Tackling the infamous g^6 term of the QCD pressure

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based on recent work with
Pablo Navarrete

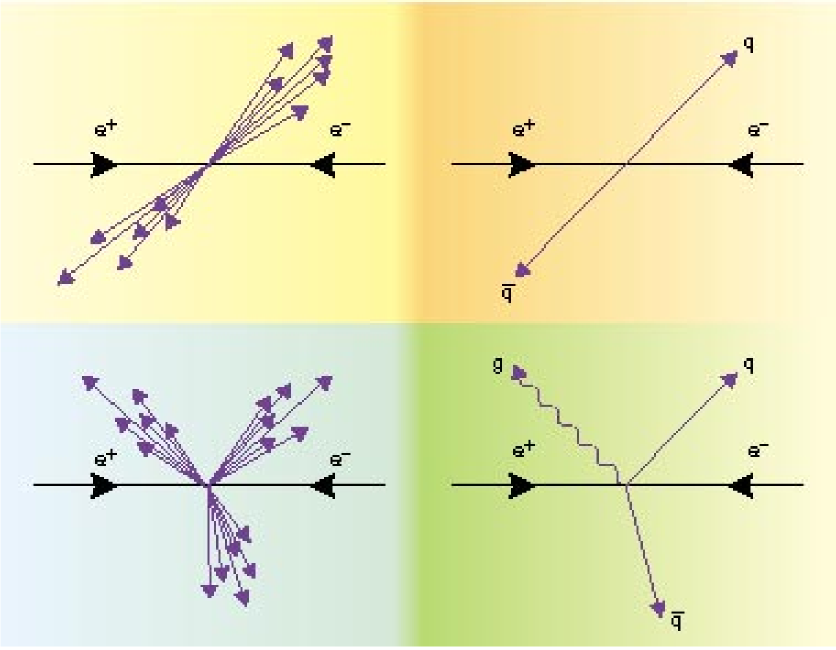
and earlier work with
A. Davydychev, I. Ghişoiu, M. Laine
(and others)

Loops and Legs 2022, Ettal

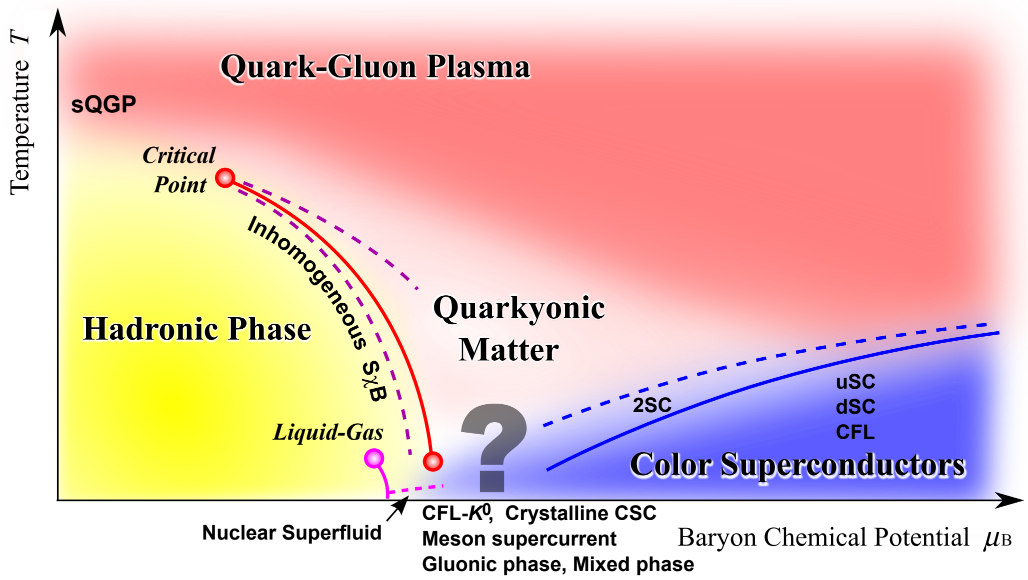
Motivation

check QCD in extreme conditions

- $E \uparrow$: collider physics



- $T \uparrow$, $\mu \uparrow$: equilibrium phase diagram



[Fukushima/Hatsuda]

- e.g. LEP, $e^+e^- \to X$
- check details of theory with jets
- nowadays: calc QCD background

- nature: early universe, n/qu stars
- $T_c \sim 170$ MeV $\sim 10\mu s$
- lab expt: SPS / RHIC / LHC HI / GSI

Setting

- Finite-temperature field theory
  - ▷ fairly mature subject; textbooks [Kapusta 89; LeBellac 00; Kapusta/Gale 06; Laine/Vuorinen 17]
  - ▷ relevant in cosmology (mostly weak int; QCD as background)
    early univ, equilibration, $T_{max} = ?$
    DM searches, relic densities
  - ▷ relevant in HIC (mainly QCD) [cf. Quark Matter 2022 meeting]
    fireball lifetime $\sim$ 10 fm/c; $T_{max} \sim 10^2$ MeV
    particle yields, jet quenching, plasma hydro

