Automated calculation of Beam functions at NNLO

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with G. Bell, K. Brune and M. Wald

Loops & Legs in Quantum Field Theories
26 April, 2022

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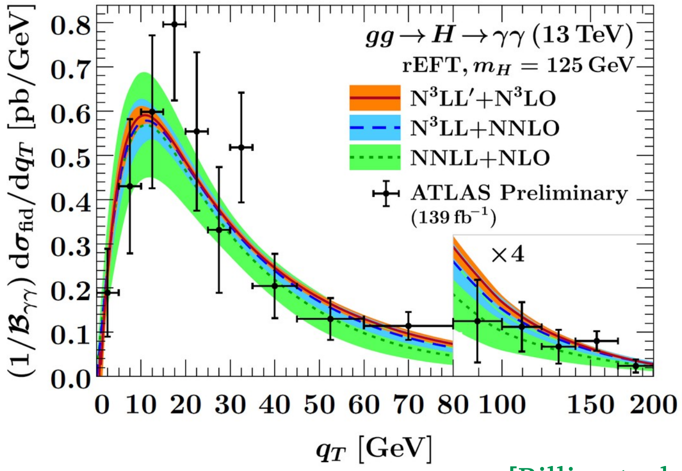
Plan

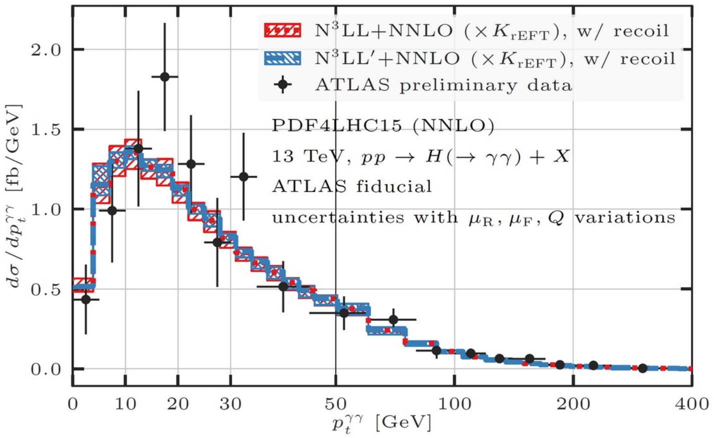
- Overview
- Automation for Beam functions
- Results
- Outlook

Motivation

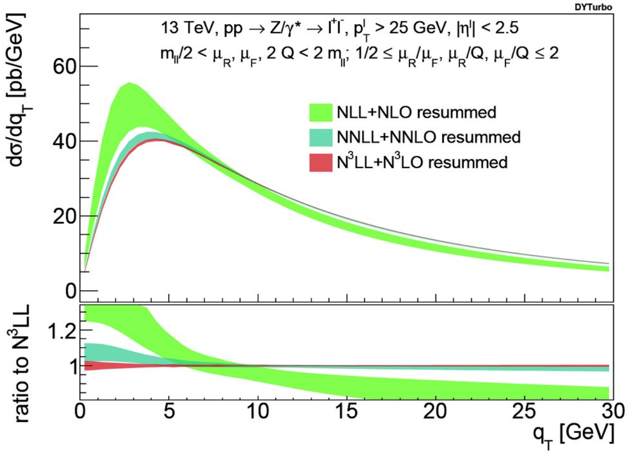
- Resummation is useful to correctly describe observables at colliders



[Billis et al `21]



[Re et al `21]



[Camarda et al `21]

- SCET has emerged as an important tool to study IR sector of QCD and resum large logarithms in a systematic framework
- The backbone relies on the underlying factorization theorems

Factorization

- Factorization in **SCET** for a wide range of observables

$$d\sigma \simeq \underbrace{H(\mu_f)}_{\text{Hard interaction}} \prod_i \underbrace{B_i(\mu_f)}_{\text{ISR}} \otimes \prod_j \underbrace{J_j(\mu_f)}_{\text{FSR}} \otimes \underbrace{S(\mu_f)}_{\text{Soft radiation}}$$

- Each function can be calculated perturbatively
- Resummation is performed by calculating them at their characteristic scales and evolving them to a common scale.

Resummation

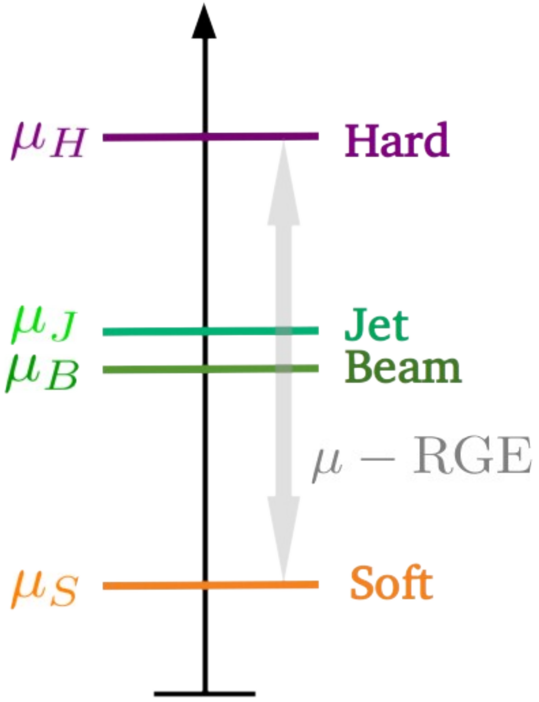
- Resummation through RGE.

- Hard function RGE :

$$\frac{d \ln H(Q,\mu)}{d \ln \mu} = \gamma_{\mathrm{H}}(Q,\mu)$$

  - Hard anomalous dimension

$$\boxed{\gamma_{\mathrm{H}}(Q,\mu) = \Gamma_{\mathrm{cusp}}(\alpha_S) \ln \frac{Q}{\mu} + \gamma^H(\alpha_S)}$$



$$H(Q,\mu) = H(Q,\mu_H) U(\mu_H,\mu)$$

Boundary term
(free from large logs)
$\mu_H \simeq Q$

Evolution kernel
(resums large logs)
$\ln^n(Q/\mu)$

$$U(\mu_H,\mu) = \exp\left[\int_{\mu_H}^{\mu} d\ln\mu' \, \gamma_{\mathrm{H}}(Q,\mu')\right]$$

Ingredients for Resummation

- Need all the anomalous dimensions & matching coefficients

$$\underbrace{\Gamma_{\rm cusp},\quad \gamma^H,\quad c_H,}_{\text{Observable-independent}}\quad \underbrace{\gamma^B,\quad c_B,\quad \gamma^J,\quad c_J,\quad \gamma^S,\quad c_S}_{\text{Observable-dependent}}$$

  - Hard anomalous dims are known to 3-loops (some cases 4-loops)

[Becher, Neubert, `10
GD, Moch, Vogt `19,`20,
Manteuffel, Panzer, Schabinger
+Huber, +Yang `20, `22
Lee, Manteuffel, Schabinger, Smirnov, Smirnov,
Steinhauser,+Huber,+Chakraborty `22]

- Beam, Jet and Soft quantities are computed on a case-by-case basis
- Need two-loop matching coefficients to achieve **NNLL′** accuracy

	$\Gamma_{\rm cusp}, \beta$	$\gamma^{H,B,J,S}$	Boundary term $(c_{H,B,J,S})$
NLL	2-loop	1-loop	1
NLL′	2-loop	1-loop	α_S
NNLL	3-loop	2-loop	α_S
NNLL′	3-loop	2-loop	α_S^2

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NNLL	3-loop	2-loop	α_S
NNLL′	3-loop	2-loop	α_S^2

This talk!

