Selected successes of sector-improved residue subtraction

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TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

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Loops & Legs 2022, 26th April 2022

Dramatic progress of recent years

ttH production at NNLO: the flavour off-diagonal channels, Catani, Stefano and Fabre, Ignacio and Grazzini, Massimiliano and Kallweit, Stefan, 2102.03256
Fully Differential Higgs Boson Production to Third Order in QCD, Chen, Gehrmann, Glover, Huss, Mistlberger and Pelloni, 2102.07607
B-hadron production in NNLO QCD: application to LHC ttbar events with leptonic decays, Czakon, Generet, Mitov and Poncelet, 2102.08267
Matching NNLO predictions to parto https://inspirehep.net/literature/1846716 sverse momentum resummation in geneva, Alioli, Bauer, Broggio, Gavardi, Kallweit, Lim, Nagar, Napoletano, Rottoli, 2102.08390
Mixed QCD-EW corrections to pp→lν+X at the LHC, Buonocore, Luca and Grazzini, Massimiliano and Kallweit, Stefan and Savoini, Chiara and Tramontano, Francesco, 2102.12539
NNLO QCD study of polarised W+W- production at the LHC, Poncelet and Popescu, 2102.13583
Next-to-next-to-leading order event generation for Z boson pair production matched to parton shower,Alioli, Broggio, Gavardi, Kallweit, Lim, Nagar, Napoletano, 2103.01214
Estimating the impact of mixed QCD-electroweak corrections on the W-mass determination at the LHC, Behring, Buccioni, Caola, Delto, Jaquier, Melnikov and Röntsch, 2103.02671
W+W- production at NNLO+PS with MINNLO_PS, Lombardi, Wiesemann and Zanderighi, 2103.12077
The pp → W(→ lν) + γ process at next-to-next-to-leading order, Campbell, De Laurentis, Ellis and Seth, 2105.00954
Exact Top-Quark Mass Dependence in Hadronic Higgs Production, Czakon, Harlander, Klappert and Niggetiedt, 2105.04436
NNLO QCD corrections to diphoton production with an additional jet at the LHC, Chawdhry, Czakon, Mitov and Poncelet, 2105.06940
A comparative study of Higgs boson production from vector-boson fusion, Buckley et al., 2105.11399
Matching N3LO QCD calculations to parton showers, Prestel, 2106.03206
Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC, Czakon, Mitov and Poncelet, 2106.05331
The qT and DeltaPhi spectra in W and Z production at the LHC at N3LL'+N2LO, Ju and Sch\"onherr, 2106.11260
Mixed Strong-Electroweak Corrections to the Drell-Yan Process, Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano and Vicini, 2106.11953
Anomalous couplings in associated VH production with Higgs boson decay to massive b quarks at NNLO in QCD, Bizon, Caola, Melnikov, Röntsch, 2106.06328
Dilepton Rapidity Distribution in Drell-Yan Production to Third Order in QCD, Chen and Gehrmann, Glover, Huss, Yang and Zhu, 2107.09085
ZZ production at nNNLO+PS with MiNNLO_PS,Buonocore, Koole, Lombardi, Rottoli, Wiesemann and Zanderighi, 2108.05337
Towards NNLO+PS Matching with Sector Showers, Campbell, Höche, Li, Preuss and Skands, 2108.07133
On non-factorisable contributions to t-channel single-top production, Bronnum-Hansen, Melnikov, Quarroz and Wang, 2108.09222
Anomalous couplings in Zγ events at NNLO+PS and improving νvγ backgrounds in dark-matter searches, Lombardi, Wiesemann, Zanderighi, 2108.11315
Next-to-leading order QCD corrections to diphoton-plus-jet production through gluon fusion at the LHC, Badger, Gehrmann, Marcoli and Moodie, 2109.12003
Polarised W+j production at the LHC: a study at NNLO QCD accuracy, Pellen, Poncelet and Popescu, 2109.14336
NNLO QCD corrections to weak boson fusion Higgs boson production in the H → bbar and H → WW* → 4l decay channels, Asteriadis, Caola, Melnikov and Röntsch, 2110.02818
VH + jet production in hadron-hadron collisions up to order as^3 in perturbative QCD, Gauld, Gehrmann-De Ridder, Glover, Huss and Majer, 2110.12992
Transverse momentum distributions in low-mass Drell-Yan lepton pair production at NNLO QCD, Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Majer and Rodriguez, 2110.15839
Fiducial cross sections for the lepton-pair-plus-photon decay mode in Higgs production up to NNLO QCD, Chen, Gehrmann, Glover and Huss, 2111.02157
Lepton-pair production at hadron colliders at N3LO in QCD, Duhr and Mistlberger, 2111.10379
Impact of jet-production data on the next-to-next-to-leading-order determination of HERAPDF2.0 parton distributions, Abt et al., 2112.01120
Next-to-next-to-leading order event generation for VH production with H → bbbar decay, Zanoli, Chiesa, Re, Wiesemann and Zanderighi, 2112.04168
Top-pair production at the LHC with MiNNLO_PS, Mazzitelli, Monni, Nason, Re, Wiesemann and Zanderighi, 2112.12135
Two-loop mixed QCD-EW corrections to neutral current Drell-Yan, Armadillo, Bonciani, Devoto, Rana, Vicini, 2201.01754
Photon Fragmentation in the Antenna Subtraction Formalism, Gehrmann and Schürmann, 2201.06982
Non-local slicing approaches for NNLO QCD in MCFM, Campbell, Ellis and Seth, 2202.07738
Third order fiducial predictions for Drell-Yan at the LHC, Chen,Gehrmann, Glover, Huss, Monni, Re, Rottoli, and Torrielli, 2203.01565
Mixed QCD-electroweak corrections to dilepton production at the LHC in the high invariant mass region, Buccioni, Caola, Chawdhry, Devoto, Heller, Manteuffel, Melnikov, Röntsch and Signorile-Signorile, 2203.11237
Automation of antenna subtraction in colour space: gluonic processes, Chen, Gehrmann, Glover, Huss and Marcoli, 2203.13531
NNLO event generation for pp → Zh → l+l- bbar production in the SM effective field theory, Haisch, Scott, Wiesemann, Zanderighi, Zanoli, 2204.00663

N3LO computations

2→3 NNLO QCD

NNLO QCD + PS

Fragmentation

Mixed EW-QCD

NNLO QCD

NNLO QCD completed for 2 → 1, 2 → 2 SM processes:

- Colour singlet production: pp → H, pp → VV (available in MATRX [Grazzini'17], MCFM [Boughezal'16])
- Massive quark production: pp → ttbar (+decays) [Czakon'15], pp → bbar [Kallweit'20], single top [Campbell'17]
- Vector plus jet: pp → V+jet, pp → A + X, flavoured jets: pp → Z+b-jets, V+c-jets [NNLOJet '16-'20,Boughezal'15,Czakon'20]
- Di-jets: pp → j + X, pp → jj + X [NNLOJet '16-'20, Czakon'19]

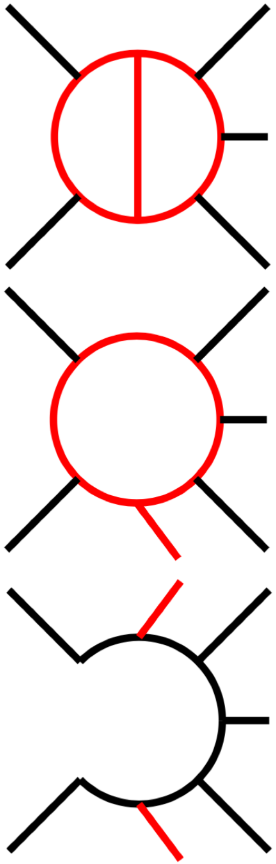
Recently first steps in the realm of 2 → 3 processes:

- Three photons [Chawdhry '19, Kallweit '20]
- Diphoton plus jet [Chawdhry '21] gg-induced @ N3LO [Badger'21]
- Three jets [Czakon '21]

Beyond fixed order:

- Dedicated resummation calculations for specific observables
- First NNLO + PS appear for colour singlet and ttbar: MiNNLOPS with MATRIX [Monni '20]
- Identified hadron production: B-hadrons in ttbar production [Czakon '21]
- Photon fragmentation [Gehrmann'21]

Requirements for two-to-three processes



2 → 3 Two-loop amplitudes:

- (Non-) planar 5 point massless 'pheno ready' [Chawdry'19'20'21,Abreu'20'21,Agarwal'21,Badger'21]
  fast progress in the last half year
  → triggered by efficient MI representation [Chicherin'20]
- 5 point with one external mass [Abreu'20,Syrrakos'20,Canko'20,Badger'21]

