Loops and Legs 2022

Renormalization of the quark and gluon energy-momentum tensors

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QCD Energy-Momentum Tensor

Canonical — Noether current for space-time translational invariance of Lagrangian

$$\partial_\mu T_C^{\mu\nu} = 0 \qquad T_C^{\mu\nu} \neq T_C^{\nu\mu}$$

[Belinfante '39, '40]
[Rosenfeld '40]

Belinfante - Rosenfeld — Symmetric $T^{\mu\nu} = T^{\nu\mu}$

$$T^{\mu\nu} = \underbrace{-F^{\mu\lambda}F^{\nu}_{\lambda} + \frac{1}{4}\eta^{\mu\nu}F^2}_{\text{“Gluonic”}\quad T_g^{\mu\nu}} + \underbrace{\sum_q i\bar{\psi}_q\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi_q}_{\text{“Quark”}\quad T_q^{\mu\nu}}$$

[Ji '95]
[Lorcé 14]

$$A^{\{\mu}B^{\nu\}} = A^\mu B^\nu + A^\nu B^\mu$$

$$\overleftrightarrow{D}^\mu \equiv \frac{1}{2}\left(D^\mu - \overleftarrow{D}^\mu\right)$$

- sum over gluon color is implicit
- Terms with gauge-fixing and ghost fields do not contribute to matrix elements between physical states

[Nielsen '77]
[Collins, Duncan, Joglekar '77]

Our Focus: Renormalization

- EM tensor is a sum of composite operators → usually divergent

[Dixon, Taylor '74]
[Kluberg-Stern, Zuber '75]
[Joglekar, Lee '76]
[Nielsen '77]

- $\partial_\mu T^{\mu\nu} = 0$ → conserved: finite & scale independent $\quad [T^{\mu\nu}]_R = T^{\mu\nu}$

[Collins, Duncan, Joglekar '77]
[Adler, Collins, Duncan '77]

- Quark and gluon parts require renormalization

$$\partial_\mu T^{\mu\nu}_{q,g} \neq 0 \quad \longrightarrow \quad [T^{\mu\nu}_q]_R \neq T^{\mu\nu}_q \qquad [T^{\mu\nu}_g]_R \neq T^{\mu\nu}_g$$

State-of-the-art for renormalization

$\overline{\text{MS}}$:

- 2-loop: Hatta, Rajan, Tanaka '18
- 3-loop: Tanaka '19

Other schemes:

- Metz, Pasquini, Rodini '20, '21
- Lorcé, Metz, Pasquini, Rodini 21

Goal Computation of renormalization constants at 4-loop

Why Studying This? - EIC

- Understanding the dynamics of quarks and gluons inside the hadrons

- How does the mass arise?

