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Top quark contribution to two-loop helicity amplitudes for W/Z boson pair production in gluon fusion

Christian Brønnum-Hansen | 26th of April | Loops and Legs 2022

Based on work with Chen-Yu Wang

Outline

1. **Motivation**

2. **Two-loop $gg \to WW/ZZ$ with full m_t dependence**
   - Numerical IBP reduction
   - Master integrals evaluated with the Auxiliary Mass Flow method

3. **Conclusion**

Motivation

- Higher experimental precision at the LHC requires more accurate theoretical predictions.
  $\rightarrow$ **more loops**
- As $\sqrt{s}$ increases, massive particles in the loops become more important.
  $\rightarrow$ **more masses**
- Precision wish list *Amoroso et al. 2020*

process	known	desired
$\vdots$	$\vdots$	$\vdots$
$pp \to H + t\bar{t}$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	NNLO_{QCD}
$\vdots$	$\vdots$	$\vdots$
$pp \to VV'$	$\text{NNLO}_{\text{QCD}} + \text{NNLO}_{\text{EW}}$ $+ \text{NLO}_{\text{QCD}}$ (gg channel)	NNLO_{QCD} (gg channel with massive loops)
$\vdots$	$\vdots$	$\vdots$
$pp \to t\bar{t}$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ NLO_{QCD} (with decays) NLO_{EW} (with decays)	NNLO_{QCD} (with decays) *Czakon, Mitov, et al. 2021*
$\vdots$	$\vdots$	$\vdots$

Motivation

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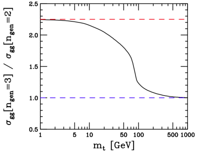
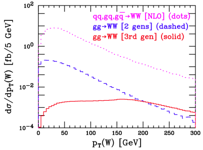
- background to $pp \to H^* \to WW/ZZ$, interference $\to$ Higgs width *Campbell et al. 2011a; Kauer and Passarino 2012; Caola and Melnikov 2013; Campbell et al. 2014; Azatov et al. 2015*
- anomalous gauge couplings

- gg channel: loop-induced: enters σ_{ppVV} at NNLO, enhanced by gluon flux & event selection *Binoth et al. 2006; Campbell et al. 2011b*
- massless NLO contribution: *Caola, Melnikov, Röntsch, et al. 2015, 2016*
  $\geq 50\%$ to σ_{ggZZ}, 50% to σ_{ggWW}
  $6-8\%$ to σ_{ppZZ}, 2% to σ_{ppWW}
- 3rd generation increases massless LO σ_{ggWW} by $10-13\%$ *Binoth et al. 2006; Campbell et al. 2011a*
- dominant contribution for high p_T *Campbell et al. 2011a*





Campbell et al. 2011a

$gg \to WW/ZZ$: Progress

- LO: one-loop *Glover and Bij 1989; Kao and Dicus 1991; Duhrssen et al. 2005; Binoth et al. 2006*
- NLO real: one-loop *Agrawal and Shivaji 2012; Melia et al. 2012; Campanario et al. 2013*
- NLO virtual: two-loop massless: $gg \to VV$ *Caola, Henn, et al. 2015; Manteuffel and Tancredi 2015*
- NLO virtual: two-loop massive:
  - region expansions: $gg \to ZZ$ *Melnikov and Dowling 2015; Gröber et al. 2019; Davies, Mishima, Steinhauser, and Wellmann 2020*
  - full m_t dependence: $gg \to ZZ$ *Agarwal et al. 2020; Brønnum-Hansen and Wang 2021b*
  - full m_t dependence: $gg \to WW$ *Brønnum-Hansen and Wang 2021a*

Analytic or alternative

Analytic

- deeper understanding (e.g. Parke-Taylor)
- fast, precise evaluation (e.g. Li_n, ${}_pF_q$, $K(\lambda)$, MPL, ...)
- wider applications (e.g. changing parameters)

