Phenomenology of next-to-soft corrections to processes at LHC

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Next-to-Soft Corrections ?

Inclusive cross-section in QCD improved parton model:

Drell-Yan(DY)/Higgs Boson production:

$$\boxed{\sigma(q^2,\tau) = \sigma_0(\mu_R^2) \int \frac{dz}{z} \Phi_{ab}\left(\frac{\tau}{z},\mu_F^2\right) \Delta_{ab}(q^2,\mu_F^2,z)}$$

Non-perturbative Flux

Convolution of f_a , f_b

Perturbatively calculable Coefficient Function

Partonic coefficient function near z -> 1,

$$\boxed{\Delta_{ab}(q^2,\mu_i^2,z) = \Delta_{ab}^{SV}(q^2,\mu_i^2,z) + \Delta_{ab}^{H}(q^2,\mu_i^2,z)}$$

Soft + Virtual

Hard

Corrections from Diagonal Channels

Corrections from Diagonal and off-Diagonal Channels

Next-Soft Corrections ?

Soft + Virtual

$$\boxed{\Delta^{SV}_{ab}(z) = \delta_{ab}\Delta_{a,\delta}\delta(1-z) + \delta_{ab}\sum_{j=0}^{\infty}\Delta_{a,\mathcal{D}_j}\left(\frac{\log^j(1-z)}{1-z}\right)_+}$$

$$\left.\begin{array}{l}\Delta_{a,\delta}\\ \Delta_{a,\mathscr{D}_j}\end{array}\right\}\ \text{Perturbatively Calculable}$$

Hard Part

Expanding around z = 1

$$\boxed{\Delta^{H}_{ab}(z) = \Delta^{NSV}_{ab}(z) + \Delta^{N^nSV}_{ab}(z)}$$

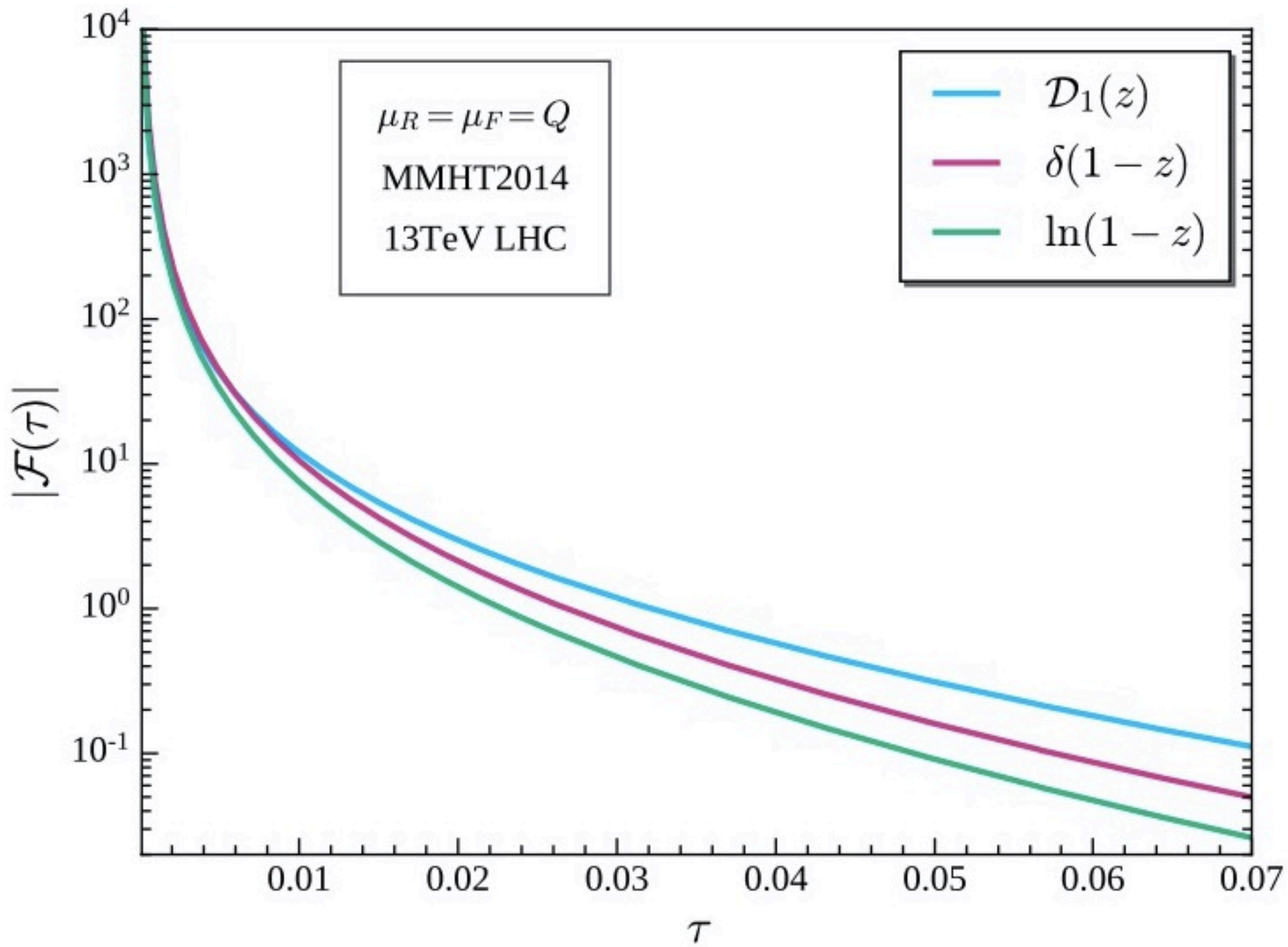
- Next to SV
- Beyond NSV

$$\Delta^{NSV}_{ab}(z) = \sum_{k=0}^{\infty} c_k \log^k(1-z)$$

$$\Delta^{N^nSV}_{ab}(z) = \sum_{k=1}^{\infty} d_k (1-z)^k \log^k(1-z)$$

Motivation

Plot of Convolutions of SV and NSV terms with the flux of incoming quark anti-quark pairs for Drell-Yan



$$\mathcal{F}(\tau) = \int_\tau^1 \frac{dz}{z} \tilde{\Phi}_{q\bar{q}} \left(\frac{\tau}{z}\right) \mathcal{G}(z)$$

$$\mathcal{G}(z) = \{\delta(1-z), \mathcal{D}_1(z), \ln(1-z)\}$$

Motivation

Significant NSV contribution due to large coefficients

Drell-Yan

SV		NSV	
$\mathcal{D}_3$	6.13%	$\ln^3(1-z)$	12.4%
$\mathcal{D}_2$	1.49%	$\ln^2(1-z)$	7.83%
$\mathcal{D}_1$	-3.24%	$\ln^1(1-z)$	-2.82%
$\mathcal{D}_0$	-4.74%	$\ln^0(1-z)$	-6.57%
$\delta(1-z)$	0.003%		
TOTAL	-0.035%		10.8%

$\mu_R = \mu_F = Q\ (200\ GeV)$

gg->H

SV		NSV	
$\mathcal{D}_3$	45.3%	$\ln^3(1-z)$	52.64%
$\mathcal{D}_2$	4.87%	$\ln^2(1-z)$	37.34%
$\mathcal{D}_1$	-10.60%	$\ln^1(1-z)$	-7.45%
$\mathcal{D}_0$	-25.51%	$\ln^0(1-z)$	-23.62%
$\delta(1-z)$	1.75%		
TOTAL	15.81%		58.91%

$\mu_R = \mu_F = m_H\ (125\ GeV)$

% contribution to the Born Cross-section
at NNLO

Previous Works

The earliest evidence that IR effects can be studied at NSV:

- Low, Burnett, Kroll

Early attempts:

- Kraemer, Laenen, Spira (98)
- Akhoury, Sotiropoulos & Sterman (98)

Important Results & Predictions using Physical Kernel Approach & explicit computation:

- Moch, Vogt et al. (09-20),
- Anastasiou, Duhr, Dulat et al. (14)

Universality of NLP effects and LL Resummation:

- Laenen, Magnea, et al. (08-21),
- Grunberg & Ravindran (09),
- Ball, Bonvini, Forte, Marzani, Ridolfi (13),
- Del Duca et al. (17).

Sub-leading Factorisation and LL Resummation at NLP using SCET:

- Larkoski, Nelli, Stewart et al. (14) ,
- Kolodrubetz, Moult, Neill, Stewart et al. (17),
- Beneke et al. (19-20).

Our Works

- Factorisation and RG invariance approach to study NSV resummation effects
  [Ajjath, Mukherjee, Ravindran , hep-ph/ 2006.06726]

- On next to soft threshold corrections to DIS and SIA processes
  [Ajjath, Mukherjee, Ravindran, Sankar, ST, JHEP 04 (2021) 131]

- Resummed Higgs boson cross section at next-to SV to NNLO + NNLL
  [Ajjath, Mukherjee, Ravindran, Sankar, ST, hep-ph/2109.12657]

- Next-to SV resummed Drell-Yan cross section beyond Leading-logarithm
  [Ajjath, Mukherjee, Ravindran, Sankar, ST, hep-ph/2107.09717]

Today's Talk !

