COLLINEAR FUNCTIONS FOR QCD RESUMMATIONS

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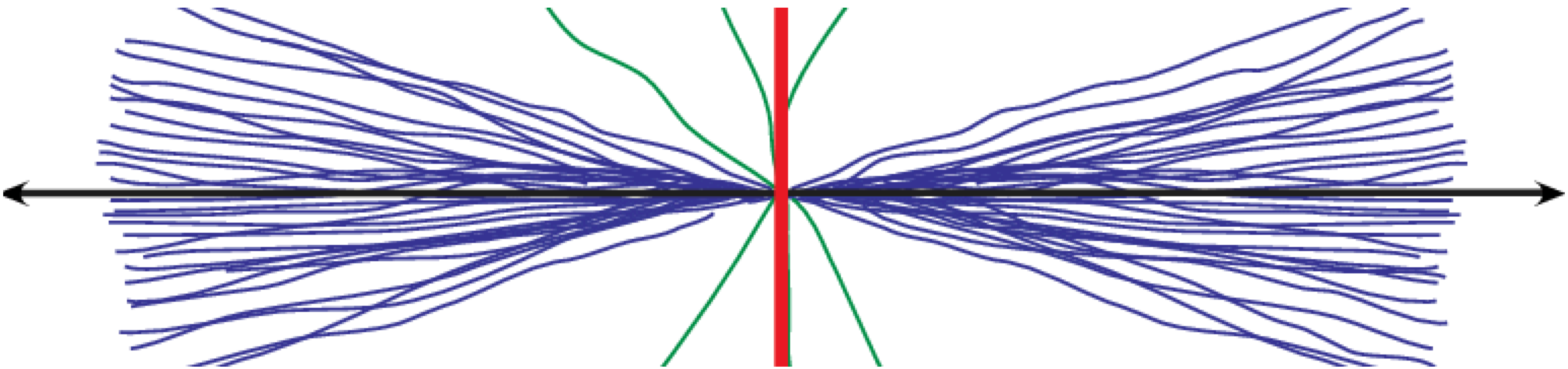

in preparation
in collaboration with Stefano Catani

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GOAL

- Computation of collinear contributions to QCD resummations starting from collinear factorisation of scattering amplitudes and defining appropriate collinear functions.

- As an example of our framework we consider two well known observables such as transverse momentum ($\mathbf{q_T}$) and N-jettiness ($\tau_\mathbf{N}$).

- For the transverse momentum case, we use a time-like auxiliary vector to avoid rapidity divergences in our calculation.

OUTLINE

- Introduction

Resumation → Transverse Momentum

Resummation → Zero-jettiness

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- Computational Framework

Collinear Factorisation → Time-Like (TL)

Collinear Factorisation → Space-Like (SL)

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Resummation
- Transverse Momentum
- Zero-jettiness

- Computational Framework

Collinear Factorisation
- Time-Like (TL)
- Space-Like (SL)

Collinear Functions
- Differential
- Integrated ($\mathbf{q_T}$, τ_0)

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- Introduction

Resummation $\rightarrow$ Transverse Momentum
Resummation $\rightarrow$ Zero-jettiness

- Computational Framework

Collinear Factorisation $\rightarrow$ Time-Like (TL)
Collinear Factorisation $\rightarrow$ Space-Like (SL)

Collinear Functions $\rightarrow$ Differential
Collinear Functions $\rightarrow$ Integrated ($\mathbf{q_T}$, τ_0)

- Perturbative Results up to NNLO

- Summary & Outlook

1. Introduction

DIFFERENTIAL CROSS SECTION: GENERAL STRUCTURE

- We consider the production of a system of colourless particles F given by

additional radiation

$$h_1(p_1) + h_2(p_2) \to F(\{q_i\}) + X$$

$$c(x_1 p_1) + \bar{c}(x_2 p_2) \to F(q \equiv \sum_i q_i)\Big|\ q_\mu q^\mu = M^2$$

- The differential cross section in a generic variable ω can be theoretically separated into following two parts

$$\lim_{\omega_0 \to 0} \int_0^{\omega_0} d\omega \frac{d\sigma_F^{\text{reg.}}}{d\omega} = 0$$

(process dependent)

$$\frac{d\sigma_F}{d\omega} = \frac{d\sigma_F^{\text{sing.}}}{d\omega} (\equiv [d\sigma_F]) + \frac{d\sigma_F^{\text{reg.}}}{d\omega}$$

$(\mathbf{q_T}, \tau_0)$

process independent

$$\left\{ \left[\frac{\ln^n \omega}{\omega}\right]_+, \delta(\omega) \right\}$$

- Singular contributions are of Soft and Collinear origin. Hence, they have some degree of universality that leads to Resummation of these terms.

TRANSVERSE MOMENTUM RESUMMATION

- Using the formalism of qT-resummation [1-2], singular part of the differential cross section in conjugate space has the following structure [2]

LO contribution

Sudakov form factor

$$[d\sigma_F] = \frac{M^2}{s} \sum_{c=q,\bar{q},g} \left[d\sigma^{(0)}_{c\bar{c},F}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}.\mathbf{q_T}} S_c(M,b)$$

$(p_1+p_2)^2$

$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[H^F C_1 C_2\right]_{c\bar{c};a_1 a_2} f_{a_1/h_1}\left(x_1/z_1, b_0^2/b^2\right) f_{a_2/h_2}\left(x_2/z_2, b_0^2/b^2\right)$$

$b_0 = 2e^{-\gamma_E}$

hard collinear factor

parton densities

- Other equivalent formulations do exist in the literature that are based either on TMD factorisation or on SCET methods [3].

[1] Collins, Soper, Sterman (1985)

[2] Catani, Cieri, de Florian, Ferrera, Grazzini (1311.1654)

[3] Becher, Neubert (1007.4005); Echevarria, Idilbi, Scimemi (1111.4996); Chiu, Jain, Neill, Rothstein (1202.0814)