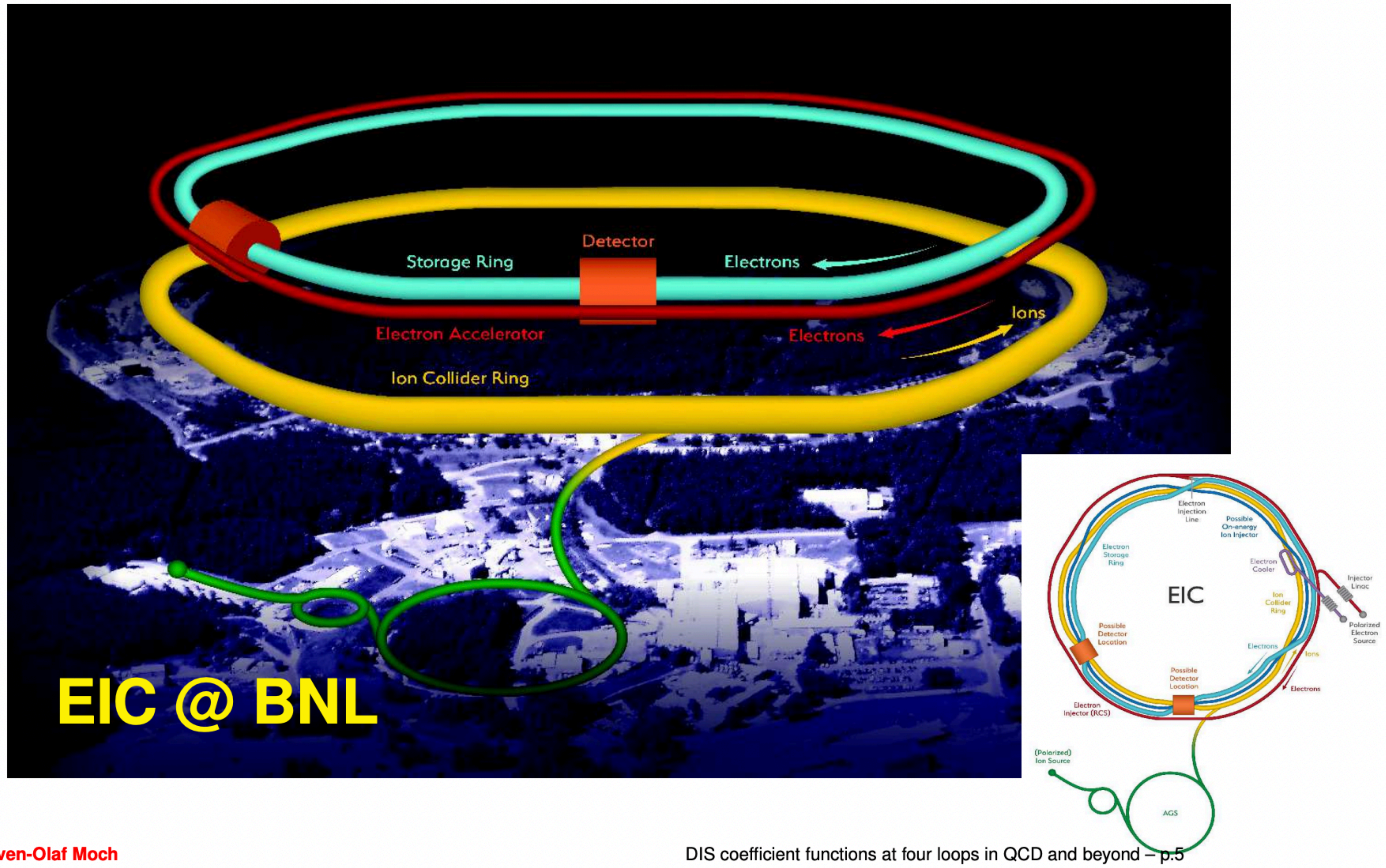
  Trace anomaly contributes

Many fundamental questions at

Electron-Ion Collider

Bright future for precision hadron physics

- Electron-Ion Collider
  A machine that will unlock the secrets of the strongest force in Nature



Sven-Olaf Moch

DIS coefficient functions at four loops in QCD and beyond – p.5

Why Studying This? - Nucleon FF

EM tensor contains important information about structures of nucleon

→ Even more important if we discuss quark and gluon parts separately

→ Partonic decomposition of nucleon mass and spin

Non-forward nucleon matrix element

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{u}(P')\Big[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}M\eta^{\mu\nu}\Big]u(P)$$

$$\Delta^\mu = P'^\mu - P^\mu$$

- All these 4 form factors are important
- D: radial pressure distribution inside a nucleon

$$\langle P|T^\mu_\mu, g|P\rangle = 2M^2\left(A_g + d\bar{C}_g\right)$$

$$\langle P|T^\mu_\mu, q|P\rangle = 2M^2\left(A_q + d\bar{C}_q\right)$$

- Ji sum rule for nucleon spin $\frac{1}{2} = J_q + J_g$ $\quad J_{q,g} = \frac{1}{2}\left(A_{q,g} + B_{q,g}\right)|_{\Delta\to 0}$

→ Related to second moment of generalized PDF

Operator Mixing

$A^{\{\mu}B^{\nu\}} = A^\mu B^\nu + A^\nu B^\mu$

$\overleftrightarrow{D}^\mu \equiv \frac{1}{2}\left(D^\mu - \overleftarrow{D}^\mu\right)$

