THRESHOLD RESUMMATION OF NEW PARTONIC CHANNELS AT NEXT-TO-LEADING POWER

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Loops and Legs 2022, Ettal, 29/04/2022

H2020 MSCA COFUND G.A. 754496

OUTLINE

- **Particle scattering near threshold**
- **Factorization and resummation at NLP**
- **New partonic channels in DIS and DY**

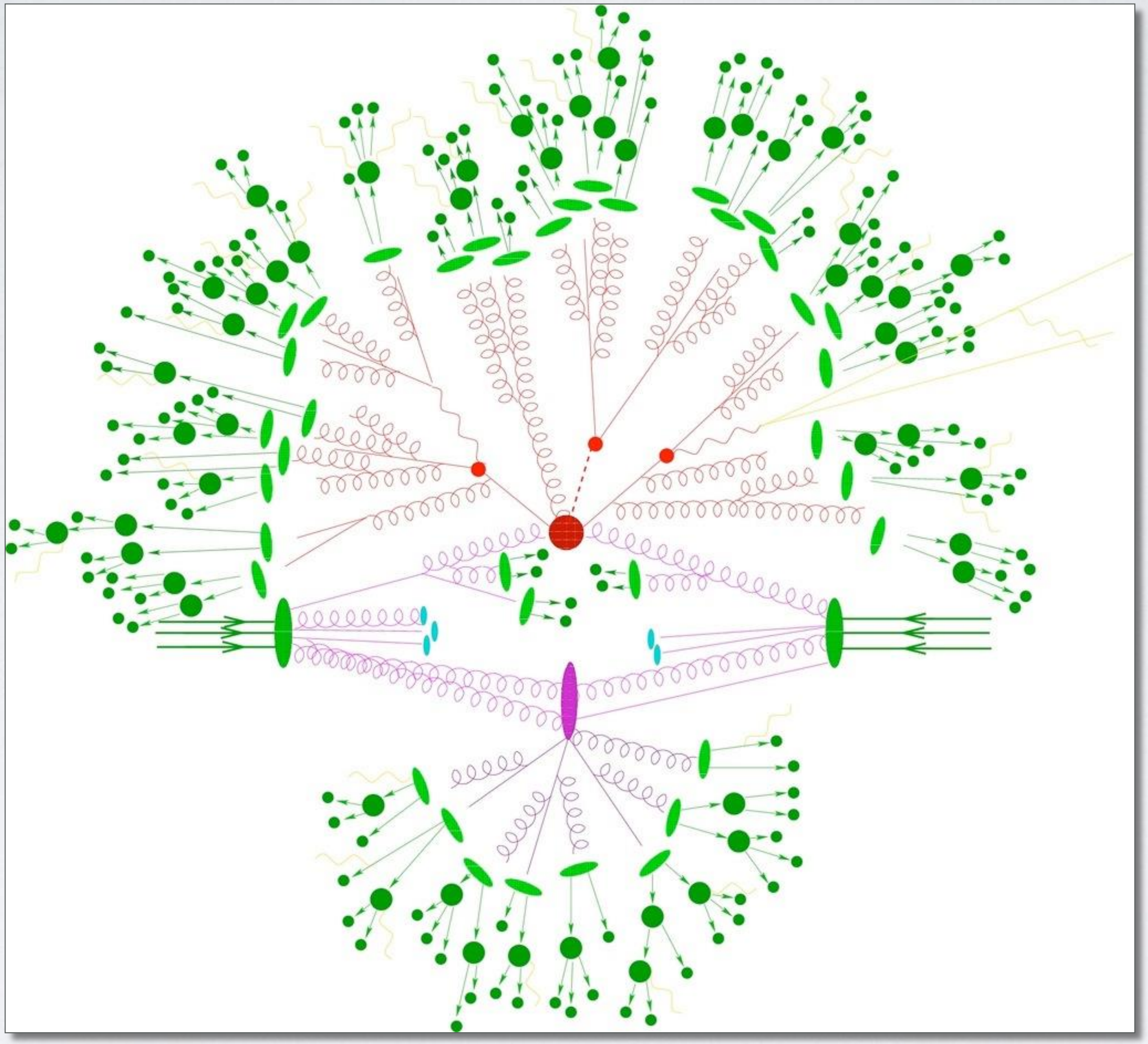
JHEP 10 (2020), 196, [arXiv:2008.04943],

with M. Beneke, M. Garny, S. Jaskiewicz, R. Szafron and J. Wang.

JHEP 12 (2021), 087, [arXiv: 2109.09752],

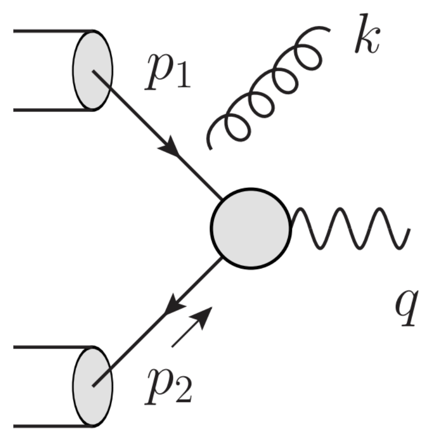
with M. van Beekveld and C. D. White.

PARTICLE SCATTERING NEAR THRESHOLD

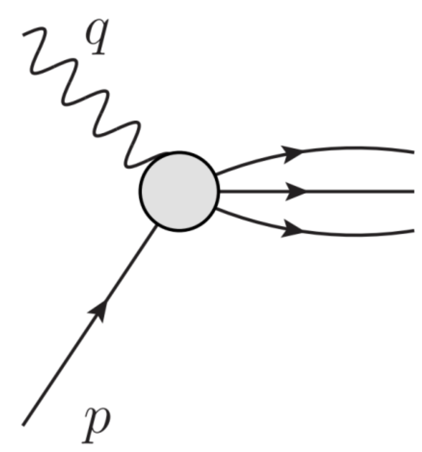


PARTICLE SCATTERING NEAR THRESHOLD

- Consider Drell-Yan and DIS near partonic threshold:



$(p_1 + p_2)^2 \equiv \hat{s}, \quad z \equiv \frac{q^2}{\hat{s}} \to 1$



$Q^2 = -q^2, \quad x \equiv \frac{Q^2}{2p\cdot q} \to 1$

- The partonic cross section has singular expansion

$$\Delta_{ab}(\xi) \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[c_n \delta(1-\xi) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-\xi)}{1-\xi}\right]_+ + d_{nm} \ln^m(1-\xi) \right) + \dots \right],$$

LP **NLP**

with $\xi = z$ for DY or x for DIS.

- Resummation of large logarithms at next-to-leading power (NLP):

$\rightarrow$ interesting theoretical challenge, relevant for precision phenomenology!

FACTORIZATION AND RESUMMATION AT NLP

- Lot of work in the past few years!
- Drell-Yan, Higgs and DIS near threshold

  Del Duca, 1990; Bonocore, Laenen, Magnea, LV, White, 2014, 2015, 2016;
  Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019;
  van Beekveld, Beenakker, Laenen, White, 2019; van Beekveld, Laenen, Sinninghe Damsté, LV, 2021;
  Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018;
  Beneke, Broggio, Jaskiewicz, LV, 2019;
  Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019, 2020.
- Operators and Anomalous dimensions

  Larkoski, Neill, Stewart 2014;
  Moult, Stewart, Vita 2017; Feige, Kolodrubetz, Moult, Stewart 2017;
  Beneke, Garny, Szafron, Wang, 2017, 2018, 2019.
- Thrust

