Zero-jettiness beam functions at N^3LO

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Based on work done in collaboration with:
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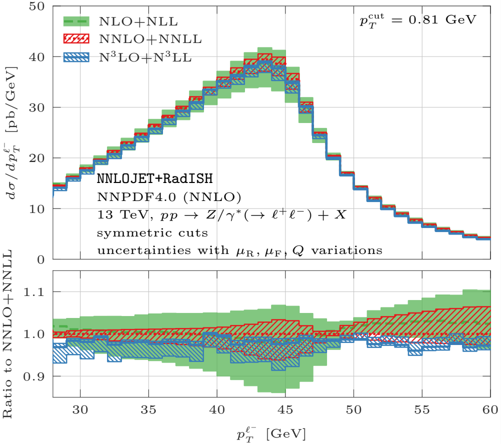
Introduction

N^3LO at hadron colliders



[Chen et al. '22]

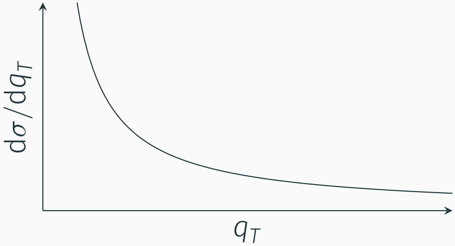
- Continuous push for higher precision
- Over the last year first N^3LO results for hadron colliders started appearing, including differential distributions
- First obvious target:
  Colour-singlet production, i.e. $pp \to H$, $pp \to \gamma^*/Z \to \ell^+\ell^-$, ...

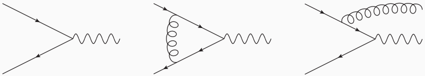
Phase space slicing

Infrared (IR) singularities

- Differential calculations require to have a handle on IR singularities
- Two broad categories: Slicing schemes and subtraction schemes
- At $\mathrm{N^3LO}$, slicing schemes seem more achievable at the moment

Example: q_T slicing for $q\bar{q} \to Z$ at NLO





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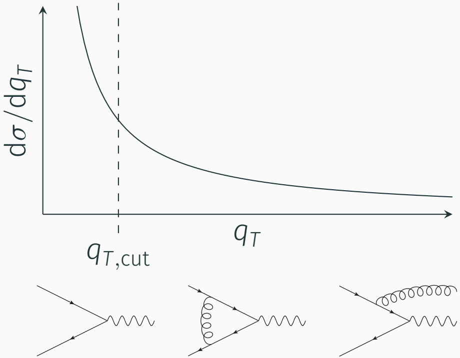

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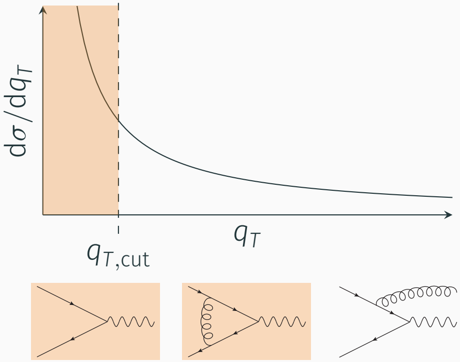

Below $q_{T,\mathrm{cut}}$:

- Born and virtual diagrams contribute
- Real diagram only contributes in soft/collinear limit

$\to$ Expand cross-section in small q_T limit

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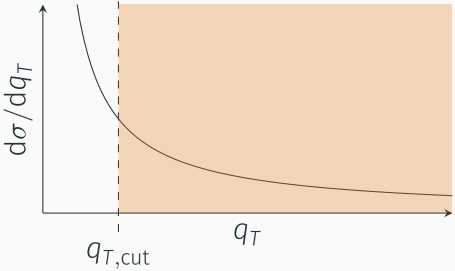

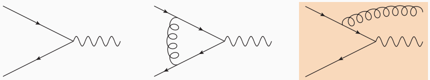

Above $q_{T,\text{cut}}$:

