Two- and three-loop QCD corrections to the width difference in the $B_s - \bar{B}_s$ system

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Loops and Legs in Quantum Field Theory
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Outline

- Neutral meson systems can oscillate between their flavor eigenstates

$$K^0 - \bar{K}^0, D^0 - \bar{D}^0 \quad \text{and} \quad B_q^0 - \bar{B}_q^0 \quad \text{with} \quad q = s, d.$$

- Loop-induced FCNC processes.
- B_q meson properties equally well accessible to theory and experiment.
- Flavor physics features multiple anomalies (LFU violation, muon $g-2$, $|V_{cb}|$, ...) challenging the SM.
- To address these challenges we need precision physics in the flavor sector.
- This implies precise *measurements* and precise *theoretical predictions*.

- $B_s^0 - \bar{B}_s^0$ oscillations between flavor eigenstates $|B_s^0\rangle$ and $|\bar{B}_s^0\rangle$

$$i\frac{d}{dt}\begin{pmatrix}|B_s^0(t)\rangle\\|\bar{B}_s^0(t)\rangle\end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix}|B_s^0(t)\rangle\\|\bar{B}_s^0(t)\rangle\end{pmatrix},$$

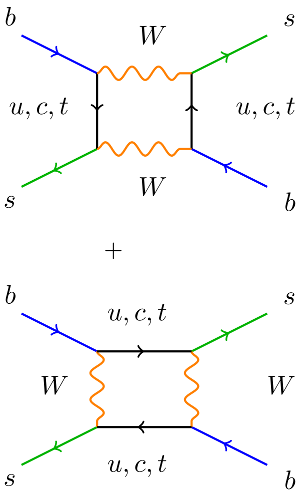
$$\hat{M} = \begin{pmatrix}M_{11} & M_{12}\\ M_{12}^* & M_{22}\end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{12}^* & \Gamma_{22}\end{pmatrix}$$

- Diagonalize the matrices, introduce mass eigenstates

$$|B_{s,L}\rangle = p\,|B_s^0\rangle + q\,|\bar{B}_s^0\rangle, \quad |B_{s,H}\rangle = p\,|B_s^0\rangle - q\,|\bar{B}_s^0\rangle$$

- The complex coefficients obey $|p|^2 + |q|^2 = 1$

$$M_{12} - \frac{i}{2}\Gamma_{12} \propto$$



- Physical observables depend on: $|M_{12}|$, $|\Gamma_{12}|$, ϕ_s
- ΔM_s: $B_s^0 - \bar{B}_s^0$ oscillation frequency: t quark is dominant in SM, sensitivity to NP in the loops

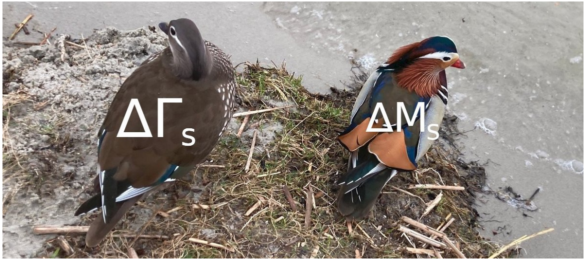
$$\Delta M_s = M_H - M_L \approx 2|M_{12}|$$

- $\Delta\Gamma_s$: $B_s^0 - \bar{B}_s^0$ width difference: only u and c contribute, precision probe of SM, little room for NP

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos\phi_s$$

- ϕ_s: CP-asymmetry in the mixing

$$a_{\text{fs}} = \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_s, \quad \phi_s = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$



- Our interest: $\Delta\Gamma_s$ from $B_s^0 - \bar{B}_s^0$
- Experimental value (HFLAV 2021 average)

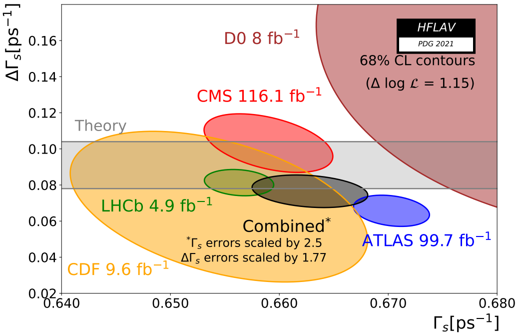
$$\Delta\Gamma^{\text{exp}} = (0.085 \pm 0.005)\ \text{ps}^{-1}$$

- Theory prediction (NLO + n_f-piece of NNLO QCD corrections) as of 2020 [Beneke, Buchalla, Greub, Lenz, & Nierste, 1999; Ciuchini, Franco, Lubicz, & Mescia, 2002; Ciuchini, Franco, Lubicz, Mescia, & Tarantino, 2003; Lenz & Nierste, 2007; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020; Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017]

$$\Delta\Gamma_s = (0.077 \pm 0.015_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.017_{\Lambda_{\text{QCD}}/m_b}) \times \text{ps}^{-1}\ \text{(pole)}$$

$$\Delta\Gamma_s = (0.088 \pm 0.011_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.014_{\Lambda_{\text{QCD}}/m_b}) \times \text{ps}^{-1}\ (\overline{\text{MS}})$$