  Moult, Stewart, Vita, Zhu 2018, 2019.
- pT and Rapidity logarithms

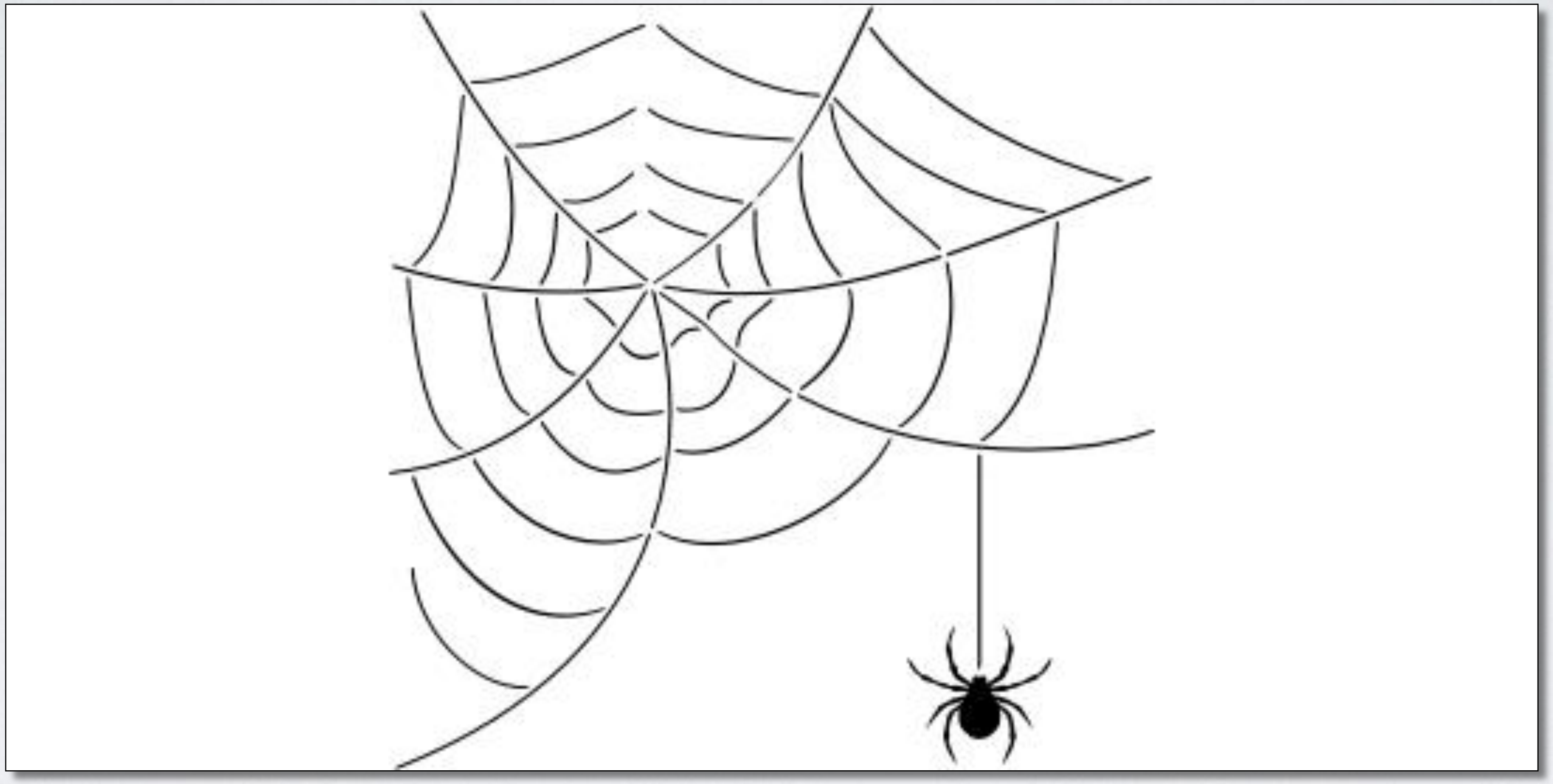
  Ebert, Moult, Stewart, Tackmann, Vita, 2018,
  Moult, Vita Yan 2019;
  Cieri, Oleari, Rocco, 2019; Oleari, Rocco 2020.
- Mass effects

  Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang, Fleming, 2020;
  Liu, Mecaj, Neubert, Wang, 2020;
  Anastasiou, Penin, 2020.
- K+G and RGE equations

  Ajjath, Mukherjee, Ravindran, Sankar, Tiwari, 2020, 2021.

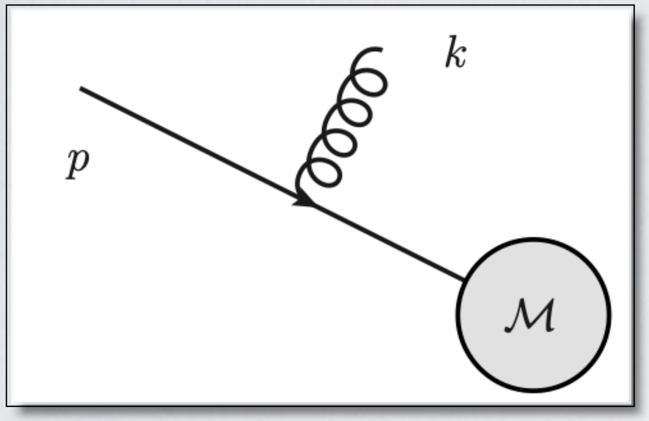
And many more!

SCATTERING NEAR THRESHOLD: LP VS NLP



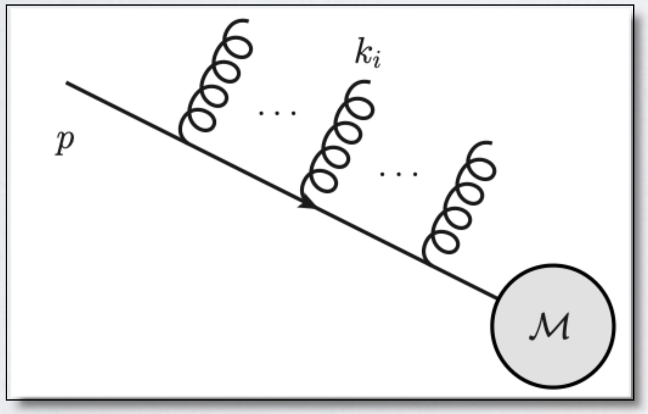
FACTORIZATION OF SOFT GLUONS AT LP

- Emission of soft gluons from an energetic parton (quark):



$$= \mathcal{M}\,\frac{\not p - \not k}{2p\cdot k}\,\gamma^\mu\, T^A\, u(p) \;\sim\; \mathcal{M}\,\frac{p^\mu}{p\cdot k}\, T^A\, u(p).$$

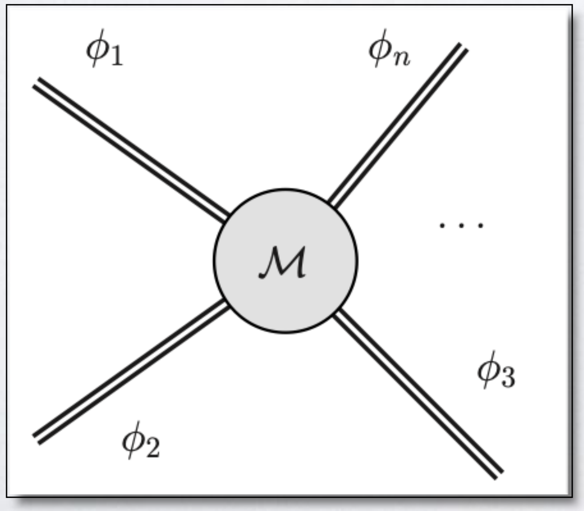
- Emission of multiple soft gluons factorises:



$$\sim \mathcal{M}\,\mathcal{S}\,u(p), \qquad \mathcal{S} = \langle 0|\Phi_\beta(-\infty,0)|0\rangle,$$

$$\Phi_\beta(\lambda_1,\lambda_2) = \mathcal{P}\exp\left\{ i\,g_s \int_{\lambda_1}^{\lambda_2} d\lambda\;\beta\cdot A(\lambda\beta)\right\}.$$

- In general



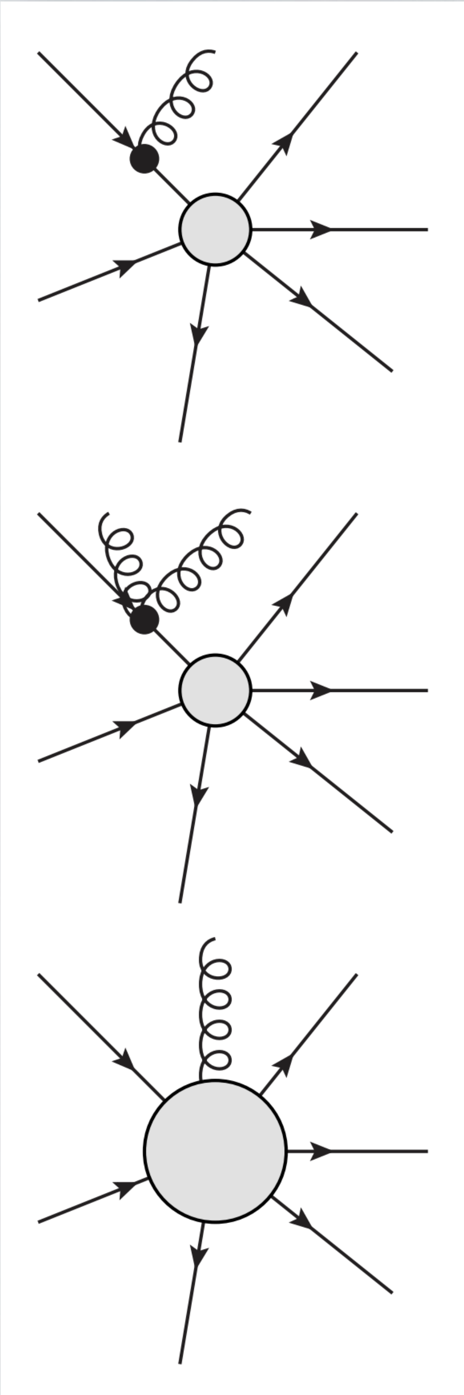
$$\sim \mathcal{M}\,\mathcal{S}\,u(p_1)\bar{v}(p_2)\ldots\bar{u}(p_n),$$

$$\mathcal{S} = \langle 0|\Phi_1\ldots\Phi_n|0\rangle \sim e^{\mathcal{W}_E}.$$

Gatheral, 1983; Frenkel, Taylor, 1984; Sterman, 1987; Catani,Trentadue, 1989; Korchemsky, Marchesini, 1992,1993; …