- Only real emission diagram contributes
- No soft or collinear singularities

$\to$ LO calculation for $q\bar{q} \to Z + j$ with $q_T > q_{T,\text{cut}}$

Phase space slicing

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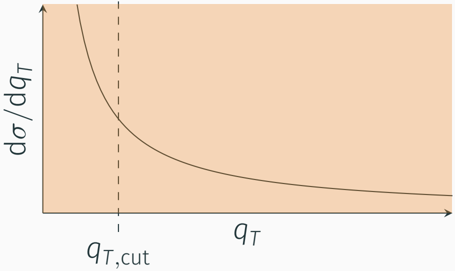
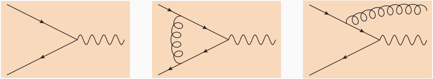




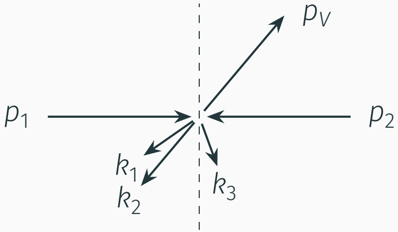
The sum of both regions is independent of $q_{T,\mathrm{cut}}$ if $q_{T,\mathrm{cut}}$ is small enough (or expansion includes sufficiently many terms)

Slicing with zero-jettiness τ

Definition: $\tau = \sum_j \min_{i\in\{1,2\}} \frac{p_i \cdot k_j}{Q_i}$

Q_i: Normalisation scales

$p_i \cdot k_j = p_i^0 k_j^0 (1 - \cos\theta_{ij})$



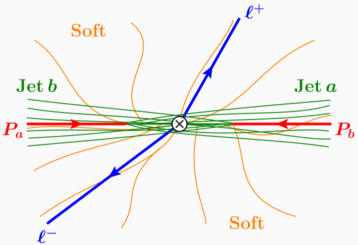
Zero-jettiness τ vanishes if *all* real emissions become soft or collinear with initial state momenta

Factorisation theorem for zero-jettiness

Aim: Find approximation for cross section below τ_{cut}

Factorisation theorem for zero-jettiness
(proven in SCET):

$$\lim_{\tau \to 0} \sigma = B \otimes B \otimes S \otimes H \otimes \sigma_{\text{LO}} + O(\tau)$$



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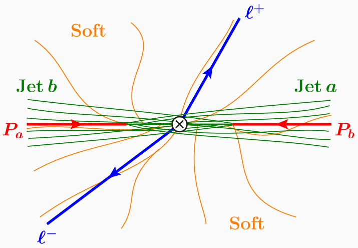

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Hard function H:

Correction to hard process

$\rightarrow$ Contains virtual corrections

$\rightarrow$ Process-dependent

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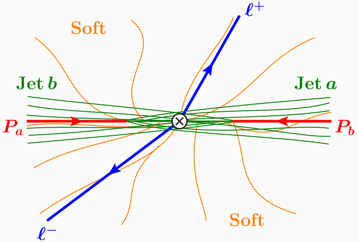


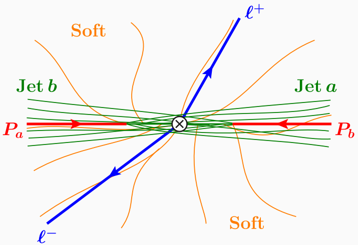
Soft function S:

Describes effects of soft, non-collinear gluons or $q\bar{q}$ pairs

$\rightarrow$ Known to NNLO

$\rightarrow$ N^{3}LO calculation currently underway, see [Baranowski et al. '22]

Aim: Find approximation for cross section below τ_{cut}



Factorisation theorem for zero-jettiness (proven in SCET):

$$\lim_{\tau \to 0} \sigma = B \otimes B \otimes S \otimes H \otimes \sigma_{\text{LO}} + O(\tau)$$

Beam functions B:

Describes collinear emissions off initial state partons

$\rightarrow$ First N^{3}LO results in large N_c/n_f limit: [Behring et al. '19]