- Next-to-soft corrections for Drell-Yan and Higgs boson rapidity distributions beyond N3LO
  [Ajjath, Mukherjee, Ravindran, Sankar, ST, Phys.Rev.D 103 (2021) L111502]

Addressing Some Questions!

- Is NSV universal like SV ?
- Are they controlled by certain IR anomalous dimensions?
- Does IR Renormalisation Group Eqn. Exist ?
- Can we exponentiate NSV logarithms?
- Can we resum and predict higher order NSV logs?
- Can we systematically build a framework for the computation of NSV corrections ?

Formalism

Mass Factorisation:

$$\boxed{\frac{1}{z}\hat{\sigma}_{ab}\left(z,\epsilon\right) = \sigma_0 \sum_{a'b'} \Gamma^T_{aa'}\left(\mu_F^2, z, \epsilon\right) \otimes \left(\frac{1}{z}\Delta_{a'b'}\left(\mu_F^2, z, \epsilon\right)\right) \otimes \Gamma_{b'b}\left(\mu_F^2, z, \epsilon\right)}$$

Collinear finite

Collinear singular

- Ultraviolet finite
- No soft and no final state collinear divergences
- Contains only initial state collinear divergences

Formalism

Diagonal Channel:

For Drell-Yan Process,

$$\boxed{\frac{\hat{\sigma}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \frac{\Delta_{qq}}{z} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{qq} \otimes \frac{\Delta_{qg}}{z} \otimes \Gamma_{g\bar{q}} + \cdots}$$

$z \to 1$

$z \to 1$

SV $\quad \left(\frac{\ln(1-z)}{(1-z)}\right)_+ \,, \quad \delta(1-z)$

Beyond NSV

$(1-z)^k \ln^k(1-z)\,, \ k = 1, \ldots \infty$

NSV $\quad \ln^k(1-z)\,, \ k = 0, \ldots \infty$

$$\boxed{\frac{\hat{\sigma}^{\mathrm{sv+nsv}}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\mathrm{sv+nsv}}_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \,.}$$

Remarkably simple form !

Formalism

off-Diagonal Channel:

For Drell-Yan Process,

$$\boxed{\frac{\hat{\sigma}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{qg} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{gg} + \cdots}$$

NSV	$\ln^k(1-z)\,,\ k = 0, \ldots \infty$
Beyond NSV	$(1-z)^k \ln^k(1-z)\,,\ k = 1, \ldots \infty$

$$\boxed{\frac{\hat{\sigma}^{\rm sv+nsv}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\rm sv+nsv}_{q\bar{q}} \otimes \Gamma_{\bar{q}g} + \Gamma^T_{qq} \otimes \Delta^{\rm nsv}_{qg} \otimes \Gamma_{gg}\,.}$$

Getting complicated due to MIXING of channels !

Factorisation

Factoring out the pure virtual contributions near z -> 1

$$\boxed{\hat{\sigma}_{c\bar{c}}(z,\epsilon) = \left(Z_{c,UV}\right)^2 |\hat{F}_c(\epsilon)|^2 S_c(z,\epsilon)}$$

UV renormalisation constant

Form Factor(virtual corrections)

Soft + next-to-soft corrections

UV finite mass-factorised partonic coefficient function for the diagonal channels :

$$\boxed{\Delta_{c\bar{c}}(z,\epsilon,q^2\mu_R^2,\mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2,z,\epsilon) \right\} \otimes \left(\Gamma\right)^{-1}}$$

Master Formula !

$$\boxed{\Delta_c(q^2, \mu_R^2, \mu_F^2, z) = \mathscr{C} \exp\Big(\Psi^c(q^2, \mu_R^2, \mu_F^2, z, \epsilon)\Big)\Big|_{\epsilon=0}}$$

where,

Ravindran et al.

$$\mathscr{C} e^{f(z)} = \delta(1-z) + \frac{1}{1!} f(z) + \frac{1}{2!} f(z) \otimes f(z) + \dots$$

$$\Psi^c(q^2, \mu_R^2, \mu_F^2, z, \varepsilon) = \left(\ln \left(Z_{UV,c}(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon) \right)^2 + \ln \left| \hat{F}_c(\hat{a}_s, \mu^2, Q^2, \varepsilon) \right|^2 \right) \delta(1-z)$$
$$+ 2\Phi^c(\hat{a}_s, \mu^2, q^2, z, \varepsilon) - 2\mathcal{C} \ln \Gamma_{cc}(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon)$$

$$\mathcal{S}_c = \mathcal{C} \exp\left(2\Phi^c\right)$$

$$\left.\begin{array}{l} \blacktriangleright\ Z_{UV,c}(\mu_R^2) \\ \blacktriangleright\ \Gamma_{cc}(\mu_F^2, z) \\ \blacktriangleright\ \hat{F}_c(Q^2) \\ \blacktriangleright\ \Phi^c(q^2, z) \end{array}\right\} \text{Building Blocks !}$$

UV Renormalisation Constant

Renormalisation Group Equation(RGE) gives,

$$\boxed{\mu_R^2 \frac{d}{d\mu_R^2} \log Z_{c,UV}(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \ \gamma_{i-1}^c}$$

UV Anomalous Dimension

$$a_s(\mu_R^2) = \frac{g_s^2(\mu_R^2)}{16\pi^2}$$

DIS and Drell-Yan process	$\gamma_i^q = 0$	i=0,1,....
b + B -> Higgs boson	γ_b^i	i=0,1-3
g + g -> Higgs boson	$\gamma_g^i = (i+1)\beta_i$	i=0,1,....

Altarelli-Parisi Kernels

Renormalisation Group Equation(RGE) gives,

AP Evolution Eqn.

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{ab}(z, \mu_F^2, \epsilon) = \frac{1}{2} \sum_{a'=q,\bar{q},g} P_{a'a}\left(z, a_s(\mu_F^2)\right) \otimes \Gamma_{a'b}(z, \mu_F^2, \epsilon), \quad a, b = q, \bar{q}, g$$

AP Splitting Function

Moch, Vogt, Vermaseren

Expanding around z -> 1

$$P_{cc}(z) = 2\left[\frac{A^c}{(1-z)_+} + B^c \delta(1-z) + C^c \log(1-z) + D^c\right] + \mathcal{O}(1-z)$$

Only Diagonal part contributes to SV+NSV

$A^c \longrightarrow$ Cusp Anomalous Dimension

$B^c, C^c, D^c \longrightarrow$ Collinear Anomalous Dimension

Form Factor

K+G/Sudakov Equation of the Form Factor,

No Poles

$$Q^2 \frac{d}{dQ^2} \log \hat{F}^c = \frac{1}{2}\left[K^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^c\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right]$$

Poles

Sen, Sterman, Magnea

$$A_q = \frac{C_F}{C_A} A_g$$

RG invariance gives,

$$\mu_R^2 \frac{d}{d\mu_R^2} K^c(a_s(\mu_R^2)) = -\mu_R^2 \frac{d}{d\mu_R^2} G^c(a_s(\mu_R^2)) = -A^c(a_s(\mu_R^2))$$

Maximally non-abelian,
Verified up to 3 loops

Solution in d = 4 + ϵ

$$\log \hat{F}^c(\hat{a}_s, Q^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \hat{\mathcal{L}}_F^{c,(i)}(\varepsilon)$$

where IR poles,

$$\hat{\mathcal{L}}_F^{c,(i)}(\varepsilon) = \sum_{j=-\infty}^{i+1} \hat{\mathcal{L}}_F^{c,(i,j)} \frac{1}{\varepsilon^j}$$

Moch, Vogt, Vermaseren;
Ravindran

Cusp ano dim

$$\{A^c, B^c, f^c, \gamma^c, g^c\}$$

Collinear ano dim

Soft ano dim

UV ano dim

Process dependent

Till Now…

Set of governing differential equations

- K+G/Sudakov Equation
- Renormalisation Group Equation(RGE)
- AP Evolution Equation

Building Blocks

- ✓ Renormalisation Constant $Z_{c,UV}$
- ✓ AP Kernel Γ_{cc}
- ✓ Form Factor $\hat{F}_c$
- ? Soft + Next-to-soft Distribution func. S_c

How to obtain ?

Guiding Principles

- Finiteness of the partonic coefficient function, $\mathbf{\Delta}_{c\bar{c}}$
- K+G/Sudakov differential equation of Form Factor
- RG evolution equation of AP kernels

Soft + Next-to-soft Distribution Func.

K+G/Sudakov equation gives,

Ravindran et al.

$$q^2 \frac{d}{dq^2} \ln S_c(q^2, z, \epsilon) = \left[\overline{K}^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon, z\right) + \overline{G}^c\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon, z\right) \right]$$

IR singular which cancels with $\hat{F}_c$, Γ_{cc}

Finite part

S_c admits an exponential solution to the K+G eqn.

$$\boxed{S_c = \mathscr{C}\, exp(2\Phi_c)}$$

$$\boxed{\mathscr{C}\, exp(2\Phi_c(z)) = \frac{\hat{\sigma}_{c\bar{c}}(z)}{Z_{c,UV}^2 |\hat{F}_c|^2}}$$

Pure virtual factored out

Real-Virtual(RV),
Real-Real(RR), etc

Soft + Next-to-soft Distribution Func.

