Massive Scattering Amplitudes

Ekta Chaubey
University of Turin
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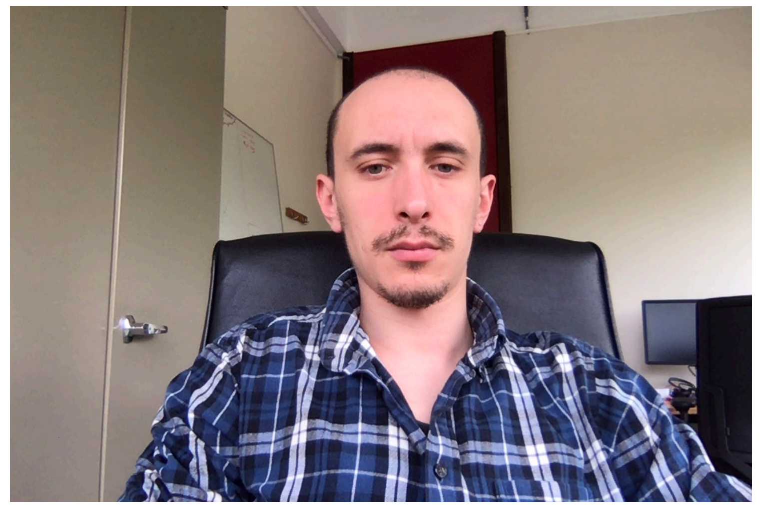
UNIVERSITÀ DEGLI STUDI DI TORINO

INFN
Istituto Nazionale di Fisica Nucleare

Collaborators: Simon Badger, Matteo Becchetti, Bayu Hartanto, Robin Marzucca, Francesco Sarandrea