$\rightarrow$ Known to N^{3}LO ([Ebert, Mistlberger, Vita '20], Bernhard's talk on Thursday)

$\rightarrow$ Topic of this talk: Ongoing independent calculation at N^{3}LO

Important given the complexity of the problem

Beam functions for zero-jettiness

Definition of the beam function

Beam function $B(t, x, \mu)$

- Non-perturbative object, similar to PDFs
- Depends on
  - momentum fraction x
  - transverse virtuality $t = -(p_*^2 - k_\perp^2)$
- Can be related to PDFs via convolution

$$B_i(t, x, \mu) = \int_0^1 \mathrm{d}z \sum_{j \in \{q, \bar{q}, g\}} \underbrace{I_{ij}(t, z, \mu)}_{\text{matching coeff.}} \underbrace{f_j\left(\frac{x}{z}, \mu\right)}_{\text{PDFs}}$$

- At leading order:

$$I_{ij}^{\mathrm{LO}}(t, z, \mu) = \delta(1 - z)\delta(t)\delta_{ij} \qquad \Rightarrow \qquad B_i^{\mathrm{LO}}(t, z, \mu) = f_i(z, \mu)\delta(t)$$

$\rightarrow$ Goal: Compute matching coefficient at $\mathrm{N^3LO}$

How to calculate matching coefficients I_{ij}?

Beam function and PDFs have operator definitions in SCET:

$$B_i \sim \langle P(p)|\mathcal{O}_i(t, xp, \mu)|P(p)\rangle$$

$$f_j \sim \langle P(p)|\mathcal{Q}_j(x'p, \mu)|P(p)\rangle$$

with $|P(p)\rangle$ proton states.

Matching relation is an operator relation (OPE)

$$B_i = \sum_j I_{ij} \otimes f_j$$

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Define partonic beam functions B_{ik} and partonic PDFs f_{jk} with parton states $|p_k(p)\rangle$

How to calculate matching coefficients I_{ij}?

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$$B_{ik} \sim \langle p_k(p)|\mathcal{O}_i(t, xp, \mu)|p_k(p)\rangle$$
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Matching relation is an operator relation (OPE)

$$B_{ik} = \sum_j I_{ij} \otimes f_{jk}$$

$\rightarrow$ Still same matching coefficients I_{ij}

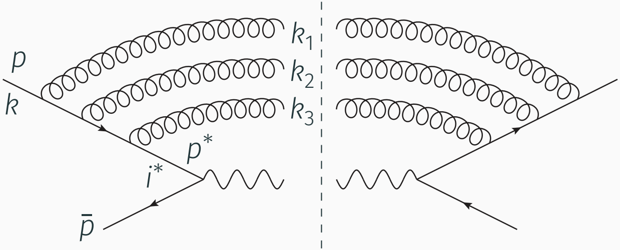
$\rightarrow$ Task becomes: Calculate B_{ik} in perturbative QCD

How to calculate partonic beam functions B_{ik}?

Observation from [Ritzmann, Waalewijn '14]:

B_{ik} can be calculated from collinear limits of QCD amplitudes

$$B_{ik}^{\mathrm{bare}} \sim \int \prod_{i=1}^{n_R} \frac{\mathrm{d}^d k_i}{(2\pi)^{d-1}} \delta_+(k_i^2)\, \delta\left(2p \cdot k_{1\ldots n_R} - \frac{t}{z}\right) \delta\left(\frac{2\bar{p} \cdot k_{1\ldots n_R}}{s} - (1-z)\right) \frac{\hat{C}_p |M(p, \bar{p}, \{k_i\})|^2}{|M_0(zp, \bar{p})|^2}$$



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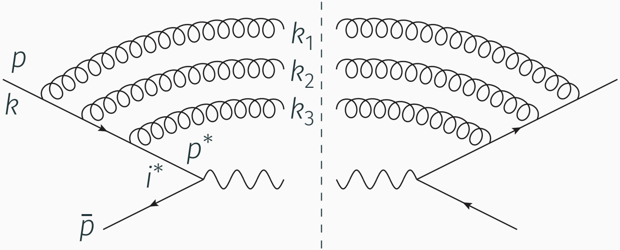