- equilibrium thermodynamics: imaginary time formalism, $t \to i\tau$
  - ▷ (grand) canonical ensemble, $Z(T,\mu) = \mathrm{Tr}[e^{-(\hat{H}-\mu\hat{N})/T}]$
  - ▷ path int quant, fields periodic: $Z = \int \mathcal{D}\phi \, e^{-\int_0^{1/T} d\tau \int d^dx \, \mathcal{L}_E}$ $\Leftarrow$ $\boxed{d = 3 - 2\varepsilon}$
  - ▷ Fourier trafo discrete; mom-space measure $T \sum_{n\in\mathbb{Z}} \int \frac{d^dp}{(2\pi)^d} \equiv \sum\!\!\!\!\!\!\int_P$
  - ▷ bosonic prop $\sim [(2n\pi T)^2 + \vec{p}^{\,2} + m^2]^{-1}$
  - ▷ Dirac prop $\sim [i\gamma_0((2n+1)\pi T + i\mu) + i\vec{\gamma}\vec{p} + m]^{-1}$

- Interplay of methods
  - ▷ QGP is strongly coupled system near $T_c \Rightarrow$ need e.g. lattice simulations
  - ▷ asymptotic freedom at high $T \Rightarrow$ weak-coupling approach in continuum
  - ▷ in general, one tries to use best of both; this talk: mostly weak-coupling

Energy scales in hot QCD

Interactions make thermal QCD a multi-scale system

- At asymptotically high T, $g \ll 1 \Rightarrow$ clean separation of 3 scales
- expansion parameter: $$g^2\, n_b(|k|) = \frac{g^2}{e^{|k|/T}-1} \overset{|k|\lesssim T}{\approx} \frac{g^2 T}{|k|}$$
- $|k| \sim \pi T / gT / g^2 T$
  aka hard/soft/ultrasoft scales
  are fully/barely/non- perturbative at high T
- no smaller momentum scales / larger length scales due to confinement

$\Rightarrow$ treatment of a multi-scale system: effective field theory !

Observable: pressure $p(T)$

- structure of strict weak-coupling expansion is non-trivial !
- $$p_{\rm QCD}(T) \equiv \lim_{V\to\infty} \frac{T}{V} \ln \int \mathcal{D}[A^a_\mu, \psi, \bar\psi] \exp\left(-\frac{1}{\hbar}\int_0^{\hbar/T} d\tau \int d^{3-2\epsilon}x\, \mathcal{L}^E_{\rm QCD}\right)$$
$$= c_0 + c_2 g^2 + c_3 g^3 + (c_4' \ln g + c_4) g^4 + c_5 g^5 + (c_6' \ln g + c_6) g^6 + \mathcal{O}(g^7)$$

[c_2 Shuryak 78, c_3 Kapusta 79, c_4' Toimela 83, c_4 Arnold/Zhai 94, c_5 Zhai/Kastening 95, Braaten/Nieto 96, c_6' KLRS 03]

- root cause of nonanalytic (in α_s) behavior well understood: above-mentioned dynamically generated scales
- clean separation best understood in effective field theory setup [here: $\mu = 0$]
  - ▷ generalizations, e.g. $\mu \neq 0$ [Vuorinen], standard model [Gynther/Vepsäläinen]

Effective theory setup: QCD $\to$ EQCD

high T: large-distance QCD dynamics contained in 3d EQCD

- integrate out hard scales $|p| \gtrsim \pi T$: ψ, $A_\mu(n \neq 0)$

$$p_{\rm QCD}(T) \equiv \frac{T}{V}\ln\int\mathcal{D}[A^a_\mu,\psi,\bar\psi]\exp\Big(-\int_0^{1/T}d\tau\int d^{3-2\epsilon}x\,\mathcal{L}^E_{\rm QCD}\Big)$$
$$= p_{\rm E}(T) + \frac{T}{V}\ln\int\mathcal{D}[A^a_k,A^a_0]\exp\Big(-\int d^{3-2\epsilon}x\,\mathcal{L}_{\rm E}\Big)$$

$$\mathcal{L}_{\rm E} = \frac{1}{2}Tr\,F^2_{kl} + Tr\,[D_k,A_0]^2 + m^2_{\rm E}\,Tr\,A_0^2 + \lambda^{(1)}_{\rm E}(Tr\,A_0^2)^2 + \lambda^{(2)}_{\rm E}\,Tr\,A_0^4 + \ldots$$

- five matching coefficients [Braaten/Nieto 95, …, Ghisoiu/Laine/Möller/YS 02-15]

$p_{\rm E} = T^4\left[\# + \#g^2 + \#g^4 + \#g^6 + \ldots\right]$, $m^2_{\rm E} = T^2\left[\#g^2 + \#g^4 + \#g^6 + \ldots\right]$,
$g^2_{\rm E} = T\left[g^2 + \#g^4 + \#g^6 + \#g^8 + \ldots\right]$, $\lambda^{(1),(2)}_{\rm E} = T\left[\#g^4 + \#g^6 + \ldots\right]$.

Effective theory setup: QCD $\to$ EQCD $\to$ MQCD

the IR of 3d EQCD is contained in 3d MQCD

- integrate out $|p| \gtrsim gT$: A_0

$$p_{\rm QCD}(T) \equiv p_{\rm E}(T) + p_{\rm M}(T) + \frac{T}{V}\ln\int\mathcal{D}[A_k^a]\exp\Big(-\int d^{3-2\epsilon}x\,\mathcal{L}_{\rm M}\Big)$$
$$\mathcal{L}_{\rm M} = \frac{1}{2}Tr\,F_{kl}^2 + \ldots$$