Alternatives

- region expansions: $gg \to ZZ$ Melnikov and Dowling 2015; Davies, Mishima, Steinhauser, and Wellmann 2020, $gg \to ZH$ Davies, Mishima, and Steinhauser 2020, $gg \to HH$ Davies, Herren, et al. 2022 [Joshua Davies]
- numerical evaluation:
  - sector decomposition: $gg \to HH$ Borowka, Greiner, et al. 2016, $gg \to ZZ$ Agarwal et al. 2020, $gg \to ZH$ Chen, Heinrich, et al. 2020
  - differential equations: $gg \to t\bar{t}$ Chen, Czakon, et al. 2018, $gg \to H$ Czakon and Niggetiedt 2020 [Marco Niggetiedt]
- combined: $gg \to HH$ Davies, Heinrich, et al. 2019, $gg \to ZH$ Chen, Davies, et al. 2022 [Stephen Jones]

Analytic or numeric?

Analytic

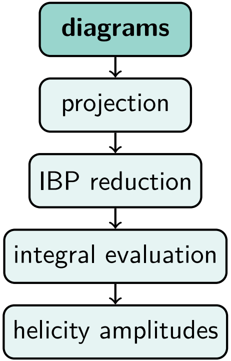
- deeper structure (e.g. Parke-Taylor)
- **fast, precise evaluation** (e.g. Li_n, ${}_pF_q$, $K(\lambda)$, MPL, ...)
- wider applications (e.g. changing parameters)

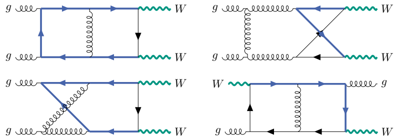
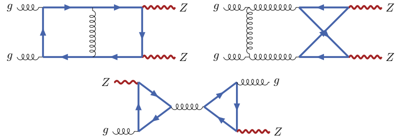
Alternatives

$$\left.\begin{array}{l}\text{fast numerical method that produces } \textbf{arbitrary precision} \text{ result} \\ \text{good } \textbf{interpolation} \text{ algorithm (or fixed grid [Jérémie Quarroz])}\end{array}\right\} \Rightarrow \begin{array}{l}\text{fast, precise evaluation} \\ \text{in the parameter space}\end{array}$$

- Numeric IBP reduction: set masses and kinematic variables to rational numbers
- Numeric DE evaluation: simple boundary condition + arbitrary precision *Lee et al. 2018; Liu, Ma, and Wang 2018; Abreu et al. 2020; Hidding 2020; Moriello 2020*

$gg \to VV$: Diagrams



	$gg \to WW$	$gg \to ZZ$
		
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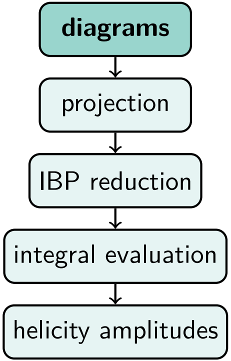
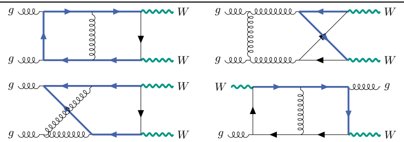
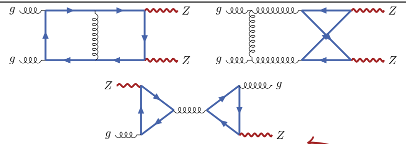
- Decomposition in colour factors: $A = \delta^{AB}\left[C_A A^{[C_A]} + C_F A^{[C_F]} + A^{[\triangle^2]}\right]$
- No intermediate Z- or Higgs-boson diagrams. Spira et al. 1995; Harlander and Kant 2005; Binoth et al. 2006; Aglietti et al. 2007; Anastasiou et al. 2007
- IR structure: $A^{(2)}(\epsilon,\mu) = \boldsymbol{I}^{(1)}(\epsilon,\mu)A^{(1)}(\epsilon,\mu) + F^{(2)}(\epsilon,\mu)$. Catani 1998

