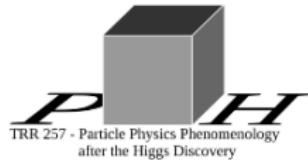


Mixed QCD-electroweak corrections to Higgs plus jet production at the LHC

Marco Bonetti

Loops & Legs in Quantum Field Theory 2022



In collaboration with
E. Panzer, V. A. Smirnov, L. Tancredi
[2007.09813] [2203.17202]

1 Motivations & Overview

2 Process

3 Master Integrals

4 Results

5 Conlcusions

Higgs boson at the LHC

[1602.00695] [1610.07922] [1802.00833]

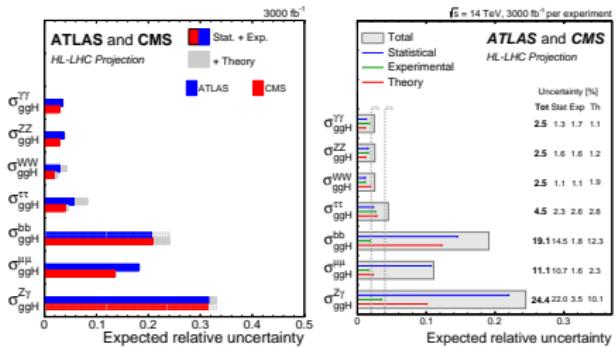
Higgs production modes					
ggH	VVH	WH	ZH	$t\bar{t}H$	Total
$44.1^{+11\%}_{-11\%}$	$3.78^{+2\%}_{-2\%}$	$1.37^{+2\%}_{-2\%}$	$0.88^{+5\%}_{-5\%}$	$0.51^{+9\%}_{-13\%}$	50.6

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HL-LHC projections

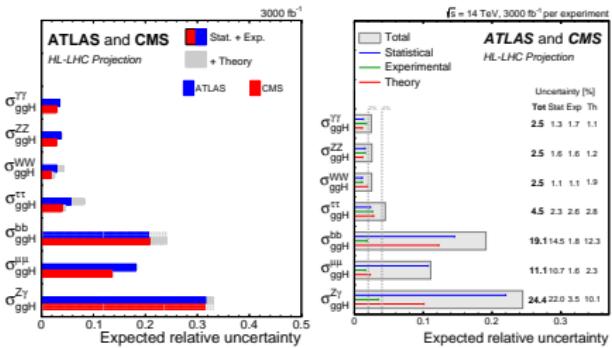


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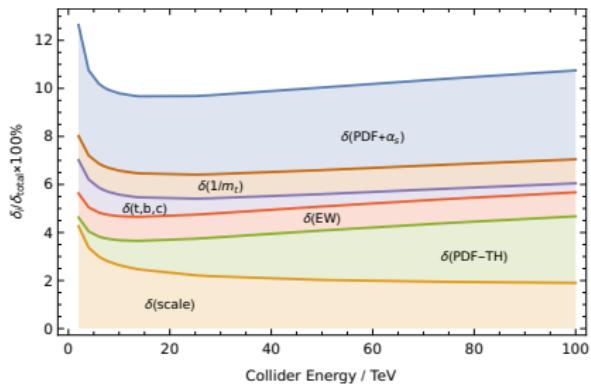
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Theoretical uncertainties

