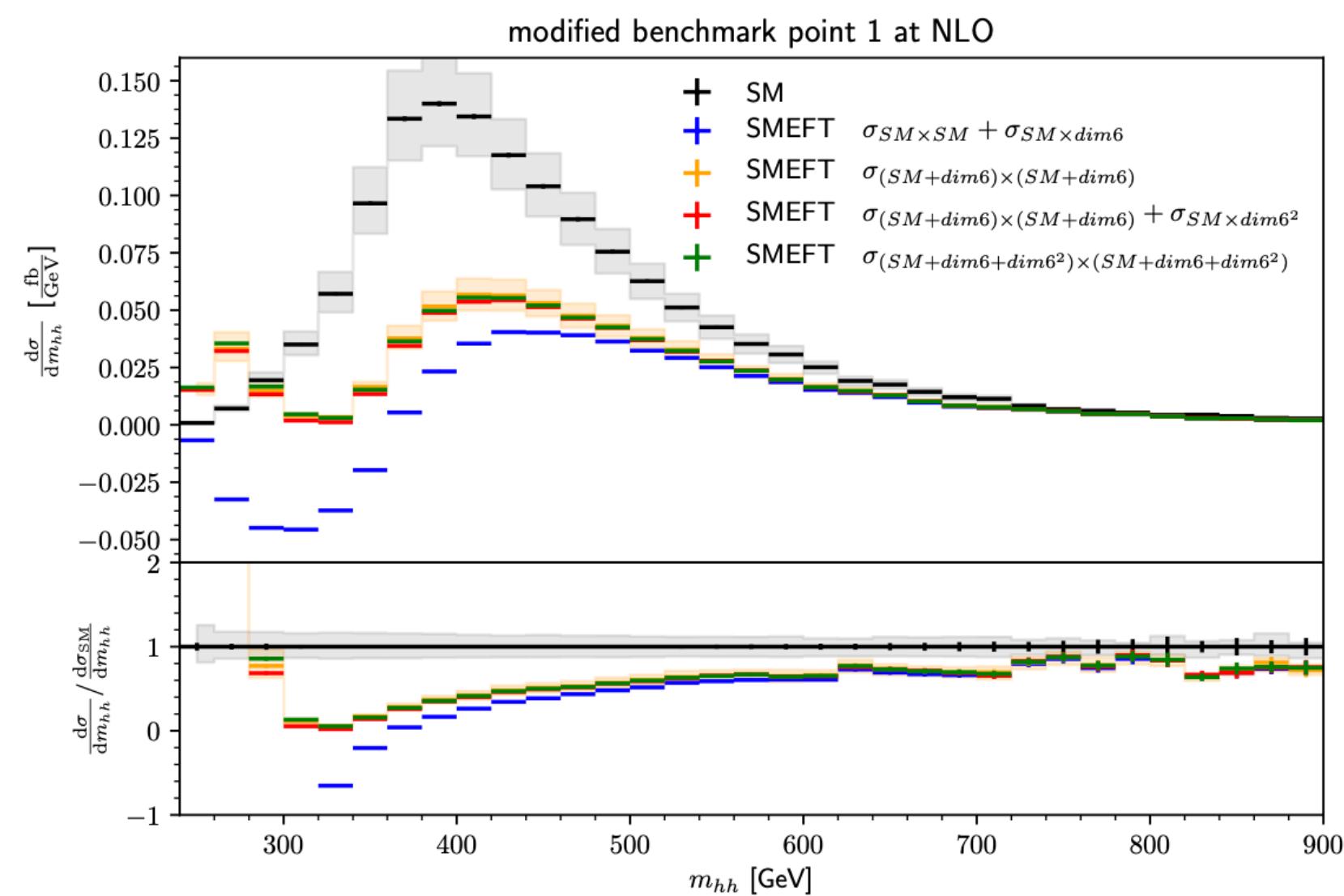
Higgs boson pair invariant mass spectrum

benchmark point 1

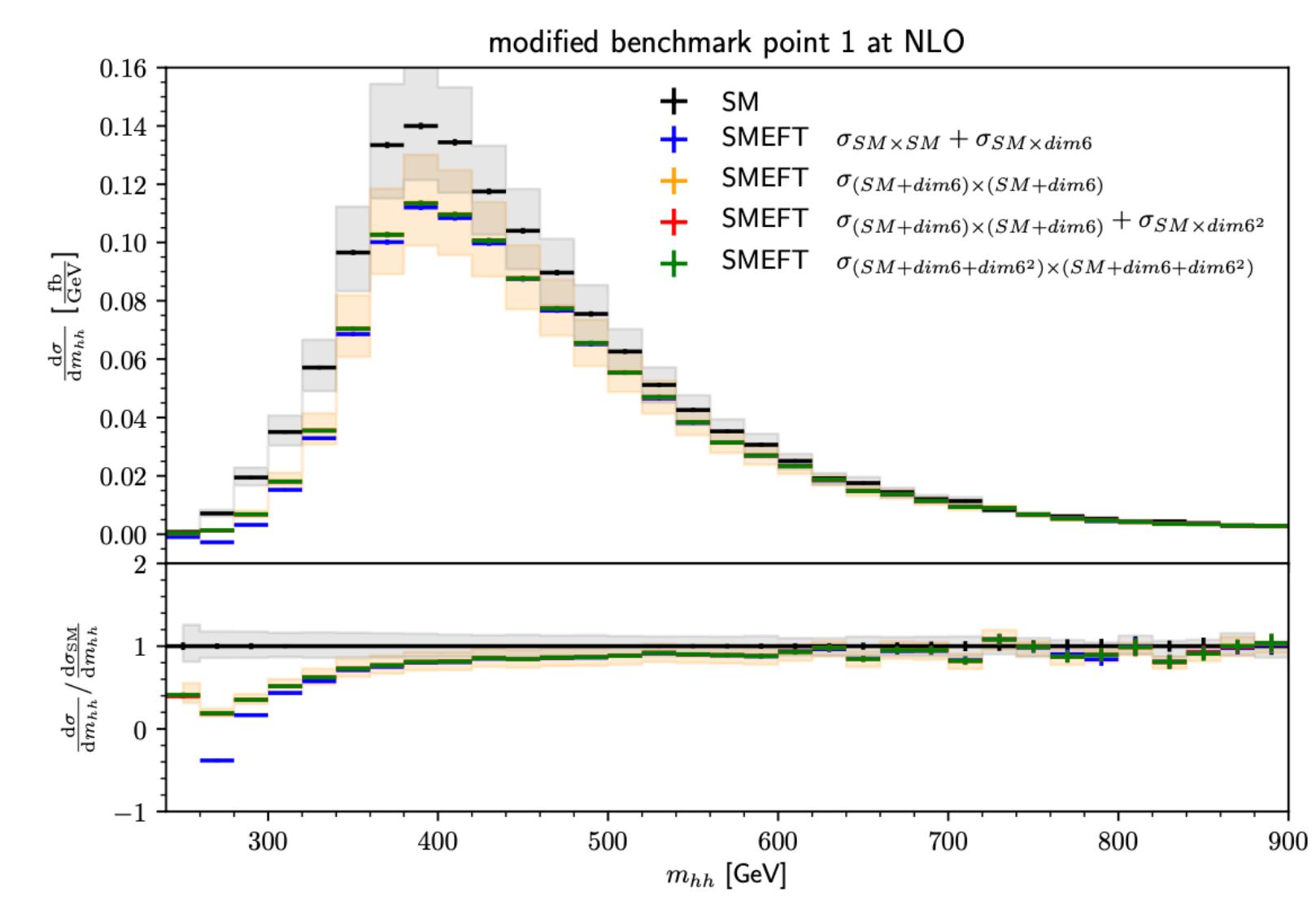
figures: Jannis Lang



$$\Lambda = 1 \text{ TeV}$$



$$\Lambda = 2 \text{ TeV}$$



$$\Lambda = 4 \text{ TeV}$$

linear dim6: negative cross sections

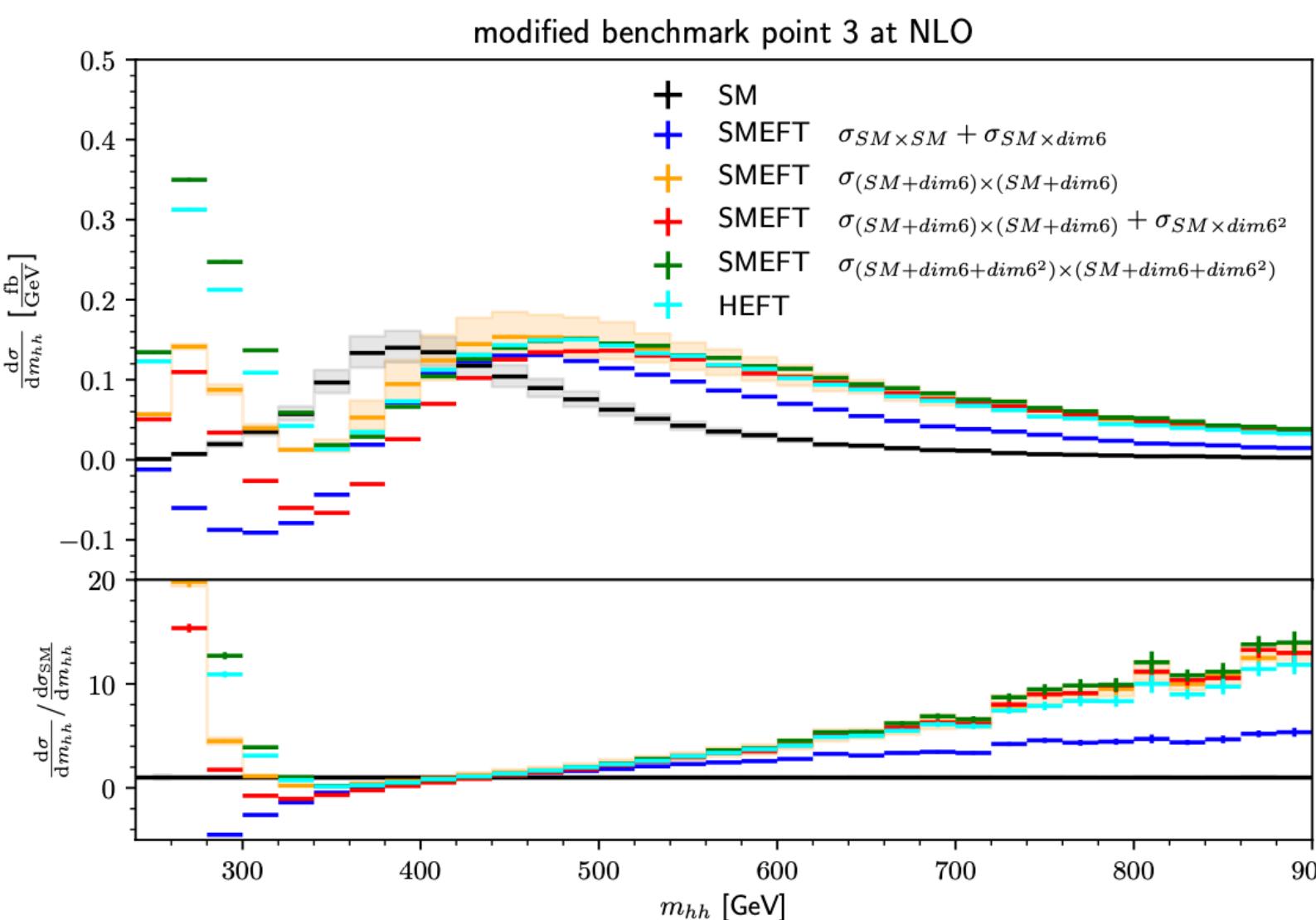
shape changes as Λ is increased (obviously, approaching SM shape)