- two matching coefficients [Kajantie et al. 03; P. Giovannangeli 04, Laine/YS 05]

$$p_{\rm M} = Tm_{\rm E}^3\left[\# + \#\frac{g_{\rm E}^2}{m_{\rm E}} + \#\frac{g_{\rm E}^4}{m_{\rm E}^2} + \#\frac{g_{\rm E}^6}{m_{\rm E}^3} + \ldots\right],\; g_{\rm M}^2 = g_{\rm E}^2\left[1 + \#\frac{g_{\rm E}^2}{m_{\rm E}} + \#\frac{g_{\rm E}^4}{m_{\rm E}^2} + \ldots\right].$$

▷ from above LO matching, expansion parameter here is $\frac{g_{\rm E}^2}{m_{\rm E}} \sim g$

Effective theory prediction for $p(T)$

$$\frac{p_{\rm QCD}(T)}{p_{\rm SB}} = \frac{p_{\rm E}(T)}{p_{\rm SB}} + \frac{p_{\rm M}(T)}{p_{\rm SB}} + \frac{p_{\rm G}(T)}{p_{\rm SB}} \quad , \quad p_{\rm SB} = \left(16 + \frac{21}{2}N_{\rm f}\right)\frac{\pi^2 T^4}{90}$$

$$= 1 + g^2 + g^4 + g^6 + \ldots \quad \Leftarrow \text{4d QCD}$$

$$+ g^3 + g^4 + g^5 + g^6 + \ldots \quad \Leftarrow \text{3d adj H}$$

$$+ \frac{1}{p_{\rm SB}}\frac{T}{V}\ln\int\mathcal{D}[A_k^a]\exp\Big(-S_{\rm M}\Big) \quad \Leftarrow \text{3d YM}$$

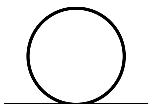
- this could be coined the physical leading-order (!) approximation
- collect contributions to $p(T)$ from all physical scales
  - weak coupling, effective field theory setup
  - faithfully adding up all Feynman diagrams
  - get long-distance input from clean lattice observable:

$$p_{\rm G}(T) \equiv \frac{T}{V}\ln\int\mathcal{D}[A_k^a]\exp\Big(-S_{\rm M}\Big) = T\#\, g_{\rm M}^6$$

only one non-perturbative (but computable!) coeff needed: 5×10^{16} flops

Brief remarks: ultrasoft contributions

- matching p_G from LAT to $\overline{\text{MS}}$ scheme needs lattice perturbation theory



$$= \int_{-\pi}^{\pi} \frac{d^3\hat{k}}{(2\pi)^3} \frac{1}{\sum_{i=1}^{3} 4\sin^2(\hat{k}_i/2) + \hat{m}^2} = \sum_{n\geq 0} \hat{m}^{2n} \left(\{\Sigma, \xi\} + \{1\}\hat{m}\right)$$

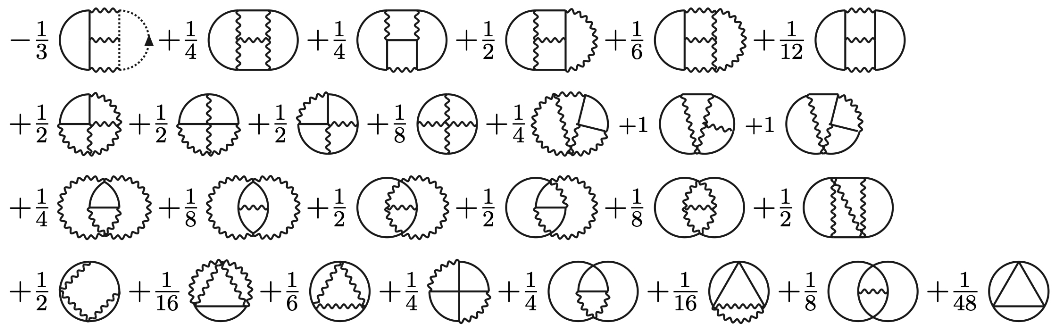
- 1loop tadpole contains elliptic integral in 3d [G.N. Watson 1939]
  - ▷ $\Sigma = 4\pi G(0) = \frac{8}{\pi}(18 + 12\sqrt{2} - 10\sqrt{3} - 7\sqrt{6})\, K^2[(2-\sqrt{3})^2(\sqrt{3}-\sqrt{2})^2]$
  - ▷ later reduced to $\Sigma = \frac{\sqrt{3}-1}{48\pi^2}\,\Gamma^2(\frac{1}{24})\,\Gamma^2(\frac{11}{24})$ [Glasser, Zucker 1977; thanx to D. Broadhurst]

- open problem: classification? very little is known systematically.

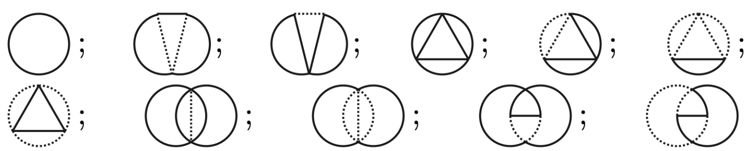
- in practice: (4-loop) Numerical Stochastic Perturbation Theory [with F. Di Renzo, 04-06]
  - ▷ no diagrams! But at fixed $N_c = 3$ only (4×10^{17} flops) $\Rightarrow$ generalization?!