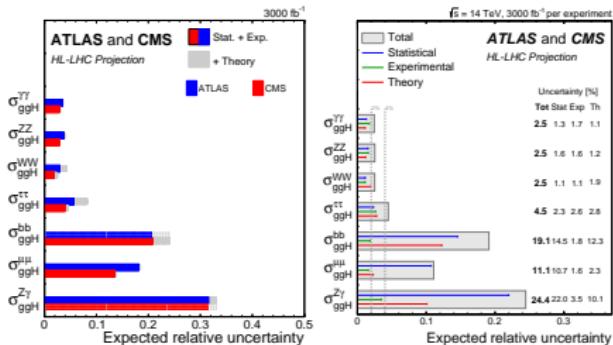


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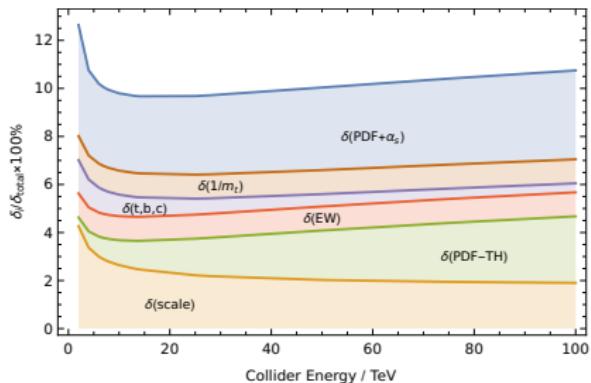
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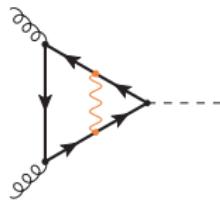
What is the source of $\delta(\text{EW})$?

QCD-EW contributions

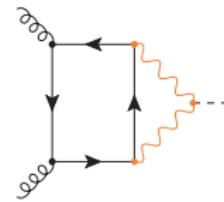
[ph0404071] [ph0407249] [ph0610033]

Exact LO Electroweak contributions

Yukawa coupling $\alpha_s \alpha Y_t$



Electroweak coupling $\alpha_s \alpha^2 v$

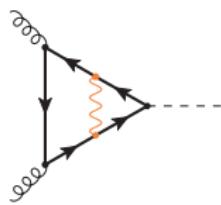


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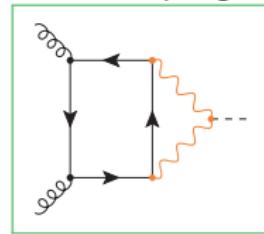
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- $\sim 0.5\%$ of $\sigma_{\text{QCD}}^{\text{LO}}$

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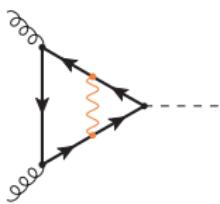
- Dominated by light quarks
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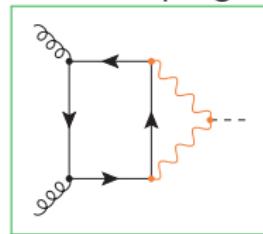
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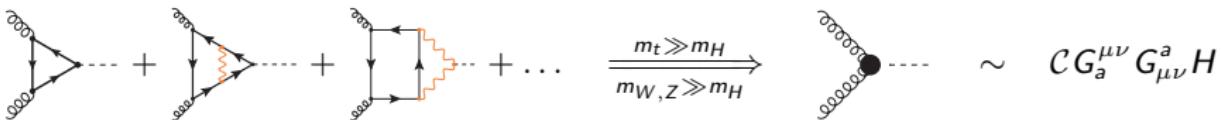
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NLO QCD corrections in HEFT

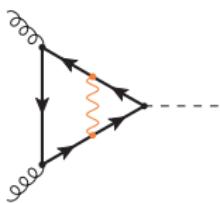


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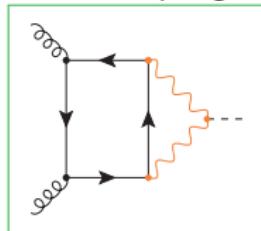
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NLO QCD corrections in HEFT

$$\text{Feynman diagram} + \text{Feynman diagram} + \text{Feynman diagram} + \dots \xrightarrow[m_t \gg m_H]{m_W, Z \gg m_H} \text{Feynman diagram} \sim \mathcal{C} G_a^{\mu\nu} G^a_{\mu\nu} H$$

- Wrong mass relations

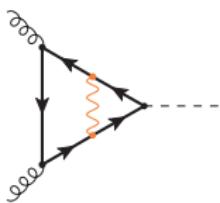
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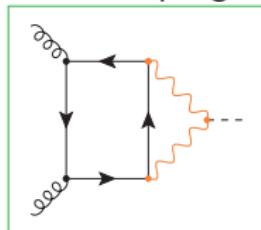
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- Wrong mass relations
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Exact NLO computation required

QCD-EW contributions at the LHC

$$\sigma_{PP \rightarrow H+j}(\mu) = \int_0^1 \int_0^1 dx_1 dx_2 f_{a/P}(x_1, \mu) f_{b/P}(x_2, \mu) \bar{\sigma}_{ab \rightarrow H+j}$$