Simon Badger



Matteo Becchetti

Robin Marzucca

Bayu Hartanto

Francesco Sarandrea

Era of precision physics

High-multiplicity scattering amplitude contributions

Impressive results for massless

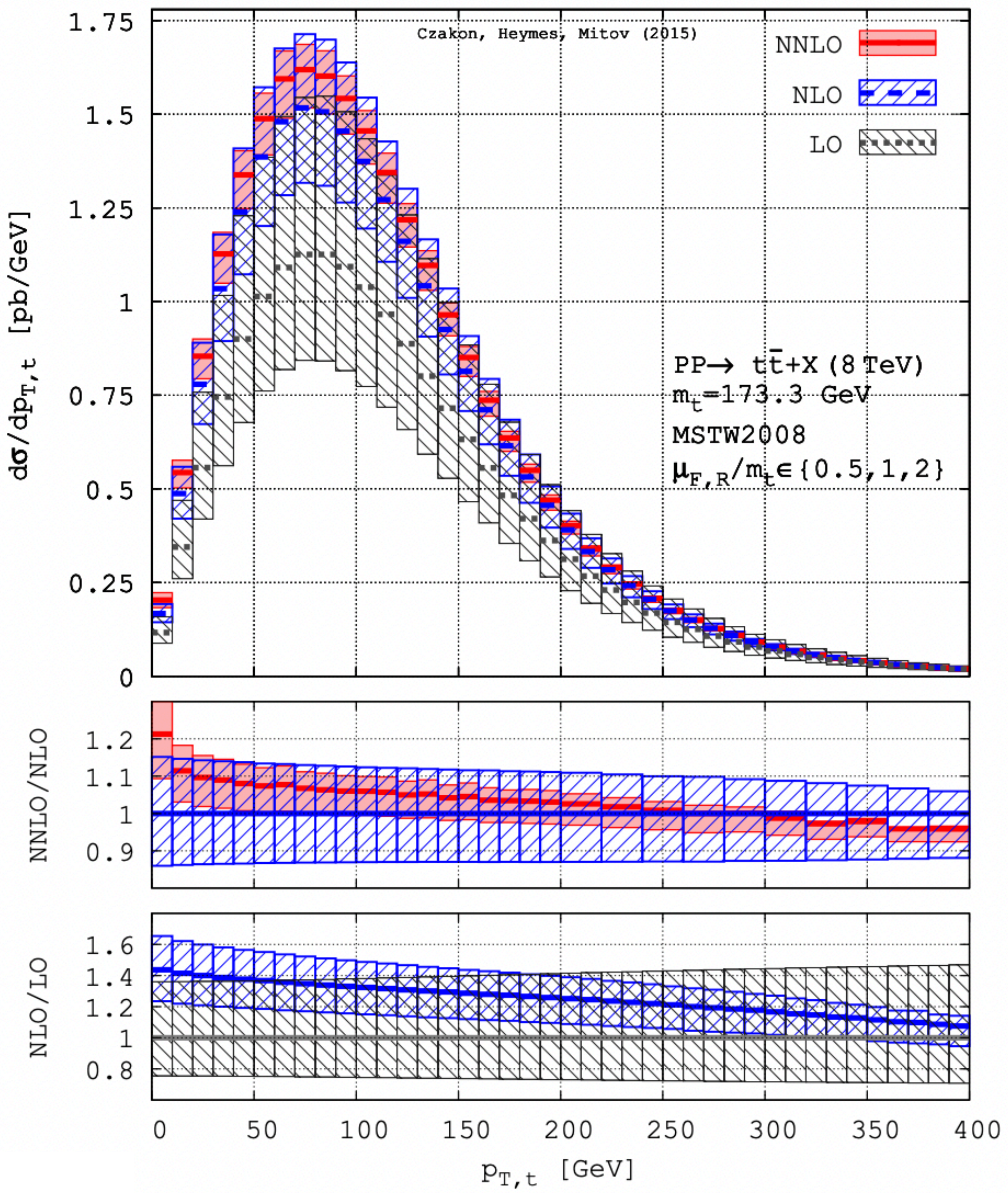
Corrections due to massive particles

Complicated analytic structure

Requires more mathematical understanding

Precision physics

- NNLO corrections important for % level predictions
- Analytic computation of multi-loop Feynman integrals still a challenging task
- Good control with many $2 \longrightarrow 2$ processes. Including more legs a leap in complexity.



Top-antitop p_T distribution in LO, NLO and NNLO QCD. Error bands from scale variation only [Czakon, Heymes, Mitov ; 2015]

Precision Wish-list

[Les Houches; 2019]

	Available	Required
$pp \to t\bar{t} + j$	NLO_{QCD} (w/ decays) NLO_{EW}	Top mass NNLO_{QCD} + NLO_{EW} (w/ decays)
$pp \to t\bar{t} + Z$	NLO_{QCD} + NLO_{EW}	NNLO_{QCD} + NLO_{EW} (w/ decays) Anomalous EW couplings
$pp \to t\bar{t} + W$	NLO_{QCD} NLO_{EW}	NNLO_{QCD} + NLO_{EW} (w/ decays)
$pp \to t\bar{t} + H$	NLO_{QCD} + NLO_{EW}	NNLO_{QCD} Top Yukawa

Questions

- Many invariants; internal masses
- Are the Feynman integrals well-known/ easily obtainable?

Analytics

- Kind of analytic structure?
- Do they help our mathematical understanding of Feynman integrals?
- Do they improve the computations for physical observables?

Practical challenges

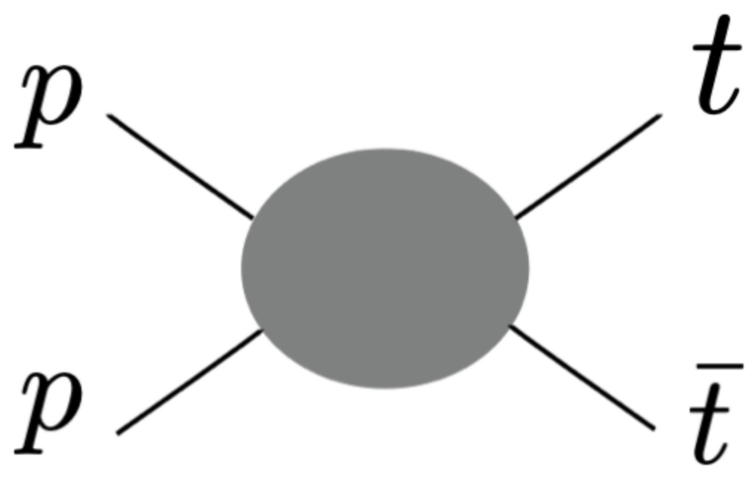
- Tools to set-up the computation/ automation?