An all order ansatz inspired from explicit results:

Ajjath, Mukherjee, Ravindran et al.

$$\Phi_c(\hat{a}_s, q^2, \mu^2, \epsilon, z) = \sum_i \hat{a}_s^i \underbrace{\left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\epsilon}{2}}}_{\text{Phase space factor}} \underbrace{S_\epsilon^i \left(\frac{i\epsilon}{1-z}\right)\left[\hat{\phi}_{SV}^{c,(i)}(\epsilon) + (1-z)\hat{\phi}_{NSV}^{c,(i)}(z,\epsilon)\right]}_{\text{From matrix elements}}$$

Cancels IR divergences from FF entirely and AP kernels partially

Cancels residual IR divergences from AP kernels

Expanding the ansatz,

$$\frac{1}{(1-z)}\left[(1-z)^2\right]^{i\frac{\epsilon}{2}} = \frac{\delta(1-z)}{i\epsilon} + \sum_{k=0}^{\infty} [i\epsilon]^k \frac{\mathcal{D}_k}{k!} \Rightarrow \hat{\phi}_{SV}^c \longrightarrow \{A^c, f^c, \overline{\mathcal{G}}^c\}$$

$$\left[(1-z)^2\right]^{i\frac{\epsilon}{2}} = \sum_{n=0}^{\infty} \frac{\left[i\epsilon\ \log(1-z)\right]^n}{n!} \Rightarrow \hat{\phi}_{NSV}^c \longrightarrow \{C^c, D^c, \phi_c(z)\}$$

Process dependent

Properties of Φ^c

- $$\Phi^q_{SV} = \left(\frac{C_F}{C_A}\right) \Phi^g_{SV}$$

- $$\Phi^q_{NSV} \neq \left(\frac{C_F}{C_A}\right) \Phi^g_{NSV}$$

- $$\Phi^b_{NSV} \neq \Phi^q_{NSV}$$

"Maximally non-abelian" verified till 3rd order

Moch, Vermaseren, et al.

Ravindran et al.

Breaks down beyond second Order

All order predictions for Δ_c

$$\begin{aligned}\Delta_c(q^2,\mu_R^2,\mu_F^2,z) &= \Delta_{c\bar{c}}^{SV+NSV}(q^2,\mu_R^2,\mu_F^2,z)\,,\\ &= \mathcal{C}\exp\left(\Psi^c(q^2,\mu_R^2,\mu_F^2,z,\varepsilon)\right)\Big|_{\varepsilon=0}\end{aligned}$$

$$D_i = \left(\frac{\log^i(1-z)}{(1-z)}\right)_+$$

Certain higher order SV and NSV terms can be predicted from lower orders completely,

$$L_z^i = \log^i(1-z)$$

GIVEN				PREDICTIONS		
$\Psi_c^{(1)}$	$\Psi_c^{(2)}$	$\Psi_c^{(3)}$	$\Psi_c^{(n)}$	$\Delta_c^{(2)}$	$\Delta_c^{(3)}$	$\Delta_c^{(i)}$
$\mathcal{D}_0,\mathcal{D}_1,\delta$ L_z^1,L_z^0				$\mathcal{D}_3,\mathcal{D}_2$ L_z^3	$\mathcal{D}_5,\mathcal{D}_4$ L_z^5	$\mathcal{D}_{(2i-1)},\mathcal{D}_{(2i-2)}$ $L_z^{(2i-1)}$
	$\mathcal{D}_0,\mathcal{D}_1,\delta$ L_z^2,L_z^1,L_z^0				$\mathcal{D}_3,\mathcal{D}_2$ L_z^4	$\mathcal{D}_{(2i-3)},\mathcal{D}_{(2i-4)}$ $L_z^{(2i-2)}$
		$\mathcal{D}_0,\mathcal{D}_1,\delta$ $L_z^3,\cdots,L_z^0$				$\mathcal{D}_{(2i-5)},\mathcal{D}_{(2i-6)}$ $L_z^{(2i-3)}$
			$\mathcal{D}_0,\mathcal{D}_1,\delta$ $L_z^n,\cdots,L_z^0$			$\mathcal{D}_{(2i-(2n-1))},\mathcal{D}_{(2i-2n)}$ $L_z^{(2i-n)}$

In general at order a_s^n,

$\log^k(1-z),\ n+1 \le k \le 2n-1$

All order predictions for Δ_c

Predictions till 7-loop for the first three NSV logs for the Higgs production in gluon fusion, using 3-loop result.

$$
\begin{aligned}
\Delta_g^{NSV} = \; & a_s \Delta_g^{NSV(1)} + a_s^2 \Delta_g^{NSV(2)} + a_s^3 \Delta_g^{NSV(3)} \\
& + a_s^4 \Bigg[\Bigg\{ -\frac{4096}{3} C_A^4 \Bigg\} L_z^7 + \Bigg\{ \frac{98560}{9} C_A^4 - \frac{7168}{9} n_f C_A^3 \Bigg\} L_z^6 + \Bigg\{ \Bigg(-\frac{298240}{9} + 23552 \zeta_2 \Bigg) C_A^4 \\
& + \frac{174208}{27} n_f C_A^3 - \frac{4096}{27} n_f^2 C_A^2 \Bigg\} L_z^5 + \mathcal{O}(L_z^4) \Bigg] + a_s^5 \Bigg[\Bigg\{ -\frac{8192}{3} C_A^5 \Bigg\} L_z^9 + \Bigg\{ \frac{96256}{3} C_A^5 \\
& - \frac{8192}{3} C_A^4 n_f \Bigg\} L_z^8 + \Bigg\{ \Bigg(-\frac{12283904}{81} + \frac{262144}{3} \zeta_2 \Bigg) C_A^5 + \frac{2569216}{81} C_A^4 n_f - \frac{81920}{81} n_f^2 C_A^3 \Bigg\} L_z^7 \\
& + \mathcal{O}(L_z^6) \Bigg] + a_s^6 \Bigg[\Bigg\{ -\frac{65536}{15} C_A^6 \Bigg\} L_z^{11} + \Bigg\{ \frac{9490432}{135} C_A^6 - \frac{180224}{27} C_A^5 n_f \Bigg\} L_z^{10} + \Bigg\{ \Bigg(\frac{671744}{3} \zeta_2 \\
& - \frac{4261888}{9} \Bigg) C_A^6 + \frac{8493056}{81} C_A^5 n_f - \frac{327680}{81} n_f^2 C_A^4 \Bigg\} L_z^9 + \mathcal{O}(L_z^8) \Bigg] \\
& + a_s^7 \Bigg[\Bigg\{ -\frac{262144}{45} C_A^7 \Bigg\} L_z^{13} + \Bigg\{ \frac{3309568}{27} C_A^7 - \frac{1703936}{135} C_A^6 n_f \Bigg\} L_z^{12} + \Bigg\{ \Bigg(-\frac{449429504}{405} \\
& + \frac{1310720}{3} \zeta_2 \Bigg) C_A^7 + \frac{11583488}{45} C_A^6 n_f - \frac{917504}{81} n_f^2 C_A^5 \Bigg\} L_z^{11} + \mathcal{O}(L_z^{10}) \Bigg] + \mathcal{O}(a_s^8) \,. \qquad (4.8)
\end{aligned}
$$

Checked upto 4th order

[Moch, Vogt, et. al], [De Florian, et al.]

[Das, et. al]

All order predictions for Δ_c

- Certain logarithms cannot be predicted completely but many color factors come from lower order result.
- For example, $\log^3(1-z)$ coefficient at 3rd order

	$gg \to H$			Drell-Yan (DY)		$b\bar{b} \to H$	
C_A^3	$\frac{-111008}{27} + 3584\zeta_2$	$\frac{-110656}{27} + 3584\zeta_2 + \chi_1$	C_F^3	$2272 + 3072\zeta_2$	$2272 + 3072\zeta_2$	$736 + 3072\zeta_2$	$736 + 3072\zeta_2$
$C_A^2 n_f$	$\frac{6560}{9}$	$\frac{19616}{27} + \chi_2$	$C_F^2 n_f$	$\frac{19456}{27}$	$\frac{6464}{9} + \chi_3$	$\frac{19456}{27}$	$\frac{6464}{9} + \chi_3$
$C_A n_f^2$	$\frac{-256}{27}$	$\frac{-256}{27}$	$C_A C_F^2$	$\frac{-111904}{27} + 512\zeta_2$	$\frac{-37184}{9} + 512\zeta_2 + \chi_4$	$\frac{-111904}{27} + 512\zeta_2$	$\frac{-37184}{9} + 512\zeta_2 + \chi_4$
			$C_F n_f^2$	$\frac{-256}{27}$	$\frac{-256}{27}$	$\frac{-256}{27}$	$\frac{-256}{27}$
			$C_A C_F n_f$	$\frac{2816}{27}$	$\frac{2816}{27}$	$\frac{2816}{27}$	$\frac{2816}{27}$
			$C_A^2 C_F$	$\frac{-7744}{27}$	$\frac{-7744}{27}$	$\frac{-7744}{27}$	$\frac{-7744}{27}$

[Anastasiou et. al]
[Duhr et. al]

Integral Representation in z-space

$$\boxed{\Delta_c(q^2, z) = C_0^c(q^2) \;\; \mathcal{C} \exp\left(2\Psi_{\mathcal{D}}^c(q^2, z)\right)}$$