→ for low values of Λ : parameter point valid in HEFT can be **invalid** in SMEFT

Higgs boson pair invariant mass spectrum

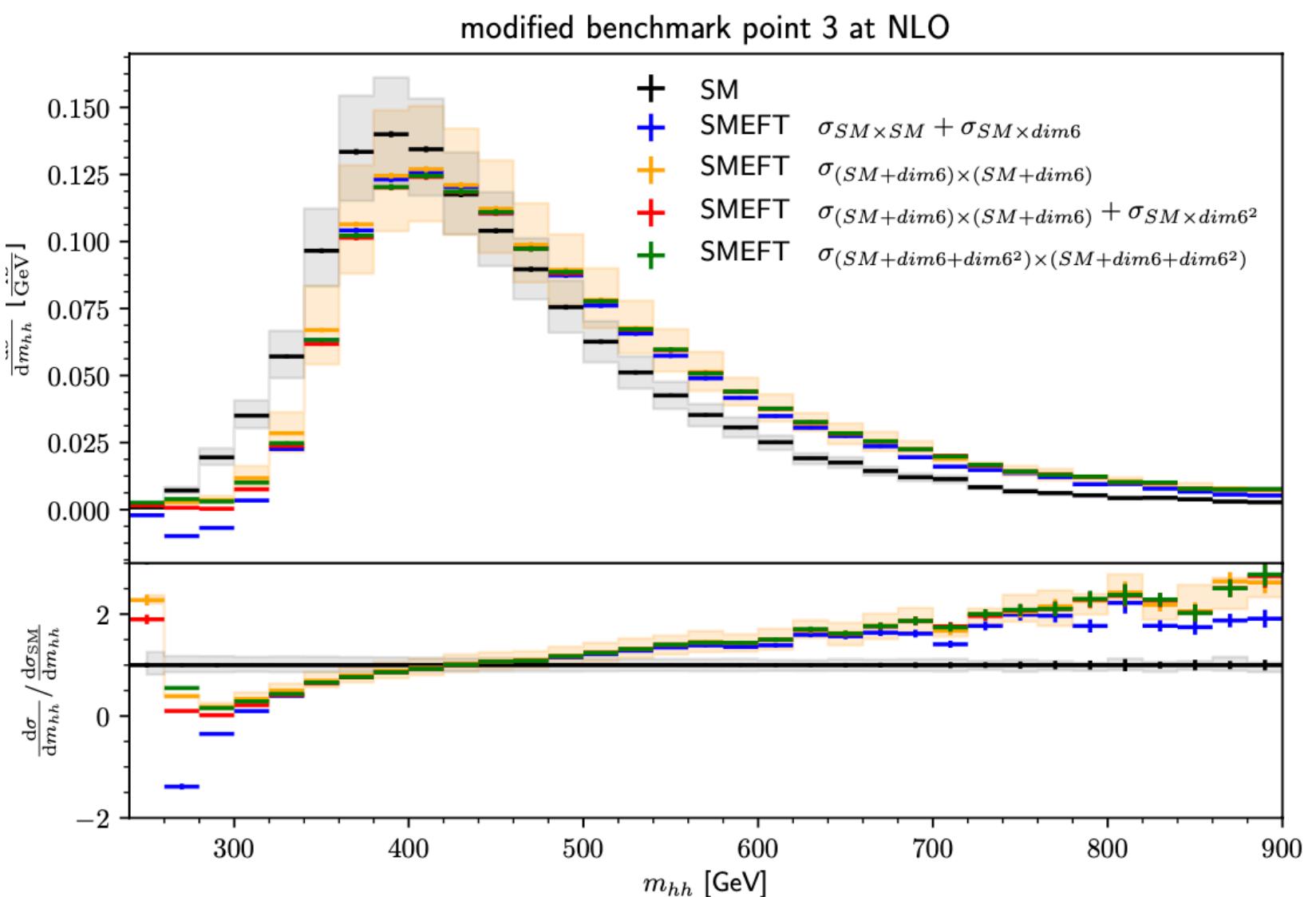
benchmark point 3

figures: Jannis Lang



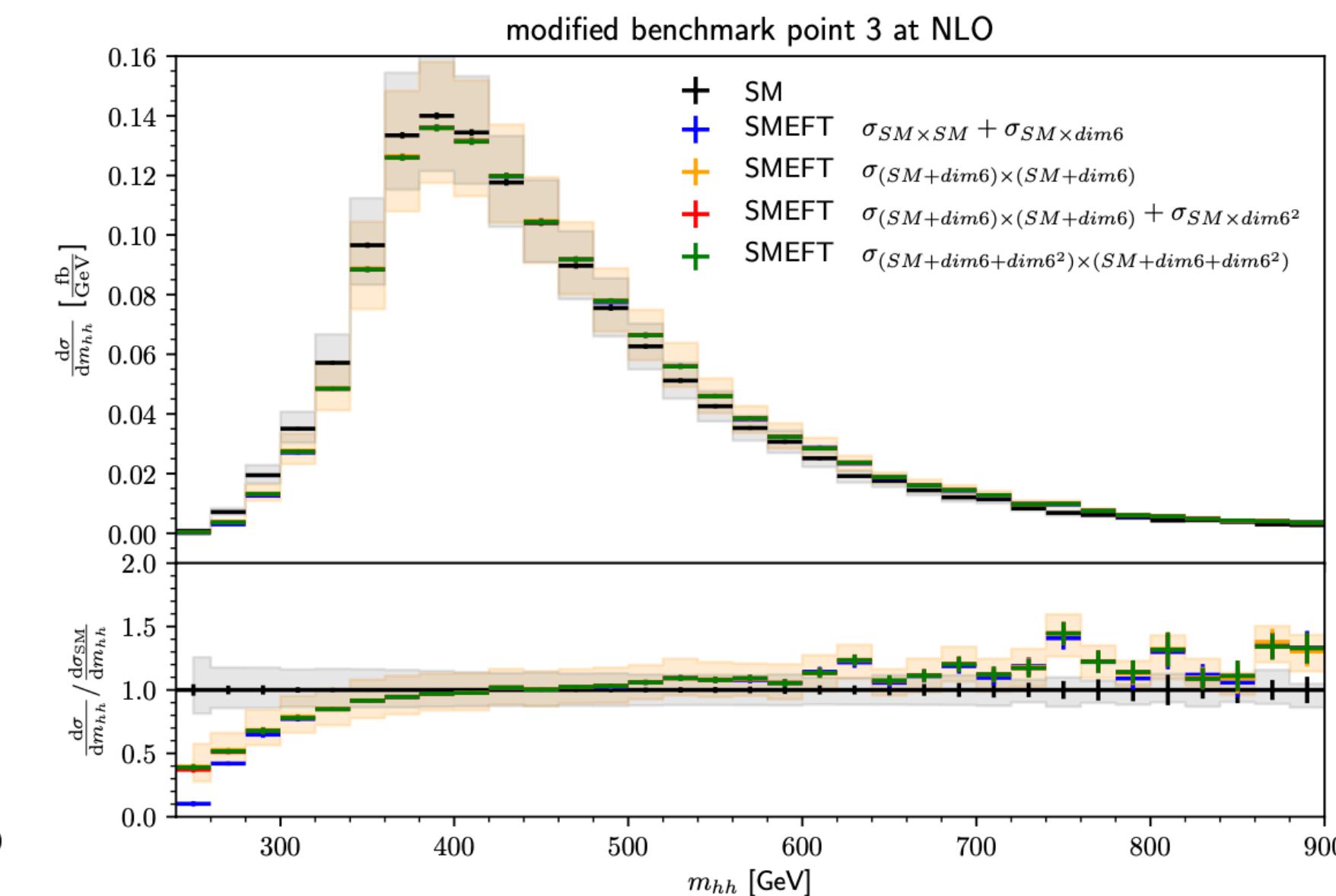
$\Lambda = 1 \text{ TeV}$

double operator insertions
have large effect



$\Lambda = 2 \text{ TeV}$

distinguishable from SM
within NLO uncertainties



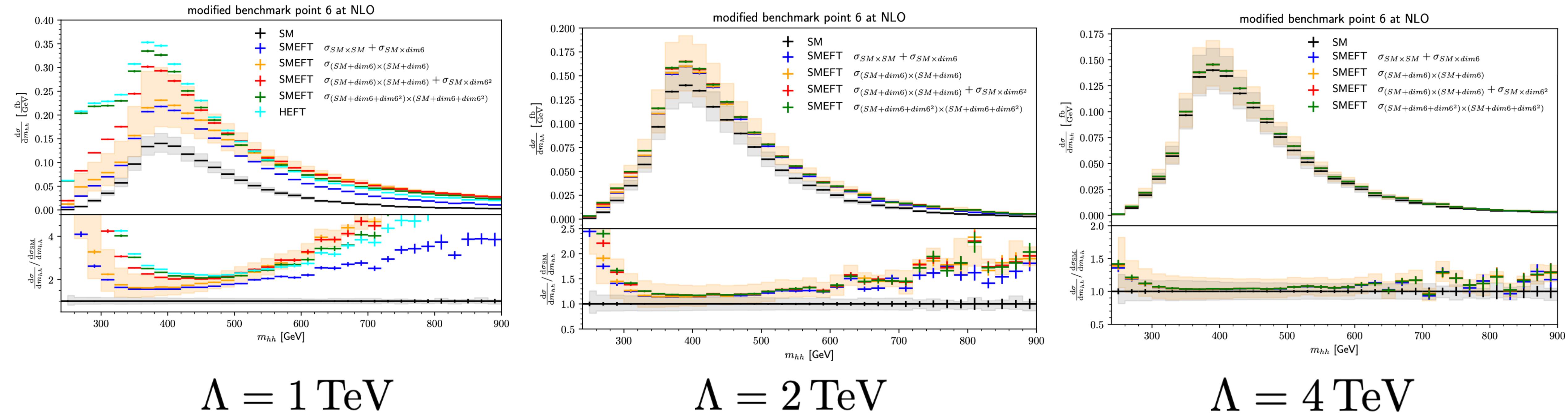
$\Lambda = 4 \text{ TeV}$

can be distinguished from SM
in low mHH region

Higgs boson pair invariant mass spectrum

benchmark point 6

figures: Jannis Lang



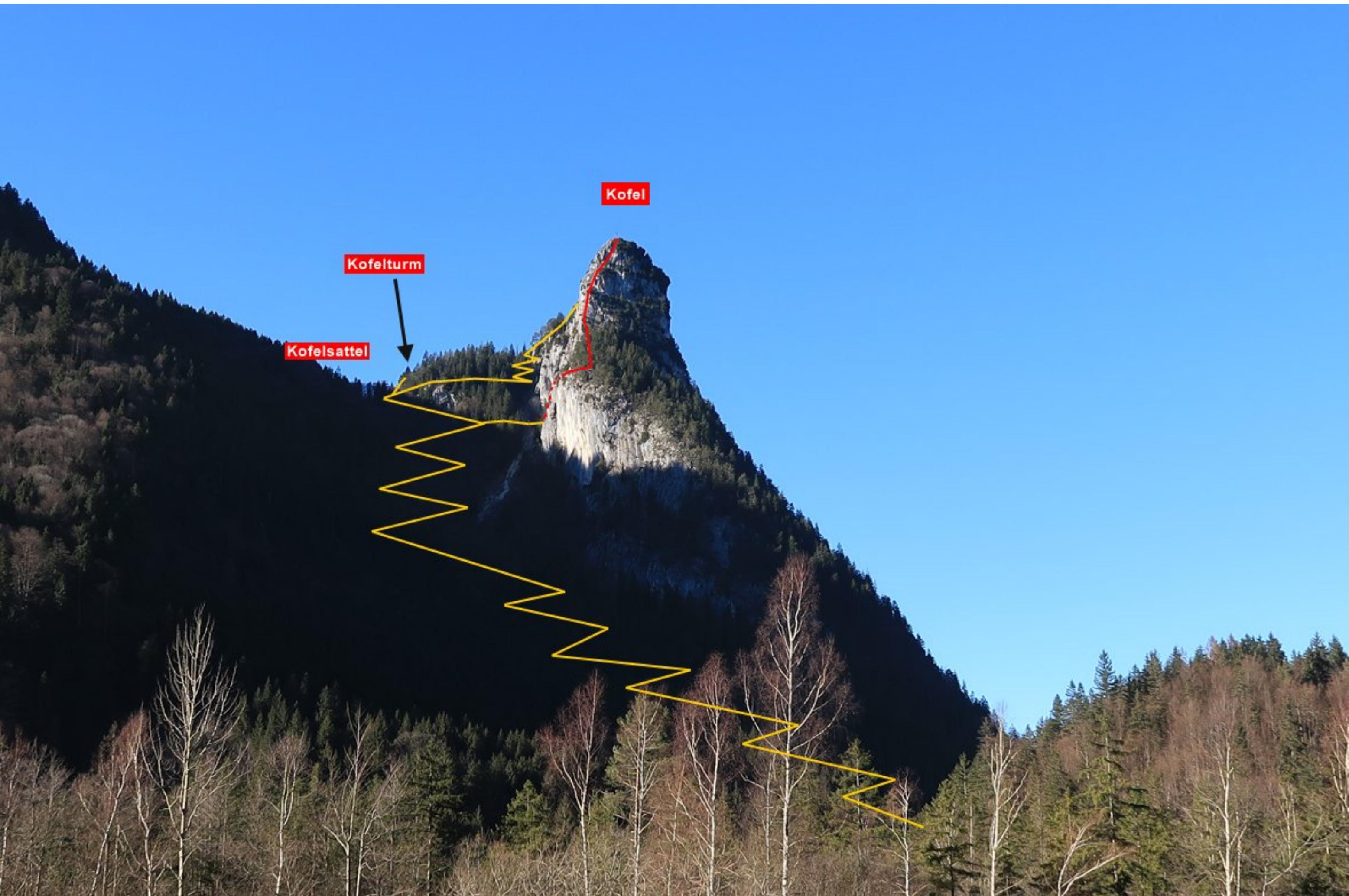
difference between green and cyan
only running of α_s

shoulder left gone

can hardly be distinguished from SM
within NLO scale uncertainties

Summary & Outlook

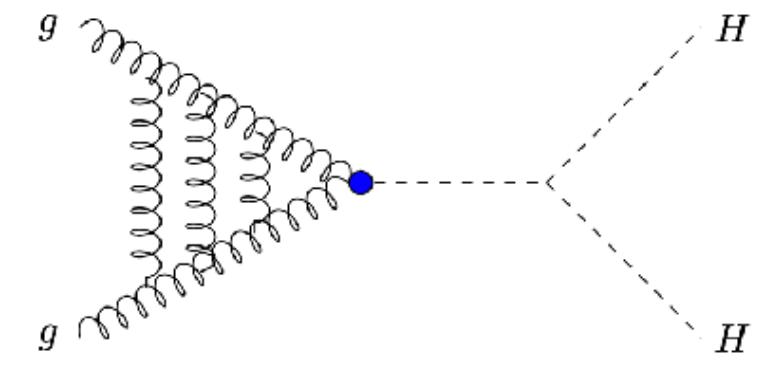
- full NLO corrections for $gg \rightarrow HH$ available within SMEFT (and HEFT)
- comparison between HEFT and SMEFT parametrisations