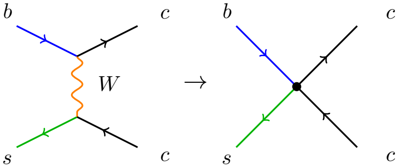
- Large perturbative uncertainty from the uncalculated NNLO corrections (pert.)
- Can be reduced by including relevant 2- and 3-loop QCD corrections
- Theory under pressure, full NNLO corrections highly desirable



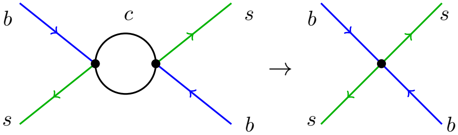
$$\Gamma_s = (\Gamma_L + \Gamma_H)/2$$

Overview of the matching calculation

- $|\Delta B| = 1$ EFT ($m_b \ll m_W, m_t$)

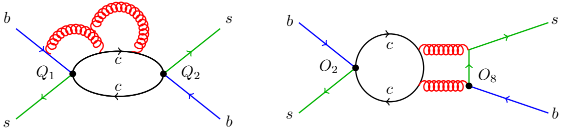


- $|\Delta B| = 2$ EFT (via HQE)

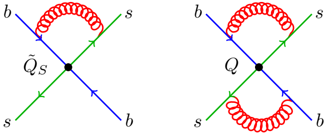


$$\Gamma_{12} \sim \frac{1}{m_b^3} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_3^{(i)} + \frac{1}{m_b^4} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_4^{(i)} + \ldots$$

- Calculation done using $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$ in the CMM operator basis for $b \to sc\bar{c}$ [Chetyrkin, Misiak, & Münz, 1998]
- Representative diagrams in the $|\Delta B| = 1$ EFT needed for the NNLO accuracy



matched to the $|\Delta B| = 2$ EFT



$|\Delta B| = 1$ side of the matching: operator basis

Effective Hamiltonian of the $|\Delta B| = 1$ theory in the CMM basis [Chetyrkin, Misiak, & Münz, 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb} \Bigg(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \Bigg) - V_{us}^* V_{ub} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}\,,$$

- Dominant current-current (cc) operators

$$Q_1 \equiv Q_1^{cc} = \bar{s}_L \gamma_\mu T^a c_L \; \bar{c}_L \gamma^\mu T^a b_L ,$$
$$Q_2 \equiv Q_1^{cc} = \bar{s}_L \gamma_\mu c_L \; \bar{c}_L \gamma^\mu b_L$$

- $Q^u_{1,2}$, $Q^{cu}_{1,2}$ and $Q^{uc}_{1,2}$ follow the same pattern

- Penguin operators

$$Q_3 = \bar{s}_L \gamma_\mu b_L \textstyle\sum_q \bar{q} \gamma^\mu q \,,$$
$$Q_4 = \bar{s}_L \gamma_\mu T^a b_L \textstyle\sum_q \bar{q} \gamma^\mu T^a q \,,$$
$$Q_5 = \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L \textstyle\sum_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q \,,$$
$$Q_6 = \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L \textstyle\sum_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q \,,$$
$$Q_8 = \frac{g_s}{16\pi^2} m_b \, \bar{s}_L \sigma^{\mu\nu} T^a b_R \, G^a_{\mu\nu} \,,$$

$|\Delta B| = 1$ side of the matching: operator basis

Effective Hamiltonian of the $|\Delta B| = 1$ theory in the CMM basis [Chetyrkin, Misiak, & Münz, 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb} \left(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \right) - V_{us}^* V_{ub} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.}\,,$$

- 4-fermion vertices generate Dirac structures with multiple insertions of γ matrices

$$(P_L)_{ij} \times (P_L)_{kl}, \quad (\gamma^\mu P_L)_{ij} \times (\gamma_\mu P_L)_{kl}, \quad (\gamma^\mu \gamma^\nu P_L)_{ij} \times (\gamma_\mu \gamma_\nu P_L)_{kl},$$
$$(\gamma^\mu \gamma^\nu \gamma^\rho P_L)_{ij} \times (\gamma_\mu \gamma_\nu \gamma_\rho P_L)_{kl}, \ldots$$

- 4-dimensions: Products of γ matrices reducible using Fierz and Chisholm identities

$$\gamma^\mu \gamma^\nu \gamma^\rho = g^{\mu\nu} \gamma^\rho + g^{\nu\rho} \gamma^\mu - g^{\mu\rho} \gamma^\nu + i \varepsilon^{\mu\nu\rho\sigma} \gamma_\sigma \gamma^5$$

- d-dimensions: Fierz and Chisholm identities become ambiguous
- Proper treatment using evanescent operators [Dugan & Grinstein, 1991; Herrlich & Nierste, 1995]