Process dependent constant

Contains $\delta(1-z)$ contribution from $\hat{F}_c$ and S_c

Exponent can be written as,

Ajjath, Mukherjee, Ravindran et al.

$$\Psi_{\mathcal{D}}^c(q^2, z) = \frac{1}{2} \int_{\mu_F^2}^{q^2(1-z)^2} \frac{d\lambda^2}{\lambda^2} P'_{cc}(a_s(\lambda^2), z) + \mathcal{Q}^c(a_s(q^2(1-z)^2), z)$$

Process independent

Finite contributions from cancellation between Γ_{cc} and S_c

$$\mathcal{Q}^c(a_s(q^2(1-z)^2), z) = \left(\frac{1}{1-z} \overline{G}_{SV}^c(a_s(q^2(1-z)^2))\right)_+ + \varphi_{f,c}(a_s(q^2(1-z)^2), z)$$

Process dependent

Finite contributions coming from S_c

NSV Resummation

- Taking Mellin moment of z-space $\Delta_c(q^2, z)$,

$$\Delta_N^c(q^2) = \int_0^1 dz\ z^{N-1} \Delta_c(q^2, z)$$

$$= C_0^c(q^2)\ \boxed{e^{\Psi^c_{sv,N}(q^2,\omega) + \Psi^c_{nsv,N}(q^2,\omega)}}$$

N-independent coefficient

- Threshold limit z -> 1 translates to N -> ∞ in N-space

- Taking SV and NSV terms till $\dfrac{1}{N}$ corrections,

$$\boxed{\left(\frac{\log(1-z)}{1-z}\right)_+ \sim \frac{\log^2 N}{2} - \frac{\log N}{2N} + \frac{1}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)}$$

$$\boxed{\log^k(1-z) \sim \frac{\log^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)}$$

NSV Resummation

$$\boxed{\Psi^c_{\mathrm{sv,N}} = \ln(g^c_0(a_s(\mu_R^2))) + g^c_1(\omega)\ln N + \sum_{i=0}^{\infty} a^i_s(\mu_R^2) g^c_{i+2}(\omega)}$$

Known since 1989
[Sterman et. al]
[Catani et. al]

$$\omega = 2\ a_s(\mu_R^2)\ \beta_0 \log N$$

$$\tilde{g}^c_0(q^2, \mu_R^2, \mu_F^2) = C^c_0(q^2, \mu_R^2, \mu_F^2)\ g^c_0(a_s(\mu_R^2))$$

Ajjath, Mukherjee, Ravindran et al.

$$\Psi^c_{\mathrm{nsv,N}} = \frac{1}{N} \sum_{i=0}^{\infty} a^i_s(\mu_R^2) \Big(\bar{g}^c_{i+1}(\omega) + h^c_i(\omega, N) \Big)$$

$$h^c_i(\omega, N) = \sum_{k=0}^{i} h^c_{ik}(\omega)\ \log^k(N)$$

New Result !

Logarithmic Accuracy

The towers of $\ln N$ that we sum over,

$$\Delta_N^c = \underbrace{\boxed{\begin{matrix} a_s \ln^2 N \\ a_s^2 \ln^4 N \\ \vdots \\ a_s^i \ln^{2i} N \\ \\ \tilde{g}_{0,0}, g_1 \end{matrix}}}_{\text{LL}} \quad \underbrace{\boxed{\begin{matrix} a_s^2 \{\ln^3 N, \ln^2 N\} \\ a_s^3 \{\ln^5 N, \ln^4 N\} \\ \vdots \\ a_s^i \{\ln^{2i-1} N, \ln^{2i-2} N\} \\ \\ \tilde{g}_{0,1}, g_2 \end{matrix}}}_{\text{NLL}} \cdots \underbrace{\boxed{\begin{matrix} a_s^n \{\ln^{n+1} N, \ln^n N\} \\ \vdots \\ a_s^i \{\ln^{2i-n+1} N, \ln^{2i-n} N\} \\ \\ \tilde{g}_{0,n-1}, g_n \end{matrix}}}_{\text{N}^\text{n}\text{LL}}$$

	LL	NLL	$\text{N}^\text{n}\text{LL}$
Exponents:	$\tilde{g}_{0,0}, g_1$	$\tilde{g}_{0,1}, g_2$	$\tilde{g}_{0,n-1}, g_n$
	Only 1-loop info	**Only 2-loop info**	**Only n-loop info**

Logarithmic Accuracy

The towers of $\ln N/N$ that we sum over,

$$\Delta_N^c = \begin{array}{ccccc} \overline{\mathrm{LL}} & & \overline{\mathrm{NLL}} & & \overline{\mathrm{N^{n-1}LL}} \\ a_s \frac{1}{N} \log N & & a_s^2 \frac{1}{N} \log^2 N & & a_s^n \frac{1}{N} \log^n N \\ a_s^2 \frac{1}{N} \log^3 N & & a_s^3 \frac{1}{N} \log^4 N & & \\ a_s^3 \frac{1}{N} \log^5 N & & a_s^4 \frac{1}{N} \log^6 N & \cdots & \\ \vdots & & \vdots & & \vdots \\ a_s^i \frac{1}{N} \log^{2i-1} N & & a_s^i \frac{1}{N} \log^{2i-2} N & & a_s^i \frac{1}{N} \log^{2i-n} N \end{array}$$

Exponents:

Logarithmic Accuracy	Resummed Exponents
$\overline{\mathrm{LL}}$	$\tilde{g}^g_{0,0}, g^g_1, \bar{g}^g_1, h^g_0$
$\overline{\mathrm{NLL}}$	$\tilde{g}^g_{0,1}, g^g_2, \bar{g}^g_2, h^g_1$
$\overline{\mathrm{NNLL}}$	$\tilde{g}^g_{0,2}, g^g_3, \bar{g}^g_3, h^g_2$

Checks on Resummation

- Expansion of the resumed result matches with the fixed order till 3-loop
- The leading logarithm for SV+NSV matches with the existing result:

$$\Delta_{LL}^{DY} = g_0\, exp\left[\ln N\, g_1(w) + \frac{1}{N} h_0(w,N)\right]$$
$$= exp\left[8C_F a_s\left(\ln^2 N + \frac{\ln N}{N}\right)\right]$$

[Beneke et. al]
[Laenen et. al]

- Next, we proceed to see the numerical impact of NSV logarithms by performing Mellin Inversion of the resumed result.

Phenomenology

- The resumed result at a given accuracy, say $N^nLO + N^nLL$ is computed by taking the difference between the resumed result and the same truncated upto order a_s^n,

$$\sigma_N^{\mathrm{H,N^nLO+\overline{N^nLL}}} = \sigma_N^{\mathrm{H,N^nLO}} + \frac{\pi G_B^2(\mu_R^2)}{4(N^2-1)} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (\tau)^{-N} f_{g,N}(\mu_F^2) f_{g,N}(\mu_F^2) \times \left(\Delta_{g,N}\Big|_{\overline{\mathrm{N^nLL}}} - \Delta_{g,N}\Big|_{tr\ \mathrm{N^nLO}} \right).$$

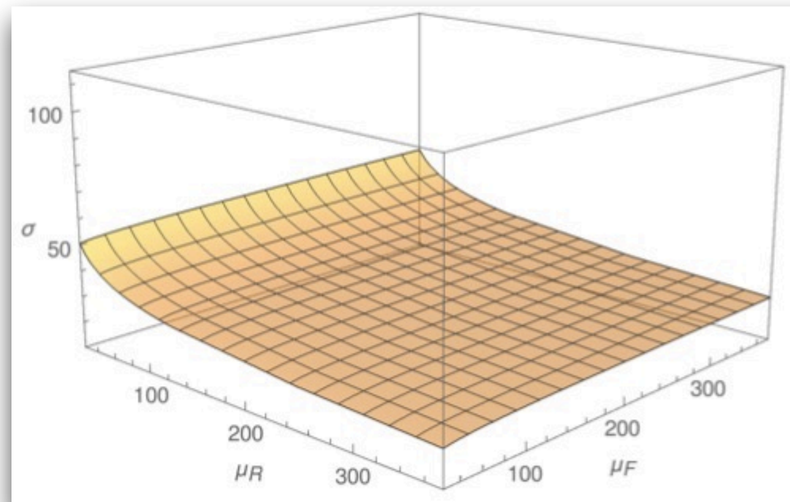
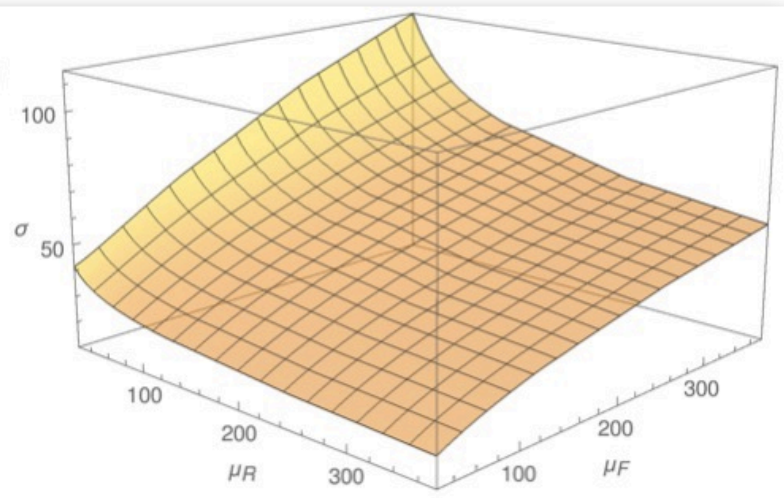
- The resumed results are matched to the fixed order result in order to avoid any double counting of threshold logarithms