FACTORIZATION OF SOFT GLUONS BEYOND LP

- One needs to consider several effects:

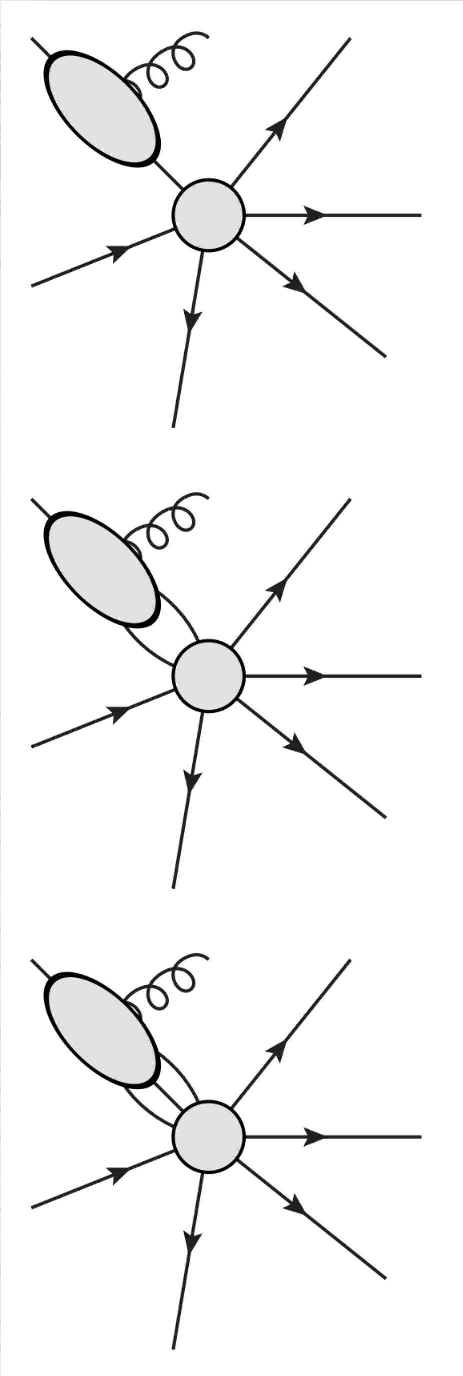


- Emission of soft gluons beyond the eikonal approximation, for instance sensitive to the spin of the emitting particle

Laenen, Magnea, Stavenga, White, 2009, 2010; Bonocore, Laenen, Magnea, LV, White, 2016.

- The soft emission resolve the hard interaction (LBK theorem)

Low 1958, Burnett,Kroll 1968



- Emission of soft gluons from a cluster of collinear particles: one finds several types of "radiative jets".

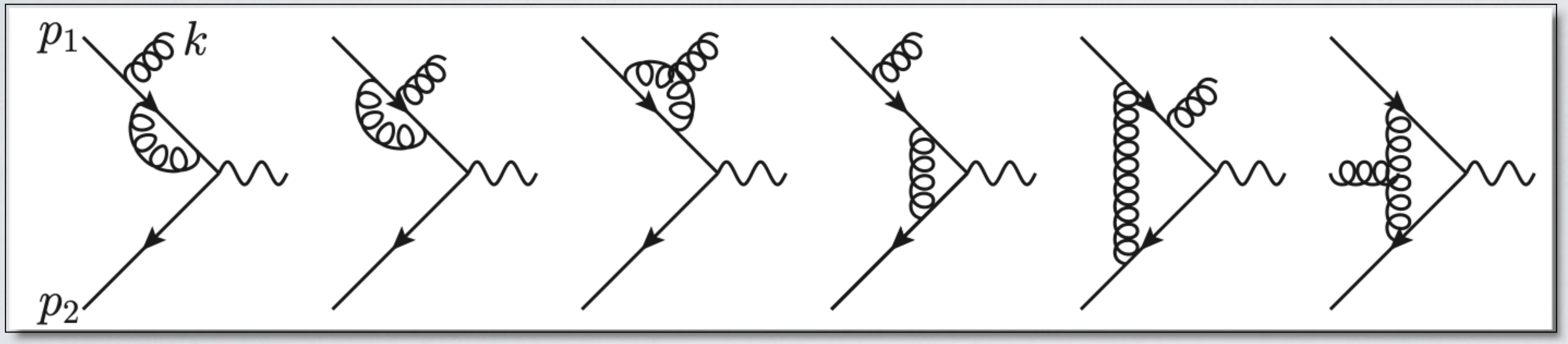
Del Duca 1990;

Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016;

Gervais 2017;

Laenen, Sinninghe-Damsté, LV, Waalewijn, Zoppi, 2020

FACTORIZATION OF SOFT GLUONS BEYOND LP



$$\hat{s} = (p_1 + p_2)^2, \quad t = (p_1 - k)^2, \quad u = (p_2 - k)^2.$$

- Goal: factorize non-analytical contributions ∝ to the scales of the problem:

$$|\mathcal{M}|^2 \propto C_F^2 \Bigg\{ \overset{\text{NLP}}{\frac{\hat{s}(t+u)}{tu}} \overset{\text{hard}}{\left(\frac{\mu^2}{-\hat{s}}\right)^{\epsilon}} \left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon} + \dots \right) + \left[\overset{\text{NLP}}{\frac{s}{t}} \overset{\text{coll.}}{\left(\frac{\mu^2}{-t}\right)^{\epsilon}} + \overset{\text{NLP}}{\frac{\hat{s}}{u}} \overset{\text{anti-coll.}}{\left(\frac{\mu^2}{-u}\right)^{\epsilon}} \right] \left(-\frac{2}{\epsilon} + \dots \right) \Bigg\}$$

$$+ C_A C_F \overset{\text{NLP}}{\frac{\hat{s}(t+u)}{tu}} \overset{\text{soft}}{\left(\frac{\hat{s}\,\mu^2}{t\,u}\right)^{\epsilon}} \left(-\frac{1}{\epsilon^2} + \dots \right) + \left[\overset{\text{NLP}}{\frac{\hat{s}}{t}} \overset{\text{coll.}}{\left(\frac{\mu^2}{-t}\right)^{\epsilon}} + \overset{\text{NLP}}{\frac{\hat{s}}{u}} \overset{\text{anti-coll.}}{\left(\frac{\mu^2}{-u}\right)^{\epsilon}} \right] \left(-\frac{5}{2} + \dots \right) \Bigg\} + \dots$$

↓

Factorization?

↓

$$S\left[\frac{\hat{s}\,\mu^2}{t\,u}, \epsilon\right] \times J\left[\frac{\mu^2}{-t}, \epsilon\right] \times \bar{J}\left[\frac{\mu^2}{-u}, \epsilon\right] \times H\left[\frac{\mu^2}{-\hat{s}}, \epsilon\right]$$

Bonocore, Laenen, Magnea, LV, White, 2014

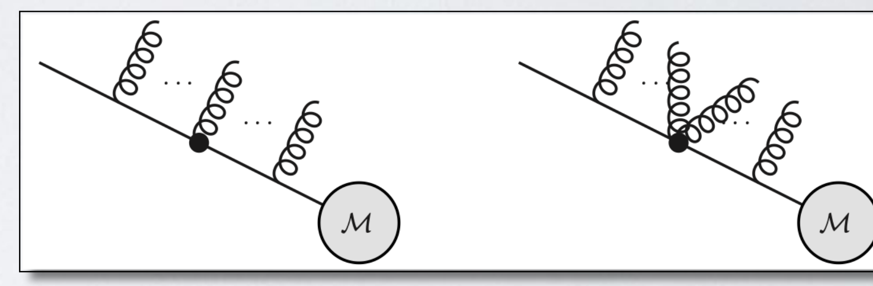
- Need an effective approach to take into account hard, collinear and soft modes.
- Two approaches: ~ Diagrammatic; ~ Soft Collinear Effective Field Theory.