$|\Delta B| = 1$ side of the matching: operator basis

Effective Hamiltonian of the $|\Delta B| = 1$ theory in the CMM basis [Chetyrkin, Misiak, & Münz, 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}}\left[-V_{ts}^* V_{tb}\left(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8\right) - V_{us}^* V_{ub}\sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb}\sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub}\sum_{i=1}^{2} C_i Q_i^{uc}\right] + \text{h.c.}\,,$$

- $|\Delta B| = 1$ LO evanescent operators

$$E_1^{(1)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a c_L \; \bar{c}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a b_L - 16 Q_1\,,$$
$$E_2^{(1)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} c_L \; \bar{c}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L - 16 Q_2\,,$$
$$E_3^{(1)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} b_L \sum_q \bar{q}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} q - 20 Q_5 + 64 Q_3\,,$$
$$E_4^{(1)} = \bar{s}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} T^a b_L \sum_q \bar{q}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} T^a q - 20 Q_6 + 64 Q_4$$

- NLO evanescent operators have up to 7 matrices ...

$|\Delta B| = 1$ side of the matching: operator basis

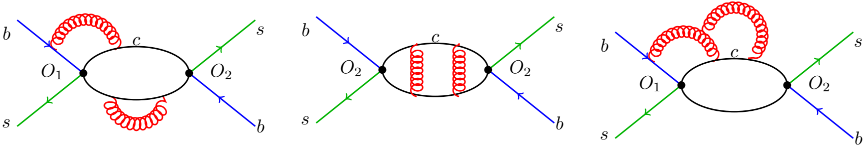
Effective Hamiltonian of the $|\Delta B| = 1$ theory in the CMM basis [Chetyrkin, Misiak, & Münz, 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Big[-V_{ts}^* V_{tb} \Big(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \Big) - V_{us}^* V_{ub} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Big] + \text{h.c.}\,,$$

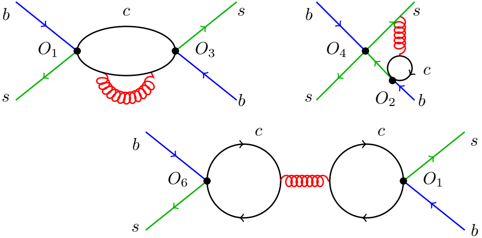
- Evanescent operators are of $\mathcal{O}(\varepsilon)$, formally vanishing in the $d \to 4$ limit
- However: A pole multiplying tree-level matrix element of an ev. operator $\langle E_i^{(j)} \rangle / \varepsilon$ is $\mathcal{O}(\varepsilon^0)$

$|\Delta B| = 1$ side of the matching: representative diagrams

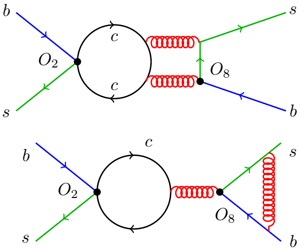
3-loop $O_{1,2} \times O_{1,2}$ correlators



2-loop $O_{1,2} \times O_{3-6}$ correlators



2-loop $O_{1,2} \times O_8$ correlators



$|\Delta B| = 2$ side of the matching: operator basis

- $\Delta\Gamma_s$ described by local $|\Delta B| = 2$ operators [Beneke, Buchalla, Greub, Lenz, & Nierste, 1999; Lenz & Nierste, 2007; Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017]
- Using Heavy Quark Expansion [Khoze & Shifman, 1983; Shifman & Voloshin, 1985; Khoze, Shifman, Uraltsev, & Voloshin, 1987; Chay, Georgi, & Grinstein, 1990; Bigi & Uraltsev, 1992; Bigi, Uraltsev, & Vainshtein, 1992; Bigi, Shifman, Uraltsev, & Vainshtein, 1993; Blok, Koyrakh, Shifman, & Vainshtein, 1994; Manohar & Wise, 1994] (expansion in $\Lambda_{\mathrm{QCD}}/m_b$) one arrives at

$$\Gamma_{12} = -(\lambda_c^q)^2\Gamma_{12}^{cc} - 2\lambda_c^q\lambda_u^q\Gamma_{12}^{uc} - (\lambda_u^q)^2\Gamma_{12}^{uu}, \quad \lambda_{q'}^q \equiv V_{q'q}^* V_{q'b}$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}\left[H^{ab}(z)\langle B_s|Q|\bar{B}_s\rangle + \widetilde{H}_S^{ab}(z)\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle\right] + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}/m_b\right)$$

- $H(z)$ and $\widetilde{H}_S(z)$: Wilson coefficients from the perturbative matching to the physical $|\Delta B| = 2$ operators

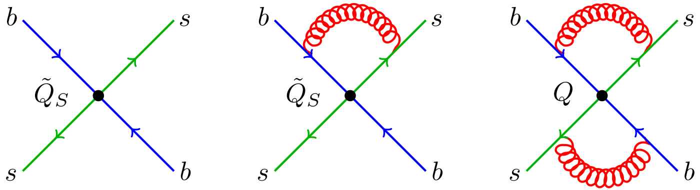
$$Q = \bar{s}_i\gamma^\mu\left(1-\gamma^5\right)b_i\ \bar{s}_j\gamma_\mu\left(1-\gamma^5\right)b_j \qquad \widetilde{Q}_S = \bar{s}_i\left(1+\gamma^5\right)b_j\ \bar{s}_j\left(1+\gamma^5\right)b_i$$