Brief remarks: soft contributions

- get $p_{\rm M}$ from weak-coupling expansion in EQCD (3d adj H) [KLRS '03]
- evaluation standard: vacuum diagrams, one mass
  - ▷ 27 skeleton (2PI) diagrams contributing to 4-loop $p_{\rm M}$



  - ▷ 377 ring diags (1PI, 2PR) with selfE insertions not shown
  - ▷ reduction to 11 master integrals

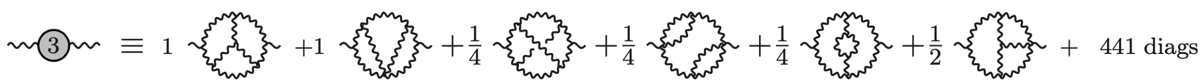


- evaluation in 3d, (ε-expansion of) all ints known analytically: MZV's
  - ▷ 3d theory is super-ren: $g_{\rm E}^2 = \mu^{-2\varepsilon}\, g_{\rm R}^2\,, \quad \lambda_{\rm E} = \mu^{-2\varepsilon}\, \lambda_{\rm R},$

$$m_{\rm E}^2 = m_{\rm R}^2 + \frac{d_{\rm A}+1}{2(4\pi)^2\,\varepsilon}\,\lambda_{\rm R}(\lambda_{\rm R} - g_{\rm R}^2 C_{\rm A}) \text{ (exact)}$$

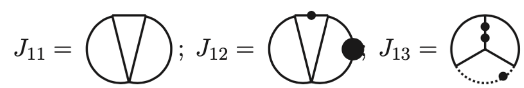
Brief remarks: matching coefficients

- to get e.g. $m_{\rm E}^2$: compare location of pole in static A_0 propagator in QCD and EQCD
- 4d QCD: $\quad 0 = P^2 + \Pi_{00}(P) \quad$ taken at $P_0 = 0$ and $|\vec{p}| = im$
  - ▷ perturbatively, $\Pi_{00}(P) = g^2\Pi_1(P) + g^4\Pi_2(P) + \ldots$
  - ▷ so $m \sim g$ small. hence $\vec{p}^{\,2} \sim g^2$ small
  - ▷ Taylor expand! $\Pi_n(P) = \Pi_n(0) + \vec{p}^{\,2}\Pi_n'(0) + \ldots$
  - ▷ all $\Pi = \Pi(0) \quad \Rightarrow \quad$ need (up to) 3-loop vacuum sum-integrals [Ghisoiu/YS '15]
- schematically:
  - ▷ 447 diags for Π



  - ▷ 10^7 vacuum sum-ints

  - ▷ 10 bosonic master sum-ints, 3 non-trivial

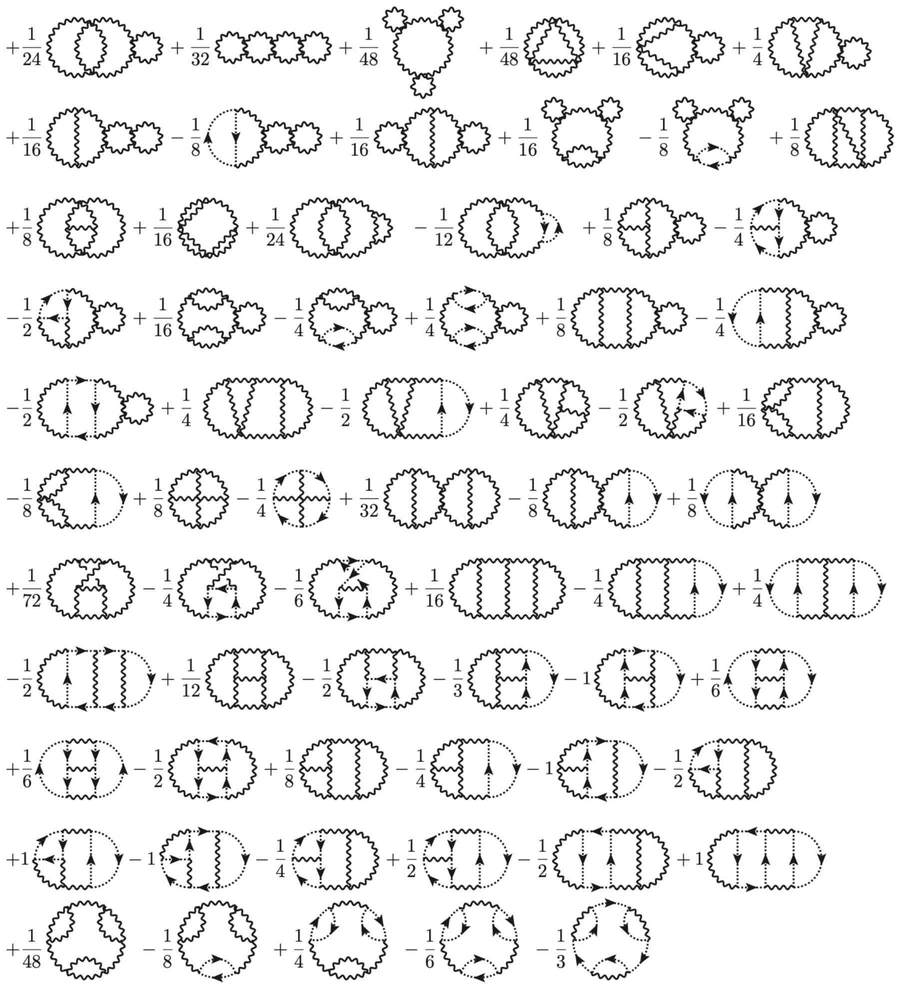


  - ▷ one example evaluation: see below
- 3d EQCD: double expansion leaves scale-free ints (no T here) $\Rightarrow 0$