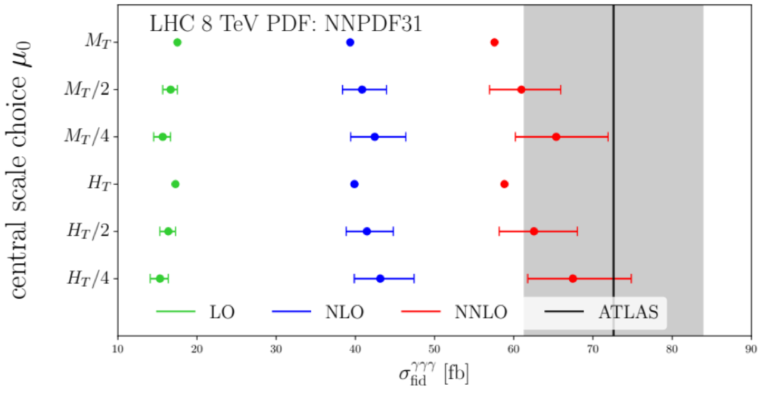
Many leg, IR stable one-loop amplitudes → OpenLoops [Buccioni'19]

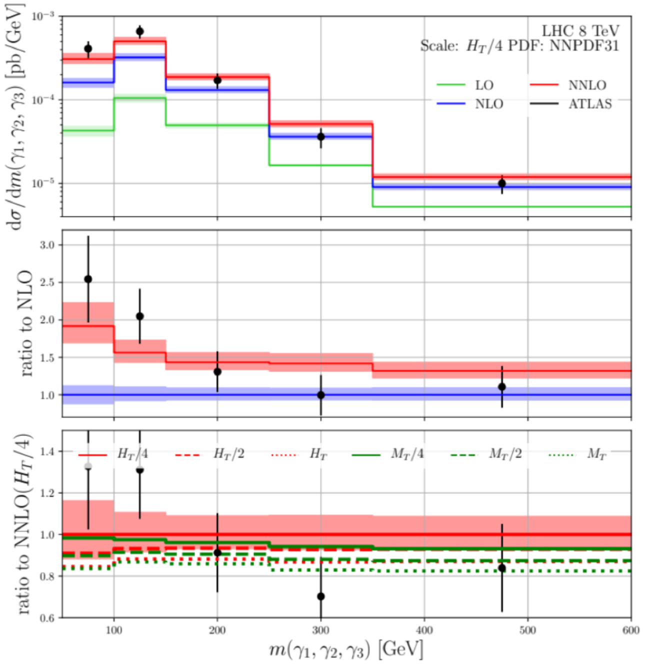
Cross sections → Combination with real radiation

- Various NNLO subtraction schemes are available:
  qT-slicing [Catain'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projection [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Sector-improved residue subtraction [Czakon'10-'14,'19]

1st example: three-photon production

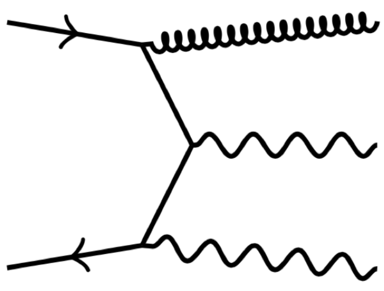
- First NNLO QCD 2 → 3 cross sections:
  NNLO QCD corrections to three-photon production at the LHC, Chawdhry, Czakon, Mitov and Poncelet, 1911.00479
  Triphoton production at hadron colliders in NNLO QCD, Kallweit, Sotnikov and Wiesemann, 2010.04681
- Simplest among the 2→3 massless cases: colour singlet
- Approximation in two-loop virtuals: only planar diagrams
  → overall small contribution
- Large NNLO/NLO K-factors
- NNLO QCD corrections essential for theory/data comparison
  Here: ATLAS



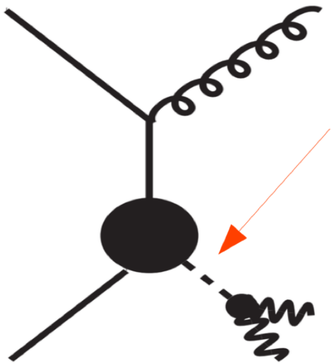


2nd example: di-photon + jet production

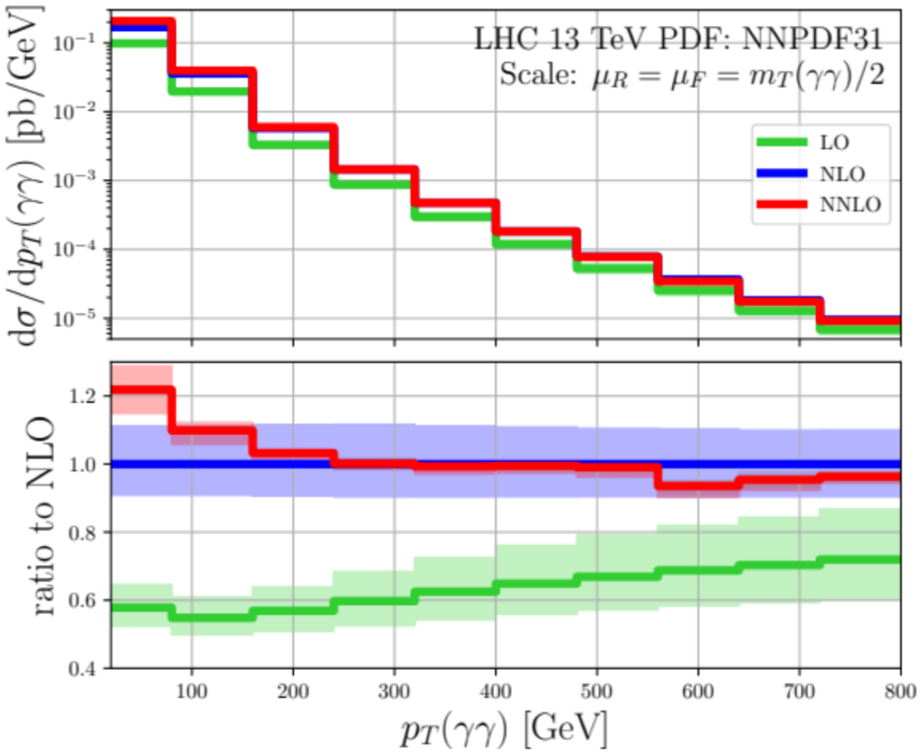
- Photon pair production @ LHC is of particular interest:
  - → Main background to cleanest Higgs decay channel
- Inclusive diphoton show large NNLO QCD corrections
  - → Perturbative convergence @ N3LO?
    First steps: [Chen's talk at RADCOR+Loopfest2021]
  - → Diphoton plus jet @ NNLO QCD ($p_T(\gamma\gamma) \to 0$ limit)
- $p_T(\gamma\gamma)$ spectrum itself interesting for Higgs → γγ :
  - → Higgs – p_T measurements resolve local Higgs couplings → BSM searches
  - → Angular diphoton observables → spin measurements



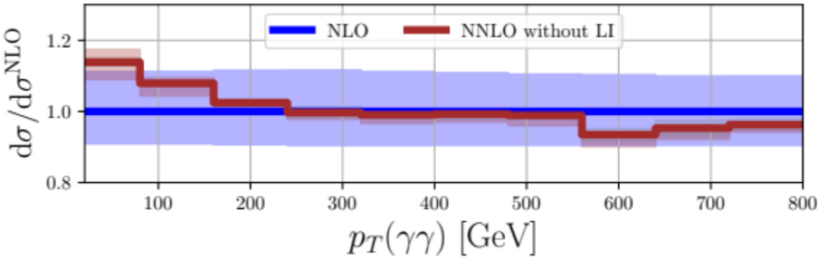
$pp \to g\gamma\gamma + X$



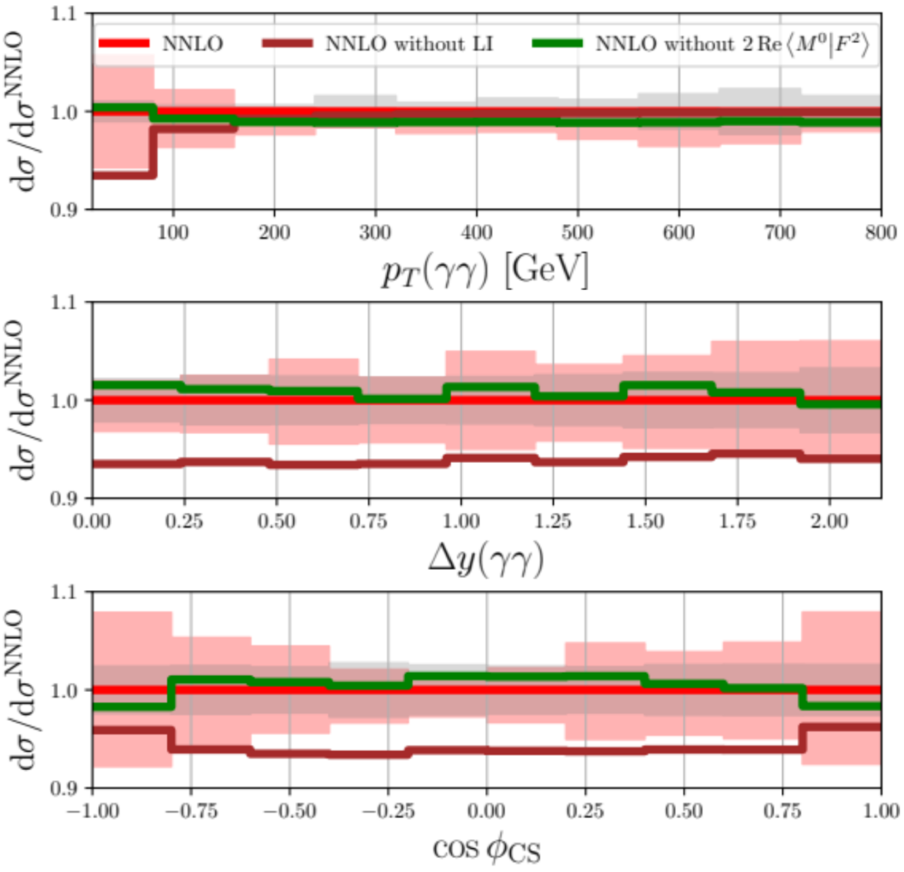
2nd example: di-photon + jet production



- Beautiful perturbative convergence
- Scale dependence:
  NLO: ~10%
  NNLO: ~1-2%
- Low p_T region:
  - ? Resummation for $p_T(\gamma\gamma)/m(\gamma\gamma) \ll 1$
  - Strong effect from the loop induced!