- Construct collinear limits (i.e. splitting functions) à la [Catani, Grazzini '99]

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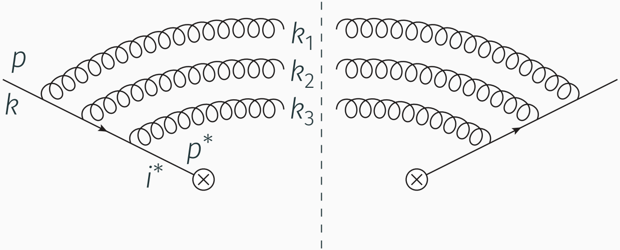

- Construct collinear limits (i.e. splitting functions) à la [Catani, Grazzini '99]
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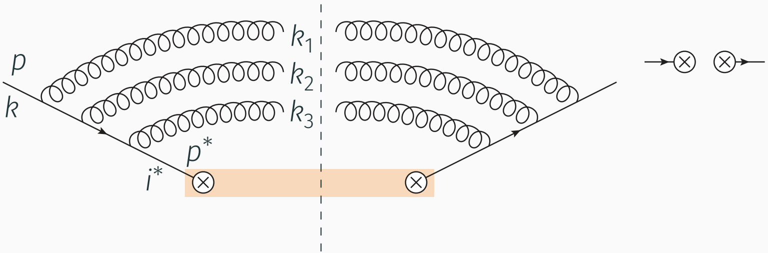

$$\rightarrow \frac{\bar{\not{p}}}{4N_c p^* \cdot \bar{p}}$$

- Construct collinear limits (i.e. splitting functions) à la [Catani, Grazzini '99]
  - Consider only one leg
  - Replace hard process by suitable projector

Observation from [Ritzmann, Waalewijn '14]:

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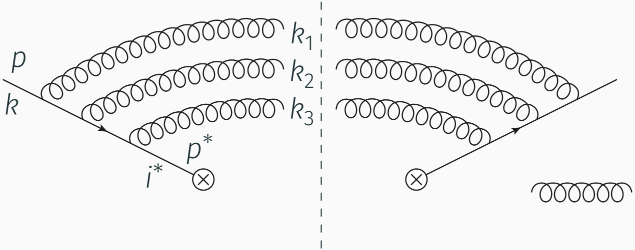

$$\rightarrow \frac{\bar{\not{p}}}{4N_c p^* \cdot \bar{p}}$$

$$\rightarrow \frac{i\left(-g^{\mu\nu} + \frac{k^\mu \bar{p}^\nu + k^\nu \bar{p}^\mu}{k \cdot \bar{p}}\right)}{k^2 + i0^+}$$

- Construct collinear limits (i.e. splitting functions) à la [Catani, Grazzini '99]
  - Consider only one leg
  - Replace hard process by suitable projector
  - Work in axial gauge

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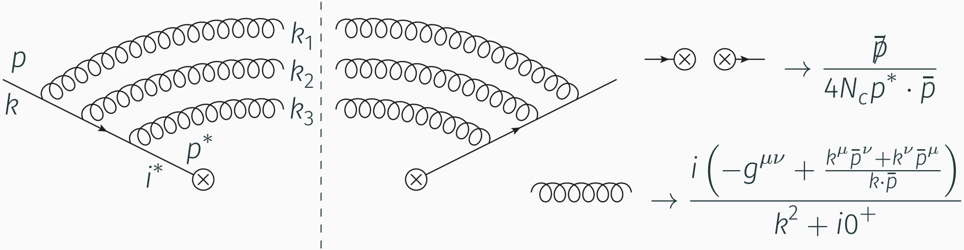
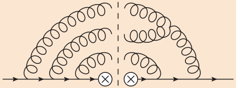