Ingredients for Resummation

- Observable-dependent anomalous dimensions & matching coefficients

$$\underbrace{\gamma^B, \quad c_B, \quad \gamma^J, \quad c_J, \quad \gamma^S, \quad c_S}_{\text{Observable-dependent}}$$

- Some can be calculated completely (semi-)analytically even at N3LL$'$

  eg. P_T, Thrust. [Luo, Yang, Zhu, Zhu `19,`20 Behring, Melnikov, Rietkerk, Tancredi, Wever `19 Ebert, Mistlberger, Vita `20]

  next talk by Arnd!

  See talk by Prasanna!

- Many are ‘difficult’ to calculate analytically, even at NNLO!
  - eg. Angularities, observables based on jet algorithms
  - Multi-differential observables
    - eg. double-differential in $\mathcal{T}_0 - \mathcal{T}_1$ [Bauer, Tackmann, Walsh, Zuberi `12, Pietrulewicz, Tackmann, Waalewijn `16 **GD**, Schunk, Tackmann (in progress)]

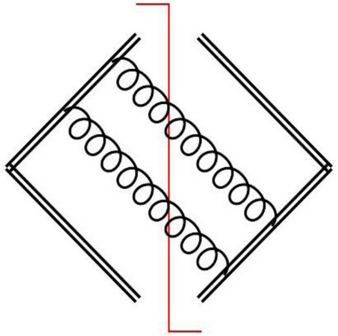
Automation

- Set up a general framework to automatically calculate general class of observables

$$d\sigma \simeq \boxed{H(\mu_f)} \prod_i B_i(\mu_f) \otimes \prod_j J_j(\mu_f) \otimes \boxed{S(\mu_f)}$$

- **Hard function** → underlying form factor LITERED, FIRE, REDUZE, AIR, KIRA ...
  [Lee, `13, Smirnov, Chukharev, `19, von Manteuffel, Studerus, `12, Anastasiou, Lazopoulos, `10 Maierhöfer, Usovitsch, Uwer, `17]

- **Soft functions** → 2-particle final state
  - NNLO di-jet soft functions: **SoftServe** [Bell, Rahn, Talbert `18, `20]
  - A large class of SCET-I and SCET-II observables has been calculated
  - Thrust, C-parameter, Angularities, Hemisphere masses, Threshold, P_T, Jet-veto resummation ...

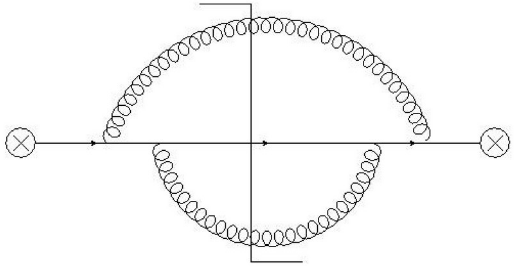


Automation

- Set up a general framework to automatically calculate general class of observables.

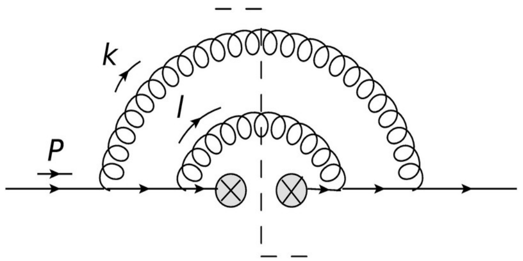
$$\mathrm{d}\sigma \simeq \boxed{H(\mu_f)} \prod_i \boxed{B_i(\mu_f)} \otimes \prod_j \boxed{J_j(\mu_j)} \otimes \boxed{S(\mu_s)}$$

- Jet functions ⟶ 3-particle final state
  - Complicated divergence structures!
  - Thrust, Angularities etc. [Bell, Brune, GD, Wald `21, (in progress)]



- Beam functions ⟶ 2-particle final state
  - Non-trivial matching onto pdfs!

This talk!



Beam function

- Proton matrix element of renormalized operator composed of partonic fields (with additional dependence on measurement)
  - Contains non-perturbative contribution – match onto PDF!

Matching also holds wrt partonic states!

Quark-Quark kernel

$$\mathcal{B}_{qq}(x,\tau,\mu) = \sum_{i\in X_c} \delta\Big((\bar{n}\cdot P)(1-x) - \bar{n}\cdot k_{X_c}\Big) \langle P|\bar{\chi}|X_c\rangle \frac{\bar{n}}{2} \langle X_c|\chi|P\rangle\, \mathcal{M}(\tau,\{k_i\})$$

- In the limit $\tau^{-1} \gg \Lambda_{QCD}$

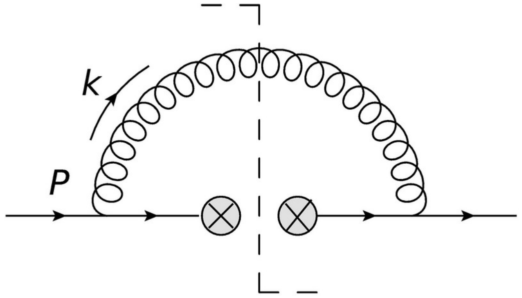
$$\mathcal{B}_{qq}(x,\tau,\mu) = \sum_i \int \frac{\mathrm{d}z}{z} \mathcal{I}_{qi}\Big(\frac{x}{z},\tau,\mu\Big) f_{iq}(z,\mu)$$

IR-divergent

IR-finite
Matching coefficients

IR-divergent
Partonic PDF

NLO



- An automated framework exists at NLO

[K. Brune's master thesis `18]

- Matrix element - LO splitting kernel

$$P^{(0)}_{q\to gq^*}(x_1) = C_F \frac{1}{x_1}\left[1+\bar{x}_1^2 - \epsilon x_1^2\right]$$

[Altarelli & Parisi `77]