HARD COLLINEAR FACTOR: STRUCTURE

- For the **quark anti-quark** annihilation channel, the hard coll. factor has the following form

$$\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2} = H_c^F\left(x_1p_1, x_2p_2; \Omega; \alpha_s(M^2)\right) C_{ca_1}\left(z_1; \alpha_s\left(b_0^2/b^2\right)\right) C_{\bar{c}a_2}\left(z_2; \alpha_s\left(b_0^2/b^2\right)\right)$$

process dependent hard function

specifying the momenta of the system F

collinear function

- For the **gluon gluon fusion** channel, it has the form given by

$$\left[H^F C_1 C_2\right]_{gg;a_1a_2} = H^F_{\mu_1\nu_1,\mu_2\nu_2}\left(x_1p_1, x_2p_2; \Omega; \alpha_s(M^2)\right)$$
$$\times C_{ga_1}^{\mu_1\nu_1}\left(z_1; p_1, p_2, \mathbf{b}; \alpha_s(b_0^2/b^2)\right) C_{ga_2}^{\mu_2\nu_2}\left(z_2; p_1, p_2, \mathbf{b}; \alpha_s(b_0^2/b^2)\right)$$

where [1]

azimuthally independent part

azimuthally correlated part

$$C_{ga}^{\mu\nu}\left(z; p_1, p_2, \mathbf{b}; \alpha_s\right) = d^{\mu\nu}(p_1,p_2) C_{ga}(z,\alpha_s) + D^{\mu\nu}(p_1,p_2;\mathbf{b}) G_{ga}(z;\alpha_s)$$

$$d^{\mu\nu}(p_1,p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1.p_2}$$

$$D^{\mu\nu}(p_1,p_2;\mathbf{b}) = d^{\mu\nu}(p_1,p_2) - 2\frac{b^\mu b^\nu}{\mathbf{b}^2} \quad \Big| \quad b^\mu = (0,\mathbf{b},0)$$

the indices μ and ν are the Lorentz indices of the gluon in hard scattering amplitude and its conjugate amplitude, respectively.

[1] Catani, Grazzini (1011.3918)

COLLINEAR FUNCTIONS: PERTURBATIVE EXPANSION

- **Azimuthally independent** collinear functions are recently known to **N3LO** in QCD coupling

$$C_{ab}\left(z;\alpha_{\mathrm{S}}\right)=\delta_{ab}\delta(1-z)+\frac{\alpha_{\mathrm{S}}}{\pi}C_{ab}^{(1)}(z)+\left(\frac{\alpha_{\mathrm{S}}}{\pi}\right)^{2}C_{ab}^{(2)}(z)+\left(\frac{\alpha_{\mathrm{S}}}{\pi}\right)^{3}C_{ab}^{(3)}(z)+\sum_{n=4}^{+\infty}\left(\frac{\alpha_{\mathrm{S}}}{\pi}\right)^{n}C_{ab}^{(n)}(z)$$

[1] [2] [3]

- **Azimuthally correlated** collinear functions are recently known to **NNLO** in QCD coupling

$$G_{ga}(z;\alpha_{s})=\frac{\alpha_{s}}{\pi}G_{ga}^{(1)}(z)+\left(\frac{\alpha_{s}}{\pi}\right)^{2}G_{ga}^{(2)}(z)+\sum_{n=3}^{+\infty}\left(\frac{\alpha_{s}}{\pi}\right)G_{ga}^{(n)}(z)$$

[4] [5]

- Similar functions do exist for the processes related by crossing such as SIDIS and production of hadrons from a pair of leptons and they are called **Time-Like collinear functions or Fragmenting Jet functions**.

[1] de Florian, Grazzini (0108273)

[2] Catani, Grazzini (1106.4652); Catani, Cieri, de Florian, Ferrera, Grazzini (1209.0158); Gehrmann, Lubbert, Yang (1209.0682, 1403.6451); Echevarria, Scimemi, Vladimirov (1604.07869); Luo, Wang, Xu, Yang, Yang, Zhu (1908.03831); Luo, Yang, Zhu, Zhu (1909.13820)

[3] Luo, Yang, Zhu, Zhu (1912.05778); Ebert, Mistlberger, Vita (2006.05329); Luo, Yang, Zhu, Zhu (2012.03256)

[4] Catani, Grazzini (1011.3918)

[5] Luo, Yang, Zhu, Zhu (1909.13820); Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov (1907.03780)

ZERO-JETTINESS RESUMMATION

- Using the formalism of resummation, the singular part is given by [1]

inverse Laplace transform

$$[d\sigma_F] = \frac{M^2}{s} \sum_{c=q,\bar{q},g} \left[d\sigma^{(0)}_{c\bar{c},F}\right] \mathcal{L}^{-1} \Bigg\{ S_c\,(M,\sigma) \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \quad e^{-\gamma_E}$$
$$\times \left[H^F I_1 I_2\right]_{c\bar{c};a_1 a_2} f_{a_1/h_1}\,(x_1/z_1, \sigma_0/\sigma)\, f_{a_2/h_2}\,(x_2/z_2, \sigma_0/\sigma) \Bigg\}$$

$$\mathcal{L} : I_{ca}\left(z, \sigma; \mu_F, \alpha_s(\mu_F^2)\right) \equiv \int_0^{+\infty} dt\, e^{-\sigma t}\, \hat{I}_{ca}\left(z, t; \mu_F, \alpha_s(\mu_F^2)\right)$$

transverse virtuality prop to 0-jettiness

- **Matching coefficients** for the beam functions

known: NLO [2]—NNLO [3]—N3LO [4]

$$\left[H^F I_1 I_2\right]_{c\bar{c};a_1 a_2} = H_c^F\left(x_1 p_1, x_2 p_2; \Omega; \alpha_s(M^2)\right) I_{ca_1}\left(z_1; \alpha_{\rm S}(\sigma_0/\sigma)\right) I_{\bar{c}a_2}\left(z_2; \alpha_{\rm S}(\sigma_0/\sigma)\right)$$

equally valid for N-jettiness resummation

- Unlike qT case, we **do not have azimuthally correlated** contributions for the 0-jettiness

$$I_{ga}^{\mu\nu}\left(z; p_1, p_2; \alpha_{\rm S}\right) = d^{\mu\nu}(p_1, p_2) I_{ga}(z, \alpha_{\rm S})$$

[1] Stewart, Tackmann, Waalewijn (1004.2489)

[2] Stewart, Tackmann, Waalewijn (1002.2213); Berger, Marcantonini, Stewart, Tackmann, Waalewijn (1012.4480); Ritzmann, Waalewijn (1407.3272)

[3] Gaunt, Stahlhofen, Tackmann (1401.5478); Gaunt, Stahlhofen, Tackmann (1405.1044)

[4] Melnikov, Rietkerk, Tancredi, Wever (1809.06300); Melnikov, Rietkerk, Tancredi, Wever (1809.02433); Behring, Melnikov, Rietkerk, Tancredi, Wever (1910.10059); Baranowski (2004.03285); Ebert, Mistlberger, Vita (2006.03056)

2. Our Framework

COLLINEAR FACTORISATION OF MATRIX ELEMENT

- The collinear factorisation of hard scattering matrix element having N collinear partons in its most general form is given by [1]

reduced matrix element, a vector in colour+spin space

auxiliary vector

$$|\mathcal{M}\left(\{q_i\}; k_1, \ldots, k_N\right)|^2 = \langle \mathcal{M}\big(\{q_i\}; \tilde{k}\big)|\, \boldsymbol{\mathcal{P}}\big(\{q_i\}; k_1, \ldots, k_N; n\big)\, |\mathcal{M}\big(\{q_i\}; \tilde{k}\big)\rangle + \ldots$$

non-coll. partons

coll. limit of $\sum_{i=1}^{N} k_i$

splitting kernel encoding singular behaviour in the coll. limit

non-singular terms

- In general the collinear splitting kernel depends on the momenta and quantum numbers (colour) of non-collinear partons in addition to its dependence on the collinear partons.