- studied truncation effects: including dim-6 operators squared, double operator insertions
- naive translation from HEFT to SMEFT can lead out of SMEFT validity range
- delicate cancellations -> small changes in treatment of anomalous couplings can have large effects
- small distortions from SM values described well by SMEFT often not distinguishable from SM within scale uncertainties





Higher order corrections: SM

N3LO: Chen, Li, Shao, Wang '19
(HTL with top mass effects)



NNLO: De Florian, Mazzitelli '13
Grigo, Melnikov, Steinhauser '14

NNLO_{FTapprox} Grazzini, Kallweit, GH, Jones,
Kerner, Lindert, Mazzitelli '18

inclusion of top quark mass dependence except in virtual $\mathcal{O}(\alpha_s^3)$

NLO full m_t

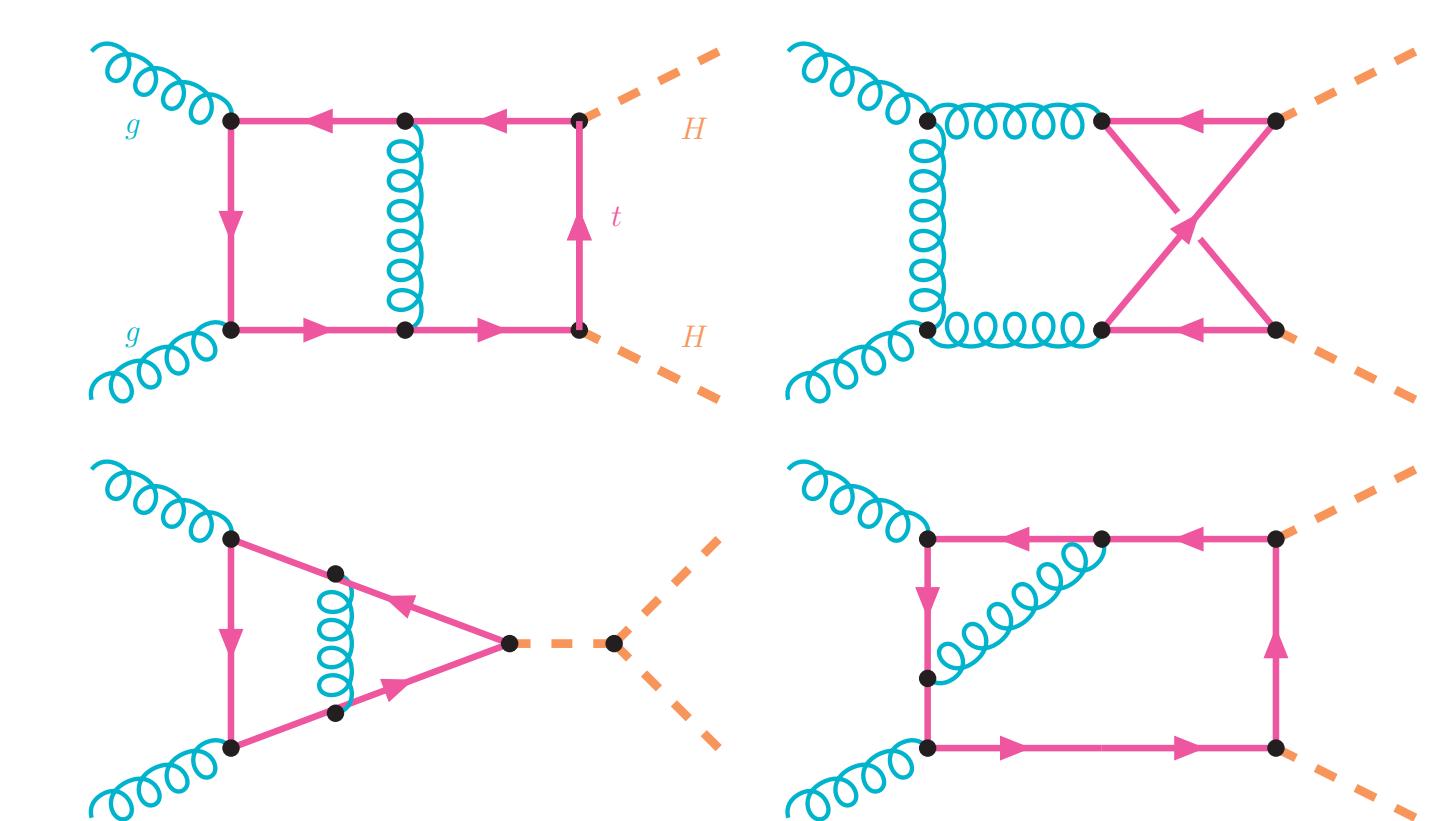
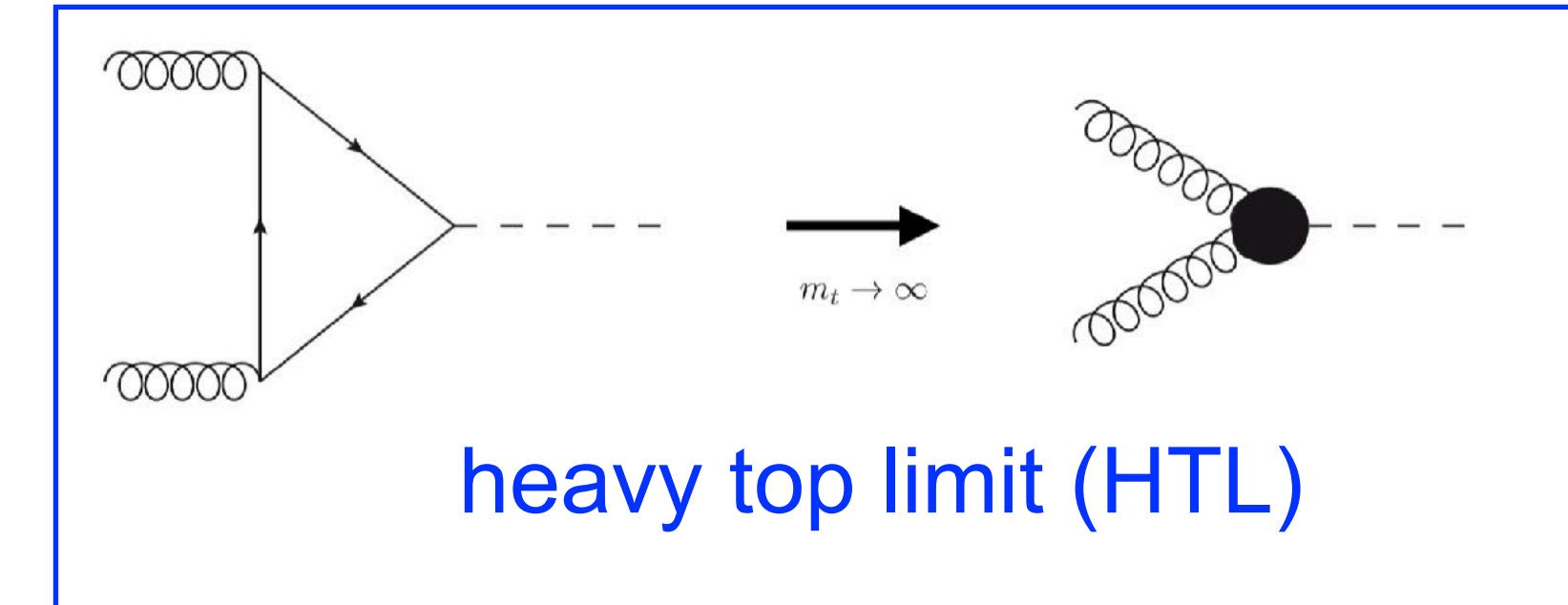
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top quark mass scheme uncertainties: pole mass versus $\overline{\text{MS}}$ mass

Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira '18, '20



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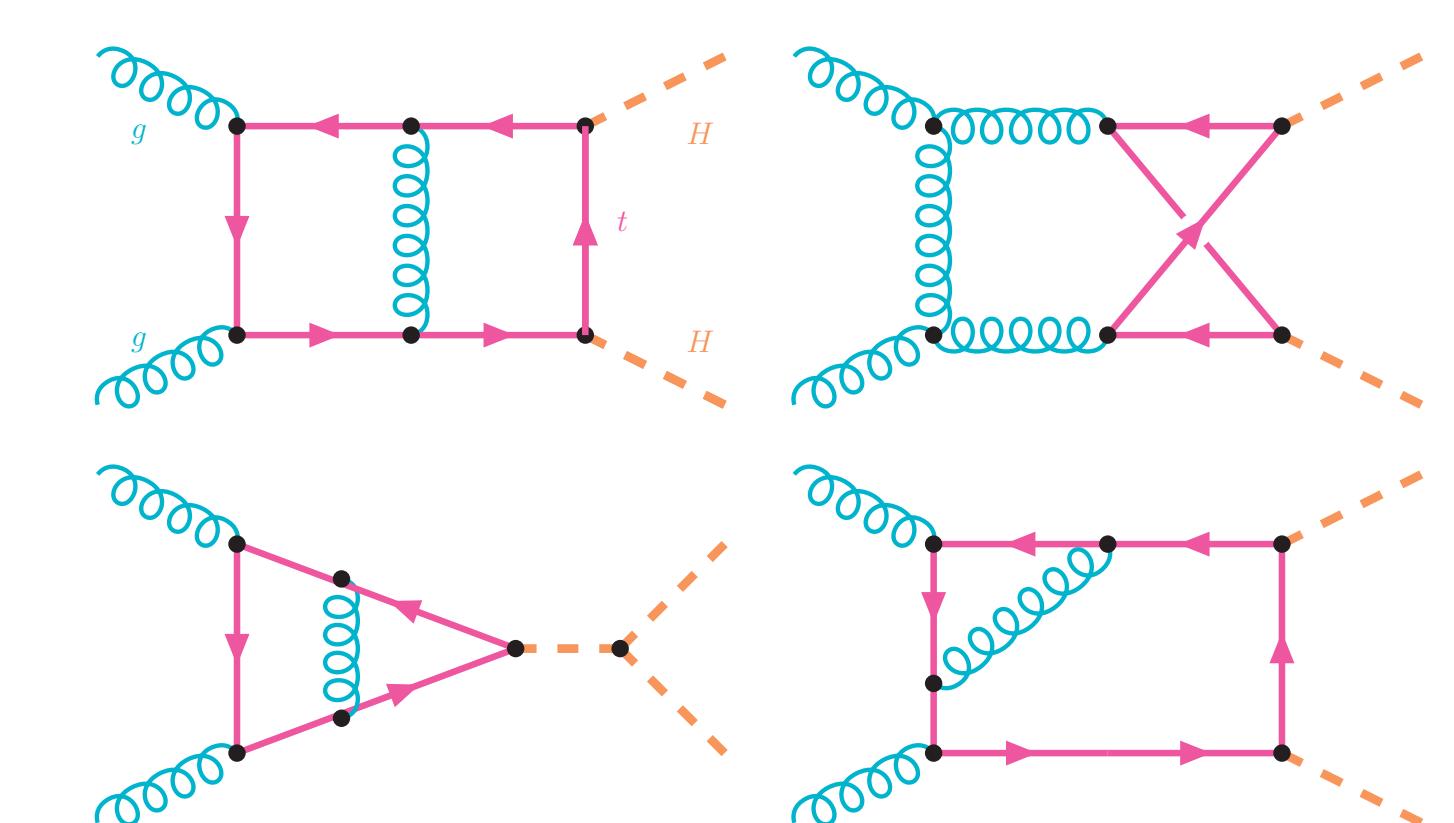
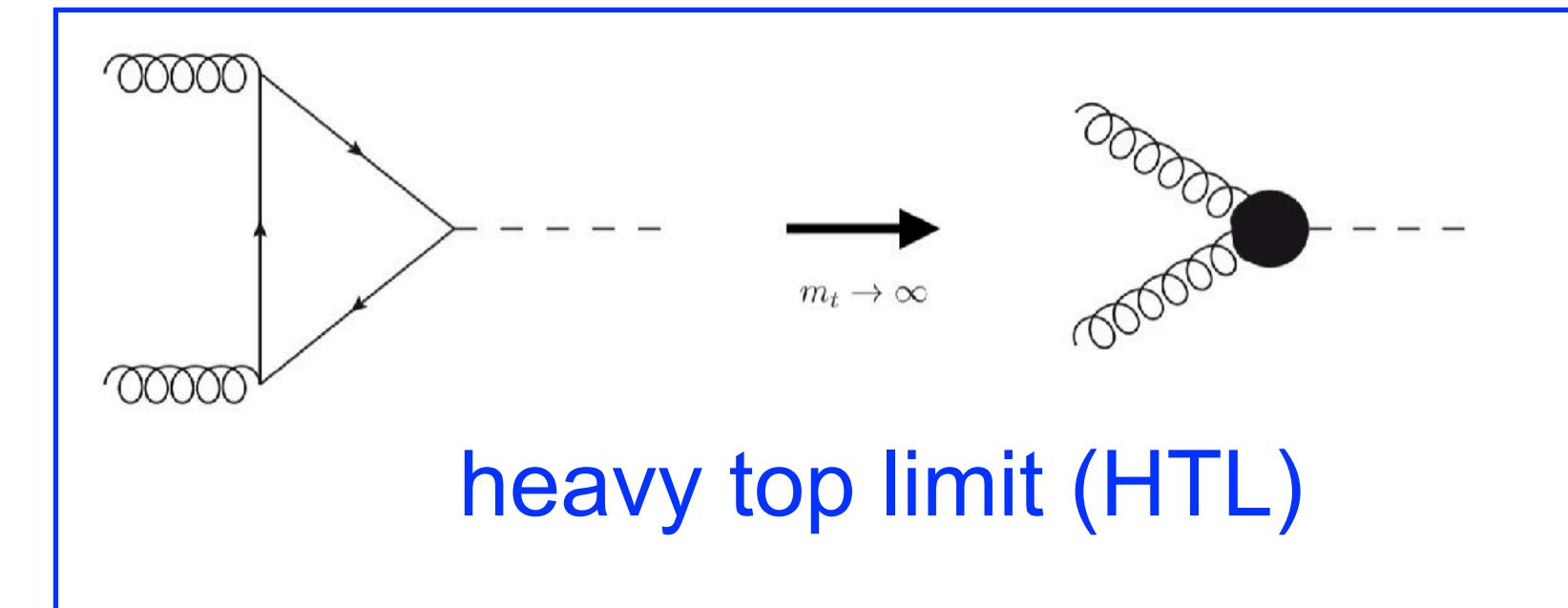
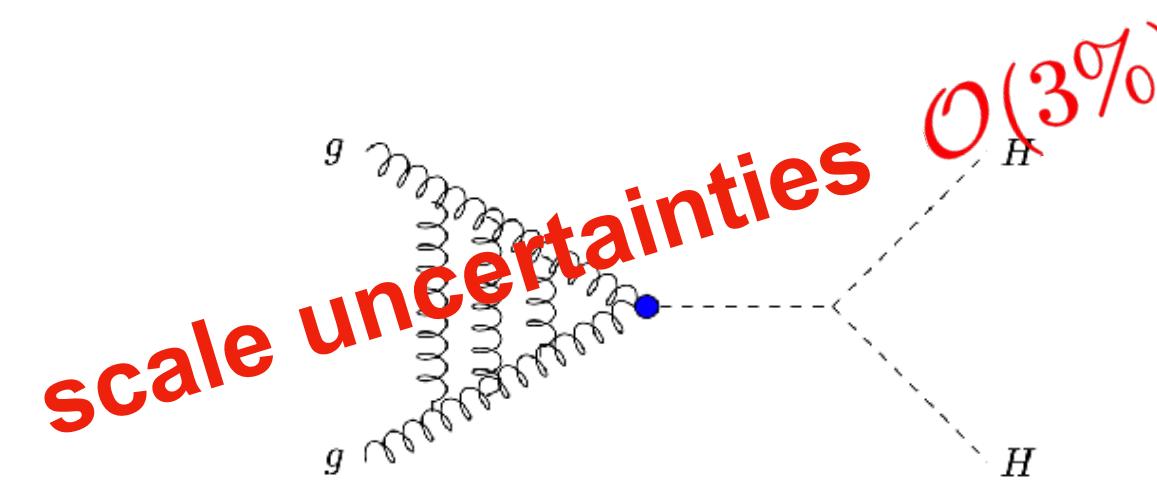
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