Karlsruher Institut für Technologie

Mixed QCD×Electroweak corrections to the Drell-Yan process in the high invariant mass region

Chiara Signorile-Signorile

Loops and Legs in Quantum Field Theory

Ettal, 29/04/2022

In collaboration with: F. Buccioni, F. Caola, H. Chawdhry, F. Devoto, M. Heller, A. von Manteuffel, K. Melnikov, R. Röntsch

Based on: arXiv 2203.11237

Outlook:

Why

- are we interested in higher-order corrections?
- are we focused on Drell-Yan?
- do we compute mixed QCD×EW corrections?
- is the computation complicated?

How

- can we overcome the intrinsic difficulties of the computation?
- can we exploit what was already known?

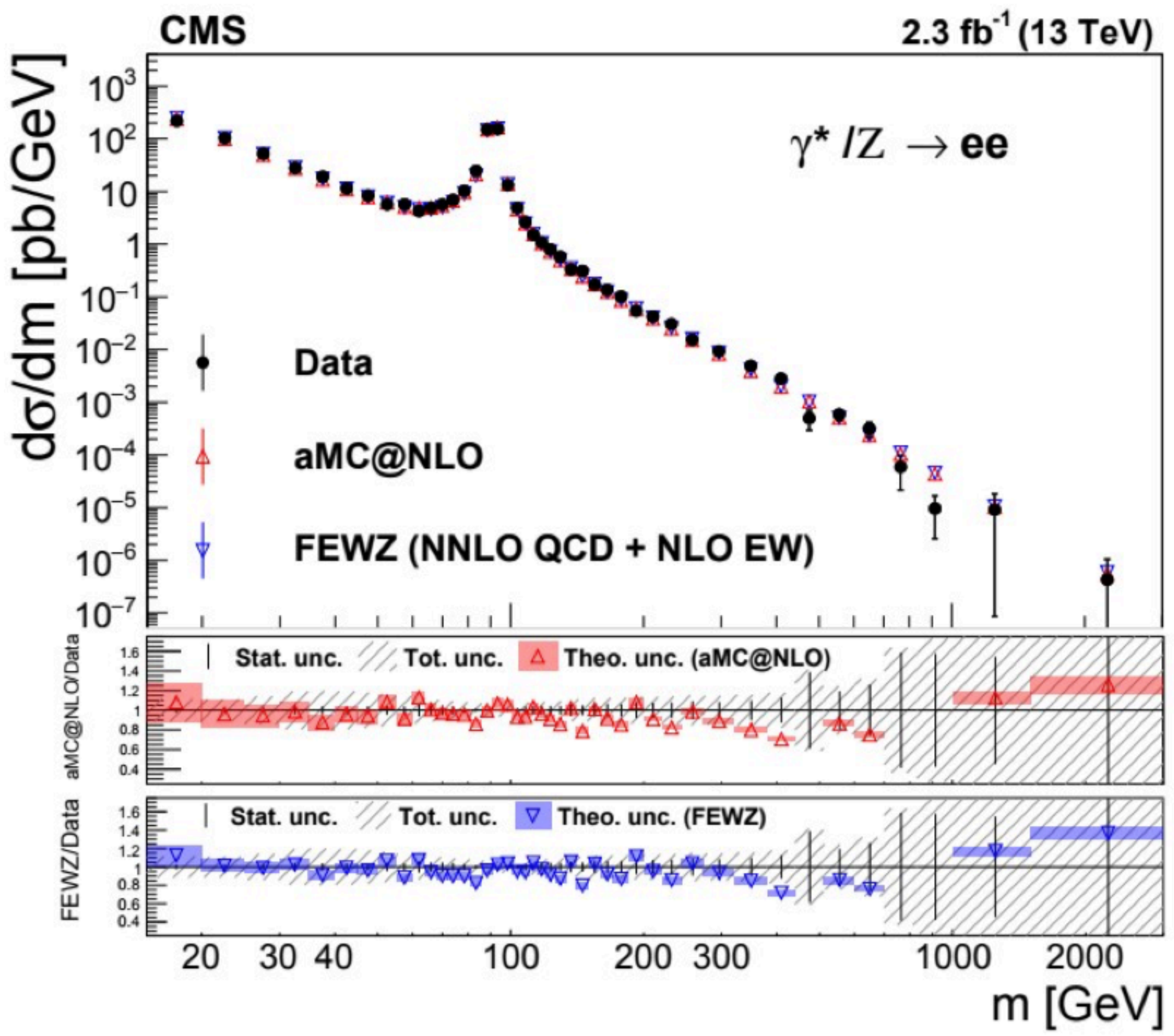
What

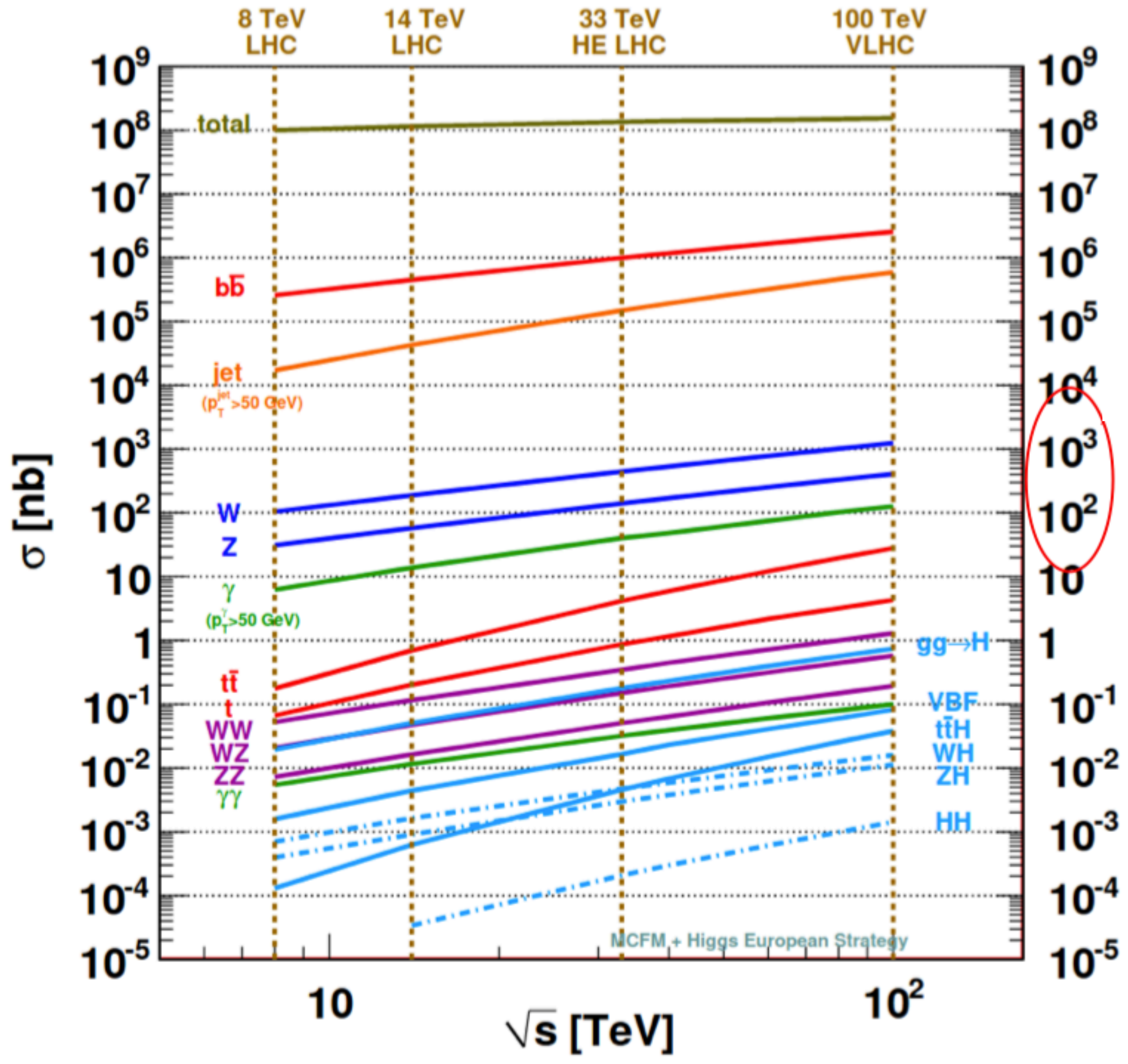
- can we learn from the results?

Motivation: why Drell-Yan?

Drell-Yan process is one of the **“standard candles”** at the LHC:

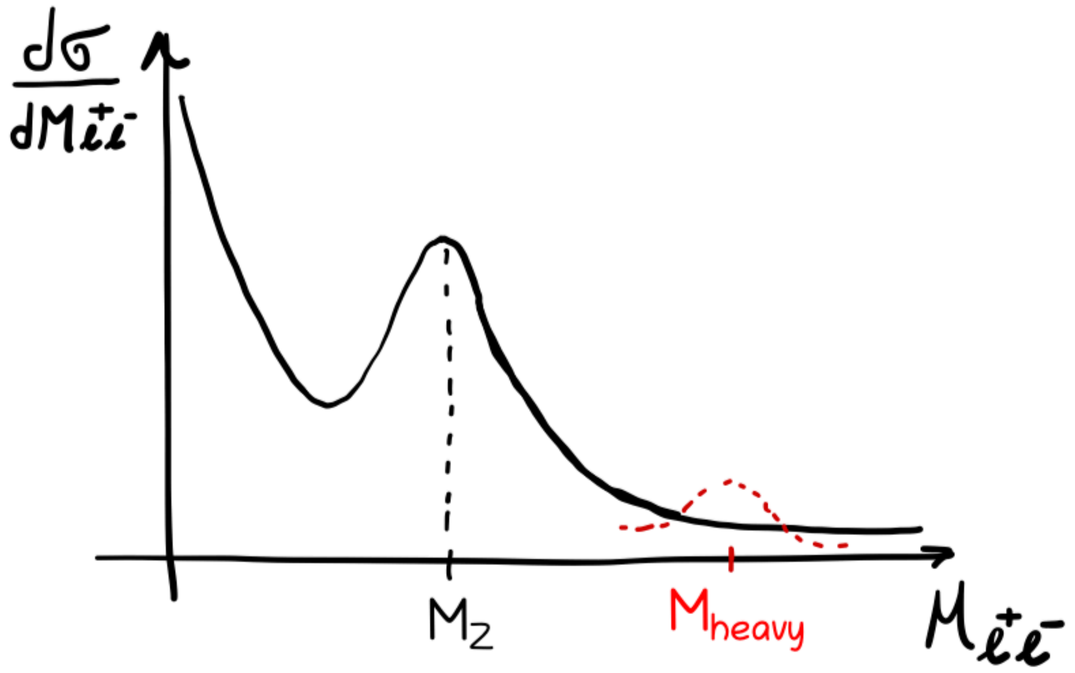
- **Clear experimental signature**, extremely well-controlled:
  - $Z \to \ell^+\ell^-$: detection of a pair of charged leptons
  - $W \to \ell\nu$: one charged lepton + missing $p_\perp$
- **Detector calibration** (e.g. detector response to lepton energy in NCDY events)
- **Large production rate:**
  - relevant background for New Physics searches also at future colliders
  - accurate theoretical predictions are necessary
- **Information about PDFs**
  - separation of valence quarks through W charge asymmetry
- Precise **determination of the SM parameters**
  - CCDY at the W resonance: determination of the W-boson mass m_W



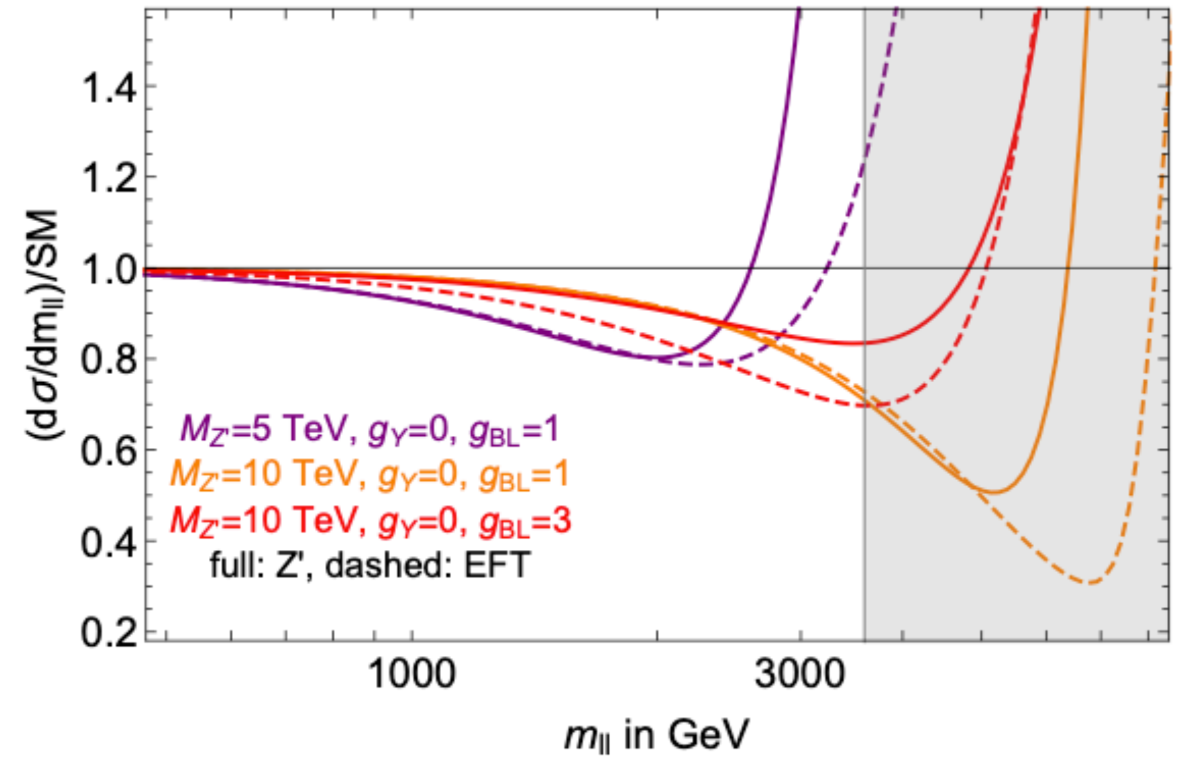


Motivation: why Drell-Yan in the high invariant mass region?

- **Hunting for New Physics** (NP)
  - Many extensions to the SM contain weakly-coupled states which can decay into leptons
    - Search for shape distortions in kinematic distributions



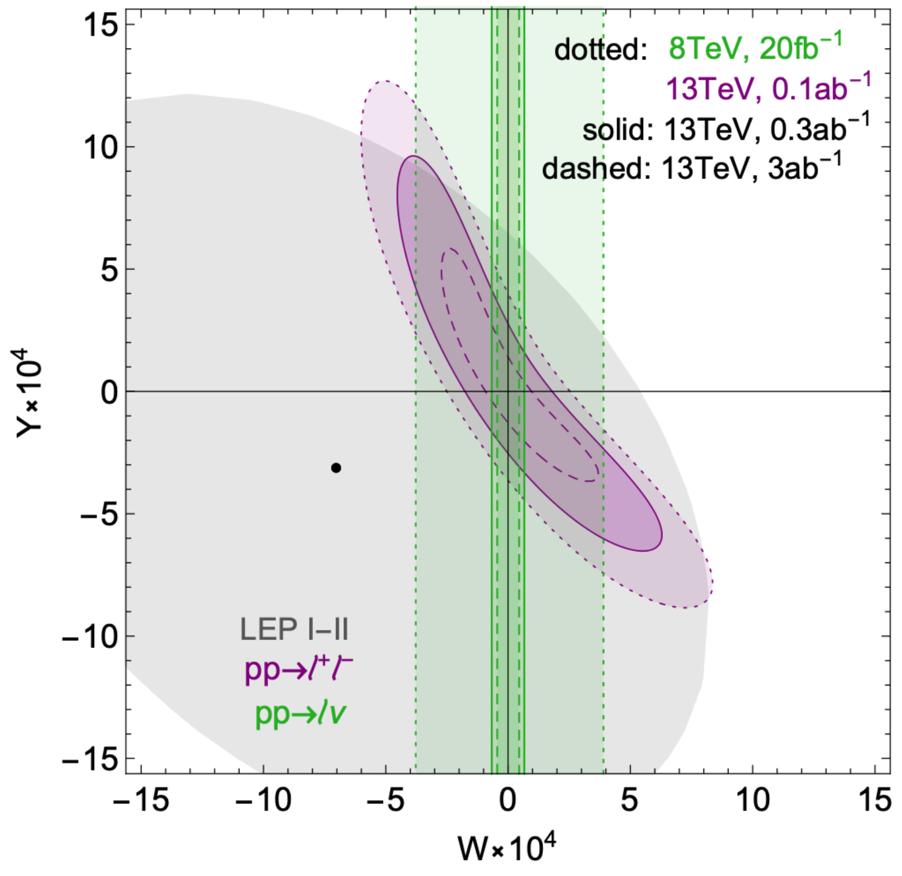
  - Constrain heavy NP in a model-independent way using **SMEFT**
    [Barbieri, Pomarol, Rattazzi, Strumia ’04] [Alioli, Farina, Pappadopulo, Ruderman ’17] [Farina et al.’17]
    - Impact on *oblique parameters* [Peskin, Takeuchi ’90], which are constrained at ***permille*** level with **LEP**
    - Contribution of **dimension-6 operators** $\rightarrow$ *quadratic growth with energy*



LHC expected to reach only ***percent****-level precision*

BUT

Higher energy can compensate for the limited precision
$\rightarrow$ enhancement factor $\sim$ 150 for $\sqrt{s} \simeq 1\text{TeV}$



Motivation: why mixed QCD×EW corrections?

• To accomplish this research program, **high-precision theoretical predictions within the SM are needed.**

$$d\sigma = \sum_{i,j} \int dx_1\, dx_2\, f_i(x_1, \mu_F) f_j(x_2, \mu_F)\, d\hat{\sigma}_{ij}\big(x_1, x_2, \mu_F, \mu_R\big)$$

Parton distributions: $f_{i,j}$

Ren./fact. scale: μ

Partonic cross section: $d\hat{\sigma}_{ij}$

Momentum fractions: x_1, x_2

Focus on fixed-order predictions

$$d\hat{\sigma}_{ij} = d\hat{\sigma}_{ij,\,\mathrm{LO}}\big(1 + \alpha_s\, \Delta^{QCD}_{ij,\,\mathrm{NLO}} + \alpha_{ew}\, \Delta^{EW}_{ij,\,\mathrm{NLO}} + \alpha_s^2\, \Delta^{QCD}_{ij,\,\mathrm{NNLO}} + \alpha_s\, \alpha_{ew}\, \Delta^{QCD\otimes EW}_{ij,\,\mathrm{NNLO}} + \alpha_s^3\, \Delta^{QCD}_{ij,\,\mathrm{N3LO}} + \ldots\big)$$

Couplings: $\alpha_s \sim 0.1\,,\ \alpha_{ew} \sim 0.01$

Target precision: $\sim$ few %

QCD corrections have to be accounted for at least up to **NNLO**

$\sigma^{\mathrm{nlo}}_{\mathrm{QCD}}$
[Altarelli, Ellis, Martinelli '79]

$\sigma^{\mathrm{nnlo}}_{\mathrm{QCD}}$
[Hamberg, van Neerven, Matsuura '91]
[Harlander, Kilgore '02]

$d\sigma^{\mathrm{nnlo}}_{\mathrm{QCD}}$ (fully diff.+leptonic decay)
[Anastasiou, Dixon, Melnikov, Petriello '03, '04]
[Melnikov, Petriello '06]
[Catani, Cieri, Ferrera, de Florian, Grazzini '09]
[Catani, Ferrera, Grazzini '10]

$\sigma^{\mathrm{n3lo}}_{\mathrm{QCD}}$
[Duhr, Dulat, Mistlberger '20]

$d\sigma^{\mathrm{n3lo}}_{\mathrm{QCD}}$
[Camarda, Cieri, Ferrera '21]
[Chen, Gehrmann, Glover, Huss, Yang '21]
[Chen, Gehrmann, Glover et al. '22]

See Xuan Chen's talk

Motivation: why mixed QCD×EW corrections?

$$d\sigma_{ij} = d\sigma_{ij,\mathrm{LO}}\left(1 + \alpha_s\,\Delta^{QCD}_{ij,\mathrm{NLO}} + \alpha_{ew}\,\Delta^{EW}_{ij,\mathrm{NLO}} + \alpha_s^2\,\Delta^{QCD}_{ij,\mathrm{NNLO}} + \alpha_s\,\alpha_{ew}\,\Delta^{QCD\otimes EW}_{ij,\mathrm{NNLO}} + \alpha_s^3\,\Delta^{QCD}_{ij,\mathrm{N3LO}} + \ldots\right)$$

Couplings: $\alpha_s \sim 0.1$, $\alpha_{ew} \sim 0.01$

Target precision: $\sim$ few %

NLO EW corrections: $\sim \mathcal{O}(\alpha_{ew}) \sim 1\,\%$

Mixed QCD×EW corrections: $\sim \mathcal{O}(\alpha_s\,\alpha_{ew}) \sim 0.1\,\%$

At high energy, this naive power counting is spoiled by **Sudakov logarithms**

In **EW theory** $m_{W/Z}$ provides a **physical cut-off**, real Z, W can be detected as distinguishable particles.

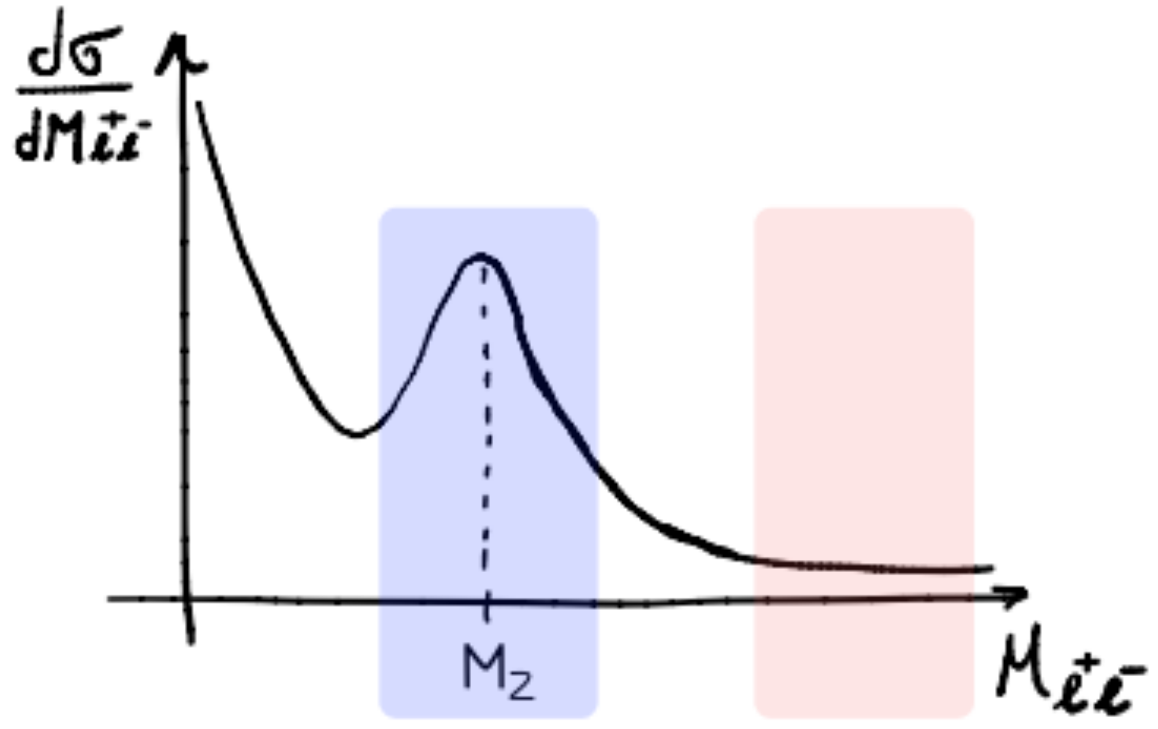
Large logs from virtual corrections are of physical significance *[Kuhn, Penin, Smirnov '00][Ciafaloni, Ciafaloni, Comelli '01] [Denner, Pozzorini '01]*

$$\frac{\alpha_{ew}}{4\pi\sin^2\theta_W}\log^2\left(\frac{s}{m_W^2}\right) \sim 6\,\%\ , \quad \frac{\alpha_{ew}}{4\pi\sin^2\theta_W}\log\left(\frac{s}{m_W^2}\right) \sim 1\,\%\ , \quad \sqrt{s} = 1\mathrm{TeV}$$

Off-shell and on-shell mixed QCD×EW corrections

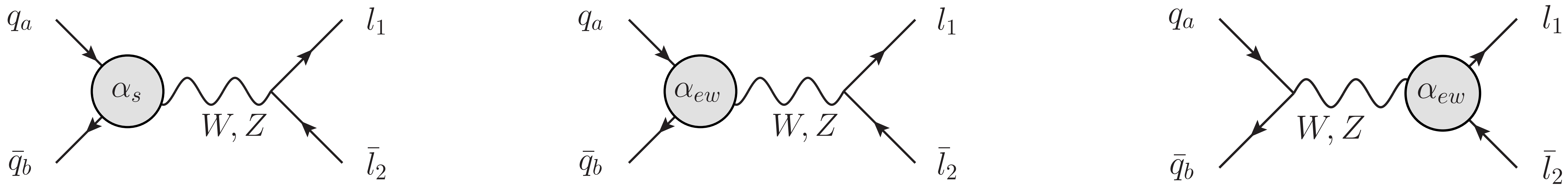
To compute mixed QCD-EW corrections in the high invariant mass region we can take advantage of the results known in the resonance region *[Delto, Jaquier, Melnikov, Röntsch '20] [Buccioni et al. '20] [Bonciani, Buccioni, Rana, Vicini '20] [Behring et al. '21] [Bonciani, Buccioni, Rana, Vicini '21]*

Resonance region: Z, W production
- Precise EW parameters determination

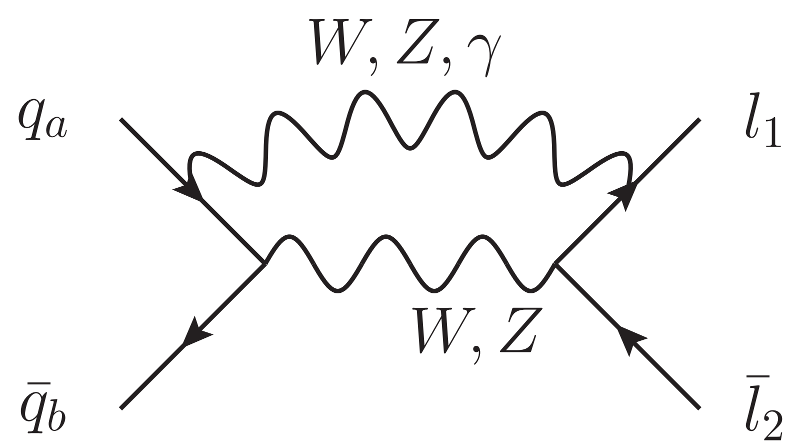


Off-shell region: l^+l^- production
- New Physics searches

Similarities:



Crucial difference:



In the resonance region *[Dittmaier, Huss, Schwinn '14]*:

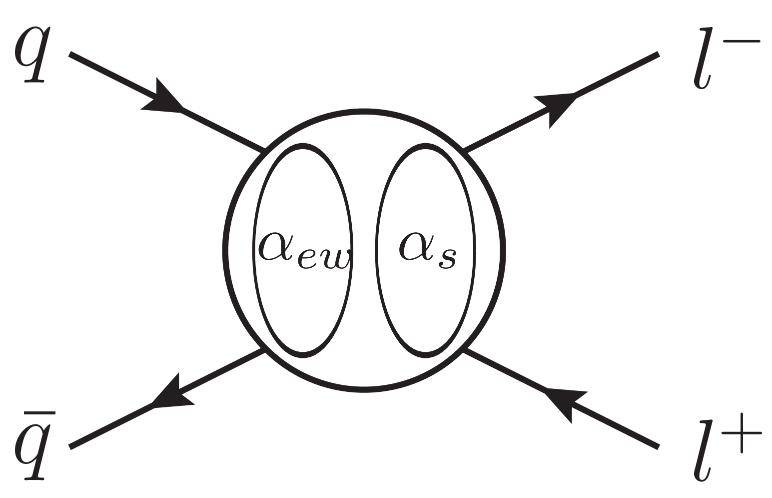
$$\mathcal{O}\left(\alpha_{ew}\frac{\Gamma_V}{M_V}\right) \sim \mathcal{O}(\alpha_{ew}^2) \longrightarrow$$

Suppressed way below percent level

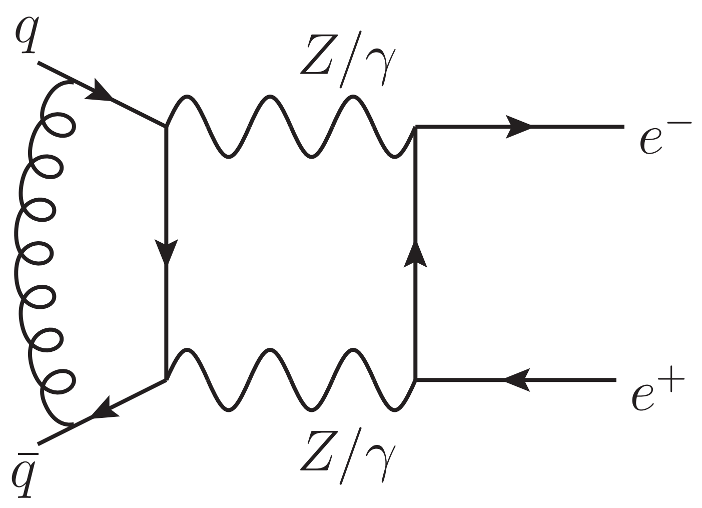
Phenomenologically negligible

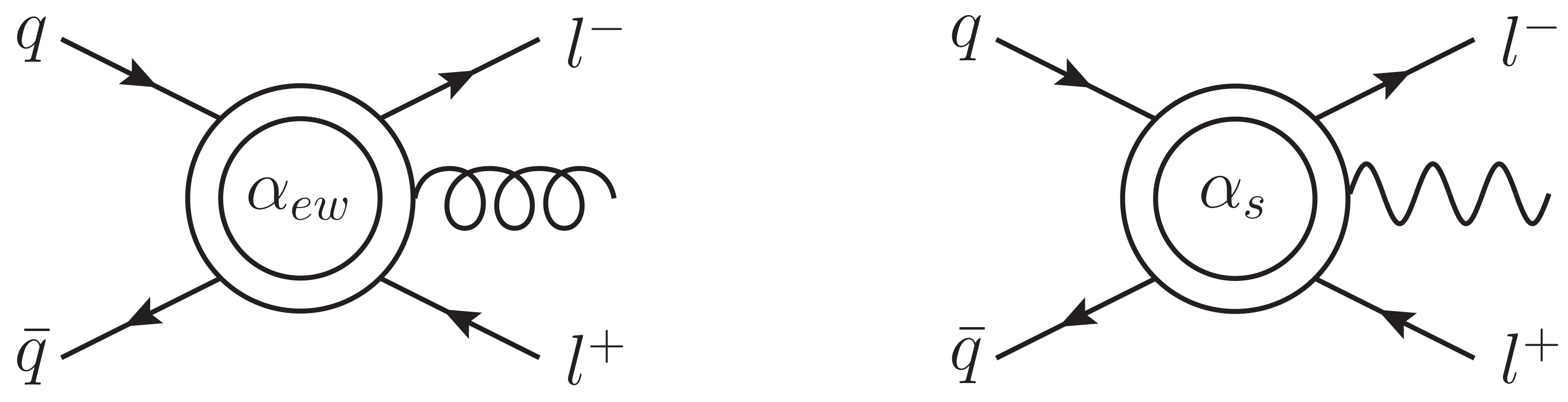
Ingredients for off-shell calculation at NNLO

Fully differential description of mixed QCD-EW effects is a complicated problem

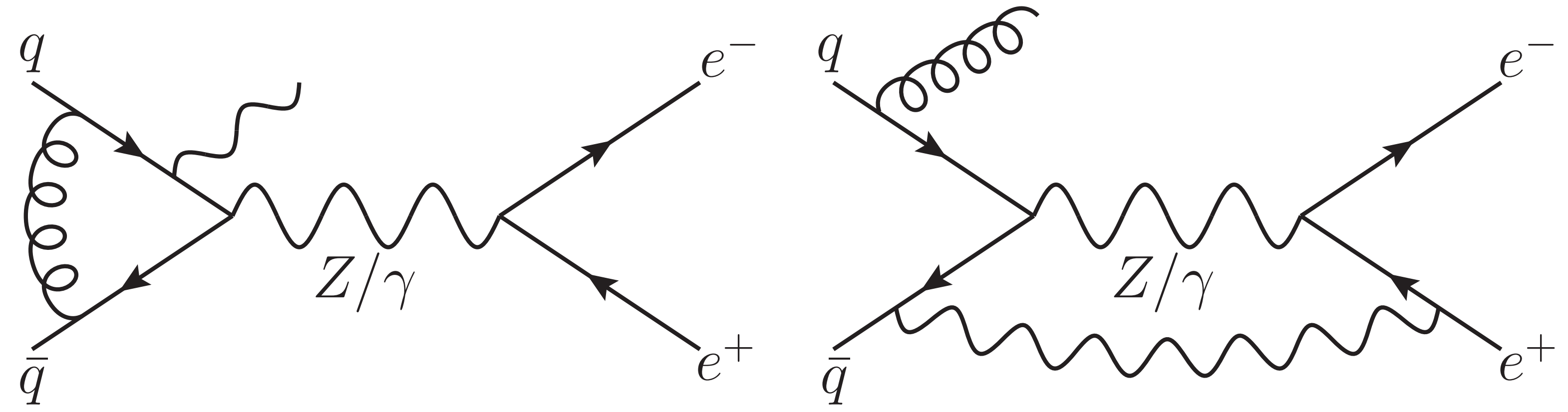


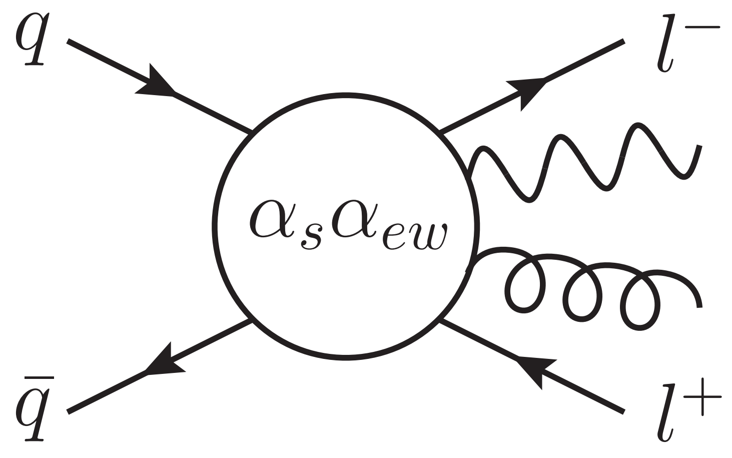
2-loop virtual + one-loop squared



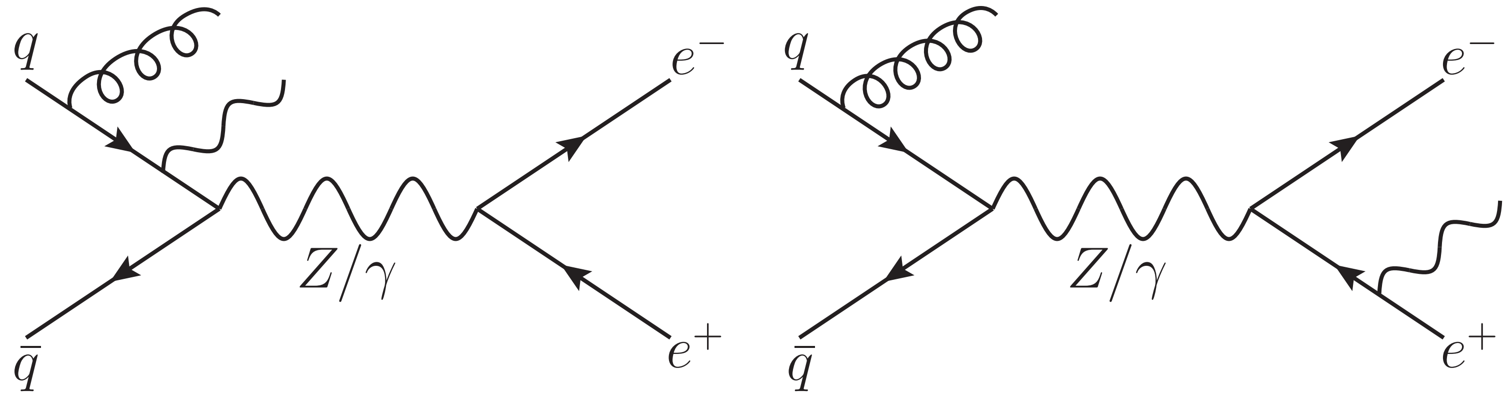


1-loop with one extra emission



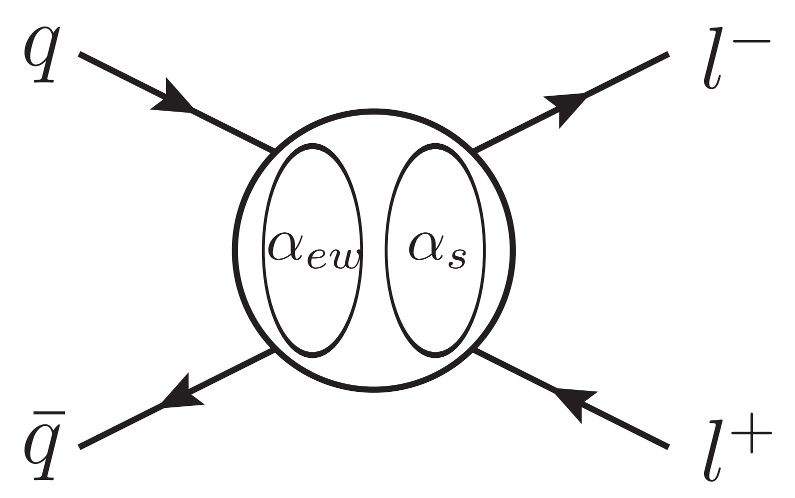


Tree-level double real



Ingredients for off-shell calculation at NNLO

Fully differential description of mixed QCD-EW effects is a complicated problem



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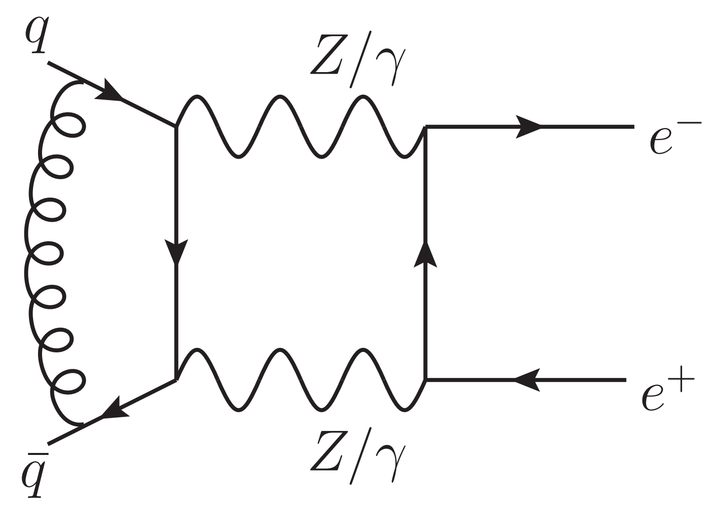
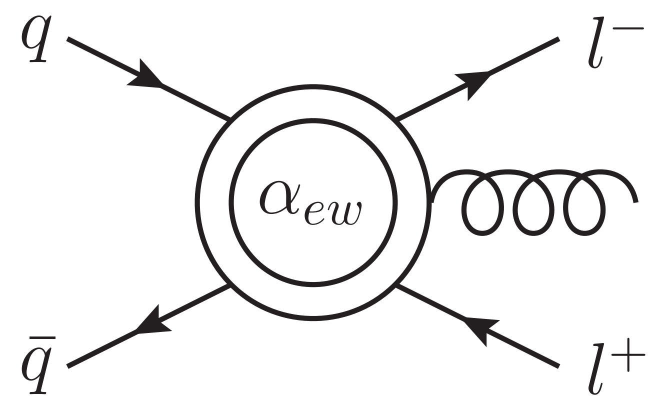
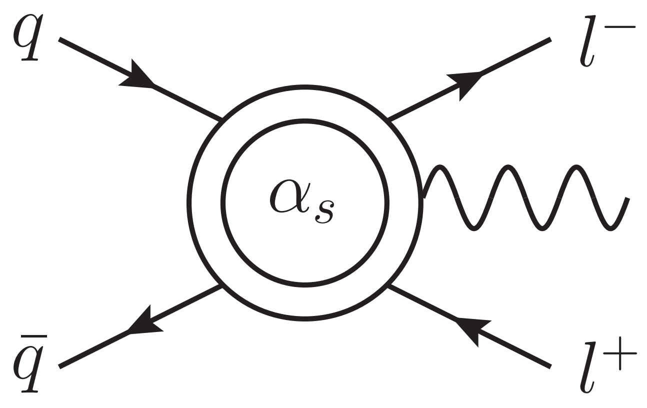


- **Several mass scales** involved
- **Fully analytic** calculation → generalised polylogarithms *[Heller, von Manteuffel, Schabinger '20] [Heller, von Manteuffel, Schabinge, Spiesberger '21]*
- Easy to evaluate and integrate
- **Factorisable** contributions **dominant at high energy** → **leading Sudakov logs**

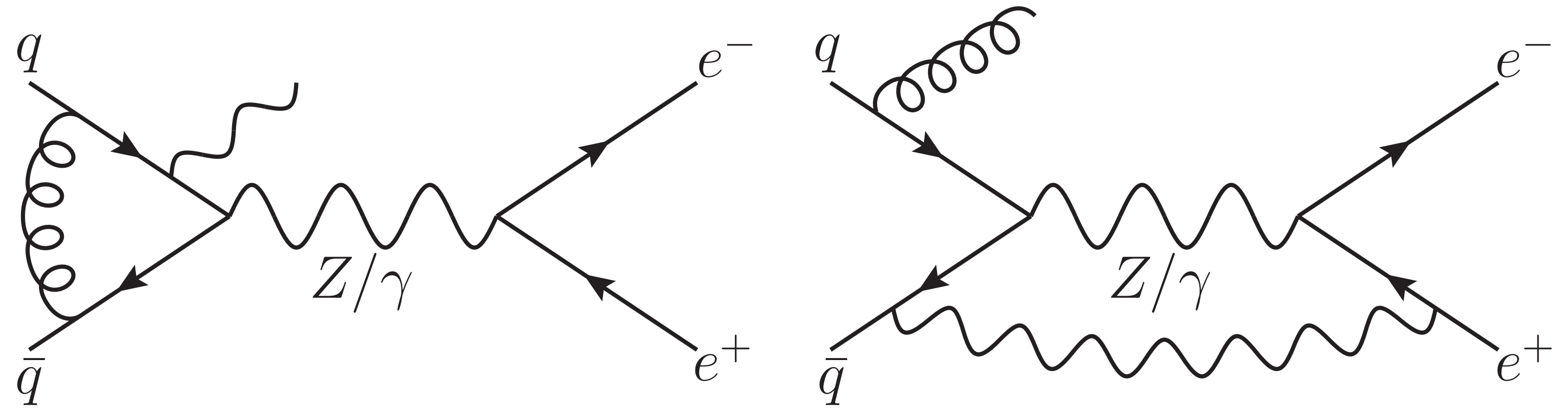
Ingredients for off-shell calculation at NNLO

Fully differential description of mixed QCD-EW effects is a complicated problem

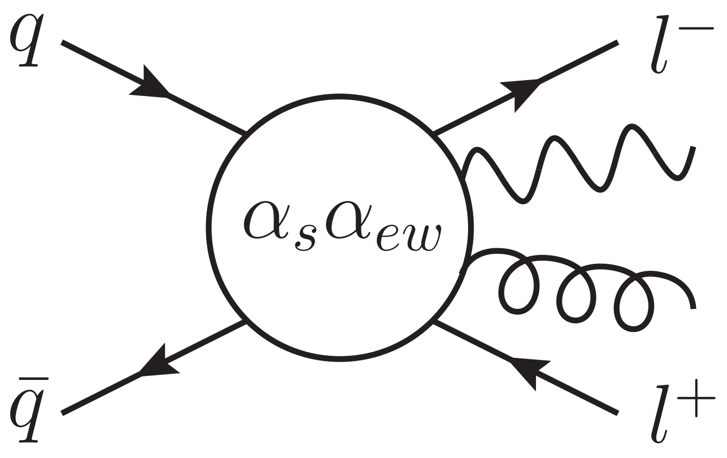




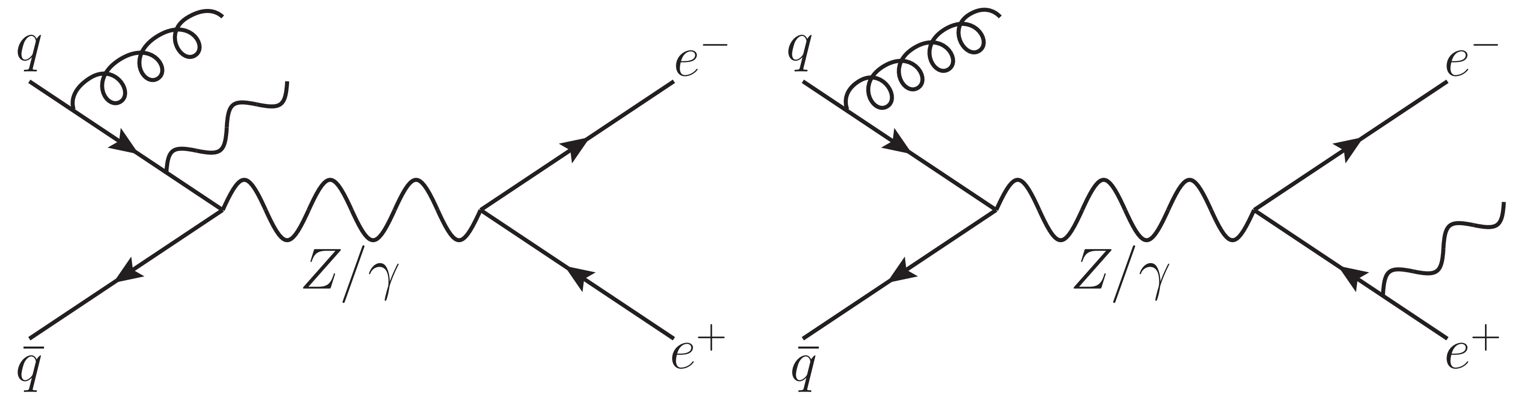
1-loop with one extra emission



- **One-loop amplitude in degenerate kinematics**
- **OpenLoops** for numerical evaluation *[Cascioli, Maierhöfer, Pozzorini '12] [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller '19]*



Tree-level double real



- Real emission amplitudes develop **soft and collinear singularities** and their contribution to differential cross sections **cannot be integrated directly**
- **Nested soft-collinear subtraction** scheme *[Caola, Melnikov, Röntsch 1702.01352]*

Ingredients for off-shell calculation at NNLO

Disclaimer

❖ First calculation of mixed corrections to charged and neutral current Drell-Yan, in the off-shell region, has been performed with a different approach:
[Buonocore, Grazzini, Kallweit, Savoini, Tramontano 2102.12539]
[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini 2106.11953]

- Massive leptons
- Semi-analytic amplitudes *[Armadillo, Bonciani, Devoto, Rana, Vicini 2201.01754]*
- qT slicing *[Catani, Grazzini 0703012] [Buonocore, Grazzini, Tramontano 1911.10166]*

See talks by:

➡ Stefan Kallweit [Tuesday]

➡ Luca Buonocore [Thursday]

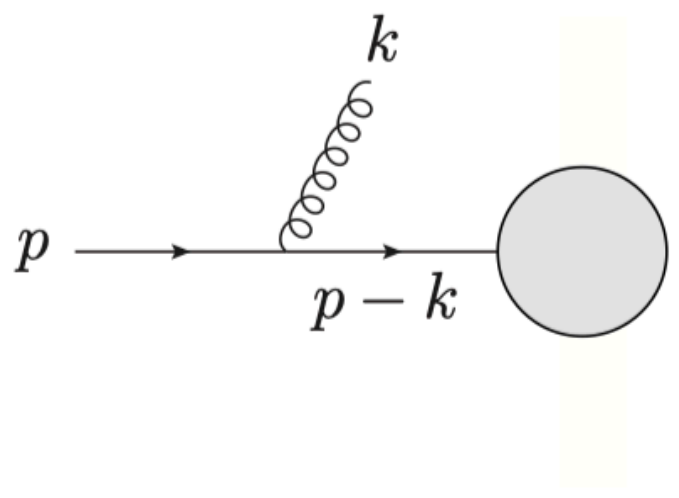
➡ Simone Devoto [Friday]

IR singularities

Higher order corrections contain **infrared singularities from soft and/or collinear radiation**.

- Virtual corrections:
  - Explicit IR singularities from loop integrations $\rightarrow$ poles in $1/\epsilon$

- Real corrections:
  - Singularities after integration over full phase space of radiated parton

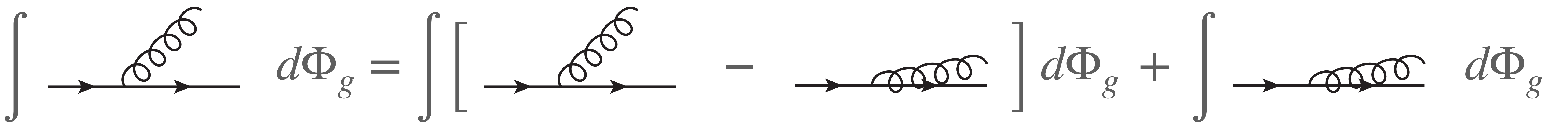


$$\sim \frac{1}{(p-k)^2} = \frac{1}{2E_p E_k (1-\cos\theta)} \xrightarrow[\substack{E_k \to 0 \\ \textbf{or} \\ \theta \to 0}]{} \infty .$$

$$\int \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1} 2E_k} |M(\{p\},k)|^2 \underset{\substack{E_k \to 0 \\ \theta \to 0}}{\sim} \int \frac{\mathrm{d}E_k}{E_k^{1+2\epsilon}} \frac{\mathrm{d}\theta}{\theta^{1+2\epsilon}} \times |M(\{p\})|^2 \sim \frac{1}{4\epsilon^2} .$$

- **Integrating implies losing kinematic information** (needed for distributions, kinematic cuts, …)

→ **Subtraction scheme**: extract singularities without integrating over full phase space of radiated partons

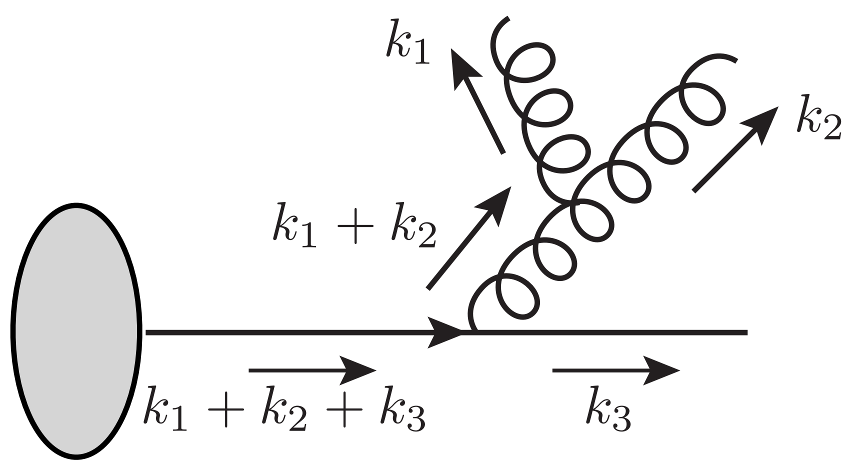


Finite in d=4, integrable numerically

exposes the same $1/\epsilon$ poles as the virtual correction

Nested soft-collinear subtraction: generalities

Extension of FKS subtraction to NNLO: **originally** introduced to **treat pure QCD processes** *[Caola, Melnikov, Röntsch 1702.01352]*



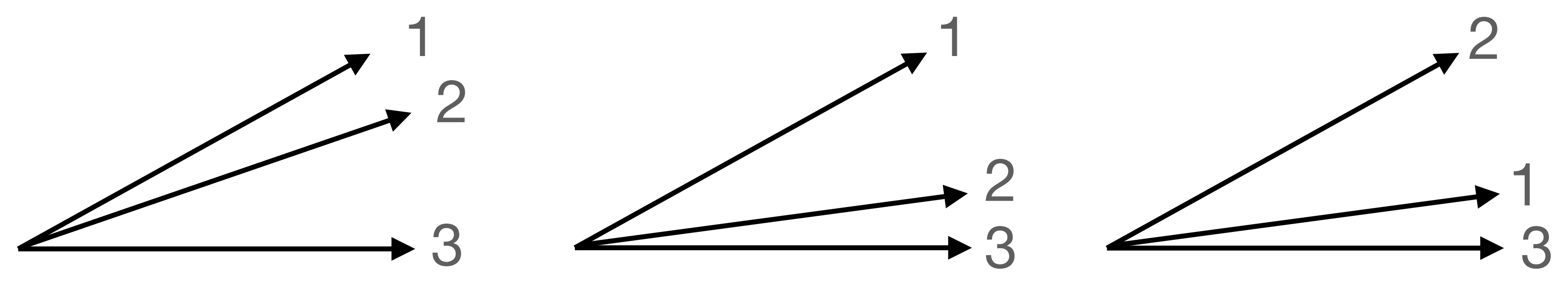
$$\sim \frac{1}{E_1 E_2 (1 - \vec{n}_1 \cdot \vec{n}_2)} \frac{1}{E_1 E_2 (1 - \vec{n}_1 \cdot \vec{n}_2) + E_1 E_3 (1 - \vec{n}_1 \cdot \vec{n}_3) + E_2 E_3 (1 - \vec{n}_2 \cdot \vec{n}_3)}$$

$E_1 \to 0 \quad E_2 \to 0 \quad E_1, E_2 \to 0$

$\vec{n}_1 \parallel \vec{n}_2 \parallel \vec{n}_3$

$\vec{n}_1 \parallel \vec{n}_2$

Strongly ordered configurations have also to be included: $E_1 \ll E_2\,, \quad E_2 \ll E_1$



$\vec{n}_1 \cdot \vec{n}_2 < \vec{n}_1 \cdot \vec{n}_3$ $\quad$ $\vec{n}_2 \cdot \vec{n}_3 < \vec{n}_1 \cdot \vec{n}_3$ $\quad$ $\vec{n}_1 \cdot \vec{n}_3 < \vec{n}_2 \cdot \vec{n}_3$