- The TL collinear region is defined by

$$\{k_i^0\} > 0$$

- The splitting kernel is process independent and this property of factorisation is called strict collinear factorisation.

- The SL collinear region is defined by

$$k_i^0 < 0$$

- Strict collinear factorisation is instead violated in SL collinear region.

[1] Catani, de Florian, Rodrigo (1112.4405)

SPLITTING KERNEL: TL COLLINEAR REGION

- The projection of TL splitting kernel for both **quark** and **gluon** splitting into colour+spin space spanned by collinear partons is given by

spin index of parent quark

denoting colour+spin of coll. partons

$$\langle s; r_i, \cdots | \boldsymbol{\mathcal{P}}_{c\to a_1\cdots a_N}(k_1,\ldots,k_N;n)\, | s'; r_i', \cdots \rangle$$
$$= \mathcal{P}_{c\to a_1\cdots a_N}(k_1,\ldots,k_N;n)\, \delta^{ss'} \, \langle r_i, \cdots | \, \mathbf{1} \, | r_i', \cdots \rangle \, , \qquad c = q, \bar{q} \, ,$$

c-number functions

trivial dependence

$$\langle \mu; r_i, \cdots | \boldsymbol{\mathcal{P}}_{g\to a_1\cdots a_N}(k_1,\ldots,k_N;n)\, | \nu; r_i', \cdots \rangle = \mathcal{P}^{\mu\nu}_{g\to a_1\cdots a_N}(k_1,\ldots,k_N;n)\, \langle r_i, \cdots | \, \mathbf{1} \, | r_i', \cdots \rangle$$

Lorentz index of parent gluon

$\propto g^{\mu\nu}, k^{\mu}_{iT} k^{\nu}_{jT}$

- The remaining dependence of the splitting kernel is due to **scalar functions** of collinear momenta of the form [1]

$$s_{ij} = 2 k_i k_j \qquad\qquad \frac{x_i}{x_j} = \frac{n k_i}{n k_j}$$

an auxiliary vector far away from the coll. region

$n^2 = 0$

- In our work, we consider a **light-like** auxiliary vector while dealing with zero-jettiness and a **time-like** auxiliary vector while dealing with transverse momentum case.

$n^2 > 0$

[1] Catani, Grazzini (9908523)

COMMENTS ON THE USE OF AUXILIARY VECTOR

- In the literature, the splitting kernels are usually calculated using a light-like auxiliary vector. Indeed this choice is very convenient for direct specific computations of the splitting kernels. However, we emphasize that one can also set

$$n^2 \neq 0$$

- By changing the auxiliary vector from light-like to time-like, we do not modify any formal expression of the splitting kernels in the literature and this is due to the fact that it depends on the auxiliary vector only through the ratio

$$\frac{x_i}{x_j} = \frac{nk_i}{nk_j}$$

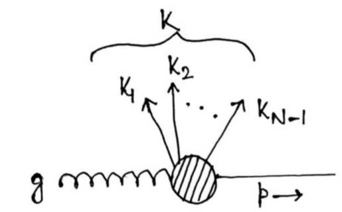
  hence, the change only affects the non singular/power suppressed terms in the collinear limit.

- Finally, we note that the projection of SL splitting kernels may not be c-number functions depending on the order of the perturbation theory. Actually, starting from N3LO it depends on the colour quantum numbers of the non-collinear partons [1].

[1] Catani, de Florian, Rodrigo (1112.4405)

DIFFERENTIAL COLLINEAR FUNCTIONS:TL REGION

- We define the differential collinear functions in the TL region as follows [1]



$$\mathcal{F}_{ga}^{\mathrm{TL}\,\mu\nu}(p,k;n) = \sum_{N=2}^{+\infty} \left[\prod_{m=1}^{N-1} \int \frac{d^d k_m}{(2\pi)^{d-1}} \delta_+\left(k_m^2\right) \right] \delta^{(d)}\!\left(k - \sum_{i=1}^{N-1} k_i \right)$$
$$\times \sum_{a_1,\dots,a_{N-1}} \frac{\widetilde{\mathcal{P}}^{\mu\nu}_{g\to a_1\dots a_N}\left(k_1,\dots,k_N;n\right)}{\mathrm{SF}\left(a_1,\dots,a_{N-1}\right)} \Bigg|_{\substack{k_N=p\\ a_N=a}}$$

Bose symmetry factor

$$\widetilde{\mathcal{P}}^{\mu\nu}_{g\to a_1\dots a_N}\left(k_1,\dots,k_N;n\right) = d^{\mu}{}_{\mu'}(p;n)\, \mathcal{P}^{\mu'\nu'}_{g\to a_1\dots a_N}\left(k_1,\dots,k_N;n\right)\, d_{\nu'}{}^{\nu}(p;n)$$

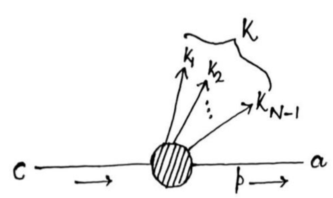
$$\mathcal{F}_{ga}^{\mathrm{TL}\,\mu\nu}(p,k;n) = d^{\mu\nu}(p;n)\, \mathcal{F}^{\mathrm{TL}}_{ga,\,\mathrm{az.in.}}(p,k;n) + D^{\mu\nu}(p,n;\mathbf{k_T},\epsilon)\, \mathcal{F}^{\mathrm{TL}}_{ga,\,\mathrm{corr.}}(p,k;n)$$

$$\mathcal{P}^{\mathrm{az.in.}}_{g\to a_1\dots a_N} = \frac{d_{\mu\nu}(p;n)}{d-2}\, \widetilde{\mathcal{P}}^{\mu\nu}_{g\to a_1\dots a_N} = \frac{d_{\mu\nu}(p;n)}{d-2}\, \mathcal{P}^{\mu\nu}_{g\to a_1\dots a_N}\,,$$
$$\mathcal{P}^{\mathrm{corr.}}_{g\to a_1\dots a_N} = \frac{D_{\mu\nu}(p,n;\mathbf{k_T},\epsilon)}{(d-2)(d-3)}\, \widetilde{\mathcal{P}}^{\mu\nu}_{g\to a_1\dots a_N} = \frac{D_{\mu\nu}(p,n;\mathbf{k_T},\epsilon)}{(d-2)(d-3)}\, \mathcal{P}^{\mu\nu}_{g\to a_1\dots a_N}$$