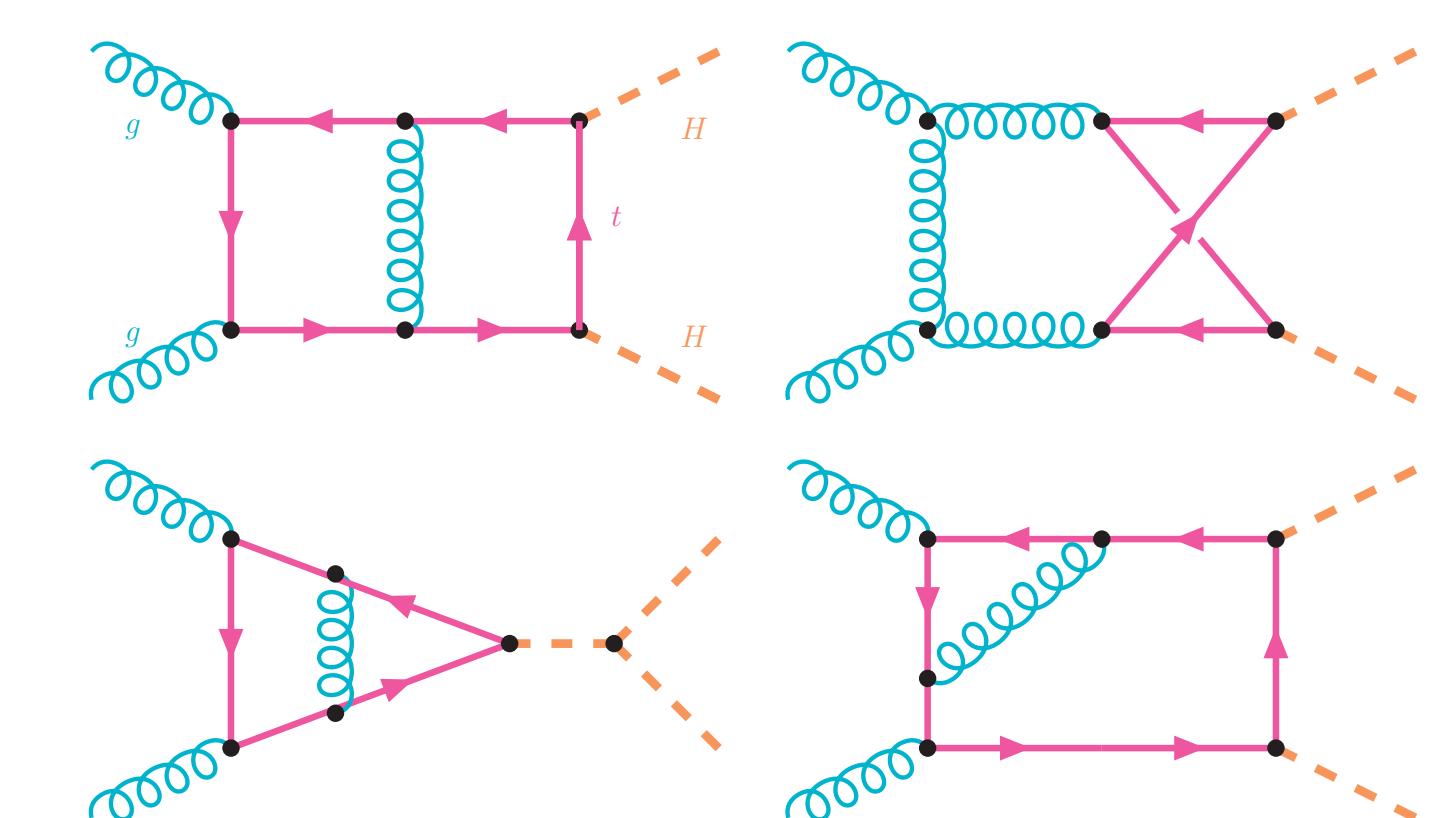
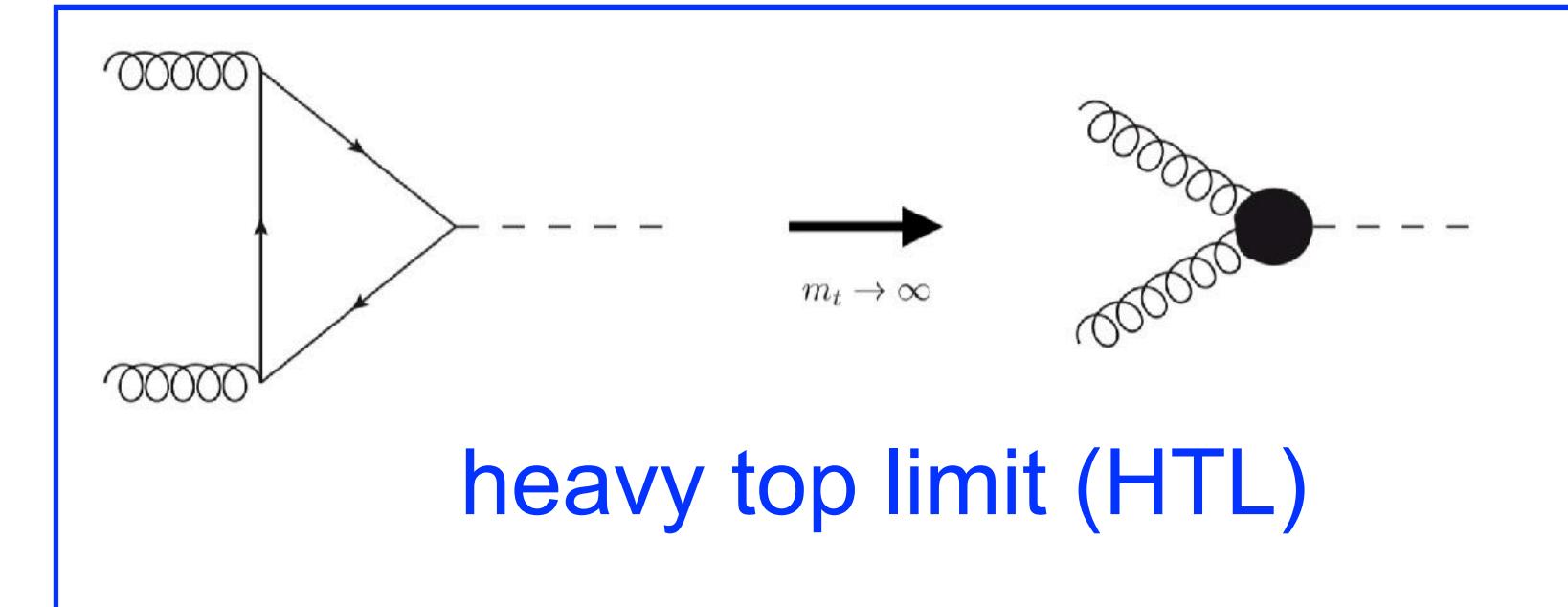
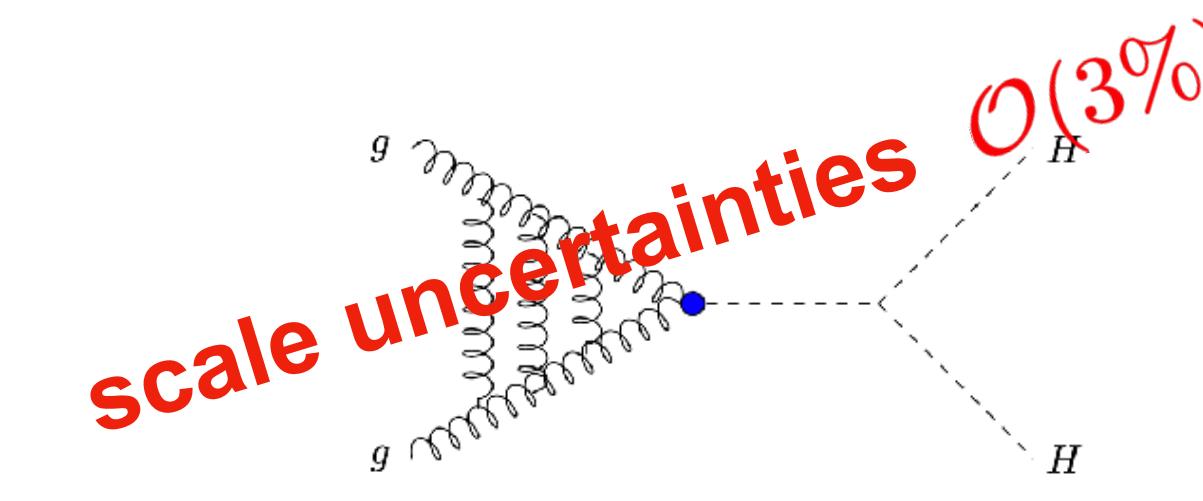
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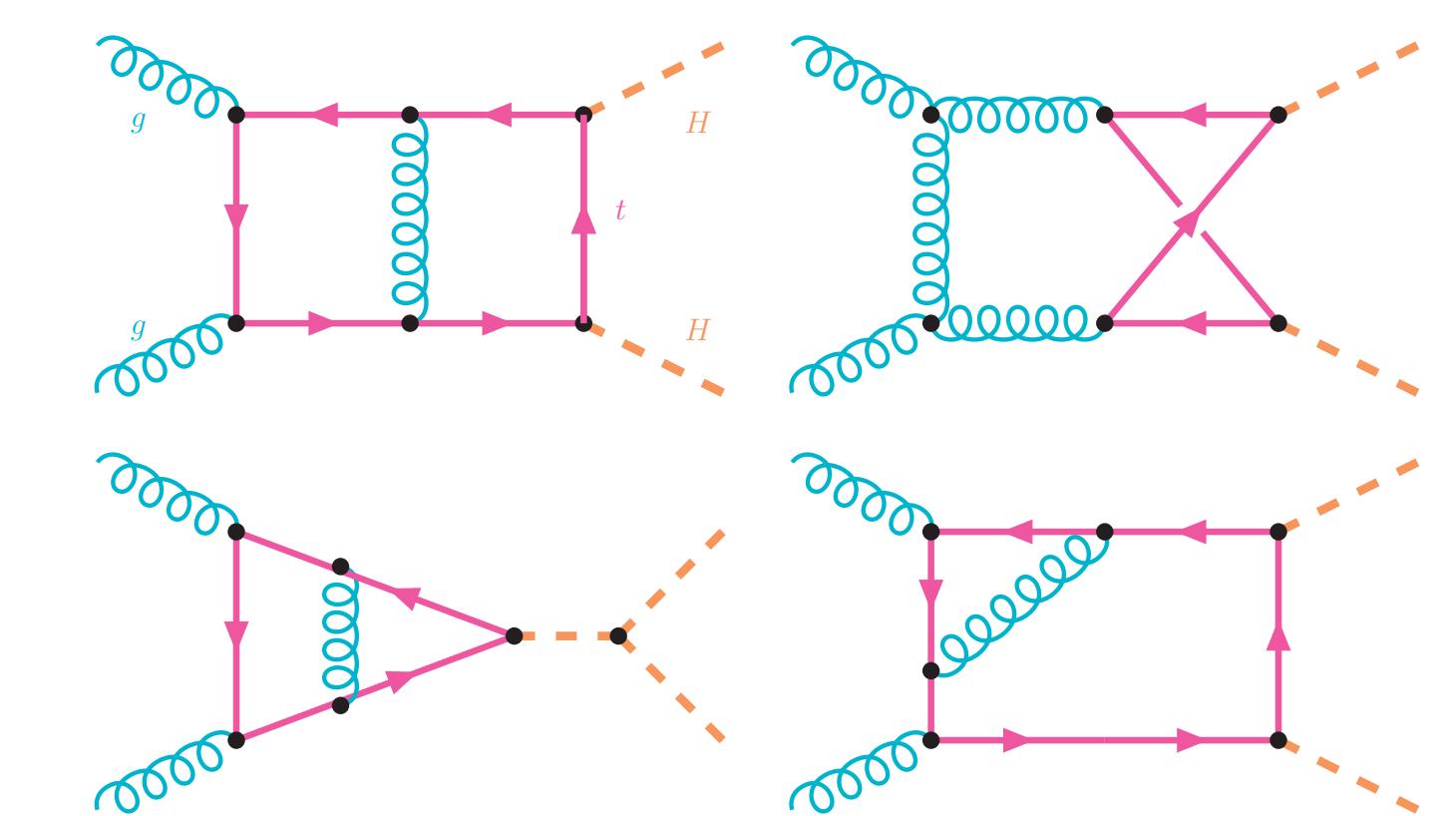
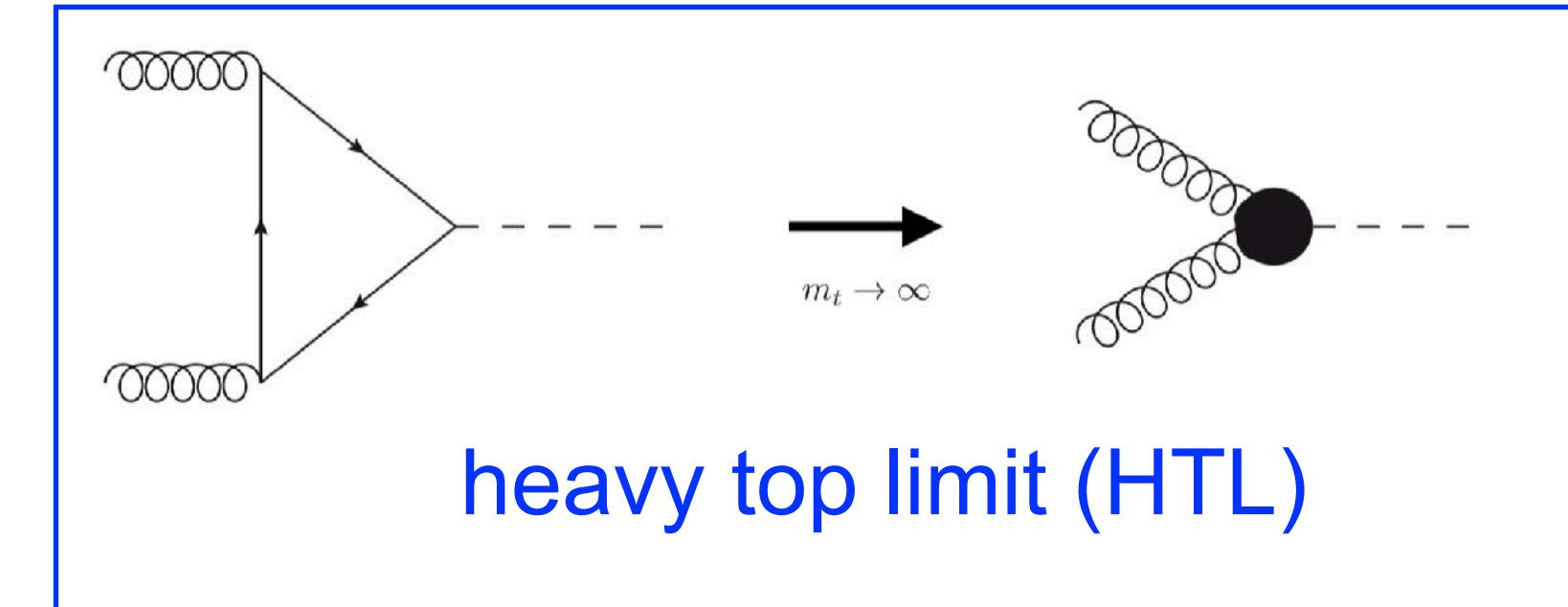
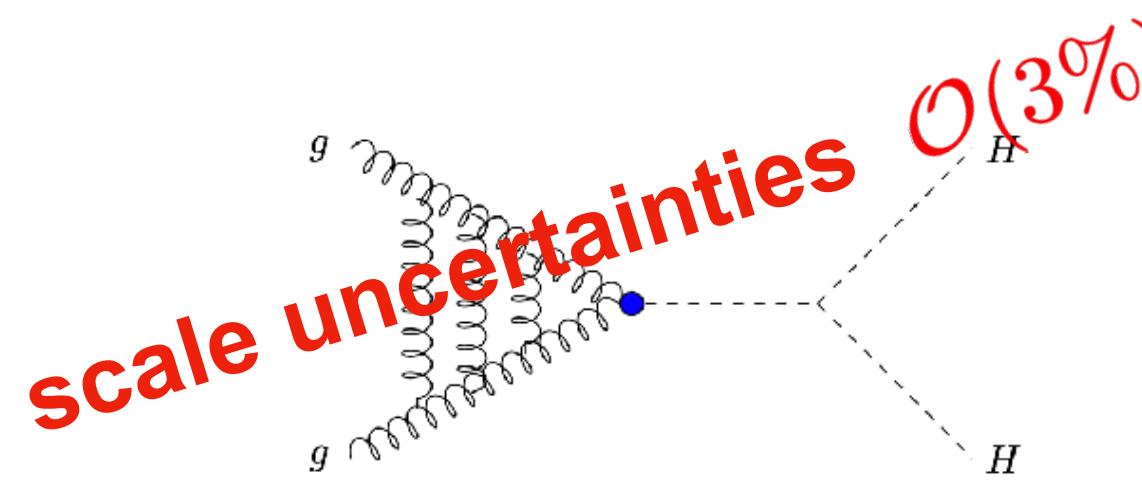