DIAGRAMMATIC APPROACH

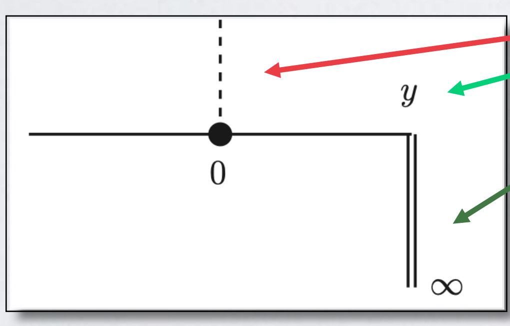
- Describe momentum regions in terms of universal functions in QCD:
- For instance, for Drell-Yan we have
  - "Derivative" of the non-radiative amplitude,
  - "Generalized" soft function,
  - "Radiative" jet function,

Low 1958, Burnett,Kroll 1968

$$\frac{k^\nu}{p_l\cdot k}\left[p_{l,\nu}\frac{\partial}{\partial p_l^\mu}-p_{l,\mu}\frac{\partial}{\partial p_l^\nu}\right]\mathcal{A}(p_j)$$



$$J_{\mu,a}(p,n,k)\,u(p)\;=\;\int d^d y\;\mathrm{e}^{-\mathrm{i}(p-k)\cdot y}\;\langle 0\,|\,\Phi_n(\infty,y)\,\psi(y)\,j_{\mu,a}(0)\,|\,p\rangle\;.$$



Del Duca 1990,

Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016

$$F_p(-\infty,0)=\mathcal{P}\exp\left[g\int\frac{d^dk}{(2\pi)^d}A_\mu(k)\left(-\frac{p^\mu}{p\cdot k}+\frac{k^\mu}{2p\cdot k}-k^2\frac{p^\mu}{2(p\cdot k)^2}-\frac{\mathrm{i}k_\nu\Sigma^{\nu\mu}}{p\cdot k}\right)+\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dl}{(2\pi)^d}A_\mu(k)A_\nu(l)\left(\frac{\eta^{\mu\nu}}{2p\cdot(k+l)}+\dots\right)\right].$$

Laenen, Magnea, Stavenga, White, 2009, 2010

Bonocore, Laenen, Magnea, LV, White,2016

- One has

$$\mathcal{A}_{\mu,a}(p_j,k)\;=\;\sum_{i=1}^{2}\left(\frac{1}{2}\,\widetilde{\mathcal{S}}_{\mu,a}(p_j,k)+g\,\mathbf{T}_{i,a}\,G^{\nu}_{i,\mu}\,\frac{\partial}{\partial p_i^\nu}+J_{\mu,a}\,(p_i,n_i,k)\right)\mathcal{A}(p_j)-\mathcal{A}^{\widetilde{\mathcal{J}}}_{\mu,a}(p_j,k)\,,$$

for $n_1 = p_2$, $n_2 = p_1$.

(Removes soft-collinear overlap in the radiative jet)

SOFT-COLLINEAR EFFECTIVE FIELD THEORY

- Effective Lagrangian and operators made of collinear and soft fields.

$$\mathcal{L}_{\mathrm{SCET}} = \sum_i \mathcal{L}_{c_i} + \mathcal{L}_s,$$

Bauer, Fleming, Pirjol, Stewart, 2000,2001; Beneke, Chapovsky, Diehl, Feldmann, 2002; Hill, Neubert 2002, Beneke, Feldmann, 2002;

$$\mathcal{O}_n = \int dt_1 \ldots dt_n \, \mathcal{C}(t_1, \ldots, t_n) \, \phi_1(t_1 n_{1+}) \ldots \phi_n(t_n n_{n+}).$$

- Constructed to reproduce a scattering process as obtained with the method of regions.
- The cross section factorizes into a hard scattering kernel, and matrix elements of soft and collinear fields.

$$\sigma \sim \mathcal{H} \otimes \mathcal{J}_1 \otimes \ldots \otimes \mathcal{J}_n \otimes \mathcal{S}.$$

Hard matching coefficient

Jet functions – matrix elements of collinear fields

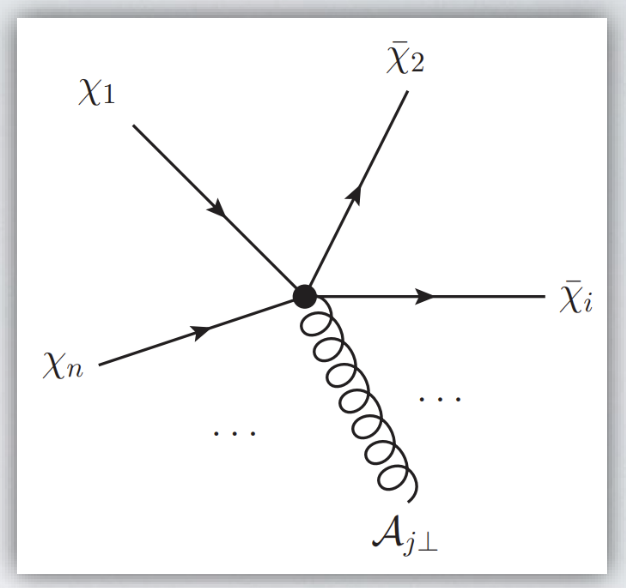
Soft function – matrix element of soft fields

- Renormalize UV divergences of EFT operators and obtain renormalization group equations.
- Each function depends on a single scale: solving the RGE resums large logarithms.

See e.g. Becher, Neubert 2006

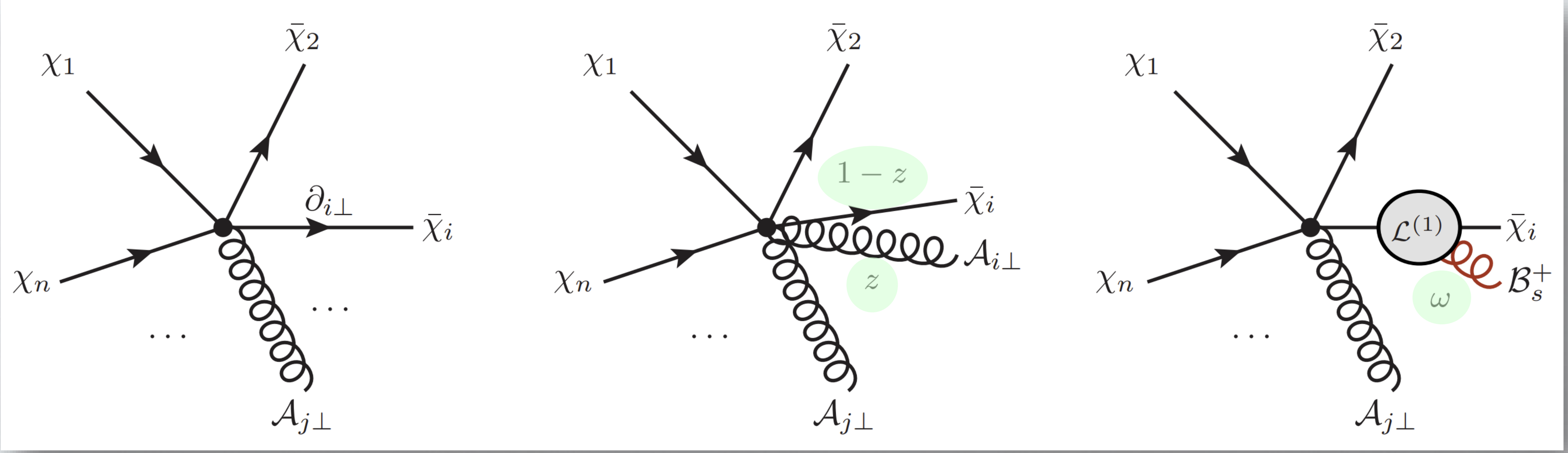
FACTORIZATION IN SCET: LP VS NLP

- Leading power (LP):
  - N-jet operators;
  - Soft-collinear decoupling.



- Next-to-leading power (NLP):
  - Kinematic suppression;
  - Multi-particle emission along the same collinear direction;
  - No soft-collinear decoupling.

Beneke, Garny, Szafron, Wang, 2017,2018



DRELL-YAN AT NLP IN SCET

- Take again Drell-Yan as an example:

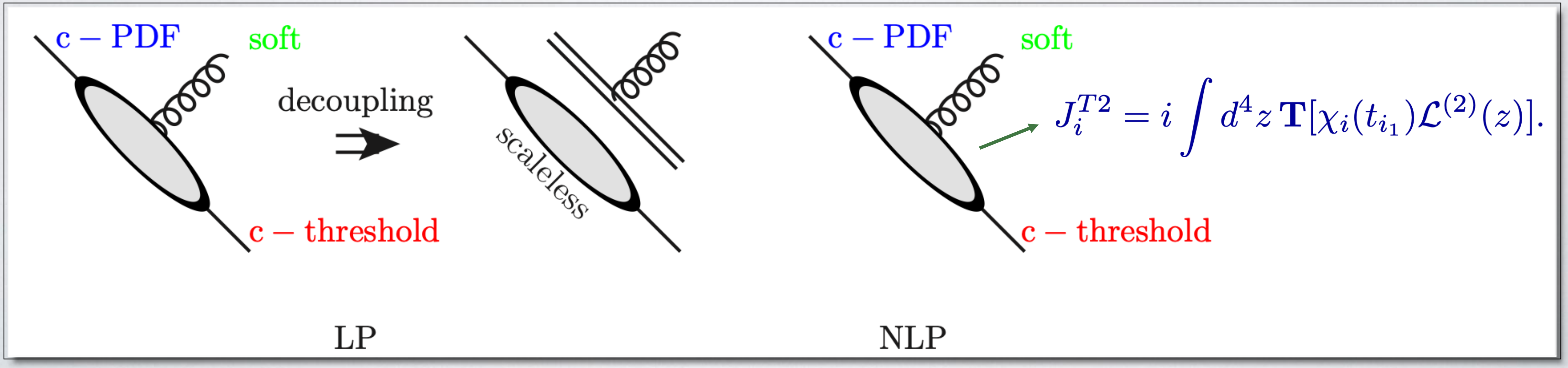
$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm EM}^2}{3N_c Q^4}\sum_{a,b}\int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\,\hat\sigma_{q\bar q}^{\rm NLP}(z),$$

- The partonic cross section factorises into

$$\hat\sigma_{q\bar q}^{\rm NLP}(z) = \sum_{\rm terms} |C|^2\, J\otimes J\otimes S,$$

  - C is the hard Wilson matching coefficient,
  - S is a *generalized* soft function,
  - J is a new collinear function.