$$d^{\mu\nu}(p;n) = -g^{\mu\nu} + \frac{p^\mu n^\nu + n^\mu p^\nu}{np} - \frac{n^2 p^\mu p^\nu}{(np)^2} \qquad D^{\mu\nu}(p,n;\mathbf{k_T},\epsilon) = d^{\mu\nu}(p;n) - (d-2)\,\frac{k_T^\mu k_T^\nu}{\mathbf{k_T}^2}$$

$$d_{\mu\nu}(p;n) D^{\mu\nu}(p,n;\mathbf{k_T},\epsilon) = 0\,,$$
$$d_{\mu\nu}(p;n)\, d^{\mu\nu}(p;n) = d-2\,,$$
$$D_{\mu\nu}(p,n;\mathbf{k_T},\epsilon) D^{\mu\nu}(p,n;\mathbf{k_T},\epsilon) = (d-2)(d-3)$$

and for the quark



$$\mathcal{F}_{ca}^{\mathrm{TL}}(p,k;n) = \sum_{N=2}^{+\infty} \left[\prod_{m=1}^{N-1} \int \frac{d^d k_m}{(2\pi)^{d-1}} \delta_+\left(k_m^2\right) \right] \delta^{(d)}\!\left(k - \sum_{i=1}^{N-1} k_i \right) \sum_{a_1,\dots,a_{N-1}} \frac{\mathcal{P}_{c\to a_1\dots a_N}\left(k_1,\dots,k_N;n\right)}{\mathrm{SF}\left(a_1,\dots,a_{N-1}\right)} \Bigg|_{\substack{k_N=p\\ a_N=a}}\,, \qquad c = q,\bar{q}$$

- TL Splitting kernels for various splitting processes are **fully known to second order** [2] and **partially known to third order** [3] in the QCD strong coupling.

[1] Stefano Catani + PKD

[2] Bern, Del Duca, Kilgore, Schmidt, Catani, Grazzini, Campbell, Glover, Kosower, Uwer, Sborlini, de Florian, Rodrigo

[3] Catani, de Florian, Rodrigo, Del Duca, Frizzo, Maltoni, Birthwright, Glover, Khoze, Marquard, Duhr, Haindl, Lazopoulos, Michel, Sborlini, Rodrigo, Badger, Buciuni, Peraro, Bern, Dixon, Kosower, Gehrmann, Jaquier, Czakon, Sapeta

DIFFERENTIAL COLLINEAR FUNCTIONS:SL REGION

- We define the differential collinear functions in the SL region as follows

$$\boldsymbol{\mathcal{F}}_{ca}(\{q_i\};p,k;n) = \sum_{N=2}^{+\infty}\left[\prod_{m=1}^{N-1}\int \frac{d^d k_m}{(2\pi)^{d-1}}\delta_+\left(k_m^2\right)\right]\delta^{(d)}\left(k-\sum_{i=1}^{N-1}k_i\right)$$

dependence on non-collinear partons

$$\times \sum_{a_1,\dots,a_{N-1}} \frac{\widetilde{\boldsymbol{\mathcal{P}}}_{\bar{c}\to a_1\dots a_N}\left(\{q_i\};k_1,\dots,k_N;n\right)}{\mathrm{SF}\left(a_1,\dots,a_{N-1}\right)}\Bigg|_{\substack{k_N=-p\\ a_N=\bar{a}}} \frac{\mathcal{N}_c(\epsilon)}{\mathcal{N}_a(\epsilon)}\;,$$

spin

$$\mathcal{N}_a(\epsilon) = (-1)^{2S_a} n_s(a,\epsilon) n_c(a)$$

# of spin polarisations

# of colours

$c = g, q, \bar{q}$

$$\mathcal{N}_g(\epsilon) = 2(1-\epsilon)(N_c^2-1)$$

$$\mathcal{N}_q(\epsilon) = \mathcal{N}_{\bar{q}}(\epsilon) = -2N_c$$

- Note that both the **SL splitting kernels and SL differential collinear functions are process dependent** through their dependence on the momenta and quantum numbers (actually, only colour) of the non-collinear partons.

- SL Splitting kernels for various splitting processes are **fully known to second order** and **partially known to third order** [1-3] in the QCD strong coupling.

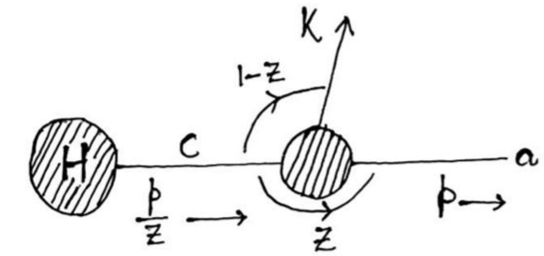
[1] Catani, de Florian, Rodrigo (1112.4405)

[2] Forshaw, Seymour, Siodmok (1206.6363)

[3] Dixon, Herrmann, Yan, Zhu (1912.09370)

INTEGRATED COLLINEAR FUNCTIONS:TL CASE

- We define the transverse momentum dependent collinear functions for the gluon case as follows



$$F_{ga}^{\mathrm{TL}\,\mu\nu}(z; p/z, \mathbf{q_T}; n) = \delta(1-z)\ \delta^{(d-2)}(\mathbf{q_T})\ \delta_{ga}\ d^{\mu\nu}(p;n) + \int d^dk\ \delta^{(d-2)}(\mathbf{k_T}+\mathbf{q_T})\ \delta\left(\frac{k^+}{p^+} - \frac{1-z}{z}\right)\ \mathcal{F}_{ga}^{\mathrm{TL}\,\mu\nu}(p,k;n)$$

Leads to logarithmically enhanced contributions

with

phase space in coll. limit

$$F_{ga}^{\mathrm{TL}\,\mu\nu}(z; p/z, \mathbf{q_T}; n) = d^{\mu\nu}(p;n)\ F_{ga,\,\mathrm{az.in.}}^{\mathrm{TL}}\left(z; \mathbf{q_T}^2, \frac{n^2\mathbf{q_T}^2}{(2np/z)^2}\right) + D^{\mu\nu}(p,n;\mathbf{q_T},\epsilon)\ F_{ga,\,\mathrm{corr.}}^{\mathrm{TL}}\left(z; \mathbf{q_T}^2, \frac{n^2\mathbf{q_T}^2}{(2np/z)^2}\right)$$