- Fully local and fully analytic

- Transparent treatment of IR singularities

  Independent subtraction of **soft and collinear divergences** > colour coherence

  Partition by means of sector functions

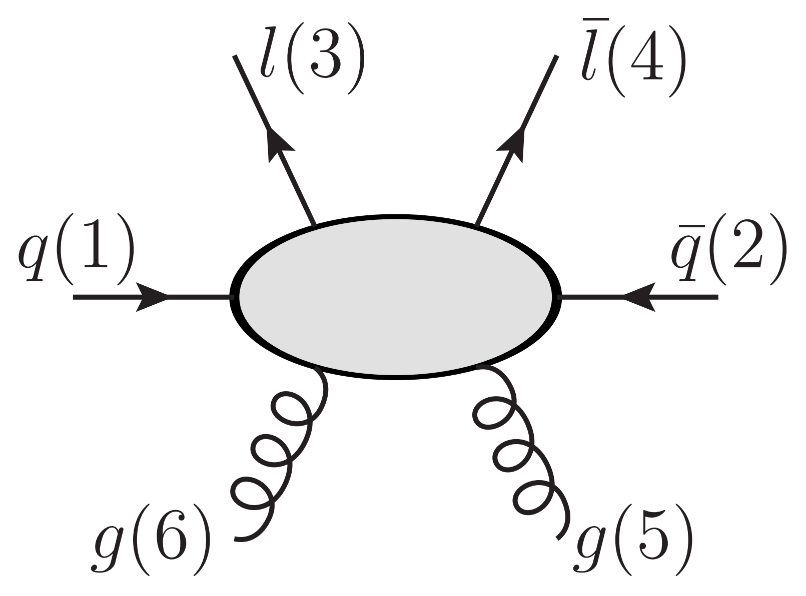
- Flexibility

  Core structure depends only on the partons contributing to the process

  Modular building blocks

NNLO QCD difficulties and solutions

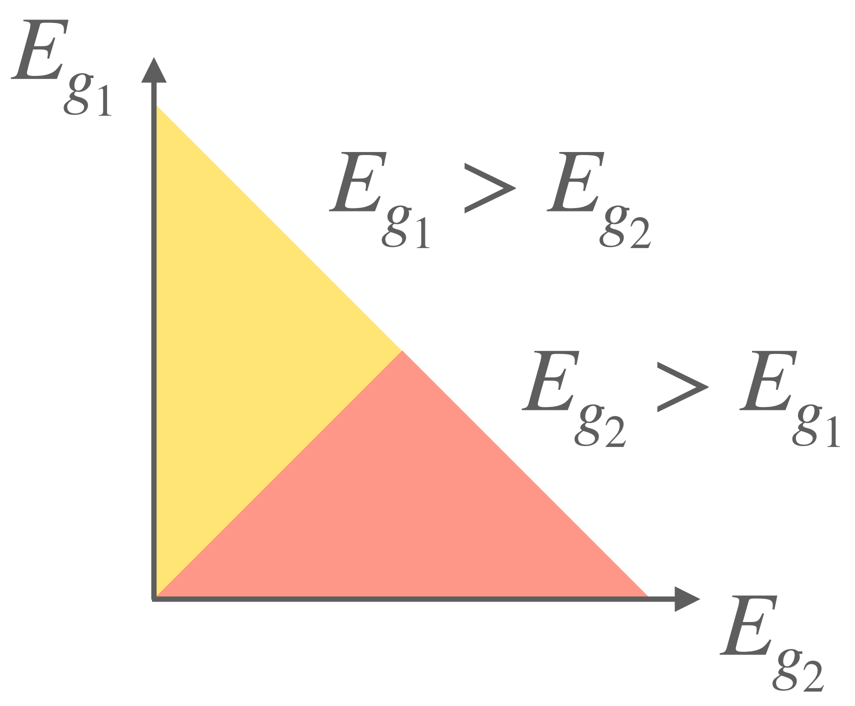
Examples: $q\bar{q} \to Z \to e^- e^+ g\, g$ [Caola, Melnikov, Röntsch 1702.01352]



Soft limits:

- Non-trivial structure of double soft eikonal
- Strongly-ordered limits to disentangle

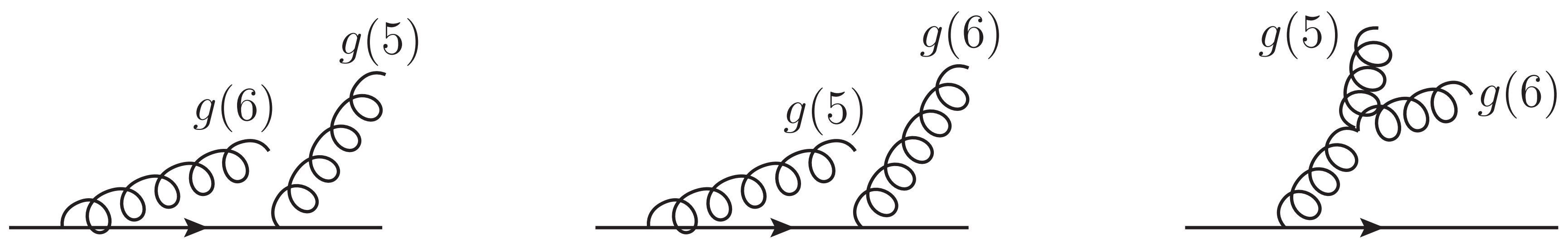
$$1 = \theta(E_{g_1} - E_{g_2}) + \theta(E_{g_2} - E_{g_1})$$



Collinear limits:

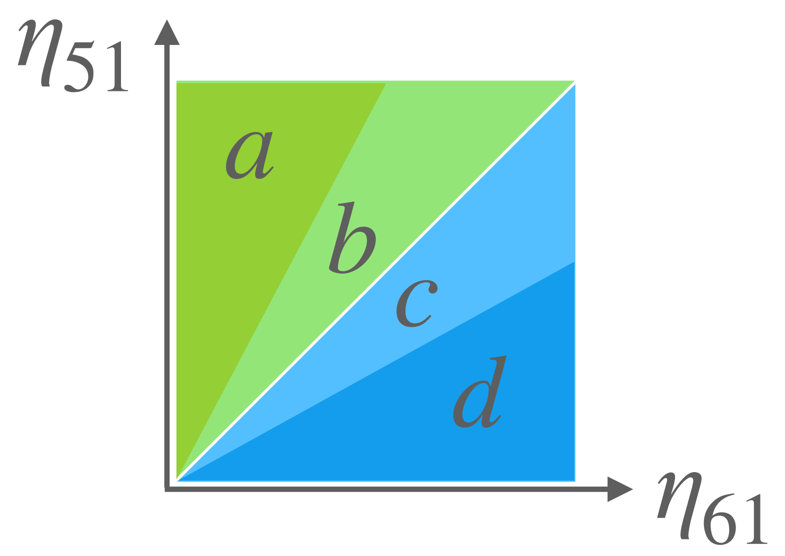
- Single, double and triple collinear limits to disentangle
- Strongly-ordered limits to disentangle in triple collinear sectors

$$1 = \sum_i \omega^i\,, \qquad i \in \{(51,61), (52,62), (51,62), (52,61)\}$$



$$\eta_{ab} = \frac{1 - \cos\vartheta_{ab}}{2}$$

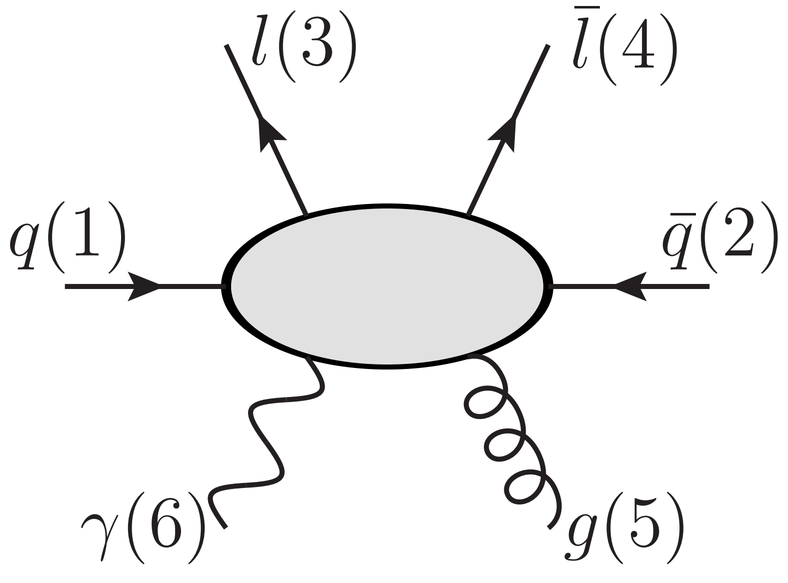
$$\omega^{51,61} = \omega^{51,61}\,(\theta_a + \theta_b + \theta_c + \theta_d)$$



➡ Non-trivial structures to integrate → Reverse unitarity

Mixed QCD-EW: differences with respect to NNLO QCD

Examples: $q\bar{q} \to Z \to e^- e^+ g\,\gamma$ *[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, CSS]*



Soft limits:

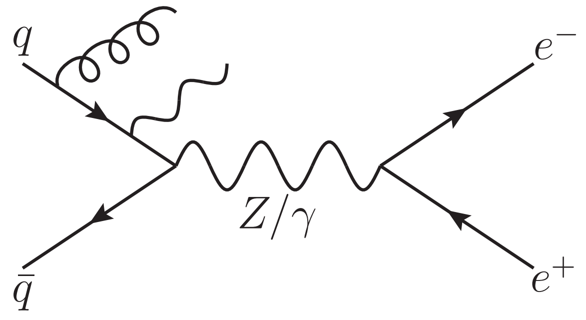
- Double soft limit factorises into NLO QCD x NLO QED → No need for energy ordering

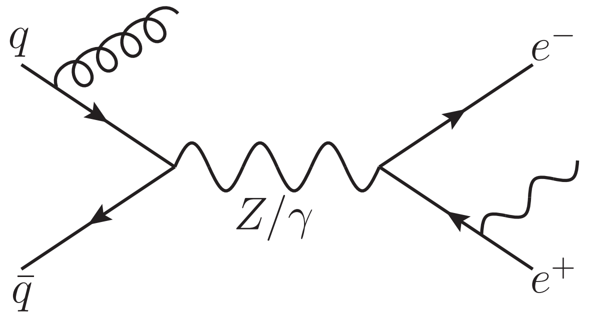
$$\lim_{E_g, E_\gamma \to 0} |\mathcal{M}_{RR}|^2 = g_s^2\, \mathrm{Eik}(p_1, p_2; p_5)\; e^2 \sum_{i,j} Q_i Q_j\, \mathrm{Eik}(p_i, p_j; p_6)\, |\mathcal{M}_B|^2$$

Collinear limits:

- Single, double and triple collinear limits to disentangle

  → More **sectors** to account for final state radiation





$$1 = \sum_{i=1}^{4} \sum_{j=1}^{2} \omega^{\gamma i, g j}$$

Mixed QCD-EW: differences with respect to NNLO QCD

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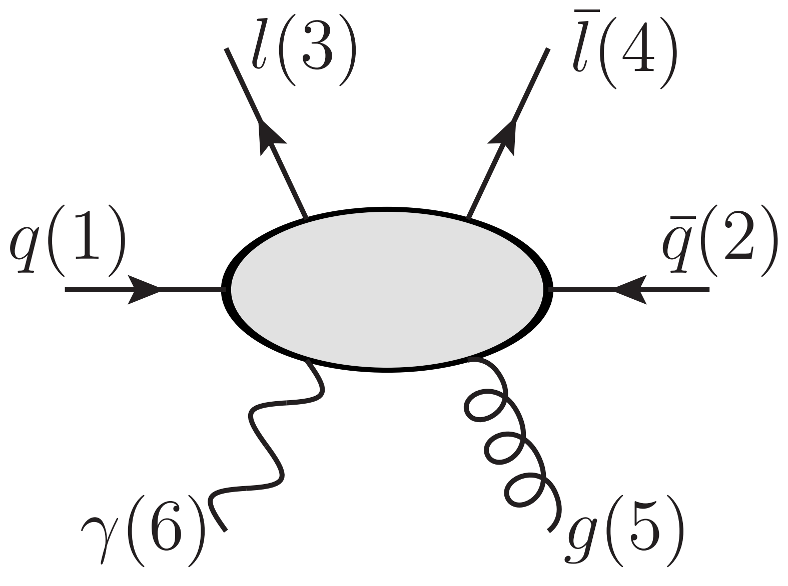

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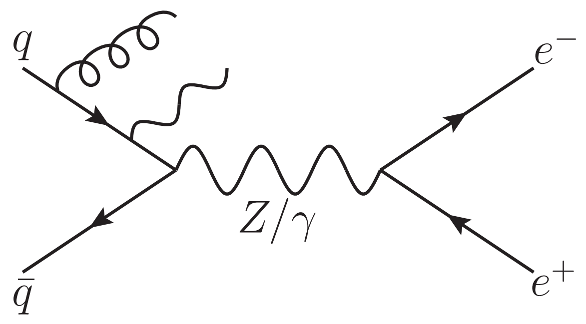
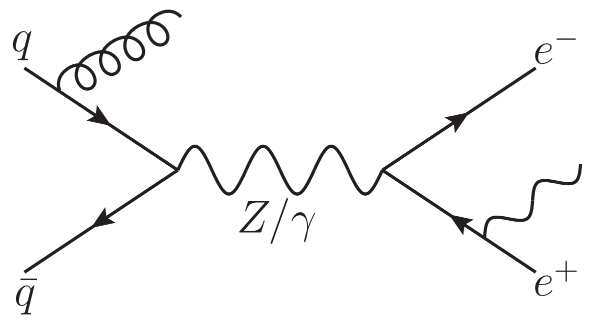
  → More **sectors** to account for final state radiation

- Strongly-ordered limits to disentangle in triple collinear sectors

  → BUT **no photon-gluon** collinear singularity





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Mixed QCD-EW: differences with respect to NNLO QCD

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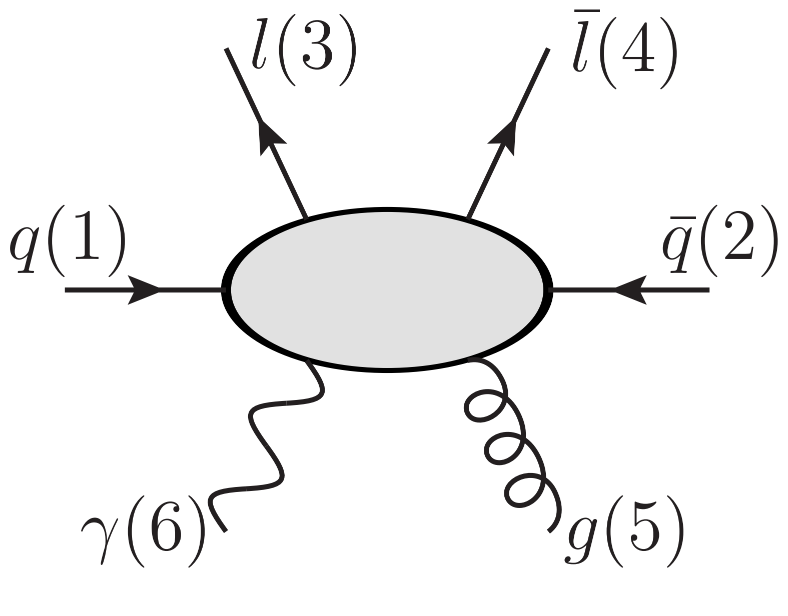

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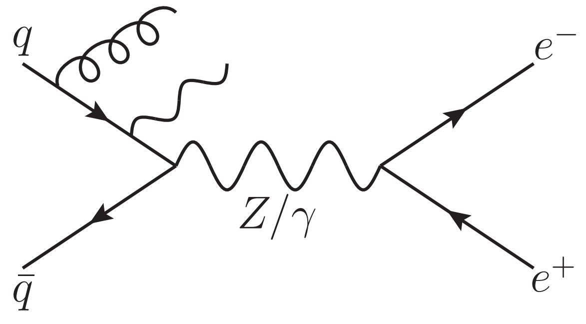
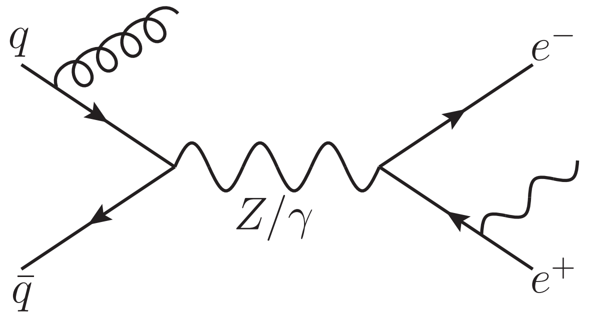
- Single, double and triple collinear limits to disentangle

  → More **sectors** to account for final state radiation

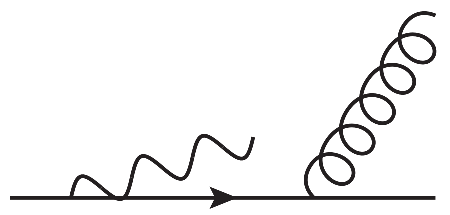


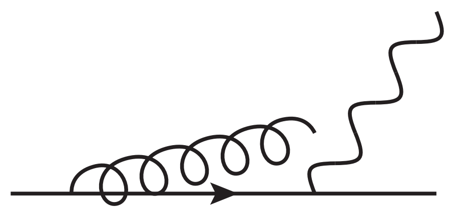


$$1 = \sum_{i=1}^{4} \sum_{j=1}^{2} \omega^{\gamma i, g j}$$

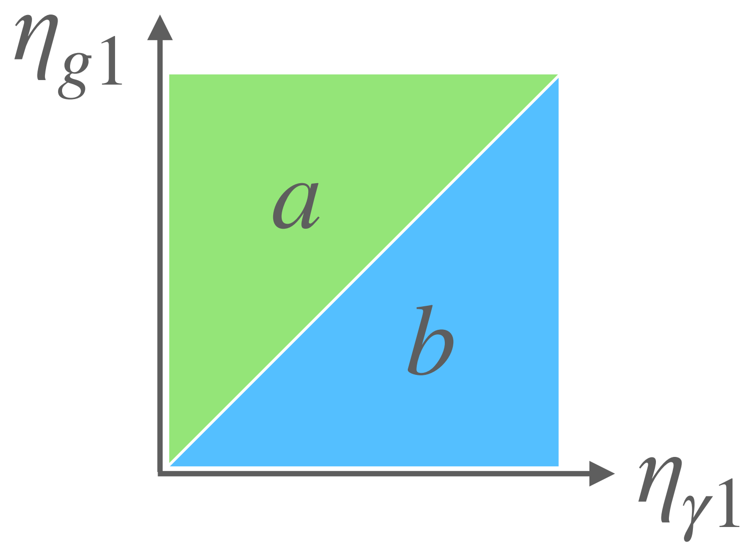
- Strongly-ordered limits to disentangle in triple collinear sectors

  → BUT **no photon-gluon** collinear singularity





$$\omega^{51,61} = \omega^{51,61}\left(\theta_a + \theta_b\right)$$



➡ Non-trivial structures to integrate

→ BUT **abelianization** of known results *[de Florian, Der, Fabre '18][Delto, Jaquier, Melnikov, Röntsch '19]*

Finite parts I

- Final result as combination of two- and one-loop corrections, double real radiation and pdf renormalisation:

✓ **Pole cancellation** proven analytically
✓ **Fully differential**
✓ **Fully analytic**
✓ **Fully local** > leads to a very stable numerical evaluation
✓ Cumbersome result for the finite parts
✓ In the **CoM** reference **frame**, $E_1 = E_2 = E_c$, **the result simplifies** remarkably
✓ Simple structures arise, **compact expressions**
✓ **Few main kinematic building blocks** can be easily identified

The considerations above hold for all the partonic channels, here we present explicit result for $q\bar{q} \to e^+e^-(g\gamma)$.
It is convenient to write the cross section in terms of contributions with different multiplicities and kinematic features

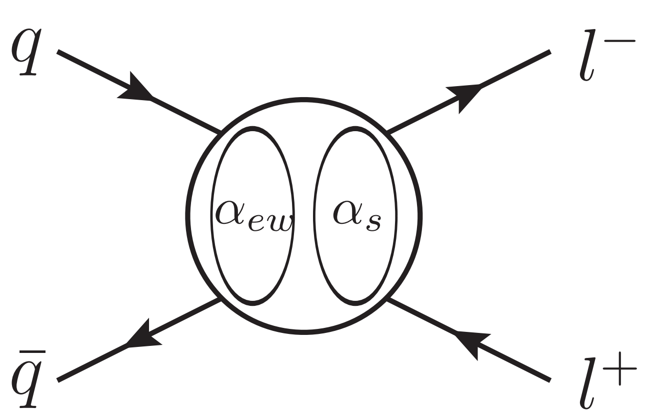
$$d\hat{\sigma}^{q\bar{q}}_{\text{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma}$$

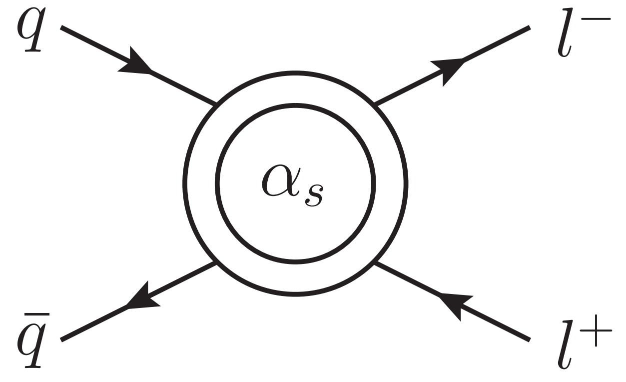
Finite parts III

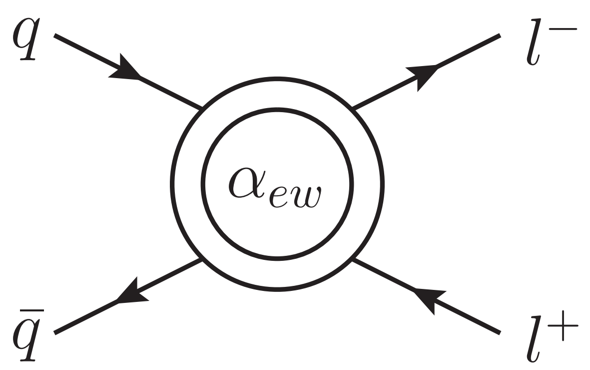
$$d\hat{\sigma}^{q\bar{q}}_{\mathrm{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\mathrm{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathrm{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\mathrm{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathrm{reg},g\gamma}$$

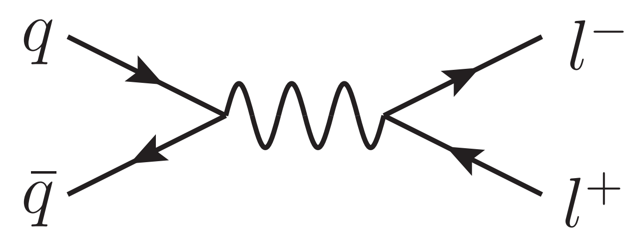
Elastic contributions arise from:

- double-unresolved real emissions of photons and gluons
- finite remainders of virtual corrections (EW and/or QCD unresolved radiation)









$$\begin{aligned}
2s \cdot d\hat{\sigma}^{q\bar{q}}_{\mathrm{el},g\gamma} = &\left\langle F^{(\mathrm{QCD}\times\mathrm{EW}),\,\mathrm{fin}}_{\mathrm{LVV+LV^2}}(1_q,2_{\bar{q}},3,4)\right\rangle \\
&+ [\alpha]\left\langle\left[\mathcal{G}_{\mathrm{EW}} + 3Q_q^2\,\log\left(\frac{s}{\mu^2}\right)\right] F^{(\mathrm{QCD}),\,\mathrm{fin}}_{\mathrm{LV}}(1_q,2_{\bar{q}},3,4)\right\rangle \\
&+ [\alpha_s]\,C_F\left[\frac{2}{3}\pi^2 + 3\log\left(\frac{s}{\mu^2}\right)\right]\left\langle F^{(\mathrm{EW}),\,\mathrm{fin}}_{\mathrm{LV}}(1_q,2_{\bar{q}},3,4)\right\rangle \\
&+ [\alpha]\,[\alpha_s]\,C_F\left\langle\left\{Q_q^2\left[-\frac{4\pi^4}{45} + (2\pi^2+32\zeta_3)\log\left(\frac{s}{\mu^2}\right)\right.\right.\right. \\
&\left.\left.\left. + \left(9-\frac{4\pi^2}{3}\right)\log^2\left(\frac{s}{\mu^2}\right)\right] + \mathcal{G}_{\mathrm{EW}}\left(\frac{2\pi^2}{3} + 3\log\left(\frac{s}{\mu^2}\right)\right)\right\} F_{\mathrm{LM}}(1_q,2_{\bar{q}},3,4)\right\rangle
\end{aligned}$$

$$F_{\mathrm{LM}}(1_q,2_{\bar{q}},3,4) \propto \sum_{\mathrm{col,pol}} \mathrm{dLips}_{34}\,(2\pi)^d\,\delta^{(d)}(p_{12}-p_{34})\,|M(\{p_i\})|^2$$

Recurring structure

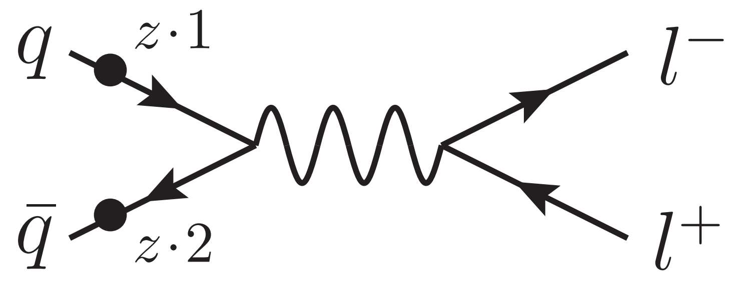
$$\mathcal{G}_{\mathrm{EW}} = Q_q^2\,\frac{2\pi^2}{3} + Q_e^2\left(13-\frac{2\pi^2}{3}\right) + 2\,Q_qQ_e\left[3\log\left(\frac{\eta_{13}}{\eta_{23}}\right) + 2\,\mathrm{Li}_2(1-\eta_{13}) - 2\,\mathrm{Li}_2(1-\eta_{23})\right]$$

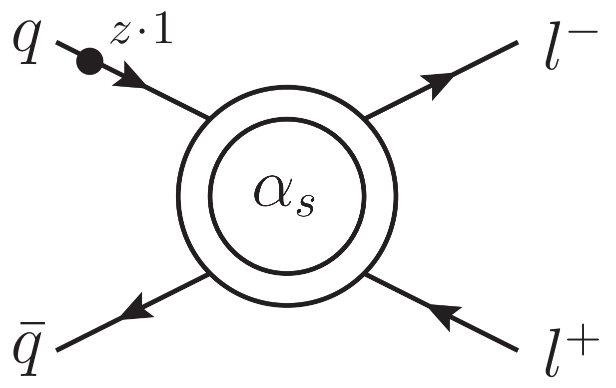
Finite parts IV

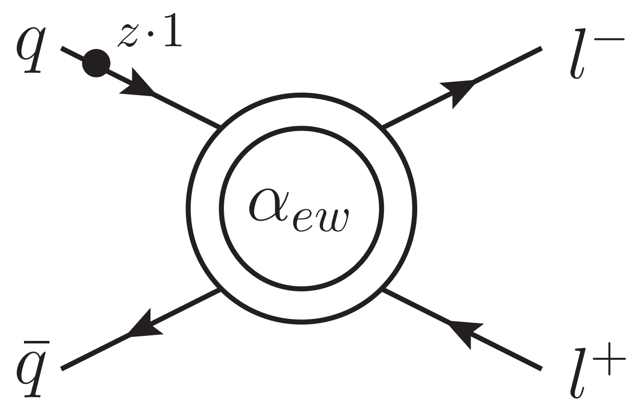
$$d\hat{\sigma}^{q\bar{q}}_{\text{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma}$$

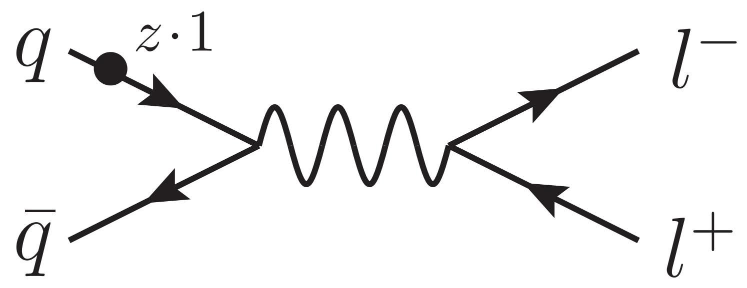
Boosted contributions arise from final states where either a gluon or a photon is collinear to incoming partons, or one of them is collinear and the other is soft.