Outline

- General methodology
- NNLO scattering amplitudes in QCD

$2 \longrightarrow 2$ scattering

Analytic form of the 2-loop amplitude

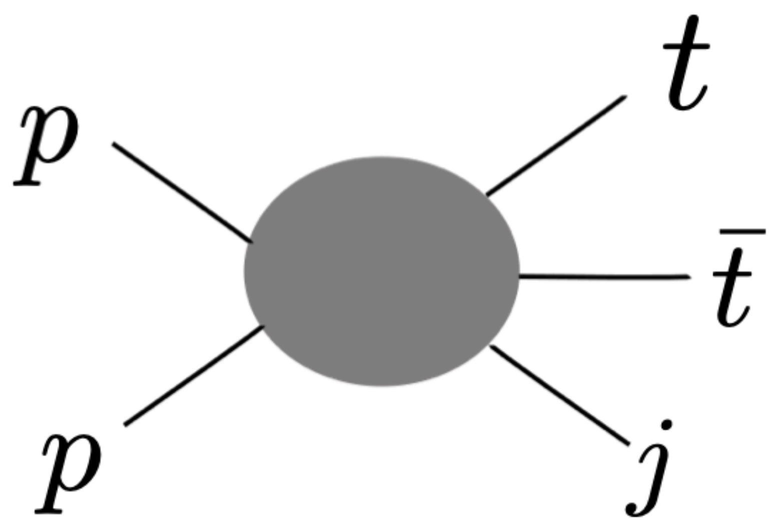


Functional relations

↕

$2 \longrightarrow 3$ scattering

1-loop amplitude; ingredient for NNLO corrections



Extension to higher-multiplicity

Workflow

Amplitude reduction

After colour ordering and helicity amplitude processing:

$$A^{(L),h} = \int \prod_{j=1}^{L} d^d k_j \frac{N_T^h}{\prod_{\alpha \in T} D_\alpha}$$

Suitable for IBP:

$$A^{(L),h} = \sum_T \sum_i c_{T,i}^h G_{T,i}$$

Modular arithmetic FiniteFlow

[Peraro, '19]

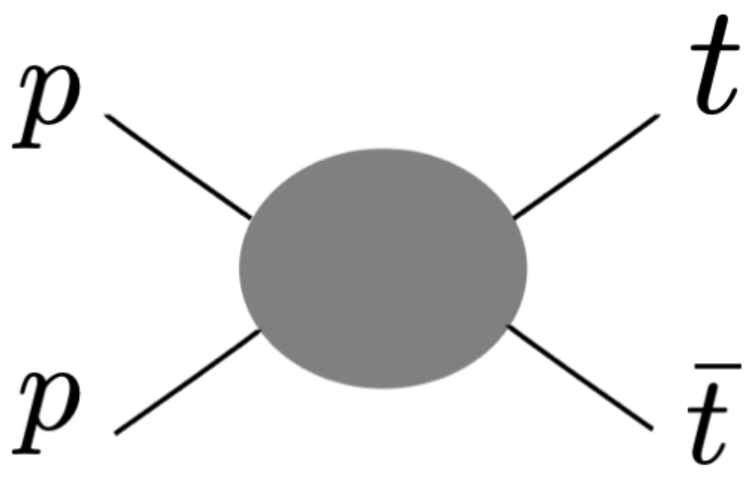
In terms of Master Integrals (MIs):

$$A^{(L),h} = \sum_k c_k^{IBP,h} MI_k$$

In terms of special functions:

$$A^{(L),h} = \sum_k \sum_{l=n(L)}^{0} \epsilon^l c_{kl}^h m_k + O(\epsilon)$$

4-point massive 2-loop $t\bar{t}$ amplitudes

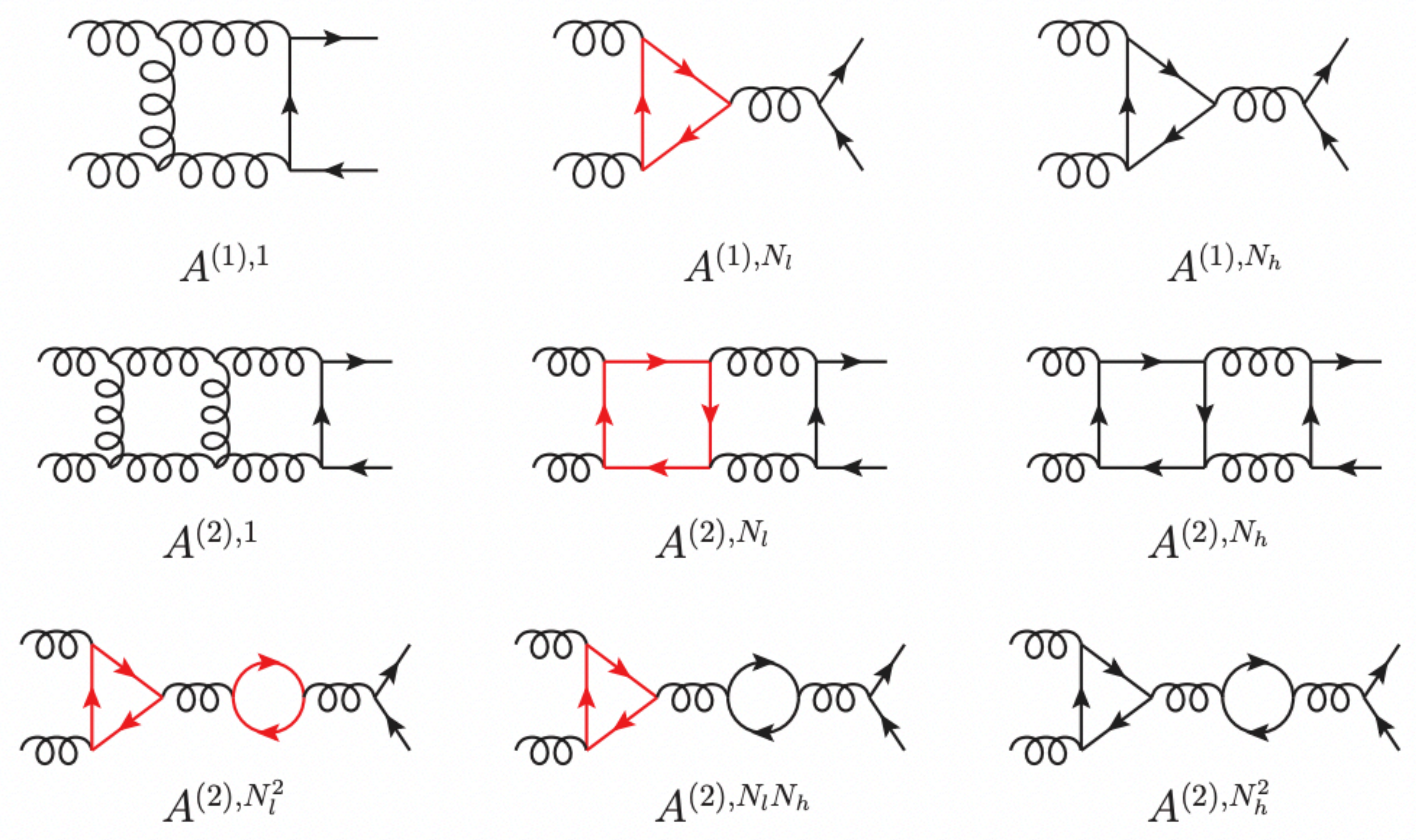


- Numerical solutions successful even if computationally intensive [Baernreuther, Czakon, Chen, Fiedler, Poncelet; 2008-2018]
- Analytic solutions for qq ⟶ tt and all non-elliptic sectors for gg ⟶ tt known [Bonciani, Ferroglia, Gehrmann, Studerus, von Manteuffel, Di Vita, Laporta, Mastrolia, Primo, Schubert, Becchetti, Casconi, Lavacca; 2009-2019]

[Mandal, Mastrolia, Ronca, J. Torres Bobadilla ;2022]

Helicity amplitudes

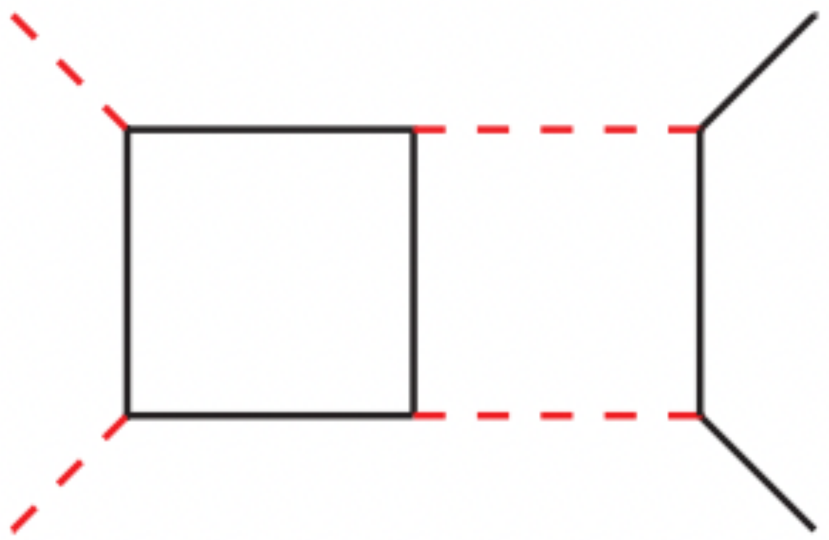
[Badger, E. Chaubey, Hartanto, Marzucca; 2020]