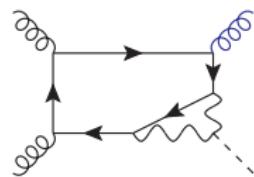
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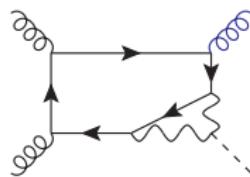
We consider $\alpha^2 v$ contributions

Partons	LO	NLO virtual	NLO real
$g \quad g$			
$g \quad q$ $g \quad \bar{q}$ $q \quad \bar{q}$			

Tensor decomposition: $gg \rightarrow Hg$



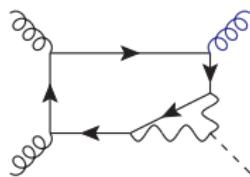
Tensor decomposition: $gg \rightarrow Hg$



Contributions from γ_5

Loop of massless quarks: sum over complete generations removes explicit γ_5 , rescaled couplings

Tensor decomposition: $gg \rightarrow Hg$



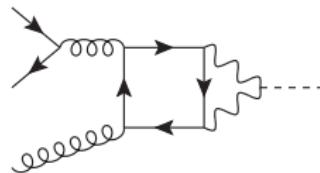
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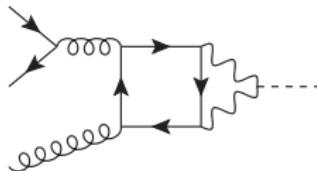
$$f^{c_1 c_2 c_3} \epsilon_{\lambda_1}^{\mu}(p_1) \epsilon_{\lambda_2}^{\nu}(p_2) \epsilon_{\lambda_3}^{\rho}(p_3) [\mathcal{F}_1 g_{\mu\nu} p_{2\rho} + \mathcal{F}_2 g_{\mu\rho} p_{2\nu} + \mathcal{F}_3 g_{\nu\rho} p_{2\mu} + \mathcal{F}_4 p_{3\mu} p_{1\nu} p_{2\rho}]$$

$$\mathcal{F}_j = 4 A_j(m_W^2) + \frac{2}{\cos^4 \theta_W} \left(\frac{5}{4} - \frac{7}{3} \sin^2 \theta_W + \frac{22}{9} \sin^4 \theta_W \right) A_j(m_Z^2)$$

Tensor decomposition $qg \rightarrow Hq$



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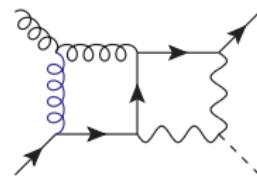
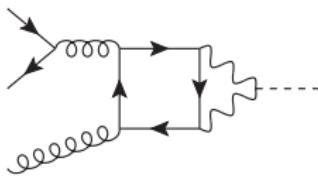
Contributions from γ_5

$\mathcal{F}_j^{\text{closed}}$ no γ_5 , rescaled couplings

$$T_{j_1 j_2}^{c_3} \bar{v}_{s_1}(p_1) \left[\tau_1^\mu \mathcal{F}_1^{\text{closed}} + \tau_2^\mu \mathcal{F}_2^{\text{closed}} \right] u_{s_2}(p_2) \epsilon_\mu^{\lambda_3}(p_3)$$

$$\mathcal{F}_j^{\text{closed}} = 4 A_j^{\text{closed}}(m_W^2) + \frac{2}{\cos^4 \theta_W} \left(\frac{5}{4} - \frac{7}{3} \sin^2 \theta_W + \frac{22}{9} \sin^4 \theta_W \right) A_j^{\text{closed}}(m_Z^2)$$

Tensor decomposition $qg \rightarrow Hq$



Contributions from γ_5

$\mathcal{F}_j^{\text{closed}}$ no γ_5 , rescaled couplings

$\mathcal{F}_j^{\text{open}}$ “polarized” coupling

$$T_{j_1 j_2}^{c_3} \bar{v}_{s_1}(p_1) \left[\tau_1^\mu (\mathcal{F}_1^{\text{closed}} + \mathbb{P}_C \mathcal{F}_{C,1}^{\text{open}}) + \tau_2^\mu (\mathcal{F}_2^{\text{closed}} + \mathbb{P}_C \mathcal{F}_{C,2}^{\text{open}}) \right] u_{s_2}(p_2) \epsilon_\mu^{\lambda_3}(p_3)$$

