Effective transverse momentum in multijet production at hadron colliders

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(based on the paper **2201.11519**, in collaboration with *L. Buonocore, M. Grazzini, J. Haag, L. Rottoli*)

Loops and Legs in Quantum Field Theory - April 25th- 30th 2022

Universität Zürich[UZH]

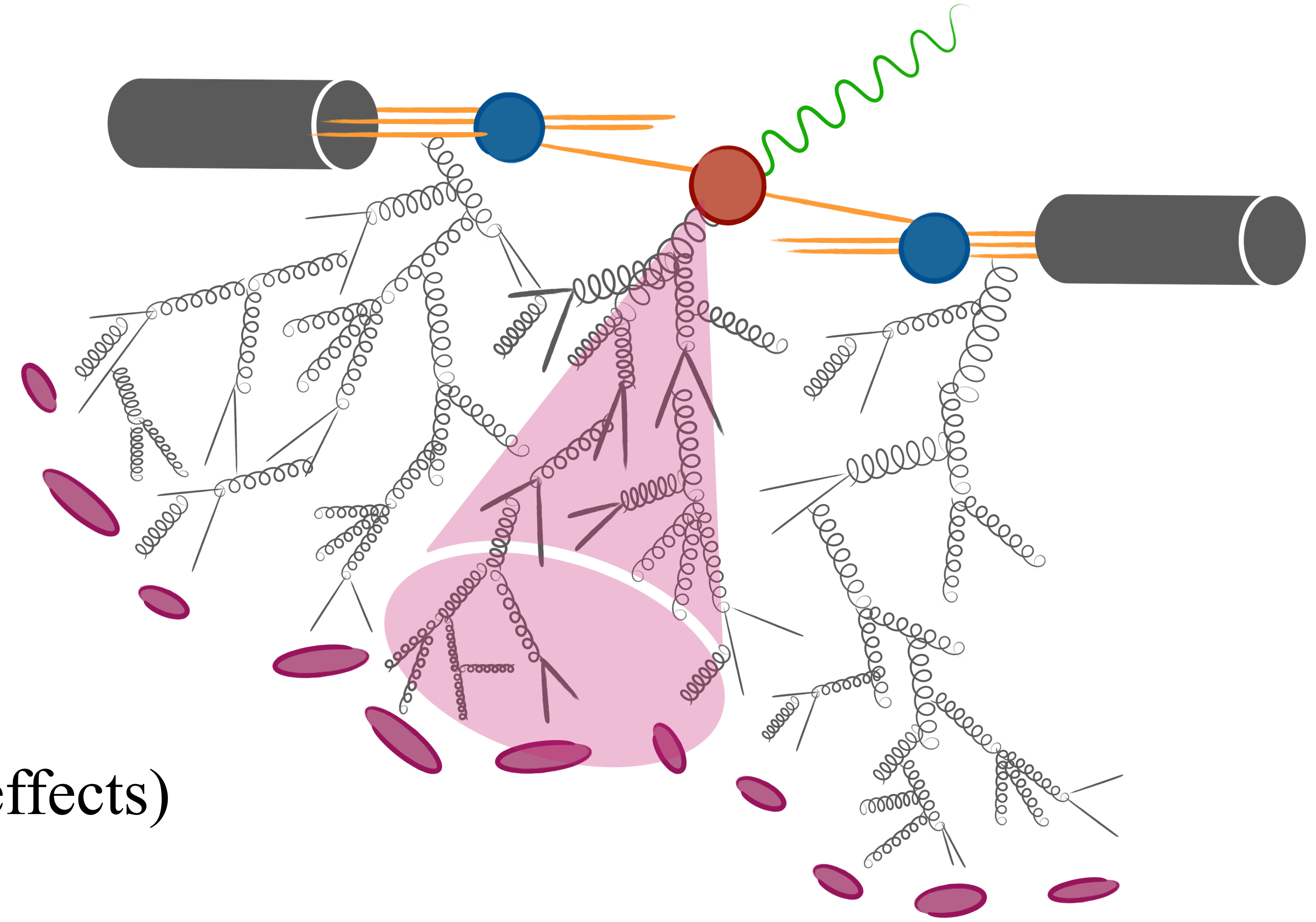
Outline

- *Introduction*
- *Exploring jet resolution variables:* q_T - imbalance, ΔE_T
- *Our proposal:* k_T^{ness}
- *Handling of IR-singularities:* k_T^{ness}-subtraction @NLO
- *Phenomenological applications:* $H+1jet$, $Z+1jet$, $Z+2jets$
- *Conclusions*

Introduction

Jet physics @ LHC :

- jets are **collimated bunches** of hadrons that represent the **fingerprints** of the high-energy partons produced in the hard-scattering interaction
- jets are **ubiquitous** at the LHC
- experimental analyses categorise events into **jet bins** according to jet multiplicity (*BSM searches*, *precision SM studies*)
- description of jet processes requires an understanding of QCD across a wide range of energy scales (**fixed-order calculations**, all-order structure and resummation, NP effects)

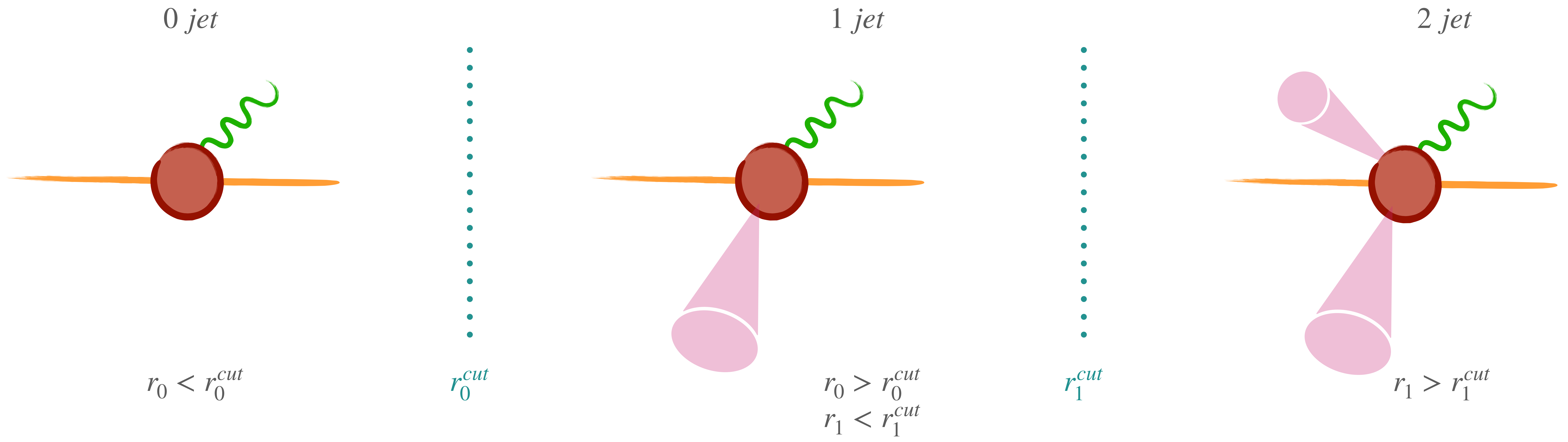


jet resolution variables:
observables capable to capture the deviation from the LO energy flow, which characterises the bulk of the events

Introduction

Jet resolution variables :

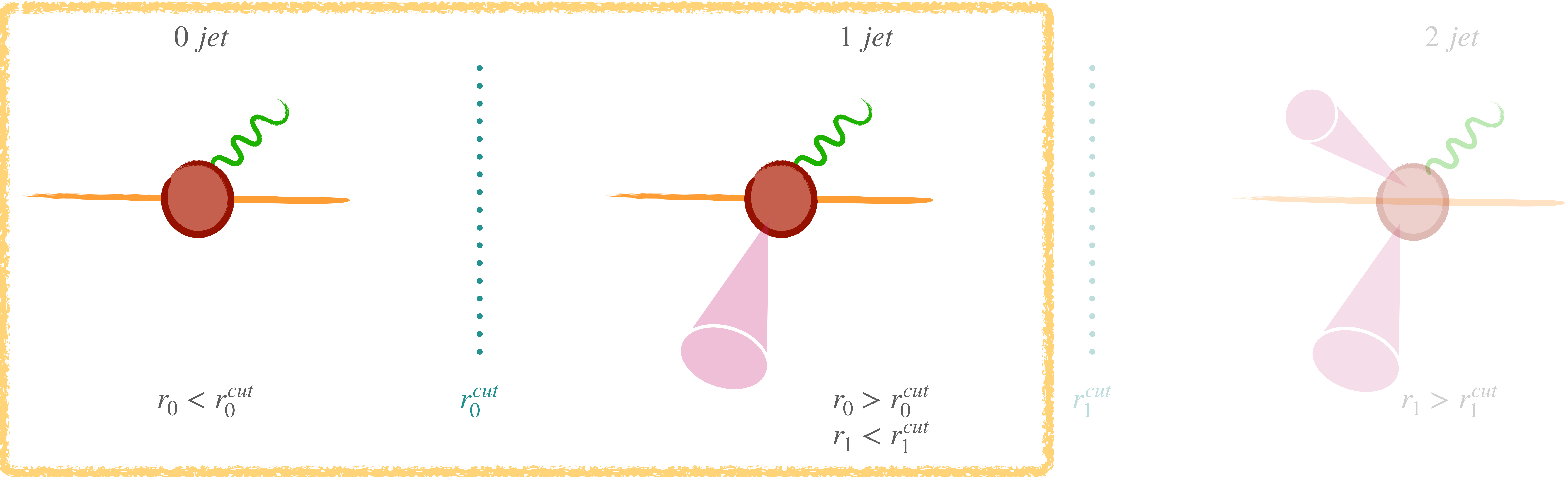
- N-jet resolution variables should smoothly capture the transition from N to $N+1$ jet configuration



Introduction

Jet resolution variables (0-jet case) :

- 0-jet resolution variables should smoothly capture the transition from 0 to 1 jet configuration



a prominent example is the **transverse momentum** of the tagged colourless system (q_T)

Introduction

q_T -subtraction (0-jet case) :

- q_T is a good resolution variable for processes that do **not** involve **jets at Born level** [Catani, Grazzini (2007)]
- it is the transverse momentum of the born-like system
- it captures the $0 \rightarrow 1$ jet transition ($q_T = 0$ if no hard jets are found, $q_T > 0$ if at least one hard jet is tagged)
- it can be used as a **slicing variable**, to regularise IR-singularities, for colour-singlet and heavy-quark production

  (for $q_T > 0$, only $N^{n-1}LO$-type singularities can appear $@N^nLO$)