Higgs production through gluon fusion

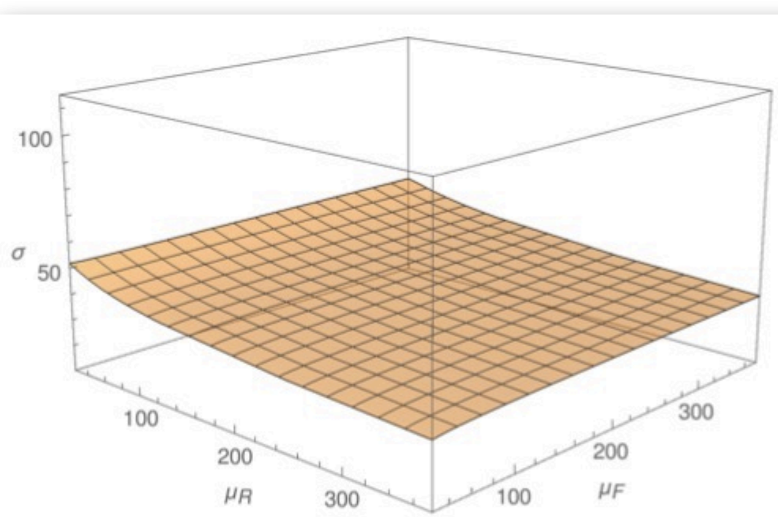
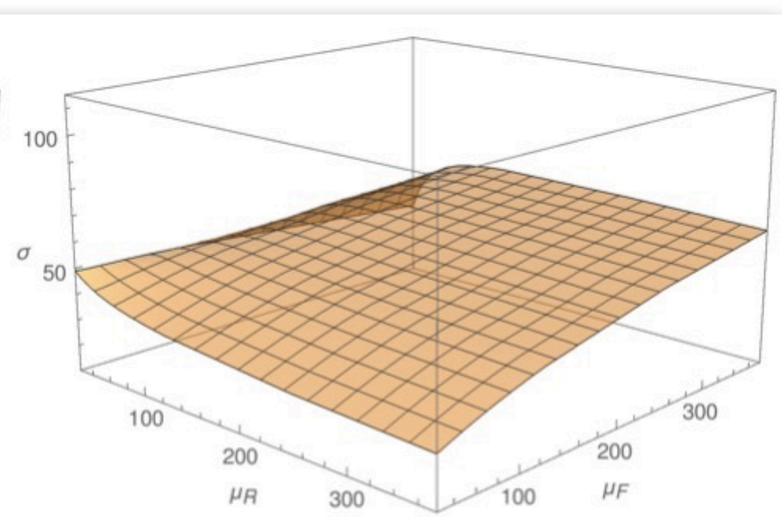
3D Plots and K-factor values

NLO	$\mathrm{NLO}+\overline{\mathrm{NLL}}$	NNLO	$\mathrm{NNLO}+\overline{\mathrm{NNLL}}$
1.6392	1.7173	1.9432	1.882

13 TeV LHC

NLO $\qquad$ $\mathrm{NLO}+\overline{\mathrm{NLL}}$

NNLO $\qquad$ $\mathrm{NNLO}+\overline{\mathrm{NNLL}}$

For the central scale

$$\mu_R = \mu_F = m_H/2$$

$$\mathrm{NNLO} \xrightarrow{-3.15\,\%} \mathrm{NNLO}+\overline{\mathrm{NNLL}}$$

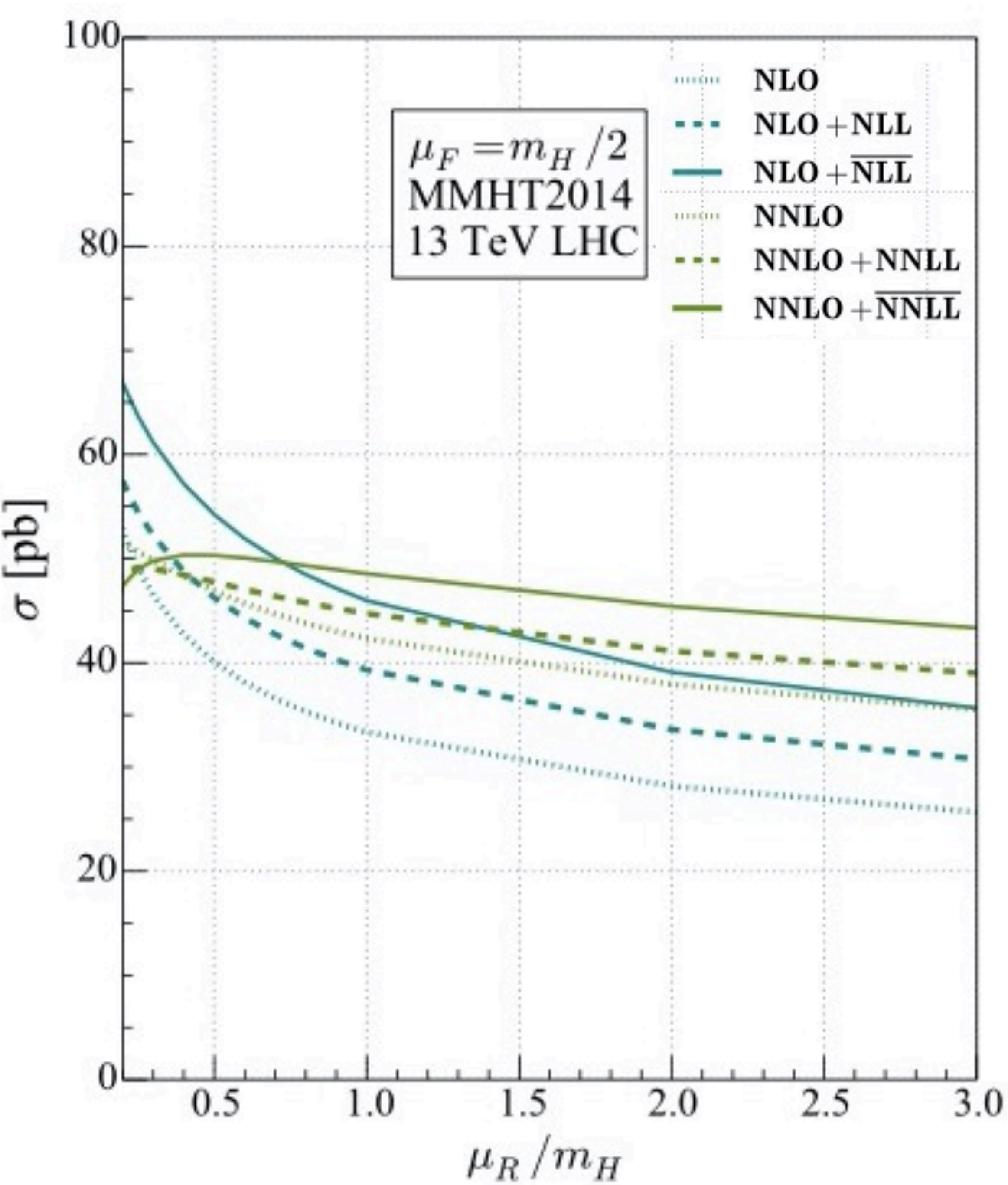
NNLO: $(+8.92\,\%\,,\,-\,10.12\%)$

$\mathrm{NNLO}+\overline{\mathrm{NNLL}}$: $(+11.90\,\%\,,\,-\,8.32\%)$

Better perturbative convergence at NNLO

Overall scale uncertainty gets closer to the corresponding fixed order result

μ_R scale variation



μ_R scale variation is less for resummed results as compared to fixed order

The scale variation is comparable for SV resumed and SV+NSV resumed at NLO accuracy