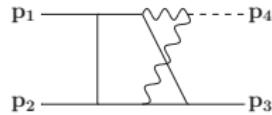
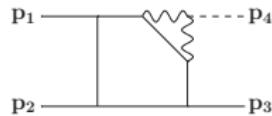
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$$\mathcal{F}_{R,j}^{\text{open}} = 1 A_j^{\text{open}}(m_W^2) + \frac{2}{\cos^4 \theta_W} (T_q - Q_q \sin^2 \theta_W)^2 A_j^{\text{open}}(m_Z^2)$$

$$\mathcal{F}_{L,j}^{\text{open}} = \frac{2}{\cos^4 \theta_W} Q_q^2 \sin^4 \theta_W A_j^{\text{open}}(m_Z^2)$$

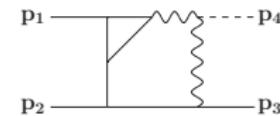
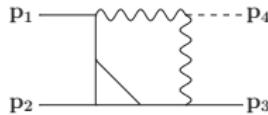
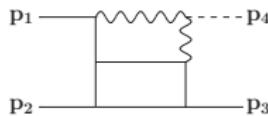
Reduction to MIs

$ggHg$



61 MIs

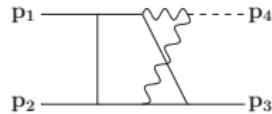
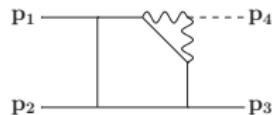
Additional for $qgH\bar{q}$



30 MIs

Reduction to MIs

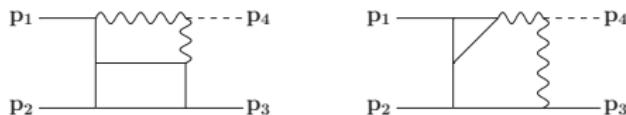
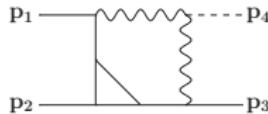
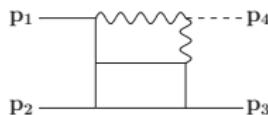
$ggHg$



61 MIs

4 square roots

Additional for $qgH\bar{q}$

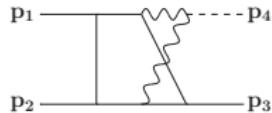
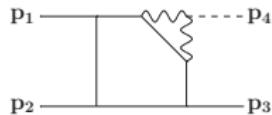


30 MIs

8 square roots

Reduction to MIs

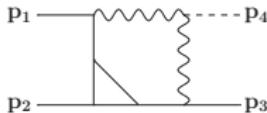
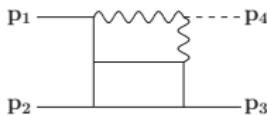
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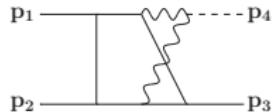
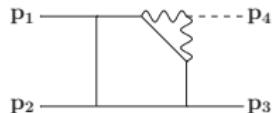
8 square roots

Differential equations for MIs

- Evolution in t , u , and m_V^2 , large-mass limit as boundary condition

Reduction to MIs

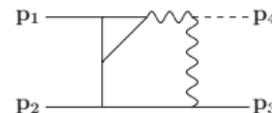
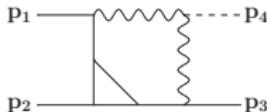
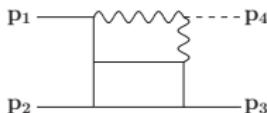
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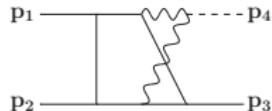
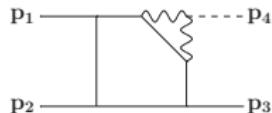
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Differential equations for MIs