  [in MATRIX, fixed-order computations publicly available up to NNLO for $pp \rightarrow V, H, VV, HH, \gamma\gamma, \gamma\gamma\gamma, V\gamma, t\bar{t}$] [Grazzini, Kallweit, Wiesemann (2017)] [Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2019,2020)]
- it has been used $@N^3LO$ for DY

  [Chen, Gehrmann, Glover, Huss, Yang and Zhu (2021)] [Chen, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli (2022)] [Camarda, Cieri, Ferrera (2022)]

  and Higgs production [Billis, Dehnadi, Ebert, Michel, Tackmann (2021)]

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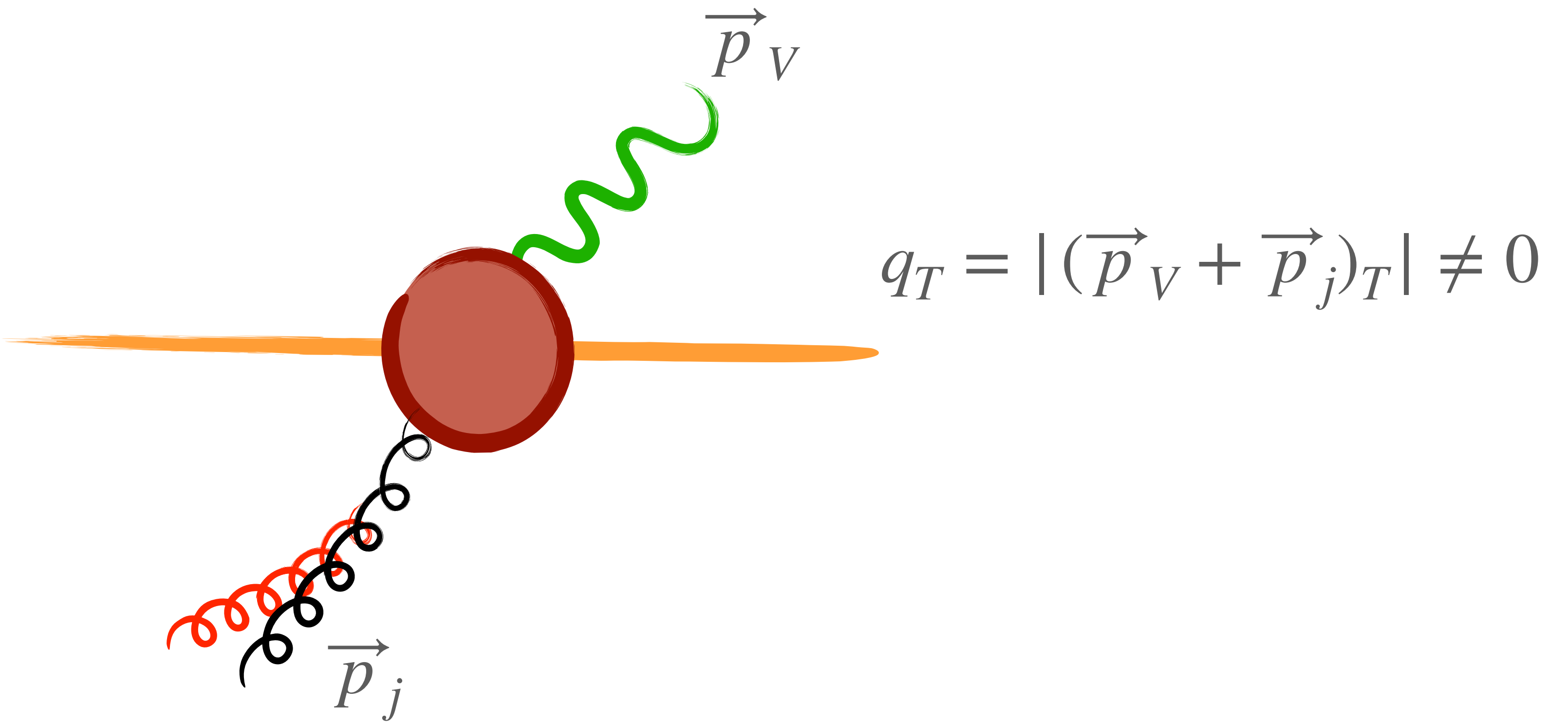
drawback:

- it cannot regularise **final-state collinear** singularities (FSR)



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Jet resolution variables (beyond 0-jet case) :

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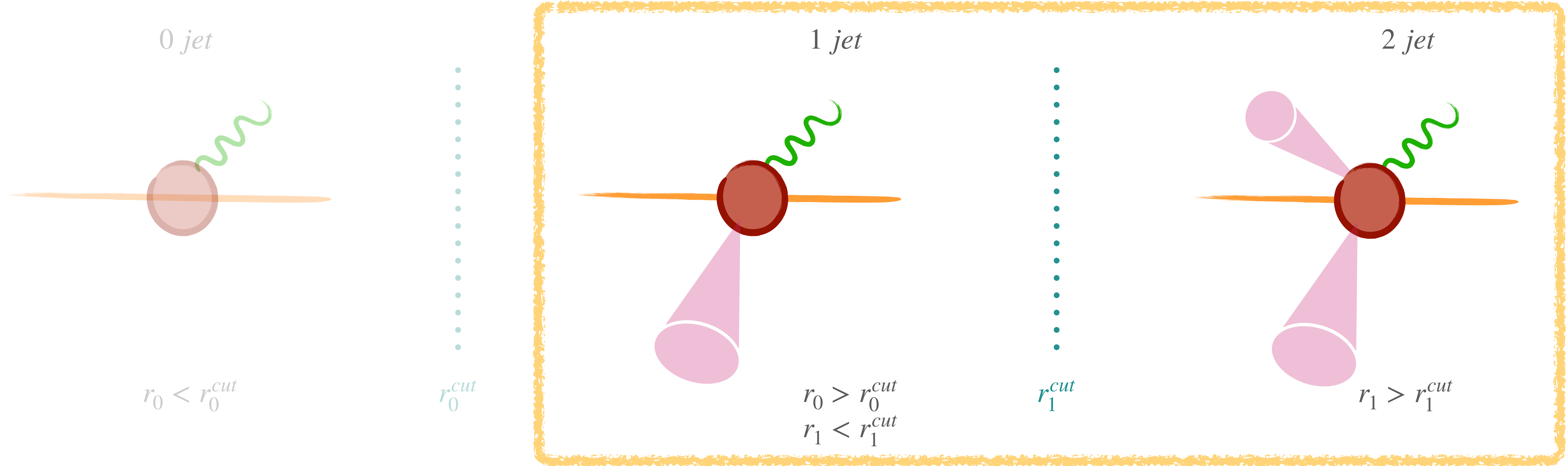

Introduction

N-jettiness (beyond 0-jet case) :

- N-jettiness (τ_N) [Stewart, Tackmann, Waalewijn (2010)] is so far the **only player in the game**!
- it has proved a successful resolution variable for processes with 1 *jet* up to NNLO

[$H + jet$: Boughezal, Focke, Giele, Liu, Petriello (2015)], [$Z + jet$: Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello (2016)], [$W + jet$: Boughezal, Liu, Petriello (2016)]

- no full computation available for 2-jettiness

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- no full computation available for 2-jettiness

drawbacks:

- **large** missing **power corrections** (logarithmically enhanced already @ NLO)
- instabilities under hadronisation and multiple-parton interactions (MPI)

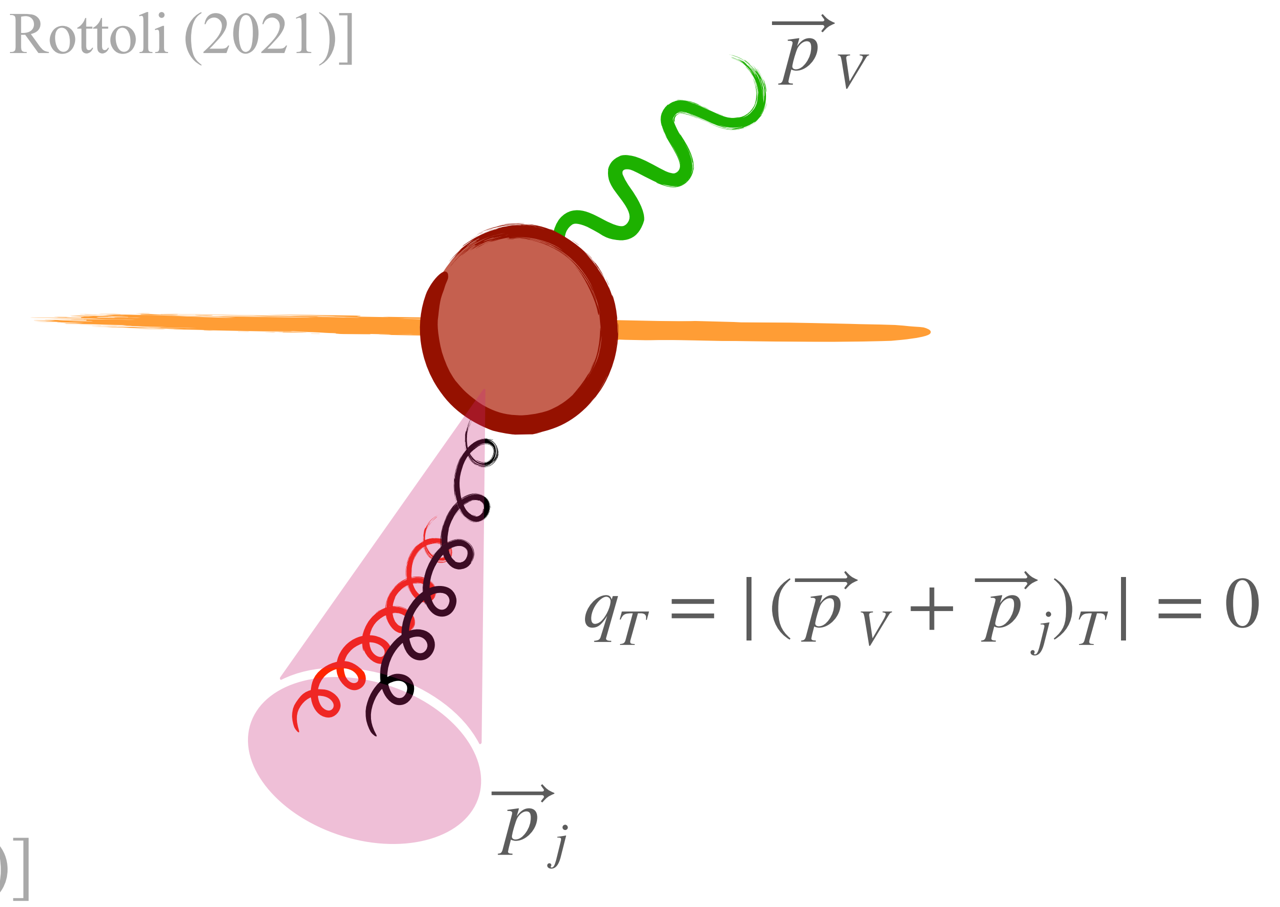
it may prove worthwhile to **explore other resolution variables** which overcome some of the shortcomings of jettiness:
- smaller power corrections
- more direct experimental evidence