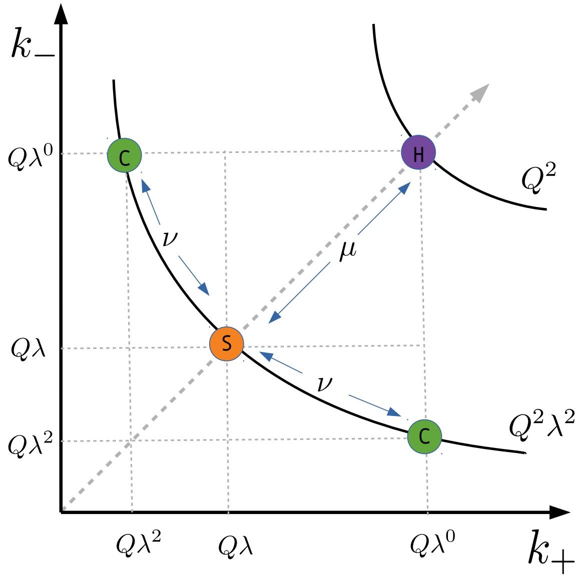
- Phase space

$$x_1 = \frac{k_-}{P_-}, \quad k_T = \sqrt{k_+ k_-}, \quad t_k = \frac{1-\cos(\theta_k)}{2}$$

$$\left\{k_-, k_T, \theta_k\right\} \to \left\{x_1, k_T, t_k\right\}$$

Rapidity divergence

- **SCET-II observables**
  - Suffers from additional rapidity divergences
  - Rapidity logs can be resummed to all orders
  - Follow the collinear-anomaly approach. [Becher, Neubert, `10]
  - Introduce analytic symmetric regulators at the phase space level [Becher, Bell, `11]



Modified PSP

$$\int \frac{d^d k}{(2\pi)^d}(2\pi)\delta_+(k^2)\left(\frac{\nu}{k_- + k_+}\right)^\alpha$$

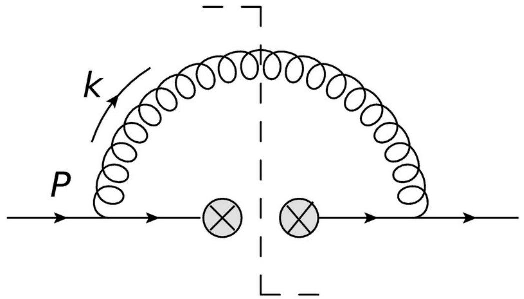
Collinear = Anti-collinear

Measurement

- Generic Measurement function for single emission (in Laplace Space)

$$\mathcal{M}_1(x_1,\tau;k) = \exp\left[-\tau k_T\left(\frac{k_T}{x_1 P_-}\right)^n f(t_k)\right]$$

Non-zero in singular limits of ME



- Example :
  - P_T – Resummation : $n=0,\ f(t_k) = -2i\ (1-2t_k)$
  - P_T – Veto : $n=0,\ f(t_k)=1$
  - Beam Thrust : $n=1,\ f(t_k)=1$

k_T – dependence is trivial! → Perform the integration analytically!

- Master Formula

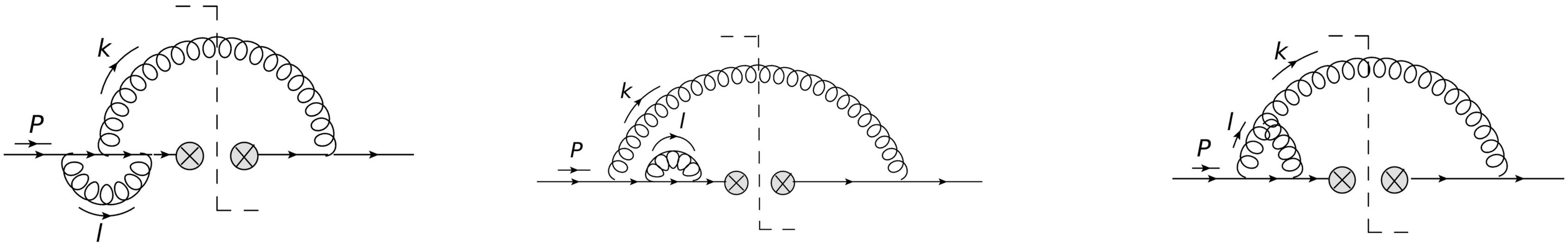
$$\mathcal{B}^{(1)}_{qq}(x_1,\tau) \sim \Gamma\left(\frac{-2\epsilon}{1+n}\right) x_1^{-1-\frac{2n\epsilon}{1+n}-\alpha}\left[x_1 P^{(0)}_{q\to gq^*}(x_1)\right]\int_0^1 dt_k (4t_k\bar{t}_k)^{-\frac{1}{2}-\epsilon} f(t_k)^{\frac{2\epsilon}{1+n}}$$

All singularities are factorised !

- Matrix Element

related to NLO collinear splitting kernel - $P^{(1)}_{q\to gq^*}(x)$

[Bern, Chalmers, +Del Duca, +Kilgore, +Schmidt `95, `99, Kosower, Uwer, `99 Sborlini, de Florian, Rodrigo `13]



- Phase space & measurement function follow NLO type

- Master formula

$$\mathcal{B}^{(2),RV}_{qq}(x_1,\tau,\mu) \sim V(\epsilon)\,\Gamma\left(\frac{-4\epsilon}{1+n}\right) x_1^{-1-\frac{4n\epsilon}{1+n}-\alpha}\mathcal{W}(x_1)\int_0^1 dt_k (4t_k\bar{t}_k)^{-\frac{1}{2}-\epsilon}\, f(t_k)^{\frac{4\epsilon}{1+n}}$$

$\sim \epsilon^{-2}+\mathcal{O}(\epsilon^{-1})$

All Phase space singularities are factorised !