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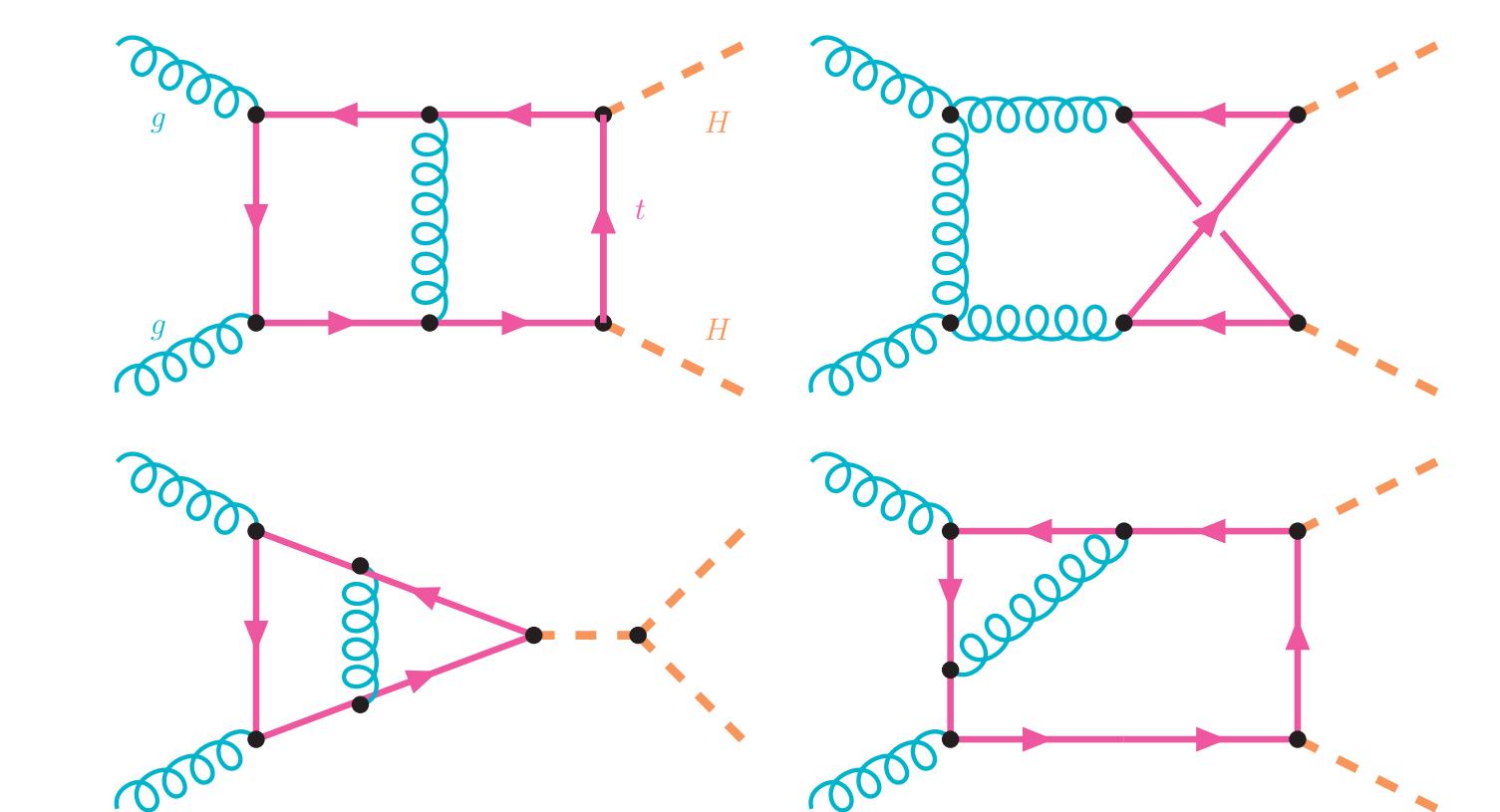
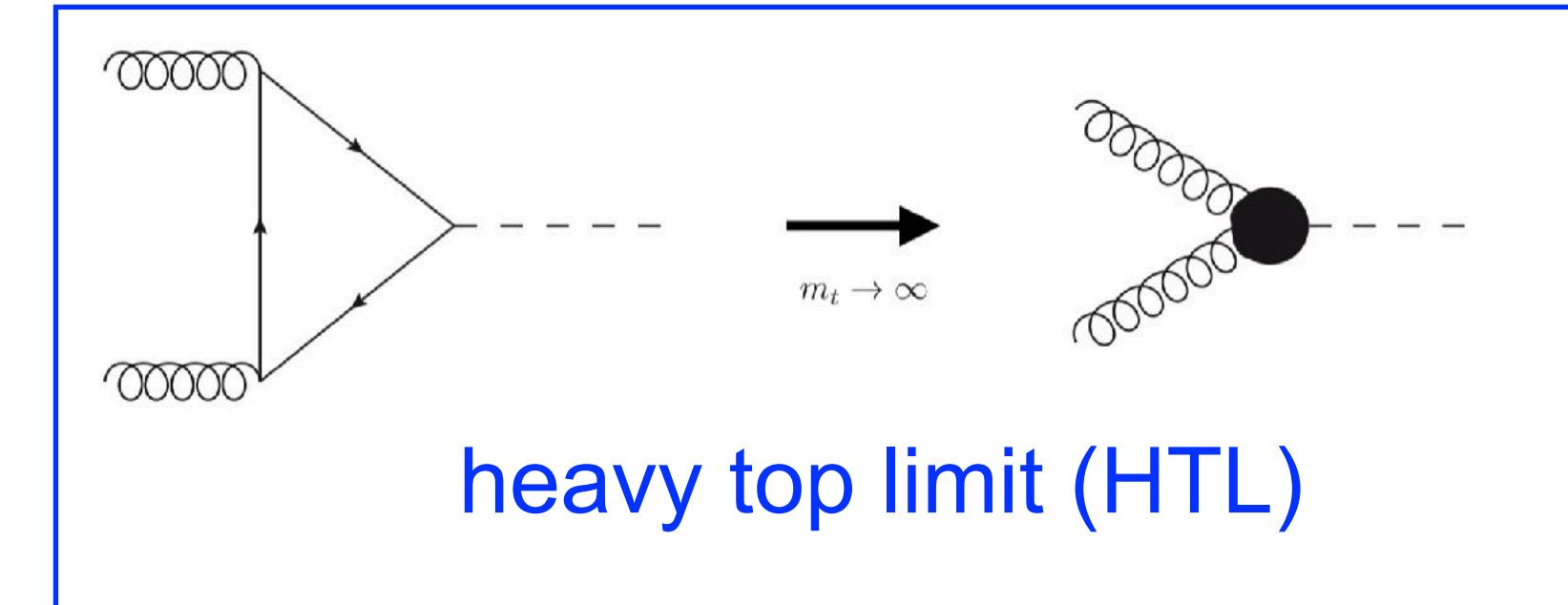
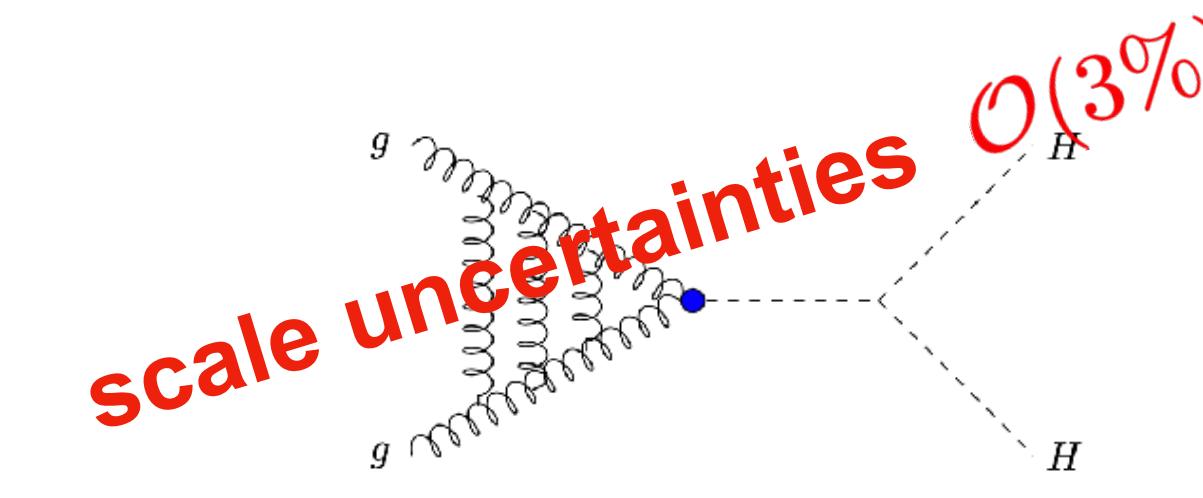
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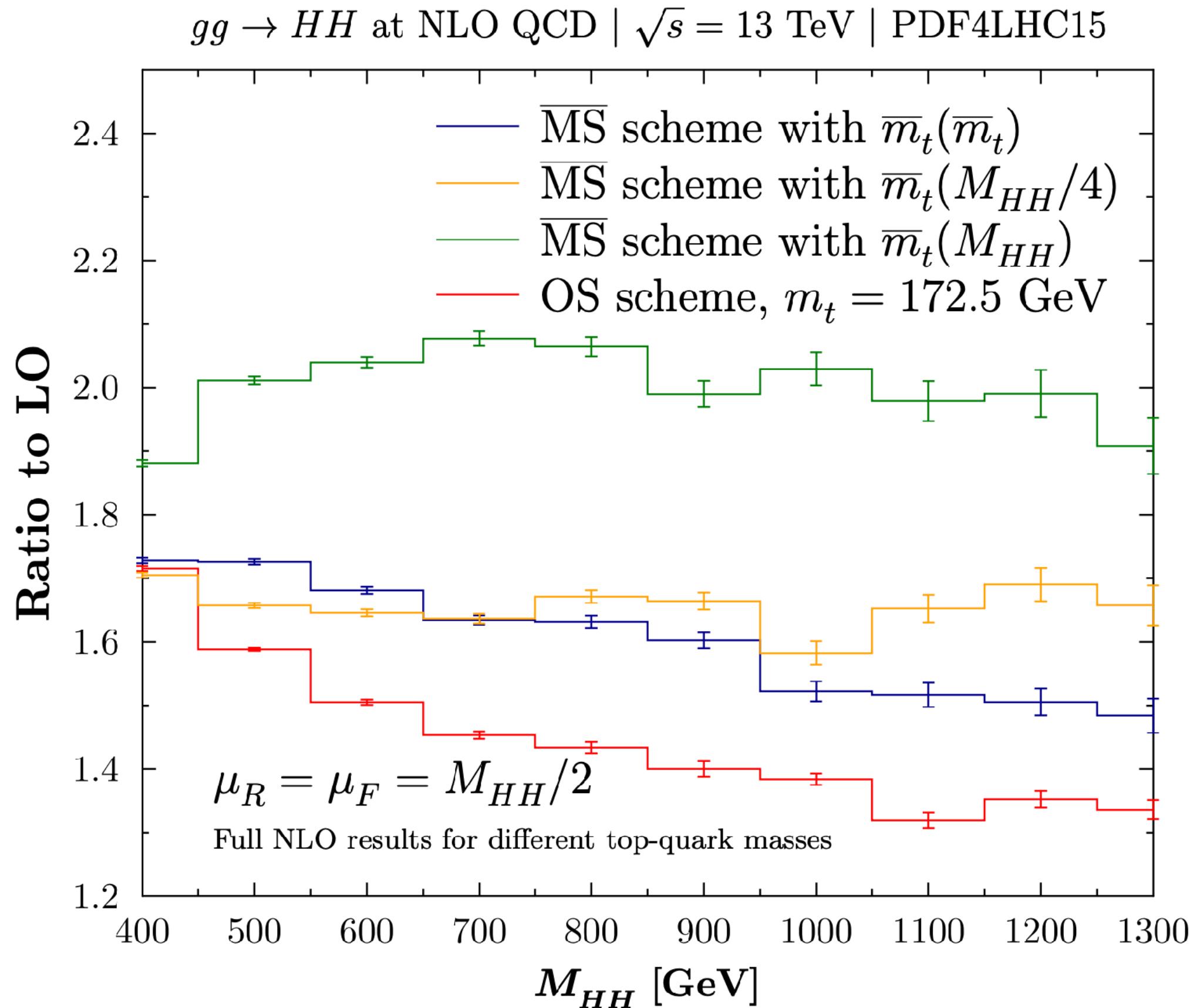