Exploring jet resolution variables

q_T-imbalance (for $V + 1jet$ processes) : [Buonocore, Grazzini, Haag, Rottoli (2021)]

- it is the **natural extension** of the transverse momentum used in the colour-singlet case
- it is defined as the transverse momentum of the $V + 1jet$ system after applying a **clustering algorithm** (anti-k_T with radius R)
- resummation formula up to NLL'
- it was applied as a slicing parameter @NLO for $H + 1jet$ [M.Costantini, Master thesis, UZH (2021)]

→ linear power corrections



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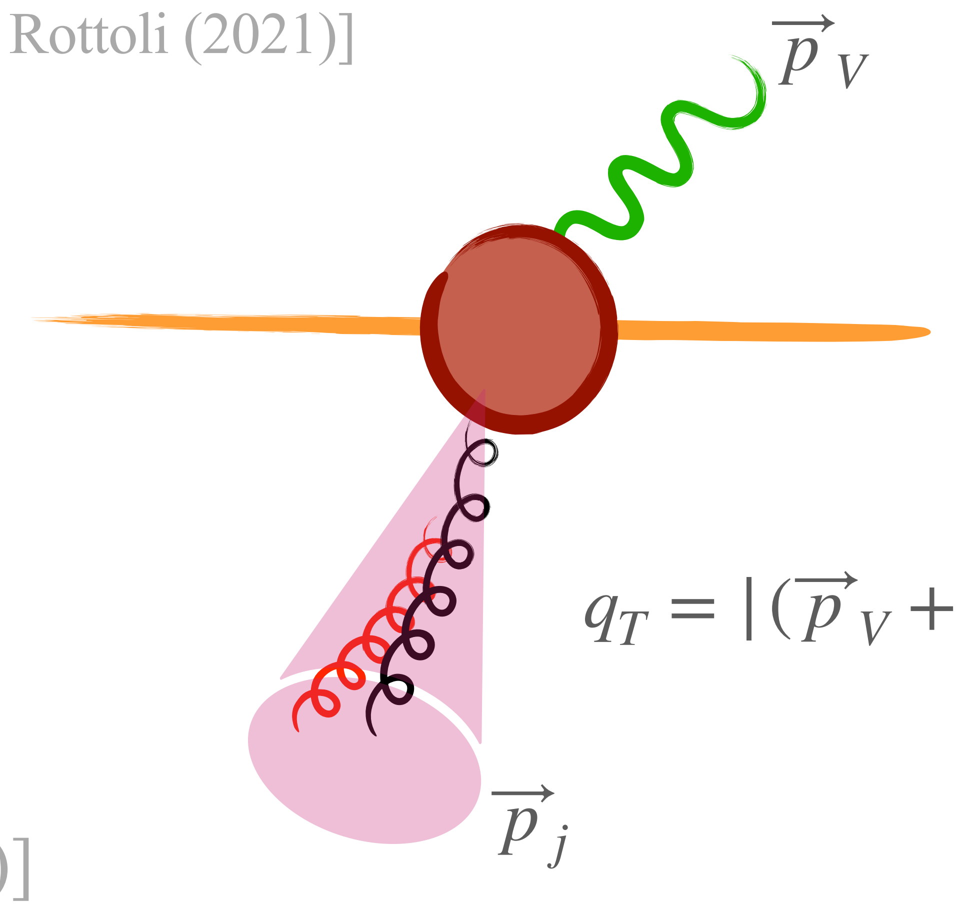

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→ linear power corrections

drawbacks:

- dependence of the observable on the additional cutting variable R (the extension to higher orders is more complicated)
- presence of **non-global logarithms** starting at NNLO

Exploring jet resolution variables

ΔE_T (for $V + 1 jet$ processes) :

- it is a **global observable** defined as the difference between the transverse energy and the transverse momentum of the vector boson

$$\Delta E_T = \sum_{i=1}^{n} |\vec{p}_{T,i}| - |\vec{p}_{T,V}|$$

- it has a more convoluted structure than q_T -imbalance due to the different scaling in each singular region

$$IS: \quad \Delta E_T \approx k_T(1 + \cos\phi) \qquad\qquad FS: \quad \Delta E_T \approx k_\perp \theta \sin^2\phi$$

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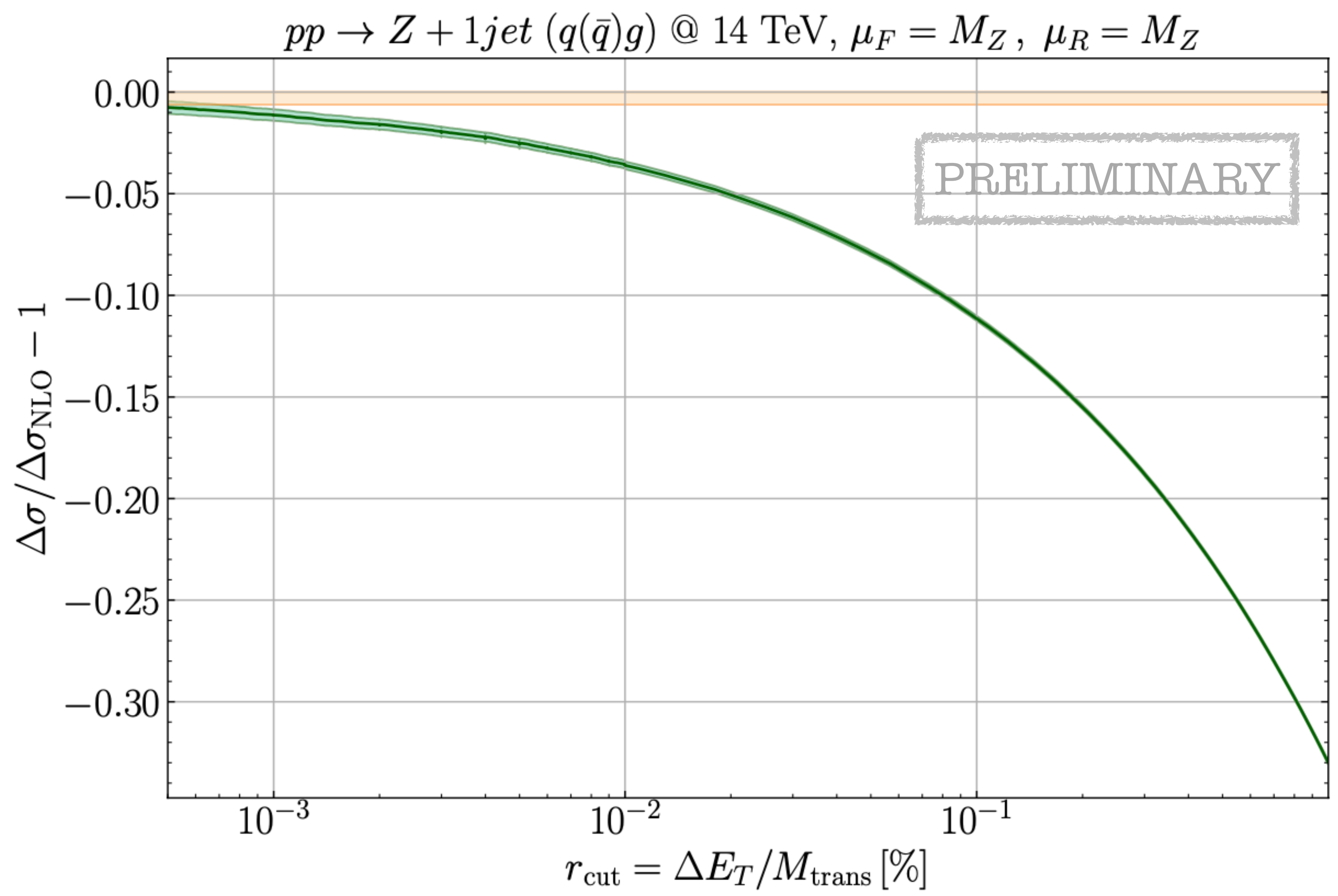
drawbacks:

- non-trivial **azimuthal dependence** of the observable:

  different beam functions @NLO and non-vanishing spin-correlations

- logarithmically enhanced linear power corrections



Exploring jet resolution variables

main goal : find an "IDEAL" jet resolution variable with the following properties

- dimensional variable able to capture the $N \to N+1$ jet transition
- global
- extendable to any number of jets
- variable that reduces to q_T in the 0-jet case and for ISC radiation
- nice convergence properties (linear scaling or better)
- stable under hadronisation and MPI

does such a variable exist ?

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our proposal is called k_T^{ness}

does such a variable exist ?