- The collinear function is trivial at LP, because all threshold collinear modes are scaleless.
- The collinear scale is induced by the injection of a soft momentum.



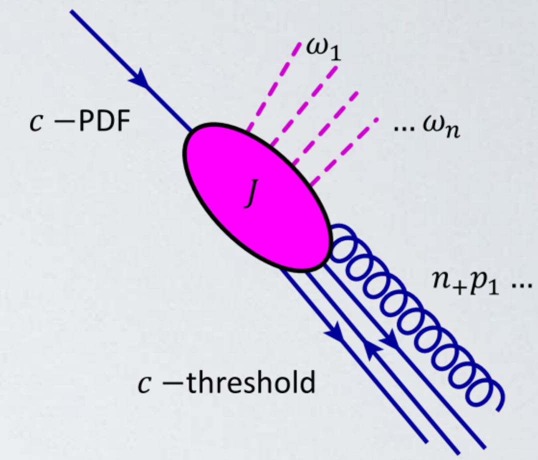
DRELL-YAN AT NLP IN SCET

- This is easily generalized at any subleading power: there can be many Lagrangian insertions, each with its own ω_i conjugate to the large component of the collinear momentum.

$$i^m \int \{d^4 z_j\}\ \mathbf{T}\Big[\{\psi_c(t_k n_+)\} \times \Big\{\mathcal{L}^{(l)}(z_j)\Big\}\Big]$$

Collinear matrix element

$$= 2\pi \sum_i \int du \int \{dz_{j-}\}\ \tilde{J}_i(\{t_k\}, u; \{z_{j-}\})\ \ \chi_c^{\mathrm{PDF}}(un_+)\ \mathfrak{s}_i(\{z_{j-}\}),$$

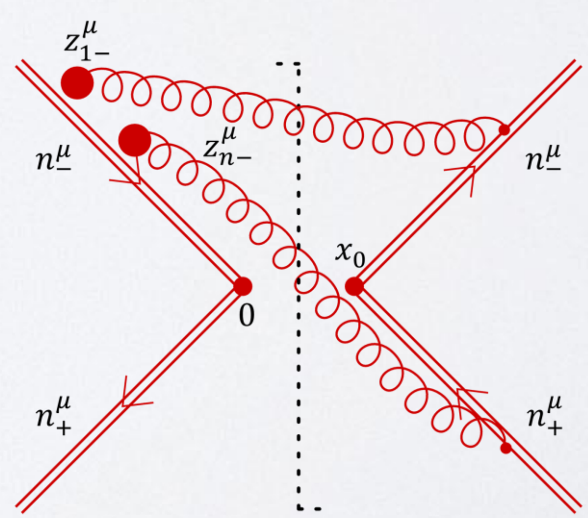


- After taking the matrix element squared, this gives a generalized soft functions:

$$S(\Omega,\omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2} \left(\prod_{j=1}^{n} \int \frac{d(z_{-j})}{4\pi} e^{-i\omega_j z_{-j}}\right)$$
$$\times\ \mathrm{Tr}\langle 0|\bar{\mathbf{T}}\big[(Y_+^\dagger Y_-)(x^0)\big]\mathbf{T}\big[(Y_-^\dagger Y_+)(x^0) \times \mathcal{L}_s^n(z_{1-}) \times \ldots \times \mathcal{L}_s^n(z_{n-})\big]|0\rangle.$$

which are equivalent to the generalized Wilson lines built in terms of NLP webs in the diagrammatic approach.

Beneke, Broggio, Jaskiewicz, LV, 2019



DRELL-YAN AT NLP IN SCET

- Up to NLP one has:

$$\Delta^{dyn}_{\rm NLP}(z) = -\frac{2}{(1-\epsilon)}\, Q \left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma_{\perp}^{\rho}\right]_{\beta\gamma}$$
$$\times \int d(n_+p)\, C^{\,A0,A0}\,(n_+p, x_b n_-p_B)\, C^{*A0A0}\,(x_a\, n_+p_A,\, x_b n_-p_B)$$
$$\times \sum_{i=1}^{5} \int \{d\omega_j\}\; J_i\,(n_+p, x_a\, n_+p_A; \{\omega_j\})\; S_i(\Omega; \{\omega_j\}) + \text{h.c.}\,.$$

Beneke, Broggio, Jaskiewicz, LV, 2019, 2020

- The convolution is regularized by dimensional regularization. For resummation, we treat the two object independently, and expand in ϵ prior to performing the convolution:

$$\int d\omega\, \underbrace{\left(n_+p\,\omega\right)^{-\epsilon}}_{\text{collinear piece}}\, \underbrace{\frac{1}{\omega^{1+\epsilon}}\frac{1}{(\Omega-\omega)^{\epsilon}}}_{\text{soft piece}}\,.$$

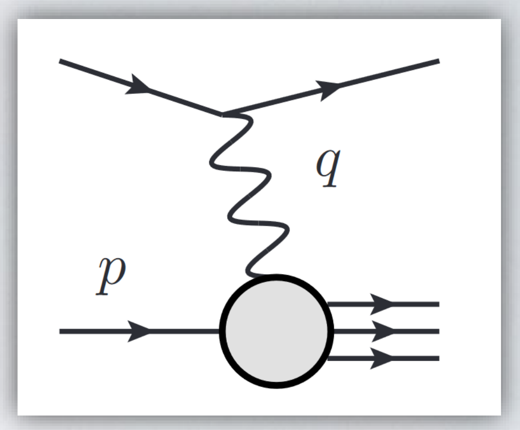
***Studies in:** Moult, Stewart, Vita, Zhu, 2019;*
Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020;
Liu, Mecaj, Neubert, Wang, Fleming, 2019, 2020;

but the convolution is endpoint divergent in d=4!

- This is actually an issue affecting in general any non-local effective field theory, such as SCET: resummation near threshold at NLP provides a perturbative, well-defined framework where to study and possibly solve the issue!

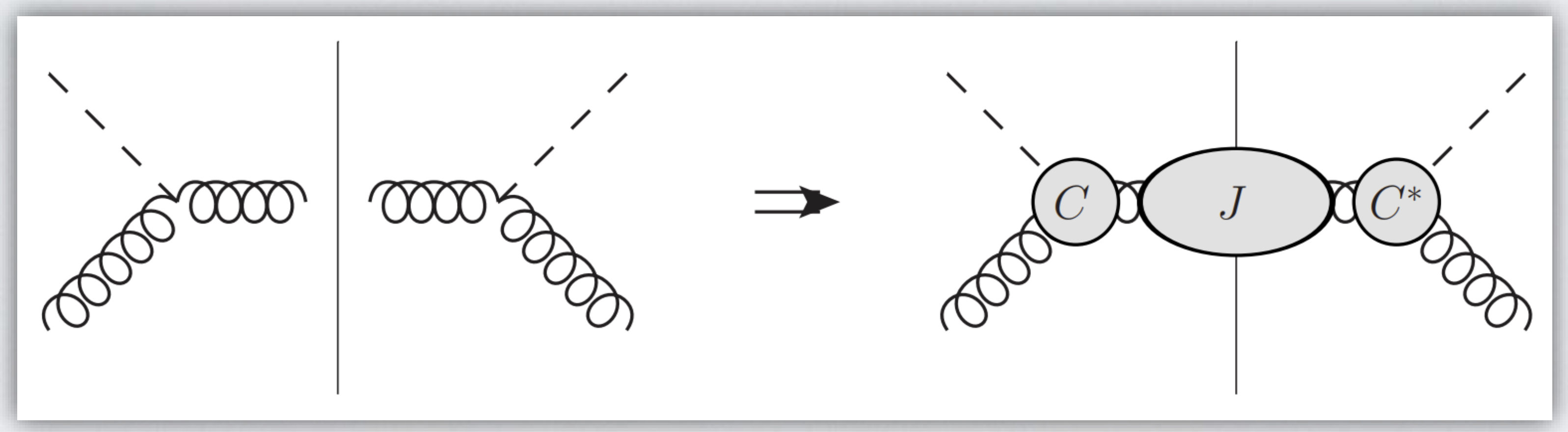
DEEP INELASTIC SCATTERING

- The problem of endpoint divergences is typical at NLP. Consider for instance Deep inelastic scattering (DIS) near threshold:



$$Q^2 \gg P_X^2 \sim Q^2(1-x), \qquad \text{with} \qquad x \equiv \frac{Q^2}{2p\cdot q} \to 1.$$

- Factorization and resummation well understood at LP:



Sterman 1987; Catani, Trentadue 1989; Korchemsky, Marchesini, 1993; Moch, Vermaseren, Vogt 2005; Becher, Neubert, Pecjak, 2007

$$W_\phi = \frac{1}{8\pi Q^2}\int d^4x\, e^{iq\cdot x}\, \langle N(P)\big|\big[G^A_{\mu\nu}G^{\mu\nu A}\big](x)\big[G^B_{\rho\sigma}G^{\rho\sigma B}\big](0)\big|N(P)\rangle$$

$$= |C(Q^2,\mu)|^2 \int_x^1 \frac{d\xi}{\xi}\, J\left(Q^2\frac{1-\xi}{\xi},\mu\right)\frac{x}{\xi} f_g\left(\frac{x}{\xi},\mu\right).$$

Short-distance coefficient and jet function are single scale object – resummation obtained by solving the corresponding RGE.