2nd example: di-photon + jet production



- Two-loop contribution (green line) <~1%,
- Loop induced contribution:
  - sizeable effects for low p_T vanishes for high p_T
  - flat effect in 'bulk' observables
  - Dominant source of scale dependence
  - NLO QCD correction (formally N3LO) relevant,
    ~~missing piece~~: $gg \to g\gamma\gamma$ two-loop [Badger'21]

Main achievement: three-jet production

Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC, Czakon, Mitov and Poncelet, 2106.05331

Computational challenges:

- Sector-improved residue subtraction for real radiation
  - Efficient c++ implementation → STRIPPER
  - Highly automated to deal with enormous amount of channels in three-jet production
    → O(1k) sectors →O(1M) individual MC integrals
- Many-leg, IR stable one-loop amplitudes → OpenLoops 2
- Double virtual amplitudes in leading-colour approximation
  - Sub-leading colour corrections expected to be small
  - Analytical expressions challenging
  - Fast numerical evaluation → very small contribution to computational cost
- The pure gluonic process evaluated within the NNLOJet framework:

A novel subtraction scheme for double-real radiation at NNLO, Czakon, 1005.0274
Four-dimensional formulation of the sector-improved residue subtraction scheme, Czakon and Heymes, 1408.2500
Single-jet inclusive rates with exact color at O(as^4) Czakon, van Hameren, Mitov and Poncelet, 1907.12911

OpenLoops 2, Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller, 1907.13071

Leading-color two-loop QCD corrections for three-jet production at hadron colliders, Abreu, Cordero, Ita, Klinkert, Page, Sotnikov, 2110.07541

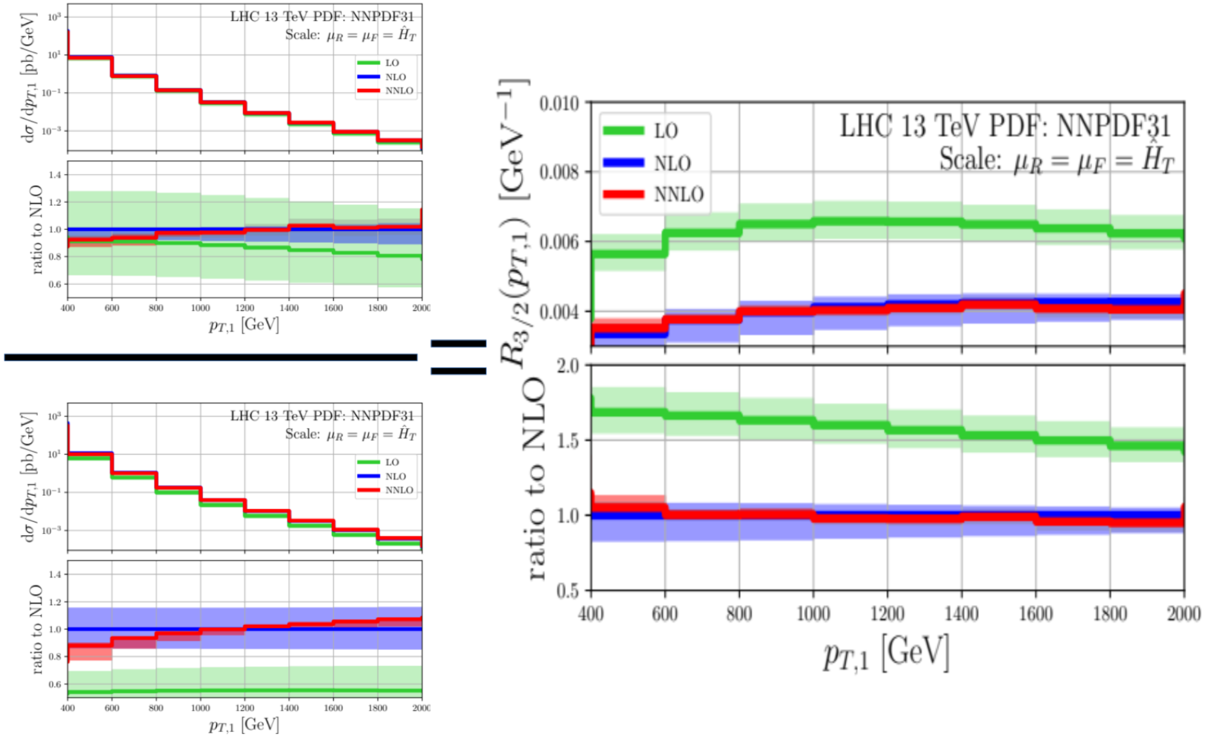
Automation of antenna subtraction in colour space: gluonic processes, Chen, Gehrmann, Glover, Huss and Marcoli, 2203.13531

Main achievement: three-jet production

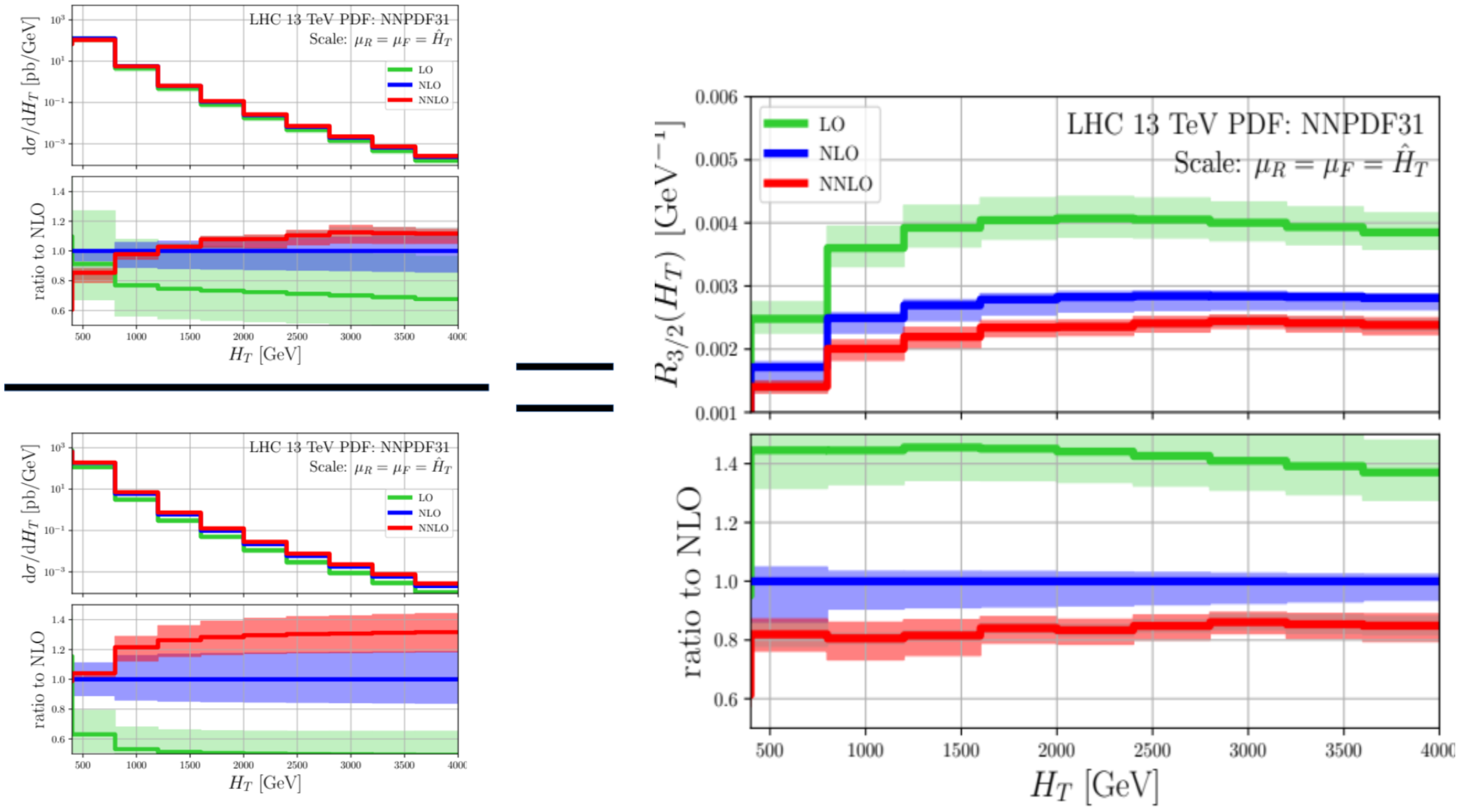
- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets:
  - $p_T(j) > 60$ GeV and $|y(j)| < 4.4$
  - $H_{T,2} = p_T(j_1) + p_T(j_2) > 250$ GeV
- Scales:

$$\mu_R = \mu_F = \hat{H}_T = \sum_{\text{partons}} p_T$$

$$R_{3/2}(X, \mu_R, \mu_F) = \frac{\mathrm{d}\sigma_3(\mu_R, \mu_F)/\mathrm{d}X}{\mathrm{d}\sigma_2(\mu_R, \mu_F)/\mathrm{d}X}$$