- Integrate over constrained phase space ($k_{1\ldots n_R} = k_1 + \cdots + k_{n_R}$)
  - Implement δ distributions via reverse unitarity

Calculation

Calculation chain

Diagrams (QGRAF)



FORM →

Amplitudes

$$B_{ij} \sim \int \frac{\mathrm{d}\Phi_{\mathcal{B}} C_A^2 C_F}{(k_1-p)^2 (k_1\cdot\bar{p})\ldots} + \ldots$$

Partial fractioning →

Map to scalar integrals

$$B_{ij} \sim C_A^2 C_F I^{\mathrm{T1}}_{1111101\ldots} + \ldots$$

↓ IBP: Reduze Kira

Master integrals

$$B_{ij} \sim C_A^2 C_F I^{\mathrm{T1}}_{1111101\ldots} + \ldots$$

Partial fractioning of MI

$$I^{\mathrm{T29}}_{3391} = I^{\mathrm{T1}}_{3263} - I^{\mathrm{T28}}_{3391}$$

Differential equations

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{pmatrix} I_1 \\ I_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} \blacksquare & & & \\ & \blacksquare & & \\ \blacksquare & & \blacksquare & \\ & \blacksquare & & \ddots \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \end{pmatrix}$$

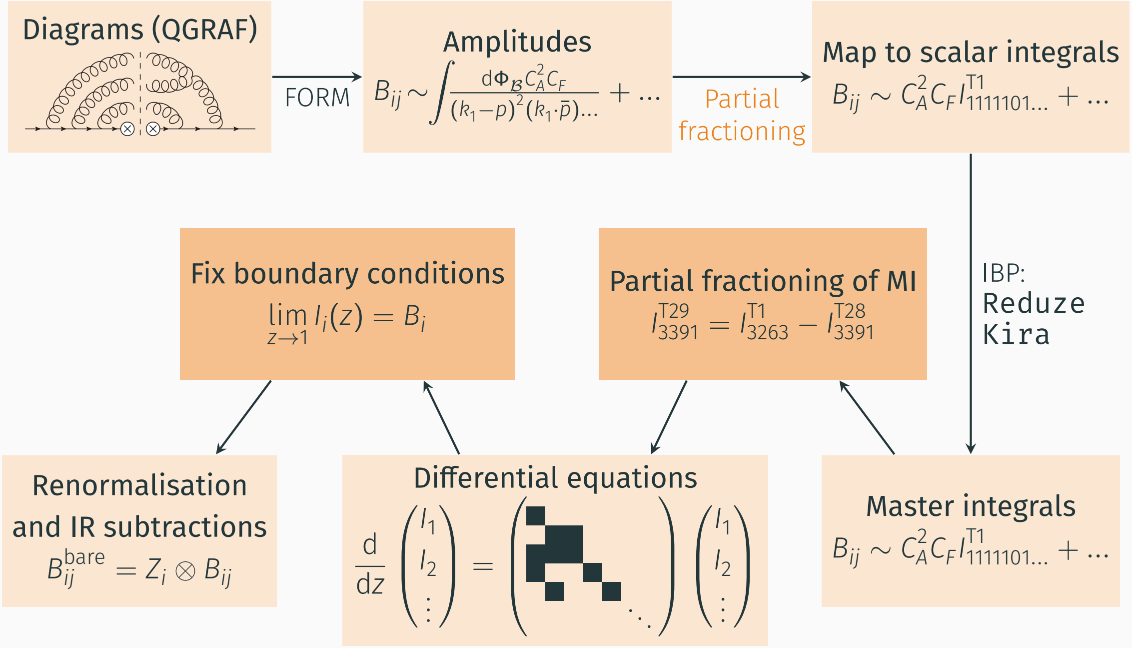
Fix boundary conditions

$$\lim_{z\to 1} I_i(z) = B_i$$

Renormalisation and IR subtractions

$$B_{ij}^{\mathrm{bare}} = Z_i \otimes B_{ij}$$

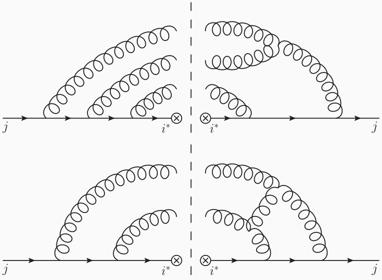
Calculation chain



Overview of the calculation

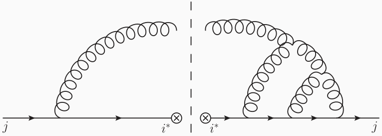
RRR and RRV

- Amounts to $\sim$ 450 three-loop master integrals
- Master integrals depend on iterated integrals over alphabet containing
  - linear letters ($f_a(z) = \frac{1}{z-a}$) with $a \in \{0, \pm1, \pm2, \pm4, \pm2i, \exp\left(\pm i\pi\frac{2}{3}\right)\}$
  - square-root valued letters $\left\{\frac{1}{\sqrt{z(4-z)}}, \frac{1}{\sqrt{z(4+z)}}, \frac{1}{\sqrt{4+z^2}}, \frac{1}{z\sqrt{4+z^2}}\right\}$



RVV

- Calculate using two-loop splitting functions published in [Duhr, Gehrmann, Jaquier '14]



Partial fractioning

- Linear relations between propagators, e.g.

$$2(k_1 + k_2) \cdot \bar{p} = s(1 - z)$$