Real-Real

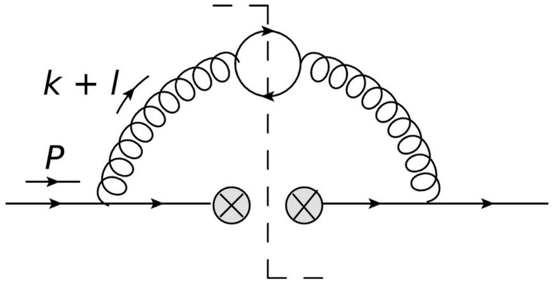
- Matrix element - LO triple collinear splitting kernels

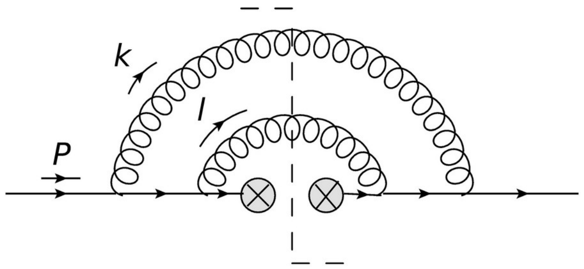
$$P^{(0),C_F T_F}_{q\to q'\bar{q}'q^*},\quad P^{(0),id}_{q\to q\bar{q}q^*},\quad P^{(0),C_F^2}_{q\to ggq^*},\quad P^{(0),C_A C_F}_{q\to ggq^*}$$

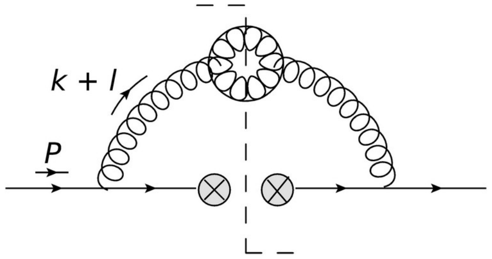
[Catani, Grazzini `98, `99
Campbell, Glover `98]

- Complicated divergence structures from ME

$$\frac{1}{s_{12}^2 s_{123}^2 (x_1+x_2)^2},\quad \frac{1}{s_{13}s_{23}x_1x_2},\quad \frac{1}{s_{13}s_{123}x_1x_2},\quad \frac{1}{s_{13}s_{123}x_2\bar{x}_3},\ \cdots$$





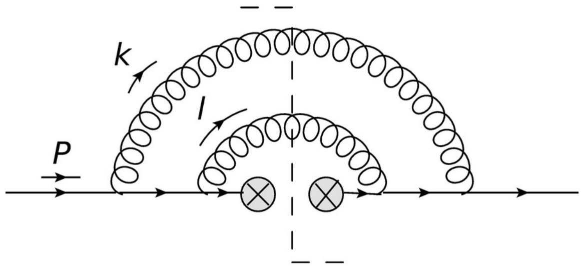


...

Real-Real

- 2-particle phase space

$$a = \frac{k_- l_T}{l_- k_T}, \quad b = \frac{k_T}{l_T}, \quad x_{12} = \frac{k_- + l_-}{P_-}, \quad q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}$$



$$\Big\{k_-, k_T, l_-, l_T, \theta_k, \theta_l, \theta_{kl}\Big\} \to \Big\{a, b, x_{12}, q_T, t_k, t_l, t_{kl}\Big\}$$

- Generic Measurement function – two real emissions

$$\mathcal{M}_2(\tau, \{k, l\}) = \exp\left[-\tau q_T \left(\frac{q_T}{x_{12} P_-}\right)^n \mathcal{F}(x_{12}, a, b, t_{kl}, t_k, t_l)\right]$$

  - Exact form depends on parametrization & ME divergences

$$\mathcal{F}(x_{12}, a, b, t_{kl}, t_k, t_l) \quad \Longrightarrow$$

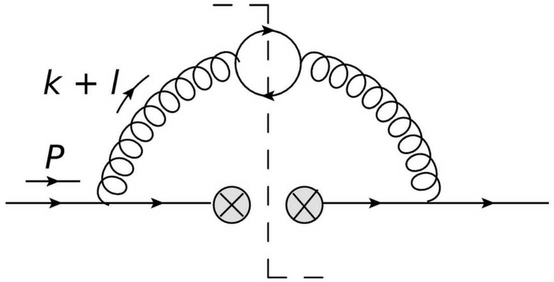
ensure it does **NOT** vanish in the singular limits of ME !

Real-Real : $C_F T_F n_f$

$$\left\{k_-, k_T, l_-, l_T, \theta_k, \theta_l, \theta_{kl}\right\} \to \left\{a, b, x_{12}, q_T, t_k, t_l, t_{kl}\right\}$$

- q_T-dependence is trivial → Perform the integration analytically
- Remap **{a,b}** to unit hypercube → **4 sectors**

Exploit k – l symmetry: **2 sectors**



- Avoid distributions in x_{12}

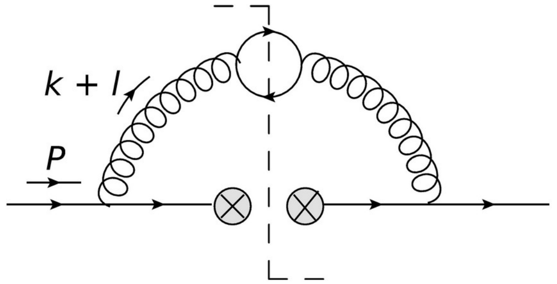
work in **Laplace-Mellin space**!

also possible to keep the distributions in x_{12}

Real-Real : $C_F T_F n_f$

- Collinear divergence `k||l` still overlaps:

$$\boxed{(\bar{a}^2 + 4at_{kl})^{-1} \xrightarrow{\text{NLT}} u^{-1}}$$



- Master formula:

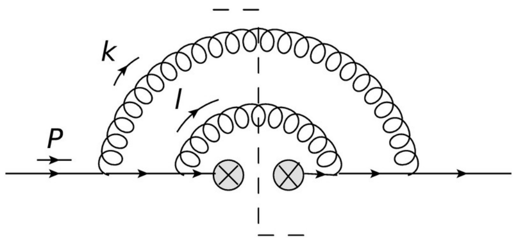
$$\mathcal{B}_{qq}^{(2),nf}(N_1,\tau) \sim \mathcal{C}(n,\epsilon)\left(\frac{\nu}{Q}\right)^{2\alpha}\int_0^1 dx_{12}\; db\; du\; dv\; dt_l\; dt_5\;\; x_{12}^{-1-2\alpha-\frac{4n\epsilon}{1+n}}\;\; u^{-1-2\epsilon}\;\; \bar{x}_{12}^{N_1-1+2\alpha+\frac{4n\epsilon}{1+n}} \times$$
$$\underbrace{\mathcal{G}(x_{12},b,u,v,t_l,t_5)}_{\text{finite function}}\underbrace{\mathcal{F}(x_{12},b,u,v,t_l,t_5)}_{\text{non-zero in the singular limit of ME}}^{\frac{4\epsilon}{1+n}}$$

All singularities factorize !