- Additional operators needed at intermediate stages (e. g. basis changes, def. of evanescent operators)

$$\widetilde{Q} = \bar{s}_i \gamma^\mu \left(1-\gamma^5\right) b_j \; \bar{s}_j \gamma_\mu \left(1-\gamma^5\right) b_i \,, \quad Q_S = \bar{s}_i \left(1+\gamma^5\right) b_i \; \bar{s}_j \left(1+\gamma^5\right) b_j \,,$$

- Not shown here: evanescent $|\Delta B| = 2$ operators and $1/m_b$ suppressed operators $|\Delta B| = 1$ to $|\Delta B| = 2$, $z \equiv m_c^2/m_b^2$
- Nonperturbative ME $\langle B_s|Q|\bar{B}_s\rangle$ and $\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle$ (also for B_d mesons) from QCD/HQET sum rules [Ovchinnikov & Pivovarov, 1988; Reinders & Yazaki, 1988; Korner, Onishchenko, Petrov, & Pivovarov, 2003; Mannel, Pecjak, & Pivovarov, 2011; Grozin, Klein, Mannel, & Pivovarov, 2016; Kirk, Lenz, & Rauh, 2017; King, Lenz, & Rauh, 2019, 2021], lattice QCD [Bazavov et al., 2016; Dowdall, Davies, Horgan, Lepage, Monahan, et al., 2019] or combined [Di Luzio, Kirk, Lenz, & Rauh, 2019]

$|\Delta B| = 2$ side of the matching: representative diagrams



Wilson coefficients of the $|\Delta B| = 2$ theory determined in the matching to $|\Delta B| = 1$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}\left(\Lambda_{\mathrm{QCD}} / m_b\right)$$

$|\Delta B| = 1$ contributions needed for NNLO (always 2 insertions from $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$)

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1,2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

- Important scale: $z \equiv m_c^2/m_b^2$
- Can expand in z, good convergence already for $\mathcal{O}(z)$
- The final result incorporates various $O_{i-j} \times O_{k-l}$ contributions at 1, 2 or 3 loops (with $i,j,k,l \in \{1-6,8\}$)
- At 3 loops we consider only $O_{1-2} \times O_{1-2}$
- Available literature results: mostly z-exact but often concern only the fermionic n_f-piece
- Our calculation: full results (n_f and non-n_f) but expanded up to $\mathcal{O}(z)$
- ✔ Many cross checks through comparisons to the existing results

$|\Delta B| = 1$ contributions needed for NNLO (always 2 insertions from $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$)

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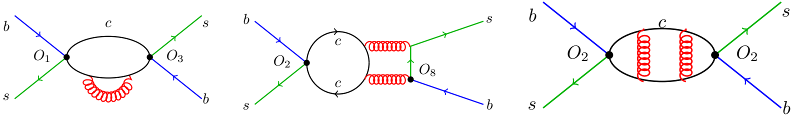
- LO contributions to $\Delta\Gamma_s$
  - 1-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Hagelin, 1981; Franco, Lusignoli, & Pugliese, 1982; Chau, 1983; Buras, Slominski, & Steger, 1984; Khoze, Shifman, Uraltsev, & Voloshin, 1987; Datta, Paschos, & Turke, 1987; Datta, Paschos, & Wu, 1988]
  - 1-loop $O_{1-2} \times O_{3-6}$ correlators (z-exact) [Beneke, Buchalla, Greub, Lenz, & Nierste, 1999]
  - 1-loop $O_{3-6} \times O_{3-6}$ correlators (z-exact) [Beneke, Buchalla, & Dunietz, 1996]
- NLO contributions to $\Delta\Gamma_s$ (z-exact)
  - 2-loop $O_{1-2} \times O_{1-2}$ correlators (z-exact) [Beneke, Buchalla, Greub, Lenz, & Nierste, 1999]
  - 2-loop $O_{1-2} \times O_{3-6}$ correlators [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] (n_f piece only, z-exact)
  - 2-loop $O_{3-6} \times O_{3-6}$ correlators [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] (n_f piece only, z-exact)
  - 1-loop $O_{1-2} \times O_8$ correlators (z-exact) [Beneke, Buchalla, Greub, Lenz, & Nierste, 1999]
  - 1-loop $O_{3-6} \times O_8$ correlators [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] (n_f piece only, z-exact)

$|\Delta B| = 1$ contributions needed for NNLO (always 2 insertions from $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$)

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1,2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