$\rightarrow$ partial fractioning identities, i.e.

$$\frac{1}{(k_1 \cdot \bar{p})(k_2 \cdot \bar{p})} = \frac{2}{s(1-z)} \left[\frac{1}{k_1 \cdot \bar{p}} + \frac{1}{k_2 \cdot \bar{p}} \right]$$

- Systematically applied using Gröbner bases [Pak '11]
- We use it twice:
  - To map to integral families
  - To find inter-family relations between master integrals

Inter-family relations from partial fractioning

$$\overbrace{\int \frac{\mathrm{d}\Phi_{\mathcal{B}}}{(k_1-p)^2(k_{13}-p)^2(k_3\cdot\bar{p})(k_{13}\cdot\bar{p})}}^{=I^{\mathrm{T29}}_{3391}} = \underbrace{\int \frac{\mathrm{d}\Phi_{\mathcal{B}}}{(k_1-p)^2(k_{12}-p)^2(k_1\cdot\bar{p})(k_2\cdot\bar{p})}}_{=I^{\mathrm{T1}}_{3263}} - \underbrace{\int \frac{\mathrm{d}\Phi_{\mathcal{B}}}{(k_1-p)^2(k_{13}-p)^2(k_1\cdot\bar{p})(k_{13}\cdot\bar{p})}}_{=I^{\mathrm{T28}}_{3391}}$$

- Partial fractioning relations reduce the number of MI:

  RRR: $356 \to 265$ RRV: $396 \to 188$
- Construction:
  - Generate seed integrals
  - Apply partial fractioning relations
  - Reduce to MI
  - Remove trivial relations and solve linear system
- Refinement possible?

Calculation of master integrals

Structure of the integrals

- Integrals depend on three variables: s, t and z
- Scale out s and t via $k_i = \tilde{k}_i\sqrt{t}$, $p = \tilde{p}\sqrt{t}$ and $\bar{p} = \tilde{\bar{p}}s/\sqrt{t}$, e.g.:

  $$I^{\text{RRV1}}_{n_1,\ldots,n_8}(s,t,z) = s^{-1-n_7-n_8}\, t^{3-3\varepsilon-(n_1+\cdots+n_6)}\, I^{\text{RRV1}}_{n_1,\ldots,n_8}(z)$$

  $\rightarrow$ Differential equations only for z-dependence

Boundary constants

- Fix integration constants in the soft limit $z \rightarrow 1$
- We used a variety of technologies to find integration constants:
  - Constraints from analytic structure of integrals
  - Direct calculation
  - Auxiliary differential equations
  - Mapping to Higgs production ($gg \rightarrow H$) threshold integrals