Real-Real : C_F^2

- Divergence structures

$$\frac{1}{s_{12}s_{13}\bar{x}_1\bar{x}_3},\quad \frac{1}{s_{12}s_{123}\bar{x}_1\bar{x}_3},\cdots\frac{1}{s_{13}s_{23}x_1x_2},\quad \frac{1}{s_{13}s_{123}x_1x_2},\cdots$$



- Complications due to many overlapping divergences in ME

- Additional complications from measurements
  - Beam Thrust $\mathcal{F} \sim (a+b)$
  - Angularity $\mathcal{F} \sim (a^n+b)$

Must stay non-zero

in the physical limits $a \to 0, b \to 0$

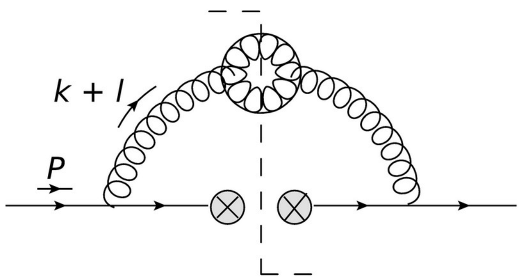
- Strategy:
  - Use of non-linear transformations
  - Sector decomposition [Heinrich `08]

~28 sectors

Real-Real : $C_A C_F$

- Divergence structures (in addition to all color structures)

$$\frac{1}{s_{12}s_{13}x_2\bar{x}_3}, \quad \frac{1}{s_{12}s_{123}x_1x_2}, \ldots$$



- Calculation follows similar to C_F^2 structures. ~ new 14 sectors
- Implementation:
  - Automated Mathematica code
  - Laurent expansion & integration in pySecDec (Cuba) [Heinrich et.al. `17 Hahn, `04, `14]

SCET-II renormalization

- Collinear anomaly approach

$$\left[\mathcal{S}(\tau,\mu,\nu)\mathcal{I}_{qq}(N_1,\tau,\mu,\nu)\mathcal{I}_{\bar{q}\bar{q}}(N_2,\tau,\mu,\nu)\right]_{q^2} \overset{\alpha=0}{=} \left(\bar{\tau}^2 q^2\right)^{-F_{q\bar{q}}(\tau,\mu)} \widehat{I}_{qq}(N_1,\tau,\mu)\widehat{I}_{\bar{q}\bar{q}}(N_2,\tau,\mu)$$

[Becher, Neubert, `10]

- RGE : Anomaly coefficients

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}F_{q\bar{q}}(\tau,\mu) = 2\Gamma_{\text{cusp}}(\alpha_S)$$

Finite non-logarithmic coefficient → d_2

- RGE: Matching coefficients

$L = \ln(\bar{\tau}\mu)$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\widehat{I}_{qq}(N_1,\tau,\mu) = \left[2\Gamma_{\text{cusp}}(\alpha_S)L + 2\gamma^B(\alpha_S)\right]\widehat{I}_{qq}(N_1,\tau,\mu) - 2\sum_i \widehat{I}_{qi}(N_1,\tau,\mu)\widehat{P}_{iq}(N_1,\mu)$$

- Solution → renormalized matching coefficients

Finite remainder coefficient $\widehat{\mathrm{I}}_{qq}(N_1)$ — Non-logarithmic piece

Results : P_T Resummation

- P_T-resummation

$$\omega_{p_T} = -2i \sum_i |k_{i,T}| \cos(\theta_i)$$

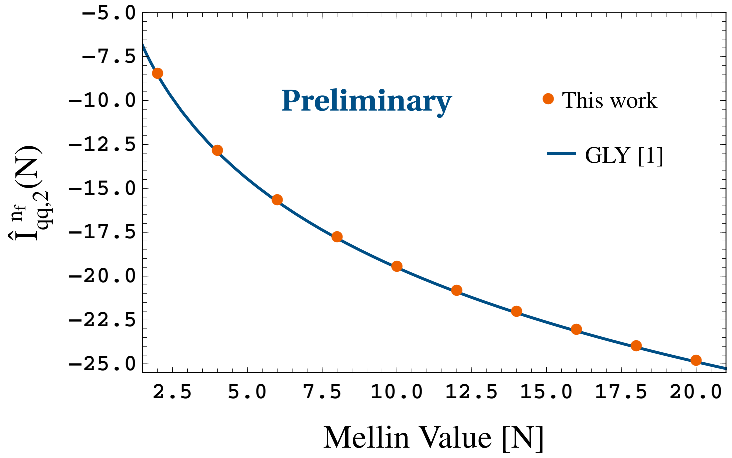
[Collin, Soper, Sterman, `85,
Catani, Grazzini,+..., `06-`14,
1. Gehrmann, Lübbert, Yang `14]

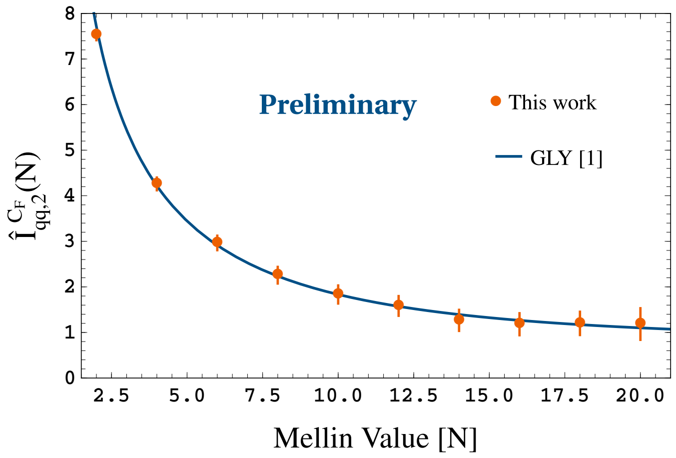
- Anomaly coefficients and non-cusp pieces are consistent with literature

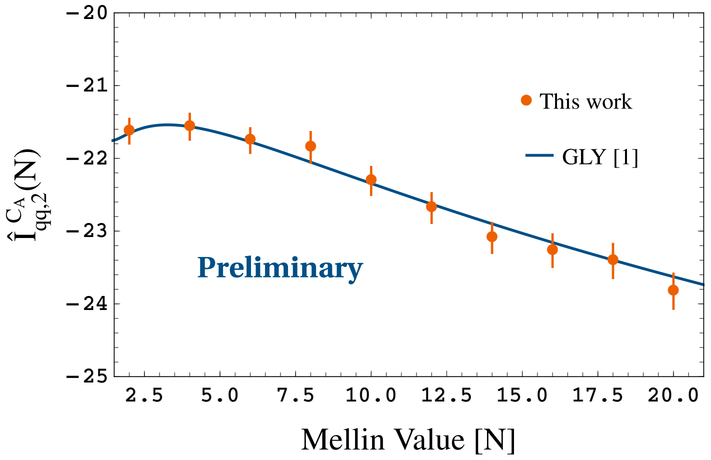
d_2	Analytical[1]	This Work
$d_2^{n_f}$	-8.296	$-8.293(4)$
$d_2^{C_F}$	0	$0.015(39)$
$d_2^{C_A}$	-3.732	$-3.723(19)$

γ_1^B	Analytical[1]	This Work
$\gamma_1^{n_f}$	-11.395	$-11.392(9)$
$\gamma_1^{C_F}$	10.610	$10.596(42)$
$\gamma_1^{C_A}$	4.637	$4.652(53)$