$gg \to VV$: Diagrams



	$gg \to WW$	$gg \to ZZ$
		
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$gg \to VV$: Projection

diagrams

↓

projection

↓

IBP reduction

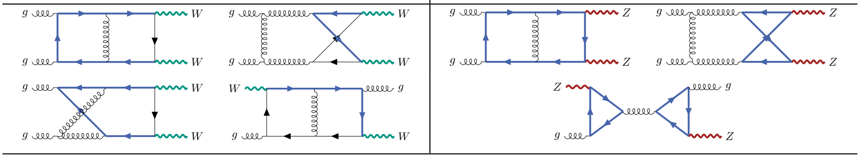
↓

integral evaluation

↓

helicity amplitudes

$gg \to WW$	$gg \to ZZ$



- γ_5 scheme:
  - *WW* and $\triangle^2$ diagrams: Larin scheme *Larin and Vermaseren 1991; Moch et al. 2015.*
  - two-loop ZZ: naive scheme, thanks to Furry's theorem.
- Projection onto 36 tensor structures *Binoth et al. 2006*

$$A = \sum_{l=1}^{18} A_l \underbrace{T_l}_{\text{parity even}} + \sum_{l=19}^{36} A_l \underbrace{S_l}_{\text{parity odd}}$$

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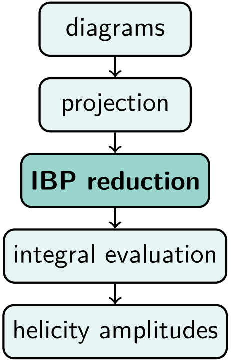
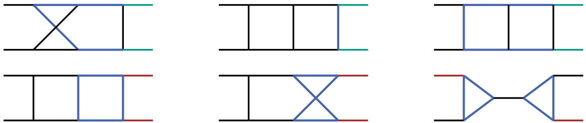
$gg \to VV$: IBP reduction



Find symmetry relations with `REDUZE 2` *Manteuffel and Studerus 2012.*



Reduction performed **at each phase space point** with `KIRA 2.2`: *Klappert et al. 2020; Lange et al. 2021:*

- parametric only in d
  - $m_t = 173$ GeV, $m_W = 80$ GeV, $m_Z = 91$ GeV
  - rational values for s, t
- avoid non-factorisable denominators *Smirnov and Smirnov 2020; Usovitsch 2020*

$gg \to VV$: IBP reduction

diagrams

↓

projection

↓

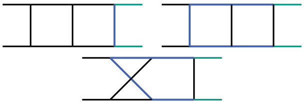
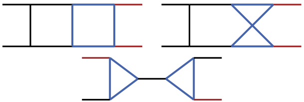
IBP reduction

↓

integral evaluation

↓

helicity amplitudes

	$gg \to WW$	$gg \to ZZ$
		
IBP families	33	21
rank	5	4
MI families	22	15
MIs	334	205
CPU h / point	8	3
GB memory	5	2

- lower the memory/storage consumption significantly
- straightforward parallelisation

$gg \to VV$: Master integral evaluation

- diagrams
- ↓
- projection
- ↓
- IBP reduction
- ↓
- **MI evaluation**
- ↓
- helicity amplitudes

- Based on the **auxiliary mass flow method** *Liu, Ma, and Wang 2018; Liu, Ma, Tao, et al. 2020; Liu and Ma 2022*

$$I \propto \lim_{\eta \to 0^+} \int \prod_{i=1}^{2} \mathrm{d}^d k_i \prod_{a=1}^{9} \frac{1}{[q_a^2 - (m_a^2 - i\eta)]^{\nu_a}}$$

- Add an imaginary part to the **top quark mass**

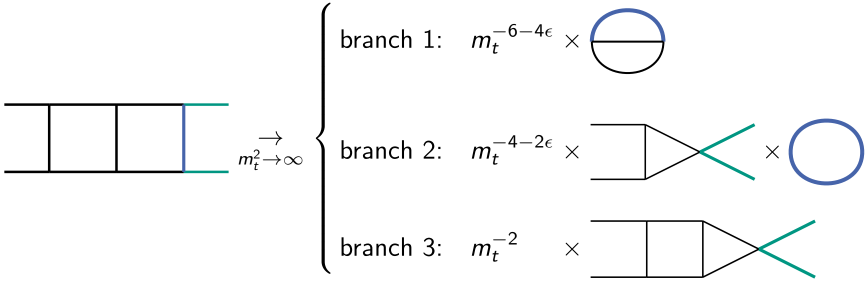
$$m_t^2 \to m_t^2 - i\eta.$$

- Solve differential equations w.r.t the mass

$$\partial_x \boldsymbol{I} = \boldsymbol{M}\boldsymbol{I}, \quad x \propto -i\eta$$