Progress report: hard contributions

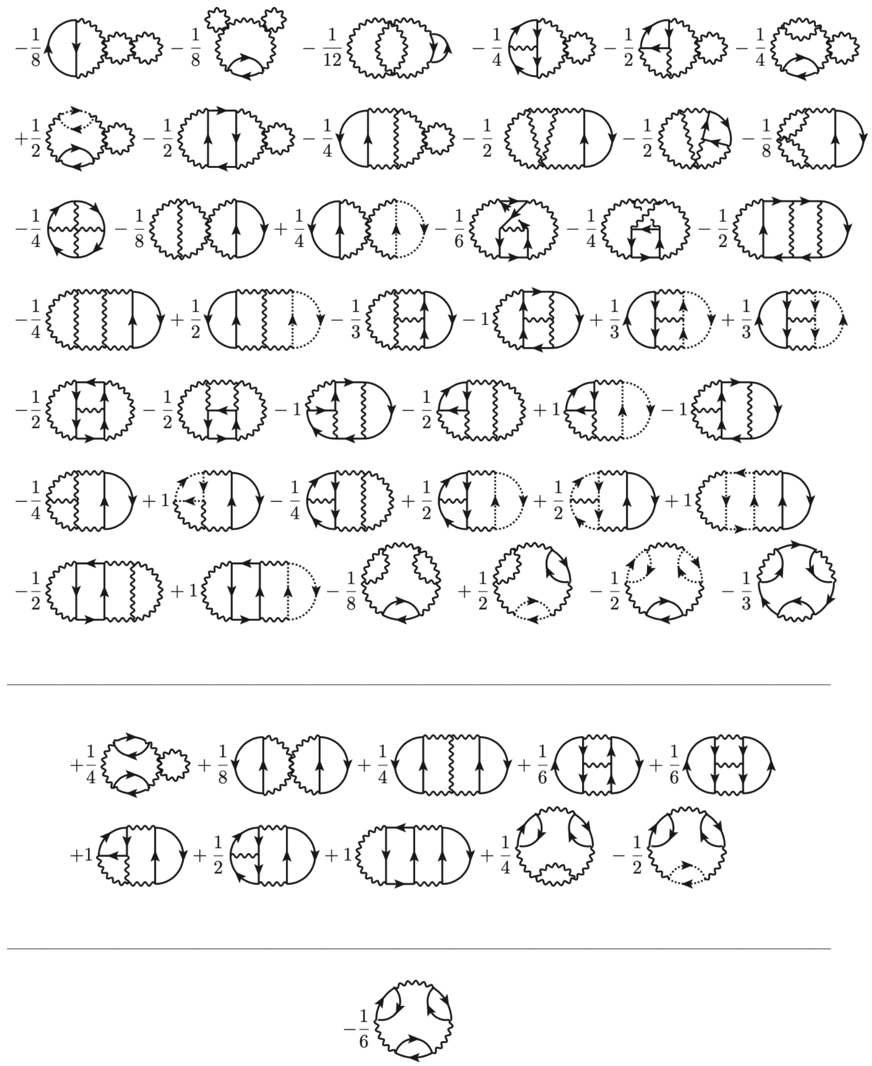
- need p_{E} to 4 loops for physical LO pressure, in (4D, hot) QCD [with Pablo Navarrete]
- look first at gauge sector; SU(N), covariant R_ξ gauge

- **QGRAF** $\longrightarrow$ 65 diags
- mapping onto family + **FORM**
- 24M $= 2^9\,6^6$ terms in hardest diag
- 176k indep sum-ints
- 25k after shifts to sector reps
- 1k after symmetries
- 21 after summing all diags
- gauge parameter drops out
- ≤ 18 after (thermal) IBP



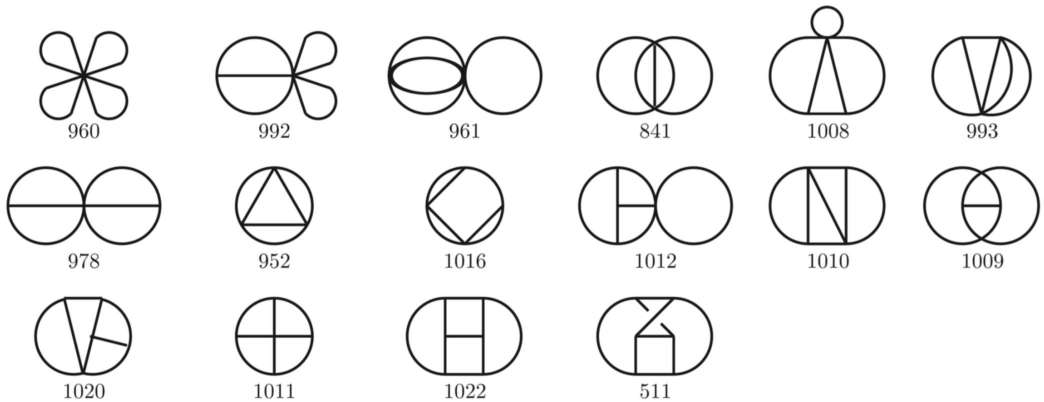
Progress report: hard contributions

- now look at fermionic sectors: $N_{\rm f}^1 \dots N_{\rm f}^3$
- strategy similar to bosonic case
  - ▷ track more indices (P_0)
- 53=42+10+1 diags
- 106k indep sum-ints
- 22k after shifts to sector reps
- 1k after symmetries
- 134 after summing all diags
- gauge parameter drops out
- $\leq$ 117 after (thermal) IBP
  - ▷ IBP still in progress
  - ▷ less powerful than at $T = 0$