- Renormalized matching coefficient at two-loop







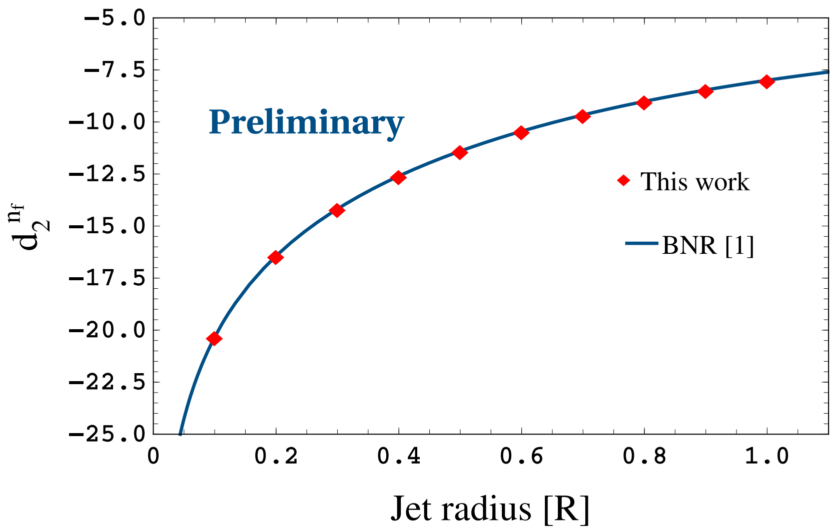
Results : P_T Veto

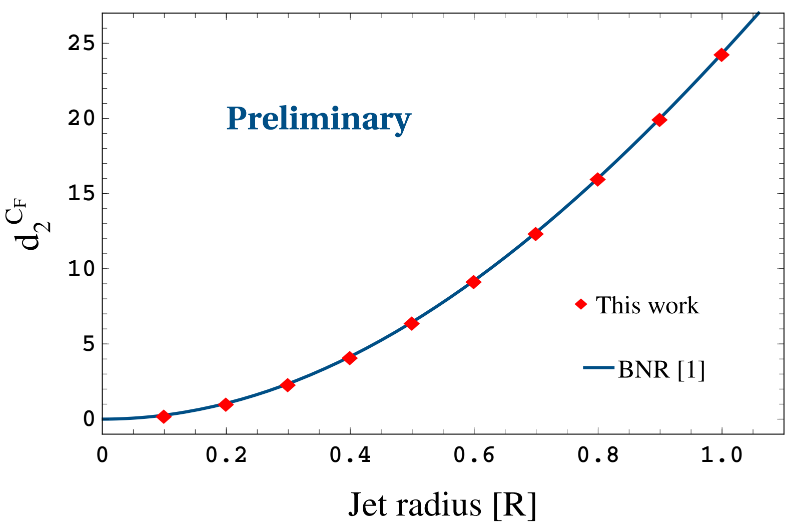
- P_T-Veto
  - Distance measure $$\Delta^2 = \frac{1}{4}\ln^2\left(\frac{l_+ k_-}{l_- k_+}\right) + \Theta^2_{lk}$$

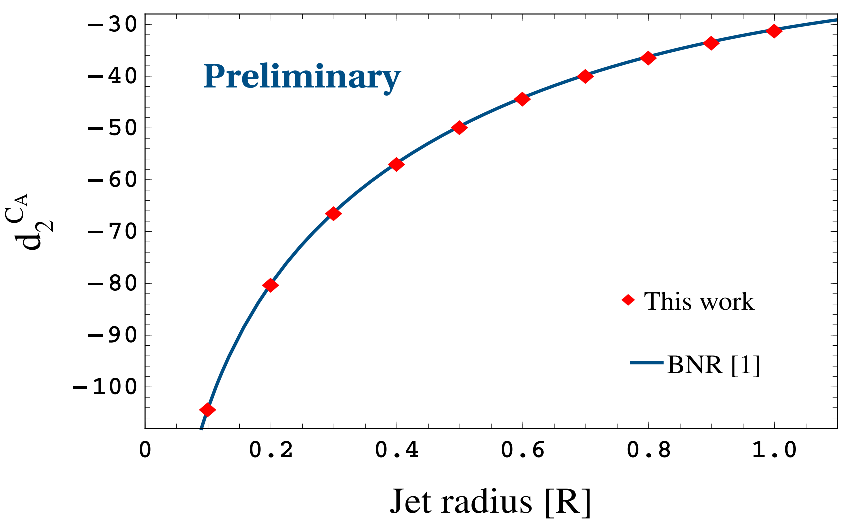
[1. Becher, Neubert, +Rothen `12,`13
2. Banfi, Monni, Salam, Zanderighi `12
3. Stewart, Tackmann, Walsh, Zuberi `13]

$$\omega_{p_T-veto} = \Theta(\Delta - R)\max(k_T, l_T) + \Theta(R-\Delta)|\vec{k}_T + \vec{l}_T|$$