SV is dominant at NLO with 73.16% contribution

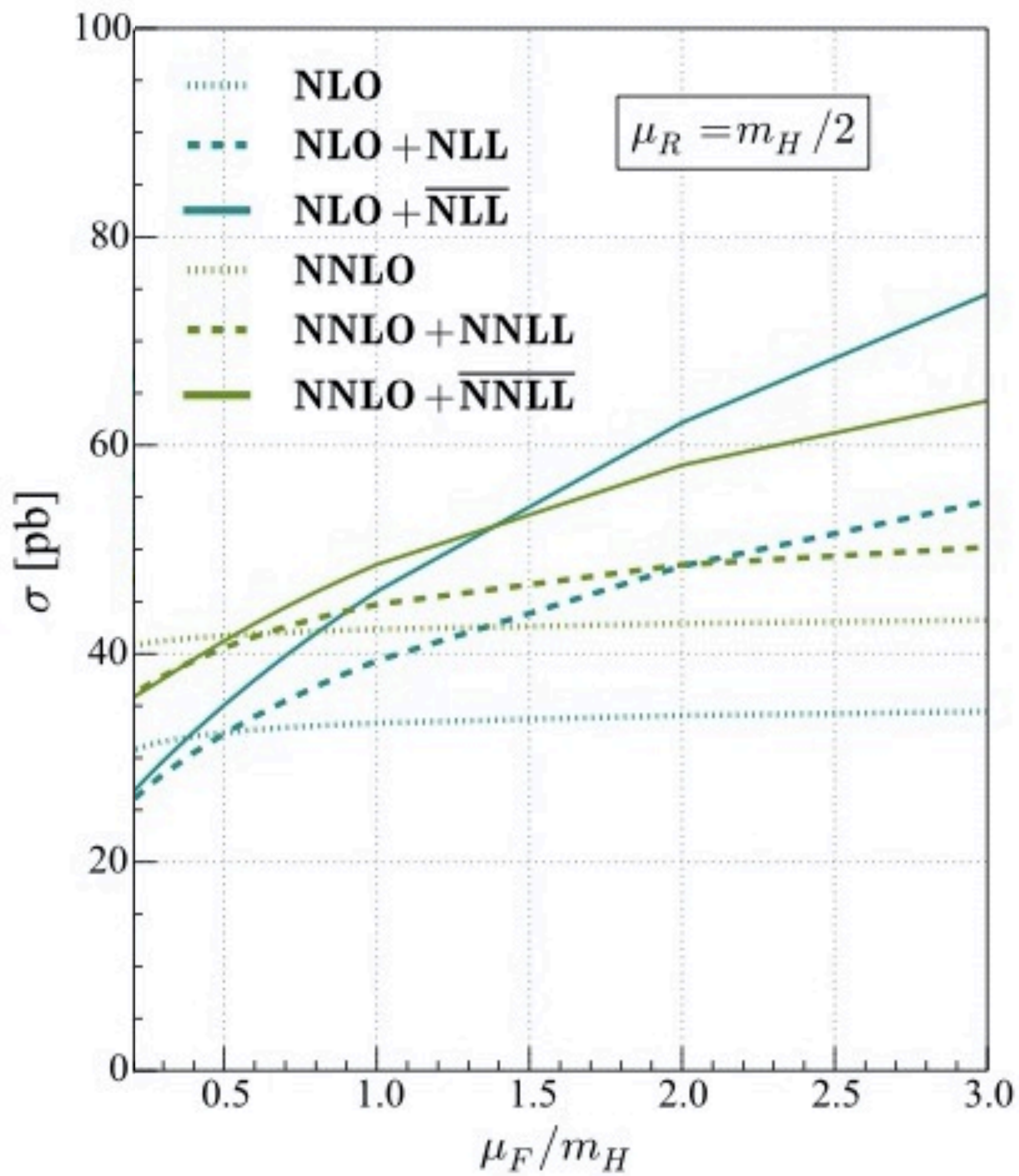
No significant improvement by the inclusion of SV but Comprehensible improvement by NSV Res results at NNLO

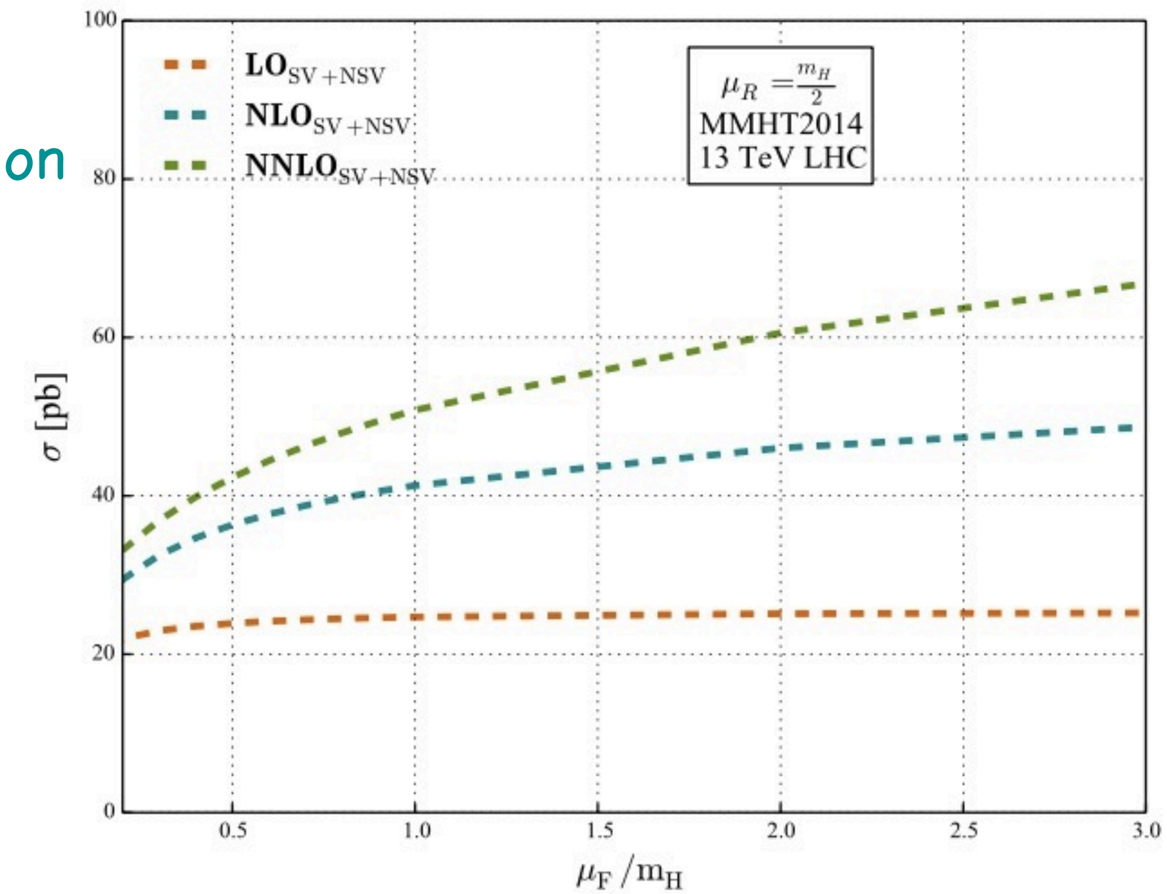
NSV is dominant at NNLO with 58.9% contribution while SV is only 15.8%

NLO	NLO+NLL	NLO+$\overline{\text{NLL}}$	NNLO	NNLO+NNLL	NNLO+$\overline{\text{NNLL}}$
$39.1681^{+9.09}_{-6.73}$	$38.0142^{+7.06}_{-5.70}$	$41.0325^{+7.06}_{-5.97}$	$46.4304^{+4.11}_{-4.70}$	$45.0904^{+4.32}_{-4.52}$	$44.9685^{+2.94}_{-3.74}$

μ_F scale variation

Fixed order is almost insensitive to μ_F variation





Fixed order truncated at SV+NSV contribution shows significant variations

μ_F dependence due to NSV contribution cancels with beyond NSV terms.

Increase in μ_F dependence with the increase in order of accuracy

% contribution of beyond NSV term increases with the order of accuracy

μ_F scale variation

NLO	NLO + NLL	NLO + $\overline{\text{NLL}}$	NNLO	NNLO + NNLL	NNLO + $\overline{\text{NNLL}}$
$39.1681^{+0.93}_{-1.23}$	$38.0142^{+8.24}_{-5.64}$	$41.0325^{+13.20}_{-7.64}$	$46.4304^{+0.44}_{-0.43}$	$45.0904^{+2.66}_{-2.79}$	$44.9685^{+5.35}_{-3.40}$