Our proposal: k_T^{ness}

- we consider a process with N**-jets at Born level**, possibly accompanied by a generic colourless system F

$$h_1(P_1) + h_2(P_2) \to j(p_1) + \ldots + j(p_N) + F(p_F) + X$$

- we introduce the usual distances between partons

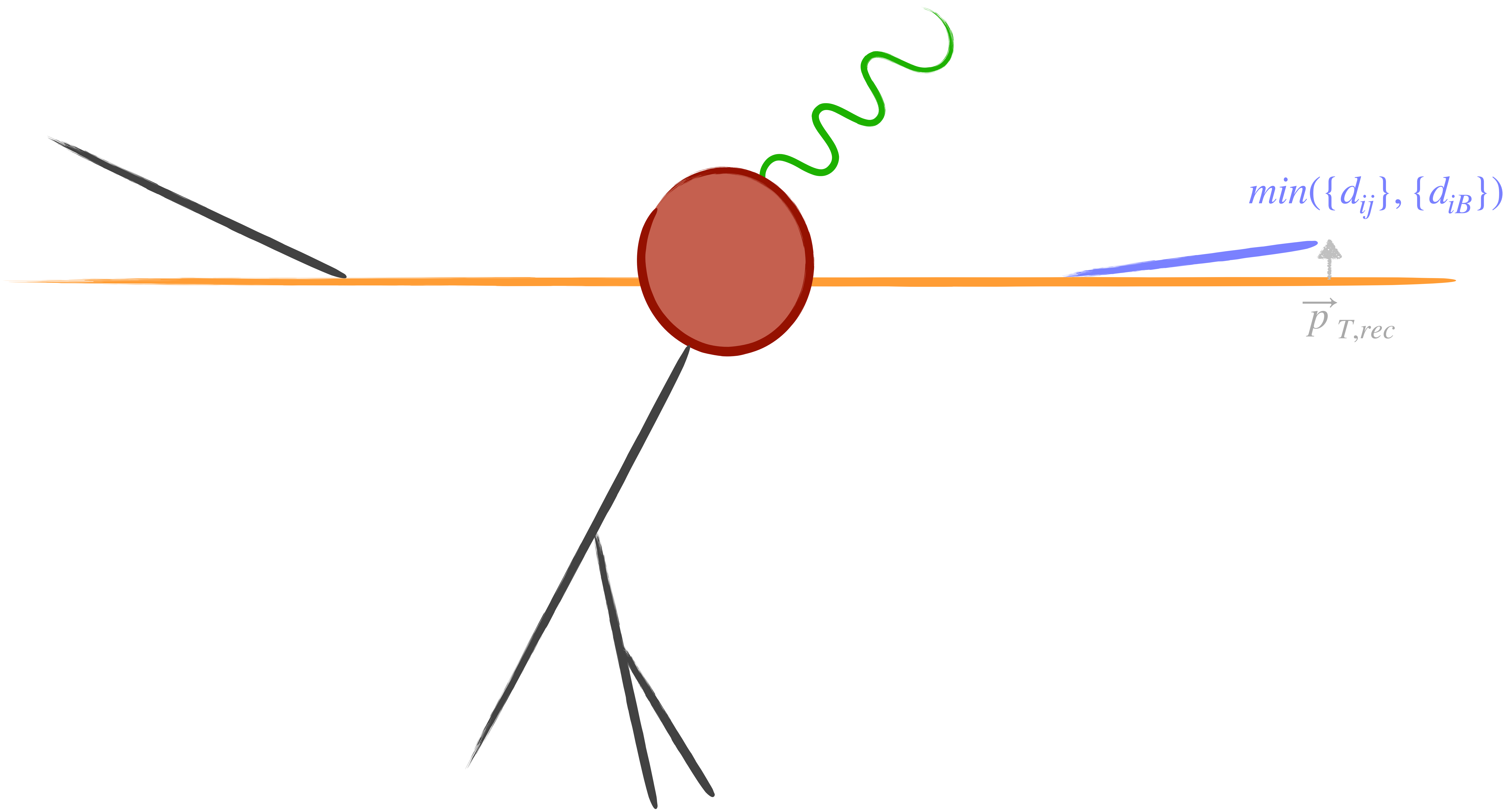
$$d_{ij} = min(p_{T,i}, p_{T,j})\frac{\Delta R_{ij}}{D} \quad \text{with} \quad \Delta R_{ij} = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2} \quad \text{and} \quad d_{iB} = p_{T,i}$$

- we can define N-k_T^{ness} via a **recursive procedure**
- physically, the variable represents an **effective transverse momentum**:
  - if the unresolved radiation is close to the beams (ISR), k_T^{ness} is the transverse momentum of the hard system
  - if the unresolved radiation is collinear to one of the final-state jets (FSR), k_T^{ness} describes the relative transverse momentum of the hard system wrt the jet direction

Our proposal: k_T^{ness}

definition : run the k_T -algorithm till $N + 1$ proto-jets are left

example of $1\text{-}k_T^{ness}$



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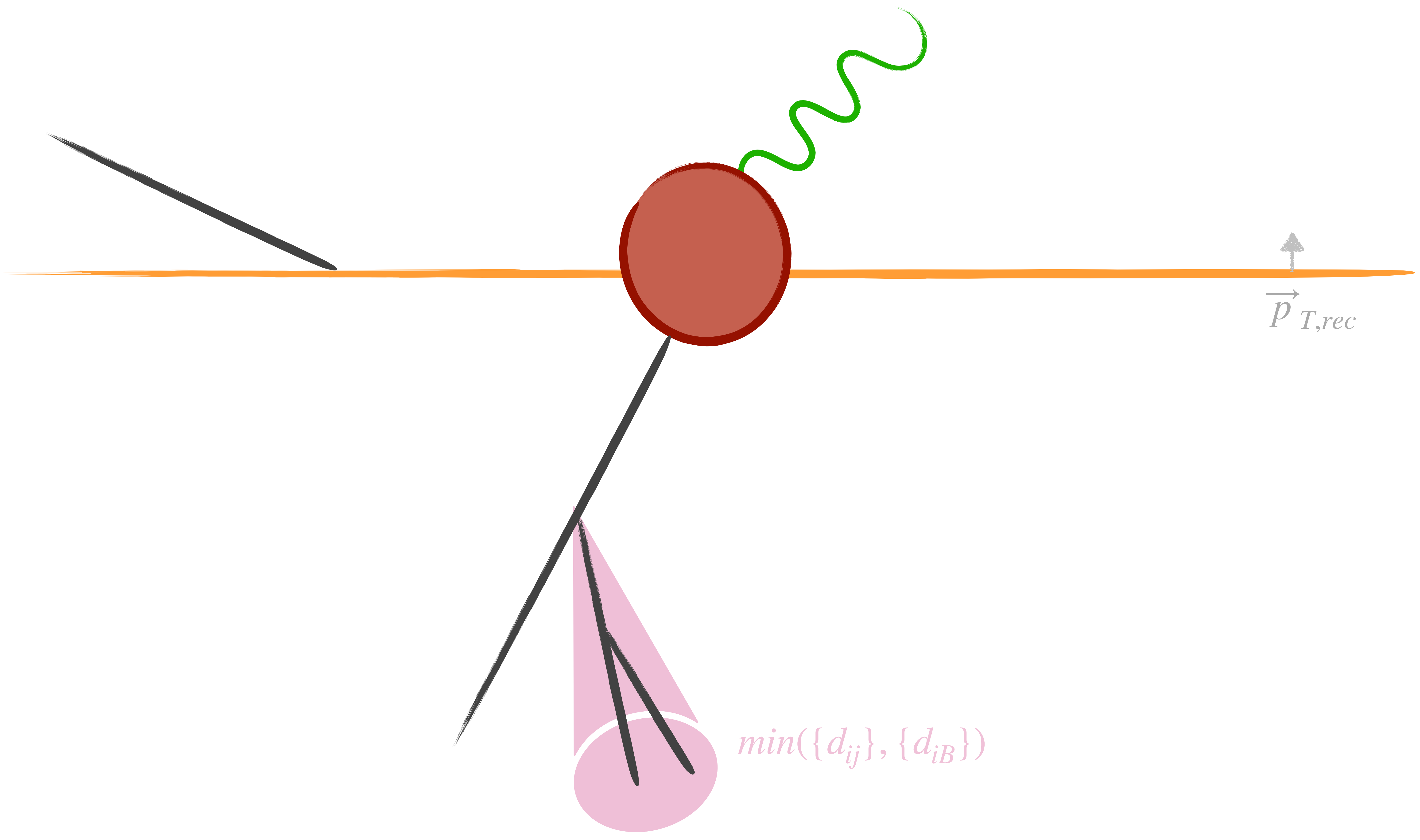

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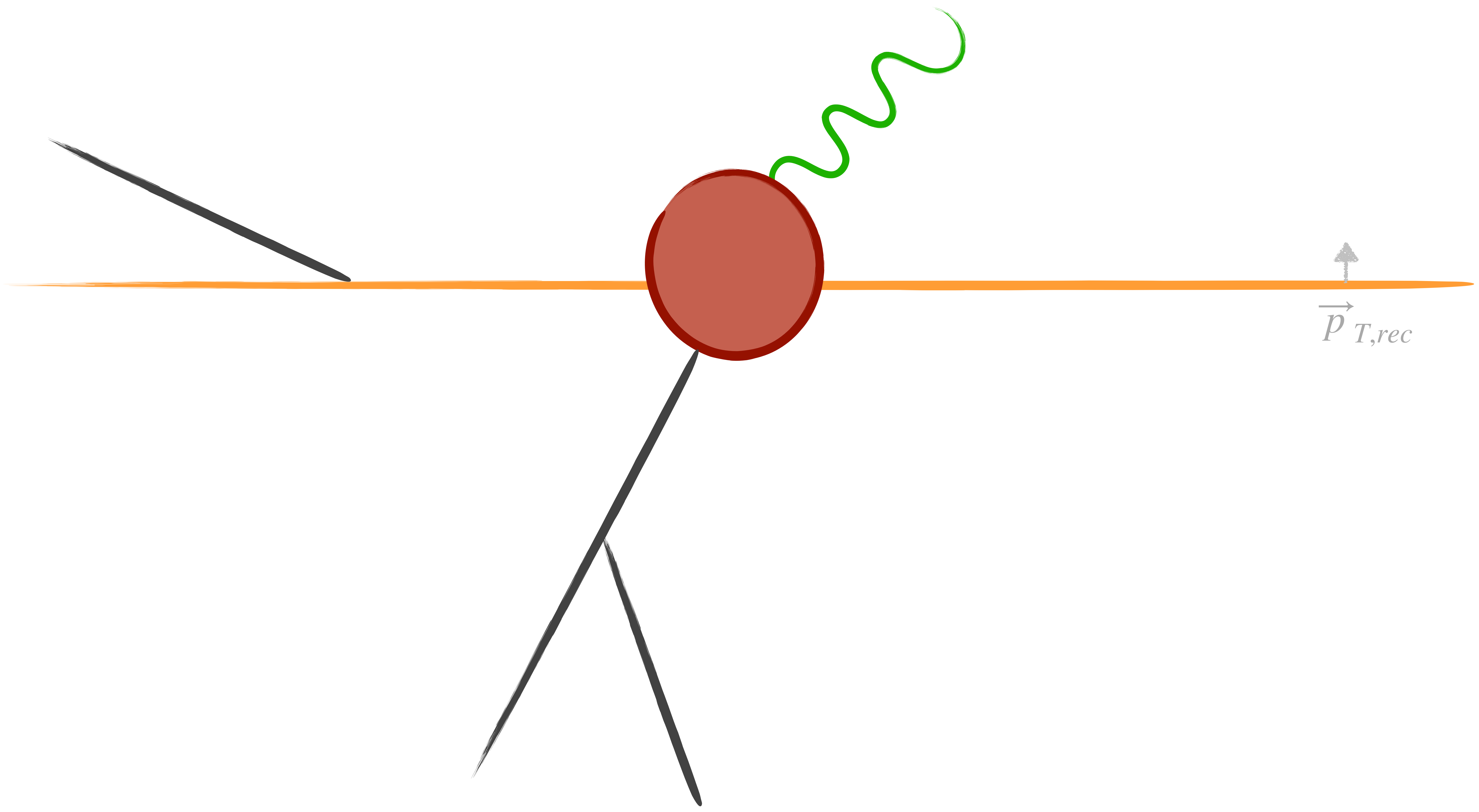