- NNLO contributions to $\Delta\Gamma_s$
  - 3-loop $O_{1-2} \times O_{1-2}$ correlators [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] (n_f piece only, $\mathcal{O}(\sqrt{z})$)
  - 2-loop $O_{1-2} \times O_8$ correlators [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] (n_f piece only, z-exact)
  - 1-loop $O_8 \times O_8$ correlators [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] (n_f piece only, z-exact)
- This work
  - Full (n_f + non-n_f) results for all 2-loop correlators at $\mathcal{O}(z)$ (including $O_8 \times O_8 \Rightarrow$ N^{3}LO)
  - Full (n_f + non-n_f) results for the 3-loop $O_{1-2} \times O_{1-2}$ at $\mathcal{O}(z)$
  - Renormalization matrix Z_{ij} for the $|\Delta B| = 2$ theory at $\mathcal{O}(\alpha_s^2)$



- Calculational strategy
  - Matching done **on-shell**: $p_b^2 = m_b^2$
  - The s-quark mass is neglected $\Rightarrow p_s = 0$
  - **Asymptotic expansion** in $z \equiv m_c^2/m_b^2$ (at first up to $\mathcal{O}(z)$)
  - Only the **imaginary part** of the $|\Delta B| = 1$ diagrams enters the matching
- Regularization
  - Dimensional regularization used **both for UV- and IR-divergences**
  - Cross-check: **massive gluons** in IR-divergent diagrams at 2 loops
  - $\varepsilon_{\text{UV}} + m_g$: renormalized amplitudes manifestly finite $\Rightarrow$ the limit $d \to 4$ is safe
  - $\varepsilon = \varepsilon_{\text{UV}} = \varepsilon_{\text{IR}}$: products of $1/\varepsilon_{\text{IR}}$ and evanescent ME are of $\mathcal{O}(\varepsilon^0)$

NLO matching with $\varepsilon = \varepsilon_{\text{IR}} = \varepsilon_{\text{UV}}$ (no gluon mass) [Ciuchini, Franco, Lubicz, & Mescia, 2002]

- Normally, only the matching coefficients of **physical** $|\Delta B| = 2$ **operators** are relevant
- Here **matching coefficients of evanescent operators** are also needed (at intermediate stages)
- $|\Delta B| = 2$ matching coefficients obtain $\mathcal{O}(\varepsilon)$ pieces

$$C = f_0^{(0)} + \varepsilon\, f_1^{(0)} + \frac{\alpha_s}{4\pi} f_0^{(1)}, \qquad C_E = f_{E,0}^{(0)} + \varepsilon\, f_{E,1}^{(0)} + \frac{\alpha_s}{4\pi} f_{E,0}^{(1)}$$

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- Finally, the difference

$$A_{\text{ren}}^{|\Delta B|=1} - A_{\text{ren}}^{|\Delta B|=2}$$

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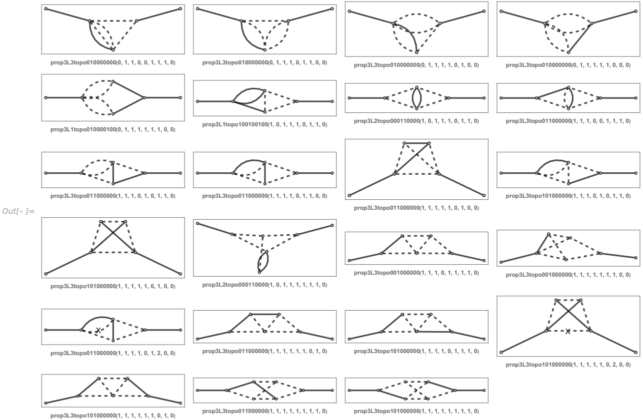
  is **manifestly finite** $\Rightarrow$ fix $f_0^{(1)}$
- Only $f_0^{(0)}$ and $f_0^{(1)}$ enter the **physical matching coefficient**
- What about $f_{E,1}^{(0)}$? Not needed at NLO, must be determined for the NNLO calculation!
- At NNLO, the LO matching must be done up $\mathcal{O}(\varepsilon^2)$, the NLO matching up to $\mathcal{O}(\varepsilon)$
- The **explicit cancellation of IR poles** (and of ξ) is a highly nontrivial cross-check of the whole calculation

- All computations done using our well-tested automatic setup
  - Diagram generation: **QGRAF** [Nogueira, 1993]
  - Feynman rules and topology identification: **Q2E/EXP** [Seidensticker, 1999; Harlander, Seidensticker, & Steinhauser, 1998] or **TAPIR** [Gerlach, Herren, & Lang, 2022]
  - Amplitude evaluation: in-house **FORM**-based [Ruijl, Ueda, & Vermaseren, 2017] **CALC** setup
  - IBPs: **FIRE** 6 [Smirnov & Chuharev, 2020] + **LiteRed** [Lee, 2014]
  - Analytic MI evaluation: **HyperInt** [Panzer, 2015], **HyperlogProcedures** [Schnetz], **PolyLogTools** [Duhr & Dulat, 2019]
  - Numerical MI evaluation: **FIESTA** [Smirnov, 2016] and **pySecDec** [Borowka, Heinrich, Jahn, Jones, Kerner, et al., 2018]
- Cross-checks of selected intermediate results: **FeynArts** [Hahn, 2001], **FeynRules** [Christensen & Duhr, 2009; Alloul, Christensen, Degrande, Duhr, & Fuks, 2014] and **FeynCalc** [Mertig, Böhm and Denner, 1991, VS, Mertig & Orellana, 2016, 2020]
- Two complementary approaches to tensor integrals in **FORM**
  - Explicit decomposition formulas (1 ext. momentum, max. rank 10), calculated using **FeynCalc** and **FERMAT** [Lewis]
  - Projections to the occurring color and 4-fermion Dirac structures