Progress report: hard contributions

- all sum-ints are from the following 16 sectors:



- g.i. sets: count of # of master sum-ints per color structure

C_A^3	$N_f \{C_A^2, C_A C_F, C_F^2\}$	$N_f^2 \{C_A, C_F\}$	N_f^3
18	{67, 49, 40}	{36, 20}	10

- simplifications
  - ▷ bosonic sector 511 vanishes after color algebra
  - ▷ IBP can remove sectors 1012, 1020 completely

- but why exactly are sum-ints hard to evaluate? [see next few slides: status 1..4-loop]
  - ▷ trivial: sectors 960, 992, 978 – known analytically (in d dim)
  - ▷ doable: sector 1008 – some cases known (up to ε^0)
  - ▷ harder: sector 952 – one genuine 4-loop master known

1-loop sum-ints

- first example: LO / 1-loop bosonic tadpole

- recall $T = 0$ case: $J_\nu(m) \equiv \int \frac{d^dp}{(2\pi)^d} \frac{1}{[p^2+m^2]^\nu} = [m^2]^{d/2-\nu} \times \frac{\Gamma(\nu-d/2)}{(4\pi)^{d/2}\,\Gamma(\nu)}$

- at $T \neq 0$ therefore [writing $P^2 = P_0^2 + \vec{p}^{\,2}$ with $P_0 = 2n\pi T$, and d-dim vector $\vec{p}$]

$$I_\nu^\eta(d) \equiv \sum\!\!\!\!\!\!\int_P \frac{(P_0)^\eta}{[P^2]^\nu} = \delta_\eta J_\nu(0) + [1+(-1)^\eta]\, T \sum_{n=1}^{\infty} (2n\pi T)^\eta J_\nu(2n\pi T)$$

$$= 0 + \frac{[1+(-1)^\eta]\, T\, \zeta(2\nu-\eta-d)}{(2\pi T)^{2\nu-\eta-d}} \frac{\Gamma(\nu-\frac{d}{2})}{(4\pi)^{d/2}\,\Gamma(\nu)}$$

  ▷ note that 'thermal part' has the form $\zeta(n_{\text{even}} - d)$

- massless sum-integral ⇔ massive (T=0) integral

- relevance: free E, selfE's, Debye screening masses, etc.

  ▷ example: blackbody radiation / Stefan-Boltzmann law at LO; $f \sim I_1^2(3) = -\frac{\pi^2 T^4}{30}$

2-loop sum-ints

- next step: NLO / 2-loop
  - ▷ a number of worked-out examples in the literature
  - ▷ general observation: factorization; confirmed by (thermal adaptation) of IBP
  - ▷ have a constructive proof of 2-loop factorization [for bos, $m = \mu = 0$, with A.Davydychev]
  - ▷ recall from 1-loop: massless sum-integral ⇔ massive (T=0) integral
- define massive 2-loop vacuum integral in d dimensions [we are interested in $d = 3 - 2\varepsilon$]

$$B^{\nu_1,\nu_2,\nu_3}_{m_1,m_2,m_3} \equiv \int\frac{d^dp}{(2\pi)^d}\int\frac{d^dq}{(2\pi)^d}\frac{1}{[m_1^2+p^2]^{\nu_1}\,[m_2^2+q^2]^{\nu_2}\,[m_3^2+(p-q)^2]^{\nu_3}}$$

- define massless bosonic 2-loop vacuum sum-integral

$$L^{\eta_1,\eta_2,\eta_3}_{\nu_1,\nu_2,\nu_3} \equiv \sum\!\!\!\!\!\!\!\int_{P,Q}\frac{(P_0)^{\eta_1}\,(Q_0)^{\eta_2}\,(P_0-Q_0)^{\eta_3}}{[P^2]^{\nu_1}\,[Q^2]^{\nu_2}\,[(P-Q)^2]^{\nu_3}} \sim \sum_{n_1,n_2\in\mathbb{Z}} n_1^{\eta_1}\,n_2^{\eta_2}\,(n_1-n_2)^{\eta_3}\,B^{\nu_1,\nu_2,\nu_3}_{n_1,n_2,n_1-n_2}$$

- remaining task: do double sum over known analytic result for B [Davydychev/Tausk 1992]
  - ▷ known result is in terms of Appell's hypergeometric function F_4
  - ▷ not practical: four infinite sums
- can do (much) better: 'masses' are linearly related ⇒ finite sums
  - ▷ examine B from scratch, at special kinematic point

2-loop sum-ints: Continuum integral B

- symms: need 2-loop massive vacuum integral $B^{\nu_1,\nu_2,\nu_3}_{m_1,m_2,m_3}$ at $\boxed{m_3 = m_1 + m_2}$ (all $m_i > 0$)
- note: under this constraint, Källén fct $\lambda(m_1^2, m_2^2, m_3^2) = 0$
- this leads to simple recurrences (IBP and dimensional) [extracted from Tarasov 1997]