Main achievement: three-jet production



$$H_T = \sum_{\text{jets}} p_T$$

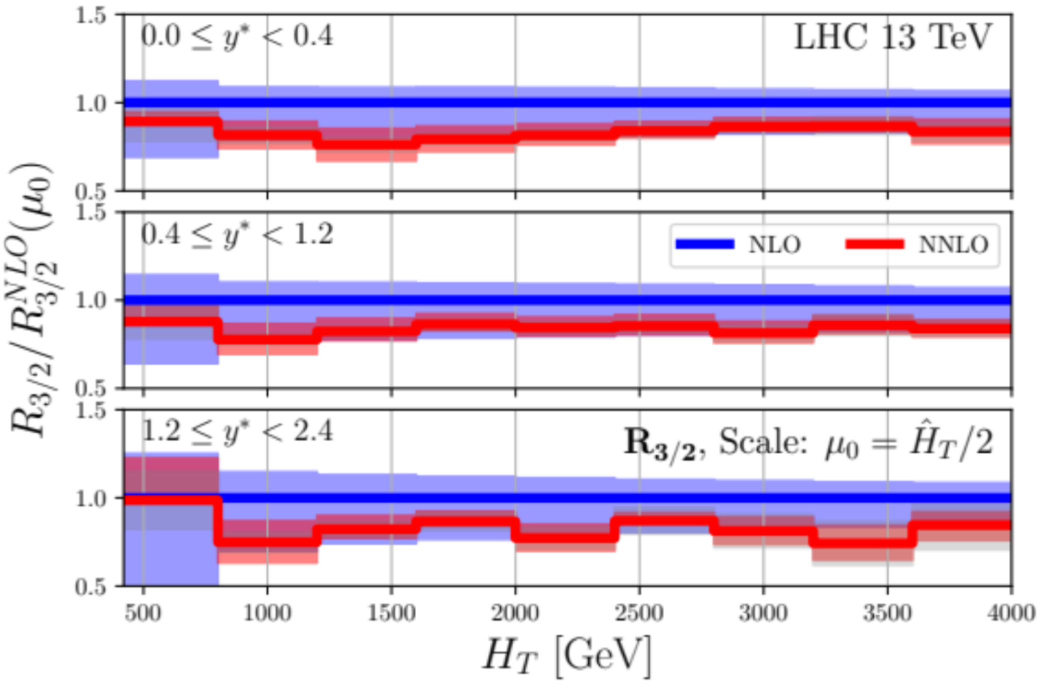
Scale dependence correlated in ratio

→ reduction of scale dependence

→ flat k-factor

→ scale bands in ratio barely overlap

Main achievement: three-jet production



Double differential w.r.t. $y^* = |y(j_1) - y(j_2)|/2$

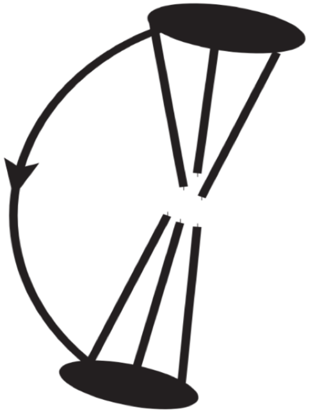
Different central scale choice: $\hat{H}_T/2$

Main achievement: three-jet production

Kinematic constraints on the azimuthal separation between the two leading jets (ϕ_{12})

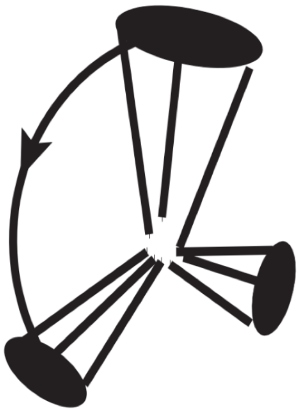
Dijet:

$\phi_{12} = \pi$



Trijet:

$\phi_{12} > \frac{2\pi}{3}$



φ12 sensitive to the jet multiplicity:

2j: $\phi_{12} = \pi$

3j: $\phi_{12} > \frac{2\pi}{3}$

4j: unconstrained

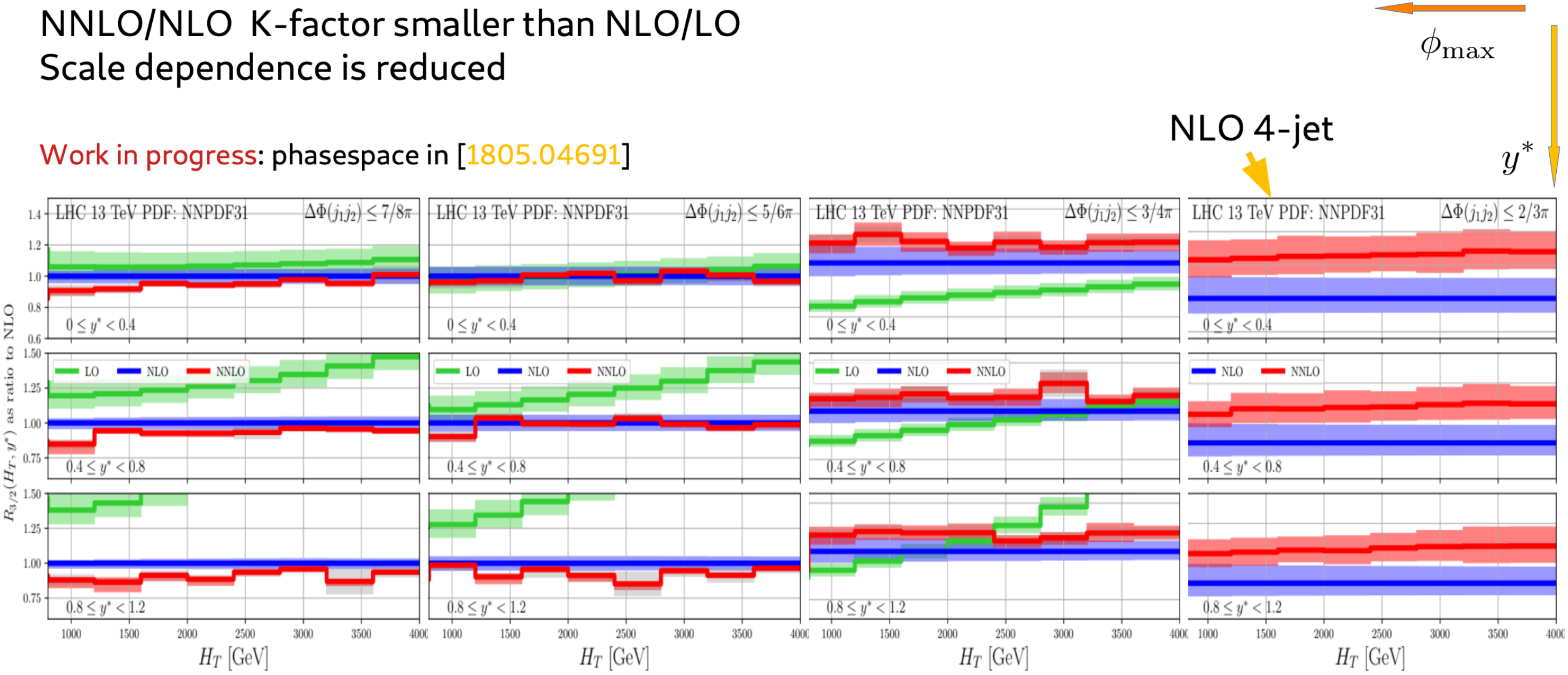
Study of the ratio:

$$R_{32}(H_T, y^*, \phi_{\max}) = \frac{\mathrm{d}\sigma_3(H_T, y^*, \phi_{12} < \phi_{\max})}{\mathrm{d}\sigma_2(H_T, y^*)}$$