- New on-shell 3-loop integrals with massive (solid) lines
- Only imaginary parts are relevant and turn out to be very simple
- Appearing constants

$$\pi, \ln(2), \zeta_2, \zeta_3, \zeta_4, \mathrm{Cl}_2(\pi/3), \sqrt{3},$$
$$\mathrm{Li}_4(1/2), \ln\left((1+\sqrt{5})/2\right)$$

- Real parts (obtained as a byproduct) more complicated but irrelevant for $\Delta\Gamma_s$
- We can directly integrate the Feynman parameter integrals using **HyperInt** [Panzer, 2015]



- Handling of master integrals highly facilitated using new **FeynCalc** functions added in the course of this project
- Using ideas and algorithms from **FIRE**, **LiteRed**, **pySecDec**, ...
- Graph representation from propagator representation: FCLoopIntegralToGraph, FCLoopGraphPlot
- Derivation of the Feynman parametrization: FCFeynmanParametrize, FCFeynmanParameterJoin
- Mappings between master integrals: FCLoopFindIntegralMappings
- Official in the upcoming **FeynCalc** 10, however already publicly available and documented
- See my talk at ACAT 2021

- New contributions to Γ_{12}^s computed in the course of this project ($z = m_c^2/m_b^2$)

Contribution	Most recent literature result	**This work**
$Q_{1,2} \times Q_{3-6}$	2 loops, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{1,2} \times Q_8$	2 loops, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{3-6} \times Q_{3-6}$	2 loop, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{3-6} \times Q_8$	1 loop, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_8 \times Q_8$	1 loop, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{1,2} \times Q_{1,2}$	3 loops, $\mathcal{O}(\sqrt{z})$, n_f-part only [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017]	3 loops, $\mathcal{O}(z)$, full

- All building blocks required for the NNLO prediction are available.

- Ingredients for the theory prediction

$$\Gamma^s_{12} = -(\lambda^s_t)^2 \left[\Gamma^{s,cc}_{12} + 2\frac{\lambda^s_u}{\lambda^s_t}\left(\Gamma^{s,cc}_{12} - \Gamma^{s,uc}_{12}\right) + \left(\frac{\lambda^s_u}{\lambda^s_t}\right)^2 \left(\Gamma^{s,uu}_{12} + \Gamma^{s,cc}_{12} - 2\Gamma^{s,uc}_{12}\right)\right]$$

$$\Gamma^{ab}_{s,12} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}\left[H^{ab}(z)\underbrace{\langle B_s|Q|\bar{B}_s\rangle}_{\frac{8}{3}M^2_{B_s} f^2_{B_s} B_{B_s}} + \widetilde{H}^{ab}_S(z)\underbrace{\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle}_{\frac{1}{3}M^2_{B_s} f^2_{B_s} \widetilde{B}'_{S,B_s}}\right] + \mathcal{O}\left(\Lambda_{\text{QCD}}/m_b\right)$$

$$M_{12} = (\lambda^s_t)^2 \frac{G_F^2 M_{B_s}}{12\pi^2} M_W^2 \hat{\eta}_B S_0\left(\frac{m_t^2}{M_W^2}\right) f^2_{B_s} B_{B_s}$$

- Cancellation of $(\lambda^s_t)^2 = (V^*_{ts}V_{tb})^2$, f_{B_s}, M_{B_s} and bag parameters (to large extent) in the ratio Γ^s_{12}/M^s_{12}
- Following [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] we calculate

$$\frac{\Delta\Gamma_s}{\Delta M_s} = -\text{Re}\left(\frac{\Gamma^s_{12}}{M^s_{12}}\right), \quad \Delta\Gamma_s = \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)\Delta M_s^{\text{exp}}$$

- $|V_{cb}|$ controversy (exclusive vs. inclusive determinations) irrelevant!