Boundary constants from Higgs threshold integrals

In the soft limit also the z-dependence scales out:

$$\mathcal{B}(s,t,z) = s^{e_s} t^{e_t} z^{-e_t} (1-z)^{e_{\bar{z}}} \mathcal{B}(1,1,1)$$

Beam function phase space

$$\mathcal{B}(s,t,z) \sim \delta\left(2k_{123}\cdot p - \frac{t}{z}\right)\delta\left(2k_{123}\cdot\bar{p} - s(1-z)\right)$$

Higgs threshold phase space

$$\mathcal{H}(s) \sim \delta\left(2k_{123}\cdot(p+\bar{p}) - s\right)$$

Idea: Choose special kinematic point $t = sz^2$ and integrate over $z \in [0,1]$

$$\int_0^1 \mathrm{d}z\, \mathcal{B}(s, sz^2, z) = \int_0^1 \mathrm{d}z\, \delta(2k_{123}\cdot p - sz)\delta(2k_{123}\cdot\bar{p} - s(1-z)) = \frac{1}{s}\mathcal{H}(s)$$

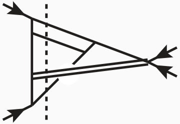
Allows to map between beam function and Higgs threshold phase space

$$\frac{1}{s}\mathcal{H}(s) = s^{e_s+e_t} B(e_t+1, e_{\bar{z}}+1)\mathcal{B}(1,1,1)$$

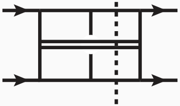
Boundary constants from Higgs threshold integrals (cont.)

This suggests a connection between our calculation and the N^{3}LO calculation of $gg \to H$ in the threshold region in [Anastasiou et al. '13] and [Anastasiou et al. '15].

RRR
Our boundary constants cover 9 of 10 MI from [Anastasiou et al. '13] including the most complicated one:

$\mathcal{F}_9 \sim$ 

RRV
Our boundary constants cover several of the soft MI from [Anastasiou et al. '15] including the most complicated one:

$\mathcal{M}_{13} \sim$ 

Along these lines it should in principle be possible to independently redo the N^{3}LO calculation of $gg \to H$ in the threshold region.

Results

Status of the calculation

Matching coefficient	$I_{q_i q_j}$	I_{qg}	I_{gq}	I_{gg}	$I_{q_i \bar{q}_j}$
Amplitudes	✓	✓	✓	✓	in progress
Boundary constants	✓	✓	✓	✓	in progress
Master integrals	✓	✓	✓	✓	in progress
Renormalisation & IR subtraction	✓	✓	✓	in progress	in progress
Pole cancellation ε^{-6}, ..., ε^{-1}	✓	✓	✓	in progress	in progress
ε^0 term matches [Ebert et al. '20]	✓	✓	✓	in progress	in progress

- Pole cancellation cross-checks three-loop DGLAP splitting functions from [Moch, Vermaseren, Vogt '04]
- Alphabet of iterated integrals in matching coefficients simplifies:

$$\left\{ \frac{1}{z}, \frac{1}{z-1}, \frac{1}{z+1}, \frac{1}{z-2}, \frac{1}{\sqrt{z(4-z)}} \right\}$$

Results: Matching coefficient $I_{qq}^{(3)}$

- Organise results according to their structure in $t \to 0$:

$$I_{qq}^{(3)} = \sum_{k=0}^{5} L_k\left(\frac{t}{\mu^2}\right) F_{+}^{(3,k)}(z) + \delta(t) F_{\delta}^{(3)}(z)$$

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plus distributions: $L_k\left(\frac{t}{\mu^2}\right) = \frac{1}{\mu^2}\left[\frac{\mu^2}{t}\ln^k\left(\frac{t}{\mu^2}\right)\right]_+$