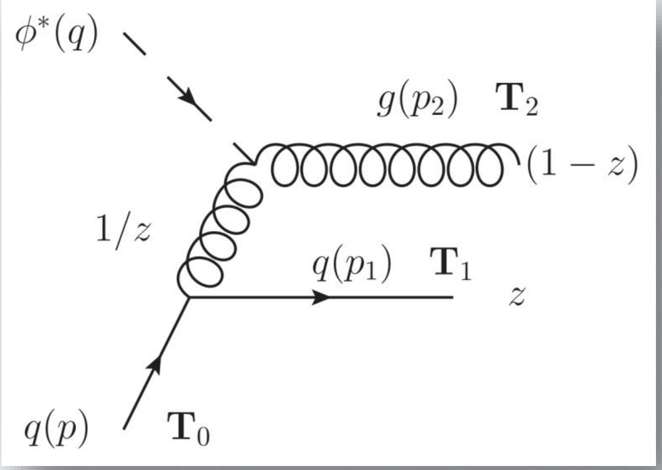
DIS: OFF-DIAGONAL CHANNEL

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

- The off-diagonal channel $q(p) + \phi^*(q) \to X(p_X)$ contributes to DIS at NLP. Consider the partonic structure function

$$W_{\phi,q}\big|_{q\phi^*\to qg} = \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^{\epsilon} \mathcal{P}_{qg}(s_{qg},z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}, \quad \mathcal{P}_{qg}(s_{qg},z) \equiv \frac{e^{\gamma_E\epsilon}Q^2}{16\pi^2\Gamma(1-\epsilon)} \frac{|\mathcal{M}_{q\phi^*\to qg}|^2}{|\mathcal{M}_0|^2}.$$

with momentum fraction $z \equiv \dfrac{n_-p_1}{n_-p_1 + n_-p_2}$, and $\bar{z} = 1 - z$.



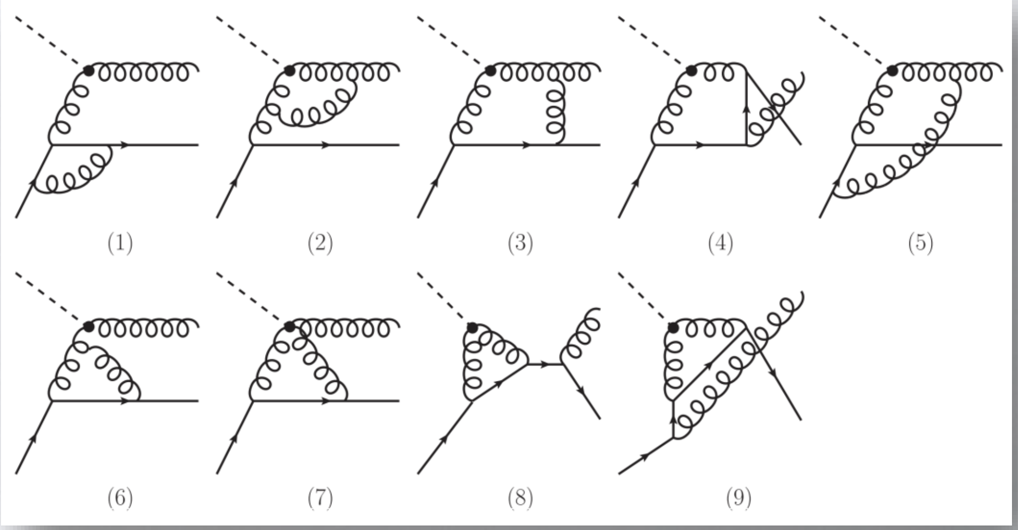
- At LO one has

$$\mathcal{P}_{qg}(s_{qg})|_{\rm tree} = \frac{\alpha_s C_F}{2\pi}\frac{\bar{z}^2}{z}, \quad \Rightarrow \quad W_{\phi,q}\Big|^{\rm NLP}_{\mathcal{O}(\alpha_s),\,\text{leading pole}} = -\frac{1}{\epsilon}\frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2}{Q^2(1-x)}\right)^{\epsilon}.$$

The single pole originate from $z \to 0$, due to the $1/z$ of the momentum distribution function.

- At NLO:

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\rm loop} = \mathcal{P}_{qg}(s_{qg},z)|_{\rm tree}\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2} \cdot \left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon} + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right) + \mathcal{O}(\epsilon^{-1}).$$



ON THE ENDPOINT DIVERGENCES

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon} + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right) + \mathcal{O}(\epsilon^{-1}).$$

- The **T1.T2** term contains a single pole, but: promoted to leading pole after integration!
- Compare exact integration:

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(1-z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3},$$

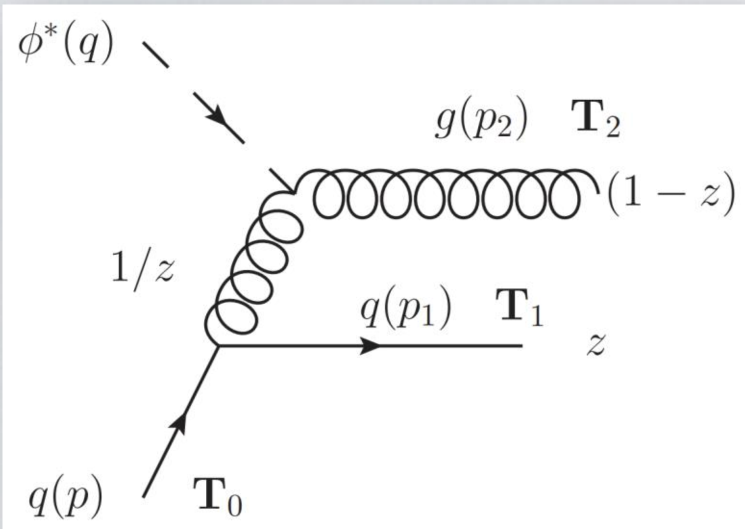
vs integration after expansion:

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right) = -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots\,.$$

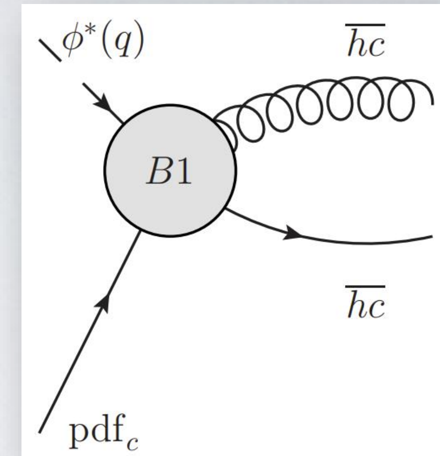
- Expansion in ε not possible before integration!
- The pole associated to **T1.T2** does not originate from the standard cups anomalous dimension.

BREACKDOWN OF FACTORIZATION NEAR THE ENDPOINT

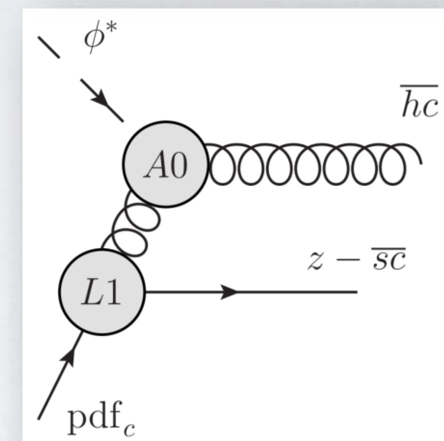
- What happens for $z \to 0$?



For z ~ 1 intermediate propagator is hard



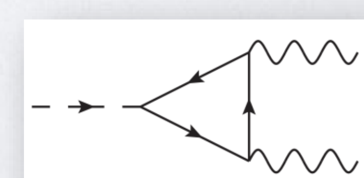
For z « 1 intermediate propagator cannot be integrated out



- Dynamic scale: zQ^2.
- In the endpoint region new counting parameter, $\lambda^2 \ll z \ll 1$.
- New modes contribute: need "z-SCET".
- z-modes are non-physical! Not related to external scales of the problem.
- Need re-factorization:

$$\underbrace{C^{B1}(Q,z)}_{\text{multi-scale function}}\, J^{B1}(z) \overset{z\to 0}{\longrightarrow} C^{A0}(Q^2)\int d^4x\, \mathbf{T}\Big[J^{A0}, \mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = \underbrace{C^{A0}(Q^2)D^{B1}(zQ^2,\mu^2)}_{\text{single-scale functions}}\, J^{B1}_{z-\overline{sc}}.$$

- Similar re-factorization proven in Liu, Mecaj, Neubert, Wang 2020.