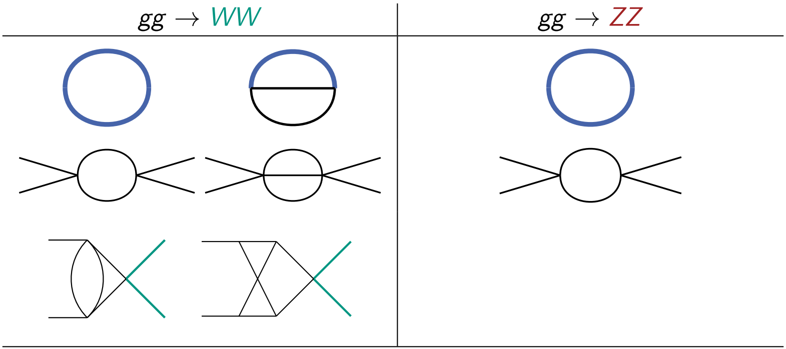
with boundary condition at $x \to -i\infty$. Physical mass at $x \to 0$.

$gg \to VV$: Master integral evaluation: boundary



- Boundary example: double box with one internal massive line
- Collect and reduce boundary integrals for all masters

$gg \to VV$: Master integral evaluation: boundary



- All boundary integrals known analytically

't Hooft and Veltman 1979; Chetyrkin et al. 1980; Scharf and Tausk 1994; Gehrmann and Remiddi 2000; Gehrmann, Huber, et al. 2005

$gg \to VV$: Master integral evaluation

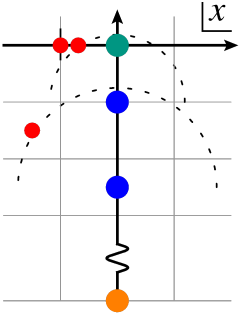
- Expand $\boldsymbol{I}$ around **boundary** in variable $y = x^{-1} = 0$:

$$\boldsymbol{I} = \sum_j^M \epsilon^j \sum_k^N \sum_l \boldsymbol{c}_{jkl} y^k \ln^l y + \ldots$$

- Evaluate and expand around **regular points**:

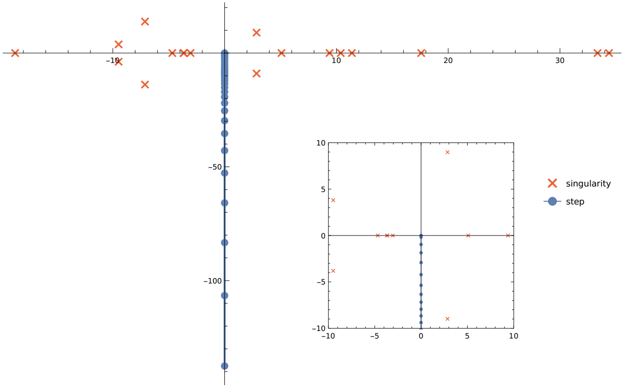
$$\boldsymbol{I} = \sum_j^M \epsilon^j \sum_{k=0}^N \boldsymbol{c}_{jk} x'^k + \ldots$$

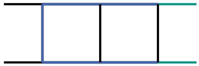
- Evaluate at the **physical point.** $x = 0 \leftarrow$ **regular point**
- **Path** is fixed by **singularities** and desired precision.



$m_t^2 \to m_t^2(1+x)$

$gg \to VV$: Master integral evaluation





- Steps for a double-box integral with five massive propagators

$gg \to VV$: Master integral evaluation

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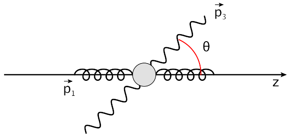
- Numerical evaluation of integrals point by point, ~ 1 CPU hour per phase space point

	$gg \to WW$	$gg \to ZZ$
accurate digits	~ 15 digits	~ 20 digits

- Expected relative error is $\left(\frac{\Delta}{R}\right)^N \Rightarrow$ *arbitrary* precision attainable
  for step size Δ respective to convergence radius R and expansion order N
- Self-consistency check with DEs in s and t
- Cross-checks with pySecDec *Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, and Zirke 2018; Borowka, Heinrich, Jahn, Jones, Kerner, and Schlenk 2019*