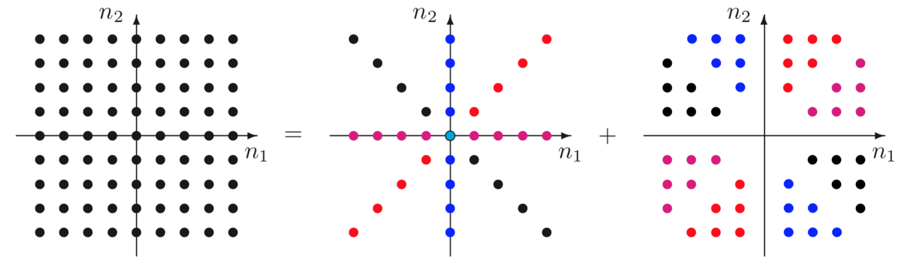
$$
\begin{aligned}
2uB^{\nu_1\nu_2\nu_3}(d) &= \left\{\frac{\mathbb{1}^-}{m_1}\left[\frac{c+\nu_2}{m_2}-\frac{c+\nu_3}{m_3}\right]+\frac{\mathbb{2}^-}{m_2}\left[\frac{c+\nu_1}{m_1}-\frac{c+\nu_3}{m_3}\right]+\frac{\mathbb{3}^-}{m_3}\left[\frac{c-\nu_1}{m_1}+\frac{c-\nu_2}{m_2}\right]\right\}B^{\nu_1\nu_2\nu_3}(d) \\
2uB^{\nu_1\nu_2\nu_3}(d) &= \frac{\lambda(\mathbb{1}^-,\mathbb{2}^-,\mathbb{3}^-)}{8\pi^2(d-2)}\,B^{\nu_1\nu_2\nu_3}(d-2) \\
\lambda(a,b,c) &= a^2+b^2+c^2-2(ab+bc+ca)
\end{aligned}
$$

[$u \equiv d + 3 - 2\nu$ and $c \equiv d + 2 - \nu$ as well as $\nu = \nu_1 + \nu_2 + \nu_3$]

- important: IBP rel asserts that B is <u>polynomial</u> in masses; allows to tackle sums
  - ▷ structure of above rels allow for closed solution (in terms finite sums)

2-loop sum-ints: back to sum-integral L

- perform the remaining (Matsubara) double sums
  - ▷ decompose double-sum into sectors where 'masses' are always positive
  - ▷ use symms of B for mapping



$$L^{\eta_1,\eta_2,\eta_3}_{\nu_1,\nu_2,\nu_3} \ni \sum_{n_1>n_2>0} n_1^{\eta_a} n_2^{\eta_b} (n_1+n_2)^{\eta_c} B^{\nu_a,\nu_b,\nu_c}_{n_1,n_2,n_1+n_2}$$

- can prove that the sums combine to
  - ▷ evaluate to single and double zeta values only
  - ▷ cancel all MZVs $\zeta(i,j)$ and single Zetas after shuffles $[\zeta(a,b)+\zeta(b,a)=\zeta(a)\zeta(b)-\zeta(a+b)]$
  - ▷ leave us with products $\zeta(i)\,\zeta(j)$ containing only $\zeta(n_{\rm even}-d)$: 1-loop sum-ints!

- obtain final result in factorized form [Davydychev/YS]

$$L^{\eta_1,\eta_2,\eta_3}_{\nu_1,\nu_2,\nu_3} = \sum rat(d)\, I^{\eta_4}_{\nu_4}\, I^{\eta_5}_{\nu_5}$$

  - ▷ for example $L^{000}_{111}=0$; or $L^{220}_{311} = -\frac{(d-4)(d^2-8d+19)}{4(d-7)(d-5)}\, I^0_2\, I^0_1$; etc.

3-loop sum-ints

- various cases of interest have been evaluated [Arnold/Zhai '95; Ghisoiu/YS '12-15]
  - ▷ disentangling (sub-)divergences, IR and UV by subtractions
  - ▷ subtraction terms typically contain limits of selfE fcts
  - ▷ obtain divergent terms analytically, finite terms numerically

- example: a non-trivial 3-loop master sum-integral

$$
\begin{aligned}
V_1 &= \sum\!\!\!\!\!\!\!\int_P \sum\!\!\!\!\!\!\!\int_Q \sum\!\!\!\!\!\!\!\int_R \frac{1}{P^2\,[Q^2]^2\,(Q-P)^2\,R^2\,(R-P)^2} \\
&= \frac{1}{(4\pi)^6}\left(\frac{e^{\gamma_{\rm E}}}{4\pi T^2}\right)^{3\epsilon}\frac{1}{6\,\epsilon^3}\Bigg[1+3\,\epsilon+\left(13-3\zeta_3+\frac{9}{2}\,\zeta_2-6\Big(\gamma_{\rm E}^2+2\gamma_1\Big)\right)\epsilon^2 \\
&\quad +\Bigg(51-42(\gamma_{\rm E}^2+2\gamma_1)+24\zeta_2\left(\frac{19}{16}+\ln(2\pi)-12\ln G\right)+2\ln 2\Big(12-12\gamma_{\rm E}^2-24\gamma_1-\zeta_3\Big) \\
&\quad +6\gamma_{\rm E}\Big(3\zeta_3-4-4\gamma_1\Big)-36\gamma_2+\frac{25}{2}\,\zeta_3-16\zeta_3'+6\,c_1+6\,c_2+6\,c_3\Bigg)\epsilon^3+\mathcal{O}(\epsilon^4)\Bigg] \\
c_2 &= \sum_{n=1}^{\infty}\int_0^\infty\!{\rm d}x\,\frac{2\,e^{-x}}{n}\left[e^x{\rm Ei}(-x)+\gamma_{\rm E}+\ln\frac{x}{4n^2}\right]\times \\
&\quad \times\left[\psi(n+1)+e^x B(e^{-x/n},n{+}1,0)+e^x{\rm Ei}(-x)-\ln(1{-}e^{-x/n})+\ln\frac{x}{n^2}-\frac{x}{12n^2}\right] \\
&\approx -3.2020672566(1)
\end{aligned}
$$