Behaviour of SV resumed result w.r.t to μ_F scale,

$$\text{NLO} + \text{NLL} \;>\; \text{NNLO} + \text{NNLL}$$

$(+21.68\,\%\,,\,-\,14.84\%)$ $(+5.90\,\%\,,\,-\,6.18\%)$

SV is 73.16% at NLO and 15.81% at NNLO

More % contribution of spurious beyond SV terms at NLO+NLL

Behaviour of SV+NSV resumed result w.r.t to μ_F scale,

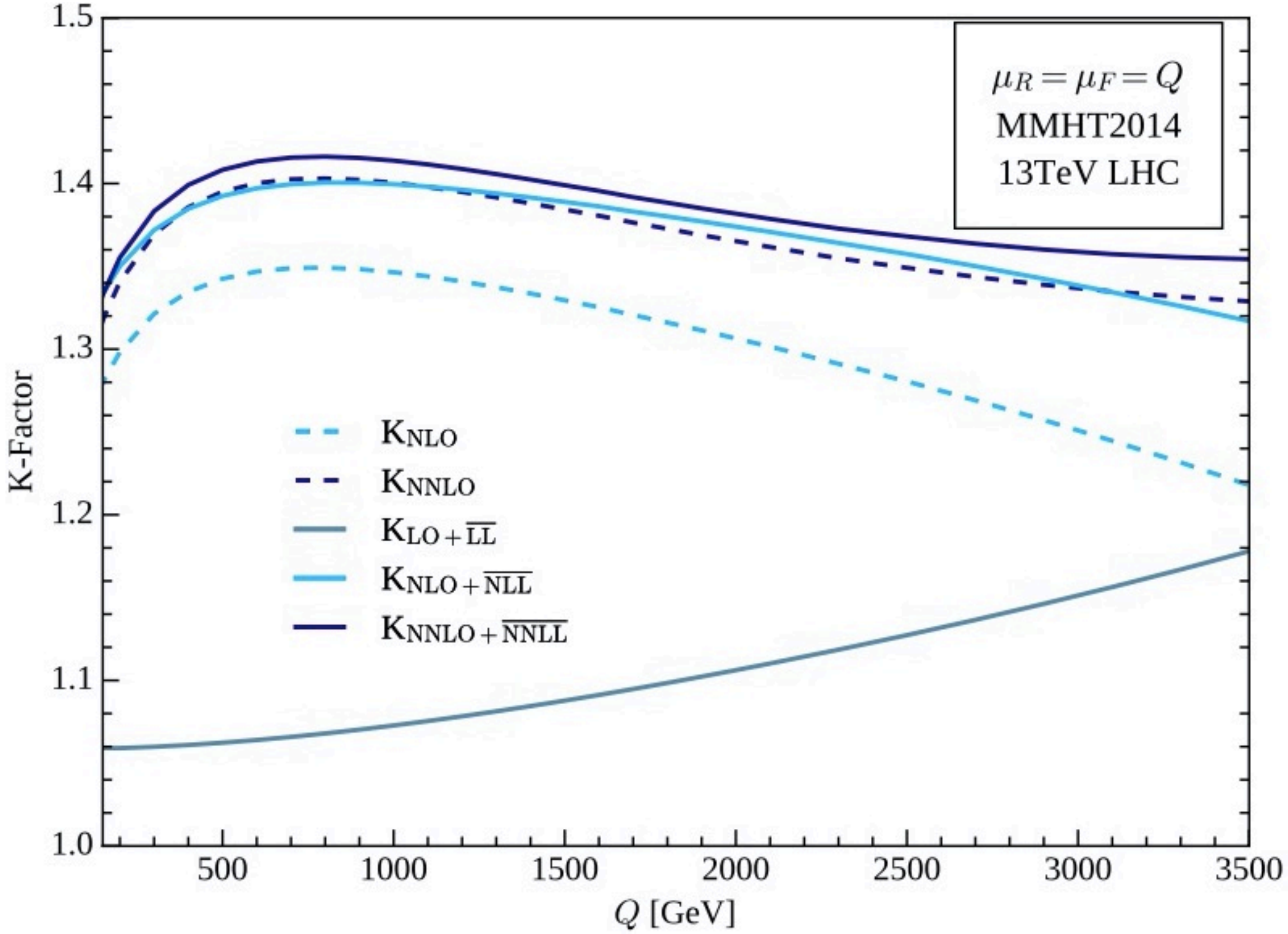
$$\text{NLO} + \overline{\text{NLL}} \;>\; \text{NNLO} + \overline{\text{NNLL}}$$

$(+32.17\,\%\,,\,-\,18.62\%)$ $(+11.90\,\%\,,\,-\,7.56\%)$

NSV is 45.81% at NLO and 58.91% at NNLO

More % contribution of spurious beyond NSV terms at NNLO+NNLL which compensates the μ_F variations

Phenomenology — Drell-Yan



Resummed curves lie above their corresponding fixed order ones

Enhancement due to the resummed corrections

Resummed curves are closer to each other as compared to the fixed order ones

Better perturbative convergence in resummed result

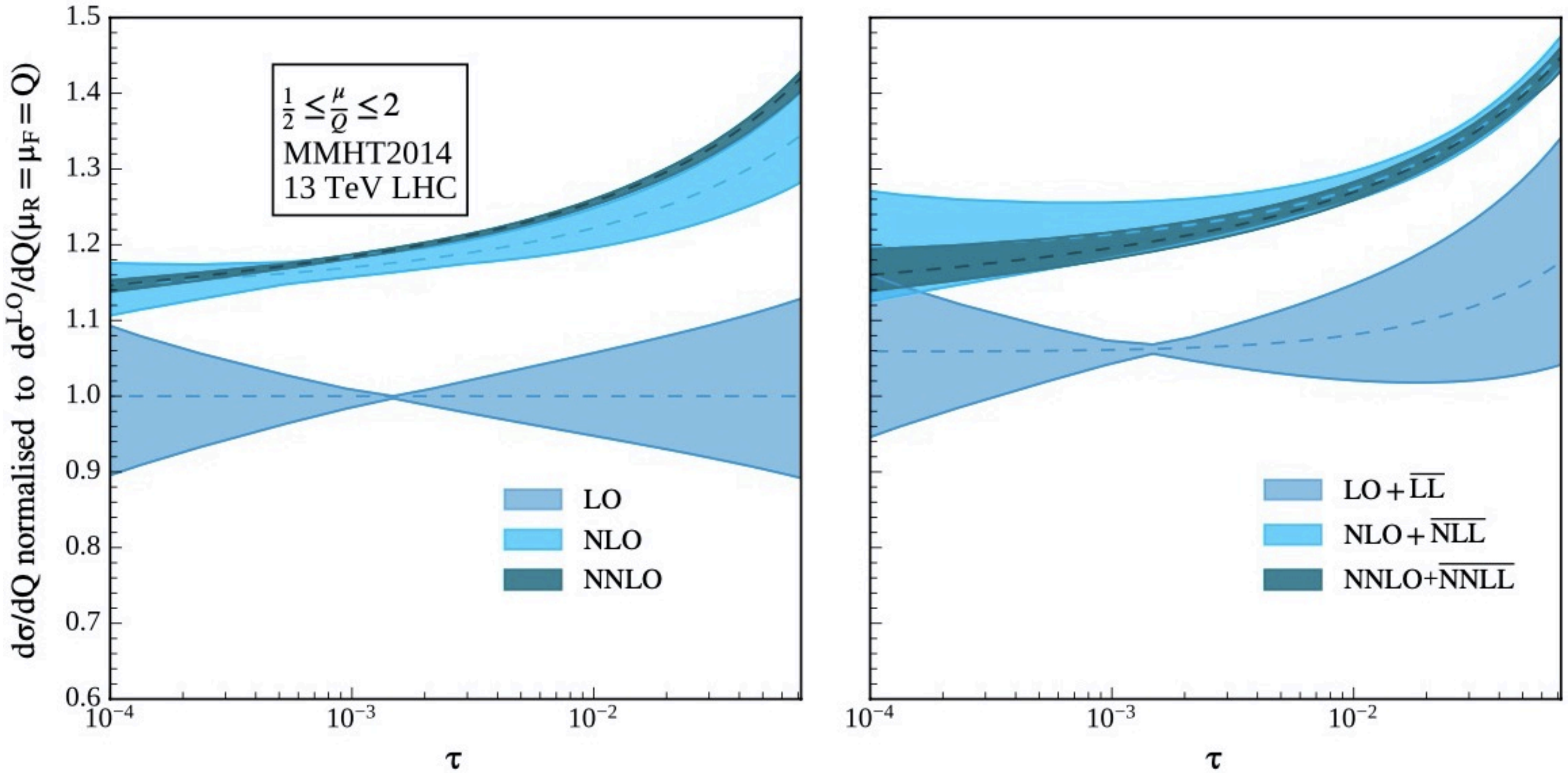
Reliability of resummed prediction is more

$\mu_R = \mu_F = Q$(GeV)	$\mathrm{LO} + \overline{\mathrm{LL}}$	NLO	$\mathrm{NLO} + \overline{\mathrm{NLL}}$	NNLO	$\mathrm{NNLO} + \overline{\mathrm{NNLL}}$
500	1.0624	1.3425	1.3925	1.3950	1.4082
1000	1.0728	1.3464	1.3995	1.4004	1.4138
2000	1.1062	1.3064	1.3739	1.3652	1.3818

$$\boxed{\mathrm{K}\,(Q) = \frac{\dfrac{d\sigma}{dQ}(\mu_R = \mu_F = Q)}{\dfrac{d\sigma^{\mathrm{LO}}}{dQ}(\mu_R = \mu_F = Q)}}$$

7-point scale uncertainty

μ = {μF , μR} is varied in the range [1/2Q, 2Q] keeping the ratio not larger than 2 and smaller than 1/2.