$gg \to VV$: Helicity amplitudes

diagrams
↓
projection
↓
IBP reduction
↓
MI evaluation
↓
helicity amplitudes



- Phase space parametrised in terms of the relative velocity β and scattering angle θ

$$s = \frac{4m_W^2}{1-\beta^2}, \qquad t = m_W^2 - \frac{s}{2}\left(1-\beta\cos\theta\right).$$

- Massive boson polarisation vectors written in terms of decay currents

$$\epsilon_{3,L}^{*\mu} = \langle 5|\gamma^\mu|6], \qquad \epsilon_{4,L}^{*\mu} = \langle 7|\gamma^\mu|8]$$

$gg \to WW$

- $\sqrt{s} \approx 367$ GeV
- $\theta \approx 36.9^\circ$
- $\mu = m_W$

$\mathbf{C_A}$		ϵ^{-2}	ϵ^{-1}
LLLL	$A^{(2)}/A^{(1)}$	$1.00000000023 - 3.0 \cdot 10^{-11} i$	$-4.94945452453 + 3.89380807761 i$
	IR pole	1.00000000000	$-4.94945452593 + 3.89380807754 i$
LRLL	$A^{(2)}/A^{(1)}$	$0.99999999815 - 1.6 \cdot 10^{-9} i$	$-4.29712348534 + 9.47440879823 i$
	IR pole	1.00000000000	$-4.29712347965 + 9.47440880940 i$

$gg \to ZZ$

- $\sqrt{s} \approx 210$ GeV
- $\theta \approx 114^\circ$
- $\mu = m_Z$

$\mathbf{C_A}$		ϵ^{-2}	ϵ^{-1}
LLLL	$A^{(2)}/A^{(1)}$	$1.0000000000008 - 7.6 \cdot 10^{-13} i$	$0.8304916142577 + 3.229874368770 i$
	IR pole	1.0000000000000	$0.8304916142539 + 3.229874368771 i$
LRLL	$A^{(2)}/A^{(1)}$	$1.0000000000009 - 1.4 \cdot 10^{-12} i$	$0.2359507533599 + 2.885154863850 i$
	IR pole	1.0000000000000	$0.2359507533772 + 2.885154863852 i$

$$\boldsymbol{I}^{(1)}(\epsilon,\mu) A^{(1)}(\epsilon,\mu) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \left(\frac{C_A}{\epsilon^2} + \frac{\frac{11}{6} C_A}{\epsilon} \right) \left(\frac{\mu^2 e^{i\pi}}{s} \right)^{\epsilon} A^{(1)}(\epsilon,\mu)$$

$gg \to WW$

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$gg \to ZZ$

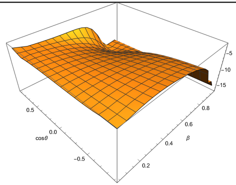
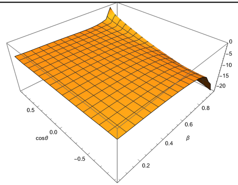
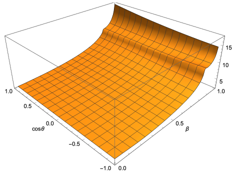
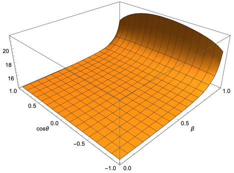
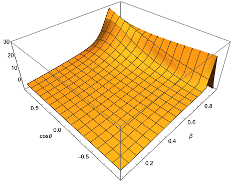
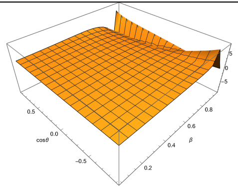
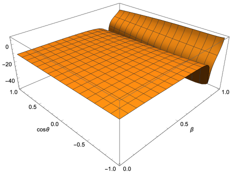
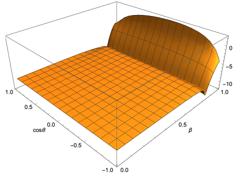
- $\sqrt{s} \approx 210$ GeV
- $\theta \approx 114^\circ$
- $\mu = m_Z$