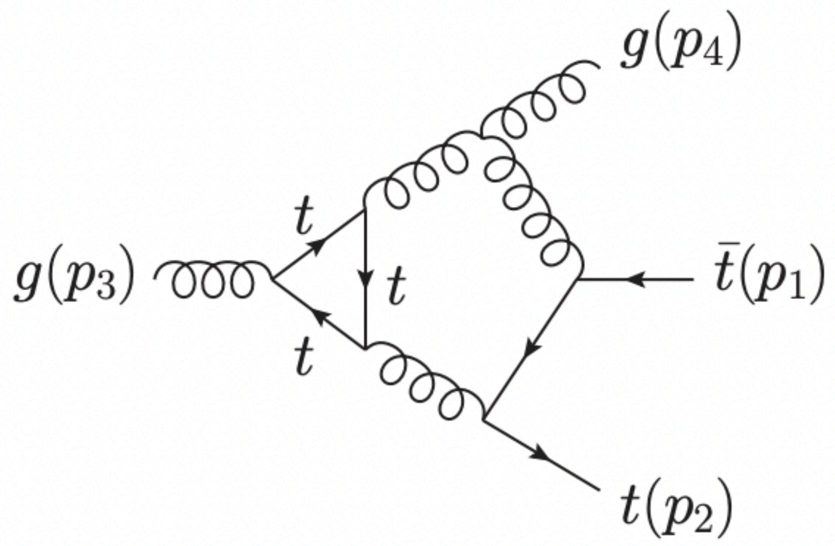
1. Include full top-quarks efficiently; set of analytic LC helicity amplitudes for corrections to top-quark pair production via gluon fusion at 2 loops in QCD
2. Obtain rational parametrisation in terms of only 2 variables, using momentum twistors; algebra performed with FF