- In addition to the leading power result for $\Delta\Gamma_s$ (our work) we also need the $1/m_b$-correction (known at LO only) [Beneke, Buchalla, & Dunietz, 1996]
- On the scheme choice: m_b and m_c in the matching coefficients in the $\overline{\text{MS}}$ scheme at $\mu_b = \mu_c = m_b$, i. e. $\bar{z} = (m_c^{\text{MS}}(m_b)/m_b^{\text{MS}}(m_b))^2$
- Freedom to choose the scheme for m_b^2 in the prefactor of Γ_{12}
- We use $\overline{\text{MS}}$, potential-subtracted (PS) [Beneke, 1998] and pole schemes in the leading power term
- m_b^2 in the subleading $1/m_b$ term (LO only) is converted to the PS scheme
- **RunDec** [Herren & Steinhauser, 2018] for the running and decoupling of quark masses and α_s.
- Numerical input [Zyla et al., 2020; Chetyrkin, Kuhn, Maier, Maierhofer, Marquard, et al., 2017; Dowdall, Davies, Horgan, Lepage, Monahan, et al., 2019; Bazavov et al., 2018; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020; Amhis et al., 2021]

$$\alpha_s(M_Z) = 0.1179 \pm 0.001, \quad m_c(3\ \text{GeV}) = 0.993 \pm 0.008\ \text{GeV},$$
$$m_b(m_b) = 4.163 \pm 0.016\ \text{GeV}, \quad m_t^{\text{pole}} = 172.9 \pm 0.4\ \text{GeV},$$
$$M_{B_s} = 5366.88\ \text{MeV} \quad f_{B_s} = (0.2307 \pm 0.0013)\ \text{GeV},$$
$$B_{B_s} = 0.813 \pm 0.034, \quad \widetilde{B}'_{S,B_s} = 1.31 \pm 0.09,$$
$$\lambda_u^s/\lambda_t^s = -(0.00865 \pm 0.00042) + (0.01832 \pm 0.00039)i$$
$$\Delta M_s^{\text{exp}} = (17.749 \pm 0.020)\ \text{ps}^{-1}$$

- Our **preliminary** results for $\Delta\Gamma_s/\Delta M_s$ and $\Delta\Gamma_s$ in the 3 schemes

$$\frac{\Delta\Gamma_s}{\Delta M_s} = (3.77^{+0.61}_{-0.55}{}_{\text{LP}\atop\text{scale}}\,{}^{+0.11}_{-0.21}{}_{\text{NLP}\atop\text{scale}} \pm 0.11_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad (\text{pole}),$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = (4.31^{+0.23}_{-0.40}{}_{\text{LP}\atop\text{scale}}\,{}^{+0.11}_{-0.21}{}_{\text{NLP}\atop\text{scale}} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad (\overline{\text{MS}}),$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = (4.19^{+0.42}_{-0.36}{}_{\text{LP}\atop\text{scale}}\,{}^{+0.11}_{-0.21}{}_{\text{NLP}\atop\text{scale}} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad (\text{PS}),$$

$$\Delta\Gamma_s = (0.067^{+0.011}_{-0.010}{}_{\text{LP}\atop\text{scale}}\,{}^{+0.004}_{-0.002}{}_{\text{NLP}\atop\text{scale}} \pm 0.002_{B\tilde{B}_S} \pm 0.014_{1/m_b} \pm 0.001_{\text{input}}) \times \text{ps}^{-1} \quad (\text{pole}),$$

$$\Delta\Gamma_s = (0.077^{+0.007}_{-0.004}{}_{\text{LP}\atop\text{scale}}\,{}^{+0.004}_{-0.002}{}_{\text{NLP}\atop\text{scale}} \pm 0.002_{B\tilde{B}_S} \pm 0.014_{1/m_b} \pm 0.001_{\text{input}}) \times \text{ps}^{-1} \quad (\overline{\text{MS}}),$$

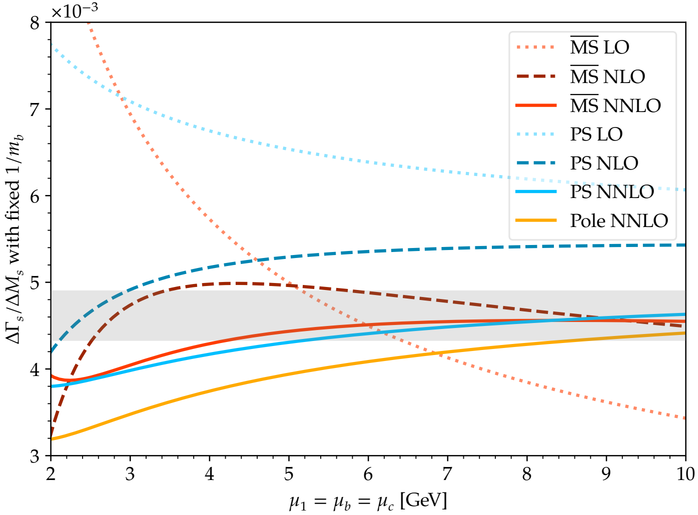
$$\Delta\Gamma_s = (0.074^{+0.007}_{-0.006}{}_{\text{LP}\atop\text{scale}}\,{}^{+0.004}_{-0.002}{}_{\text{NLP}\atop\text{scale}} \pm 0.002_{B\tilde{B}_S} \pm 0.014_{1/m_b} \pm 0.001_{\text{input}}) \times \text{ps}^{-1} \quad (\text{PS})$$