$$2s \cdot d\sigma^{q\bar{q}}_{\text{bt},g\gamma} = [\alpha]\,[\alpha_s]\,2C_F\,Q_q^2 \int_0^1 dz_1 dz_2\,\tilde{P}^{\text{NLO}}_{qq}(z_1,E_c)\left\langle \frac{F_{\text{LM}}(z_1\cdot 1, z_2\cdot 2, 3, 4)}{z_1\,z_2}\right\rangle \tilde{P}^{\text{NLO}}_{qq}(z_2,E_c)$$
$$+\sum_{i=1}^{2}\int_0^1 dz\,\tilde{P}^{\text{NLO}}_{qq}(z,E_c)\Big[[\alpha]\,Q_q^2\left\langle F^{(i),(\text{QCD}),\text{fin}}_{\text{LV}}(1_q,2_{\bar{q}},3,4;z)\right\rangle$$
$$+[\alpha_s]\,C_F\left\langle F^{(i),(\text{EW}),\text{fin}}_{\text{LV}}(1_q,2_{\bar{q}},3,4;z)\right\rangle\Big]$$
$$+[\alpha]\,[\alpha_s]\,C_F\sum_{i=1}^{2}\int_0^1 dz\left\langle\Big\{Q_q^2\,P^{\text{NNLO}}_{qq}(z,E_c)+\tilde{P}^{\text{NLO}}_{qq}(z,E_c)\right.$$
$$\left.\times\left[Q_e^2\,G_{e^2}+2Q_qQ_e\left(G^{(1,2)}_{eq}+(-1)^i\log\left(\frac{s_{i3}}{s_{i4}}\right)\log(z)\right)\right]\Big\}F^{(i)}_{\text{LM}}(1_q,2_{\bar{q}},3,4;z)\right\rangle$$









Recurring structure

$$G^{(i,j)}_{eq} = \text{Li}_2(1-\eta_{i3}) - \text{Li}_2(1-\eta_{i4}) - \text{Li}_2(1-\eta_{j3}) + \text{Li}_2(1-\eta_{j4})$$
$$+\left[\frac{3}{2}-\log\left(\frac{E_3}{E_c}\right)\right]\log\left(\frac{\eta_{i3}}{\eta_{j3}}\right) - \left[\frac{3}{2}-\log\left(\frac{E_4}{E_c}\right)\right]\log\left(\frac{\eta_{i4}}{\eta_{j4}}\right),$$

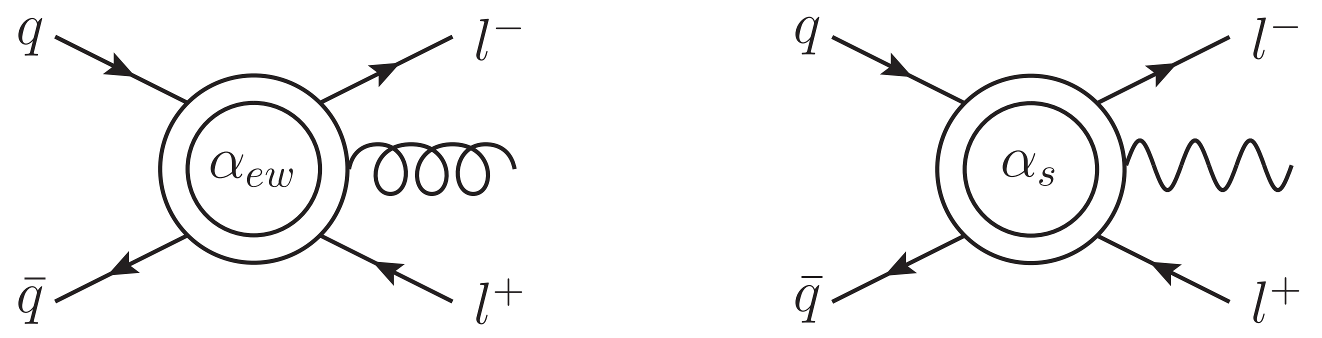
$$G_{e^2} = 13 - \frac{2}{3}\pi^2 + \log^2\left(\frac{E_3}{E_4}\right) + \left[3 - 2\log\left(\frac{E_3E_4}{E_c^2}\right)\right]\log(\eta_{34}) + 2\text{Li}_2(1-\eta_{34})$$

$$\tilde{P}^{\text{NLO}}_{qq}(z,E) = 4\mathcal{D}_1(z) - 2(1+z)\log(1-z) + (1-z) + 2\log\left(\frac{2E_c}{\mu}\right)(2\mathcal{D}_0(z) - (1+z))$$

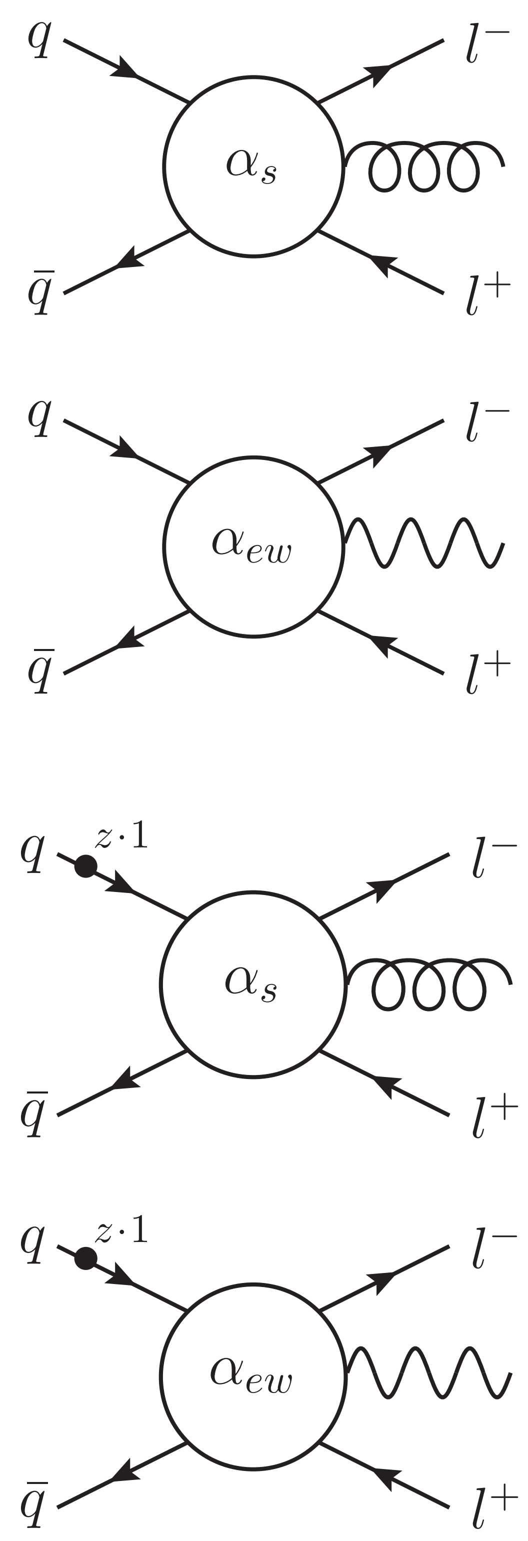
Finite parts V

$$d\hat{\sigma}^{q\bar{q}}_{\text{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma}$$

The $\mathcal{O}_{nlo}$ terms describes NLO corrections to processes with an additional gluon or photon in the final state. They arise from virtual corrections to these final states and from remnants of $l^-l^+g\gamma$ state in case either a gluon or a photon becomes unresolved.



$$\begin{aligned}
2s \cdot d\sigma^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} = &\left\langle \mathcal{O}^{g}_{\text{nlo}}\, F^{(\text{EW}),\,\text{fin}}_{\text{LV}}(1_q, 2_{\bar{q}}, 3, 4|5_g)\right\rangle \\
&+ \left\langle \mathcal{O}^{\gamma}_{\text{nlo}}\, F^{(\text{QCD}),\,\text{fin}}_{\text{LV}}(1_q, 2_{\bar{q}}, 3, 4|5_\gamma)\right\rangle \\
&+ [\alpha] \left\langle \mathcal{O}^{g}_{\text{nlo}} \left[Q_q^2 \left(\frac{2}{3}\pi^2 + 3\log\left(\frac{s}{\mu^2}\right)\right) + 2Q_qQ_e\, G^{(1,2)}_{eq} + Q_e^2\, G_{e^2}\right] F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_g)\right\rangle \\
&+ [\alpha_s]\, C_F \left[\frac{2}{3}\pi^2 + 3\log\left(\frac{s}{\mu^2}\right)\right] \left\langle \mathcal{O}^{\gamma}_{\text{nlo}}\, F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_\gamma)\right\rangle \\
&+ [\alpha]\, Q_q^2 \sum_{i=1}^{2} \int_0^1 dz \left\langle \mathcal{O}^{g}_{\text{nlo}} \left[\tilde{P}^{\text{NLO}}_{qq}(z, E_c) + \tilde{\omega}^{\gamma i, gi}_{\gamma \| i}\, \log\eta_{i5}\, \bar{P}^{\text{AP},0}_{qq,R}(z)\right] F^{(i)}_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_g; z)\right\rangle \\
&+ [\alpha_s]\, C_F \sum_{i=1}^{2} \int_0^1 dz \left\langle \mathcal{O}^{\gamma}_{\text{nlo}} \left[\tilde{P}^{\text{NLO}}_{qq}(z, E_c) + \tilde{\omega}^{\gamma i, gi}_{g \| i}\, \log\eta_{i5}\, \bar{P}^{\text{AP},0}_{qq,R}(z)\right] F^{(i)}_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_\gamma; z)\right\rangle
\end{aligned}$$



Recurring structure $\quad \bar{P}^{\text{AP},0}_{qq,R}(z) = 2\mathcal{D}_0(z) - (1+z)\,,$

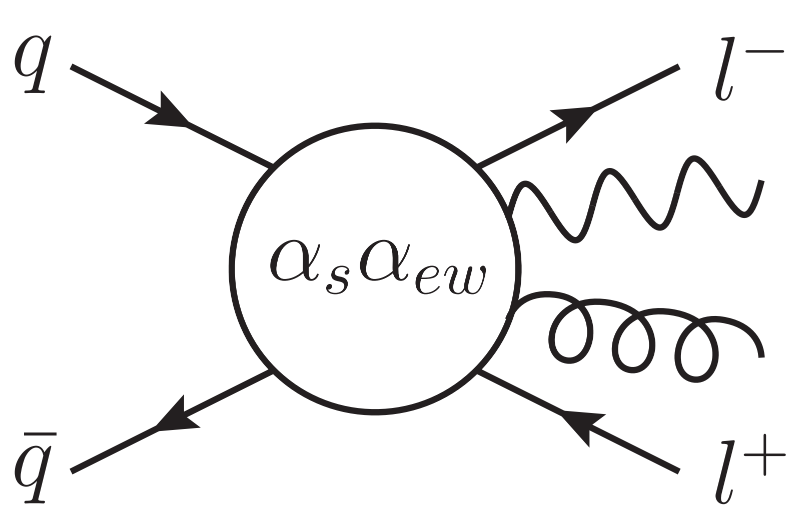
Finite parts VI

$$d\hat{\sigma}^{q\bar{q}}_{\text{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma}$$

The regulated term is fully resolved and can be implemented numerically in $d = 4$.
Subtraction terms involve known eikonal contributions and splitting functions and are organised through partition functions

$$2s \cdot d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma} = \left\langle (I - S_g)(I - S_\gamma)\, \Omega_1^{q\bar{q}}\, F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_g, 6_\gamma) \right\rangle$$

$$\begin{aligned}
\Omega_1^{q\bar{q}} = &(1 - C_{g\gamma,1})(1 - C_{g1})\, \omega^{\gamma 1, g1}\, \theta_A + (1 - C_{g\gamma,1})(1 - C_{\gamma 1})\, \omega^{\gamma 1, g1}\, \theta_B \\
&+ (1 - C_{g\gamma,2})(1 - C_{g2})\, \omega^{\gamma 2, g2}\, \theta_A + (1 - C_{g\gamma,2})(1 - C_{\gamma 2})\, \omega^{\gamma 2, g2}\, \theta_B \\
&+ (1 - C_{g2})(1 - C_{\gamma 1})\, \omega^{\gamma 1, g2} + (1 - C_{g1})(1 - C_{\gamma 2})\, \omega^{\gamma 2, g1} \\
&+ (1 - C_{g2})(1 - C_{\gamma 3})\, \omega^{\gamma 3, g2} + (1 - C_{g2})(1 - C_{\gamma 4})\, \omega^{\gamma 4, g2} \\
&+ (1 - C_{g1})(1 - C_{\gamma 3})\, \omega^{\gamma 3, g1} + (1 - C_{g1})(1 - C_{\gamma 4})\, \omega^{\gamma 4, g1},
\end{aligned}$$



Phenomenology: fiducial cross section

Definition of the fiducial cross section:

$\sqrt{s} = 13.6\,\text{TeV}$

$m_l = 0$

$m_{ll} > 200\,\text{GeV}$ (dressed leptons)

$R_{l\gamma} = 0.1$ (dressed leptons)

$p_\perp^l > 30\,\text{GeV}$

$\sqrt{p_\perp^{l^+} p_\perp^{l^-}} > 35\,\text{GeV}$

$|y_l| < 2.5$

NNPDF31_nnlo_as_0118_luxqed

$\mu_F = \mu_R = m_{ll}/2$ (dressed leptons)

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LO — NLO QCD — NLO EW — NNLO QCD — NNLO QCDxEW

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σ[fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
$q\bar{q}$	1561.42	340.31	−49.907	44.60	−16.80
$\gamma\gamma$	59.645		3.166		
qg		0.060		−32.66	1.03
$q\gamma$			−0.305		−0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	−47.046	13.88	−15.71

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What do we learn?

✓ **NLO QCD** increases LO by **~20%**

✓ **NLO EW de**creases LO by **~3%**

✓ **NNLO QCD in**creases LO by **~0.8%**

✓ **QCD×EW de**creases LO by **~1%**

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$$\sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \ldots$$

LO, NLO QCD, NLO EW, NNLO QCD, NNLO QCDxEW

σ[fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
$q\bar{q}$	1561.42	340.31	-49.907	44.60	-16.80
$\gamma\gamma$	59.645		3.166		
qg		0.060		-32.66	1.03
$q\gamma$			-0.305		-0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	-47.046	13.88	-15.71

What do we learn?

✓ **NLO QCD** increases LO by **~20%** → $\delta^{\text{QCD}} \sim 8C_F\alpha_s/(2\pi) \sim 0.2$

✓ **NLO EW decreases** LO by **~3%** → $\delta^{\text{ew}} \sim \alpha_{ew}/sin^2\theta_W \sim 0.03$

✓ **NNLO QCD increases** LO by **~0.8%** → unexpected : $\alpha_s^2 \sim 0.014$

strong qq and qg cancellation

✓ **QCD×EW decreases** LO by **~1%**

Phenomenology: fiducial cross section

Definition of the fiducial cross section:

$\sqrt{s} = 13.6\,\mathrm{TeV}$

$m_l = 0$

$m_{ll} > 200\,\mathrm{GeV}$ (dressed leptons)

$R_{l\gamma} = 0.1$ (dressed leptons)

$p_\perp^l > 30\,\mathrm{GeV}$

$\sqrt{p_\perp^{l^+} p_\perp^{l^-}} > 35\,\mathrm{GeV}$

$|y_l| < 2.5$

NNPDF31_nnlo_as_0118_luxqed

$\mu_F = \mu_R = m_{ll}/2$ (dressed leptons)

$$\sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \ldots$$

LO, NLO QCD, NLO EW, NNLO QCD, NNLO QCDxEW

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LO, NLO QCD, NLO EW, NNLO QCD, NNLO QCDxEW

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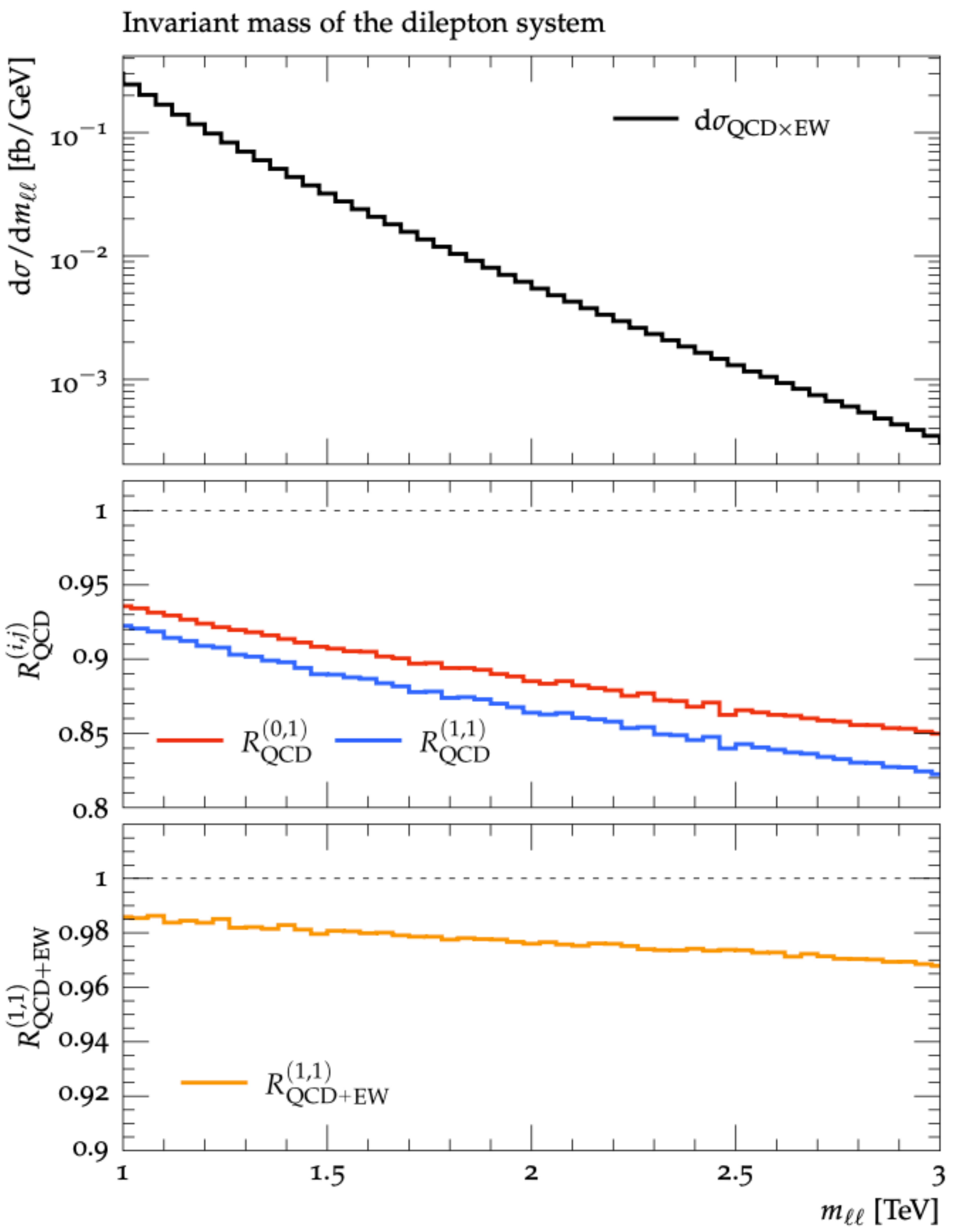
→ QCDxEW corrections larger than NNLO QCD ones!

Kinematic distributions



$$d\sigma_{QCD\times EW} = d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(2,0)} + d\sigma^{(1,1)}$$

What do we learn in the $m_{ll} \in [1,3]\,\mathrm{TeV}$ region?

✓ The **cross section drops** more than 3 orders of magnitude

✓ **Impact of NLO EW** corrections on NLO QCD results:

- Large corrections growing from $\mathcal{O}(-6\%)$ to $\mathcal{O}(-15\%)$

✓ **Impact of NLO EW+mixed QCD×EW** corrections on NLO QCD results:

- Grows from $\mathcal{O}(-8\%)$ to $\mathcal{O}(-18\%)$

✓ **Impact of mixed QCD×EW** corrections on NLO QCD+NLO EW results:

- Grows from $\mathcal{O}(-1.5\%)$ to $\mathcal{O}(-3\%)$

Phenomenology: m_{ll} windows

$$\sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \ldots$$

LO, NLO QCD, NLO EW, NNLO QCD, QCDxEW

Question: can we capture NNLO QCDxEW by only computing (NLO QCD)·(NLO EW) $\rightarrow \delta\sigma^{(1,1)}_{\text{fact}}$?

$$\Phi^{(1)}: \; 200\text{ GeV} < m_{\ell\ell} < 300\text{ GeV},$$
$$\Phi^{(2)}: \; 300\text{ GeV} < m_{\ell\ell} < 500\text{ GeV},$$
$$\Phi^{(3)}: \; 500\text{ GeV} < m_{\ell\ell} < 1.5\text{ TeV},$$
$$\Phi^{(4)}: \; 1.5\text{ TeV} < m_{\ell\ell} < \infty.$$

σ [fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma^{(1,1)}_{\text{fact.}}$
$\Phi^{(1)}$	1169.8	254.3	−30.98	10.18	−10.74	−6.734
$\Phi^{(2)}$	368.29	71.91	−11.891	2.85	−4.05	−2.321
$\Phi^{(3)}$	82.08	14.31	−4.094	0.691	−1.01	−0.7137
$\Phi^{(4)} \times 10$	9.107	1.577	−1.124	0.146	−0.206	−0.1946

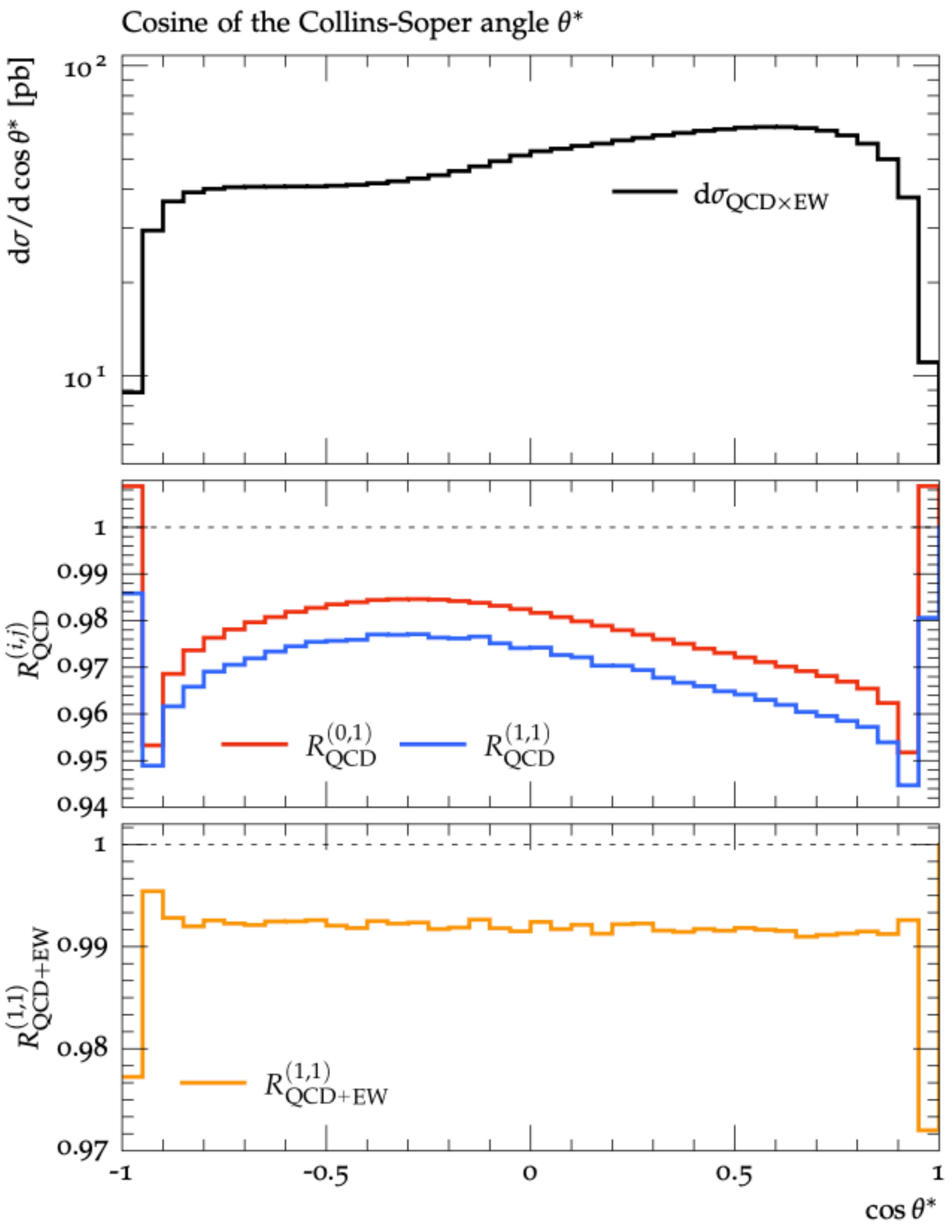
What do we learn?

✓ **At high invariant mass** ($m_{ll} > 1.5\,\text{TeV}$) the **factorised approx.** captures more than **90%** of the **exact result**

$$\frac{\delta\sigma^{(1,1)}_{\text{fact.}}}{\sigma^{(0,0)}} = \left[\frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}} \sim 0.17\right] \cdot \left[\frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}} \sim -0.12\right] \sim -0.021 \qquad \frac{\delta\sigma^{(1,1)}}{\sigma^{(0,0)}} \sim -0.023$$

⟶ Expected: factorised approx. correctly reproduces the leading Sudakov logs, which dominate at high invariant masses

Kinematic distributions: an example



Angular distributions can test quark to lepton interactions

Forward-backward asymmetry has been measured *[CMS 2202.12327]*

$$m_{ll} > 200\text{GeV we found:} \qquad A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = 0.1580^{+0.15\%}_{-0.07\%}$$

Mixed QCD-EW corrections changes the value by about **2 permille**
→ comparable with the uncertainties

	$\tilde{A}_{FB}$	A_{FB}
$\Phi^{(1)}$	$0.1442^{+0.05\%}_{-0.31\%}$	$0.1440^{+0.11\%}_{-0.09\%}$
$\Phi^{(2)}$	$0.1852^{+0.08\%}_{-0.40\%}$	$0.1847^{+0.10\%}_{-0.19\%}$
$\Phi^{(3)}$	$0.2401^{+0.13\%}_{-0.64\%}$	$0.2388^{+0.06\%}_{-0.47\%}$
$\Phi^{(4)}$	$0.3070^{+0.49\%}_{-1.5\%}$	$0.3031^{+0.19\%}_{-1.2\%}$

Excluding mixed corrections

Including mixed corrections

Mixed QCD-EW corrections affect A_{FB} at the **percent** level for $m_{ll} \gtrsim 1\text{TeV}$

→ this shift should become observable at HL-LHC

Conclusions

1. **Mixed QCDxEW** corrections to Drell-Yan are important to **search for NP in the high energy regime**

2. The **results** show a remarkably **simple structure**

3. **Mixed QCD-EW** amount to about **-1%** of the fiducial **LO** cross-section

   → **larger** than expected from **coupling magnitude**

   → **even with relative low cut on** m_{ll}

4. **Good approximation** by the **product** of **QCD and EW** corrections in the **TeV region**

Thank you for your attention!