- For the quark case, it is given by

$$F_{ca}^{\mathrm{TL}}\left(z; \mathbf{q_T}^2, \frac{n^2\mathbf{q_T}^2}{(2np/z)^2}\right) = \delta(1-z)\ \delta^{(d-2)}(\mathbf{q_T})\ \delta_{ca} + \int d^dk\ \delta^{(d-2)}(\mathbf{k_T}+\mathbf{q_T})\ \delta\left(\frac{k^+}{p^+} - \frac{1-z}{z}\right)\ \mathcal{F}_{ca}^{\mathrm{TL}}(p,k;n)\ ,\ c = q, \bar{q}$$

- We also define zero-jettiness partonic beam functions as follows

$$\mathcal{B}_{ca}^{\mathrm{TL}}\left(z; t, \frac{n^2 t}{(2np/z)^2}\right) = \delta(1-z)\delta(t)\delta_{ca} + \int d^dk\, \delta(t - 2zpk)\, \delta\left(\frac{k^+}{p^+} - \frac{1-z}{z}\right) \mathcal{F}_{ca}^{\mathrm{TL}}(p,k;n)$$

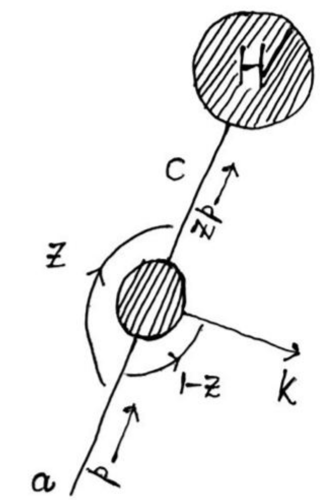
INTEGRATED COLLINEAR FUNCTIONS:SL CASE

- We define the transverse momentum dependent collinear functions for both quark and gluon splitting as follows

$$\mathbf{F}_{ca}(\{q_i\}; z; zp, \mathbf{q_T}; n) = \mathbf{1}\, \delta(1-z)\, \delta^{(d-2)}(\mathbf{q_T})\, \delta_{ca} + z \int d^d k\; \delta^{(d-2)}(\mathbf{k_T} + \mathbf{q_T})\, \delta\!\left(\frac{k^+}{p^+} - 1 + z\right) \boldsymbol{\mathcal{F}}_{ca}(\{q_i\}; p, k; n)$$

dependence on non-collinear partons

phase space in coll. limit



- We define the zero-jettiness partonic beam functions for both quark and gluon splitting as follows

$$\boldsymbol{\mathcal{B}}_{ca}(\{q_i\}; z; zp, t; n) = \mathbf{1}\, \delta(1-z)\, \delta(t)\, \delta_{ca} + z \int d^d k\; \delta(t - 2zpk)\, \delta\!\left(\frac{k^+}{p^+} - 1 + z\right) \boldsymbol{\mathcal{F}}_{ca}(\{q_i\}; p, k; n)$$

process dependent

transverse virtuality

- Note that until now our framework is quite general i.e. without restricting to any perturbative order. For the purpose presenting our perturbative results, from now on we are restricting ourselves to NNLO in perturbation theory. Up to this order SL collinear and beam functions are also process independent. Hence, like TL region the functions we will be dealing with are as follows

$$F^{\mu\nu}_{ga}(z; zp, \mathbf{q_T}; n)\,, \quad F_{ga,\,\mathrm{az.in.}}\!\left(z; \mathbf{q_T}^2, \frac{n^2 \mathbf{q_T}^2}{(2zpn)^2}\right), \quad F_{ga,\,\mathrm{corr.}}\!\left(z; \mathbf{q_T}^2, \frac{n^2 \mathbf{q_T}^2}{(2zpn)^2}\right), \quad F_{ca}\!\left(z; \mathbf{q_T}^2, \frac{n^2 \mathbf{q_T}^2}{(2zpn)^2}\right), \quad \mathcal{B}_{ca}\!\left(z; t, \frac{n^2\, t}{(2zpn)^2}\right)$$

COMMENTS ON COLLINEAR FUNCTIONS IN SCET

- There are related definitions of TMD collinear functions and zero-jettiness beam functions from SCET. These functions are defined in a process independent way using auxiliary Wilson line operators along light-like directions.

- For TL case at the partonic level, SCET functions are equivalent to our collinear functions by using a light-like auxiliary vector [1].

- Same equivalence holds true for the SL region is only up to second order in strong coupling. This is due to the fact that ours results in general process dependent and this dependency goes away if we are within NNLO in perturbation theory.

[1] Ritzmann, Waalewijn (1407.3272)

TMD FUNCTIONS & RAPIDITY DIVERGENCES

- While defining collinear functions at the integrated level, we approximated the phase space in the collinear limit. While this approximation greatly simplifies the calculation, it forces one to enter non-collinear region ($k^+ \leq k^- < +\infty$). As a result one encounters divergences from the upper limit of the integration called rapidity divergences.

$$\text{Collinear phase space} \Rightarrow \frac{k_T^2}{2k^+} \leq k^- < \infty \Rightarrow \text{Rapidity divergences} \left(\frac{1}{2}\ln\frac{k^+}{k^-}\right)$$

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- Any specific observable is free from rapidity divergences and the cancellation happens among the components of the resummation formula.

- For the computation of individual components such as soft functions, collinear functions etc. there are very many rapidity regulators that exist in the literature and they are introduced at the integrated level.

- We avoid rapidity divergences in our computation by introducing a time-like auxiliary vector at the matrix element level.

- Finally, N-jettiness coefficient functions are free from these divergences as the minus component of the total momentum is fixed and small from the observable's definition.

3. Perturbative Results

DIFFERENTIAL COLLINEAR FUNCTIONS

- SL differential collinear functions up to NNLO have the following perturbative expansion [1]

$$\mathcal{F}(p,k;n) = \mathcal{F}^{(1R)}(p,k;n) + \left[\, \mathcal{F}^{(2R)}(p,k;n) + \mathcal{F}^{(1R1V)}(p,k;n)\,\right] + \mathcal{O}(\alpha_{\rm S}^3)$$

single real contribution

double real contribution

1-loop real-virtual contribution

- Azimuthally independent contributions @ $\mathcal{O}(\alpha_{\rm S})$

$$S_\epsilon = (4\pi\, e^{-\gamma_E})^\epsilon$$

notice the explicit n-dependence

$$\mathcal{F}^{(1R)}_{ca,\,\text{az.in.}}(p,k;n) = \frac{\alpha_{\rm S}^u\,\mu_0^{2\epsilon}\,S_\epsilon}{\pi}\;\frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}}\;\frac{\delta_+(k^2)}{pk}\;\frac{1}{z_n}\;\widehat{P}_{ca}(z_n;\epsilon) \qquad z_n = \frac{n(p-k)}{np}$$