- Evolution in t , u , and m_V^2 , large-mass limit as boundary condition
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- Single MIs contain at most 3 square roots and are rationalizable
- No global rationalization found

Reduction to MIs

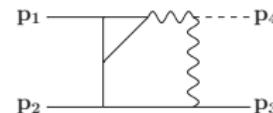
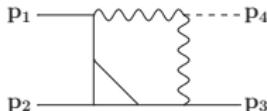
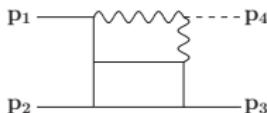
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DEs uneffective for $ggHg$ & $qgH\bar{q}$

Direct integration & linear reducibility

[Panzer,2014]

Direct integration over Feynman parameters

Direct integration & linear reducibility

[Panzer,2014]

Direct integration over Feynman parameters

$$\begin{aligned}
 \text{Diagram} &\propto \int_0^1 dx_2 \int_0^1 dx_1 \int_0^1 dx_3 \int_0^1 dx_4 \frac{\delta(1-X)}{[\sum x_j \mathcal{D}_j]^A} \\
 &\propto \int_0^1 dx_2 G(x_2 + \sqrt{\alpha x_2 + \beta} \dots; x_2) + \dots
 \end{aligned}$$

Direct integration & linear reducibility

[Panzer,2014]

Direct integration over Feynman parameters

$$\text{Diagram } \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \text{ with edges labeled } \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4 \quad \propto \int_0^1 dx_2 \int_0^1 dx_1 \int_0^1 dx_3 \int_0^1 dx_4 \frac{\delta(1-X)}{[\sum x_j \mathcal{D}_j]^A}$$

$$\propto \int_0^1 dx_2 G(x_2 + \sqrt{\alpha x_2 + \beta} \dots; x_2) + \dots$$

Linear reducibility

There exists an integration order for the kernel f_0

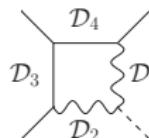
$$\int_0^{+\infty} dz_1 \cdots \int_0^{+\infty} dz_k f_0$$

such that each integral is a hyperlog of the next integration variable.

Direct integration & linear reducibility

[Panzer,2014]

Direct integration over Feynman parameters



$$\propto \int_0^1 dx_3 \int_0^1 dx_2 \int_0^1 dx_1 \int_0^1 dx_4 \frac{\delta(1-X)}{[\sum x_j \mathcal{D}_j]^A}$$

$$\propto \int_0^1 dx_3 G(\alpha + \sqrt{\beta} \dots; x_3) + \dots$$

Linear reducibility

There exists an integration order for the kernel f_0

$$\int_0^{+\infty} dz_1 \cdots \int_0^{+\infty} dz_k f_0$$

such that each integral is a hyperlog of the next integration variable.

- Integration over $d \log$ s: result as GPLs
- No integration variables under square roots: no rationalization needed

A quasi-finite basis

[Tarasov,1996][Lee,2010][von Manteuffel. . . ,2015]

- 2-loop MIs highly divergent: up to ϵ^{-4}
- Amplitudes well behaved: $ggHg$: ϵ^0 $qgH\bar{q}$: ϵ^{-2}

A quasi-finite basis

[Tarasov,1996][Lee,2010][von Manteuffel. . . ,2015]

- 2-loop MIs highly divergent: up to ϵ^{-4}
- Amplitudes well behaved: $ggHg: \epsilon^0$ $qgH\bar{q}: \epsilon^{-2}$

Quasi-finite basis

$$\mathcal{I}^{D+2}(a_1, \dots, a_7) = \frac{16}{s t u (D-4)(D-3)} \int \tilde{d}^D k_1 \tilde{d}^D k_1 \frac{G(k_1, k_2, p_1, p_2, p_3)}{\mathcal{D}_1^{a_1} \dots \mathcal{D}_7^{a_7}}$$

- **UV finiteness:** negative SDD by rising powers of (massive) propagators
- **IR finiteness:** Gram determinant cures soft & collinear divergences