Define 4-operators

$$\mathcal{O}_1 \equiv -F^{\mu\lambda}F^{\nu}{}_{\lambda}$$
$$\mathcal{O}_2 \equiv \eta^{\mu\nu}F^2$$
$$\mathcal{O}_3 \equiv \overline{\psi}\gamma^{\{\mu}\frac{i}{2}\overleftrightarrow{D}^{\nu\}}\psi$$
$$\mathcal{O}_4 \equiv \eta^{\mu\nu}\overline{\psi}m\psi$$

sum over quark flavors is implicit

$$T_g^{\mu\nu} = \mathcal{O}_1 + \frac{1}{4}\mathcal{O}_2$$
$$T_q^{\mu\nu} = \mathcal{O}_3$$

Lorentz covariance analysis: Mix under renormalization

$$[\mathcal{O}_i]_R = \sum_j Z_{ij}\mathcal{O}_j$$

$$\begin{pmatrix}\mathcal{O}_3\\ \mathcal{O}_1\\ \mathcal{O}_2\\ \mathcal{O}_4\end{pmatrix}_R = \begin{pmatrix} Z_{qq} & Z_{qg} & Z_{qF} & Z_{qm}\\ Z_{gq} & Z_{gg} & Z_{gF} & Z_{gm}\\ 0 & 0 & Z_{FF} & Z_{Fm}\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\mathcal{O}_3\\ \mathcal{O}_1\\ \mathcal{O}_2\\ \mathcal{O}_4\end{pmatrix}$$

Mixing with equation-of-motion and BRST exact operators are omitted

[Talk by Giulio Falcioni]

And: [Falcioni, Herzog '22]

→ matrix elements between physical states vanish

[Talk by Tongzhi Yang]

→ Our interest: renormalization of T_q and T_g between physical states

Goal and Approach

- Determine these 10 renormalization constants at 4-loop in QCD

- Calculable through evaluating Green correlation function involving these operators: Feynman diagrams

  - Two-point Green function with zero momentum operator insertion
  - Sudakov form factor

$$\mathcal{L}_{\text{int}} = h_{\mu\nu}(x) \left(\kappa_q T_q^{\mu\nu} + \kappa_g T_g^{\mu\nu} \right)$$

Exploit RG evolution

Three-loop: [TA, Banerjee, Dhani, Mathews, Rana, Ravindran '16]

- An alternative method without explicit computation

[Hatta, Rajan, Tanaka '18]

[Tanaka '18]

Symmetric Rank-Two Tensor

- Can be separated into traceless and trace components

- Traceless & trace parts are in different irreducible representations of Lorentz group

→ renormalization should preserve Lorentz symmetry

→ do not mix

- Regroup the operators to traceless tensor representation of Lorentz group

$$\bar{\mathcal{O}}^{\mu\nu} \equiv \mathcal{O}^{\mu\nu} - \frac{1}{d}\eta^{\mu\nu}\eta_{\alpha\beta}\mathcal{O}^{\alpha\beta}$$

trace ($\eta_{\alpha\beta}\mathcal{O}^{\alpha\beta}$)

$$\text{trace}(\bar{\mathcal{O}}^{\mu\nu}) = 0$$

apply this on $\mathcal{O}_1$ and $\mathcal{O}_3$ (& EOM for operator)

$$\bar{\mathcal{O}}_1 = \mathcal{O}_1 + \frac{1}{d}\mathcal{O}_2$$

$$\bar{\mathcal{O}}_3 = \mathcal{O}_3 - \frac{1}{d}\mathcal{O}_4$$

$$\begin{pmatrix} \bar{\mathcal{O}}_3 \\ \bar{\mathcal{O}}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_4 \end{pmatrix}_R = \begin{pmatrix} Z_{qq} & Z_{qg} & 0 & 0 \\ Z_{gq} & Z_{gg} & 0 & 0 \\ 0 & 0 & Z_{FF} & Z_{Fm} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{\mathcal{O}}_3 \\ \bar{\mathcal{O}}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_4 \end{pmatrix}$$

Block diagonal renormalization constant matrix

RG of Twist-Two Spin-Two

$\mathcal{O}_1$ and $\mathcal{O}_3$ twist-2 + twist-4

$\bar{\mathcal{O}}_1$ and $\bar{\mathcal{O}}_3$ twist-2 + ~~twist-4~~ twist-4 components can be expressed in terms of $\mathcal{O}_2$ and $\mathcal{O}_4$