Backup

Mixed QCD-EW: differences with respect to NNLO QCD

$$1 = \underbrace{\omega^{\gamma 1, g1} + \omega^{\gamma 2, g2}}_{} + \underbrace{\omega^{\gamma 1, g2} + \omega^{\gamma 2, g1} + \omega^{\gamma 3, g2} + \omega^{\gamma 3, g1} + \omega^{\gamma 4, g2} + \omega^{\gamma 4, g1}}_{}$$

Triple collinear partition

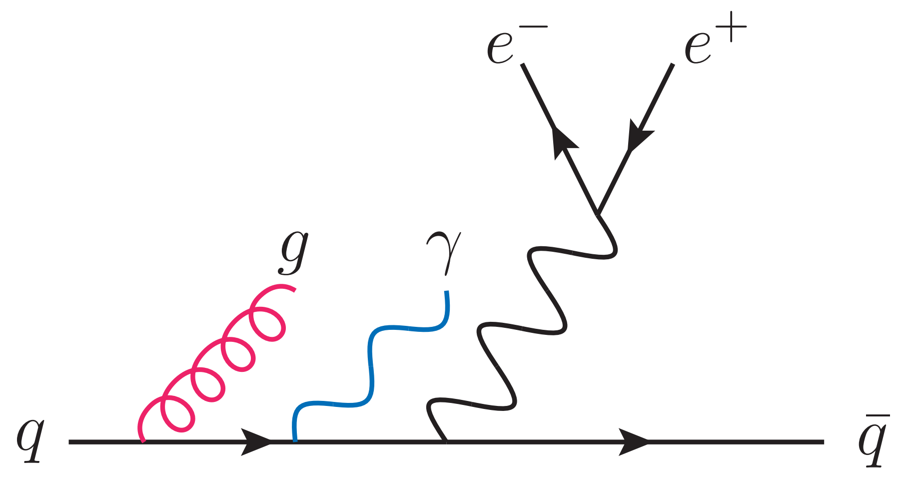
$\sim \mathrm{NNLO} \rightarrow$ **difficult!**

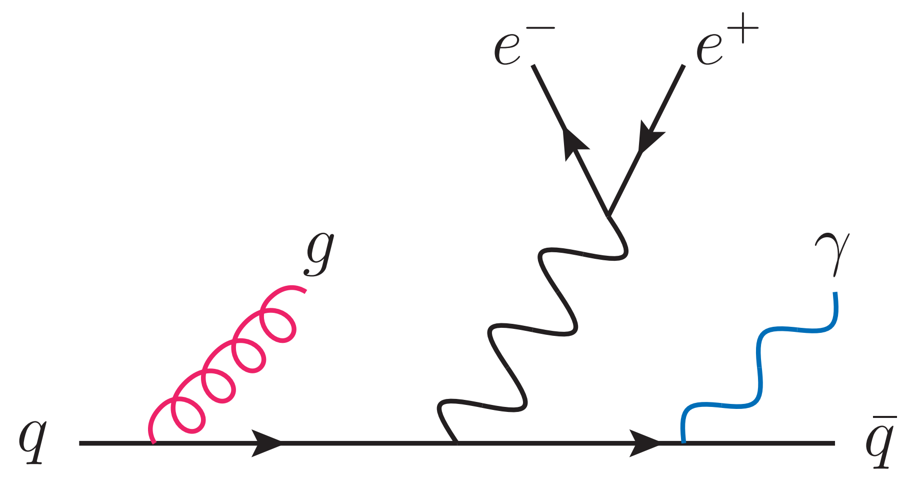
Overlapping singularities remain

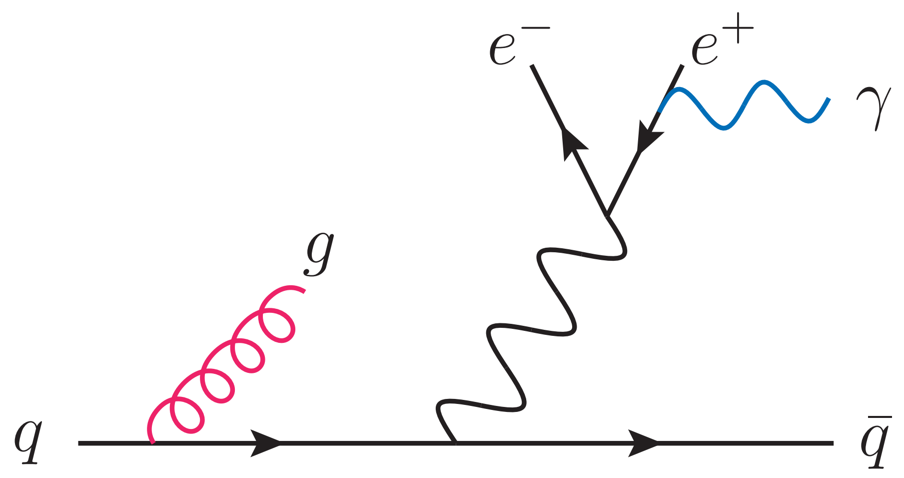
Single collinear partition

$\sim \mathrm{NLO} \times \mathrm{NLO} \rightarrow$ **simple!**

Large rapidity difference in each sector







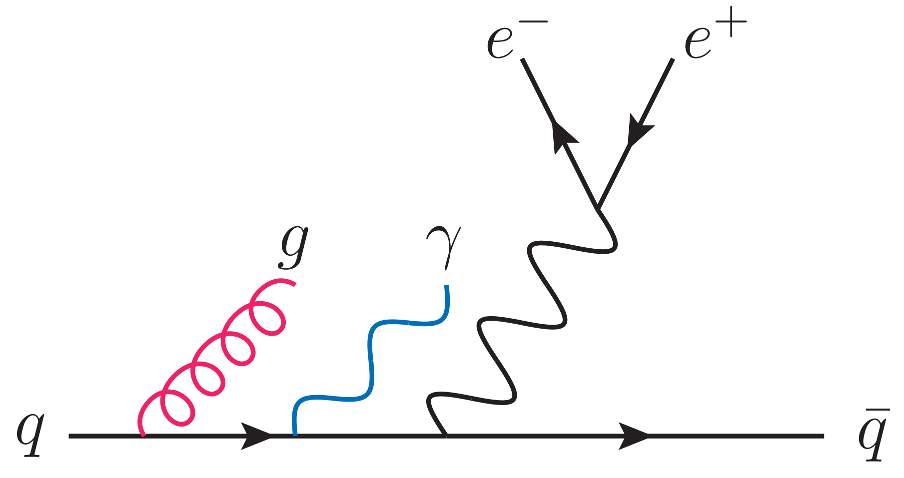
Mixed QCD-EW: differences with respect to NNLO QCD

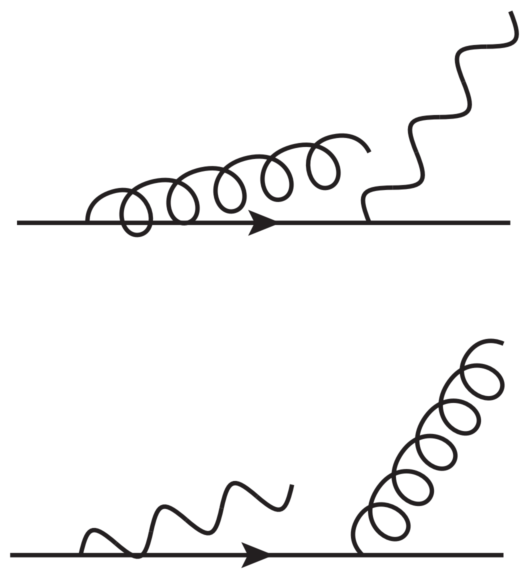
$$1 = \underbrace{\omega^{\gamma 1, g1} + \omega^{\gamma 2, g2}}_{\text{Triple collinear partition}} + \omega^{\gamma 1, g2} + \omega^{\gamma 2, g1} + \omega^{\gamma 3, g2} + \omega^{\gamma 3, g1} + \omega^{\gamma 4, g2} + \omega^{\gamma 4, g1}$$

Triple collinear partition

$\sim$ NNLO $\rightarrow$ **difficult!**

Overlapping singularities remain



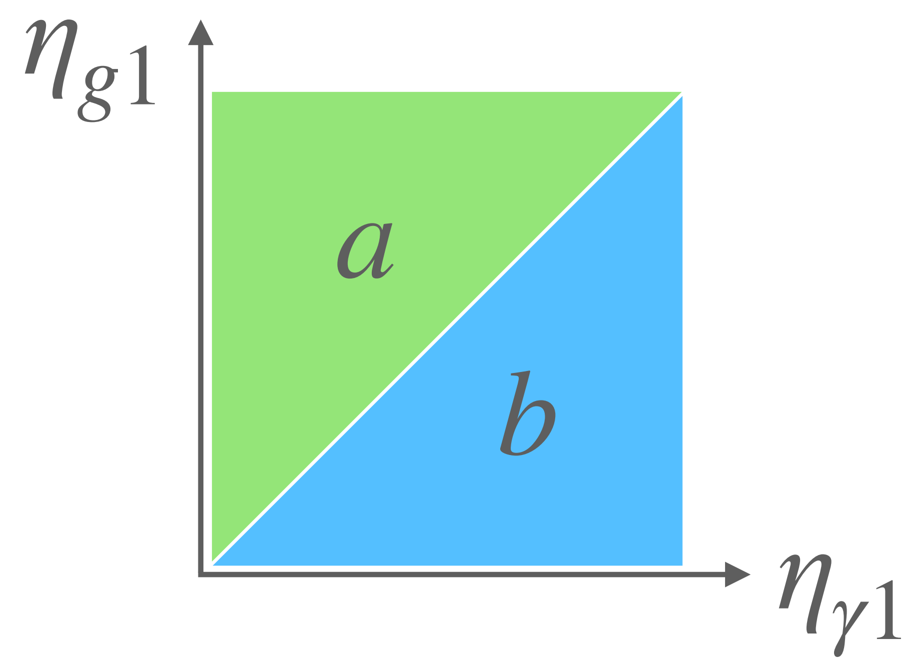


Triple collinear: $1 \parallel g \parallel \gamma$
Strongly-ordered: $1 \parallel g$

Triple collinear: $1 \parallel g \parallel \gamma$
Strongly-ordered: $1 \parallel \gamma$

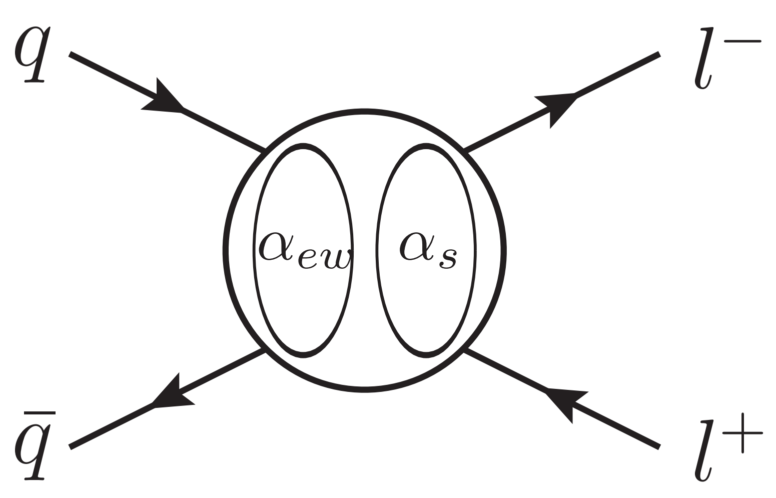
Define **angular ordering** to separate singularities

$$1 = \theta_a + \theta_b = \theta(\eta_{g1} - \eta_{\gamma 1}) + \theta(\eta_{\gamma 1} - \eta_{g1})$$



Ingredients for off-shell calculation at NNLO

❖ Fully differential description of mixed QCD-EW effects is a complicated problem



2-loop virtual + one-loop squared

$$\left\langle F^{\mathrm{QCD\times EW}}_{LVV+LV^2}(1,2,3,4)\right\rangle = \frac{\alpha_s(\mu)}{2\pi}\frac{\alpha(\mu)}{2\pi}\left[I^{\mathrm{QCD}}\cdot I^{\mathrm{QED}} + \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}H_{QCD\times EW}\right]\left\langle F_{LM}(1,2,3,4)\right\rangle$$
$$+\frac{\alpha_s(\mu)}{2\pi}I^{\mathrm{QCD}}\left\langle F^{\mathrm{EW,\,fin}}_{LV}(1,2,3,4)\right\rangle + \frac{\alpha(\mu)}{2\pi}I^{\mathrm{QED}}\left\langle F^{\mathrm{QCD,\,fin}}_{LV}(1,2,3,4)\right\rangle + \left\langle F^{\mathrm{QCD\times EW,\,fin}}_{LVV+LV^2}(1,2,3,4)\right\rangle$$

Genuine NNLO hard-triple collinear contribution

$$H_{QCD\times EW} = 2C_F Q_q^2\left(\frac{\pi^2}{2} - 6\zeta_3 - \frac{3}{8}\right)$$

Universal IR operators
[Catani '98]

Finite part one-loop EW/QCD correction

Finite part of the two-loop mixed QCDxEW correction

- Handle chiral couplings in dimensional regularisation
  - > γ_5 really a 4-dimentional object > need for a prescription in d-dimension
- Solving master integrals
- Adapt the result for a fast Monte-Carlo integration

Ingredients for off-shell calculation at NNLO

❖ Fully differential description of mixed QCD-EW effects is a complicated problem

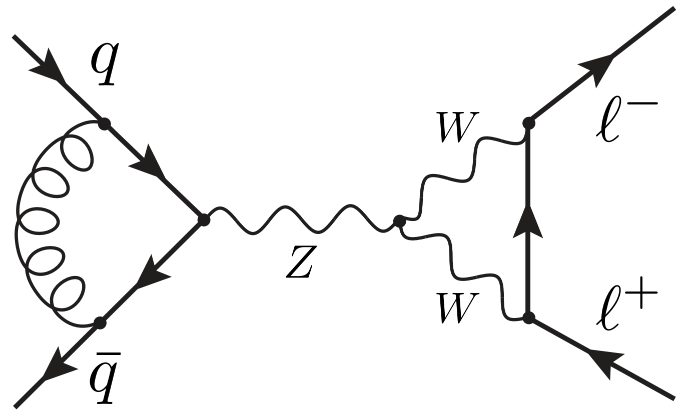
- Given the Catani operator, the finite 1-loop QCD remainder reads

$$\left\langle F_{LV}^{\mathrm{QCD,\,fin}}(1,2,3,4)\right\rangle = C^{\mathrm{QCD}}\left\langle F_{LM}(1,2,3,4)\right\rangle, \qquad C^{\mathrm{QCD}} = -\,8C_F\,\frac{\alpha_s(\mu)}{2\pi}$$

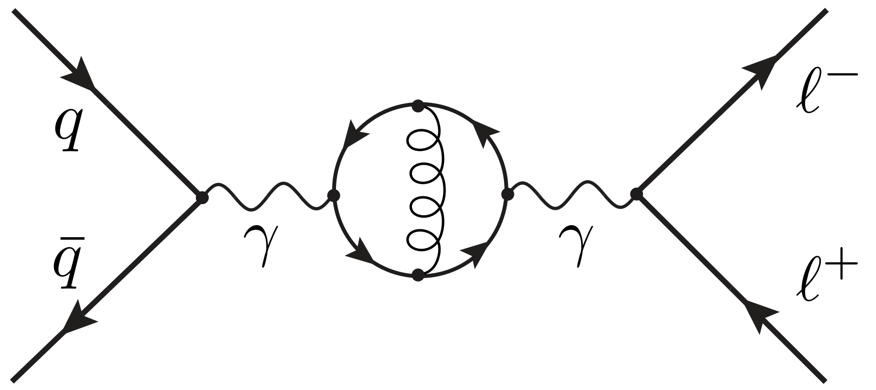
- At **2-loop** we find it convenient to split the virtual correction into **factorisable** and **non-factorisable** contributions

$$\left\langle F_{LVV+LV^2}^{\mathrm{QCD\times EW,\,fin}}(1,2,3,4)\right\rangle = \left\langle F_{LVV+LV^2}^{\mathrm{fact}}(1,2,3,4)\right\rangle + \left\langle F_{LVV+LV^2}^{\mathrm{non-fact}}(1,2,3,4)\right\rangle$$

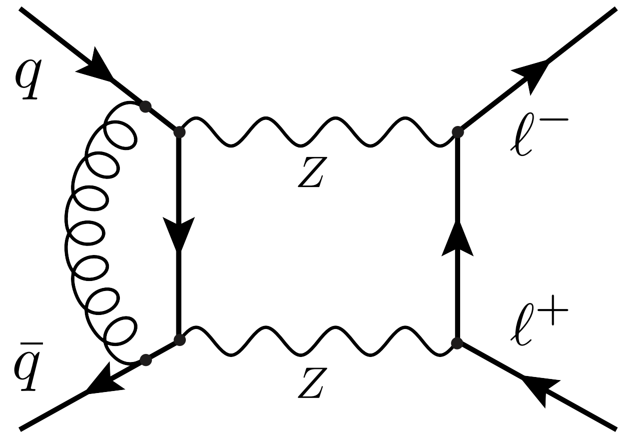
$$\left\langle F_{LVV+LV^2}^{\mathrm{fact}}(1,2,3,4)\right\rangle \equiv C^{\mathrm{QCD}}\left\langle F_{LV}^{\mathrm{EW,\,fin}}(1,2,3,4)\right\rangle$$



factorisable



fermionic non-fact.



bosonic non-fact.

Heller, von Manteuffel, Schabinger '20
Heller, von Manteuffel, Schabinge, Spiesberger '21

- **Non-factorisable** part is **CPU expensive**, BUT typically **1 order** of magnitude **smaller** than the **factorisable part**, across the entire phase space → can be determined to a much lower accuracy to obtain the cross section with a target precision

Ingredients for off-shell calculation at NNLO

❖ Fully differential description of mixed QCD-EW effects is a complicated problem

- Separation into factorisable/non-factorisable allows us to capture the **bulk of the virtual top-quarks contributions**
  → **neglected** in the **finite part** of the **bosonic non-factorisable** contribution
  → **kept in all the other contributions**

- **0.7s** per phase space point

Theoretical uncertainties

Theoretical uncertainties can be estimated by varying the central scale by a factor of 2, and changing the input scheme for the electroweak parameters ($\alpha(m_Z)$-scheme: $\alpha(m_Z) = 1/128$, the other input parameters are kept fixed)

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8\%}_{-0.15\%} \text{ fb.}$$

The main source of theoretical uncertainty is the input-scheme change which, however, is reduced from about 6% at leading order to about 2% when NLO EW corrections are included.

The mixed QCD-EW corrections are about -1% and therefore they are comparable in size to the theoretical uncertainties.

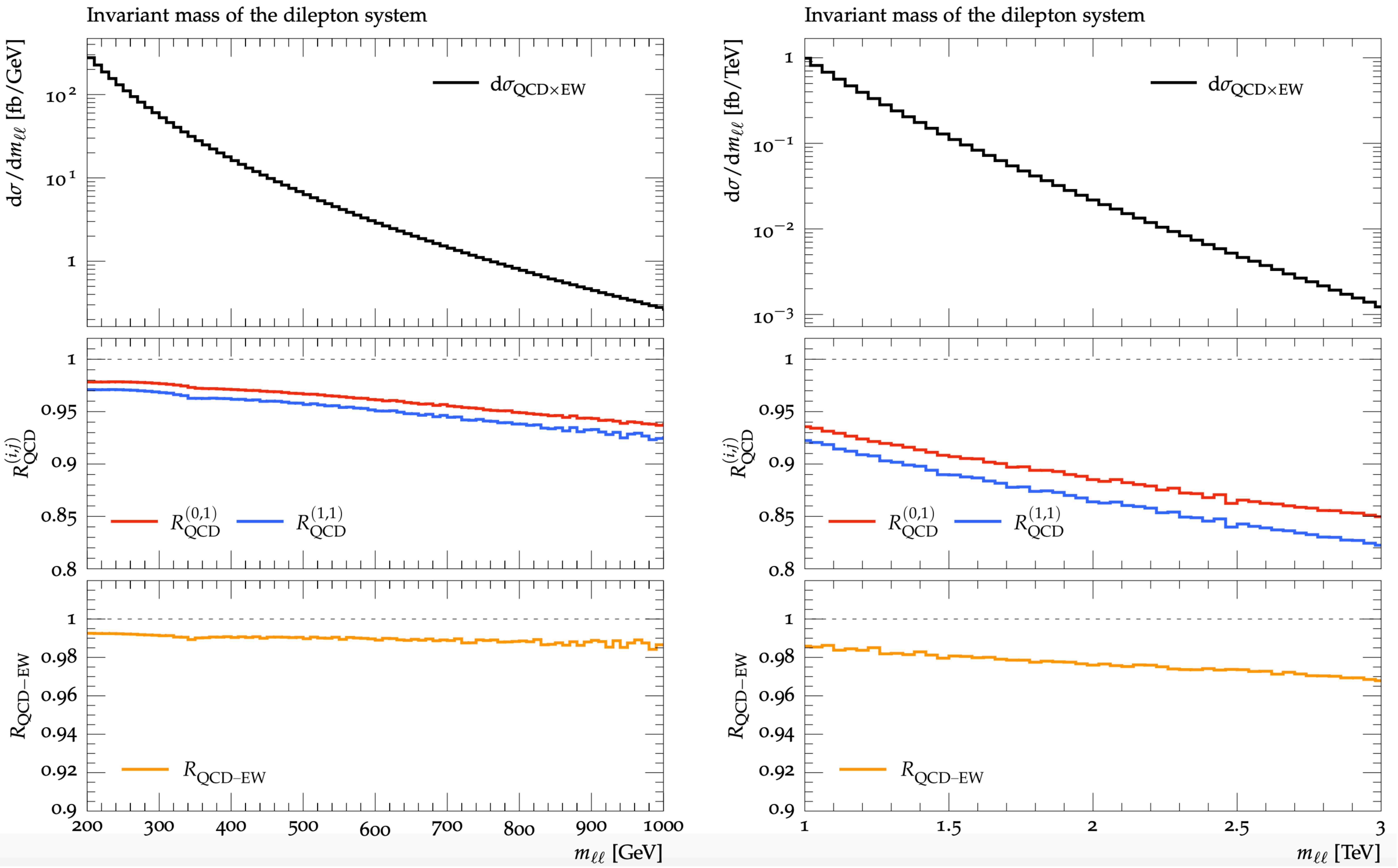
$$\sigma_{\mathrm{QCD}\times\mathrm{EW}} \equiv \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65\%}_{-0\%} \text{ fb.}$$

The mixed QCD-EW corrections remove a large source of input-scheme dependence coming from NLO QCD contribution.
Pure EW scheme uncertainty is reduced from about 1% to about 0.5% after the inclusion of mixed corrections

The results above do not include uncertainties from PDFs, which are known to be significant.
The uncertainty on the $q\bar{q}$ luminosity ranges from about 2% for $m_{ll} \lesssim 1$TeV to about 5% for $m_{ll} \sim 2$TeV.

Kinematic distributions

$$d\sigma_{\text{QCD}\times\text{EW}} = d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(2,0)} + d\sigma^{(1,1)}$$



$$R^{(0,1)}_{\text{QCD}} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$

$$R^{(1,1)}_{\text{QCD}} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(1,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$

$$R^{(1,1)}_{\text{QCD+EW}} = \frac{R^{(1,1)}_{\text{QCD}}}{R^{(0,1)}_{\text{QCD}}}$$

Brief historical recap

$\sigma_{\rm QCD}^{\rm nlo}$

[Altarelli, Ellis, Martinelli ’79]

$d\sigma_{\rm QCD}^{\rm nnlo}$ (fully diff.+leptonic decay)

[Anastasiou, Dixon, Melnikov, Petriello ’03, ‘04]

[Melnikov, Petriello ’06]

[Catani, Cieri, Ferrera, de Florian, Grazzini ’09]

[Catani, Ferrera, Grazzini ’10]

Complete EW corr. to Z prod.

[Bauer, Brein, Hollik, Schappacher, Wackeroth ’02]

[Zykunov ’07]

[Carloni, Calame, Montagna, Nicrosini, Vicini ’07]

[Arbuzov, Bardin, Bondarenko, Christina, Kalinovskaya, Nanava, Sadykov ’08]

[Dittmaier, Huber ’09]

$\sigma_{\rm QCD}^{\rm nnlo}$

[Hamberg, van Neerven, Matsuura ‘91]

[Harlander, Kilgore ‘02]

Complete EW corr. to W prod.

[Dittmaier, Krämer ’02]

[Bauer, Wackerste ’04]

[Zykunov ’06]

[Arbuzov, Bardin, Bondarenko, Christina, Kalinovskaya, Nanava, Sadykov ’06]

[Carloni, Calame, Montagna, Nicrosini, Vicini ’16]

$\sigma_{\rm QCD}^{\rm n3lo}$

[Duhr, Dulat, Mistlberger ‘20]

$d\sigma_{\rm QCD}^{\rm n3lo}$

[Camarda, Cieri, Ferrera ’21]

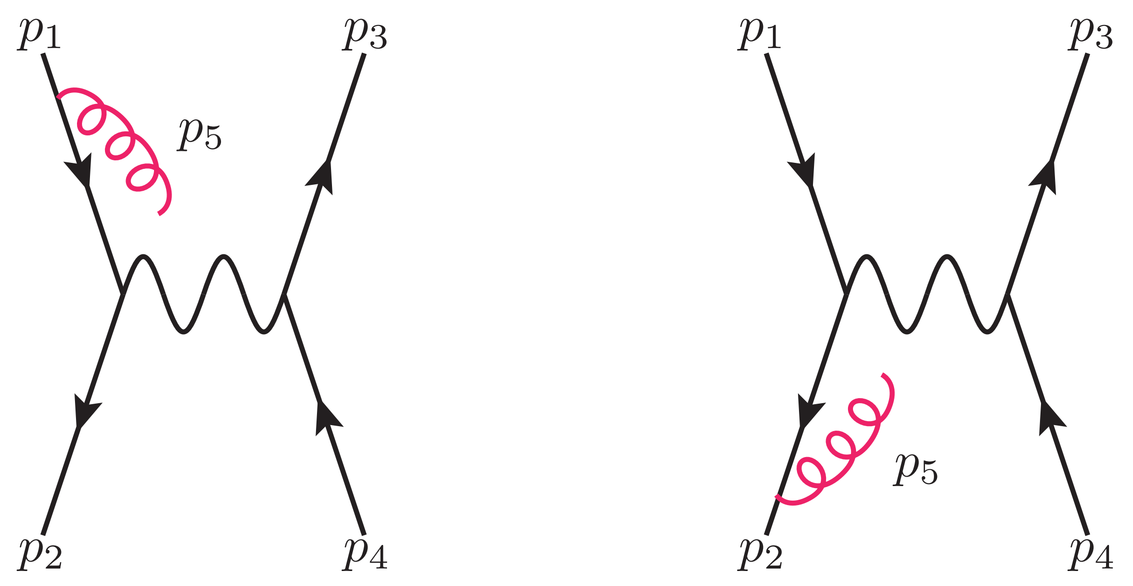
[Chen, Gehrmann, Glover, Huss, Yang ’21]

[Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli ’22]

Nested soft-collinear subtraction: NLO QCD

What do we need to know to define the counterterm?