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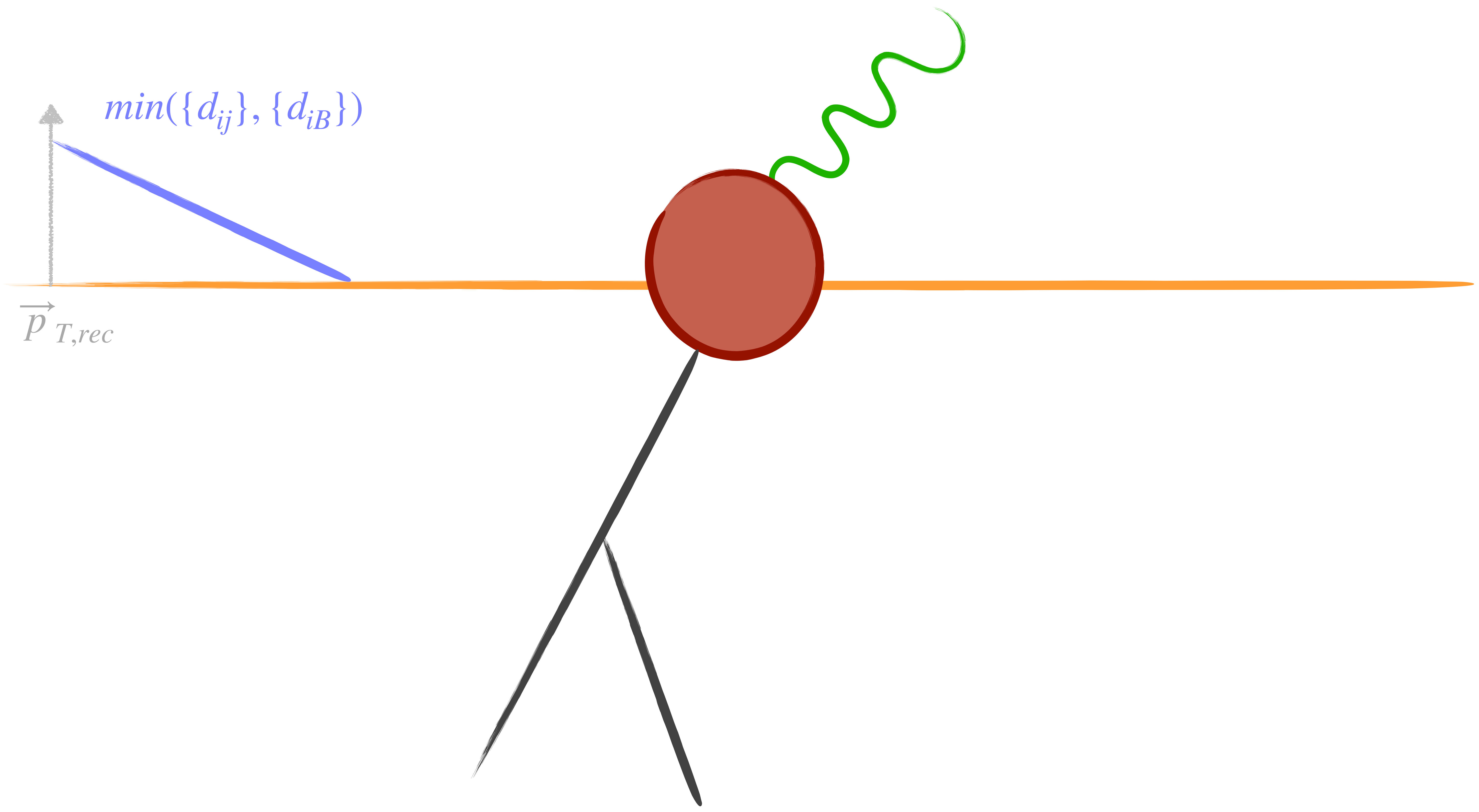

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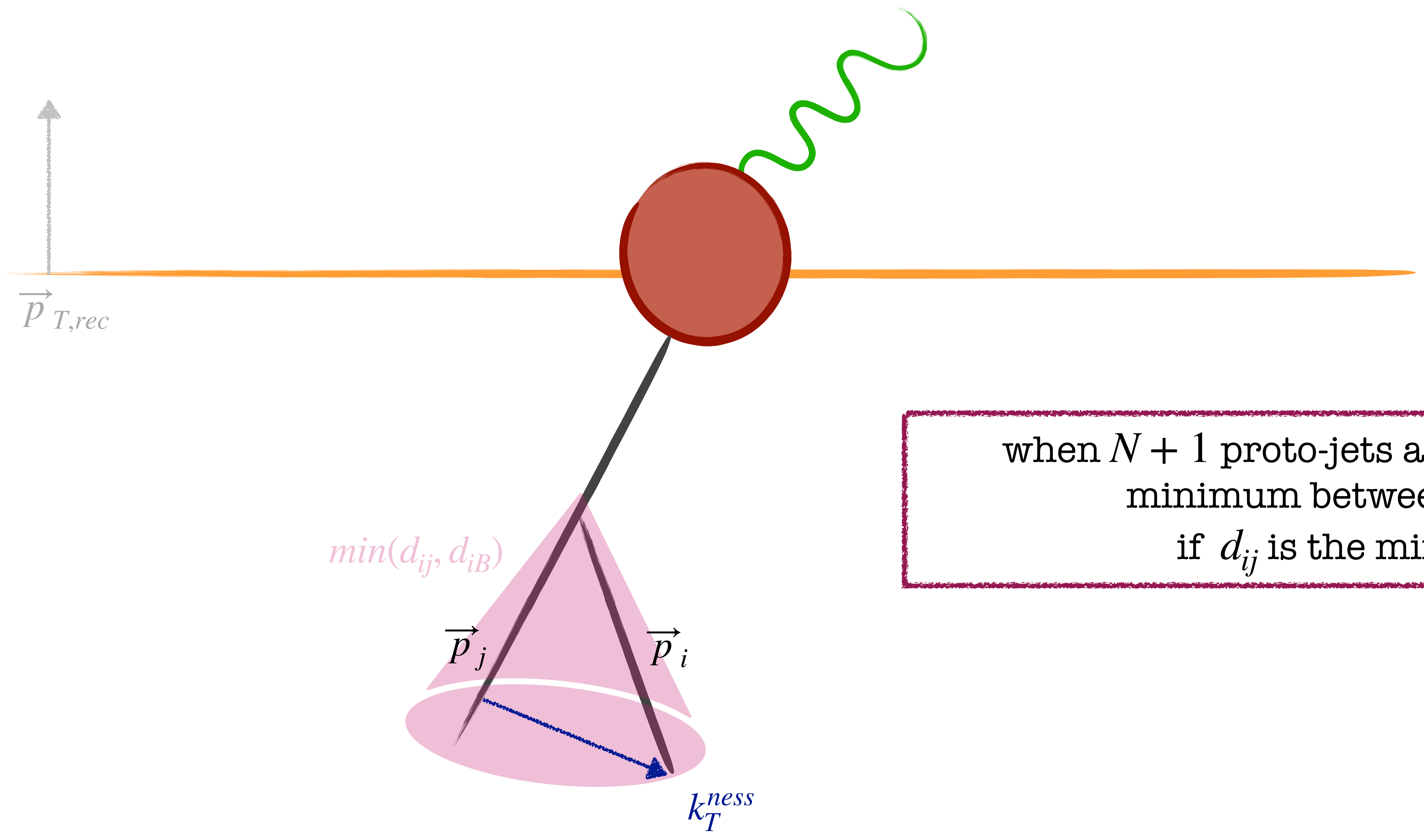

when $N+1$ proto-jets are left, compare the minimum between d_{iB} and d_{ij}:
if d_{ij} is the min → $k_T^{ness} = d_{ij}$

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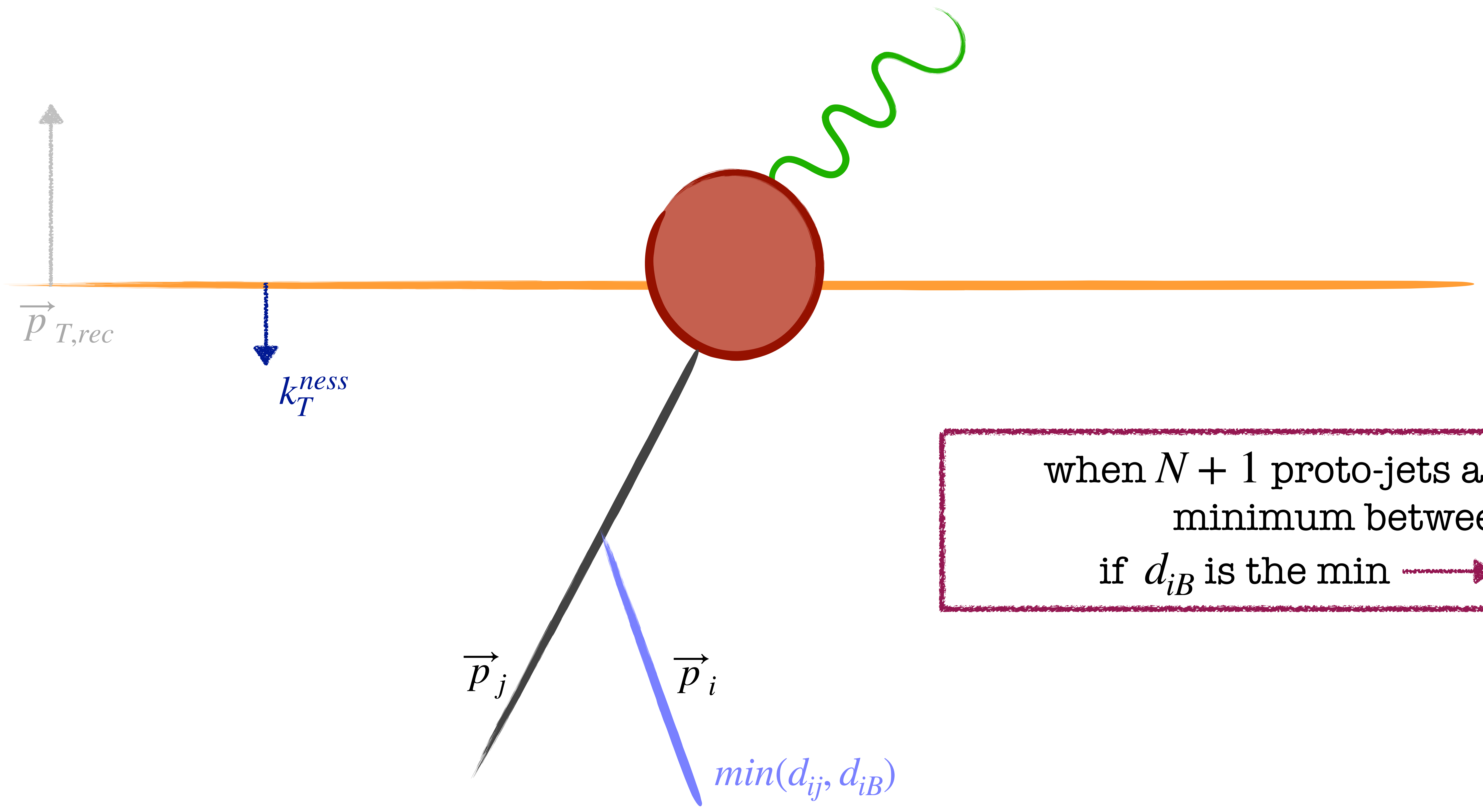

when $N+1$ proto-jets are left, compare the minimum between d_{iB} and d_{ij}:
if d_{iB} is the min $\longrightarrow$ $k_T^{ness} = |\vec{p}_{T,i} + \vec{p}_{T,rec}|$