- Azimuthally correlated contributions @ $\mathcal{O}(\alpha_{\rm S})$

real contributions to Altarelli-Parisi splitting functions

$$\mathcal{F}^{(1R)}_{gg,\,\text{corr.}}(p,k;n) = -\,\frac{\alpha_{\rm S}^u\,\mu_0^{2\epsilon}\,S_\epsilon}{\pi}\;\frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}}\;\frac{\delta_+(k^2)\,C_A}{pk}\;\frac{1-z_n}{z_n^2}\,,$$

$$\mathcal{F}^{(1R)}_{ga,\,\text{corr.}}(p,k;n) = -\,\frac{\alpha_{\rm S}^u\,\mu_0^{2\epsilon}\,S_\epsilon}{\pi}\;\frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}}\;\frac{\delta_+(k^2)\,C_F}{pk}\;\frac{1-z_n}{z_n^2}\,, \qquad a = q,\bar{q}$$

$$\widehat{P}_{qq}(x;\epsilon) = \frac{1}{2}\,C_F\left[\frac{1+x^2}{1-x} - \epsilon(1-x)\right]\,, \qquad \widehat{P}_{gg}(x;\epsilon) = C_A\left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]\,,$$

$$\widehat{P}_{qg}(x;\epsilon) = \frac{1}{2}\,T_R\left[1 - \frac{2x(1-x)}{1-\epsilon}\right]\,, \qquad \widehat{P}_{gq}(x;\epsilon) = \frac{1}{2}\,C_F\left[\frac{1+(1-x)^2}{x} - \epsilon x\right]\,,$$

$$\widehat{P}_{q\bar{q}}(x;\epsilon) = \widehat{P}_{qq'}(x;\epsilon) = \widehat{P}_{q\bar{q}'}(x;\epsilon) = 0$$

[1] Stefano Catani + PKD

TMD COLLINEAR FUNCTIONS IN MOMENTUM SPACE

- At the bare level, azimuthally independent collinear functions at NLO are obtained as follows

explicit **n-dependence** and it is proportional to **delta function**

$$F^{(1R)}_{gg,\,\mathrm{az.in.}} = \frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}} \frac{C_A}{\mathbf{q_T}^2} \left[\frac{z}{(1-z)_+} + z(1-z) + \frac{1-z}{z} - \frac{1}{2}\delta(1-z)\ln\left(\frac{n^2 \mathbf{q_T}^2}{(2zpn)^2}\right)\right]$$

$$F^{(1R)}_{qq,\,\mathrm{az.in.}} = \frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}} \frac{C_F}{\mathbf{q_T}^2} \left[\frac{z}{(1-z)_+} + \frac{(1-\epsilon)(1-z)}{2} - \frac{1}{2}\delta(1-z)\ln\left(\frac{n^2 \mathbf{q_T}^2}{(2zpn)^2}\right)\right]$$

$$F^{(1R)}_{gq,\,\mathrm{az.in.}} = \frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}} \frac{C_F}{\mathbf{q_T}^2} \left[\frac{1-z}{z} + \frac{1-\epsilon}{2} z\right] \quad F^{(1R)}_{qg,\,\mathrm{az.in.}} = \frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}} \frac{T_R}{\mathbf{q_T}^2} \left[\frac{1}{2} - \frac{z(1-z)}{1-\epsilon}\right]$$

- Azimuthally correlated collinear functions at $\mathcal{O}(\alpha_S)$ are obtained as follows

$$F^{(1R)}_{gq,\,\mathrm{corr.}} = -\frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}} \frac{C_F}{\mathbf{q_T}^2} \frac{1-z}{z}\,, \qquad F^{(1R)}_{gg,\,\mathrm{corr.}} = -\frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\pi^{1-\epsilon}} \frac{C_A}{\mathbf{q_T}^2} \frac{1-z}{z}\,.$$

- General formulae for the Fourier transformation from momentum space to conjugate impact parameter space

$$\int d^{d-2}\mathbf{q_T}\, e^{-i\mathbf{b}.\mathbf{q_T}} \frac{\ln^m(\mathbf{q_T}^2)}{(\mathbf{q_T}^2)^{1+\delta}} \xrightarrow{d=4-2\epsilon} \left.\frac{d^m}{d\rho^m}\right|_{\rho=0} \pi^{1-\epsilon}\left(\frac{\mathbf{b}^2}{4}\right)^{\epsilon+\delta-\rho} \frac{\Gamma(\rho-\epsilon-\delta)}{\Gamma(1+\delta-\rho)}\,,$$

$$\int d^{d-2}\mathbf{q_T}\, e^{-i\mathbf{b}.\mathbf{q_T}} \frac{\ln^m(\mathbf{q_T}^2)}{(\mathbf{q_T}^2)^{1+\delta}} D^{\mu\nu}(p,n;\mathbf{q_T}) \xrightarrow{d=4-2\epsilon} -\left.\frac{d^m}{d\rho^m}\right|_{\rho=0} \pi^{1-\epsilon}\left(\frac{\mathbf{b}^2}{4}\right)^{\epsilon+\delta-\rho} \frac{\Gamma(1+\rho-\epsilon-\delta)}{\Gamma(2+\delta-\rho)} D^{\mu\nu}(p,n;\mathbf{b})\,.$$

ZERO-JETTINESS BEAM FUNCTIONS IN t-SPACE

- At the bare level, the partonic beam functions at NLO [1] are obtained as follows

$$\mathcal{B}_{gg} = \frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} C_A t^{-1-\epsilon} \times \left[\left(\frac{z}{1-z}\right)^{\epsilon} \left(\frac{1-2z}{z} + z(1-z) + \frac{1-z^{-\epsilon}}{1-z} \right) + \left(\frac{(1-z)^{-\epsilon}}{1-z} \right)_+ - \frac{1}{\epsilon}\delta(1-z) \right] ,$$

$$\mathcal{B}_{qq} = \frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} C_F t^{-1-\epsilon} \times \left[\left(\frac{z}{1-z}\right)^{\epsilon} \left(\frac{(1-\epsilon)(1-z)}{2} - 1 + \frac{1-z^{-\epsilon}}{1-z} \right) + \left(\frac{(1-z)^{-\epsilon}}{1-z} \right)_+ - \frac{1}{\epsilon}\delta(1-z) \right]$$

$$\mathcal{B}_{gq} = \frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} C_F t^{-1-\epsilon} z^{1+\epsilon} (1-z)^{-\epsilon} \left[\frac{1-z}{z^2} + \frac{1-\epsilon}{2} \right] , \qquad \mathcal{B}_{qg} = \frac{\alpha_s^u \mu_0^{2\epsilon} S_\epsilon}{\pi} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} T_R t^{-1-\epsilon} z^{1+\epsilon} (1-z)^{-\epsilon} \left[\frac{1}{2z} - \frac{1-z}{1-\epsilon} \right] ,$$