4-loop sum-ints

- a single one has already been computed
  - ▷ following 3-loop evaluation strategy
    - ▷ disentangled (sub-)divergences by hand
    - ▷ constant term only numerically
  - ▷ gave the g^6 term in scalar ϕ^4 [GLSTV 08]

$$\sum\!\!\!\!\!\!\!\int_{PQRS} \frac{1}{P^2(P+S)^2\,Q^2(Q+S)^2\,R^2(R+S)^2} = \frac{T^4}{(4\pi)^4}\frac{1}{16\epsilon^2}\left[1+\epsilon t_{11}+\epsilon^2 t_{12}+\dots\right]$$

with $t_{11} = \frac{44}{5} - 4\gamma_E + 12\frac{\zeta'(-1)}{\zeta(-1)} - 4\zeta(3) - \zeta(2)$

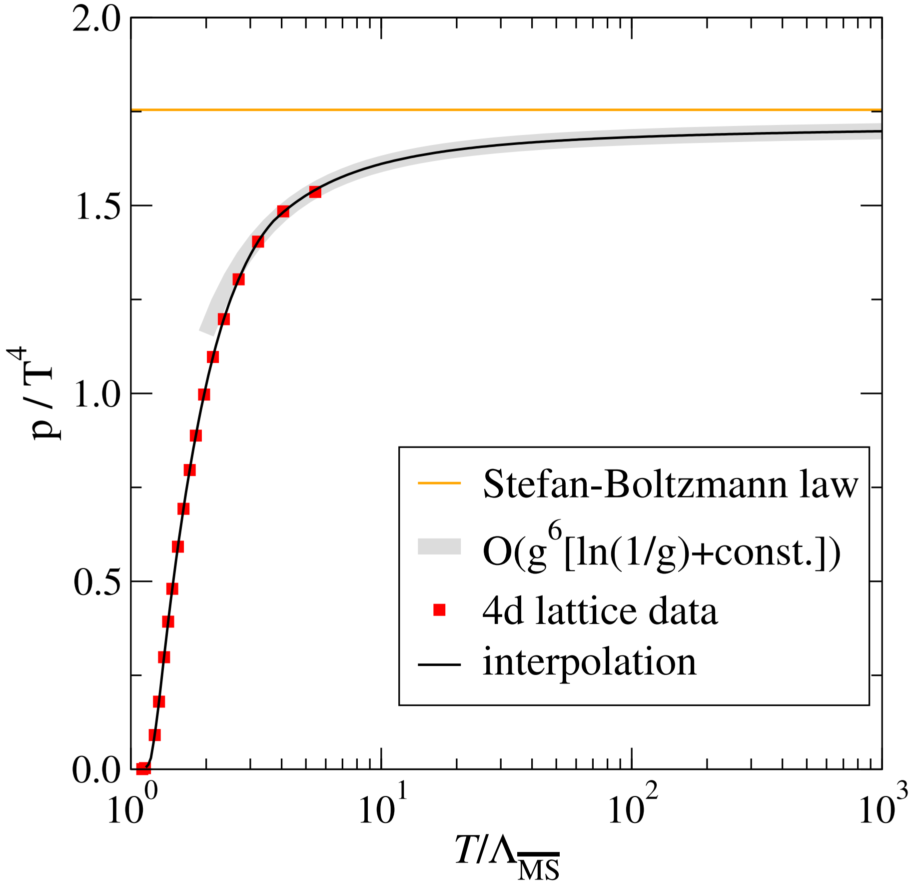
- fermionic generalization for $g^6 N_\mathrm{f}^3$ in QCD known [Gynther et al. 09]
- further progress badly needed [with Pablo Navarrete]

Summary

- thermal field theory: results phenomenologically relevant for cosmology and HIC
  - ▷ perturbative tools (by far) not as well developed/automatized as for collider physics
- nagging problem: physical LO QCD pressure not yet known!
  - ▷ for last missing piece $p_{\rm E}$, have g.i. expression, with few masters
  - ▷ NLO is then simple: entails 5-loop massive vacuum ints (in 3d)
- bottleneck: evaluation of master sum-integrals
  - ▷ 1-loop: trivial analytic soln, for general case
  - ▷ 2-loop: reducible to trivial case (aided by IBP on Kallen zero)
  - ▷ 3-loop: all physics cases computed (ε^0 piece typically numerical)
  - ▷ 4-loop [$\Leftarrow$ current challenge]: some isolated cases known (Φ^4 theory)
- interesting interplay of methods
  - ▷ lattice and continuum field theory
  - ▷ multiloop expansions, EFT, dim-6 operators
  - ▷ sum-integrals, (3d) integrals, lattice sums
- generalizations? e.g. $m_{\rm q}$, μ [Vuorinen et al.]; SYM [Strickland et al.]

Estimating $p_{\mathsf{QCD}}(T, N_f{=}0)$ at LO

while working on the open problems at physical LO ...



- fix unknown perturbative $\mathcal{O}(g^6)$ coeff
- match to lattice data [Boyd et al. 96]
  at intermediate T $\sim$ 3-5T_c
  translate via $T_c/\Lambda_{\overline{\rm MS}} \approx 1.20$
- precision on $\mathcal{O}(g^6)$ coeff?
  data to $1000T_c$
  [Wuppertal group 12])