A quasi-finite basis

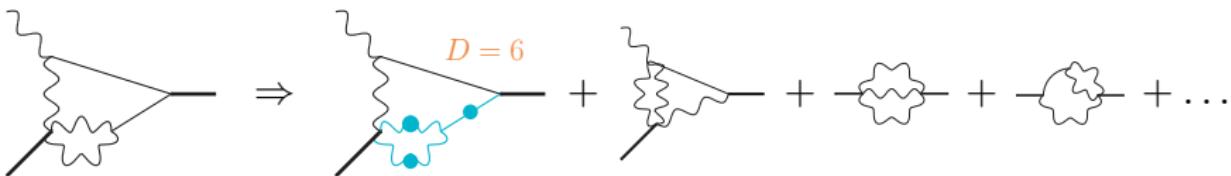
[Tarasov,1996][Lee,2010][von Manteuffel... ,2015]

- 2-loop MIs highly divergent: up to ϵ^{-4}
- Amplitudes well behaved: $ggHg: \epsilon^0$ $qgH\bar{q}: \epsilon^{-2}$

Quasi-finite basis

$$\mathcal{I}^{D+2}(a_1, \dots, a_7) = \frac{16}{s t u (D-4)(D-3)} \int \tilde{d}^D k_1 \tilde{d}^D k_1 \frac{G(k_1, k_2, p_1, p_2, p_3)}{\mathcal{D}_1^{a_1} \dots \mathcal{D}_7^{a_7}}$$

- **UV finiteness:** negative SDD by rising powers of (massive) propagators
- **IR finiteness:** Gram determinant cures soft & collinear divergences



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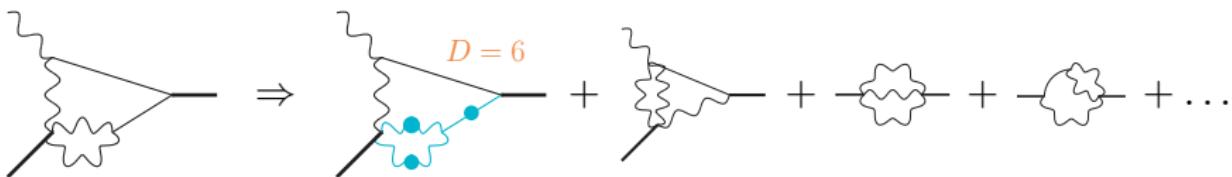
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- **UV finiteness:** negative SDD by rising powers of (massive) propagators
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- MIs are shifted into finite integrals and divergent sub-graphs
- Good choices do not worsen the poles in the coefficients

Simplifying the amplitude

[Duhr. . . ,2019][Heller. . . ,2021]

$\sim 1 \text{ GiB}$

$$\begin{aligned} A = & \frac{2y - x}{y^3 - x^2y} G_1 + \\ & \frac{x - 1}{y(y - x)} G_2 + \\ & \frac{-x^2 - xy + 2x - y}{y(x - y)(x + y)} G_3 + \dots \end{aligned}$$

Simplifying the amplitude

[Duhr... ,2019][Heller... ,2021]

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5526 (6789)

① Basis of algebraic prefactors

- Partial fraction decomposition (without extra poles)
- Basis of algebraic monomials
- Basis of algebraic prefactors

$$\begin{aligned}
 A = & \left[\frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{2} \frac{1}{y(x-y)} \right] G_1 + \\
 & \left[\frac{1}{y(x-y)} - \frac{1}{x-y} - \frac{1}{y} \right] G_2 + \\
 & \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] G_3 + \dots
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\downarrow
 5526 (6789)
 \downarrow
 690 (3823)

$$A = \left[\frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{2} \frac{1}{y(x-y)} \right] (G_1 + G_3) + \\ \left[\frac{1}{y(x-y)} - \frac{1}{x-y} - \frac{1}{y} \right] (G_2 + G_3) + \dots$$

Simplifying the amplitude

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5526 (6789)



690 (3823)

② Linearly independent transcendental expressions

- Numerical checks for zeroes & implementation
- PSLQ reduction



325 (983)

$\sim 1 \text{ MiB}$

$$\begin{aligned}
 A &= \left[\frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{2} \frac{1}{y(x-y)} \right] (G_1 + G_3) + \\
 &\quad \left[\frac{1}{y(x-y)} - \frac{1}{x-y} - \frac{1}{y} \right] (G_1 + G_3) + \dots \\
 &= \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] (G_1 + G_3) + \dots
 \end{aligned}$$