C_A		ϵ^{-2}	ϵ^{-1}
LLLL	$A^{(2)}/A^{(1)}$	1.0000000000008 –	
	IR pole	1.0000000000000	
LRLL	$A^{(2)}/A^{(1)}$	1.0000000000009 –	
	IR pole	1.0000000000000	

$$I^{(1)}(\epsilon,\mu)A^{(1)}(\epsilon,\mu) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left(\frac{C_A}{\epsilon^2} + \frac{\frac{11}{6}C_{A}}{\epsilon}\right.$$

$gg \to ZZ$ finite terms cross-checked against *Agarwal et al. 2020*
- analytic IBP reduction
- sector decomposition

and against *Davies, Mishima, Steinhauser, and Wellmann 2020*
- low- and high-energy expansions

	$gg \to WW$ $LLLL$	$gg \to WW$ $LRLL$	$gg \to ZZ$ $LLLL$	$gg \to ZZ$ $LRLL$
$\mathbf{C_A}$				
$\mathbf{C_F}$				

$$\frac{2\,\mathrm{Re}\left[F^{(2)}A^{(1)\star}\right]}{|A^{(1)}|^2}$$ as a function of $\beta = \sqrt{1 - 4m_V^2/s}$ and scattering angle $\cos\theta$

Conclusion

- We calculated two-loop helicity amplitudes for $gg \to WW/ZZ$ with full m_t dependence.
- **Numeric IBP reduction** is efficient in practice for complicated multi-scale processes.
- **Auxiliary mass flow** method provides an efficient and precise way to evaluate multi-loop integrals.
- Future work: Impact on the cross section as well as interference with Higgs.

Thank you for your attention!

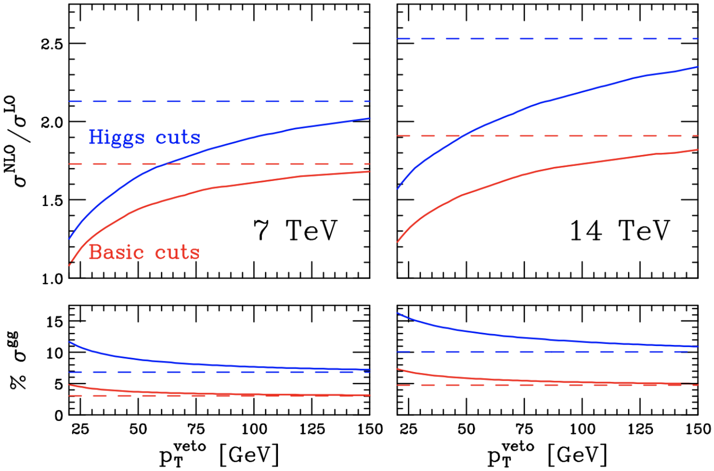


Figure 10: The ratio of NLO to LO (upper) and the percentage of the NLO cross section from the gg initial state (lower) for $WW \to e^+\mu^-\nu_e\overline{\nu}_\mu$ production, as a function of the jet-veto p_T^{veto}. Results are shown using the basic cuts of Eq. (7.6) (upper, blue curves) and the Higgs search cuts of Eq. (7.7) (lower, red curves). The NLO to LO ratio and gluon percentage with no veto applied are shown as dashed lines on the plot. The dashed lines are thus the asymptotic values of the solid curves.

$\overline{q}_1 q_2 W$ vertex introduces axial current

$$i\overline{q}_1\gamma_\mu \frac{1-\gamma_5}{2} q_2 W^\mu. \tag{1}$$

γ_5 in d-dimensions through Larin-scheme *Larin and Vermaseren 1991; Moch et al. 2015*

$$\gamma_\mu\gamma_5 = -\frac{1}{3!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\nu\gamma^\rho\gamma^\sigma \tag{2}$$