DIS FACTORIZATION

- Re-factorization is nontrivial: needs to be embedded in a complete EFT description of DIS:
- Physical modes:

"z-SCET" is here

Perturbative modes

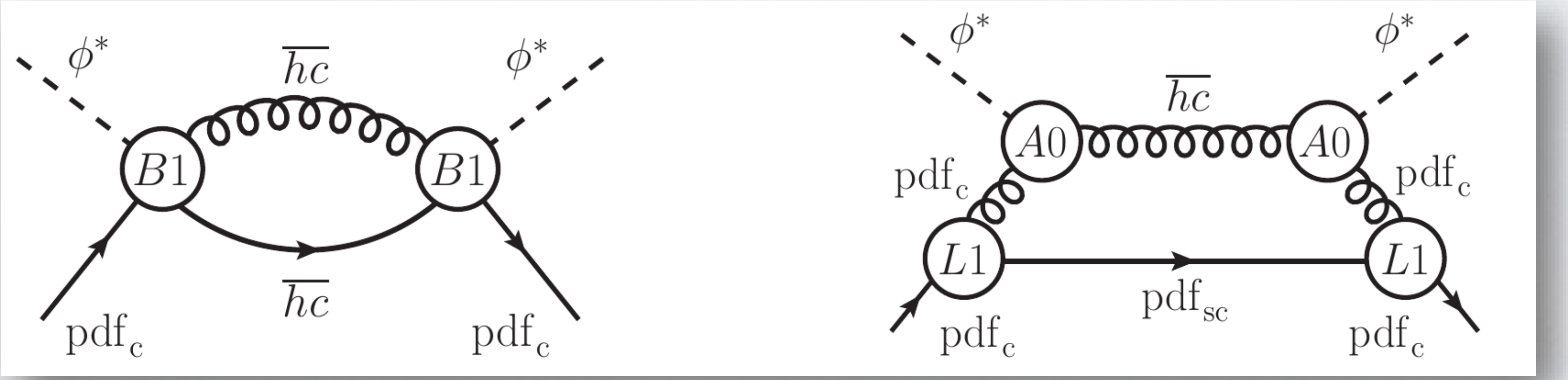
$$\begin{aligned} &\text{Hard:} && p^2 = Q^2, \\ &\text{Hard-collinear:} && p^2 = Q^2\lambda^2 = Q^2/N, \end{aligned}$$

Non-perturbative modes

$$\begin{aligned} &\text{Collinear:} && p^2 = \Lambda^2, \\ &\text{Soft-collinear:} && p^2 = \Lambda^2\lambda^2 = \Lambda^2/N. \end{aligned}$$

$$g(N) \equiv \int_0^1 dx\, x^{N-1}\, g(x),$$

$$x \to 1 \;\Leftrightarrow\; N \to \infty.$$



Time-ordered product contribution

B-type current contribution

See talk by M. Beneke for the explicit construction of refactorization in SCET I

- Both terms contain endpoint divergences in the convolution integral.
- But we can already gain significant information by implementing d-dimensional consistency conditions.

→ ***Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020***

D-DIMENSIONAL CONSISTENCY CONDITIONS

- Hadronic structure function is finite:

$$W = \sum_i W_{\phi,i}\, f_i = \sum_i \tilde{C}_{\phi,k} \tilde{f}_k\,, \qquad \text{with} \qquad \tilde{f}_k = Z_{ki} f_i, \quad W_{\phi,i} = \tilde{C}_{\phi,k} Z_{ki}.$$

- Focus on the bare functions: at NLP one has:

$$\sum_i (W_{\phi,i} f_i)^{NLP} = W^{NLP}_{\phi,q} f^{LP}_q + W^{NLP}_{\phi,\bar{q}} f^{LP}_{\bar{q}} + W^{NLP}_{\phi,g} f^{LP}_g + W^{LP}_{\phi,g} f^{NLP}_g\,.$$

- In d-dimensions: the general expansion of the cross section reads

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c^{(n)}_{kj}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}}\right)^\epsilon + f_{\bar{q}}(\Lambda),\; f_g(\Lambda) \text{ terms}\,.$$

- In this equation:

Each hard loop gives $\left(\frac{\mu^2}{Q^2}\right)^\epsilon$, each hard-collinear loop gives $\left(\frac{\mu^2}{Q^2} N\right)^\epsilon$,

Each collinear loop gives $\left(\frac{\mu^2}{\Lambda^2}\right)^\epsilon$, each soft-collinear loop gives $\left(\frac{\mu^2}{\Lambda^2} N\right)^\epsilon$.

- Invoking cancellation of poles gives a series of constraints on the coefficients $c_{kj}^{(n)}$.

D-DIMENSIONAL CONSISTENCY CONDITIONS

- One finds that there are only *n* independent coefficients, one per loop in a given region!
- Consider $c_{n1}^{(n)}$: this is the n-loop hard region. Assume exponentiation of 1-loop result:

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon}\right.$$
$$\left. + \mathbf{T}_1\cdot\mathbf{T}_2\left[\boxed{\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right) + \mathcal{O}(\epsilon^{-1}).$$

Similar conjecture "soft quark Sudakov" in Moult, Stewart, Vita, Zhu, 2019.

- Restricting to the hard region and substituting color operators one has

$$\mathcal{P}_{qg,\text{hard}}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (C_A - C_F)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right].$$

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

- With $f_i(\mu) = U_{ij}(\mu) f_j(\Lambda)$ one has

$$\sum_i (W_{\phi,i} f_i)^{NLP}\Big|_{\propto f_q(\Lambda)} = \left(W^{NLP}_{\phi,q}U^{LP}_{qq} + W^{LP}_{\phi,g}U^{NLP}_{gq}\right) f_q(\Lambda)\,.$$

(Reproduces earlier conjecture by Vogt, 2010)

- Inserting the result above in the end one has

$$W^{NLP,LP}_{\phi,q} = -\frac{1}{2N}\frac{C_F}{C_F - C_A}\frac{\epsilon N^{\epsilon}}{N^{\epsilon}-1}\left(\exp\left[\frac{\alpha_s C_F}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}(N^{\epsilon}-1)\right] - \exp\left[\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}(N^{\epsilon}-1)\right]\right),$$

$$U^{NLP,LP}_{gq} = -\frac{1}{2N}\frac{C_F}{C_F - C_A}\frac{\epsilon N^{\epsilon}}{N^{\epsilon}-1}\left(\exp\left[-\frac{\alpha_s C_F}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}(N^{\epsilon}-1)\right] - \exp\left[-\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}(N^{\epsilon}-1)\right]\right).$$

OFF-DIAGONAL DIS: FINITE STRUCTURE FUNCTION

- Furthermore, recall

$$W = \sum_i W_{\phi,i} f_i = \sum_k \tilde{C}_{\phi,k} \tilde{f}_k, \quad \Rightarrow \quad W^{NLP}_{\phi,q} = \tilde{C}^{NLP}_{\phi,q} Z^{LP}_{qq} + \tilde{C}^{LP}_{\phi,g} Z^{NLP}_{gq} .$$

- We obtain a solution for $\tilde{C}$:

$$\tilde{C}^{NLP,LL}_{\phi,q}\Big|_{\epsilon\to 0} = \frac{1}{2N\ln N}\frac{C_F}{C_F - C_A}\Bigg(\mathcal{B}_0(a)\exp\left[C_A\frac{\alpha_s}{\pi}\left(\frac{1}{2}\ln^2 N + \ln N \ln\frac{\mu^2}{Q^2}\right)\right] - \exp\left[\frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}\ln^2 N + \ln N\ln\frac{\mu^2}{Q^2}\right)\right]\Bigg), \qquad \text{with} \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N,$$

and

$$P_{ij} = -\gamma_{ij} = \frac{dZ_{ik}}{d\ln\mu}(Z^{-1})_{kj}\,, \qquad \gamma^{NLP,LL}_{gq}(N) = -\frac{1}{N}\,\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a)\,, \qquad \mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n,$$

with Bernoulli numbers $B_0 = 1$, $B_1 = -1/2$, …

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

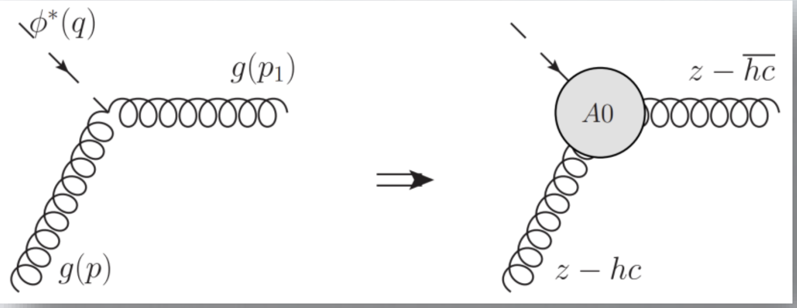
- Reproduces earlier conjecture by *Vogt, 2010*.