Main achievement: three-jet production

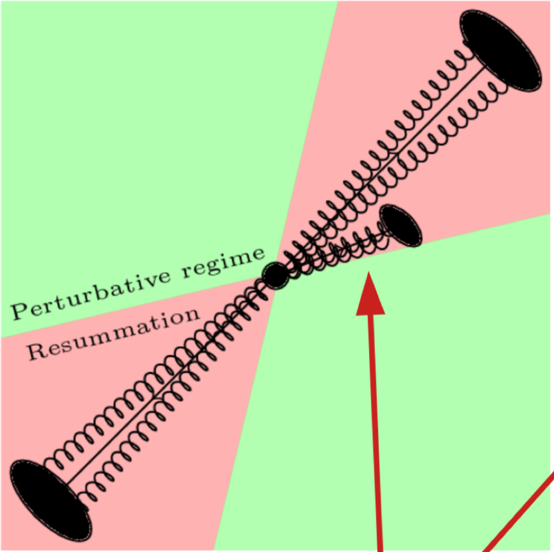
NNLO/NLO K-factor smaller than NLO/LO
Scale dependence is reduced

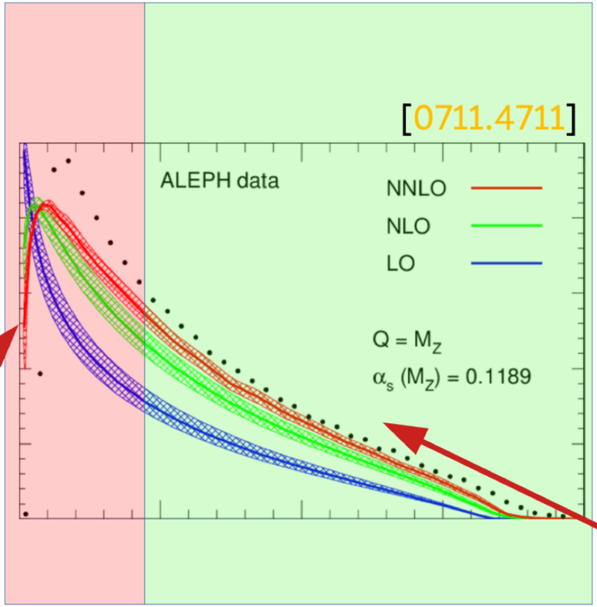
Work in progress: phasespace in [1805.04691]

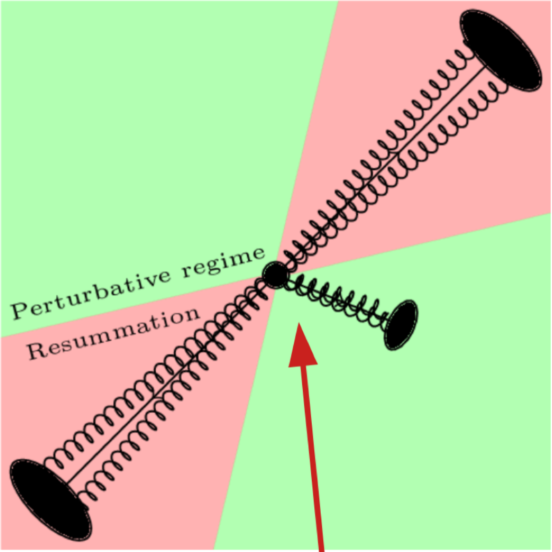


Main achievement: three-jet production

Typically event-shapes measure departure from dijet topologies







Anisotropic, dijet like

Sensitivity to resummation, non-perturbative effects

Example: 1-Thrust at LEP

Isotropic, multi-jet

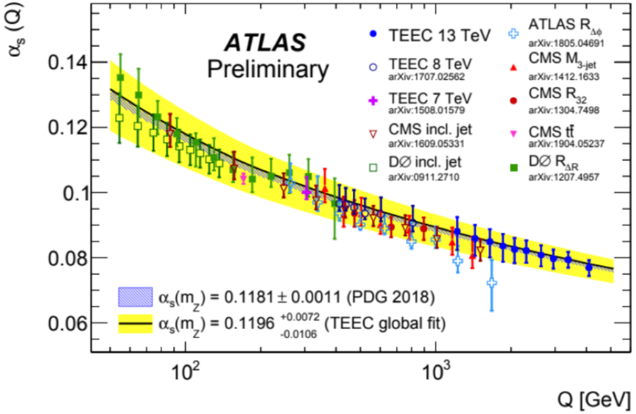
Sensitive to hard matrix elements

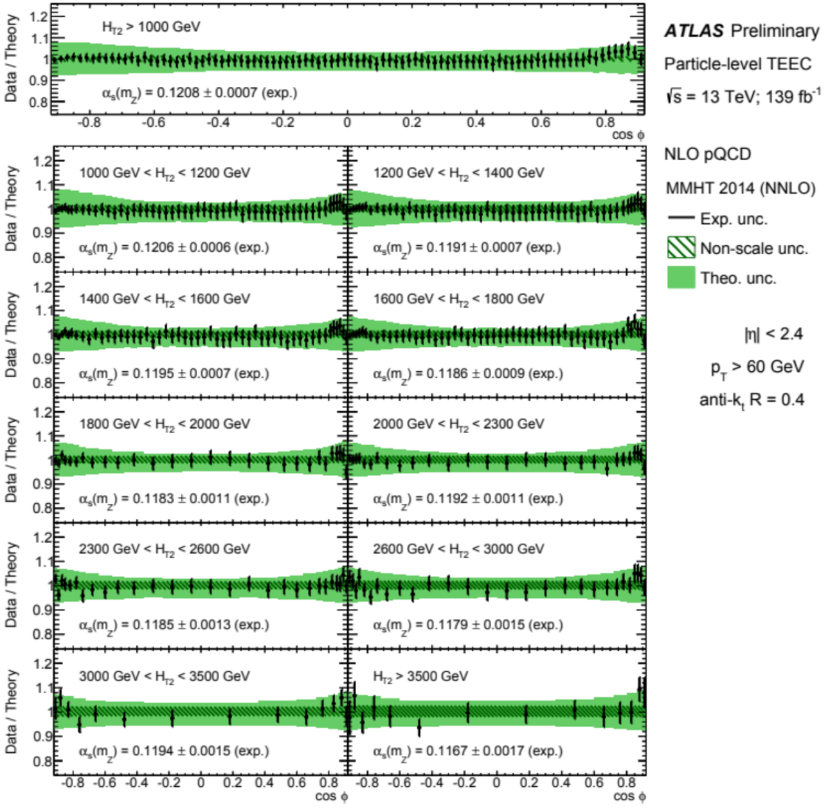
Main achievement: three-jet production

Strong coupling measurements from event-shapes:
→ three jet is leading contribution
→ normalization through dijet rates

TEEC: Transverse Energy-Energy Correlation

$$\frac{1}{\sigma}\frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi} = \frac{1}{N}\sum_{A=1}^{N}\sum_{ij}\frac{E^{A}_{\perp,i}E^{A}_{\perp,j}}{\left(\sum_k E^{A}_{T,k}\right)^2}\delta(\cos\phi - \cos\phi_{ij})$$





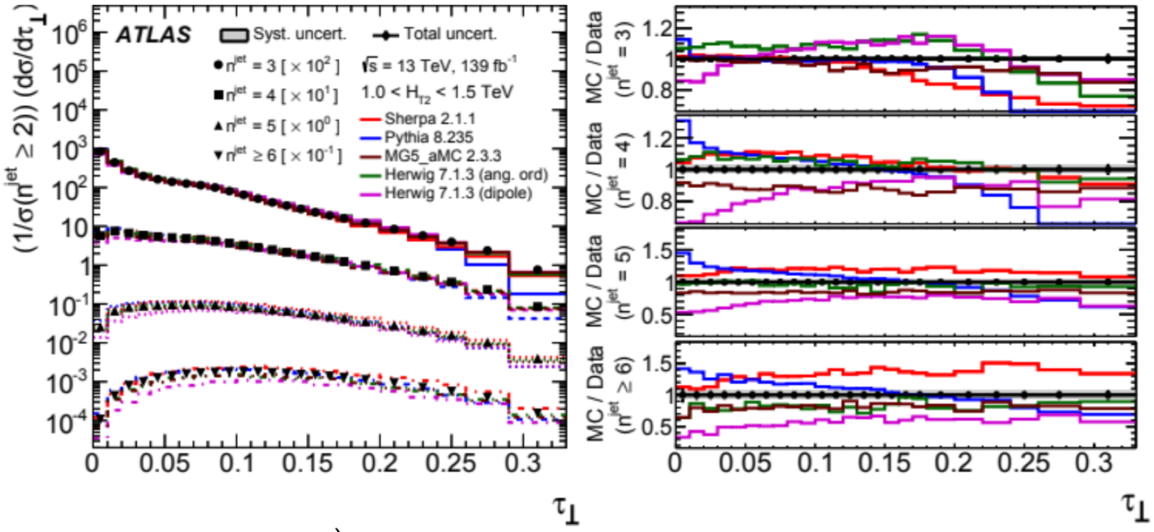
[ATLAS-CONF-2020-025]