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\alpha\beta\kappa\lambda} = \begin{vmatrix} \delta^\mu_\alpha & \delta^\mu_\beta & \delta^\mu_\kappa & \delta^\mu_\lambda \\ \delta^\nu_\alpha & \delta^\nu_\beta & \delta^\nu_\kappa & \delta^\nu_\lambda \\ \delta^\rho_\alpha & \delta^\rho_\beta & \delta^\rho_\kappa & \delta^\rho_\lambda \\ \delta^\sigma_\alpha & \delta^\sigma_\beta & \delta^\sigma_\kappa & \delta^\sigma_\lambda \end{vmatrix} \tag{3}$$

finite renormalisation to restore axial Ward identities *Larin 1993*

$$J^A_\mu = Z_5 J^A_{\mu,\,b} = \left[1 - \frac{\alpha_s}{2\pi} 2C_F + \mathcal{O}(\alpha_s^2)\right] J^A_{\mu,\,b} \tag{4}$$

projection onto 38 tensor structures T_I and S_I Binoth et al. 2006; Manteuffel and Tancredi 2015

$$A(\{p_i\},\{\epsilon_j\},m_t) = \sum_{I=1}^{20} A_I(s,t,m_W,m_t)\ T_I(\{p_i\},\{\epsilon_j\}) + \sum_{I=21}^{38} A_I(s,t,m_W,m_t)\ S_I(\{p_i\},\{\epsilon_j\}). \tag{5}$$

define projectors to obtain form factors A_I

$$P_J = \sum_{I=1}^{20} a_{JI}\ T_I^*(\{p_i\},\{\epsilon_j\}) + \sum_{I=21}^{38} b_{JI}\ S_I^*(\{p_i\},\{\epsilon_j\})$$
$$A_I = \sum_{\text{pol}} P_I A(\{p_i\},\{\epsilon_j\},m_t) \tag{6}$$
$$\text{e.g.}\ \sum_{\text{pol}} \epsilon_1^\mu \epsilon_1^{\nu *} = -\eta^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{p_1 \cdot p_2}$$

expected relative error: $\left(\frac{\Delta}{R}\right)^N$

- N, order of expansion
- Δ/R size of step Δ in terms of radius of convergence R

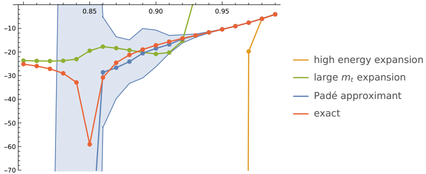
table of correct digits compared to analytic result:

Δ/R, # — N	4	8	16	32
1, 1	0	1	1	3
$\frac{1}{2}$, 3	2	3	5	11
$\frac{1}{4}$, 9	3	6	10	20
$\frac{1}{8}$, 23	4	8	17	31
$\frac{1}{16}$, 58	5	11	20	41
$\frac{1}{32}$, 139	7	13	25	49

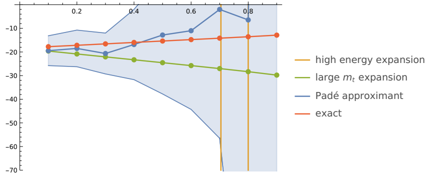
- # number of intermediate steps required

$gg \to ZZ$: comparison to series expansion results

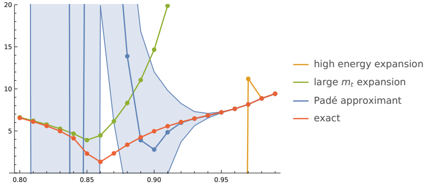
Karlsruhe Institute of Technology



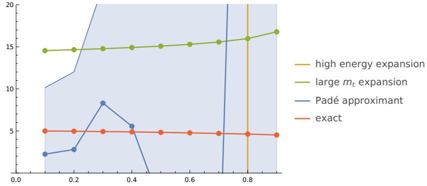
$0.8 \leq \beta \leq 0.99$, $\cos\theta = 0.2$, LLLL



$\beta = 0.9$, $0.1 \leq \cos\theta \leq 0.9$, LLLL



$0.8 \leq \beta \leq 0.99$, $\cos\theta = 0.2$, LRLL



$\beta = 0.9$, $0.1 \leq \cos\theta \leq 0.9$, LRLL

Backup

Comparison with *Davies, Mishima, Steinhauser, and Wellmann 2020*