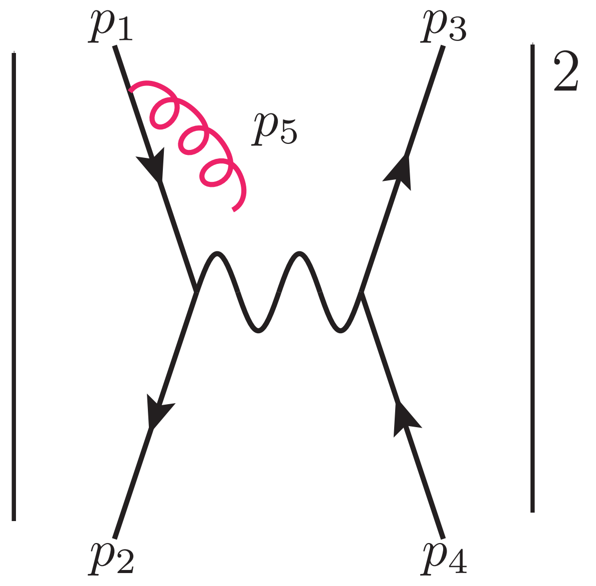
- Which singular limits do contribute



$$\mathcal{M} \sim \frac{1}{(k_1 - k_5)^2} + \frac{1}{(k_2 - k_5)^2} \sim \frac{1}{E_1 E_5 (1 - \vec{n}_1 \cdot \vec{n}_5)} + \frac{1}{E_2 E_5 (1 - \vec{n}_2 \cdot \vec{n}_5)}$$

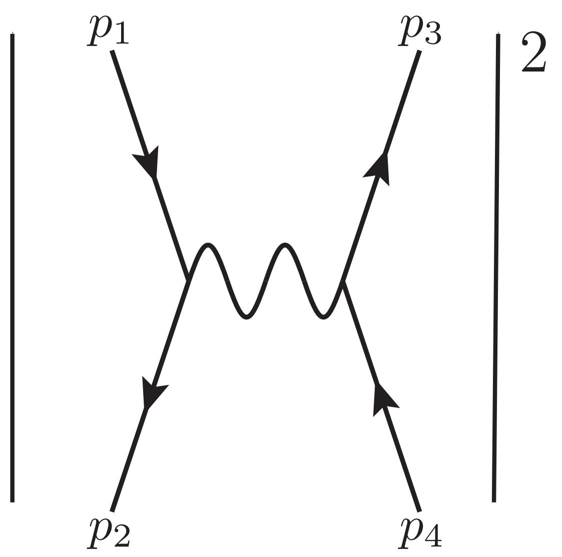
$$\mathcal{M} \to \infty \quad \begin{cases} E_5 \to 0 & \to S_5 \\ \vec{n}_1 \parallel \vec{n}_5 & \to C_{15} \\ \vec{n}_2 \parallel \vec{n}_5 & \to C_{25} \end{cases}$$

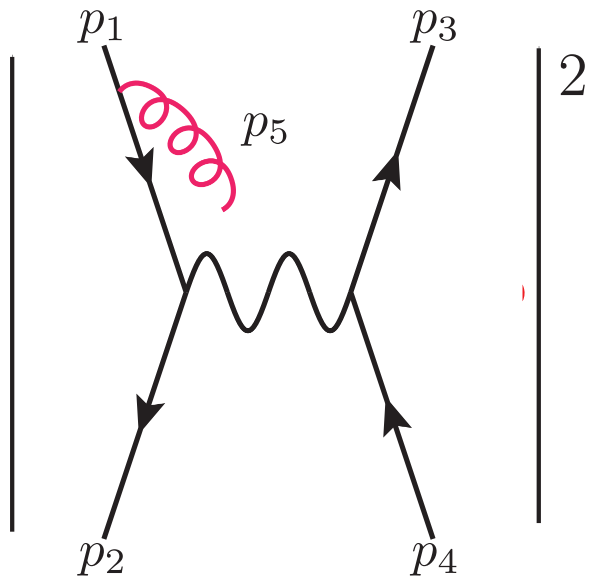
- How they act on the relevant contributions



$$\underset{p_1 \parallel p_5}{\sim} \; g_s^2 \, \frac{1}{p_1 \cdot p_5} \, \boxed{P_{qg}(z)}$$

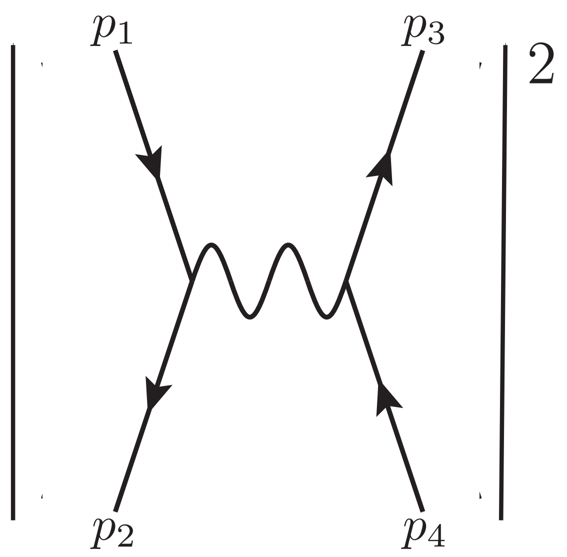
Splitting kernel





$$\underset{E_5 \to 0}{\sim} \; g_s^2 \, \boxed{Eik(p_1, p_2; p_5)}$$

Eikonal kernel



$$P_{qg}(z) = \frac{1 + z^2}{1 - z} - \epsilon(1 - z)\,, \qquad z = \frac{s_{12}}{s_{12} + s_{23}}$$

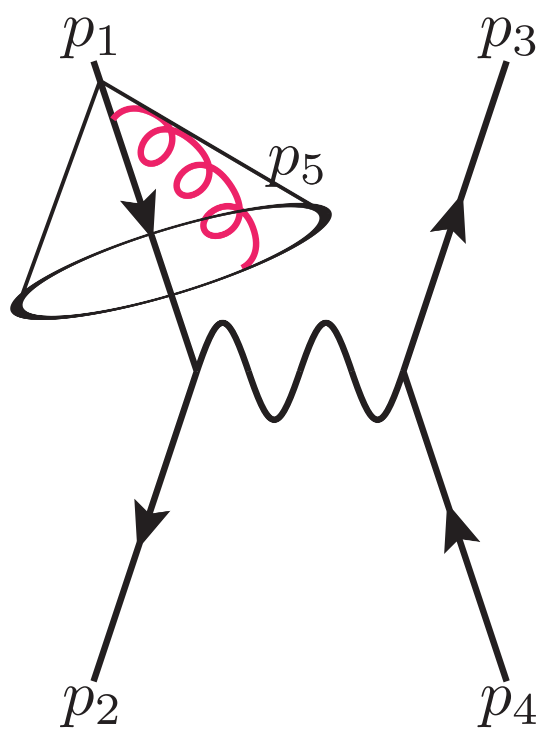
$$Eik(p_1, p_2; p_5) = \frac{p_1 \cdot p_2}{(p_1 \cdot p_5)\,(p_2 \cdot p_5)}$$

Nested soft-collinear subtraction: NLO EW

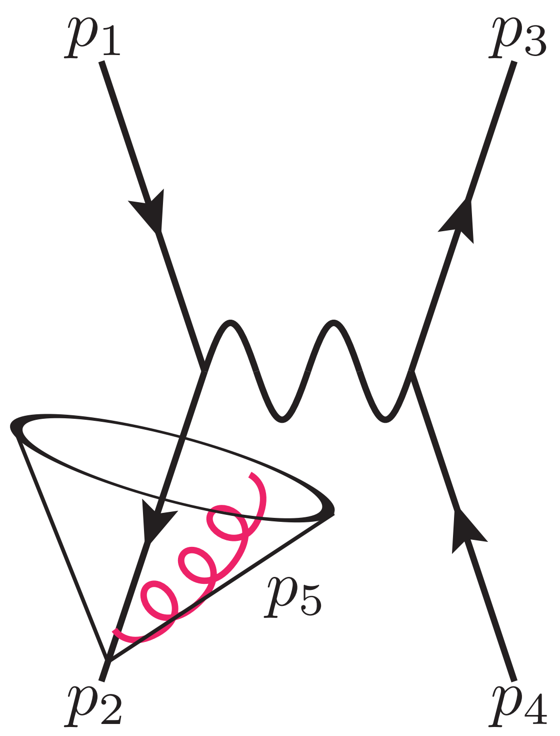
- How to simplify their treatment
  - Phase space partition: useful to isolate collinear singularities (as in FKS), full freedom in their definition

$$\omega^{5j} \left| \mathcal{M}(1_q, 2_{\bar{q}}, 3_{l^-}, 4_{l^+}; 5_g) \right|^2$$

Partition function



ω^{51} Damp: $\vec{n}_2 \parallel \vec{n}_5$
Enhance: $\vec{n}_1 \parallel \vec{n}_5$



ω^{52} Damp: $\vec{n}_1 \parallel \vec{n}_5$
Enhance: $\vec{n}_2 \parallel \vec{n}_5$

- Sector-by-sector subtraction:
  - Each sector contains a minimum number of singularities
  - Simple parametrisation

$$\omega^{5j} = \left[\left(I - C_{5j}\right) + C_{5j} \right] \omega^{5j} = \left(I - C_{5j}\right) \omega^{5j} + C_{5j}$$

Distributions for QCDxEW corrections to dilepton production

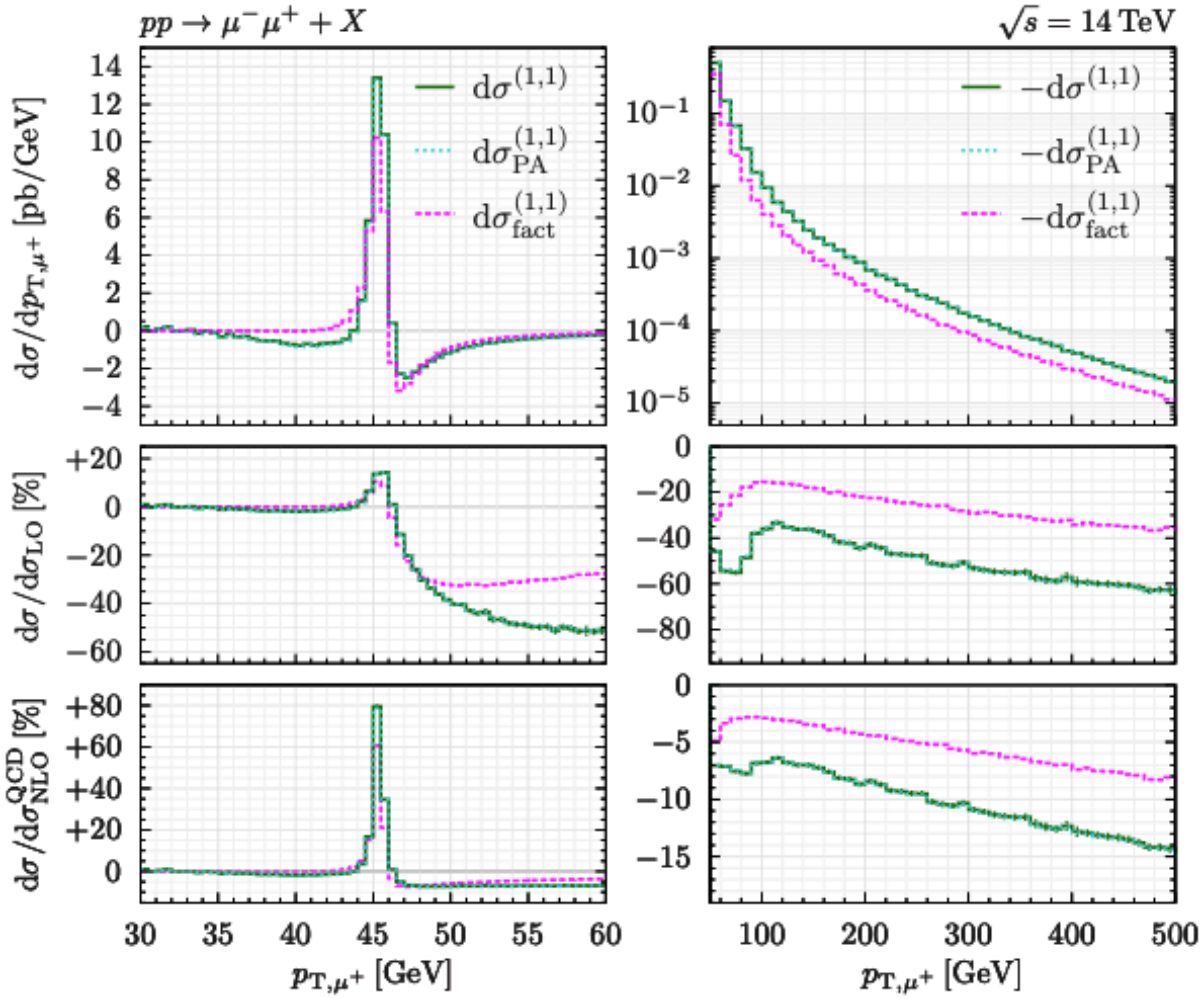
- First complete QCDxEW corrections presented by *[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini '21]*

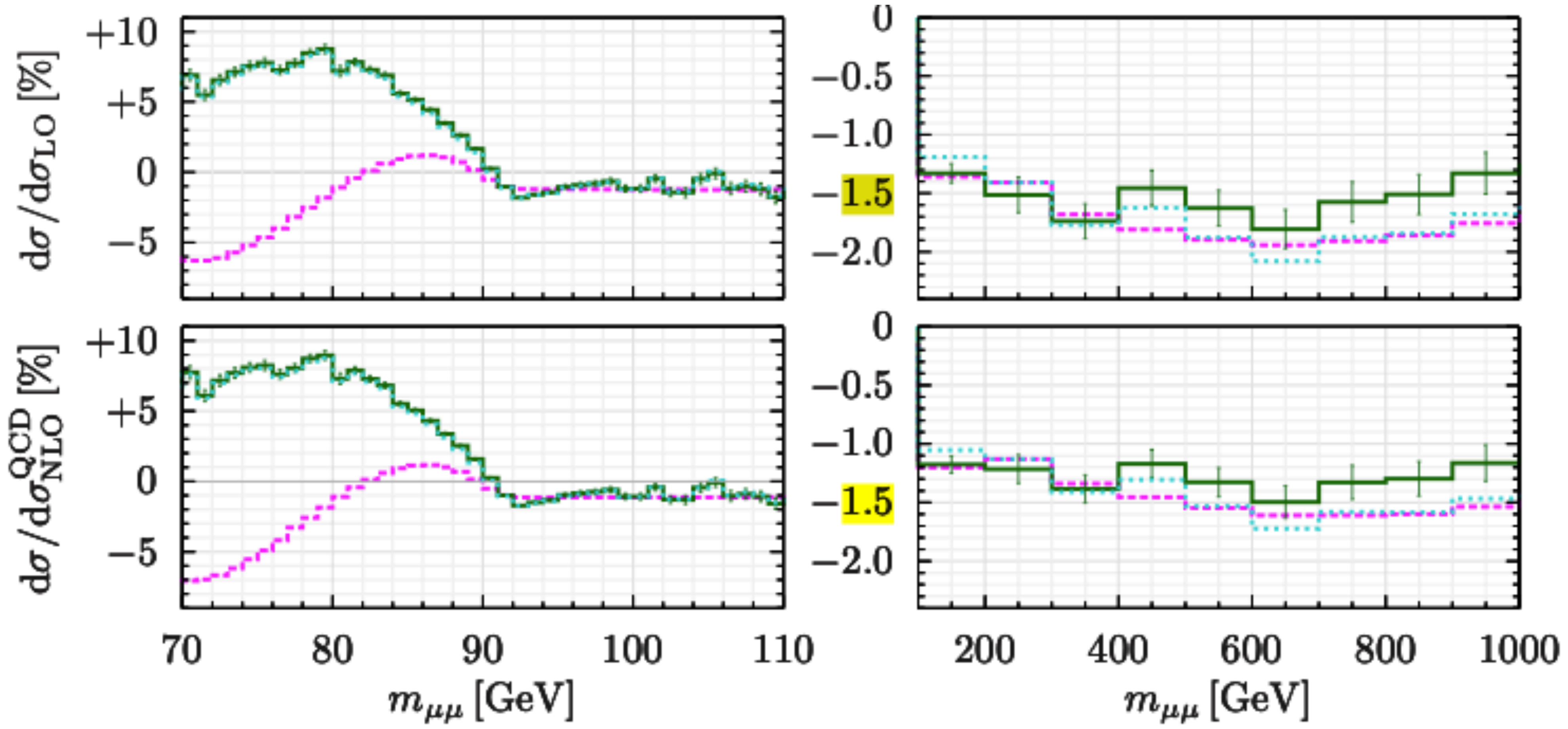
- **Two-loop amplitudes** evaluated with help of **semi-analytic method**

- **IR singularities** regulated by $q_\perp$ **subtraction** as implemented in MATRIX.

- Results for **massive leptons** (mass as IR regulator)

  - **Fiducial cross section** increased by $\sim 0.5\,\%$ relative to LO
  - Larger impact at **high-pT**: -60% correction





- High **invariant mass**: correction $\sim -1.5\,\%$ w.r.t. NLO QCD.
- Factorised approximation works well at the Jacobian peak, fails at higher pT.

IR regularisation: subtraction vs slicing

$F(x)$ arbitrary complicated function

$$I = \lim_{\epsilon\to 0}\left[\int_0^1 \frac{dx}{x}\, x^\epsilon F(x) - \frac{1}{\epsilon} F(0)\right]$$

Goal: compute *I* without relying on the analytic evaluation of the integral

Slicing

$$I \sim \lim_{\epsilon\to 0}\left[F(0)\int_0^\delta \frac{dx}{x}\, x^\epsilon + \int_\delta^1 \frac{dx}{x}\, x^\epsilon F(x) - \frac{1}{\epsilon} F(0)\right] = F(0)\,\log\delta + \int_\delta^1 \frac{dx}{x}\, x^\epsilon F(x)$$

Slicing parameter $\delta \ll 1$ $\rightarrow$ power dependence on the slicing parameter in the result

Subtraction

$$I = \lim_{\epsilon\to 0}\Bigg[\underbrace{\int_0^1 \frac{dx}{x}\, x^\epsilon\Big(F(x) - F(0)\Big)}_{\text{Regulated, finite for } \epsilon\to 0} + \underbrace{\int_0^1 \frac{dx}{x}\, x^\epsilon F(0)}_{\text{Extract } 1/\epsilon \text{ pole}} - \frac{1}{\epsilon} F(0)\Bigg]$$

Regulated, finite for $\epsilon \to 0$

Extract $1/\epsilon$ pole

Counterterm: the definition may be involved!

Why is NNLO so difficult?

At NLO two main strategies have been implemented

Catani Seymour:

- Counterterm contribution: reproduces the **IR singularities** related to a dipole in **all of the phase space** [complicated structure]
- Full counterterm: sum of **contributions**, each **parametrised differently**
- **Analytic integration** of each term [non trivial, complicated structure of the counterterm]

FKS:

- **Partition** of the radiative phase space with sector functions
- **Different parametrisation** for each sector
- **Analytic integration**, after getting rid of sector functions [non trivial, non optimised parametrisation]

Detail informations of NNLO kernels also available ~ 20 years ago

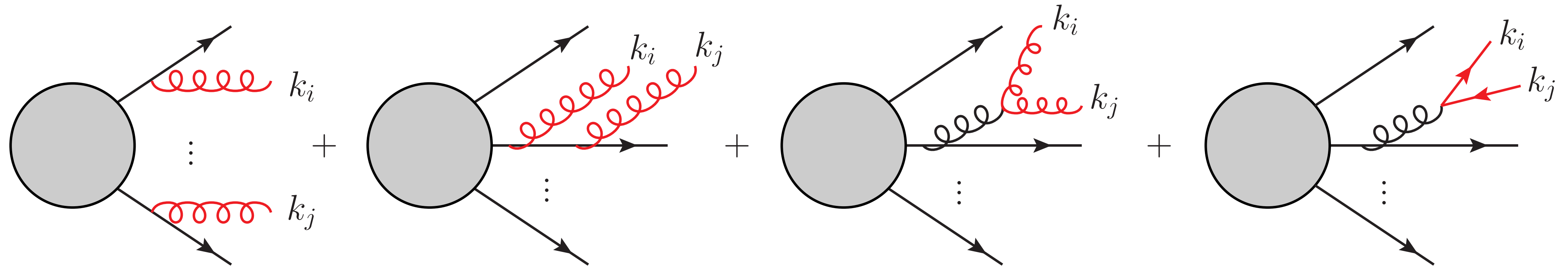
(N3LO kernels partially available *[Catani, Colferai, Torrini 1908.01616, Del Duca, Duhr, Haindl, Lazopoulos Michel 1912.06425, Dixon, Herrmann, Kai Yan, Hua Xing Zhu 1912.09370Yu Jiao Zhu 2009.08919]*)

Why is NNLO so difficult?

Under IR singular limits, the RR factorise into: (universal kernel) x (lower multiplicity matrix elements)

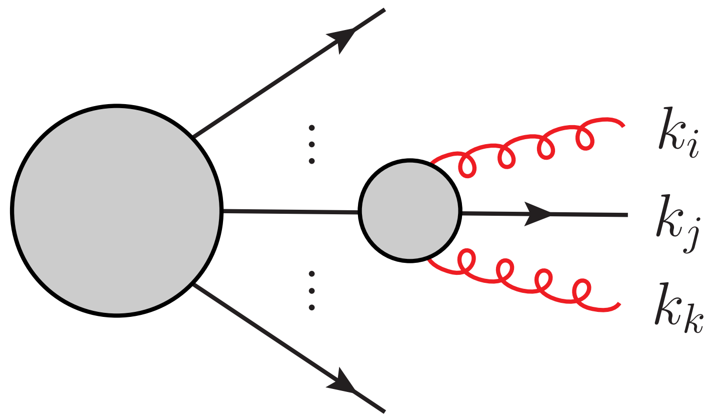
Double soft limit *[Catani, Grazzini 9903516,9810389]*

$$\lim_{k_i, k_j \to 0} RR_{n+2}\left(\{k\}_n, k_i, k_j\right) \sim \mathrm{Eik}\left(\{k\}_n, k_i, k_j\right) \otimes B_n\left(\{k\}_n\right)$$



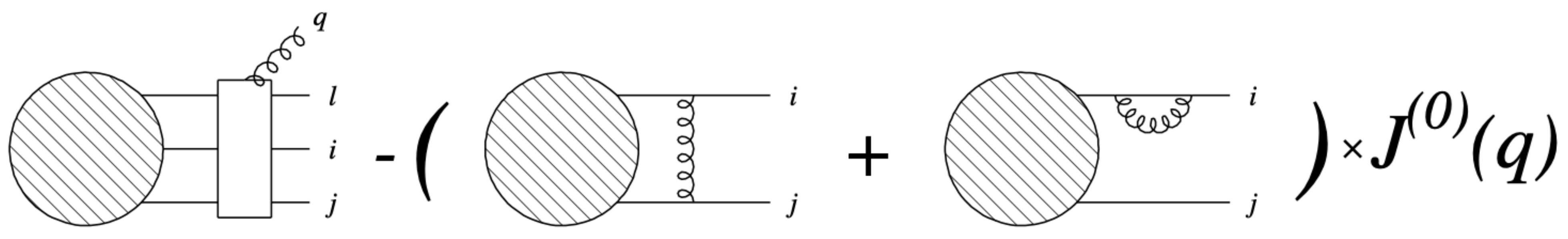
Triple collinear limit *[Catani, Grazzini 9903516,9810389]*

$$\lim_{k_i \| k_j \| k_k} RR_{n+2}\left(\{k\}_{n-1}, k_i, k_j, k_k\right) \sim \frac{1}{s_{ijk}^2} P\left(k_i, k_j, k_k\right) \otimes B_n\left(\{k\}_{n-1}, k_{ijk}\right)$$



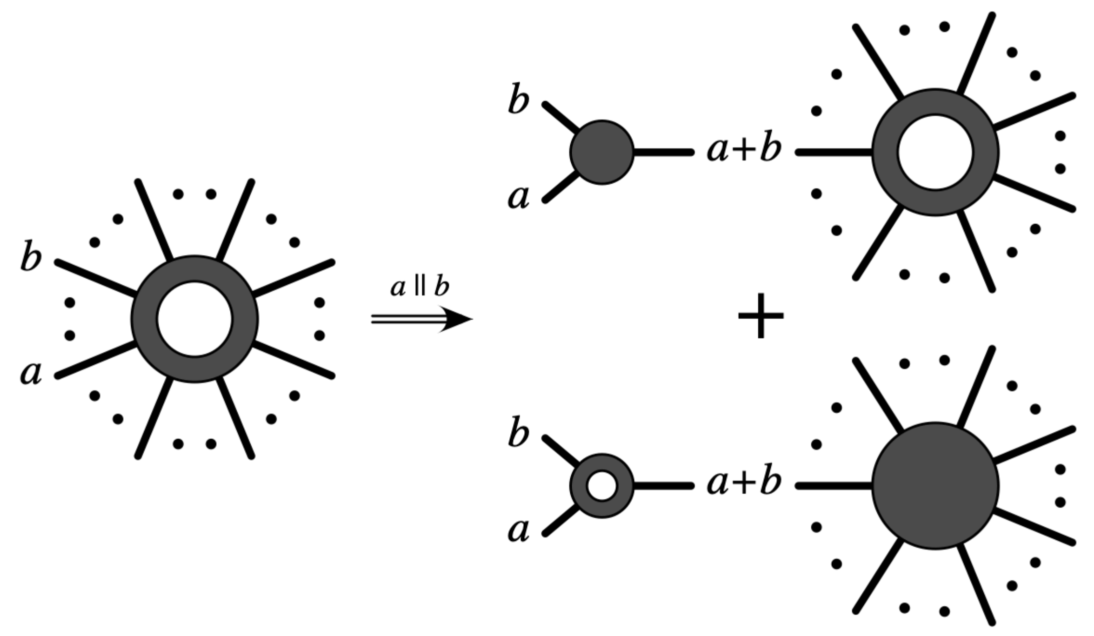
One loop single soft limit *[Catani, Grazzini 0007142]*

$$\lim_{k_i \to 0} RV_{n+1}\left(\{k\}_n, k_i\right) \sim \mathrm{Eik}\left(\{k\}_n, k_i\right) \otimes V_n\left(\{k\}_n\right) + \widetilde{\mathrm{Eik}}\left(\{k\}_n, k_i\right) \otimes B_n\left(\{k\}_n\right)$$



One loop single collinear limit *[Kosower 9901201, Bern, Del Duca, Kilgore, Schmidt 9903516]*

$$\lim_{k_i \| k_{\rightarrow} 0} RV_{n+1}\left(\{k\}_n, k_i\right) \sim \frac{1}{s_{ij}} \left[P\left(k_i, k_j\right) \otimes V_n\left(\{k\}_n\right) + \widetilde{P}\left(k_i, k_j\right) \otimes B_n\left(\{k\}_n\right) \right]$$



$$\mathbf{S}_{ij}\, RR(\{k\}) \propto \sum_{c,d\neq i,j}\left[\sum_{e,f\neq i,j} I^{(i)}_{cd}\, I^{(j)}_{ef}\, B_{cdef}\big(\{k\}_{\not{i}\not{j}}\big) + I^{(ij)}_{cd}\, B_{cd}\big(\{k\}_{\not{i}\not{j}}\big)\right]$$

$$I^{(i)}_{cd} = \frac{s_{cd}}{s_{ic}\, s_{id}} \qquad I^{(ij)}_{cd} = 2\, T_R\, I^{(q\bar{q})(ij)}_{cd} - 2\, C_A\, I^{(gg)(ij)}_{cd} \qquad s_{ab} = 2p_a \cdot p_b$$

$$I^{(q\bar{q})(ij)}_{cd} = \frac{s_{ic}s_{jd} + s_{id}s_{jc} - s_{ij}s_{cd}}{s^2_{ij}(s_{ic} + s_{jc})(s_{id} + s_{jd})} \qquad I^{(gg)(ij)}_{cd} = \frac{(1-\epsilon)(s_{ic}s_{jd} + s_{id}s_{jc}) - 2s_{ij}s_{cd}}{s^2_{ij}(s_{ic} + s_{jc})(s_{id} + s_{jd})} + s_{cd}\,\frac{s_{ic}s_{jd} + s_{id}s_{jc} - s_{ij}s_{cd}}{s_{ij}s_{ic}s_{jd}s_{id}s_{jc}}\left[1 - \frac{1}{2}\,\frac{s_{ic}s_{jd} + s_{id}s_{jc}}{(s_{ic} + s_{jc})(s_{id} + s_{jd})}\right]$$

$$\mathbf{C}_{ijk}\, RR(\{k\}) \propto \frac{1}{s^2_{ijk}}\, P^{\mu\nu}_{ijk}\big(s_{ir}, s_{jr}, s_{kr}\big)\, B_{\mu\nu}\big(\{k\}_{\not{i}\not{j}\not{k}}, k_{ijk}\big) \qquad P^{\mu\nu}_{ijk}B_{\mu\nu} = P_{ijk}\, B + Q^{\mu\nu}_{ijk}\, B_{\mu\nu}$$

$$P^{(3g)}_{ijk} = C_A^2 \left\{ \frac{(1-\epsilon)s^2_{ijk}}{4s^2_{ij}}\left(\frac{s_{jk}}{s_{ijk}} - \frac{s_{ik}}{s_{ijk}} + \frac{z_i - z_j}{z_{ij}}\right)^2 + \frac{s_{ijk}}{s_{ij}}\left[4\,\frac{z_iz_j - 1}{z_{ij}} + \frac{z_iz_j - 2}{z_k} + \frac{(1 - z_kz_{ij})^2}{z_iz_kz_{jk}} + \frac{5}{2}\, z_k + \frac{3}{2}\right]\right.$$