Main achievement: three-jet production

ATLAS measurement of event shapes [2007.12600]

Transverse Thrust:

$$\tau_T = 1 - \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}|}{\sum |\vec{p}_{T,i}|}$$

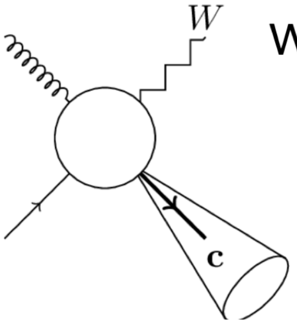


Linearised sphericity tensor:

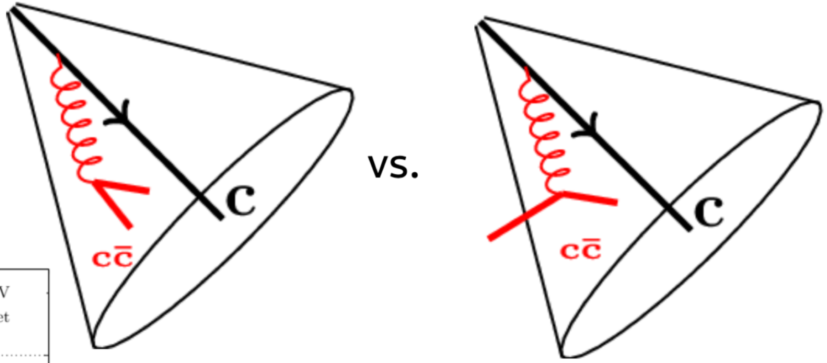
$$\mathcal{M}_{xyz} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{1}{|\vec{p}_i|} \begin{pmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} & p_{x,i}p_{z,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 & p_{y,i}p_{z,i} \\ p_{z,i}p_{x,i} & p_{z,i}p_{y,i} & p_{z,i}^2 \end{pmatrix}$$

Something different: flavor-sensitive anti-kT jets

Example:

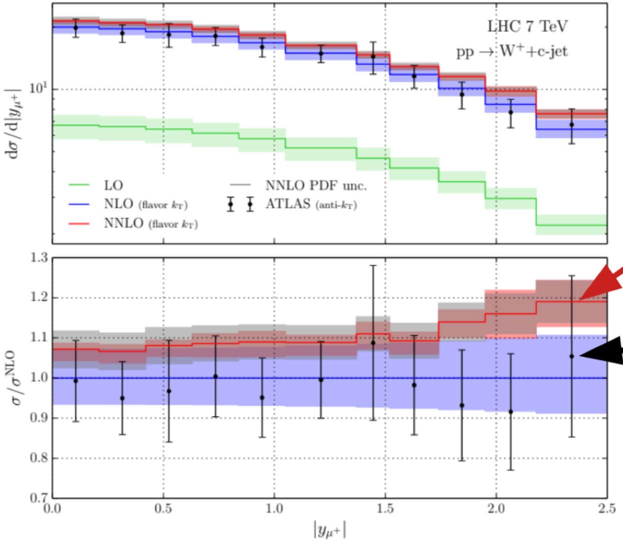


Well known problem in massless NNLO QCD [Banfi'06]:



A possible solution: change the clustering
→ Flavour – KT algorithm

[Czakon'20]



NNLO QCD with flavour kT

ATLAS data with standard anti-kT

A proper comparison would require to unfold experimental data

Something different: flavor-sensitive anti-kT jets

Anti-kT: $d_{ij} = \min(k_{T,i}^{-2}, k_{T,j}^{-2}) R_{ij}^2 \quad d_i = k_{T,i}^{-2}$

The energy ordering in anti-kT prevents correct recombination of flavoured pairs in the double soft limit.

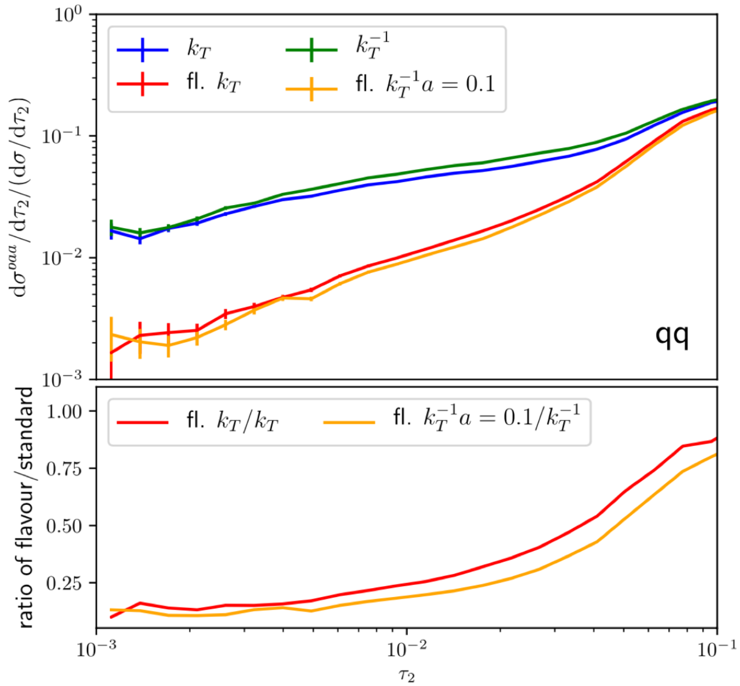
Proposed modification [to be published soon]:

A soft term designed to modify the distance of flavoured pairs.

$$d_{i,j}^{(F)} = d_{i,j} \begin{cases} \mathcal{S}_{ij} & \text{i,j is flavoured pair} \\ 1 & \text{else} \end{cases}$$

$$\mathcal{S}_{ij} = 1 - \theta(1-x)\cos\left(\frac{\pi}{2}x\right) \quad \text{with} \quad x = \frac{k_{T,i}^2 + k_{T,j}^2}{2ak_{T,\max}^2}$$

IR safety check:



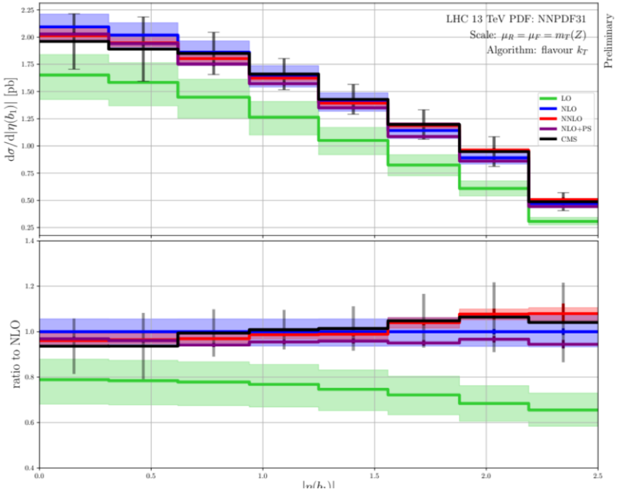
Something different: flavor-sensitive anti-kT jets

Benchmark process: pp → Z(ll) + b-jet

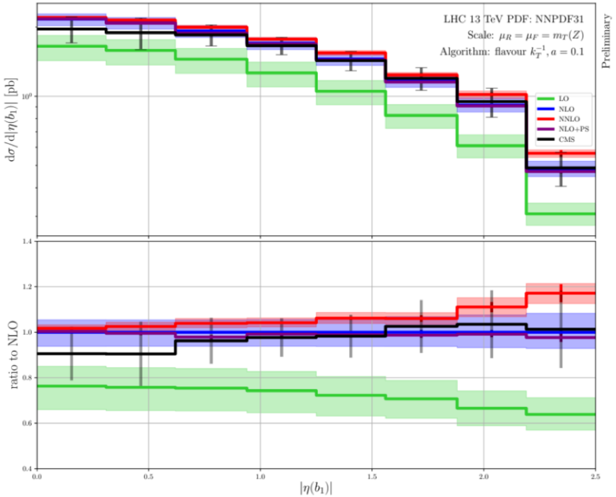
Tunable parameter a:

- Limit a → 0 <=> original anti-kT (IR unsafe)
- Large a <=> large modification of cluster sequence

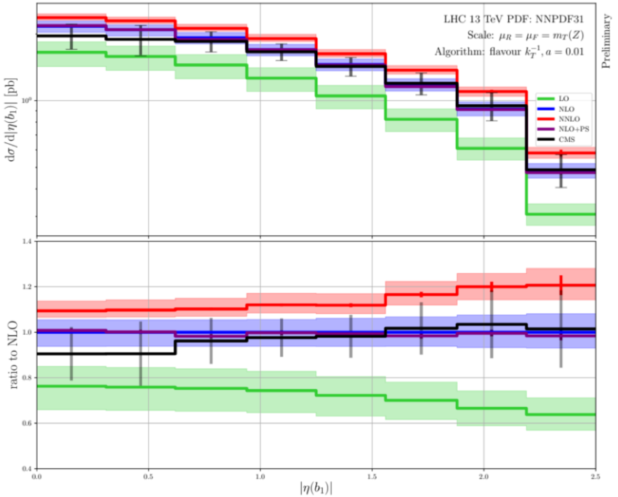
Flavour kT:



Flavour anti-kT: $a = 0.1$



Flavour anti-kT: $a = 0.01$



Beyond perturbation: fragmentation

Fixed order QCD predictions with a final state hadron

Considering partonic computation
+ transition of parton to hadron (collinear fragmentation of massless partons)

Advantage is that the hadrons momentum is measurable while the quark's is not

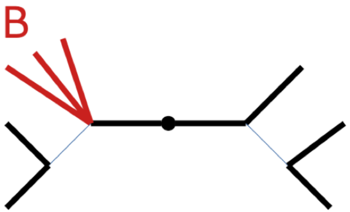
Fragmentation function (similar to PDFs)
Probability to find a hadron with a fraction x of the quarks momentum: $D_{i \to h}(x)$

No Parton-shower needed

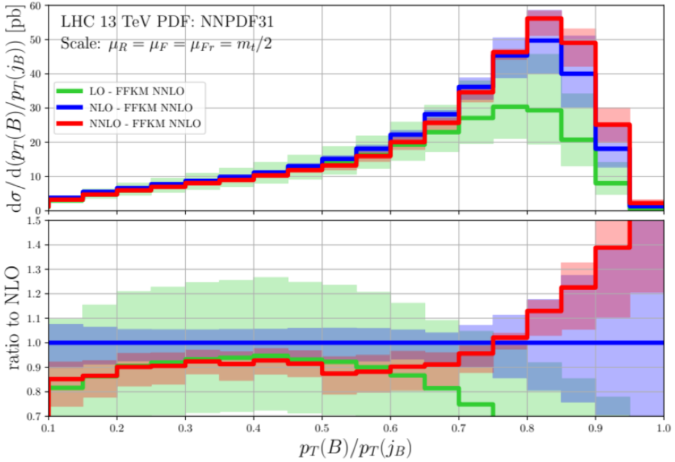
Implementation in the STRIPPER framework

Beyond perturbation: fragmentation

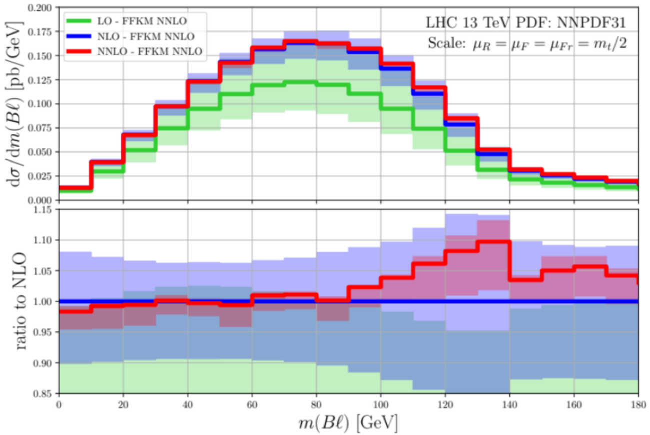
$$pp \to t\bar{t} \to B\ell\bar{\ell}\nu\bar{\nu}\bar{b} + X$$

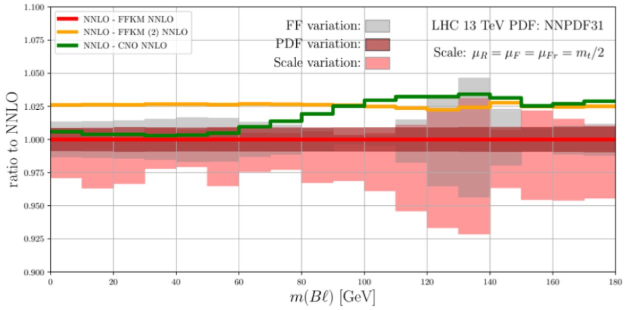


pT(B)/pT(jB): sensitive to B-hadron fraction x



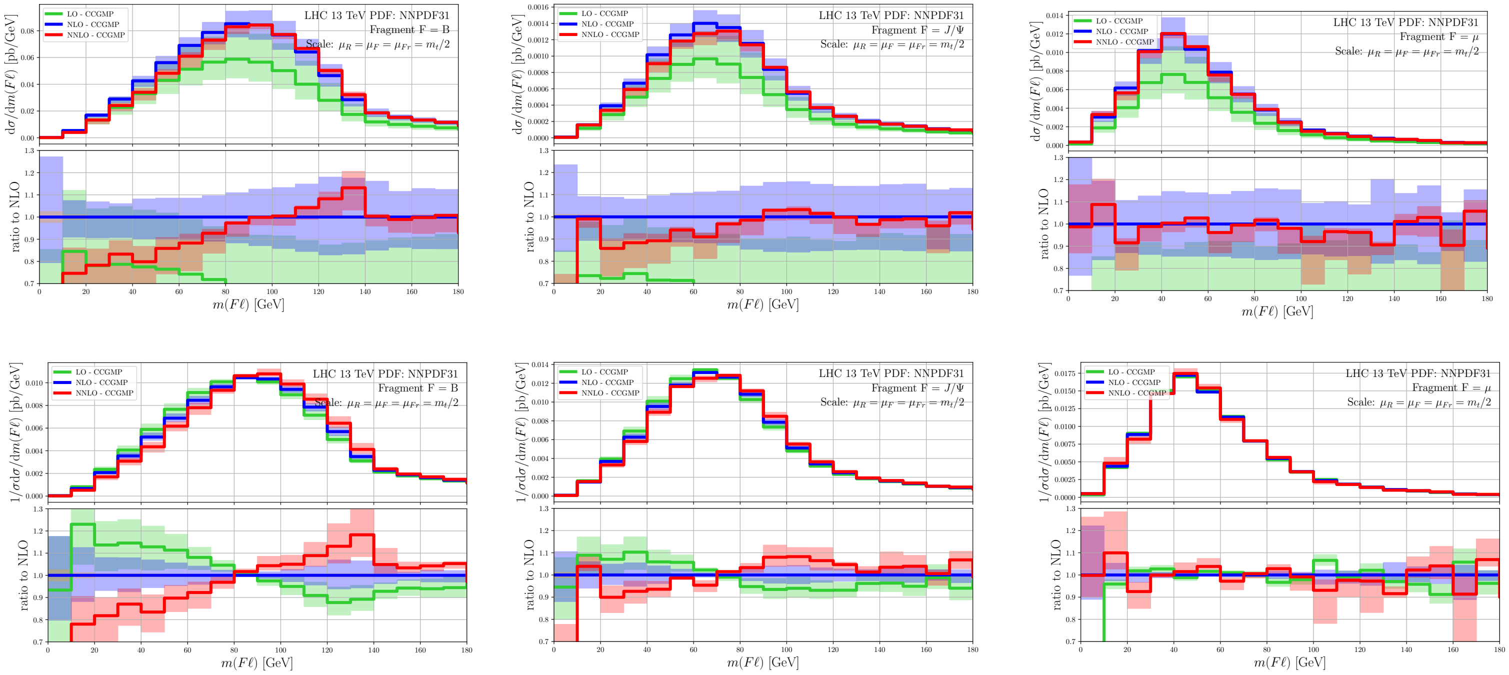
m(lB): sensitive to top-quark mass





Beyond perturbation: fragmentation

A step further and we can also describe B-hadron decays



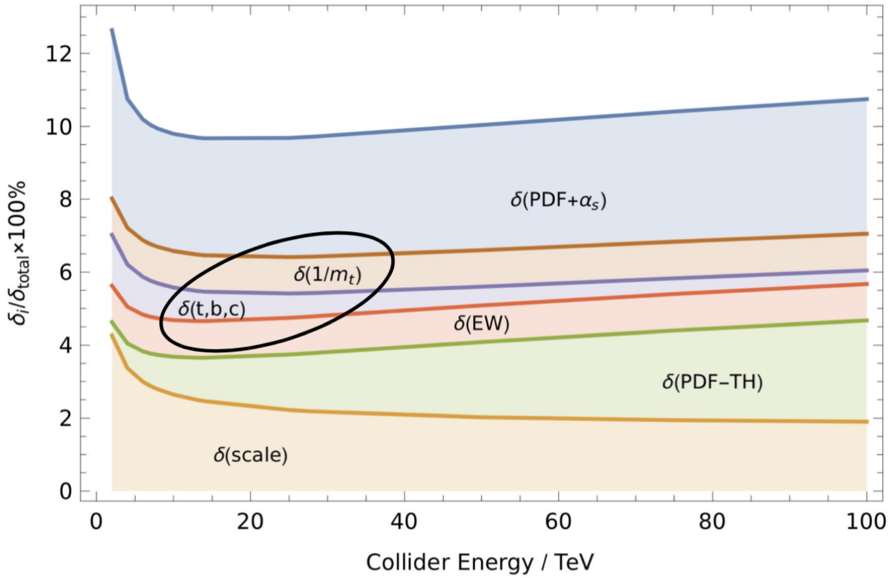
Corcella, Czakon, Generet, Mitov and Poncelet, Preliminary