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$$F_\delta^{(3)}(z) = C_{-1}^{(3)}\delta(1-z) + \sum_{k=0}^{5} C_k^{(3)} D_k(z) + F_{\delta,h}^{(3)}(z)$$

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- "Hard" contributions $F_{\delta,h}^{(3)}(z)$

$$F_{\delta,h}^{(3)}(z) = N_f^2 T_F^2 N_c F_1(z) + N_f T_F N_c^2 F_2(z) + N_c^3 F_3(z) + \ldots$$

Results: Matching coefficient $I_{qq}^{(3)}$

- Organise res

$$I_{qq}^{(3)} = \sum_{k=0}^{5} L_k$$

- Isolate soft c

$$F_\delta^{(3)}(z) = C_{-}^{(3}$$

- "Hard" contri $_{\delta,h}(z)$

$$F_{\delta,h}^{(3)}(z) = N_f^2 T_F^2 N_c F_1(z) + N_f T_F N_c^2 F_2(z) + N_c^3 F_3(z) + \ldots$$

$$\begin{aligned}
F_3(z) = {} & \frac{1}{2916}(715565z - 197242) + \frac{35}{108}(698z - 69)H_1 + \frac{181}{27}(31z+1)H_{1,1} + \frac{1}{9}(1403z + 662)H_{1,1,1} \\
& + \frac{8}{3}(32z+23)H_{1,1,1,1} + 60(z+1)H_{1,1,1,1,1} + \frac{1}{1-z}\left[\frac{1}{648}\left(-217440z^2 + 191022z - 186085\right)H_0\right] \\
& + \frac{1}{1-z}\left[\frac{1}{162}\left(-52174z^2 + 38784z - 38101\right)H_2 + \frac{1}{162}\left(-32914z^2 + 29415z - 33625\right)H_{1,0}\right. \\
& \left. + \frac{1}{972}\left(50848z^2 - 34734z - 1747\right)\pi^2\right] + \frac{1}{1-z}\left[\frac{1}{18}\left(-4800z^2 + 1759z - 3599\right)H_3\right. \\
& + \frac{1}{18}\left(-3843z^2 + 2024z - 3645\right)H_{2,1} + \frac{1}{54}\left(-8357z^2 + 3903z - 8099\right)H_{2,0} \\
& + \frac{1}{9}\left(-1704z^2 + 795z - 1793\right)H_{1,2} - \frac{2}{9}\left(554z^2 - 277z + 541\right)H_{1,1,0} \\
& + \frac{1}{54}\left(-7033z^2 + 2574z - 7429\right)H_{1,0,0} - \frac{13}{27}\left(407z^2 - 96z + 185\right)H_{0,0,0} \\
& + \frac{1}{108}\left(4442z^2 - 2067z - 243\right)\pi^2 H_1 + \frac{1}{108}\left(6139z^2 - 2356z + 4431\right)\pi^2 H_0 \\
& \left. + \frac{1}{54}\left(15898z^2 - 5313z - 10099\right)\zeta_3\right] + \frac{1}{1-z}\left[\frac{1}{18}\left(-3653z^2 + 726z - 1559\right)H_4\right. \\
& + \frac{1}{3}\left(-572z^2 + 186z - 327\right)H_{3,1} + \frac{1}{9}\left(-1388z^2 + 477z - 656\right)H_{3,0}
\end{aligned}$$

Conclusions

Conclusions and outlook

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- We calculate the beam functions for zero-jettiness at N^3LO via phase space integrals over splitting functions in QCD
- We have completed the matching coefficients $I_{q_i q_j}$, I_{qg} and I_{gq} and confirm the results from [Ebert, Mistlberger, Vita '20]

Outlook

- Completion of final matching coefficients $I_{q_i \bar{q}_j}$ and I_{gg} is underway
- Calculation of zero-jettiness soft function is in progress, see [Baranowski, Delto, Melnikov, Wang '22]
- It will be interesting to put the beam functions to use in slicing calculations for colour-singlet production, as well as in resummation applications.