- LP: scale variation in the leading power piece; NLP: scale variation in the $1/m_b$ term
- Scales μ_1 ($|\Delta B| = 1$ theory), and μ_b, μ_c (quark masses) simultaneously varied between 2GeV and 10GeV.
- Experiment:

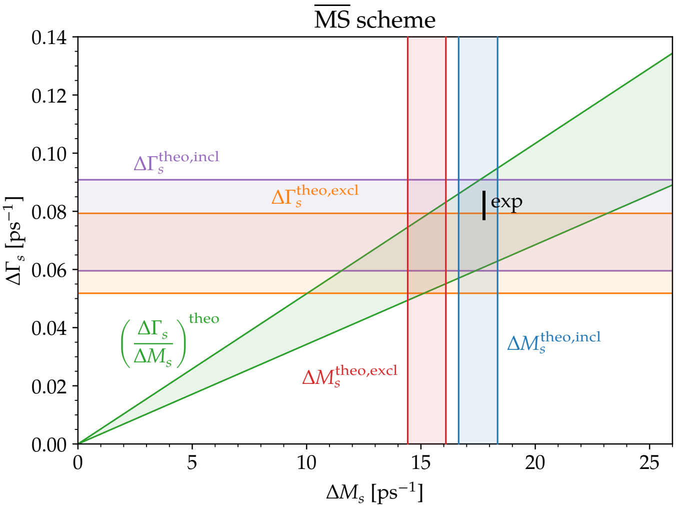
$$\Delta\Gamma_s^{\text{exp}} = (0.082 \pm 0.005) \times \text{ps}^{-1}$$

- The pole scheme seems to be inadequate for the B-meson mixing observables!

- Renormalization scale dependence at LO, NLO and NNLO (no μ-variations in the subleading $1/m_b$-terms)
- μ_b, μ_c: renormalization scales of the quark masses
- μ-dependence at NNLO better than at NLO for $\overline{\text{MS}}$ and PS
- $\overline{\text{MS}}$- and PS-predictions close together: reduction of scheme uncertainty
- For $\mu \approx 9$ GeV the NNLO correction vanishes



- $\overline{\text{MS}}$-prediction for $\Delta\Gamma_s/\Delta M_s$ against individual predictions
- Individual predictions dominated by the uncertainties in $|V_{cb}|$
- Uncertainties from $\langle B_S|Q|\bar{B}_S\rangle$ less important in the ratio
- Currently cannot distinguish between $|V_{cb}|^{\text{excl.}}$ and $|V_{cb}|^{\text{incl.}}$



Summary

- Experimental precision of $\Delta\Gamma_s$ necessitates the NNLO calculation!
- We calculated all building blocks needed to obtain the NNLO correction to $B_s^0 - \bar{B}_s^0$ mixing
- All the occurring 3-loop MI from the current-current contribution calculated analytically (for $m_c = 0$)
- The result for the 2-loop current-penguin contribution already published [Gerlach, Nierste, VS, Steinhauser, 2021, 2022]
- The scale uncertainty of leading power results reduces from 35% to 11% in the $\overline{\text{MS}}$ and from 21% auf 13% in the PS schemes

Outlook

- Analytic results for the remaining 3-loop current-current matching coefficients to be published soon
- New theory predictions for $\Delta\Gamma_s$ and the CP asymmetry a_{fs}^s
- Higher order expansions in $z \equiv m_c^2/m_b^2$, ideally z-exact results at least for the 2-loop contributions
- Could 3-loop penguin contributions help reducing the NNLO scale dependence even further?
- The theoretical precision of the $1/m_b$-term must be increased to the NLO accuracy

Anomalies in the flavor physics (incomplete list)

- Hints for Lepton Flavor Universality (LFU) violation in semileptonic decays
  - Loop-level (FCNC): $b \to s\ell\ell$ (e. g. $B_d \to K^* \mu^+ \mu^-$) [Aaij et al., 2017, 2019; Abdesselam et al., 2021; Choudhury et al., 2021; Aaij et al., 2022]
  - Tree-level: $b \to c\tau\nu$ (e. g. $\bar{B}^0 \to D^{*+} \tau^- \bar{\nu}_\tau$) [Lees et al., 2019; Hirose et al., 2017; Aaij et al., 2015, 2018]
- Dimuon charge asymmetry (related to CP-asymmetry in B-mixing) [Abazov et al., 2006, 2010a, 2010b, 2011, 2014]
- $g-2$ of the muon [Abi et al., 2021]
- CP-violation in the neutral kaon system (ε'/ε) [Alavi-Harati et al., 1999; Fanti et al., 1999]
- a_{fb} in $Zb\bar{b}$ decays [Abbaneo et al., 1996]
- LFU violation in leptonic τ decays [Aubert et al., 2010; Anastassov et al., 1997; Albrecht et al., 1992]
- $B \to \pi K$ puzzle [Buras, Fleischer, Recksiegel, & Schwab, 2003, 2004b, 2004a]
- Inclusive vs exclusive determinations of V_{cb} [Waheed et al., 2019; Lees et al., 2019]