RESUMMATION FROM RE-FACTORIZATION: A GLIMPSE

- Is it possible to achieve this in SCET? Another look at re-factorization:

$$C^{B1}(Q,z)J^{B1}(z) \stackrel{z\to 0}{\longrightarrow} C^{A0}(Q^2)\int d^4x\,\mathbf{T}\Big[J^{A0},\mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = C^{A0}(Q^2)D^{B1}(zQ^2,\mu^2)J^{B1}_{z-\overline{sc}}.$$

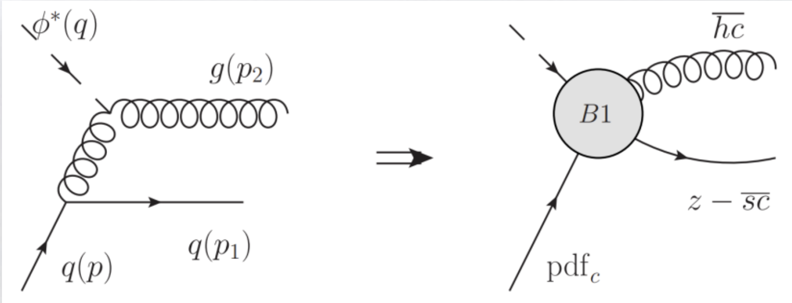
- Integrate out hard modes (solve RGEs in d-dimensions)



$$\frac{d}{d\ln\mu}C^{A0}(Q^2,\mu^2) = \frac{\alpha_s C_A}{\pi}\ln\frac{Q^2}{\mu^2}\,C^{A0}(Q^2,\mu^2)\,.$$

$$\Rightarrow \qquad \left[C^{A0}\left(Q^2,\mu^2\right)\right]_{\text{bare}} = C^{A0}\left(Q^2,Q^2\right)\exp\left[-\frac{\alpha_s C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right].$$

- Integrate out z-hardcollinear modes



$$\frac{d}{d\ln\mu}D^{B1}\left(zQ^2,\mu^2\right) = \frac{\alpha_s}{\pi}\left(C_F - C_A\right)\ln\frac{zQ^2}{\mu^2}\,D^{B1}\left(zQ^2,\mu^2\right).$$

$$\Rightarrow \qquad \left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F - C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right].$$

- This reproduces

$$\mathcal{P}_{qg,\text{hard}}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + \left(C_A - C_F\right)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right].$$

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

OFF-DIAGONAL DIS: THE DIAGRAMMATIC WAY

- The tower of coefficient in the soft real emissions is particularly suitable to be determined with diagrammatic methods. It can be determined based on the following considerations:

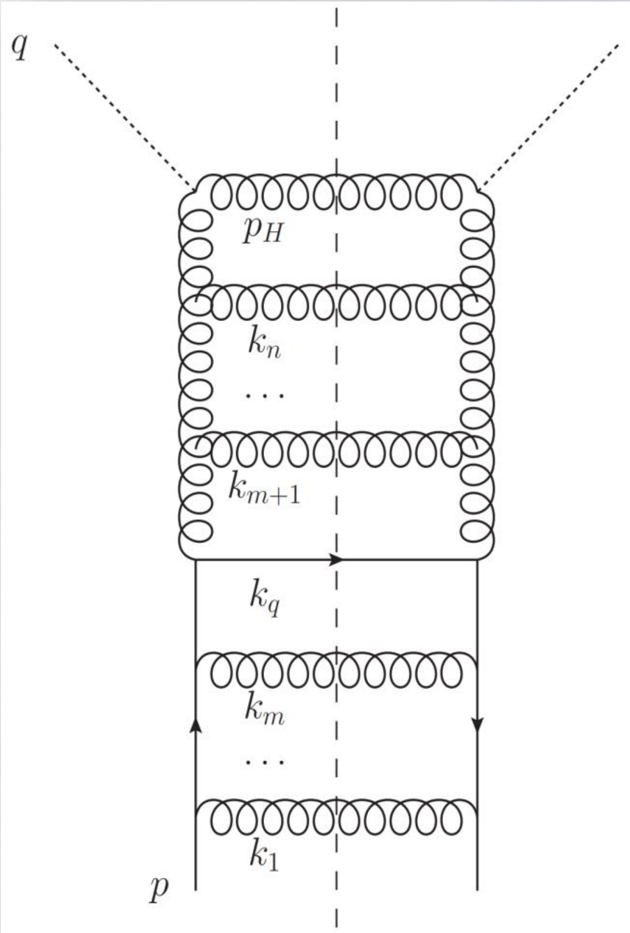
- In a physical polarization gauge in which

Gribov, Lipatov, 1972; Dokshitzer, Diakonov, Troian, 1980; Dokshitzer, Khoze, Mueller, Troian, 1991.

$$\sum_{\text{pols.}} \epsilon^{\dagger}_{\mu}(k)\epsilon_{\nu}(k) = -\eta_{\mu\nu} + \frac{k_\mu c_\nu + k_\nu c_\mu}{c \cdot k}\,, \qquad c = q + xp.$$

only ladder diagrams contribute to the LLs.

- The power suppression is given by the soft quark polarization sum; gluon emissions are eikonal (LP).
- Phase space can be also approximated to LP, and factorizes in Laplace space.
- The full result is found requiring that virtual corrections modify the real emission contributions at each order, removing singularities which are simultaneously soft and collinear.
- In the end one recover the previous result



$$W_{\phi,q}\Big|_{\text{LL}} = -\frac{2a_s C_F}{\epsilon}\frac{N^\epsilon}{N}\frac{1}{C_F - C_A}\left(\frac{4a_s(N^\epsilon - 1)}{\epsilon^2}\right)^{-1}\left\{\exp\left[\frac{4a_s C_F(N^\epsilon - 1)}{\epsilon^2}\right] - \exp\left[\frac{4a_s C_A(N^\epsilon - 1)}{\epsilon^2}\right]\right\},$$

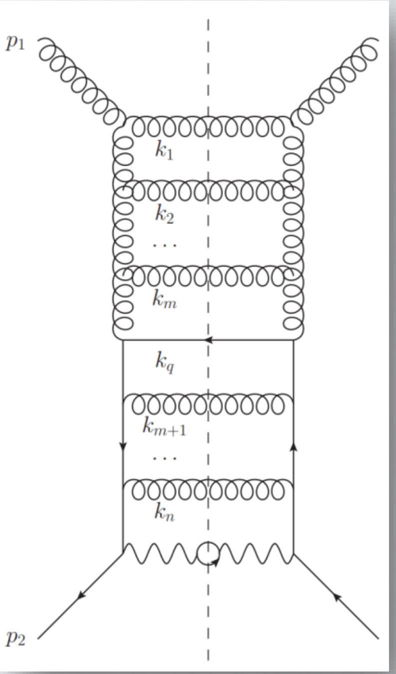
van Beekveld, LV, White 2021

OFF-DIAGONAL DIS: THE DIAGRAMMATIC WAY

- The same procedures can be easily adapted to the subleading qg channel in Drell-Yan (and Higgs production).
- Consistency conditions can be studied to determine the smallest set of parameters necessary to determine the whole partonic cross section;
- The set of parameters can be determined
  - by assuming exponentiation of a given region, justified within a refactorization approach.
  - by direct calculation of the ladder diagrams contributing to the real emission.
- Either way, one in the end reproduces an earlier conjecture in Lo Presti, Almasy, Vogt 2014:

Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang, 2020 (unpublished)

van Beekveld, LV, White 2021



$$W_{\mathrm{DY},g\bar{q}}\Big|_{\mathrm{LL}} = -\frac{T_R}{2(C_F - C_A)}\frac{1}{N}\frac{\epsilon(N^{\epsilon-1})}{N^\epsilon - 1}\exp\left[\frac{4a_s C_F(N^\epsilon - 1)}{\epsilon^2}\right] \times \left\{\exp\left[\frac{4a_s C_F N^\epsilon(N^\epsilon - 1)}{\epsilon^2}\right] - \exp\left[\frac{4a_s C_A N^\epsilon(N^\epsilon - 1)}{\epsilon^2}\right]\right\},$$

$$\widetilde{C}_{\mathrm{DY},g\bar{q}}\Big|_{\mathrm{LL}} = \frac{T_R}{C_A - C_F}\frac{1}{2N\ln N}\left[e^{8C_F a_s \ln^2 N}\mathcal{B}_0[4a_s(C_A - C_F)\ln^2 N] - e^{(2C_F + 6C_A)a_s \ln^2 N}\right].$$

CONCLUSION

- The resummation of large leading logarithms at NLP is now under control, both in the diagonal (quark-antiquark, gluon-gluon) and off-diagonal (quark-gluon) channels, in electroweak annihilation processes (Drell-Yan, Higgs production, etc) and DIS.

- The next step is to formalize the refactorization process, such as to allow for a systematic resummation at NLP, beyond leading logarithmic accuracy.

  → ***See talk by M. Beneke***

- These result will be useful to improve the accuracy of relevant processes for the LHC and may provide useful information to extend resummation at NLP to other kinematic limits (small pT, small β, etc).