Twist-2 spin-2 flavor singlet quark/gluon operators involved in Wilson OPE for DIS in Bjorken limit

RG equation $$\frac{\mathrm{d}}{\mathrm{d}\ \ln \mu_R^2} \begin{pmatrix} [\bar{O}_3]_R \\ [\bar{O}_1]_R \end{pmatrix} = \begin{pmatrix} \gamma_{qq} & \gamma_{qg} \\ \gamma_{gq} & \gamma_{gg} \end{pmatrix} \cdot \begin{pmatrix} [\bar{O}_3]_R \\ [\bar{O}_1]_R \end{pmatrix}$$

→ coincide with 2nd moment of DGLAP evolution for flavor-singlet unpolarized PDFs

Recent 4-loop anomalous dimensions in $\overline{\mathrm{MS}}$

Moch, Ruijl, Ueda, Vermaseren, Vogt '21

[Talk by Sven Moch]

[Talk by Johannes Blümlein]

Three loop: [Larin, Nogueira, van Ritbergen, Vermaseren '97]
[Moch, Vogt, Vermaseren '04]

Renormalized Symmetric Traceless Operators

Definition in d-dimensions

$$\bar{\mathcal{O}}^{\mu\nu} \equiv \mathcal{O}^{\{\mu\nu\}} - \frac{1}{d}\eta^{\mu\nu}\eta_{\alpha\beta}\mathcal{O}^{\alpha\beta}$$

Bare

$$\left[\bar{\mathcal{O}}^{\mu\nu}\right]_R \equiv \left[\mathcal{O}^{\{\mu\nu\}}\right]_R - \frac{1}{d}\eta^{\mu\nu}\eta_{\alpha\beta}\left[\mathcal{O}^{\alpha\beta}\right]_R$$

Renormalized

[Wilson '70]

$$\neq \left[\mathcal{O}^{\{\mu\nu\}}\right]_R - \frac{1}{d}\eta^{\mu\nu}\left[\eta_{\alpha\beta}\mathcal{O}^{\alpha\beta}\right]_R$$

In DR, trace and renormalization do not commute

$$\left[\eta_{\alpha\beta}\mathcal{O}^{\alpha\beta}\right]_R \neq \eta_{\alpha\beta}\left[\mathcal{O}^{\alpha\beta}\right]_R$$

Book on Renormalization: [Collins '86]

Starts arising from tensor of rank 2

finite difference

$$\frac{\eta_{\mu\nu}}{\epsilon} \xrightarrow{\text{Tr}} \frac{4}{\epsilon} - 2 \xrightarrow{\overline{\text{MS}}} \frac{4}{\epsilon}$$

$$\frac{\eta_{\mu\nu}}{\epsilon} \xrightarrow{\overline{\text{MS}}} \frac{\eta_{\mu\nu}}{\epsilon} \xrightarrow{\text{Tr}} \frac{4}{\epsilon} - 2$$

Renormalized EM Tensor

$$\left[T^{\mu\nu}_{q,g}\right]_R \quad = \quad \text{Traceless} \quad + \quad \text{Trace}$$

$$[\mathcal{O}_i]_R = \sum_j Z_{ij}\mathcal{O}_j$$

DGLAP

Trace Anomaly

Trace Anomaly

$$T^{\mu\nu} = -F^{\mu\lambda}F^{\nu}_{\lambda} + \frac{1}{4}\eta^{\mu\nu}F^2 + \sum_q i\bar{\psi}_q\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi_q$$

Trace

Classical $\quad T^{\mu}_{\mu,cl} \overset{\text{EOM}}{=} \sum_q m_q\bar{\psi}_q\psi_q$ $\quad$ vanishing in chiral limit

[Nielsen '77]
[Tarrach '82]

Quantum $\quad T^{\mu}_{\mu} = \frac{\beta(a_s,\epsilon)}{2a_s}\left[F^2\right]_R + \sum_q (1+\gamma_m)\left[m_q\bar{\psi}_q\psi_q\right]_R$ $\quad$ NON-vanishing even in chiral limit

[Chanowitz, Ellis '72, '73]
[Crewther '72]
[Collins, Duncan, Joglekar '77]