Master integrals

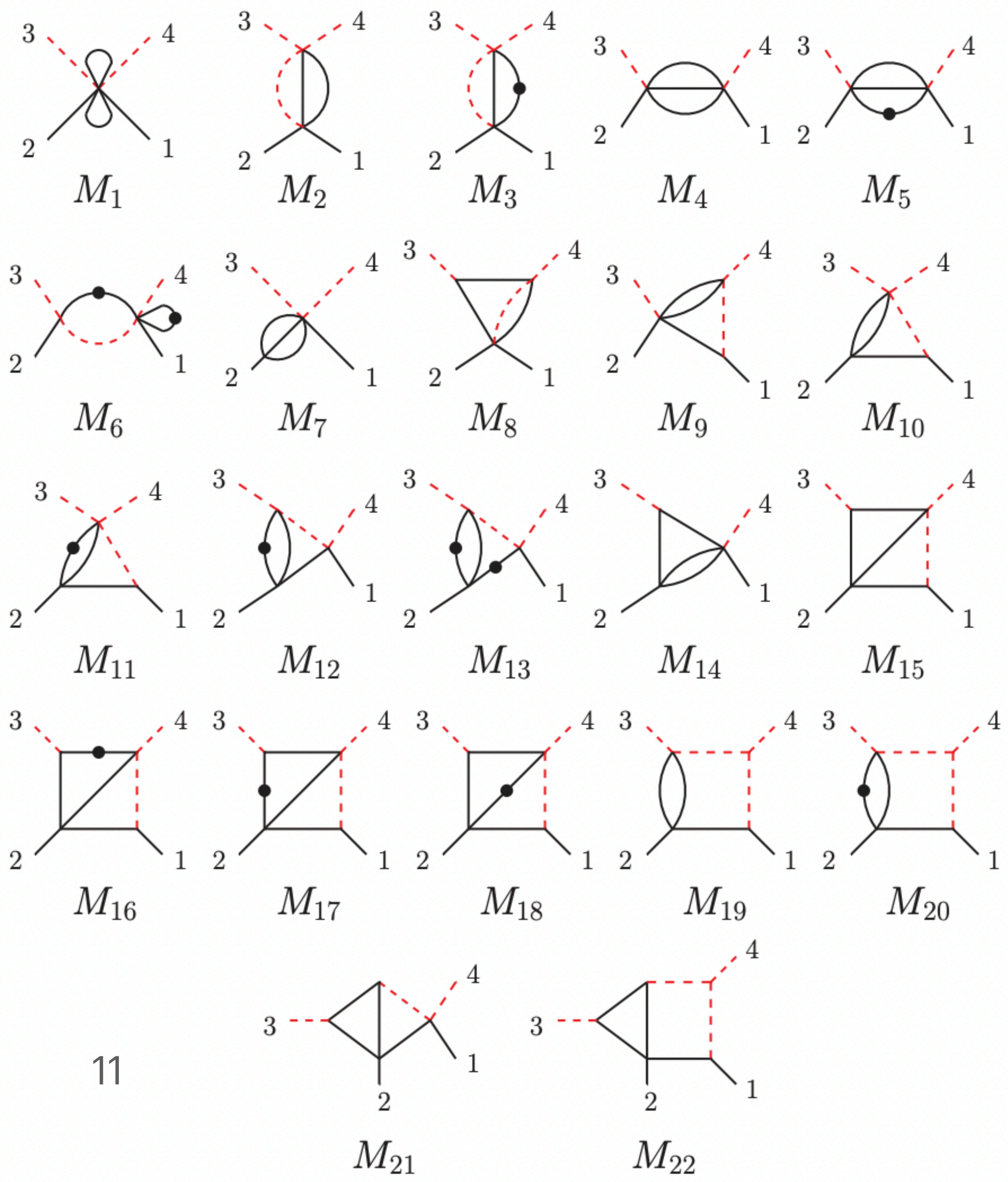
- Top-box integral computed analytically in terms of iterated integrals over 3 **elliptic curves**



[Adams, E. Chaubey, Weinzierl; 2018]



One previously unknown integral in the amplitude



Canonical form DE derived, obtained iterated integral form and boundary constants

Analytic finite remainders

Additional function relations necessary to cancel IR poles - beyond shuffle relations

$$
\begin{aligned}
I(a^{(b)}_{3,3}, f, \dots) &= \int a^{(b)}_{3,3} I(f, \dots) \\
&= \int d\left(\psi_1^{(b)} \frac{x(-1+y)}{\pi(-1+x)^2}\right) \cdot I(f, \dots) \\
&= \left[\psi_1^{(b)} \frac{x(-1+y)}{\pi(-1+x)^2} I(f, \dots)\right]_{(0,1)}^{(x,y)} - I\left(\psi_1^{(b)} \frac{x(-1+y)}{\pi(-1+x)^2} f, \dots\right)
\end{aligned}
$$

Stefan's talk!

Direct reconstruction of finite remainders

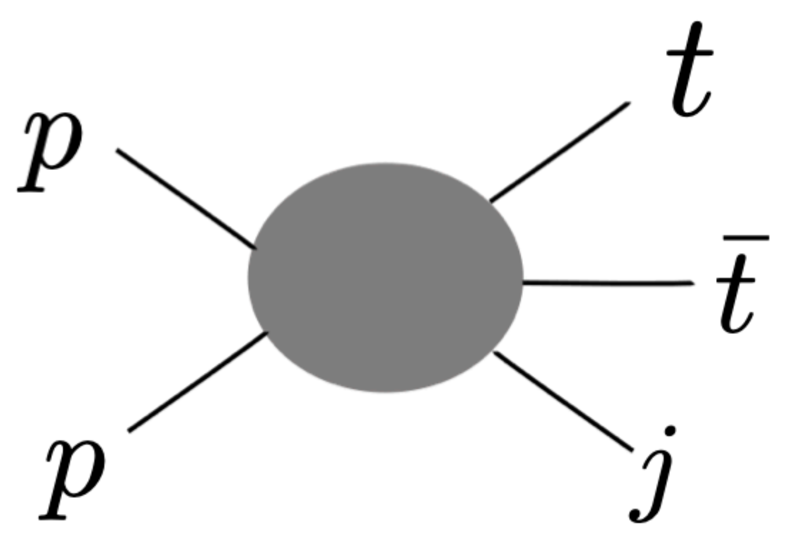
	Monomials	Monomials with relations
Amplitude	12025	11791
Finite remainder	3586	3158

Test evaluations match with previous numerical results [Baernreuther, Czakon, Fiedler; 13]

Analytic continuation of iterated integrals in physical region needs further investigation

5-pt scattering

Best approach?