- Anomaly coefficients - depends on the jet radius at 2-loop







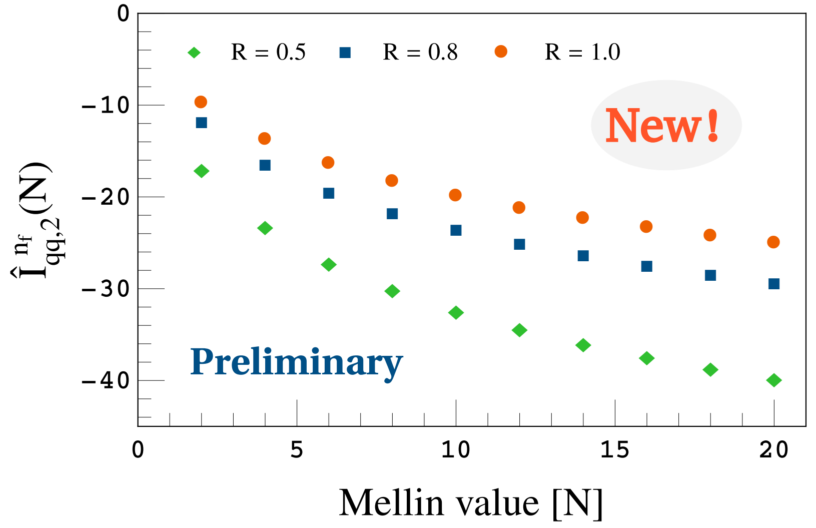
Results : P_T Veto

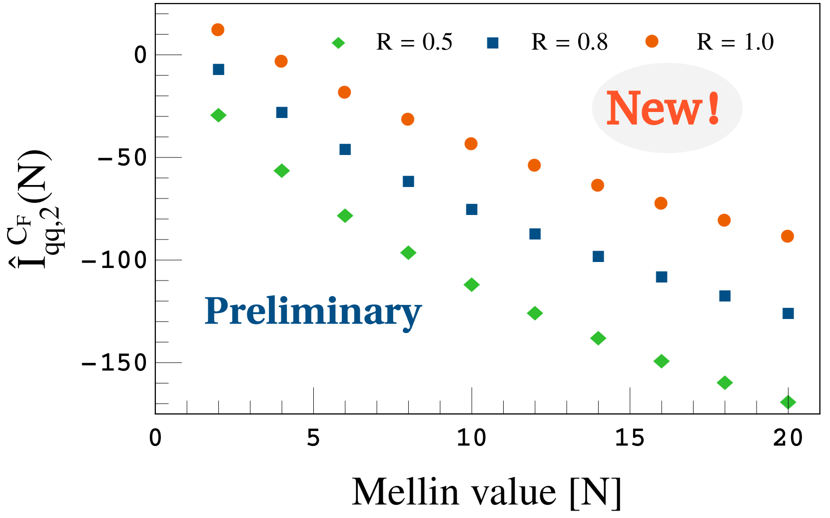
- Non-cusp pieces are in agreement

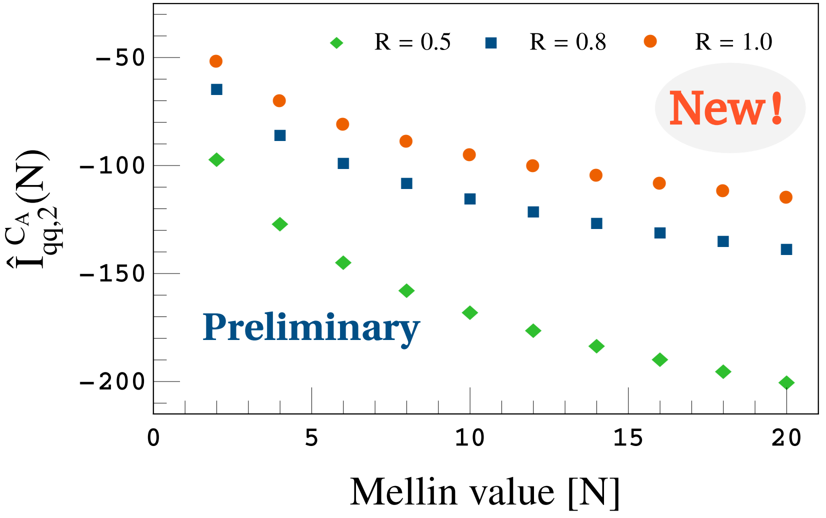
- Matching coefficients at two loops also depend on the Jet radius R

γ_1^B	Analytical[1]	This Work
$\gamma_1^{n_f}$	-11.395	$-11.395(2)$
$\gamma_1^{C_F}$	10.610	$10.611(19)$
$\gamma_1^{C_A}$	4.637	$4.640(17)$

[1. Becher, Neubert, Rothen `13]







Results : Transverse Thrust

- Transverse Thrust

$$\omega_{TT} = 1 - \max_{\vec{n}_\perp} \frac{\sum_i |\vec{p}_{i,\perp} \cdot \vec{n}_\perp|}{\sum_i |\vec{p}_{i,\perp}|}$$

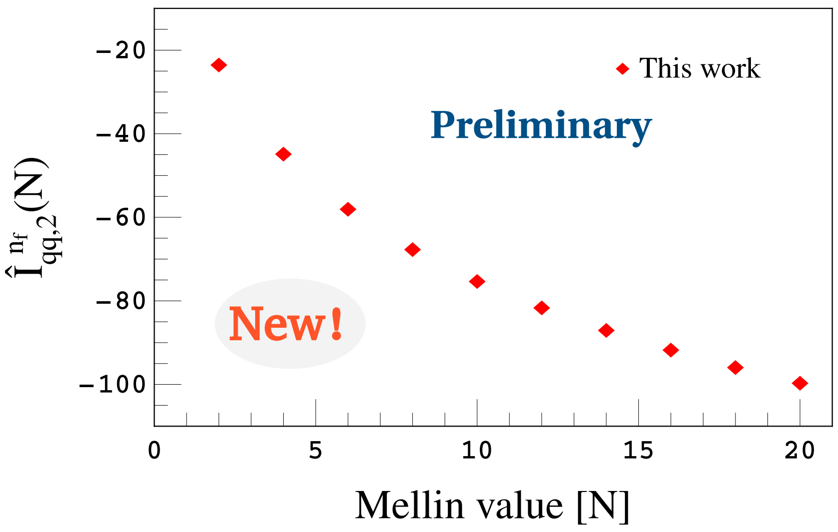
[Banfi, Salam, Zanderighi, `04,`10]

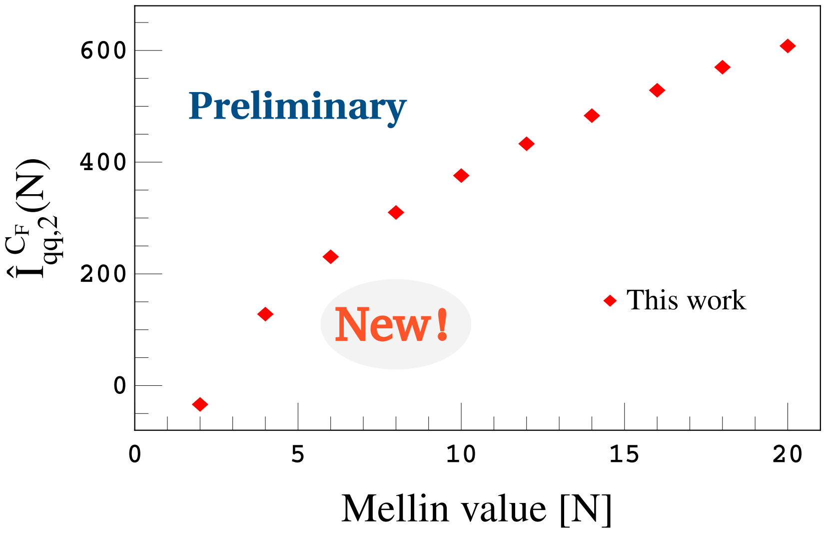
d_2	Numerical[1]	Numerical[2]	This Work
$d_2^{n_f}$	$-37.191(6)$	$-37.174(1)$	$-37.174(8)$
$d_2^{C_F}$	0	0	$0.046(73)$
$d_2^{C_A}$	$208.0(1)$	$208.105(1)$	$208.068(107)$