- Trace anomaly: breakdown of approximate conformal symmetry
- $T^{\mu}_{\mu} = \left[T^{\mu}_{\mu}\right]_R$ ➡ $\left(T^{\mu}_{\mu} - T^{\mu}_{\mu,cl}\right)$ is RG invariant

$$\beta(a_s,\epsilon) = -\epsilon a_s - \sum_{n=0}\beta_n a_s^{n+2} \qquad a_s = \frac{\alpha_s}{4\pi}$$

Parametrization of Trace

Anomaly can be distributed

$$\eta_{\mu\nu}\left[T_q^{\mu\nu}\right]_R = x_q\left[F^2\right]_R + (1+y_q)\left[m\bar{\psi}\psi\right]_R$$

$$\eta_{\mu\nu}\left[T_g^{\mu\nu}\right]_R = x_g\left[F^2\right]_R + y_g\left[m\bar{\psi}\psi\right]_R$$

Constraints

$$x_q + x_g = \frac{1}{2a_s}\beta(a_s,\epsilon) \qquad y_q + y_g = \gamma_m$$

x & y : finite and expandable in α_s

↓

We will determine

Z_{FF} and Z_{Fm}

Exploit RG invariance

$$\begin{pmatrix} \blacksquare & \blacksquare & Z_{qF} & Z_{qm} \\ \blacksquare & \blacksquare & Z_{gF} & Z_{gm} \\ 0 & 0 & Z_{FF} & Z_{Fm} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\eta_{\mu\nu}\left[T_q^{\mu\nu}\right]_R + \eta_{\mu\nu}\left[T_g^{\mu\nu}\right]_R$$

$$[\mathcal{O}_i]_R = \sum_j Z_{ij}\mathcal{O}_j \qquad T^{\mu\nu} = (T^{\mu\nu})_R$$

$$\epsilon\left(F^2\right)_R \to 0 \quad \text{as } \epsilon \to 0$$

$$-\frac{\epsilon}{2}F^2 + m\bar{\psi}\psi$$

Trace anomaly

$$[\mathcal{O}_i]_R = \sum_j Z_{ij}\mathcal{O}_j$$

$$\frac{\beta(a_s,\epsilon)}{2a_s}Z_{FF}F^2 + \left(1 + \gamma_m(a_s) + \frac{\beta(a_s,\epsilon)}{2a_s}Z_{Fm}\right)m\bar{\psi}\psi$$

$$Z_{FF} = -\epsilon\frac{a_s}{\beta(a_s,\epsilon)}$$

$$Z_{Fm} = -\frac{2\gamma_m(a_s)a_s}{\beta(a_s,\epsilon)}$$

- Consistent with explicitly (diagrammatic) computed results to 2-loops [Tarrach '82]
- All order results exist: follow from general theory of renormalization of gauge-invariant operators [Nielsen '75] [Spiridonov '84]

4-loop mass ano dim: [Chetyrkin '97] [Larin, Ritbergen, Vermaseren '97] 4-loop beta: [Czakon '04]

Relating Renormalization Constants

$$\left[T_q^{\mu\nu}\right]_R = [\mathcal{O}_3]_R \quad = \quad \text{Traceless} \quad + \quad \text{Trace}$$

$$[\mathcal{O}_i]_R = \sum_j Z_{ij}\mathcal{O}_j \qquad \text{DGLAP} \qquad \text{Trace Anomaly}$$

$$\begin{pmatrix} Z_{qq} & Z_{qg} & Z_{qF} & Z_{qm} \\ Z_{gq} & Z_{gg} & Z_{gF} & Z_{gm} \\ 0 & 0 & Z_{FF} & Z_{Fm} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Z_{qF} = \frac{1}{d}Z_{qg} + \frac{x_q}{d}Z_{FF}$$