for top mass scheme uncertainties
see also talk of Stephen Jones

Top quark mass renormalisation scheme uncertainties

$$\overline{m}_t(m_t) = \frac{m_t}{1 + \frac{4}{3} \frac{\alpha_s(m_t)}{\pi} + K_2 \left(\frac{\alpha_s(m_t)}{\pi} \right)^2 + K_3 \left(\frac{\alpha_s(m_t)}{\pi} \right)^3 + \dots}$$

relation between pole mass and $\overline{\text{MS}}$ mass



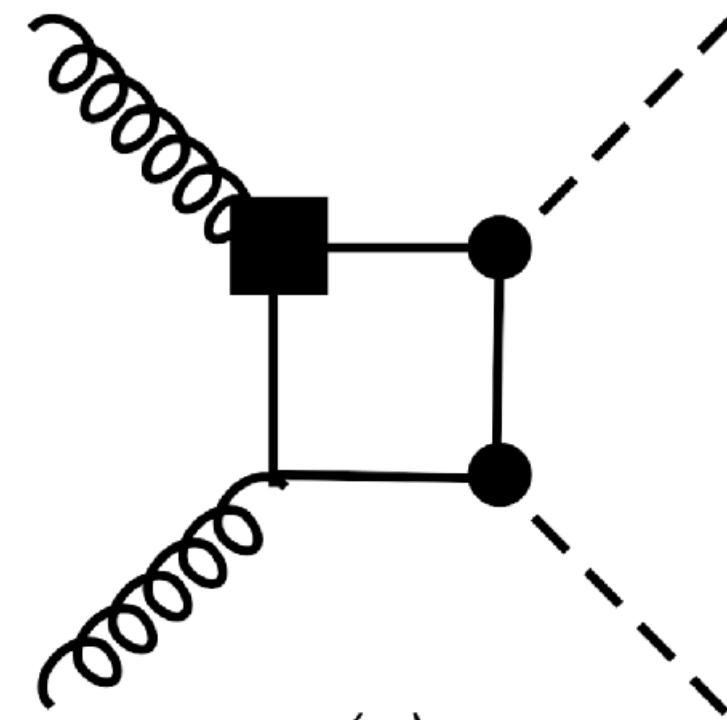
Baglio, Campanario, Glaus Mühlleitner,
Ronca, Spira 2003.03227, 2008.11626

also present in other heavy quark
loop induced processes

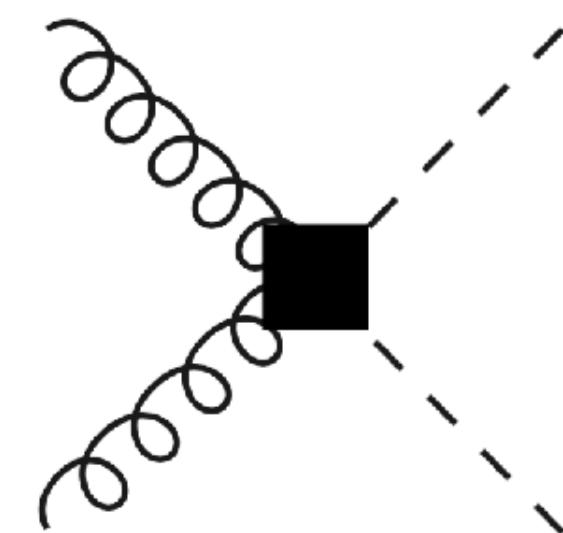
Chromomagnetic operator

$$O_{tG} = y_t g_s \bar{t}_L \sigma_{\mu\nu} G^{\mu\nu} t_R$$

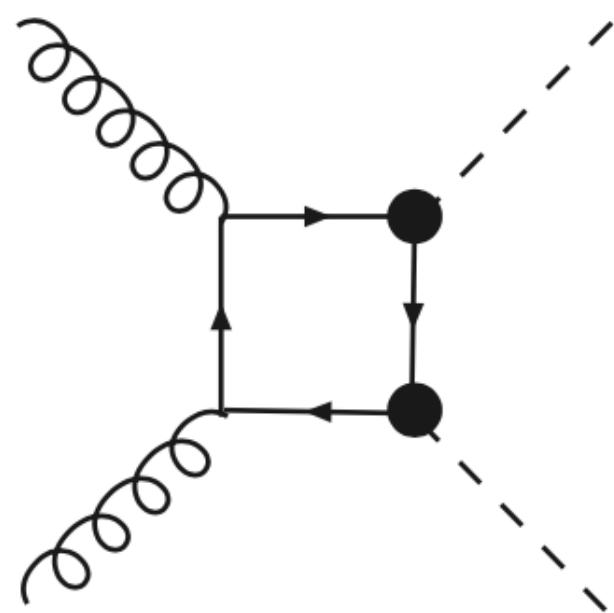
suppressed by loop factor $1/(16\pi^2)$



relative to



or



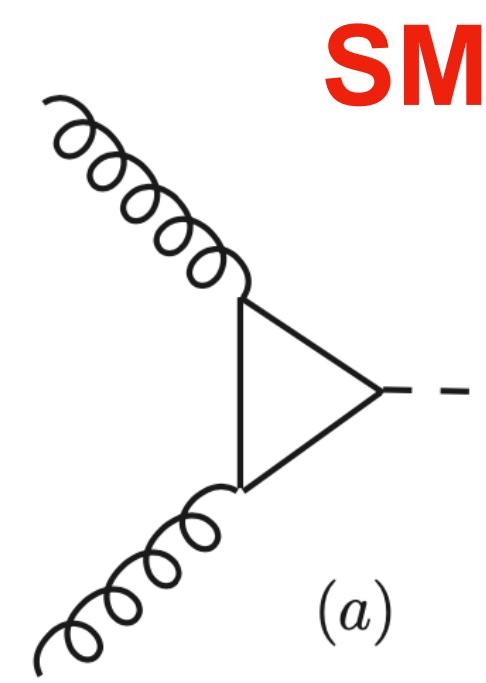
Buchalla, GH, Müller-Salditt, Pandler
arXiv:2204.11808

in weakly coupled UV theories operators coupling to field strength tensors must come from a contracted loop

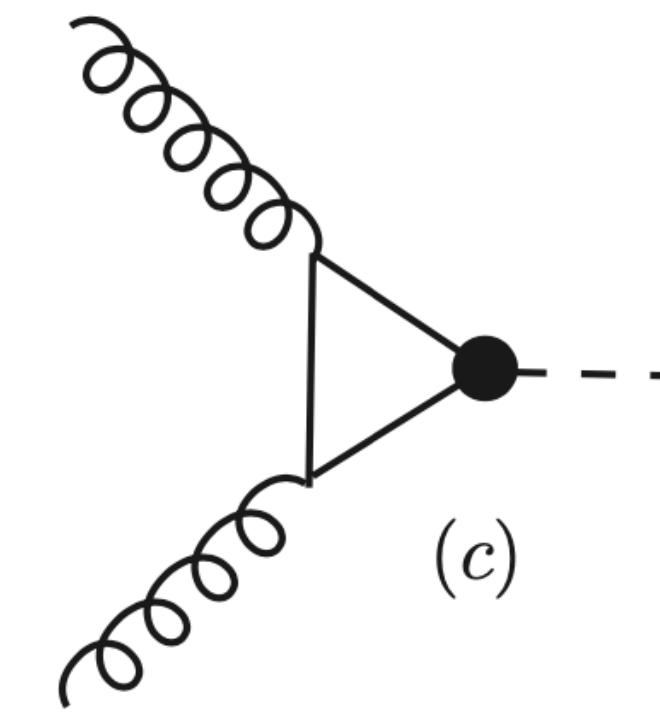
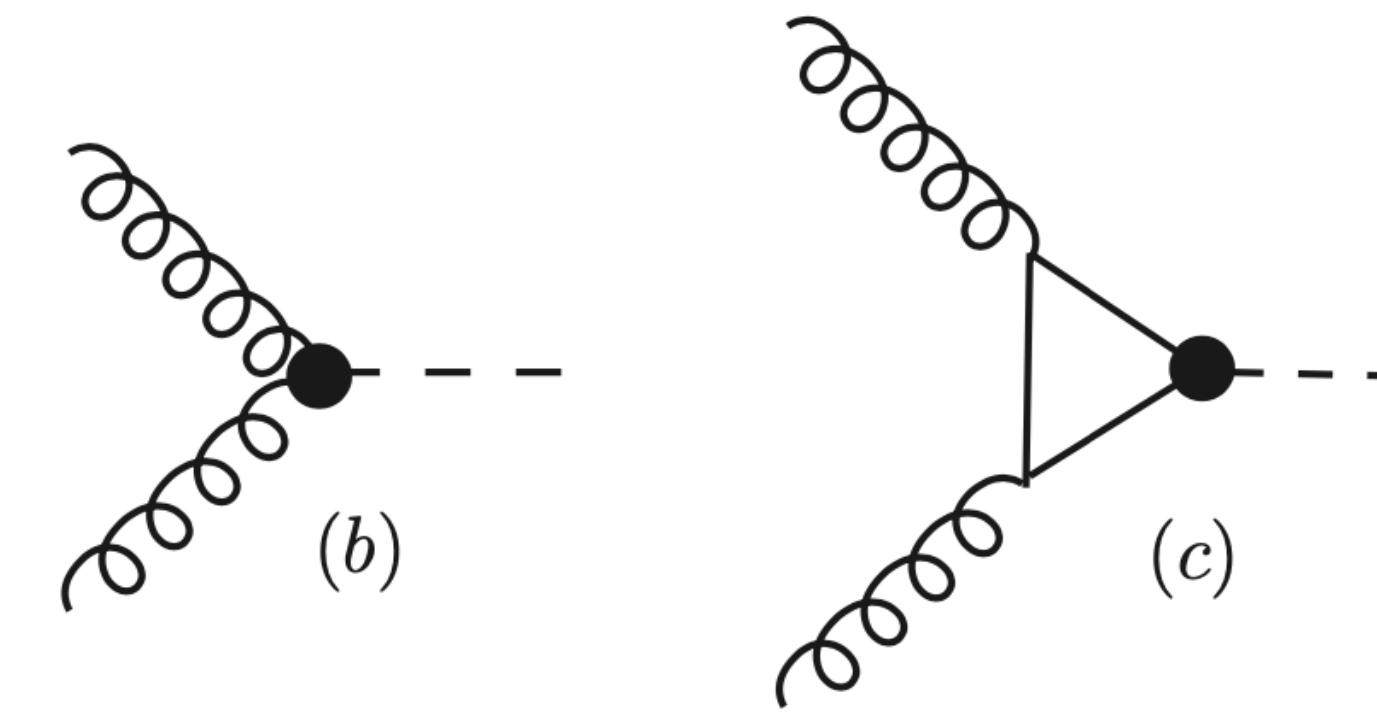
see also Buchalla et al 1806.05162;
Arzt, Einhorn, Wudka, hep-ph/9405214