Inclusion of masses $\longrightarrow$ step-up in analytic complexity compared to previously considered 2-loop 5-point amplitudes

Non-elliptic $\not\longrightarrow$ "simple" using standard methods.

Complicated alphabet, many square roots, branch cuts

Expressible in terms of polylogarithms? [Duhr, Brown '20]

How difficult to analytically continue?

The semi-numerical approach

Obtain results quickly and with high precision along with learning about nature of the problem

Analytic property

Using a canonical form for the MIs and using DiffExp [Hidding; 2020] to compute them numerically still provides opportunity to extract poles analytically

1-loop amplitude for tt-jet

[Badger, Becchetti, E. Chaubey, Marzucca, Sarandrea; 2021]

- Analytic helicity amplitudes for this process not present. $O(\epsilon)$ known numerically.
- Gauge the sense of complexity arising in an analytic two-loop computation of $pp \to t\bar{t}j$

- Analytic helicity amplitudes for 1-loop QCD corrections ⟶ Previously missing ingredient for NNLO, expansion of the 1-loop helicity amplitudes up to $O(\epsilon^2)$

  Already much more complicated than what is done at 1-loop
- Includes decay information for top-quark pair in narrow-width approximation

See also [N. Syrrakos '202

Helicity amplitudes

$$u_+(p,m) = \frac{(\not{p} + m)|n\rangle}{\langle p^\flat n\rangle}$$

$$A_x^{(L)}(1_t^+, 2_{\bar{t}}^+, 3^{h_3}, 4^{h_4}, 5^{h_5}; n_1, n_2) = m_t \Phi(3^{h_3}, 4^{h_4}, 5^{h_5})$$
$$\sum_{i=1}^{4} \Theta_i(1, 2; n_1, n_2) A_x^{(L),[i]}(1_t^+, 2_{\bar{t}}^+, 3^{h_3}, 4^{h_4}, 5^{h_5})$$

- Rational phase-space parametrisation using momentum twistors
- Modular arithmetic in FF reconstruction for coefficients of the partial colour amplitudes written in terms of MIs

Amplitude reconstruction

Fast reconstruction with FF. Application of

linear relations and

univariate partial fractioning

to MI coefficients before reconstruction for further improvement

$$
\begin{aligned}
s_{34} &= (p_3 + p_4)^2, \\
t_{12} &= s_{12}/s_{34}, \\
t_{23} &= (s_{23} - m_t^2)/s_{34}, \\
t_{45} &= s_{45}/s_{34}, \\
t_{15} &= (s_{15} - m_t^2)/s_{34}, \\
x_{5123} &= -\frac{\langle 5|p_1 p_{45}|3\rangle}{\langle 53\rangle s_{12}}.
\end{aligned}
$$

[Badger et al. '2021]