$$Z_{qm} = -\frac{1}{d}Z_{qq} + \frac{x_q}{d}Z_{Fm} + \frac{1+y_q}{d}$$

$\left[T_q^{\mu\nu}\right]_R$

$$Z_{gF} = \frac{1}{d}Z_{gg} + \frac{x_g-1}{d}Z_{FF}$$

$$Z_{gm} = -\frac{1}{d}Z_{gq} + \frac{x_g-1}{d}Z_{Fm} + \frac{y_g}{d}$$

$\left[T_g^{\mu\nu}\right]_R$

Determining x & y and Unknown Z

Explicit results of 6 renormalization constants in $\overline{\text{MS}}$

$$Z_X = (\delta_{X,gg} + \delta_{X,qq} + \delta_{X,FF}) + \sum_{n=1} \frac{a_{X,n}}{\epsilon^n}$$

$$X = \{qq, qg, gq, gg, FF, Fm\}$$

$$Z_{qF} = \frac{1}{d} Z_{qg} + \frac{x_q}{d} Z_{FF}$$

$$Z_{qm} = -\frac{1}{d} Z_{qq} + \frac{x_q}{d} Z_{Fm} + \frac{1+y_q}{d}$$

$$Z_{gF} = \frac{1}{d} Z_{gg} + \frac{x_g - 1}{d} Z_{FF}$$

$$Z_{gm} = -\frac{1}{d} Z_{gq} + \frac{x_g - 1}{d} Z_{Fm} + \frac{y_g}{d}$$

Laurent expansion of RHS around $\epsilon = 0$

$\mathcal{O}(\epsilon^0)$ must vanish in $\overline{\text{MS}}$

$$x_{q,g}, y_{q,g} \sim \sum_{n=1} a_s^n \quad \text{and finite}$$

same accuracy as the renormalization constants

- New result $\mathcal{O}(a_s^4)$
- Consistent with 3-loop results

[Tanaka '18]

$$\begin{aligned}
x_g^{(4)} = {} & \frac{d_A^{abcd} d_A^{abcd}}{N_A}\Big(\frac{40}{9} - \frac{352}{3}\zeta_3\Big) + n_f \frac{d_F^{abcd} d_A^{abcd}}{N_A}\Big(-\frac{70}{3} - \frac{320}{9}\zeta_5 + \frac{2372}{9}\zeta_3\Big) \\
& + n_f^2 \frac{d_F^{abcd} d_F^{abcd}}{N_A}\Big(\frac{236}{9} + \frac{320}{9}\zeta_5 - \frac{848}{9}\zeta_3\Big) + C_F n_f^3\Big(-\frac{6949}{8748} + \frac{16}{27}\zeta_3\Big) \\
& + C_F^2 n_f^2\Big(-\frac{377003}{52488} + \frac{28}{9}\zeta_4 + \frac{14}{3}\zeta_3\Big) + C_F^3 n_f\Big(-\frac{151453}{17496} - \frac{40}{3}\zeta_5 + \frac{32}{9}\zeta_4 - \frac{4}{81}\zeta_3\Big) \\
& + C_A n_f^3\Big(\frac{4919}{34992} - \frac{22}{81}\zeta_3\Big) + C_A C_F n_f^2\Big(-\frac{39775}{26244} - \frac{55}{9}\zeta_4 + \frac{227}{81}\zeta_3\Big) \\
& + C_A C_F^2 n_f\Big(\frac{1471673}{52488} + \frac{250}{9}\zeta_5 - \frac{142}{9}\zeta_4 + \frac{445}{162}\zeta_3\Big) + C_A^2 n_f^2\Big(-\frac{83215}{11664} + \frac{20}{27}\zeta_5 + 3\zeta_4 \\
& - \frac{1235}{162}\zeta_3\Big) + C_A^2 C_F n_f\Big(\frac{73613}{52488} + \frac{5}{9}\zeta_5 + \frac{517}{18}\zeta_4 - \frac{779}{18}\zeta_3\Big) + C_A^3 n_f\Big(\frac{2275213}{23328} - \frac{445}{27}\zeta_5 \\
& - \frac{33}{2}\zeta_4 + \frac{6239}{162}\zeta_3\Big) + C_A^4\Big(-\frac{150653}{972} + \frac{22}{9}\zeta_3\Big)
\end{aligned}$$