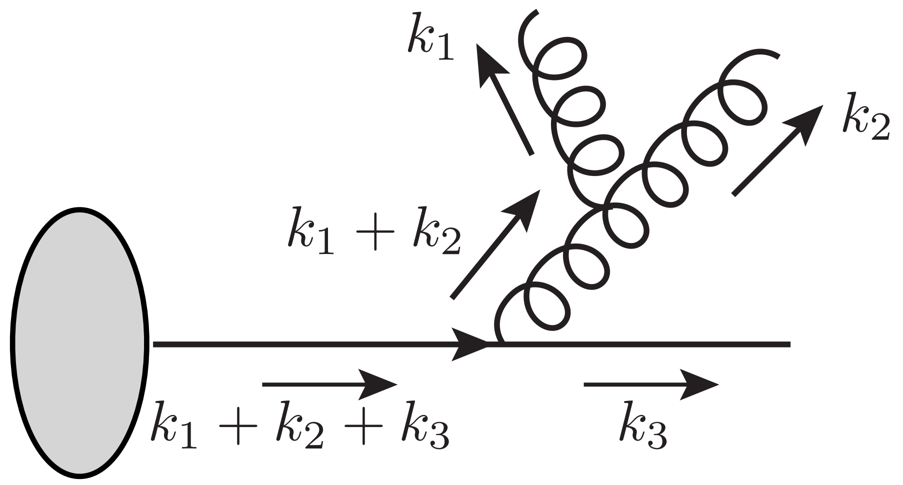
$$\left. + \frac{s^2_{ijk}}{2s_{ij}s_{ik}}\left[\frac{2z_iz_jz_{ik}(1-2z_k)}{z_kz_{ij}} + \frac{1 + 2z_i(1+z_i)}{z_{ik}z_{ij}} + \frac{1 - 2z_iz_{jk}}{z_jz_k} + 2z_jz_k + z_i(1+2z_i) - 4\right] + \frac{3(1-\epsilon)}{4}\right\} + perm\,.$$

$$z_a = \frac{s_{ar}}{s_{ir} + s_{jr} + s_{kr}}\,, \; z_{ab} = z_a + z_b$$

$$Q^{(3g)\mu\nu}_{ijk} = C_A^2\, \frac{s_{ijk}}{s_{ij}} \left\{ \left[\frac{2z_j}{z_k}\frac{1}{s_{ij}} + \left(\frac{z_jz_{ik}}{z_kz_{ij}} - \frac{3}{2}\right)\frac{1}{s_{ik}}\right]\tilde{k}_i^2\, q_i^{\mu\nu} + \left[\frac{2z_i}{z_k}\frac{1}{s_{ij}} - \left(\frac{z_jz_{ik}}{z_kz_{ij}} - \frac{3}{2} - \frac{z_i}{z_k} + \frac{z_i}{z_{ij}}\right)\frac{1}{s_{ik}}\right]\tilde{k}_j^2\, q_j^{\mu\nu} - \left[\frac{2z_iz_j}{z_{ij}z_k}\frac{1}{s_{ij}} + \left(\frac{z_jz_{ik}}{z_kz_{ij}} - \frac{3}{2} - \frac{z_i}{z_j} + \frac{z_i}{z_{ik}}\right)\frac{1}{s_{ik}}\right]\tilde{k}_k^2\, q_k^{\mu\nu}\right\} + perm\,.$$

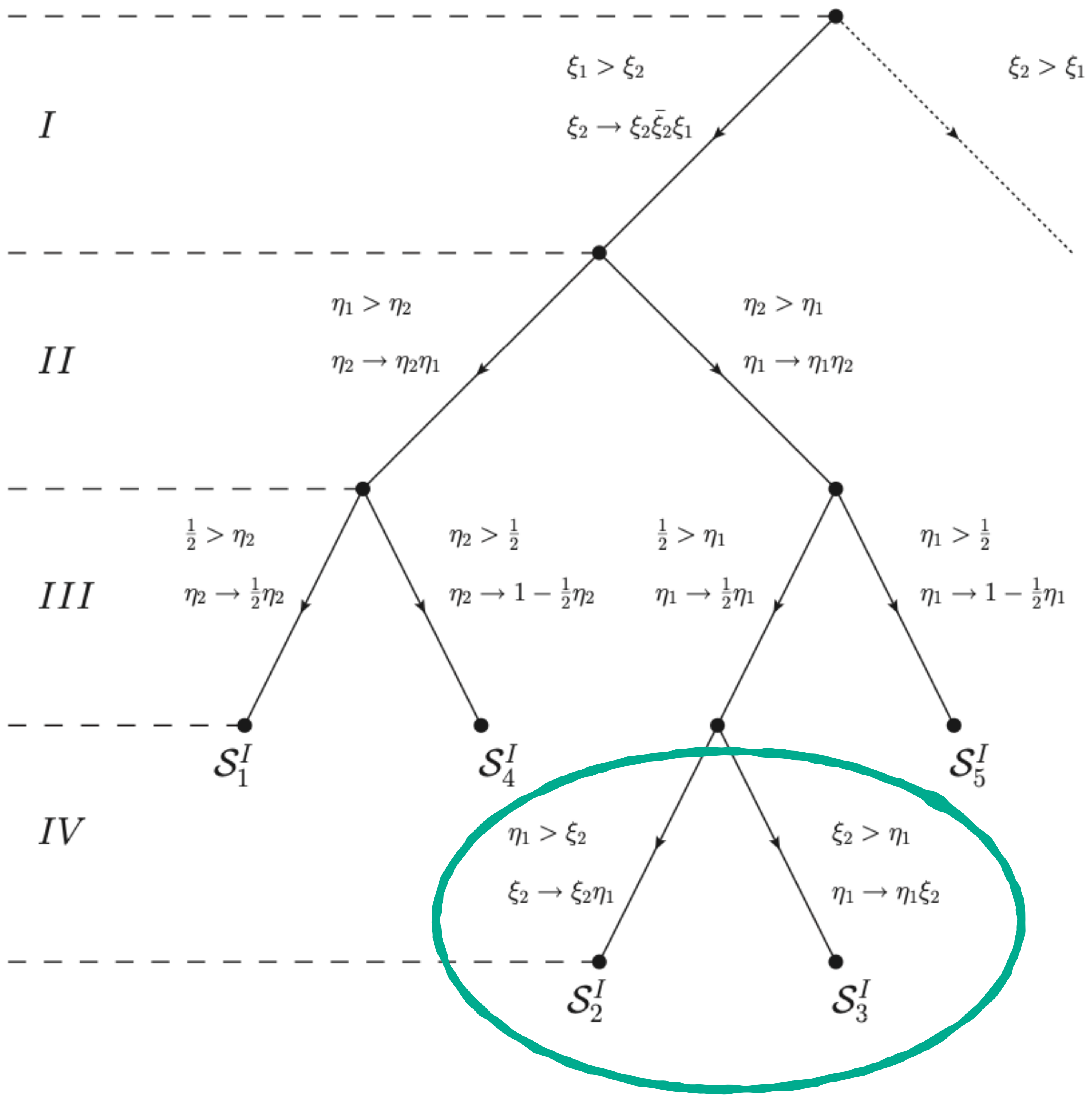
Key problem: several **different invariants** combined into **non-trivial** and various **structures**, to be integrated over a **6-dim PS.**

Clear understanding of which singular configurations do actually contribute



$$\sim \frac{1}{(k_1+k_2)^2}\,\frac{1}{(k_1+k_2+k_3)^2} = \frac{1}{2k_1\cdot k_2}\,\frac{1}{2k_1\cdot k_2 + 2k_1\cdot k_3 + 2k_2\cdot k_3} \iff k_1 \to 0 \text{ and } k_2 \parallel k_3$$

Entangled soft-collinear limits of diagrams can not be treated in a process-independent way.
Do non-commutative limits actually contribute?



STRIPPER was implemented taking into account all the possible choices of soft and collinear limits order -> redundant configurations were included

Gauge invariant amplitudes are free of entangled singularities
thanks to color coherence: soft parton does not resolve angles of the collinear partons

Soft-collinear limits can be described by taking the known soft and collinear limits sequentially

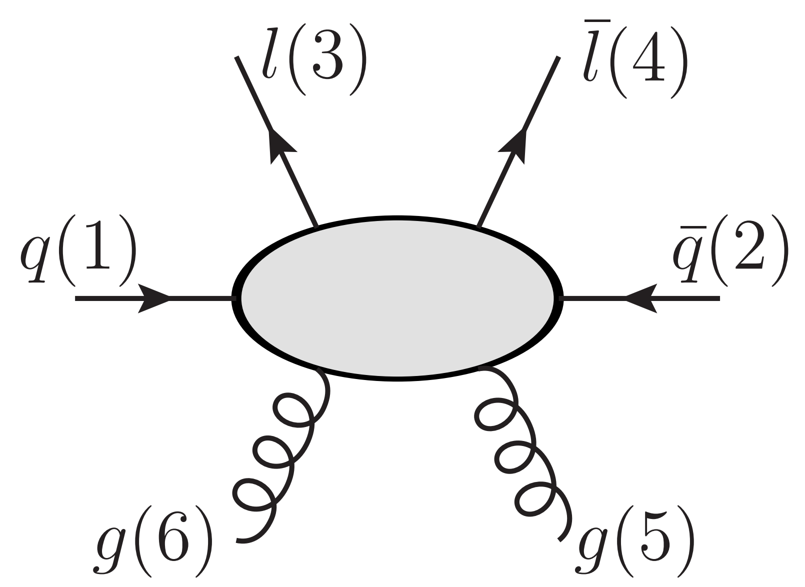
Phase space partitions

Efficient way to simplify the problem: introduce **partition functions** (following FKS philosophy):

- **Unitary partition**
- Select a **minimum number of singularities** in each sector
- Do **not affect** the **analytic integration** of the counterterms

Definition of partition functions benefits from remarkable degree of **freedom**: different approaches can be implemented

Examples: Nested soft-collinear subtraction $q\bar{q} \to Z \to e^- e^+ g\, g$ [Caola, Melnikov, Röntsch 1702.01352]



$$1 = \omega^{51,61} + \omega^{52,62} + \omega^{51,62} + \omega^{52,61}$$

$$\omega^{51,61} = \frac{\rho_{25}\,\rho_{26}}{d_5\, d_6}\left(1 + \frac{\rho_{15}}{d_{5621}} + \frac{\rho_{16}}{d_{5612}}\right) \qquad \omega^{51,62} = \frac{\rho_{25}\,\rho_{16}\,\rho_{56}}{d_5\, d_6\, d_{5612}}$$

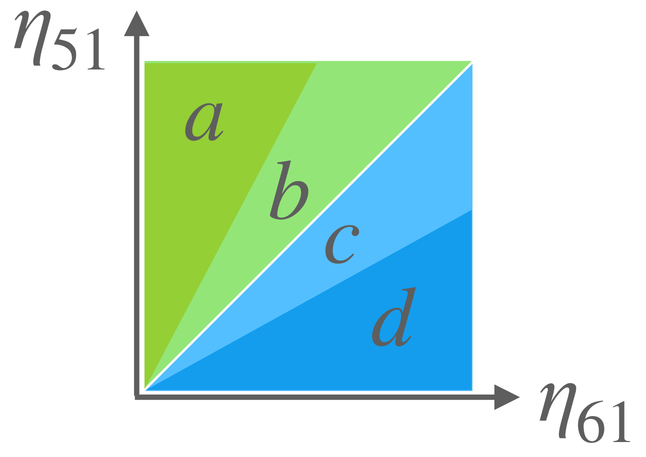
$$\omega^{52,62} = \frac{\rho_{15}\,\rho_{16}}{d_5\, d_6}\left(1 + \frac{\rho_{25}}{d_{5621}} + \frac{\rho_{26}}{d_{5612}}\right) \qquad \omega^{52,61} = \frac{\rho_{15}\,\rho_{26}\,\rho_{56}}{d_5\, d_6\, d_{5621}}$$

$$\rho_{ab} = 1 - \cos\vartheta_{ab}\,,\; \eta_{ab} = \rho_{ab}/2$$

$$d_{i=5,6} = \rho_{1i} + \rho_{2i} = 2$$

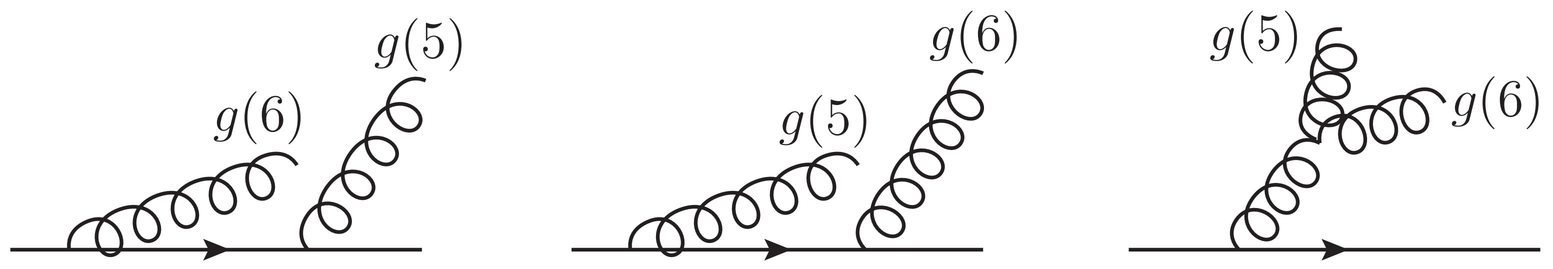
$$d_{5621} = \rho_{56} + \rho_{52} + \rho_{61}$$

$$d_{5612} = \rho_{56} + \rho_{51} + \rho_{62}$$



$$1 = \theta\!\left(\eta_{61} < \frac{\eta_{51}}{2}\right) + \theta\!\left(\frac{\eta_{51}}{2} < \eta_{61} < \eta_{51}\right) + \theta\!\left(\eta_{51} < \frac{\eta_{61}}{2}\right) + \theta\!\left(\frac{\eta_{61}}{2} < \eta_{51} < \eta_{61}\right)$$

$$= \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}$$



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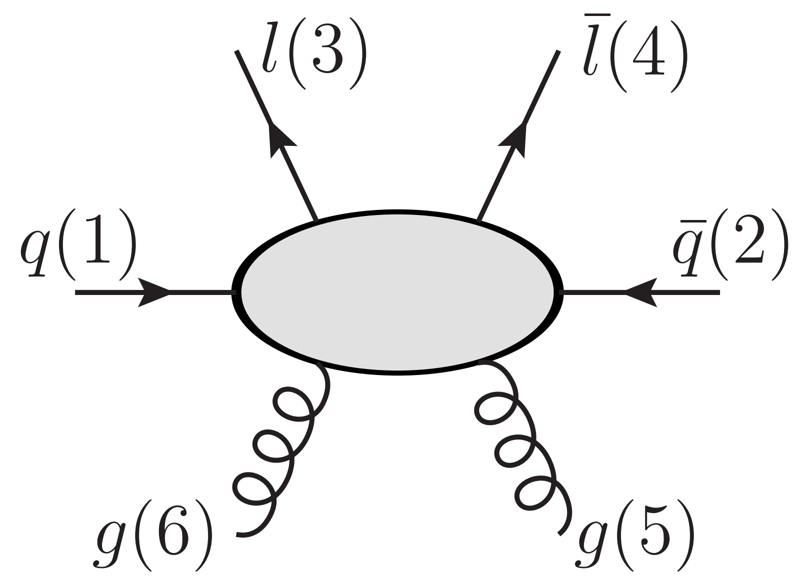
Examples: Nested soft-collinear subtraction $q\bar{q} \to Z \to e^- e^+ \, g \, g$ *[Caola, Melnikov, Röntsch 1702.01352]*



Advantages:

1. Simple definition
2. Structure of collinear singularities fully defined
3. Same strategy holds for NNLO mixed QCDxEW processes
4. **Minimum number of sector**

Disadvantages:

1. Partition based on angular ordering -> Lorentz invariance not preserved
   -> angles defined in a given reference frame
2. Theta function

3. Solve the PS integrals

The problem is now well defined:

A. **Singular kernels** and their nested limits have to be **subtracted from the double real correction** to get integrable object

$$\int d\Phi_{n+2}\, RR_{n+2} = \int d\Phi_{n+2} \underbrace{\left[RR_{n+2} - K_{n+2}\right]} + \int d\Phi_{n+2}\, K_{n+2} \qquad K_{n+2} \supset C_{ij},\, C_{kl},\, S_i,\, S_{ij},\, C_{ijk}$$

Fully regulated real emission contribution → Numerical evaluation

Examples: Nested soft-collinear subtraction $q\bar{q} \to Z \to e^- e^+ g\, g$ [Caola, Melnikov, Röntsch]

$$[\,\mathrm{d}f_i\,] = \frac{\mathrm{d}^d k_i}{(2\pi)^d}(2\pi)\,\delta_+(k_i^2)$$

$$\begin{aligned}
\mathrm{d}\hat{\sigma}^{NNLO}_{\text{resolv.}} = \int \theta(E_5 - E_6)\,\theta(E_{\max} - E_5) \Bigg\{ &\sum_{i,j\in\{1,2\}, i\neq j} \left(1 - C_{5i}\right)\left(1 - C_{6j}\right)\left(1 - S_{56}\right)\left(1 - S_6\right)[\mathrm{d}k_5]\,[\mathrm{d}k_6]\,\omega^{5i,6j}\, B\big(\{k\}_{1\ldots 6}\big) \\
&+ \sum_{i\in\{1,2\}} \Big[\theta^{(a)}\left(1 - C_{i56}\right)\left(1 - C_{6i}\right) + \theta^{(b)}\left(1 - C_{i56}\right)\left(1 - C_{56}\right) \\
&\quad + \theta^{(c)}\left(1 - C_{i56}\right)\left(1 - C_{5i}\right) + \theta^{(d)}\left(1 - C_{i56}\right)\left(1 - C_{56}\right)\Big][\mathrm{d}k_5]\,[\mathrm{d}k_6]\,\omega^{5i,6i}\, B\big(\{k\}_{1\ldots 6}\big)\Bigg\}
\end{aligned}$$

Explicit expression depends on the scheme

3. Solve the PS integrals

The problem is now well defined:

A. **Singular kernels** and their nested limits have to be **subtracted from the double real correction** to get integrable object

$$\int d\Phi_{n+2}\, RR_{n+2} = \int d\Phi_{n+2} \left[RR_{n+2} - K_{n+2} \right] + \int d\Phi_{n+2}\, K_{n+2} \qquad K_{n+2} \supset C_{ij},\, C_{kl},\, S_i,\, S_{ij},\, C_{ijk}$$

B. **Counterterms** have to be **integrated over the unresolved phase space**

$$I = \int \mathrm{PS}_{\text{unres.}} \otimes \text{Limit} \otimes \text{Constraints}$$

The 'Limit' component is universal and known. The phase space is well defined. Constraints may vary depending on the scheme.

Several kinematic structures have to be integrated **analytically** over a 6-dim PS.

Different approximations and techniques can be applied: the results assume different form depending on the adopted strategy

Two main structure are the most complicated ones and affect most of the physical processes:

- **Double soft**
- **Triple collinear**

Kernels integration

Examples: Nested soft-collinear subtraction $q\bar{q} \to Z \to e^- e^+ g\, g$ *[Caola, Delto, Frellesvig, Melnikov 1807.05835, Delto, Melnikov 1901.05213]*

Two soft parton (5,6) and two hard massless radiator (1,2): arbitrary relative angle between the three-momenta of the radiators

$$I_{12}^{(gg)(56)} = \frac{(1-\epsilon)(s_{51}s_{62} + s_{52}s_{61}) - 2s_{56}s_{12}}{s_{56}^2(s_{51} + s_{61})(s_{52} + s_{62})} + s_{12}\,\frac{s_{51}s_{62} + s_{52}s_{61} - s_{56}s_{12}}{s_{56}s_{51}s_{62}s_{52}s_{61}}\left[1 - \frac{1}{2}\,\frac{s_{51}s_{62} + s_{52}s_{61}}{(s_{51} + s_{61})(s_{52} + s_{62})}\right]$$

$$I_{S_{56}}^{(gg)} = \int [\mathrm{d}k_5]\,[\mathrm{d}k_6]\,\theta(E_{\max} - E_5)\,\theta(E_5 - E_6)\,I_{12}^{(gg)(56)}\big(k_1, k_2, k_5, k_6\big) \qquad [\,\mathrm{d}f_i\,] = \frac{\mathrm{d}^d k_i}{(2\pi)^d}\,(2\pi)\,\delta_+(k_i^2)$$

$$E_5 = E_{\max}\,\xi \qquad E_6 = E_{\max}\,\xi\,z \qquad 0 < \xi < 1\,,\, 0 < z < 1$$

Reverse unitarity: **map phase space integrals onto loop integrals** *[Anastasiou, Melnikov 0207004]*
after defining integral families, integration-by-part identities. Differential equations w.r.t. the ratio of energies of emitted gluons at fixed angle.
Boundary conditions for z=0, and arbitrary angle

Kernels integration

Examples: Nested soft-collinear subtraction $q\bar{q} \to Z \to e^- e^+ g\, g$ *[Caola, Delto, Frellesvig, Melnikov 1807.05835, Delto, Melnikov 1901.05213]*

$$
\begin{aligned}
I_{S_{56}}^{(gg)} &= (2E_{\max})^{-4\epsilon}\left[\frac{1}{8\pi^2}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\right]^2\left\{\frac{1}{2\epsilon^4}+\frac{1}{\epsilon^3}\left[\frac{11}{12}-\ln(s^2)\right]\right.\\
&+\frac{1}{\epsilon^2}\left[2\mathrm{Li}_2(c^2)+\ln^2(s^2)-\frac{11}{6}\ln(s^2)+\frac{11}{3}\ln 2-\frac{\pi^2}{4}-\frac{16}{9}\right]\\
&+\frac{1}{\epsilon}\left[6\mathrm{Li}_3(s^2)+2\mathrm{Li}_3(c^2)+\left(2\ln(s^2)+\frac{11}{3}\right)\mathrm{Li}_2(c^2)-\frac{2}{3}\ln^3(s^2)\right.\\
&\quad+\left(3\ln(c^2)+\frac{11}{6}\right)\ln^2(s^2)-\left(\frac{22}{3}\ln 2+\frac{\pi^2}{2}-\frac{32}{9}\right)\ln(s^2)\\
&\quad\left.-\frac{45}{4}\zeta_3-\frac{11}{3}\ln^2 2-\frac{11}{36}\pi^2-\frac{137}{18}\ln 2+\frac{217}{54}\right]\\
&+4\mathrm{G}_{-1,0,0,1}(s^2)-7\mathrm{G}_{0,1,0,1}(s^2)+\frac{22}{3}\mathrm{Ci}_3(2\delta)+\frac{1}{3\tan(\delta)}\mathrm{Si}_2(2\delta)\\
&+2\mathrm{Li}_4(c^2)-14\mathrm{Li}_4(s^2)+4\mathrm{Li}_4\left(\frac{1}{1+s^2}\right)-2\mathrm{Li}_4\left(\frac{1-s^2}{1+s^2}\right)\\
&+2\mathrm{Li}_4\left(\frac{s^2-1}{1+s^2}\right)+\mathrm{Li}_4(1-s^4)+\left[10\ln(s^2)-4\ln\left(1+s^2\right)\right.\\
&\left.+\frac{11}{3}\right]\mathrm{Li}_3(c^2)+\left[14\ln(c^2)+2\ln(s^2)+4\ln\left(1+s^2\right)+\frac{22}{3}\right]\mathrm{Li}_3(s^2)\\
&+4\ln(c^2)\mathrm{Li}_3(-s^2)+\frac{9}{2}\mathrm{Li}_2^2(c^2)-4\mathrm{Li}_2(c^2)\mathrm{Li}_2(-s^2)+\left[7\ln(c^2)\ln(s^2)\right.\\
&\left.-\ln^2(s^2)-\frac{5}{2}\pi^2+\frac{22}{3}\ln 2-\frac{131}{18}\right]\mathrm{Li}_2(c^2)+\left[\frac{2}{3}\pi^2-4\ln(c^2)\ln(s^2)\right]\times\\
&\mathrm{Li}_2(-s^2)+\frac{\ln^4(s^2)}{3}+\frac{\ln^4\left(1+s^2\right)}{6}-\ln^3(s^2)\left[\frac{4}{3}\ln(c^2)+\frac{11}{9}\right]\\
&+\ln^2(s^2)\left[7\ln^2(c^2)+\frac{11}{3}\ln(c^2)+\frac{\pi^2}{3}+\frac{22}{3}\ln 2-\frac{32}{9}\right]-\frac{\pi^2}{6}\ln^2\left(1+s^2\right)\\
&+\zeta_3\left[\frac{17}{2}\ln(s^2)-11\ln(c^2)+\frac{7}{2}\ln\left(1+s^2\right)-\frac{21}{2}\ln 2-\frac{99}{4}\right]+\ln(s^2)\times\\
&\left[-\frac{7\pi^2}{2}\ln(c^2)+\frac{22}{3}\ln^2 2-\frac{11}{18}\pi^2+\frac{137}{9}\ln 2-\frac{208}{27}\right]-12\mathrm{Li}_4\left(\frac{1}{2}\right)\\
&+\frac{143}{720}\pi^4-\frac{\ln^4 2}{2}+\frac{\pi^2}{2}\ln^2 2-\frac{11}{6}\pi^2\ln 2+\frac{125}{216}\pi^2+\frac{22}{9}\ln^3 2\\
&\left.+\frac{137}{18}\ln^2 2+\frac{434}{27}\ln 2-\frac{649}{81}+\mathcal{O}(\epsilon)\right\},
\end{aligned}
$$

$$\delta = \frac{\delta_{12}}{2},\; s = \sin\frac{\delta_{12}}{2},\; c = \cos\frac{\delta_{12}}{2}$$

$$\mathrm{Ci}_n(z) = \frac{\mathrm{Li}_n(e^{iz})+\mathrm{Li}_n(e^{-iz})}{2},\; \mathrm{Si}_n(z) = \frac{\mathrm{Li}_n(e^{iz})-\mathrm{Li}_n(e^{-iz})}{2i}$$

Singular structure of the RR

Under fundamental limits, the RR factorise into: (universal kernel) x (lower multiplicity matrix elements)

$$\mathbf{S}_{ij}\, RR(\{k\}) \propto \sum_{c,d\neq i,j} \left[\sum_{e,f\neq i,j} I_{cd}^{(i)}\, I_{ef}^{(j)}\, B_{cdef}\big(\{k\}_{\not{i}\not{j}}\big) + I_{cd}^{(ij)}\, B_{cd}\big(\{k\}_{\not{i}\not{j}}\big) \right]$$

$$\mathbf{C}_{ijk}\, RR(\{k\}) \propto \frac{1}{s_{ijk}^2}\, P_{ijk}^{\mu\nu}\big(s_{ir}, s_{jr}, s_{kr}\big)\, B_{\mu\nu}\big(\{k\}_{\not{i}\not{j}\not{k}}, k_{ijk}\big)$$

$$\mathbf{C}_{ijkl}\, RR(\{k\}) \propto \frac{1}{s_{ij}\, s_{kl}}\, P_{ij}^{\mu\nu}\big(s_{ir}, s_{jr}\big)\, P_{kl}^{\rho\sigma}\big(s_{kr'}, s_{lr'}\big)\, B_{\mu\nu\rho\sigma}\big(\{k\}_{\not{i}\not{j}\not{k}\not{l}}, k_{ij}, k_{kl}\big)$$

$$\mathbf{SC}_{ijk}\, RR(\{k\}) = \mathbf{CS}_{jki} RR(\{k\}) \propto \frac{1}{s_{jk}} \sum_{c,d\neq i} P_{jk}^{\mu\nu}\, I_{cd}^{(i)}\, B_{\mu\nu}^{cd}\big(\{k\}_{\not{i}\not{j}\not{k}}, k_{jk}\big)$$