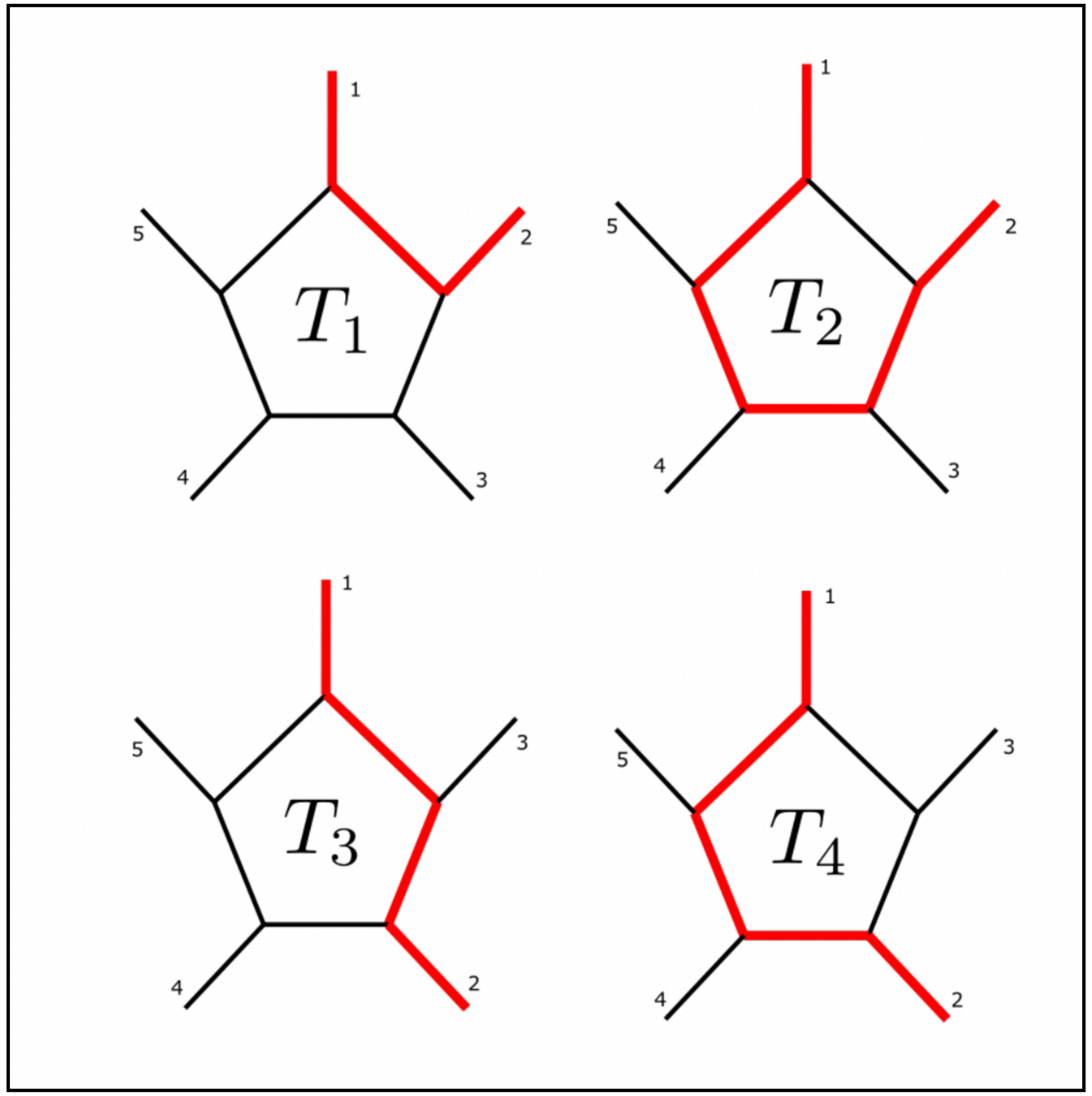
Vasily's talk

Ben's talk

Simone Zoia's missing talk

Master integrals

- 4 pentagon topologies
- Canonical form DE for all 130 MIs
- Numerical solution using generalised power series [Moriello; 2019] expansion in DiffExp
- Analytic result of (all but 1) boundary constants



Summary & Outlook

- First analytic results of 2-loop amplitude with top quark loops.
- For higher multiplicities, semi-numerical approach efficient.
- One-loop helicity amplitudes for $t\bar{t}j$ up to $O(\epsilon^2)$.
- Elliptic functions require more thought in terms of a minimal basis; nevertheless already exposes simplifications.
- Optimistic prospects for higher-loops!

Thank you 🙏

Back-up slide

- Performances for 5 pt:

Timings of all MIs for all topologies & permutations within a range of 30min- 1 hr for phase-space spoint, on a laptop for 16 digits; can be improved by building a precomputed grid of points as BC.

- Effect of relations for 4 pt:

Mapping the MI in A^(2,N_h) to a basis of special functions, 12025 monomials; applying relations reduce to 11791 monomials.

Much fewer monomials appearing in 2-loop finite remainder. In one of the sub amplitudes with helicity +++-, 3586 monomials appear; applying relations bring them to 3158.

1-2 days (~30 cores). Not completely trivial but not worth serious optimisation either.

Helicity amplitudes

- Rational phase-space parametrisation using momentum twistors & sampling of Feynman diagrams using modular arithmetic
- Parametrisation applied to each projected amplitudes ⟶ reduced to MIs and reconstructed using FF
- Sub-amplitudes constructed from the projected amplitudes, again with the help of reconstruction over finite fields, linear relations and univariate partial fractioning