Finally: a contribution to Higgs physics

Theory uncertainties

- δ(scale) and δ(PDF-TH) due to missing higher-order terms in $\hat{\sigma}$ and PDFs Anastasiou, et al. `15
- δ(trunc) has been removed Mistlberger `18
- δ(EW) was addressed recently
  Bonetti, Melnikov, Tancredi `18
  Anastasiou, del Duca, et al. `19
  Becchetti, Bonciani, et al. `21
- δ(t,b,c) and $\delta(1/m_t)$ related to quark mass effects

Higgs Physics at the HL-LHC and HE-LHC
Report from Working Group 2 on the Physics of the HL-LHC, and Perspectives at the HE-LHC `19



δ(scale)	δ(trunc)	δ(PDF-TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$
$^{+0.10}_{-1.15}$ pb	±0.18 pb	±0.56 pb	±0.49 pb	±0.40 pb	±0.49 pb
$^{+0.21\%}_{-2.37\%}$	$\pm0.37\%$	$\pm1.16\%$	$\pm1\%$	$\pm0.83\%$	$\pm1\%$

Handbook of LHC Higgs cross sections:
4. Deciphering the nature of the Higgs sector
Report of the LHC Higgs Cross Section Working Group `16

Finally: a contribution to Higgs physics

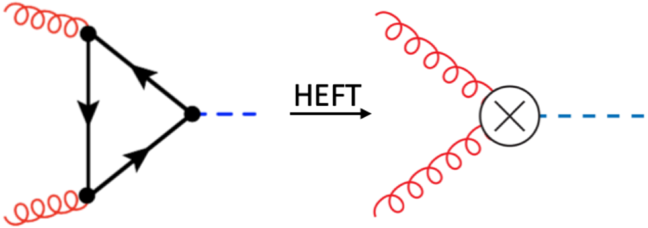
- Effects of a finite top-quark mass on the total hadronic Higgs-boson production cross section for the LHC
  - PDF set: NNPDF31_nnlo_as_0118
  - $\mu_R = \mu_F = m_H/2$
  - $M_H = 125$ GeV $\Rightarrow M_t \approx 173.055$ GeV

channel	$\sigma^{\rm NNLO}_{\rm HEFT}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma^{\rm NNLO}_{\rm exact}-\sigma^{\rm NNLO}_{\rm HEFT})$ [pb] $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$(\sigma^{\rm NNLO}_{\rm exact}/\sigma^{\rm NNLO}_{\rm HEFT}-1)$ [%]
		$\sqrt{s}=8\,\mathrm{TeV}$		
gg	$7.39+8.58+3.88$	$+0.0353$	$+0.0879\pm0.0005$	$+0.62$
qg	$0.55+0.26$	-0.1397	-0.0021 ± 0.0005	-18
qq	$0.01+0.04$	$+0.0171$	-0.0191 ± 0.0002	-4
total	$7.39+9.15+4.18$	-0.0873	$+0.0667\pm0.0007$	-0.10
		$\sqrt{s}=13\,\mathrm{TeV}$		
gg	$16.30+19.64+8.76$	$+0.0345$	$+0.2431\pm0.0020$	$+0.62$
qg	$1.49+0.84$	-0.3696	-0.0115 ± 0.0010	-16
qq	$0.02+0.10$	$+0.0322$	-0.0501 ± 0.0006	-15
total	$16.30+21.15+9.79$	-0.3029	$+0.1815\pm0.0023$	-0.26

Czakon, Harlander, Klappert, Niggetiedt `21

Finally: a contribution to Higgs physics

Contributions to σ_{tot}



$48.58\,\text{pb} =$	$16.00\,\text{pb}$	$(+32.9\%)$	(LO, rEFT)
	$+\,20.84\,\text{pb}$	$(+42.9\%)$	(NLO, rEFT)
	$-\;\;2.05\,\text{pb}$	(-4.2%)	((t,b,c), exact NLO)
	$+\;\;9.56\,\text{pb}$	$(+19.7\%)$	(NNLO, rEFT)
	$+\;\;0.34\,\text{pb}$	$(+0.7\%)$	(NNLO, $1/m_t$)
	$+\;\;2.40\,\text{pb}$	$(+4.9\%)$	(EW, QCD-EW)
	$+\;\;1.49\,\text{pb}$	$(+3.1\%)$	(N^3LO, rEFT)

"Born-improved" total cross section:

$$\sigma^{\text{HO}}_{\text{HEFT}} = \left(\frac{\sigma^{\text{HO}}}{\sigma^{\text{LO}}}\right)_{M_t\to\infty} \sigma^{\text{LO}}$$

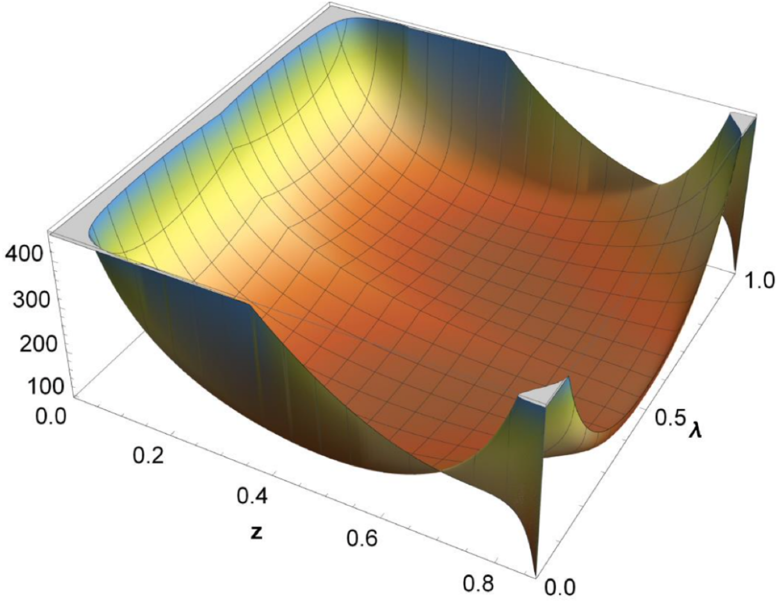
- Gluon-fusion is induced by quark loops
  - NLO result available for arbitrary quark masses — Graudenz, Spira, Zerwas `93
  - Radiative corrections beyond NLO restricted to top-loop induced terms — Anastasiou, Melnikov `02; Harlander, Kilgore `02; Ravindran, Smith, van Neerven `03
- Dominant effect of top-loop induced terms can be accounted for in HEFT approximation

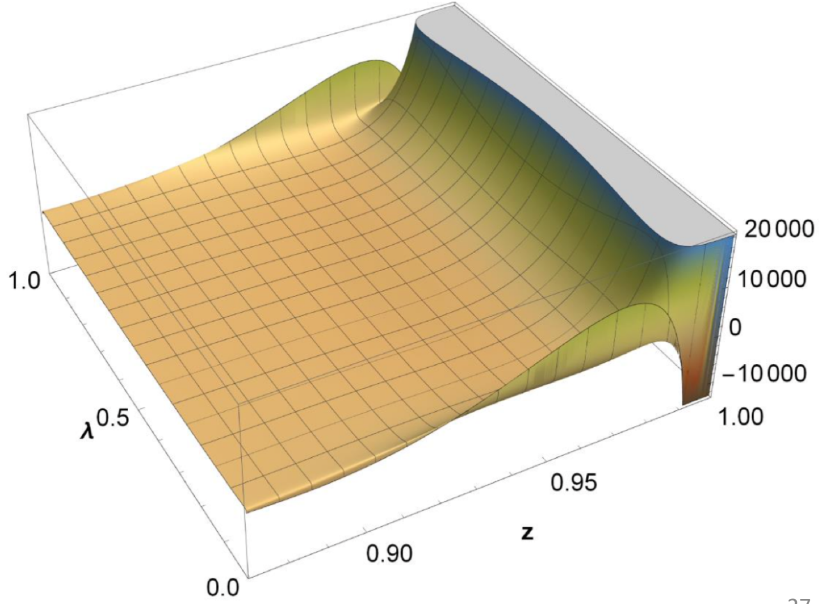
Finally: a contribution to Higgs physics

Not only thanks to STRIPPER but also to the evaluation of non-trivial amplitudes

$$\langle M^{(1)}_{\rm exact}|M^{(2)}_{\rm exact}\rangle|_{\rm regulated} \equiv \langle M^{(1)}_{\rm exact}|M^{(2)}_{\rm exact}\rangle - \left[\langle M^{(1)}_{\rm HEFT}|M^{(2)}_{\rm HEFT}\rangle + \frac{8\pi\alpha_s}{\hat{t}}\left\langle P^{(0)}_{gg}\left(\frac{\hat{s}}{\hat{s}+\hat{u}}\right)\right\rangle\langle F^{(1)}|(F^{(2)}_{\rm exact} - F^{(2)}_{\rm HEFT})\rangle\right]$$

- Real part of the regulated quantity at $\mu_R = m_H/2$:





Summary

Many non-trivial applications

The framework has demonstrated its generality

Can we obtain N3LO predictions ?