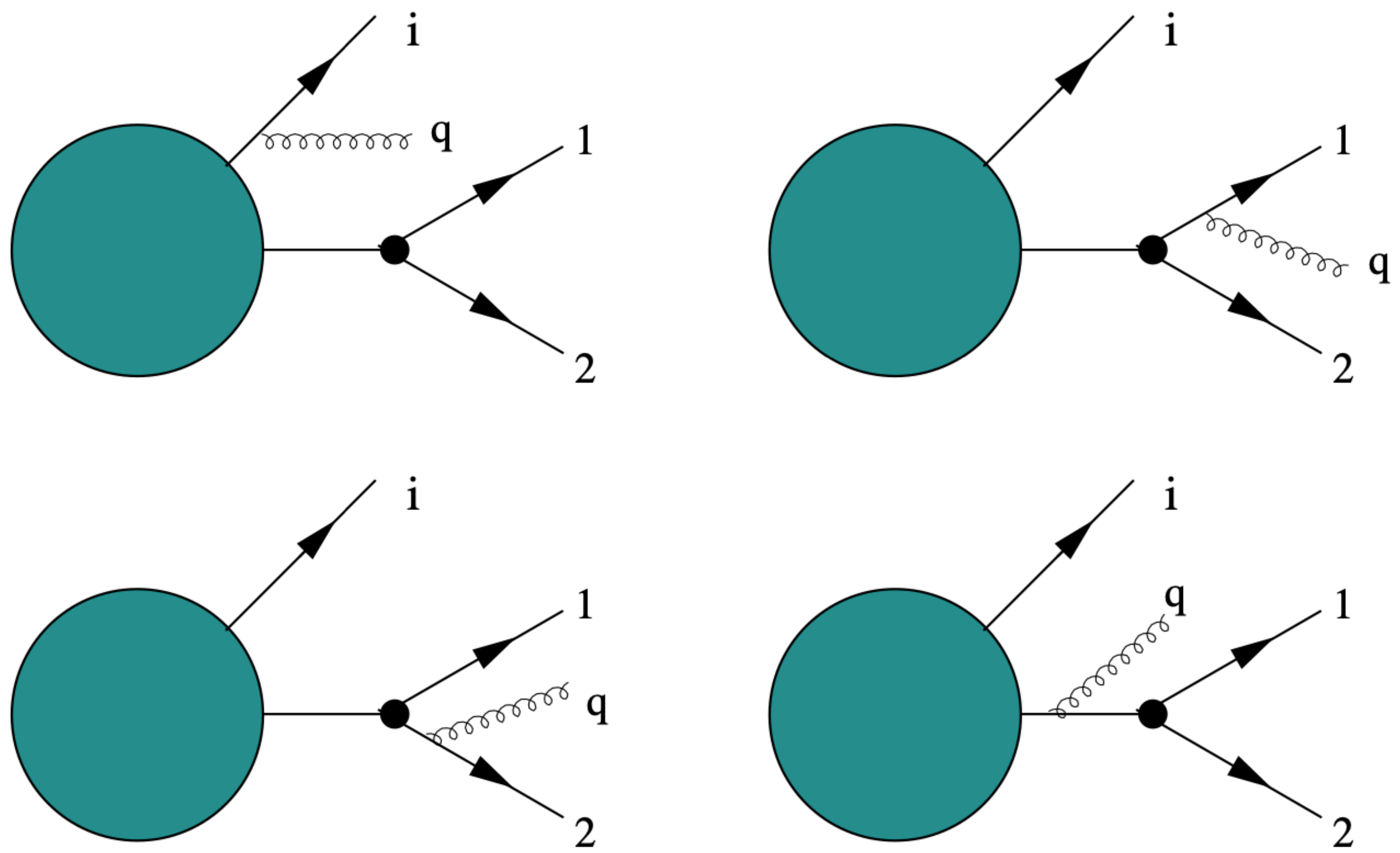
Born-level kinematics does not satisfy the mass-shell condition and momentum conservation

→ Momentum mapping needed!

Colour coherence and disentangled soft-collinear singularities

Parton q is soft and partons $1{,}2$ are collinear *[Catani, Grazzini 9908523]*

$$\left|\mathcal{M}_{g,a_1,a_2,\ldots,a_n}(q,p_1,p_2,\ldots,p_n)\right|^2 \simeq -\frac{2}{s_{12}}(4\pi\mu^{2\epsilon}\alpha_s)^2 \left\langle \mathcal{M}_{a,a_3,\ldots,a_n}(p,p_3,\ldots,p_n)\right| \hat{\mathbf{P}}_{a_1a_2}\,\left[\mathbf{J}^\dagger_{(12)\mu}(q)\,\mathbf{J}^\mu_{(12)}(q)\right]\left|\mathcal{M}_{a,a_3,\ldots,a_n}(p,p_3,\ldots,p_n)\right\rangle$$



Mother parton: $a \to a_1 + a_2$

Altarelli-Parisi splitting functions: spin correlations

Soft current: colour correlations

$$\mathbf{J}^\dagger_{(12)\mu}(q)\,\mathbf{J}^\mu_{(12)}(q) \simeq \sum_{i,j=3}^{n} \mathbf{T}_i\cdot\mathbf{T}_j\,\mathcal{S}_{ij}(q) + 2\sum_{i=3}^{n}\mathbf{T}_i\cdot\mathbf{T}_{(12)}\,\mathcal{S}_{i\,(12)}(q)$$

$$\mathbf{T}_{(12)} = \mathbf{T}_1 + \mathbf{T}_2 \qquad \mathcal{S}_{ij}(q) = \frac{2\,s_{ij}}{s_{iq}\,s_{jq}} \qquad \mathcal{S}_{i\,(12)}(q) = \frac{2\,(s_{i1}+s_{i2})}{s_{iq}\,(s_{1q}+s_{2q})}$$

The soft-collinear limit at $\mathcal{O}\left(\alpha_s^2\right)$ is fully described in a factorised way, where the factors are the soft eikonal function and the Altarelli-Parisi splitting functions that control IR limits at $\mathcal{O}\left(\alpha_s\right)$.

This simplification, which is due to colour coherence, was not performed in FKS.

Mixed QCDxEW corrections: the real-virtual component

1-loop correction to $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)\gamma(p_5) + e^+(p_3)\,e^-(p_4)g(p_5)$

IR subtraction proceeds as for NLO:

- soft singularities are extracted first
- Collinear singularities are extracted from the the soft-regulated term

$$
\begin{aligned}
2s \cdot d\sigma^{\rm RV} = &\left\langle S_g\, F^{\rm EW}_{LRV}\,(1{,}2{,}3{,}4\,|\,5_g)\right\rangle + \left\langle S_\gamma\, F^{\rm QCD}_{LRV}\,(1{,}2{,}3{,}4\,|\,5_\gamma)\right\rangle \\
&+\left\langle (I - S_g)\,(C_{g1} + C_{g2})\, F^{\rm EW}_{LRV}\,(1{,}2{,}3{,}4\,|\,5_g)\right\rangle + \sum_{k=1}^{4}\left\langle (I - S_\gamma)\, C_{\gamma k}\, F^{\rm QCD}_{LRV}\,(1{,}2{,}3{,}4\,|\,5_\gamma)\right\rangle \\
&+\left\langle \mathcal{O}_{\rm nlo}\, F^{\rm EW}_{LRV}(1{,}2{,}3{,}4\,|\,5_g)\right\rangle + \left\langle \mathcal{O}_{\rm nlo}\, F^{\rm QCD}_{LRV}(1{,}2{,}3{,}4\,|\,5_\gamma)\right\rangle
\end{aligned}
$$

Several simplifications occur with respect to the pure QCD case:

- soft limit simplify into abelian-like contributions

$$
S_g\, F^{\rm EW}_{LRV}(1{,}2{,}3{,}4\,|\,5_g) = 2C_F g^2_{s,b}\,\frac{\rho_{12}}{E_5^2\,\rho_{15}\,\rho_{25}}\, F^{EW}_{LV}(1{,}2{,}3{,}4)
$$

- One-loop collinear kernel can be obtain via abelianization > fewer terms

$$
C_{g1}\, F^{\rm EW}_{LRV}(1{,}2{,}3{,}4\,|\,5_g) = \frac{g^2_{s,b}}{E_5^2\,\rho_{51}}\left[(1-z)\,C_F\,P_{qq}(z)\,\frac{F^{\rm EW}_{LV}(z\cdot 1{,}2{,}3{,}4)}{z} + [\alpha]\,C_F\,Q_q^2\,\frac{\Gamma^3(1-\epsilon)\,\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\,\frac{(2E_1^2)^{-\epsilon}}{\rho^\epsilon_{15}}\,P^{\rm loop,1}_{qq}(z)\,\frac{F_{LM}(z\cdot 1{,}2{,}3{,}4)}{z}\right]
$$

Mixed QCDxEW corrections: the double real component

Mixed QCD×EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

$$\left\langle (I - S_g)(I - S_\gamma)\,F_{LM}\,(1,2,3,4\,|\,5,6) \right\rangle = \left\langle F_{LM}^{S_r C_s}\,(1,2,3,4\,|\,5,6) \right\rangle + \left\langle F_{LM}^{S_r C_t}\,(1,2,3,4\,|\,5,6) \right\rangle + \left\langle F_{LM}^{S_r C_r}\,(1,2,3,4\,|\,5,6) \right\rangle$$

Contains **soft-regulated single collinear singularities**
> matrix elements of lower multiplicity

Contains **soft-regulated triple collinear singularities**
> matrix elements of lower multiplicity
> partition functions select 1 strongly-ordered limit each

All singularities removed through nested subtractions:
> evaluable in 4-dimensions
> only terms involving fully-resolved real-real matrix element

Analytic results for non trivial integrals from triple collinear (and for double soft) limit calculated in *[Caola, Delto, Frellesvig, Melnikov '18, Delto, Melnikov '19]*

After integration subtraction terms return lower multiplicity terms multiplied by explicit poles

$\left\langle F_{LM}\,(1,2,3,4) \right\rangle$

Elastic LO-like

$\left\langle F_{LM}\,(z_1 \cdot 1, z_2 \cdot 2,3,4) \right\rangle$

$\left\langle F_{LM}\,(z \cdot 1,2,3,4) \right\rangle$

$\left\langle F_{LM}\,(1, z \cdot 2,3,4) \right\rangle$

Boosted LO-like

$\left\langle \mathcal{O}^g_{\rm nlo}\,F_{LM}\,(1,2,3,4\,|\,5_g) \right\rangle$

$\left\langle \mathcal{O}^\gamma_{\rm nlo}\,F_{LM}\,(1,2,3,4\,|\,5_\gamma) \right\rangle$

Elastic NLO-like

$\left\langle \mathcal{O}^g_{\rm nlo}F_{LM}\,(z \cdot 1,2,3,4\,|\,5_g) \right\rangle$ $\left\langle \mathcal{O}^\gamma_{\rm nlo}F_{LM}\,(1, z \cdot 2,3,4\,|\,5_\gamma) \right\rangle$

$\left\langle \mathcal{O}^g_{\rm nlo}F_{LM}\,(1, z \cdot 2,3,4\,|\,5_g) \right\rangle$ $\left\langle \mathcal{O}^\gamma_{\rm nlo}F_{LM}\,(z \cdot 1,2,3,4\,|\,5_\gamma) \right\rangle$

Boosted NLO-like

Mixed QCDxEW corrections: the double real component

Mixed QCD×EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

$$\left\langle (I - S_g)(I - S_\gamma)\, F_{LM}(1,2,3,4\,|\,5,6) \right\rangle = \sum_{i=1}^{4} \left\langle (I - S_g)(I - S_\gamma)\, \Omega_i^{q\bar{q}}\, F_{LM}(1,2,3,4\,|\,5,6) \right\rangle$$

$$\Omega_1^{q\bar{q}} = (1 - C_{g\gamma,1})(1 - C_{g1})\,\omega^{\gamma 1, g1}\,\theta_a + (1 - C_{g\gamma,1})(1 - C_{\gamma 1})\,\omega^{\gamma 1, g1}\,\theta_b + \ldots$$

All singularities removed via nested subtractions:
> evaluable in 4-dimensions
> only terms involving fully-resolved RR matrix element

Triple-collinear counterterm:

$$\Omega_2^{q\bar{q}} = C_{g\gamma,1}(1 - C_{g1})\,\omega^{\gamma 1, g1}\,\theta_A + C_{g\gamma,1}(1 - C_{\gamma 1})\,\omega^{\gamma 1, g1}\,\theta_B + (1 \leftrightarrow 2)$$

Analytic results for non trivial integrals from triple collinear limit calculated in *[Caola, Delto, Frellesvig, Melnikov ’18, Delto, Melnikov ’19]*

After integration subtraction terms return lower multiplicity terms multiplied by explicit poles

$\left\langle F_{LM}(1,2,3,4) \right\rangle$

Elastic LO-like

$\left\langle F_{LM}(z_1 \cdot 1, z_2 \cdot 2,3,4) \right\rangle$
$\left\langle F_{LM}(z \cdot 1,2,3,4) \right\rangle$
$\left\langle F_{LM}(1, z \cdot 2,3,4) \right\rangle$

Boosted LO-like

$\left\langle \mathcal{O}^g_{\rm nlo}\, F_{LM}(1,2,3,4\,|\,5_g) \right\rangle$
$\left\langle \mathcal{O}^\gamma_{\rm nlo}\, F_{LM}(1,2,3,4\,|\,5_\gamma) \right\rangle$

Elastic NLO-like

$\left\langle \mathcal{O}^g_{\rm nlo} F_{LM}(z \cdot 1,2,3,4\,|\,5_g) \right\rangle$ $\left\langle \mathcal{O}^\gamma_{\rm nlo} F_{LM}(1, z \cdot 2,3,4\,|\,5_\gamma) \right\rangle$
$\left\langle \mathcal{O}^g_{\rm nlo} F_{LM}(1, z \cdot 2,3,4\,|\,5_g) \right\rangle$ $\left\langle \mathcal{O}^\gamma_{\rm nlo} F_{LM}(z \cdot 1,2,3,4\,|\,5_\gamma) \right\rangle$

Boosted NLO-like

Nested soft-collinear subtraction: NLO EW

NLO EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

- Basic ingredients

$$\mathrm{d}\sigma^{\mathrm{nlo}} = \mathrm{d}\sigma^{\mathrm{R}} + \mathrm{d}\sigma^{\mathrm{V}} + \mathrm{d}\sigma^{\mathrm{PDF}}$$

Real contribution — Virtual correction — PDF renormalisation

- Real radiation

$$\mathrm{d}\sigma^{\mathrm{R}} = \frac{1}{2s}\int [\mathrm{d}p_\gamma]\, F_{LM}(1,2,3,4;5) \equiv \left\langle F_{LM}(1,2,3,4;5)\right\rangle$$

$$[\,\mathrm{d}f_i\,] = \frac{\mathrm{d}^{d-1}k_i}{(2\pi)^{d-1}\,2E_k}\,\theta(E_{\max} - E_k)$$

Unresolved phase space (partonic CoM) — Arbitrary parameter

$$F_{LM}(1,2,3,4;5) = \mathrm{d}\,\mathrm{PS}_{3,4}\left|\mathcal{M}(1,2,3,4;5)\right|^2 \mathcal{O}_{\mathrm{kin}}(1,2,3,4;5)$$

Lorentz-invariant phase space for leptons (incl. conservation δ) — Matrix element squared — IR-safe observable

- Soft and collinear operators

$$S_i\,A = \lim_{E_i \to 0} A\,, \qquad C_{ij}\,A = \lim_{\rho_{ij}\to 0} A\,.$$

$$\rho_{ij} = 1 - \cos\vartheta_{ij}\,, \quad \eta_{ij} = \rho_{ij}/2$$

Nested soft-collinear subtraction: NLO EW

NLO EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

- Complete IR subtraction

$$\langle F_{LM}(1,2,3,4\,|\,5)\rangle = \langle S_5\, F_{LM}(1,2,3,4\,|\,5)\rangle + \langle (I - S_5)\, F_{LM}(1,2,3,4\,|\,5)\rangle$$

$$= \langle S_5\, F_{LM}(1,2,3,4\,|\,5)\rangle + \sum_{i=1}^{4} \langle C_{5i}\,(I - S_5)\, F_{LM}(1,2,3,4\,|\,5)\rangle + \sum_{i=1}^{4} \langle \mathcal{O}_{\rm nlo}^{(i)}\, \omega^{5i}\, F_{LM}(1,2,3,4\,|\,5)\rangle$$

Soft subtraction term: complete decoupling of the unresolved photon

Hard-collinear subtraction term: decoupling of the unresolved photon

Fully subtracted term is finite: numerical integration in 4-dimensions

$$\mathcal{O}_{\rm nlo} \equiv \sum_{i=1}^{4} \mathcal{O}_{\rm nlo}^{(i)}\, \omega^{5i} \equiv \sum_{i=1}^{4} \left(I - C_{5i}\right)\left(I - S_5\right) \omega^{5i}$$

Phase space singularities are make explicit by analytically integrating the subtraction contributions

Partition functions do not affect the integration of the singular kernels!

Nested soft-collinear subtraction: NLO EW

NLO EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

- Final result as combination of virtual corrections and pdf renormalisation:
  - ✓ Poles cancellation proven independently on E_{max} and ω^{5i} functions
  - ✓ Simple structure arise, compact expressions
  - ✓ Fully differential

$$2s \cdot \mathrm{d}\sigma^{\mathrm{nlo}} = \langle \mathcal{O}_{\mathrm{nlo}}\, F_{LM}(1,2,3,4\,|\,5)\rangle + \langle F_{LV}^{\mathrm{fin}}(1,2,3,4)\rangle$$

Finite remainder of virtual corrections

$$+\frac{\alpha(\mu)}{2\pi}\Bigg\{ Q_q^2 \int_0^1 dz \Big(P_{qq}^{(0)}(z)\,\log\Big(\frac{s}{\mu^2}\Big) + P_{qq}^{'}(z)\Big)\Bigg[\Big\langle \frac{F_{LM}(z\cdot 1,2,3,4)}{z}\Big\rangle + \Big\langle \frac{F_{LM}(1,z\cdot 2,3,4)}{z}\Big\rangle\Bigg]$$

Boosted LO-like terms, convoluted with splitting functions

$$+\Bigg[Q_q^2\,\frac{2\pi^2}{3} + Q_e^2\Big(13 - \frac{2\pi^2}{3}\Big)$$

$$+2\,Q_q\,Q_e\Bigg[\Big(\log\Big(\frac{E_{max}^2}{E_4\,E_1}\Big) + \frac{3}{2}\Big)\log\Big(\frac{\eta_{13}\,\eta_{24}}{\eta_{23}\,\eta_{14}}\Big)$$

$$+\mathrm{Li}_2(1-\eta_{13}) + \mathrm{Li}_2(1-\eta_{24}) - \mathrm{Li}_2(1-\eta_{14}) - \mathrm{Li}_2(1-\eta_{23})\Bigg]\Big\langle F_{LM}(1,2,3,4)\Big\rangle\Bigg\}$$

Elastic LO-like terms

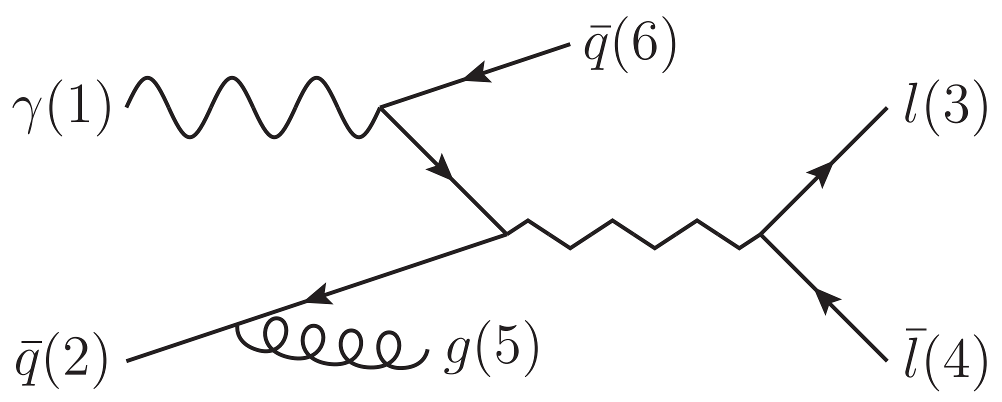
$$P_{qq}^{(0)}(z) = 2\,D_0(z) - (1+z) + \frac{3}{2}\,\delta(1-z)\,, \qquad P_{qq}^{'}(z) = 4\,D_1(z) + (1-z) - 2\,(1+z)\,\log(1-z)\,.$$

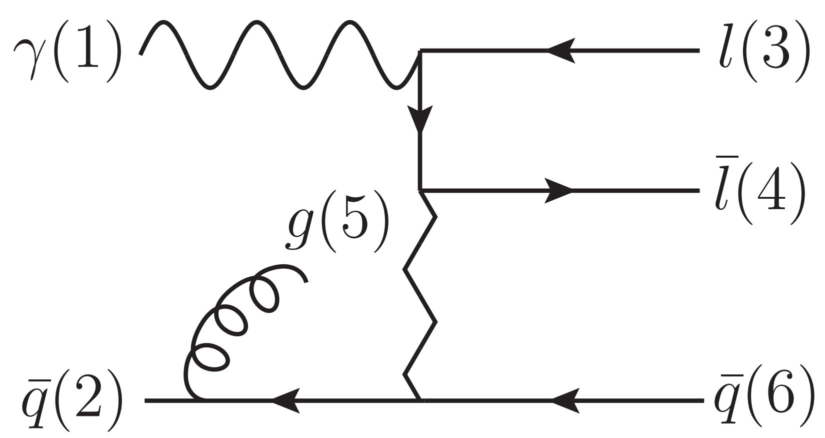
The RR component: generalities

The differences between the on-shell and the off-shell calculation

In the **off-shell case** the **simplifications** mentioned for the on-shell computation are valid, but they are **not enough**:

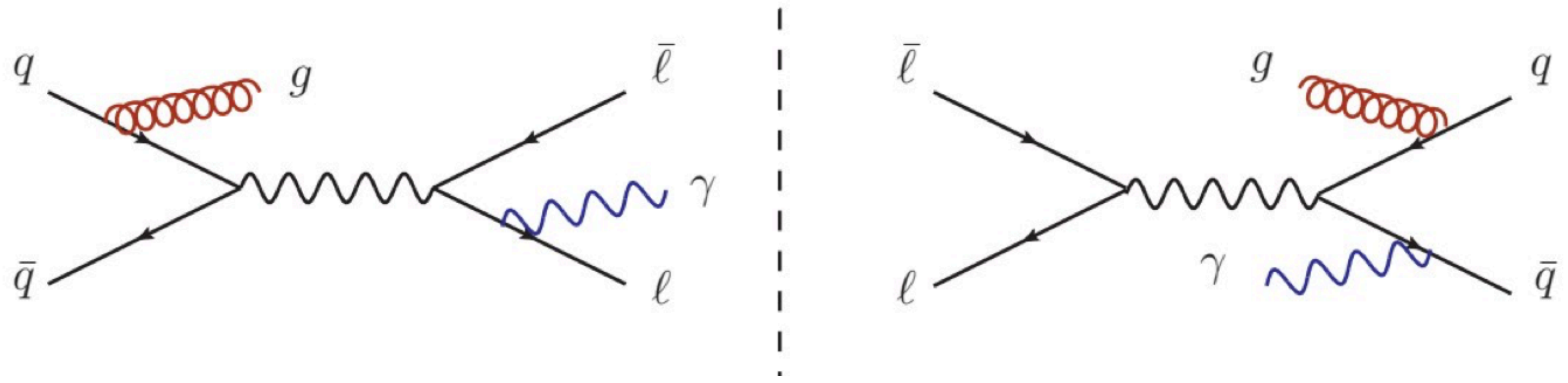
- new diagrams from initial state photons





Extend the partition to isolate singularities from configurations with collinear photons and leptons

- interference between "initial-initial" and "initial-final" corrections



- Only singular for soft photons
- Soft singularities can be treated as iterated NLO-like singularities

The RR component: generalities

From NNLO QCD to mixed EWxQCD: the on-shell case as intermediate step

> **Final state radiation** introduces additional soft and collinear singularities leading to new kinematic structure

> **The phase space partition** has to be **extended**

BUT

Absence of gluon-gluon interactions

> no true double-soft singularities

> no spin correlations

> pure abelian contributions

> **abelianization** of known results

- Adapting the NNLO QCD computation is practically all you need to obtain the mixed QCDxEW correction in the resonance region.

[de Florian, Der, Fabre '18][Delto, Jaquier, Melnikov, Röntsch '19] [Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

- In the **high energy region**, **full control of the subtraction procedure** is crucial to be able to adapt the results from NNLO QCD. A straightforward abelianization is not sufficient.

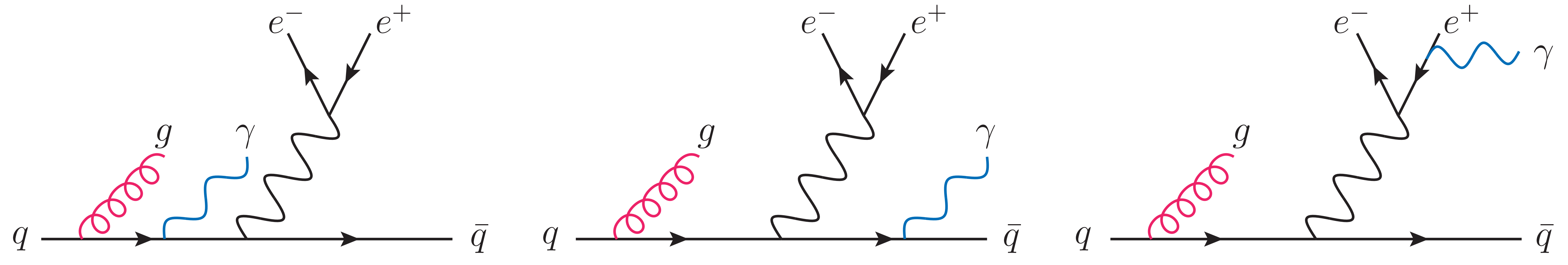
Mixed QCDxEW corrections: the double real component

Mixed QCD×EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \rightarrow e^+(p_3)\,e^-(p_4)$

- Several singular limits:

$$S_i\,A = \lim_{E_i \to 0} A\,, \qquad C_{ij}\,A = \lim_{\rho_{ij} \to 0} A\,, \qquad C_{ijk}\,A = \lim_{\rho_{ij},\,\rho_{jk},\,\rho_{ik} \to 0} A$$

- Difficulty: numerous collinear singularities to regulate



- Simplification: trivial soft singularities structure: no strongly-ordered soft limits,
  double soft limit completely factorised into abelian-like terms

$$\left\langle F_{LM}(1,2,3,4\,|\,5,6) \right\rangle = \left\langle S_g\,S_\gamma\,F_{LM}(\ldots\,|\,5,6) \right\rangle + \left\langle \left[(I - S_g)\,S_\gamma + (I - S_\gamma)\,S_g \right] F_{LM}(\ldots\,|\,5,6) \right\rangle + \left\langle (I - S_g)\,(I - S_\gamma)\,F_{LM}(\ldots\,|\,5,6) \right\rangle$$

Double soft subtraction term — Single soft subtraction term — Soft-finite term

Second and third term contain (potentially **overlapping**) **collinear singularities**

> as for NLO we introduce **phase space partitioning**