Handling of IR-singularities

k_T^{ness} -subtraction :

- we consider the NLO cross section, at the partonic level, $d\hat{\sigma}_{NLO} = d\sigma_V + d\sigma_R$ and we introduce a slicing cut r_{cut} on the variable $r = k_T^{ness}/M$:

$$d\hat{\sigma}_{NLO} = (d\sigma_V + d\sigma_R)\Theta(r_{cut} - r) + d\sigma_R\Theta(r - r_{cut})$$

- in the region **above the cut** the additional radiation is resolved and the real integration can be performed in $d = 4$
- it is divergent for $r_{cut} \to 0$

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- in the region **below the cut**, the computation must be performed in $d = 4 - 2\epsilon$ and we can rely on the IR-factorisation of the real matrix element
- the real computation is performed by organising the relevant terms in each singular region and removing double counting (*beam*, *jet* and *soft* functions)
- soft and collinear poles cancel between real and virtual contributions
- the sensitivity to IR kinematics appears as $\log(r_{cut})$

Handling of IR-singularities

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$$d\hat{\sigma}_{NLO} = (d\sigma_V + d\sigma_R)\Theta(r_{cut} - r) + d\sigma_R\Theta(r - r_{cut})$$

- the **master formula** for NLO subtraction is

$$d\hat{\sigma}_{NLO}^{F+Njets+X} = \mathscr{H}_{NLO}^{F+Njets} \otimes d\hat{\sigma}_{LO}^{F+Njets} + [d\hat{\sigma}_{LO}^{F+(N+1)jets} - d\hat{\sigma}_{NLO}^{CT,F+Njets}]_{r>r_{cut}}$$

where the **counterterm** is

$$d\hat{\sigma}^{\mathrm{CT,F+Njets}}_{\mathrm{NLO}\,ab} = \frac{\alpha_S}{\pi}\frac{dk_T^{\mathrm{ness}}}{k_T^{\mathrm{ness}}}\Bigg\{\Bigg[\ln\frac{Q^2}{(k_T^{\mathrm{ness}})^2}\sum_\alpha C_\alpha - \sum_\alpha \gamma_\alpha - \sum_i C_i \ln\left(D^2\right) - \sum_{\alpha\neq\beta}\langle \mathbf{T}_\alpha\cdot\mathbf{T}_\beta\rangle \ln\left(\frac{2p_\alpha\cdot p_\beta}{Q^2}\right)\Bigg]\times$$
$$\times\,\delta_{ac}\delta_{bd}\delta(1-z_1)\delta(1-z_2) + 2\delta(1-z_2)\delta_{bd}P^{(1)}_{ca}(z_1) + 2\delta(1-z_1)\delta_{ac}P^{(1)}_{db}(z_2)\Bigg\}\otimes d\hat{\sigma}^{\mathrm{F+N\,jets}}_{\mathrm{LO}\,cd}$$

with $\gamma_q = \dfrac{3}{2}C_F$ and $\gamma_g = \dfrac{11C_A - 2n_f}{6}$

Handling of IR-singularities

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where the **finite piece** is

$$\mathscr{H}^{F+Njets}_{cd;ab} = (\mathbf{HS})_{cd} C_{ca} C_{db} \prod_{i=1}^{N} J_i$$

with jet function

$$J_i^f = \begin{cases} 1 + \frac{\alpha_S(\mu_R)}{\pi}\Big\{C_A\Big[\frac{131}{72} - \frac{\pi^2}{4} - \frac{11}{6}\ln(2D) - \ln(D)\ln\Big(\frac{Q^2 D}{4E_i^2}\Big)\Big] \\ \qquad + T_R n_f\Big[-\frac{17}{36} + \frac{2}{3}\ln(2D)\Big]\Big\} + \mathcal{O}(\alpha_S^2) & \text{if } f = g \\ 1 + \frac{\alpha_S(\mu_R)}{\pi} C_F\Big[\frac{7}{4} - \frac{\pi^2}{4} - \frac{3}{2}\ln(2D) - \ln(D)\ln\Big(\frac{Q^2}{4E_i^2}\Big) - \ln^2(D)\Big] + \mathcal{O}(\alpha_S^2) & \text{if } f = q, \bar{q} \end{cases}$$

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with soft factor

$$(\mathbf{HS})_{cd} = \frac{\langle \mathcal{M}_{cd} | \mathbf{S} | \mathcal{M}_{cd} \rangle}{|\mathcal{M}^{(0)}_{cd}|^2} \qquad \mathbf{S} = 1 + \frac{\alpha_S(\mu_R)}{\pi}\mathbf{S}^{(1)} + \mathcal{O}(\alpha_S^2)$$

Phenomenological applications

Setup and implementation

- we implemented k_T^{ness} -subtraction @NLO, for processes with 1 and 2 jets, in a **private Fortran code**. The subtraction is now implemented, for an arbitrary number of jets, also in the MATRIX framework [work done by J.Haag and S.Kallweit]
- we consider **proton-proton collisions** @ LHC with $\sqrt{s} = 13$ TeV
- the parameters common to all studied processes are:

  - $G_F = 1.16639 \times 10^{-5}$ GeV^{-2} (G_μ *scheme* to treat the EW input parameters)
  - $m_W = 80.385$ GeV
  - $m_Z = 91.1876$ GeV $\Gamma_Z = 2.4952$ GeV
  - $m_H = 125$ GeV
  - anti-k_T algorithm with $R = 0.4$
  - `NNPDF_nlo_as_0118` with $\alpha_s(m_Z) = 0.118$

Phenomenological applications

H + 1jet production @NLO

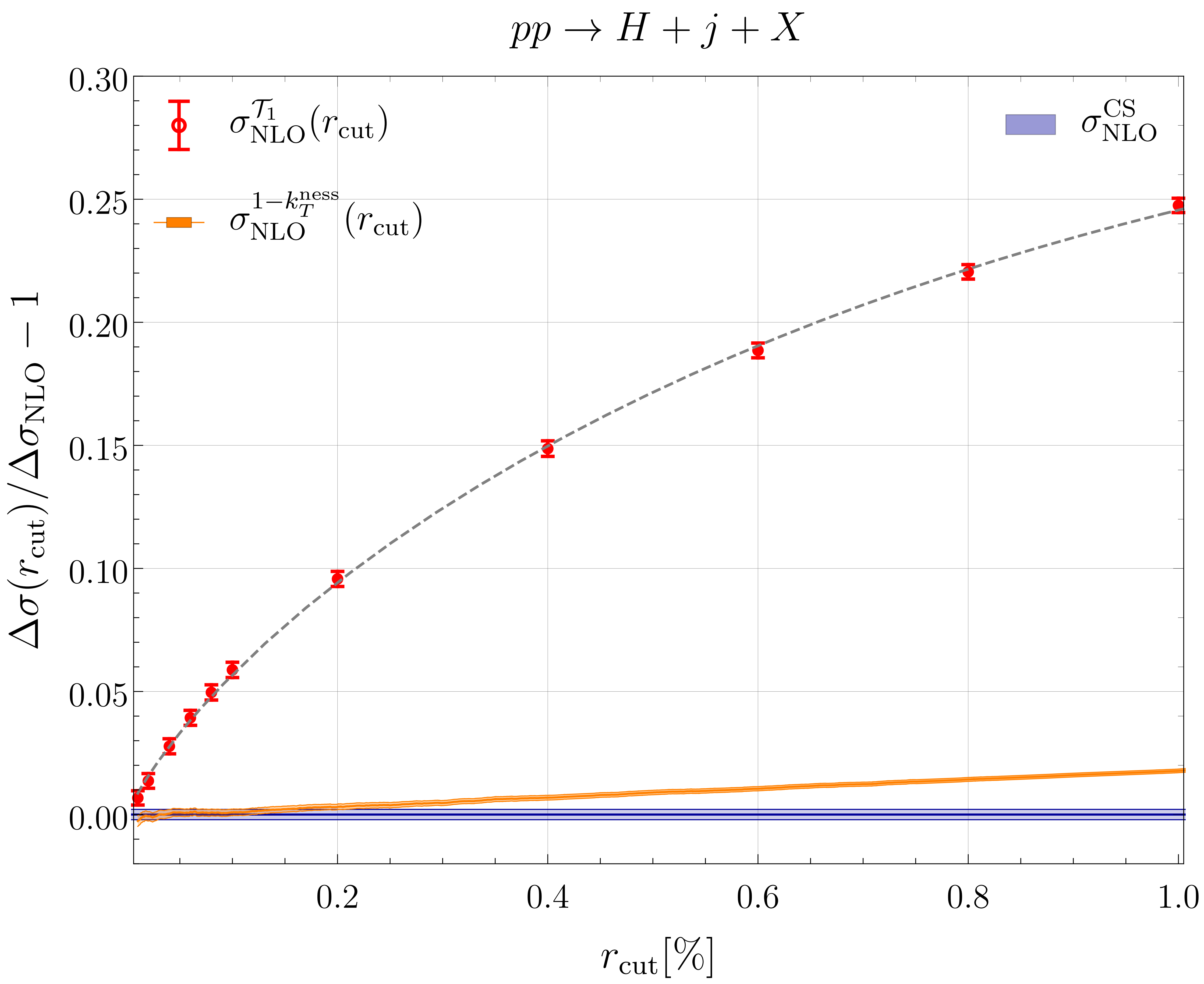
$p_T^j > 30$ GeV
$D = 1$
$\mu_R = \mu_F = m_H$