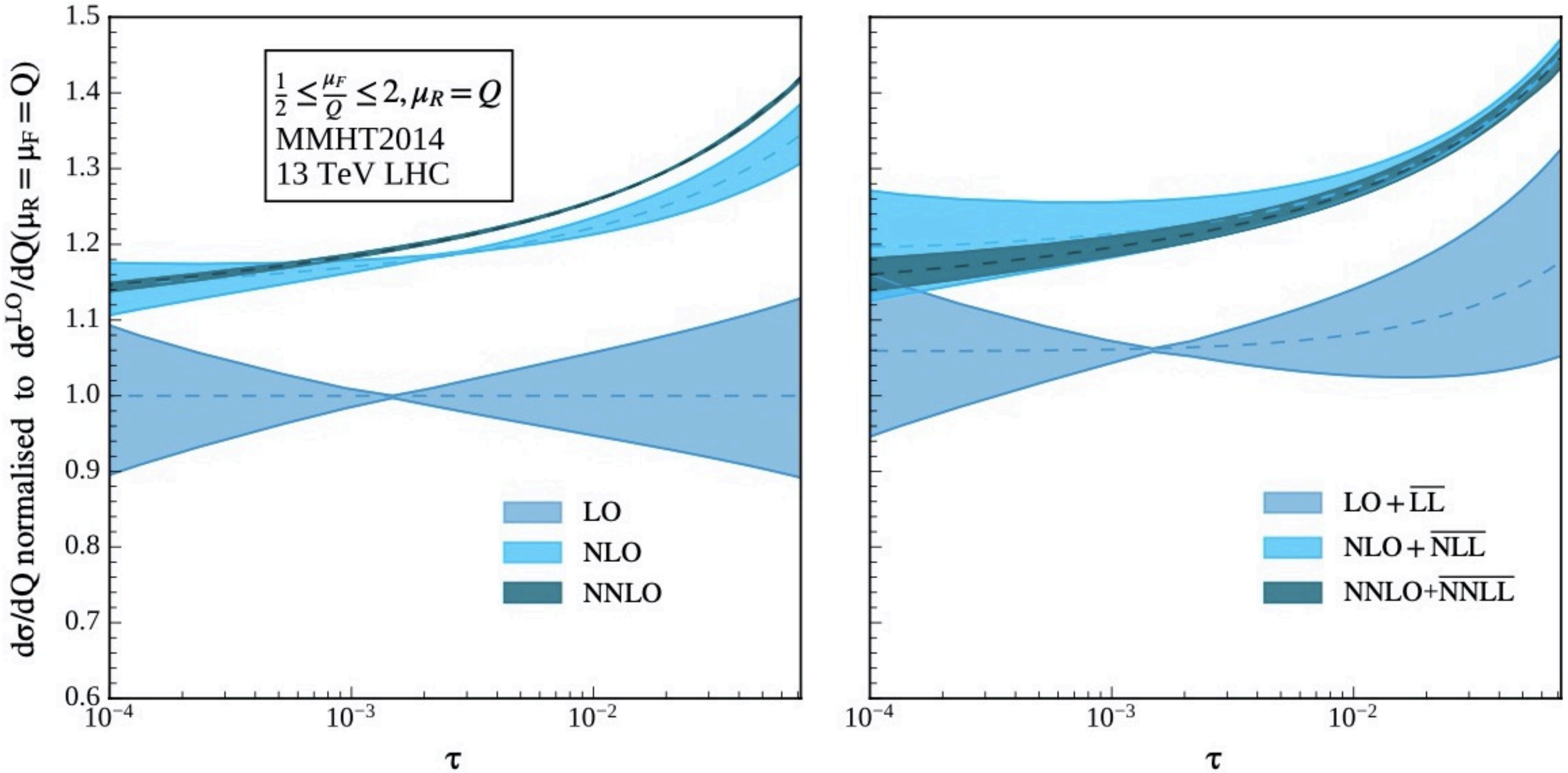


Resummed result shows a systematic reduction in uncertainty with the inclusion of each logarithmic accuracy

Improvement in uncertainty at NLO with the inclusion of $\overline{\mathrm{NLL}}$ is more as compared to the inclusion of $\overline{\mathrm{NNLL}}$ at NNLO

Q	LO	LO+$\overline{\mathrm{LL}}$	NLO	NLO+$\overline{\mathrm{NLL}}$	NNLO	NNLO+$\overline{\mathrm{NNLL}}$
1000	$2.3476^{+4.10\%}_{-3.92\%}$	$2.5184^{+4.49\%}_{-4.25\%}$	$3.1609^{+1.79\%}_{-1.69\%}$	$3.2857^{+2.08\%}_{-1.18\%}$	$3.2876^{+0.20\%}_{-0.31\%}$	$3.3191^{+1.13\%}_{-0.86\%}$
2000	$0.0501^{+8.50\%}_{-7.46\%}$	$0.0554^{+9.10\%}_{-7.91\%}$	$0.0654^{+2.83\%}_{-2.98\%}$	$0.0688^{+1.43\%}_{-1.23\%}$	$0.0684^{+0.37\%}_{-0.62\%}$	$0.0692^{+0.89\%}_{-0.78\%}$

μ_F scale variation



Resummed bands here, look similar to their corresponding 7-point bands

width of the 7-point band mainly comes from the μ_F uncertainties

NLO band gets improved with the inclusion of $\overline{\text{NLL}}$ but it is not the case at NNLO

NLO : $q\bar{q}$ -> 22 %
qg -> -5 %

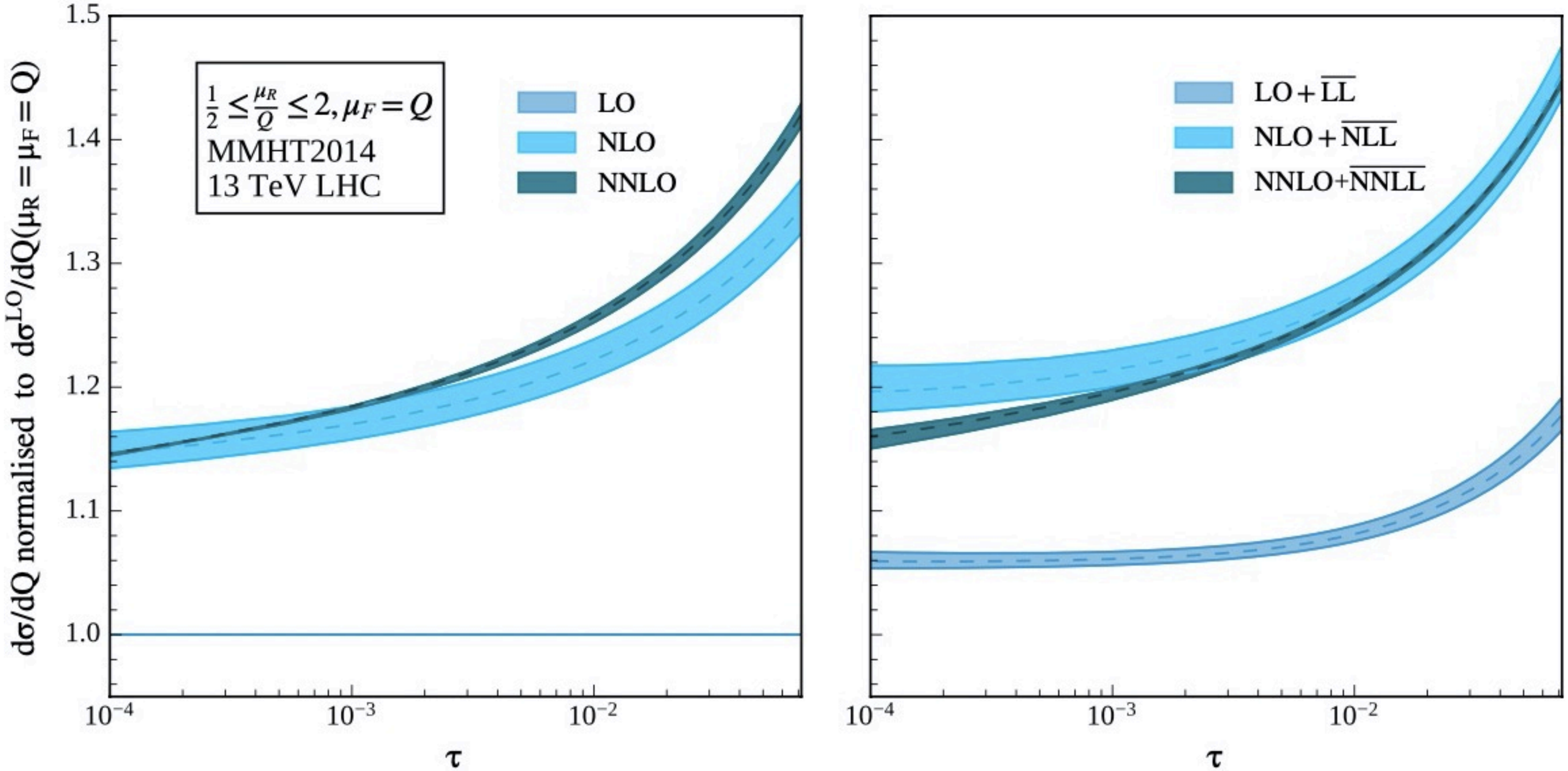
NNLO : $q\bar{q}$ -> 4.9 %
qg -> -2.8 %

$q\bar{q}$ dominating

Bigger cancellation b/w $q\bar{q}$ & qg

Missing qg-channel resummed contribution leads to more uncertainty at $\text{NNLO} + \overline{\text{NNLL}}$

μ_R scale variation



The uncertainty band becomes substantially thiner at $\mathrm{NNLO} + \overline{\mathrm{NNLL}}$

Each partonic channel is invariant under μ_R variation and hence inclusion of more corrections within a channel is expected to reduce the uncertainty

Conclusion

- **Can we exponentiate NSV logarithms ?**

  Yes. It comes out as a consequence of K+G differential equation.

- **Is NSV universal like SV ?**

  No. Unlike the SV coefficients, these terms contain the vertex informations and thereby do not possess the universality.

- **Can we resum and predict higher order NSV logs ?**

  Yes. The exponentiation of the correct exponents give the interference terms between SV $\otimes$ NSV and SV $\otimes$ SV of the lower orders, which is resumed to all orders in perturbation theory

- **Can we systematically build a framework for the computation of NSV corrections ?**

  Yes. We have systematically build a framework on the basis of two building blocks: Factorisation & RG invariance for NSV corrections