- The general transform formula from t-space to conjugate Laplace space is given by

$$\int_0^\infty dt\, e^{-\sigma t} \frac{\ln^m t}{t^{1+\delta}} = \frac{d^m}{d\rho^m}\bigg|_{\rho=0} \sigma^{\delta-\rho}\,\Gamma(\rho-\delta) \,.$$

[1] Ritzmann, Waalewijn (1407.3272)

RENORMALIASTION: TMD COLLINEAR FUNCTIONS

- QCD strong coupling renormalisation

$$\alpha_S^u \mu_0^{2\epsilon} S_\epsilon = \alpha_S\left(b_0^2/b^2\right)\left(\frac{b_0^2}{b^2}\right)^{\epsilon}\left[1-\frac{\alpha_S\left(b_0^2/b^2\right)}{\pi}\frac{\beta_0}{\epsilon}+\mathcal{O}\left(\alpha_S^2\left(b_0^2/b^2\right)\right)\right]$$

$$\equiv \alpha_S\left(\frac{b_0^2}{b^2}\right)^{\epsilon}\left[1-\frac{\alpha_S}{\pi}\frac{\beta_0}{\epsilon}+\mathcal{O}\left(\alpha_S^2\right)\right]$$

$$\beta_0 = \frac{11}{12}C_A - \frac{n_f}{6}$$

- Infrared factorisation in b-space

$$F_{ca}\left(z;\frac{n^2b_0^2}{(2zpn)^2b^2};\alpha_S;\epsilon\right) = \mathcal{Z}_c\left(\frac{n^2b_0^2}{(2zpn)^2b^2};\alpha_S;\epsilon\right)\sum_b \tilde{C}_{cb}\left(z;\frac{n^2b_0^2}{(2zpn)^2b^2};\alpha_S;\epsilon\right)\otimes\Gamma_{ba}\left(z;\alpha_S;\epsilon\right)$$

$$\tilde{C}_{cb}\left(z;\frac{n^2b_0^2}{(2zpn)^2b^2};\alpha_S;\epsilon=0\right) = C_{cb}\left(z;\alpha_S\right)$$

AP splitting functions

$$\Gamma_{ij}\left(z;\alpha_S\right) = \delta_{ij}\delta(1-z) - \frac{\alpha_S}{\pi}\frac{P_{ij}^{(0)}(z)}{\epsilon}+\mathcal{O}\left(\alpha_S^2\right)$$

- Infrared factorisation factors for both gluon and quark are as follows

$$\mathcal{Z}_g = 1+\frac{\alpha_S}{\pi}\left[\frac{C_A}{2}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{n^2b_0^2}{(2znp)^2b^2}\right)\right)+\frac{\beta_0}{\epsilon}-C_A\frac{\pi^2}{24}\right]+\mathcal{O}(\alpha_S^2)$$

$$\mathcal{Z}_q = 1+\frac{\alpha_S}{\pi}\left[\frac{C_F}{2}\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{n^2b_0^2}{(2znp)^2b^2}\right)\right)+\frac{3}{4}\frac{C_F}{\epsilon}-C_F\frac{\pi^2}{24}\right]+\mathcal{O}(\alpha_S^2)$$

Resummation scheme-dependent

RENORMALIASTION: BEAM FUNCTIONS

- QCD strong coupling renormalisation

$$\alpha_{\rm S}^{u}\mu_0^{2\epsilon}S_\epsilon = \alpha_{\rm S}\left(\sigma_0/\sigma\right)\left(\frac{\sigma_0}{\sigma}\right)^{\epsilon}\left[1-\frac{\alpha_{\rm S}\left(\sigma_0/\sigma\right)}{\pi}\frac{\beta_0}{\epsilon}+\mathcal{O}\left(\alpha_{\rm S}^2\left(\sigma_0/\sigma\right)\right)\right]$$

$$\equiv \alpha_{\rm S}\left(\frac{\sigma_0}{\sigma}\right)^{\epsilon}\left[1-\frac{\alpha_{\rm S}}{\pi}\frac{\beta_0}{\epsilon}+\mathcal{O}\left(\alpha_{\rm S}^2\right)\right]$$

- Infrared factorisation

$$\mathcal{B}_{ca}\left(z;\alpha_{\rm S};\epsilon\right) = \tilde{\mathcal{Z}}_c\left(\alpha_{\rm S};\epsilon\right)\sum_b\int_z^1\frac{dx}{x}\,\tilde{I}_{cb}\left(x;\alpha_{\rm S};\epsilon\right)\Gamma_{ba}\left(\frac{z}{x};\alpha_{\rm S};\epsilon\right)$$

$$\tilde{I}_{cb}\left(z;\alpha_{\rm S};\epsilon=0\right) = I_{cb}\left(z;\alpha_{\rm S}\right)$$

- Infrared factorisation factors for both gluon and quark are as follows

$$\tilde{\mathcal{Z}}_g = 1+\frac{\alpha_{\rm S}}{\pi}\left(\frac{C_A}{\epsilon^2}+\frac{\beta_0}{\epsilon}\right)+\mathcal{O}(\alpha_{\rm S}^2)\,.$$

$$\tilde{\mathcal{Z}}_q = 1+\frac{\alpha_{\rm S}}{\pi}\left(\frac{C_F}{\epsilon^2}+\frac{3}{4}\frac{C_F}{\epsilon}\right)+\mathcal{O}(\alpha_{\rm S}^2)\,,$$

PERTURBATIVE RESULTS @ $\mathcal{O}(\alpha_S)$

- TMD collinear functions [1]

$$C_{gq}^{(1)}(z) = \frac{1}{2} C_F z, \qquad G_{gq}^{(1)}(z) = C_F \frac{1-z}{z}$$
$$C_{qg}^{(1)}(z) = T_R z (1-z), \qquad G_{gg}^{(1)}(z) = C_A \frac{1-z}{z}$$
$$C_{qq}^{(1)}(z) = \frac{1}{2} C_F (1-z),$$

- Beam function matching coefficients [2]

$$I_{gq}^{(1)}(z) = \frac{C_F}{2} \left\{ z + \frac{1+(1-z)^2}{z} \ln\left(\frac{1-z}{z}\right) \right\},$$
$$I_{gg}^{(1)}(z) = C_A \left\{ \left[\frac{\ln(1-z)}{1-z}\right]_+ + \left(\frac{1}{z} - 2 + z - z^2\right) \ln(1-z) - \frac{(1-z+z^2)^2}{z(1-z)} \ln(z) \right\},$$
$$I_{qg}^{(1)}(z) = T_R \left\{ z(1-z) + \frac{z^2+(1-z)^2}{2} \ln\left(\frac{1-z}{z}\right) \right\},$$
$$I_{qq}^{(1)}(z) = \frac{C_F}{2} \left\{ 1 - z + 2\left[\frac{\ln(1-z)}{1-z}\right]_+ - (1+z)\ln(1-z) - \frac{1+z^2}{1-z} \ln(z) \right\}.$$

- Our results are in full agreement with those in the literature.