- comparison between the results obtained with 1-k_T^{ness} and 1-jettiness.
- the cut is applied on the dimensionless variable

$$r = \mathcal{T}_1/\sqrt{m_H^2 + (p_T^j)^2} \qquad \text{with} \qquad \mathcal{T}_1 = \sum_i \min_l \left\{ \frac{2q_l \cdot p_i}{Q_l} \right\}$$

$$\text{and} \qquad r = k_T^{\rm ness}/\sqrt{m_H^2 + (p_T^j)^2}$$

- **linear** missing power corrections vs logarithmically-enhanced ones



1-jettines results obtained from a suitably modified version of MCFM

Phenomenological applications

Z + 2jets production @NLO

$p_T^j > 30$ GeV $\quad |\eta^j| < 4.5$

$p_T^l > 20$ GeV $\quad |\eta^l| < 2.5$

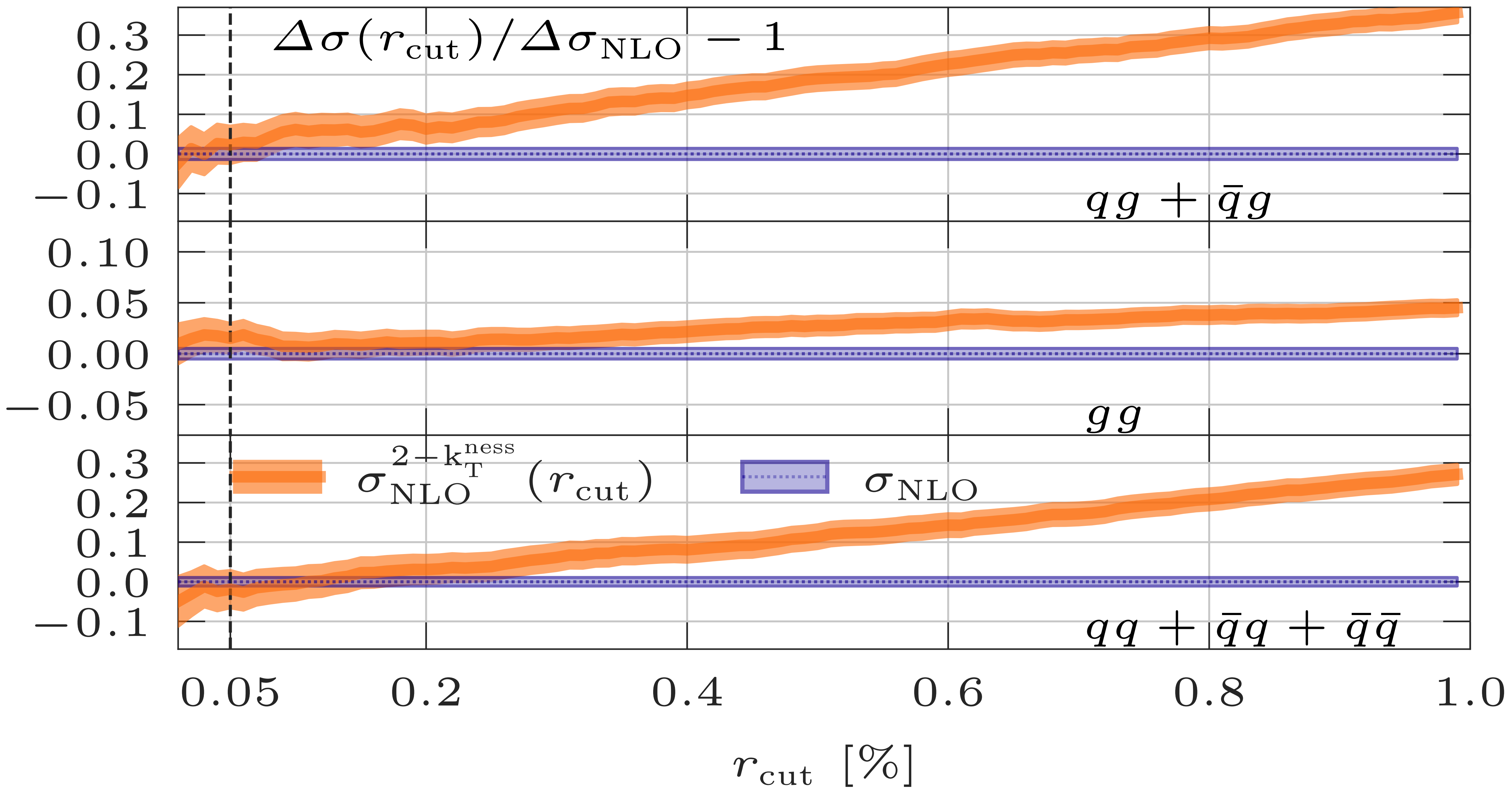
$66\ GeV < m_{ll} < 116\ GeV$

$\Delta R_{lj} > 0.5 \quad \Delta R_{ll} > 0.2$

$D = 0.1$

$\mu_R = \mu_F = m_Z$

- comparison between the results obtained with 2-k_T^{ness} and FKS local subtraction, at the level of the **total cross section**
- the cut is applied on k_T^{ness} normalised over the hard scale M which is the transverse mass of the dilepton system
- **linear** power behaviour in all channels
- control of the NLO correction ($r_{cut} \to 0$) at the **percent level**



Colour-correlated amplitudes from OPENLOOPS.
The reference NLO result is obtained with POWHEG
(FKS local subtraction).

Phenomenological applications

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$D = 0.1$

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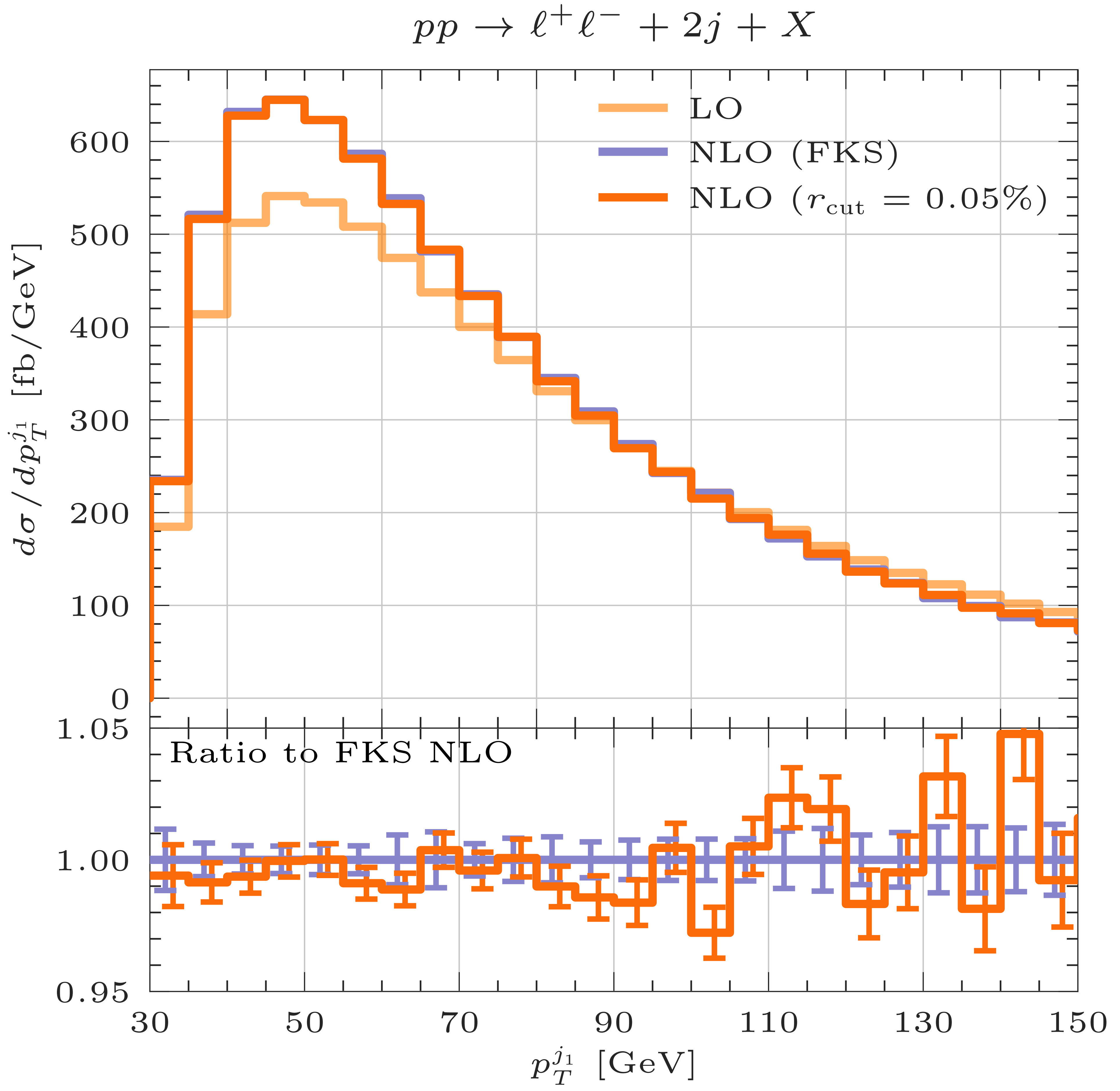
- comparison between the results obtained with 2-k_T^{ness} and FKS local subtraction, at the level of the **differential distributions**
- the cut is applied on k_T^{ness} normalised over the hard scale M which is the transverse mass of the dilepton system (fixed $r_{cut} = 0.05\,\%$)
- **nice agreement** in all range within uncertainties



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Phenomenological applications