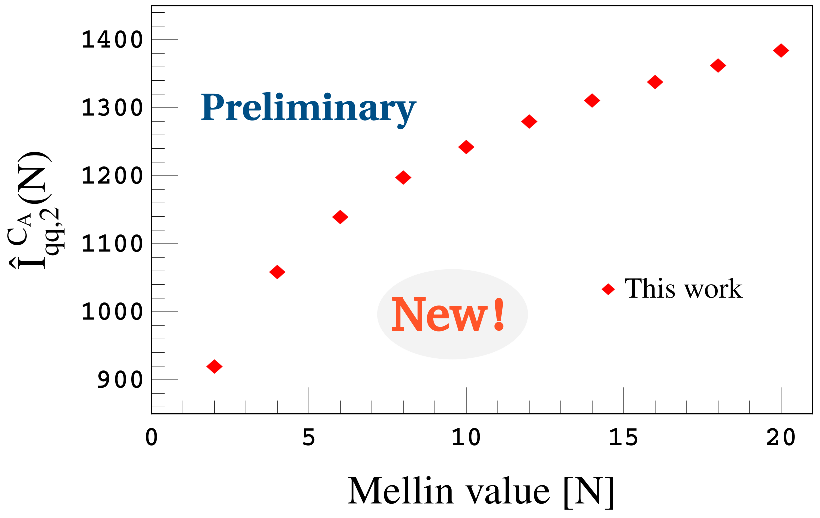
γ_1^B	Analytical[1]	This Work
$\gamma_1^{n_f}$	-11.395	$-11.395(4)$
$\gamma_1^{C_F}$	10.610	$10.625(54)$
$\gamma_1^{C_A}$	4.637	$4.645(62)$

- New two-loop matching coefficients!

[1. Becher, Tormo, +Piclum `15
2. Bell, Rahn, Talbert `18,`20]







Results : Beam Thrust

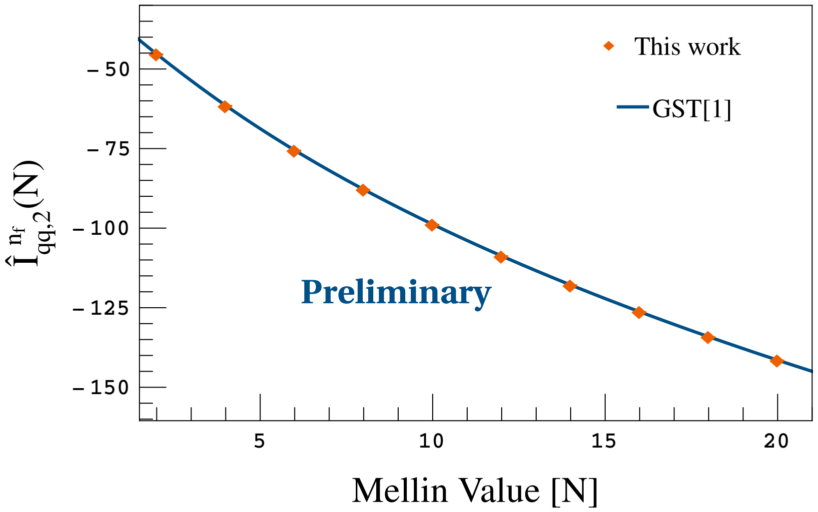
- Beam Thrust

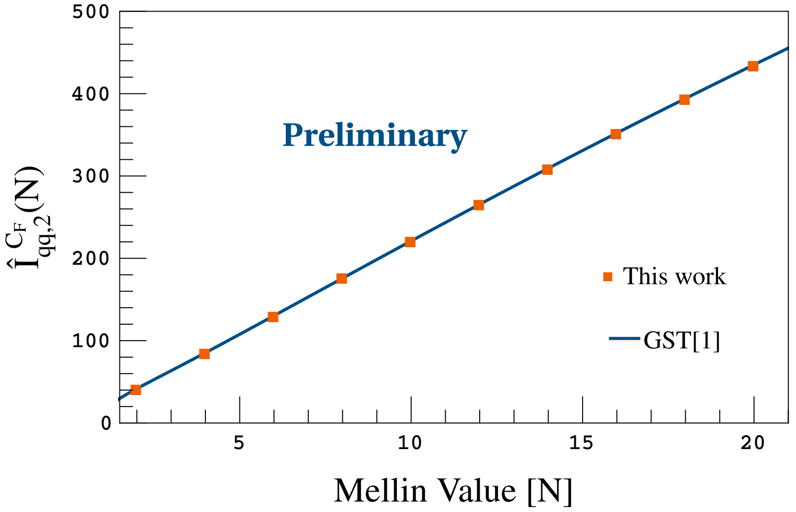
$$\omega_{BT} = \sum_i k_{+,i}$$

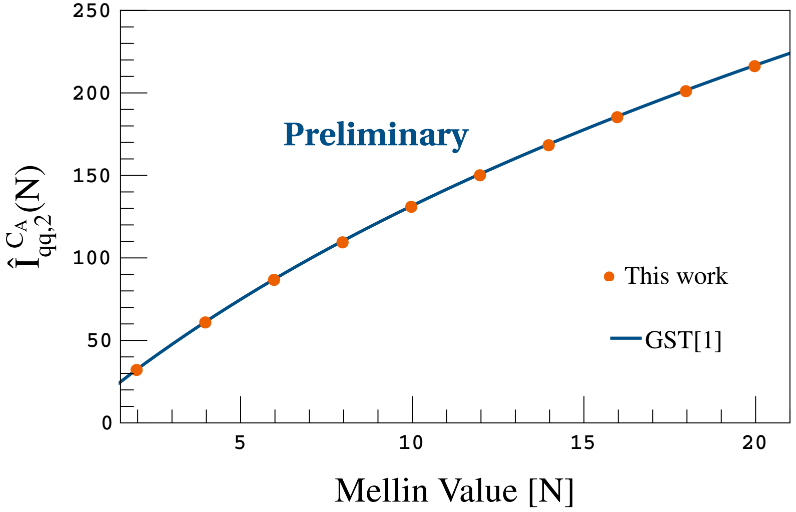
[Stewart, Tackmann, Waalewijn `09]

γ_1^B	Analytical[1]	This Work
$\gamma_1^{n_f}$	-13.35	$-13.35(1)$
$\gamma_1^{C_F}$	10.61	$10.61(7)$
$\gamma_1^{C_A}$	-3.26	$-3.27(8)$

[1. Gaunt, Stahlhofen, Tackmann `14]







Outlook

- Developed an automated way to calculate **Beam** functions for a wide class of obervables at NNLO.
- Framework already works for

  P_T-resummation, P_T-veto, Transverse Thrust, Beam Thrust

- More kernels & observables are on the way!
- Future plans:
  - Development of an automated C++ code.
  - Application to phenomenology!

Thank you for your attention!