Anomaly Induced Mass: Proton

QCD Trace Anomaly $\leftrightarrow$ Nucleon Mass $2M^2 = \langle P|T^\mu_\mu|P\rangle = \langle P|\eta_{\nu\sigma}\left[T^{\nu\sigma}_q\right]_R|P\rangle + \langle P|\eta_{\nu\sigma}\left[T^{\nu\sigma}_g\right]_R|P\rangle$

$$\eta_{\nu\sigma}\left[T^{\nu\sigma}_q\right]_R = \left(0.07957747155\alpha_s(\mu_R) + 0.05886948401\alpha_s^2(\mu_R) + 0.02160365423\alpha_s^3(\mu_R) + 0.01367502244\alpha_s^4(\mu_R)\right)\left[F^2\right]_R$$
$$+\left(1 + 0.1414710605\alpha_s(\mu_R) - 0.008234952168\alpha_s^2(\mu_R) - 0.06435111168\alpha_s^3(\mu_R) - 0.09861260742\alpha_s^4(\mu_R)\right)\left[m\bar{\psi}\psi\right]_R$$

$$\eta_{\nu\sigma}\left[T^{\nu\sigma}_g\right]_R = \left(-0.4376760935\alpha_s(\mu_R) - 0.2615118513\alpha_s^2(\mu_R) - 0.1838271927\alpha_s^3(\mu_R) - 0.2560961305\alpha_s^4(\mu_R)\right)\left[F^2\right]_R$$
$$+\left(0.4951487118\alpha_s(\mu_R) + 0.7765872615\alpha_s^2(\mu_R) + 0.8654924478\alpha_s^3(\mu_R) + 1.007417073\alpha_s^4(\mu_R)\right)\left[m\bar{\psi}\psi\right]_R$$

$$\eta_{\nu\sigma}\left[T^{\nu\sigma}_q\right]_R = 0.0538719\left[F^2\right]_R + 1.05336\left[m\bar{\psi}\psi\right]_R$$
$$\eta_{\nu\sigma}\left[T^{\nu\sigma}_g\right]_R = -0.298333\left[F^2\right]_R + 0.551265\left[m\bar{\psi}\psi\right]_R$$

$\alpha_s(\mu_R = 1\ \text{GeV}) \sim 0.47358$

RG invariant $\left[m\bar{\psi}\psi\right]_R = m\bar{\psi}\psi$

Agree with literature to $\mathcal{O}(\alpha_s^3)$

RG evolution $\dfrac{d}{d\ln\mu_R}\left[F^2\right]_R = -2\left(\dfrac{d(\beta/2a_s)}{d\ln a_s}\left[F^2\right]_R + \dfrac{d\gamma_m}{d\ln a_s}m\bar{\psi}\psi\right)$

[Tanaka '18]

Anomaly Induced Mass: Pion

$$m_\pi^2 = \langle \pi | m\bar{\psi}\psi | \pi \rangle$$

$$(1-\gamma_m) m_\pi^2 = \langle \pi | \frac{\beta}{2a_s} [F^2]_R | \pi \rangle$$

$$\frac{1}{2m_\pi^2} \langle \pi(p) | \eta_{\mu\nu} \left[T_q^{\mu\nu} \right]_R | \pi(p) \rangle = 0.388889 + 0.12215\alpha_s(\mu_R) + 0.124659\alpha_s^2(\mu_R) + 0.0991604\alpha_s^3(\mu_R) = 0.485227$$

$$\frac{1}{2m_\pi^2} \langle \pi(p) | \eta_{\mu\nu} \left[T_g^{\mu\nu} \right]_R | \pi(p) \rangle = 0.611111 - 0.12215\alpha_s(\mu_R) - 0.124659\alpha_s^2(\mu_R) - 0.0991604\alpha_s^3(\mu_R) = 0.514773$$

Disagree with literature at $\mathcal{O}(\alpha_s^3)$

[Tanaka '18]

Quark Gluon

$$m_\pi^2 \approx 0.485227 m_\pi^2 + 0.514773 m_\pi^2$$ almost equal contributions

(in chiral limit) $m_P^2 \approx -0.1 m_P^2 + 1.1 m_P^2$ quark contribution is negative

Summary

- Understanding the renormalization of quark/gluon EM tensor is important in hadronic physics
- EIC: understanding mass, spin, pressure of hadrons
- Exploiting RG invariance and other properties: renormalization to 4-loop
- Trace anomaly and its effect on proton's and pion's mass

Thank you!