[1] de Florian, Grazzini (0108273); Catani, Grazzini (1106.4652)

[2] Stewart, Tackmann, Waalewijn (1002.2213); Ritzmann, Waalewijn (1407.3272)

PERTURBATIVE RESULTS @ $\mathcal{O}\left(\alpha_S^2\right)$

- Azimuthally correlated TMD collinear functions are obtained as follows

$$G_{gq}^{(2)} = C_F^2 \left\{ -\frac{1-z}{2} + \frac{5}{4}\ln(z) - \frac{1}{4}\ln^2(z) - \frac{1-z}{2z}\left[\ln(1-z) + \ln^2(1-z)\right]\right\} + C_F n_f \left\{ -\frac{1-z}{3z}\left[\frac{2}{3} + \ln(1-z)\right]\right\} + C_A C_F \left\{ -\frac{11}{18z} + \frac{10}{9} - \frac{z}{2} - \ln(z)\left[\frac{1}{z} + \frac{5}{2}\right] + \frac{1}{2}\ln^2(z) + \frac{1-z}{z}\left[\frac{5}{6}\ln(1-z) + \frac{1}{2}\ln^2(1-z) + \mathrm{Li}_2(z) - \frac{\pi^2}{6}\right]\right\},$$

$$G_{gg}^{(2)} = C_F n_f \left\{ \frac{(1-z)^3}{2z} - \frac{1}{4}\ln^2(z)\right\} + C_A n_f \left\{ -\frac{17}{36z} + \frac{4}{9} + \frac{z}{12} + \frac{z^2}{36} - \frac{1}{6}\ln(z)\right\} + C_A^2 \left\{ -\frac{37}{36z} + \frac{31}{18} - \frac{13z}{12} + \frac{11z^2}{36} - \ln(z)\left[\frac{1}{z} + \frac{19}{12}\right] + \frac{1}{2}\ln^2(z) + \frac{1-z}{z}\left[\mathrm{Li}_2(z) - \frac{\pi^2}{6}\right]\right\}$$

- Our results are in full agreement with those in the literature [1].

[1] Luo, Yang, Zhu, Zhu (1909.13820); Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov (1907.03780)

PERTURBATIVE RESULTS @ $\mathcal{O}\left(\alpha_S^2\right)$

- Azimuthally correlated TMD collinear functions in the TL region are obtained as follows

$$
\begin{aligned}
G_{gq}^{TL(2)} =& C_F\Big\{-\frac{1}{8}+\frac{3z}{4}-\frac{5z^2}{8}-\ln(z)\left[\frac{1}{4}+\frac{3z}{8}-\frac{z^2}{4}\right]+z(1-z)\Big[\frac{1}{4}\ln(1-z) \\
&+\frac{3}{8}\ln^2(z)-\frac{3}{2}\ln(z)\ln(1-z)+\frac{1}{4}\ln^2(1-z)-\mathrm{Li}_2(z)-\frac{\pi^2}{12}\Big]\Big\} \\
&+N_f\left\{z(1-z)\left[\frac{1}{9}-\frac{1}{6}\ln(z)+\frac{1}{6}\ln(1-z)\right]\right\}+C_A\Big\{\ln(z)\left[\frac{1}{4}+\frac{13z}{6}-\frac{17z^2}{12}\right] \\
&+\ln^2(z)\left[\frac{3z}{4}+\frac{z^2}{2}\right]+z(1-z)\Big[-\frac{25}{36}-\frac{5}{12}\ln(1-z)+\frac{1}{2}\ln(z)\ln(1-z) \\
&-\frac{1}{4}\ln^2(1-z)+\frac{1}{2}\mathrm{Li}_2(z)+\frac{\pi^2}{6}\Big]\Big\},
\end{aligned}
$$

$$
\begin{aligned}
G_{gg}^{TL(2)} =& C_F n_f\left\{\frac{1}{18z}+\frac{1}{2}+z-\frac{14z^2}{9}+\ln(z)\left[-\frac{1}{3z}+1+\frac{3z}{2}\right]+\frac{3z}{4}\ln^2(z)\right\} \\
&+C_A n_f\left\{-\frac{1}{36z}-\frac{1}{12}-\frac{4z}{9}+\frac{17z^2}{36}-\frac{z}{6}\ln(z)\right\}+C_A^2\Big\{-\frac{1}{36z}-\frac{5}{12}-\frac{20z}{9}+\frac{11z^2}{4} \\
&+\ln(z)\left[\frac{1}{3z}-1-\frac{67z}{12}+z^2\right]+z(1-z)\Big[\ln(z)\ln(1-z)-\mathrm{Li}_2(1-z)\Big] \\
&-\ln^2(z)\left[3z-\frac{z^2}{2}\right]\Big\}.
\end{aligned}
$$

- Our results are in full agreement with those in the literature [1].

[1] Luo, Yang, Zhu, Zhu (1909.13820)

SUMMARY & OUTLOOK

- I have presented an alternative way to compute both SL and TL collinear functions for QCD resummations using respective splitting kernels for the scattering amplitude.

- To compute these functions, we defined a differential version at the intermediate level and integrate them using proper observable definition to obtain specific collinear functions for both transverse momentum and zero-jettiness case.

- For the azimuthally independent collinear functions, we have presented results up to NLO and for the azimuthally correlated case, we have results up to NNLO in perturbation theory.

- In our computation, we have stressed on the point that SL collinear functions, in general, can be process dependent and this dependency smoothly flows from splitting kernels to collinear functions.

- Instead of using a regulator to cure rapidity divergences those are present in the transverse momentum case, we use a time-like auxiliary vector to avoid them at the matrix element level.

SUMMARY & OUTLOOK

- NNLO results for the azimuthally independent collinear functions are under completion.

- Residual soft functions at NNLO are also under completion.

- Our formalism can be extended and applied to other observables such as jet mass distributions, energy-energy correlation functions etc.

- It would be interesting to extend our procedures to the next order i.e. N3LO in perturbation theory to understand the process dependence nature of SL splitting kernels and how it affects the corresponding collinear functions.

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Thank you for your attention