Loop counting matters in SMEFT

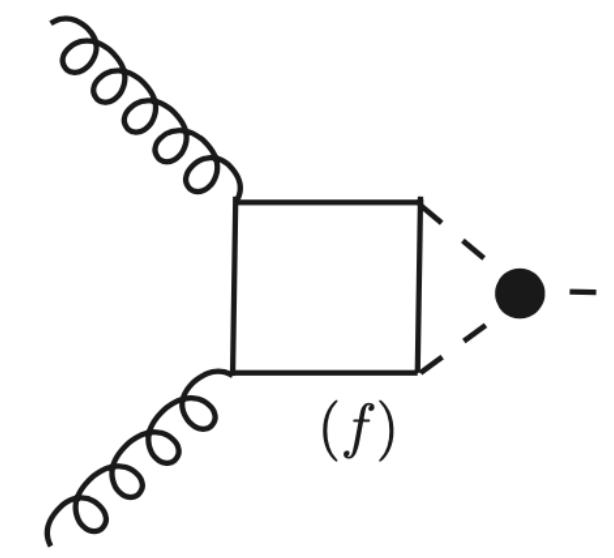
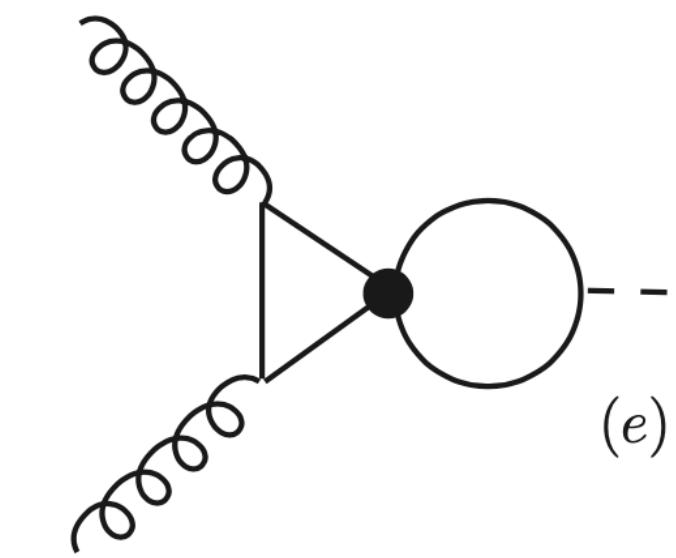
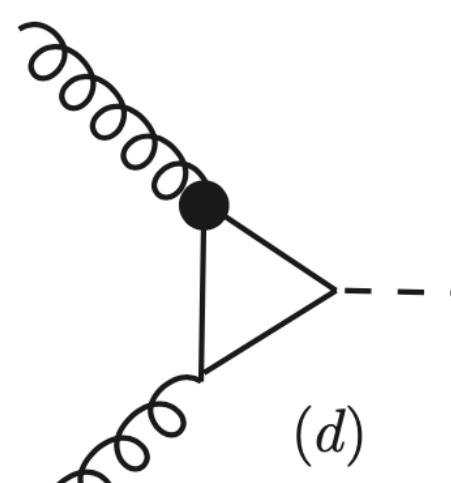
example single Higgs production Buchalla, GH, Müller-Salditt, Pandler, arXiv:2204.11808



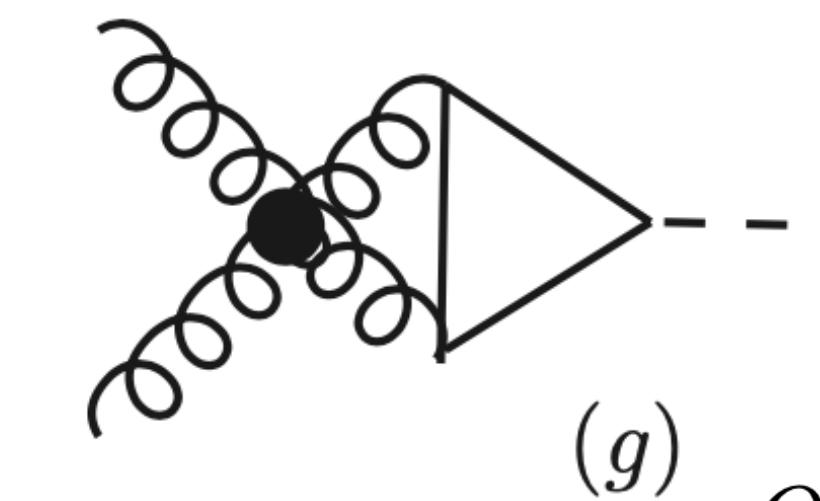
$$\frac{1}{16\pi^2}$$



$$\frac{1}{\Lambda^2} \frac{1}{16\pi^2}$$



$$\frac{1}{\Lambda^2} \frac{1}{(16\pi^2)^2}$$



$$\frac{1}{\Lambda^2} \frac{1}{(16\pi^2)^3}$$

$$Q_{uG} = \bar{q} \sigma^{\mu\nu} T^A u \tilde{\phi} G_{\mu\nu}^A$$

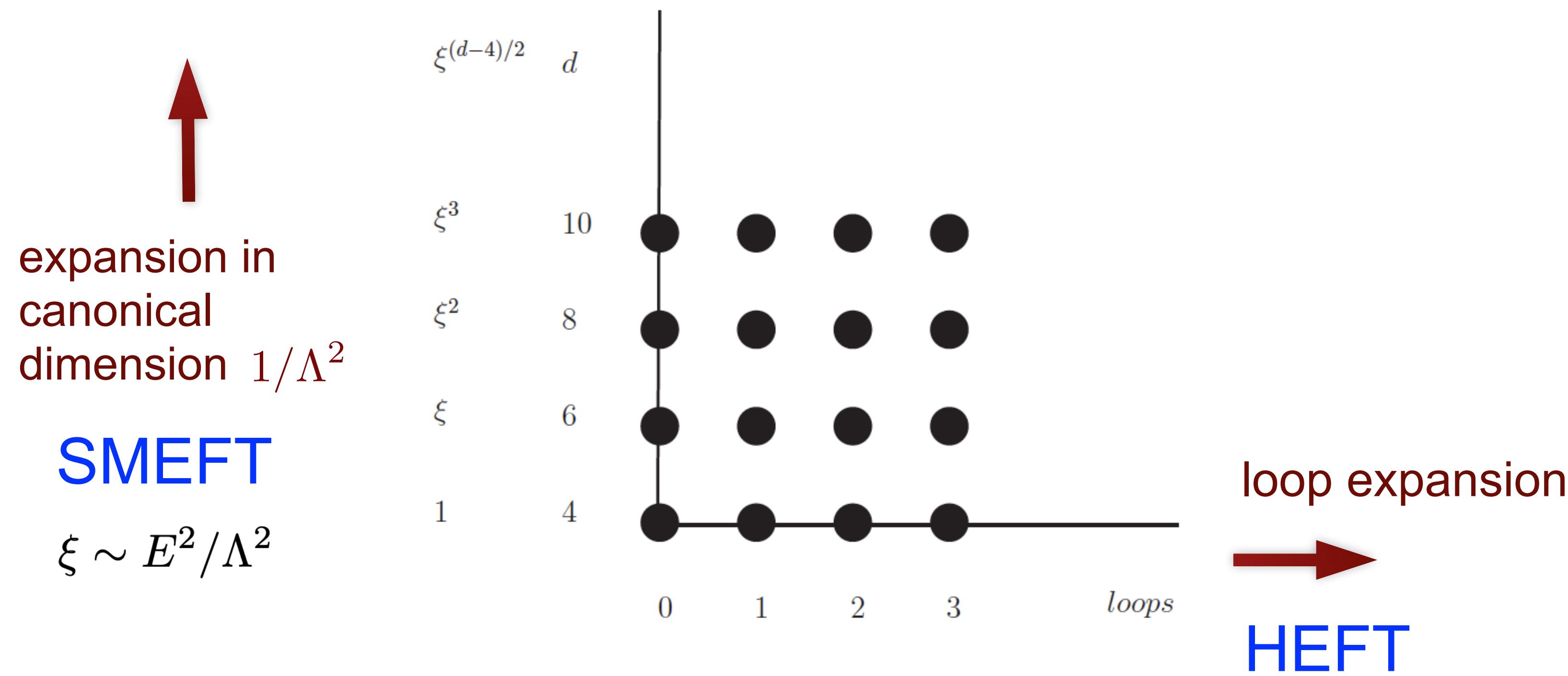
if only canonical dimension is counted, (b) - (g) would all contribute at the same order (dim 6) !

Counting schemes

HEFT (EWChL): “loop expansion”

based on chiral dimension $d_\chi = 2L + 2$ L : “Loop”

with $d_\chi(A_\mu, \varphi, h) = 0$, $d_\chi(\partial, \bar{\psi}\psi, g, y) = 1$



SMEFT and HEFT

both respect the SM gauge symmetries

- **SMEFT:** Higgs field $\Phi(x)$ is complex doublet, transforms linearly under $SU(2) \times U(1)$

$$\Phi(x) \rightarrow \exp \left[-i\alpha^a(x) \frac{\sigma^a}{2} - i\beta(x) \frac{1}{2} \right] \Phi(x)$$

- **HEFT:** Higgs field is EW singlet

Goldstone boson fields $\pi^a(x)$, represented as $U(x) = \exp(i\pi^a(x)\sigma^a/f)$
 linear transformations on $U(x)$ act non-linearly on $\pi^a(x)$

$$U(x) \rightarrow \exp \left[-i\alpha^a(x) \frac{\sigma^a}{2} \right] U(x) \exp \left[i\beta(x) \frac{\sigma^3}{2} \right]$$