Dijet production @NLO

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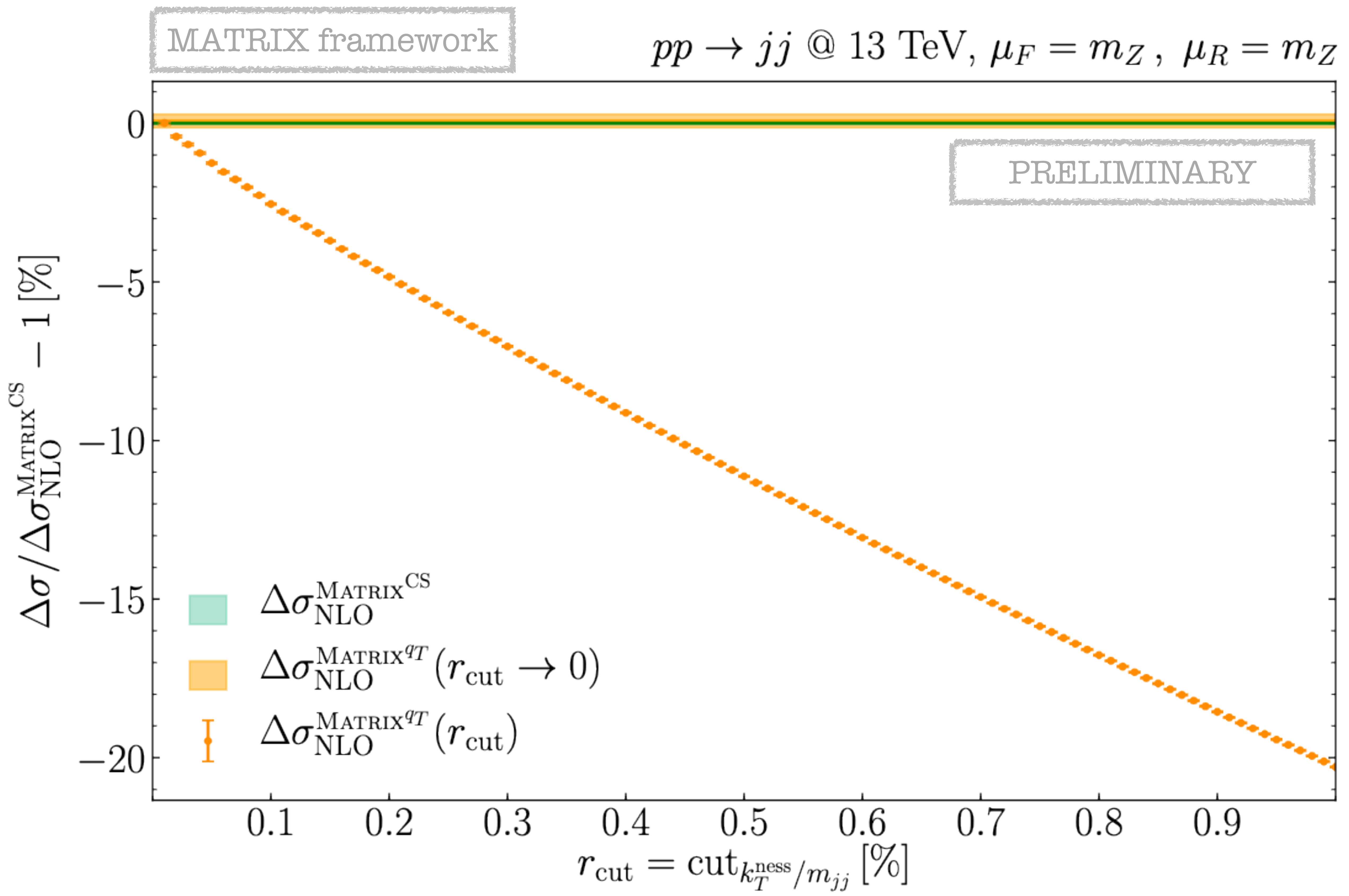
$\mu_R = \mu_F = m_Z$



- comparison between the results obtained with 2-k_T^{ness} and CS local subtraction, at the level of the **total cross section**
- the cut is applied on k_T^{ness} normalised over the hard scale M which is the invariant mass of the dijet system
- **linear** power behaviour

Phenomenological applications

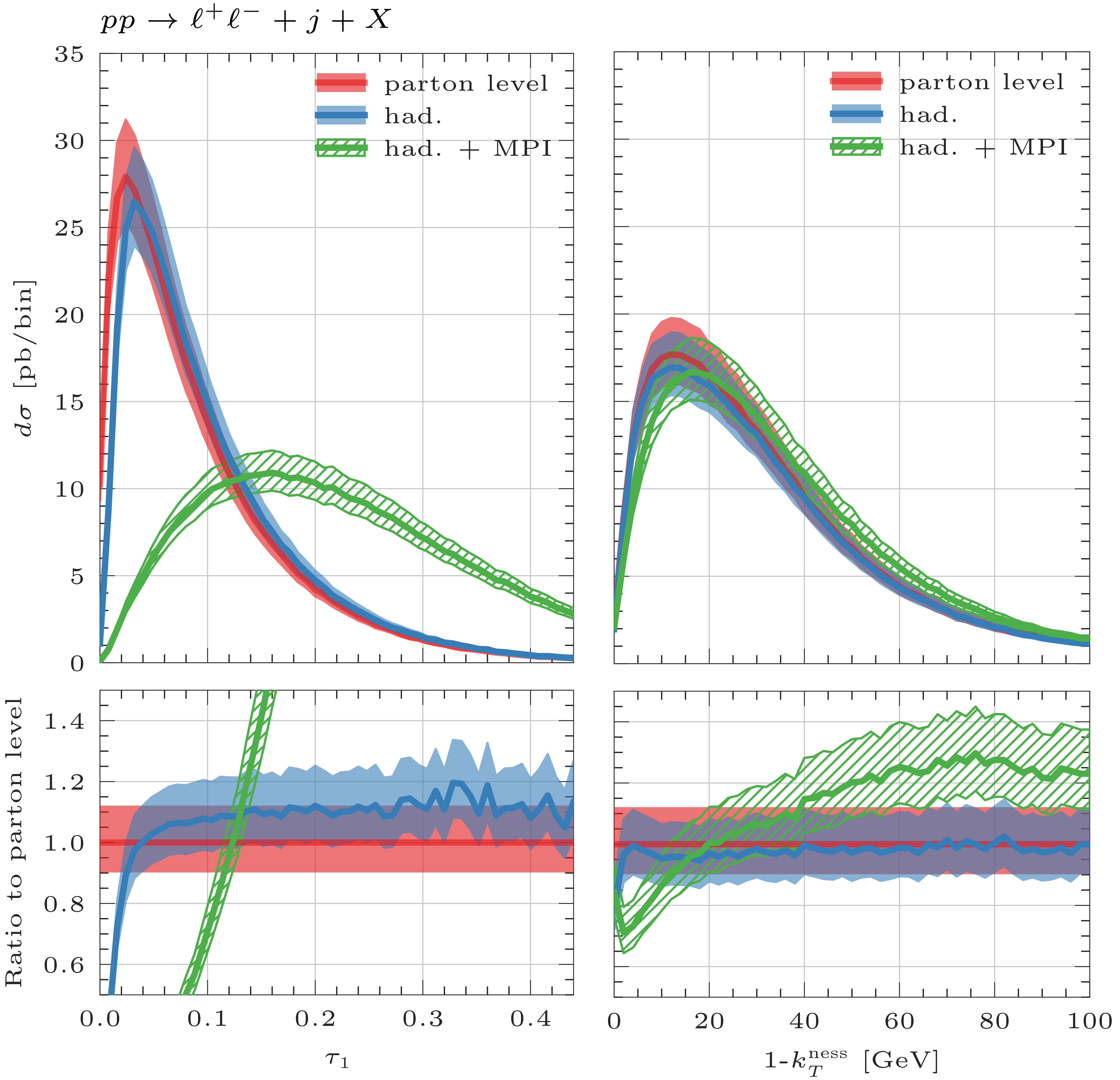
Z + 1jet production: hadronisation and MPI effects

$p_T^j > 30$ GeV

$|y^{j_1}| < 2.5$

$D = 1$

$\mu_R = \mu_F = m_Z$



- comparison between the results obtained with 1-k_T^{ness} and 1-jettiness (treated as **event shape variables**)

$$\tau_1 = \mathcal{T}_1 / Q \qquad \mathcal{T}_1 = \sum_i \min_l \left\{ \frac{2 q_l \cdot p_i}{Q} \right\}$$

- 1-k_T^{ness} shows a peak at $\sim$ 15 GeV which remains stable under hadronisation and MPI
- 1-k_T^{ness} is overall much **more stable** than 1-jettiness

LO events obtained with POWHEG and showered with PYTHIA8 (A14 tune).
FASTJET was used to reconstruct the leading jet and to define k_T^{ness}.

Conclusions

- we introduced a new variable k_T^{ness} able to capture the $N \to N+1$ jet transition
- we computed all relevant ingredients necessary for the construction of a subtraction formula @NLO
- we showed that k_T^{ness} has promising properties: pure linear power corrections, stability under NP effects

- future goals:
  - extension of the subtraction method @NNLO [ongoing work on Z+1jet]
  - inclusion of processes with heavy quarks plus jets
  - resummation (factorisation in b-space ?)
  - implementation in MATRIX framework

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THANKS !

BACKUP SLIDES

k_T^{ness}-subtraction

finite piece :

- the **master formula** for NLO subtraction is

$$d\hat{\sigma}_{NLO}^{F+Njets+X} = \mathcal{H}_{NLO}^{F+Njets} \otimes d\hat{\sigma}_{LO}^{F+Njets} + [d\hat{\sigma}_{LO}^{F+(N+1)jets} - d\hat{\sigma}_{NLO}^{CT,F+Njets}]_{r>r_{cut}}$$

where the **finite piece** is

$$\mathcal{H}_{cd;ab}^{F+Njets} = (\mathbf{HS})_{cd} C_{ca} C_{db} \prod_{i=1}^{N} J_i$$

with soft factor

$$(\mathbf{HS})_{cd} = \frac{\langle \mathcal{M}_{cd} | \, \mathbf{S} \, | \mathcal{M}_{cd} \rangle}{|\mathcal{M}_{cd}^{(0)}|^2} \qquad \mathbf{S} = 1 + \frac{\alpha_S(\mu_R)}{\pi} \mathbf{S}^{(1)} + \mathcal{O}(\alpha_S^2)$$

- the soft factor $\mathbf{S}^{(1)}$ is an operator in colour space and it is related to the integral, over the radiation phase space, of the soft-subtracted current

$$\mathbf{J}_{\rm sub}^2 = \left(-\mathbf{T}_c \cdot \mathbf{T}_d \, \omega_{cd} - \sum_i (\mathbf{T}_c \cdot \mathbf{T}_i \, \omega_{ci} + (c \leftrightarrow d)) - \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j \, \omega_{ij} \right) \Theta(r_{\rm cut} - k_{T,S}^{\rm ness}/Q)$$
$$- \left(\mathbf{T}_c^2 \, \omega_d^c + (c \leftrightarrow d) \right) \Theta(r_{\rm cut} - k_t/Q) - \sum_i \mathbf{T}_i^2 \, \omega_{CS,i} \Theta(r_{\rm cut} - k_{T,CS_i}^{\rm ness}/Q)$$