Simplifying the amplitude

[Duhr... ,2019][Heller... ,2021]

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③ Manipulation of GPLs

- GPLs into \log , Li_2 , Li_3 ...

$$A = \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] (C_1 \log a_1 + C_2 \log a_2) + \\ \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] (C_4 \text{Li}_2(b_1, b_2) + C_5 \log b_3 \log b_4) + \\ \left[\frac{1}{2} \frac{1}{y(x-y)} + \frac{3}{2} \frac{1}{y(x+y)} - \frac{1}{x-y} - \frac{1}{y} \right] (C_6 \log^3 c_1 + C_7 \zeta(3)) + \dots$$

$qgH\bar{q}$: finite remainder

$$\mathcal{F}^{\text{closed}} = \left(\frac{\alpha_S^0}{2\pi} \right) \mathcal{F}_1^{\text{closed}} \quad \mathcal{F}^{\text{open}} = \mathcal{F}_0^{\text{open}} + \left(\frac{\alpha_S^0}{2\pi} \right) [\mathcal{B} N_C + \mathcal{C} N_C^{-1}]$$

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- ① UV div. α_S renormalization only

$$\alpha_S^0 = \alpha_S \left(\frac{\mu_R^2}{\mu_0^2} \right)^\epsilon S_\epsilon^{-1} \left[1 + \alpha_S \frac{\beta_0}{\epsilon} + O(\alpha_S^2) \right]$$

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$$\tilde{\mathcal{F}}_1^{\text{open}} = I_{q\bar{q}g}^{(1)} \tilde{\mathcal{F}}_0^{\text{open}} + \mathcal{F}_{1,\text{fin}}$$

$$I_{q\bar{q}g}^{(1)} = \frac{-e^{\epsilon\gamma}}{4\Gamma(1-\epsilon)} \left\{ \mathcal{V}_q \frac{4C_F - 2C_A}{C_F} \left(-\frac{\mu_R^2}{s} \right)^\epsilon + \left(\mathcal{V}_g + \frac{C_A}{C_F} \mathcal{V}_q \right) \left[\left(-\frac{\mu_R^2}{t} \right)^\epsilon + \left(-\frac{\mu_R^2}{u} \right)^\epsilon \right] \right\}$$

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Helicity amplitudes

- Physical (and simpler) result

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$ggHg$

$$\mathcal{A}_{+++}^{ggHg} = \frac{m_H^2}{\sqrt{2}\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \frac{su}{m_H^2} \left(\mathcal{F}_1 + \frac{t}{u} \mathcal{F}_2 + \frac{t}{s} \mathcal{F}_3 + \frac{t}{2} \mathcal{F}_4 \right)$$

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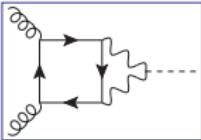
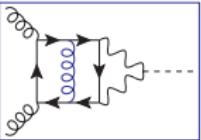
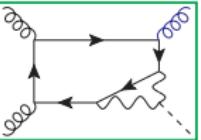
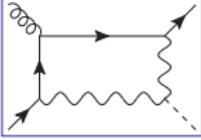
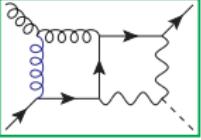
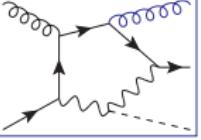
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- $\mathcal{F}_{2,m_V}(t, u) = \mathcal{F}_{1,m_V}(u, t)$

Conclusions & Outlook

Complete analytic results

Partons	LO	NLO virtual	NLO real
$g \ g$			
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$g \ \bar{q}$			
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The next steps

- Full $\sigma_{PP \rightarrow H+X}^{(\alpha_S^3 \alpha^2)}$ evaluation
- Top quark inclusion

$\sigma_{gg \rightarrow H+X}^{(\alpha_S^2 \alpha^2 + \alpha_S^3 \alpha^2)}$: [Beccetti... , 2020]



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New challenges

- Expression optimization

- Non-vanishing γ_5 contributions & masses

Thank you for your attention



Institute for
Theoretical
Particle Physics
and Cosmology

