Antenna subtraction at NNLO with identified hadrons

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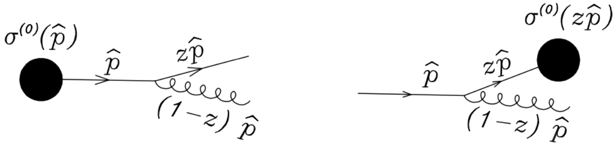
Introduction

The identification of hadrons require the introduction of a **fragmentation function** (FF) to describe the fragmentation of the high-energy quark or gluon into the actually detected hadron:

$$\mathrm{d}\sigma^H = \sum_p \int \mathrm{d}\eta \, D_{H\leftarrow p}(\eta, \mu_a^2) \, \mathrm{d}\hat{\sigma}_p(\eta, \mu_a^2) \, .$$

Whenever we identify a QCD particle, we spoil the cancellation of collinear divergences!

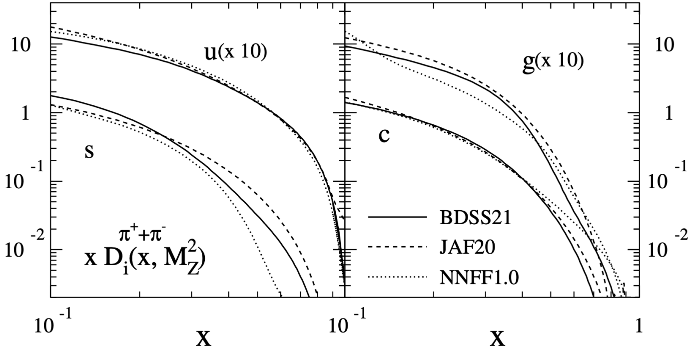


[from P. Nason's lectures]

Such divergences are absorbed in the bare $D^{(b)}_{H\leftarrow p}(\eta)$ to result in the physical $D_{H\leftarrow p}(\eta, \mu_a^2)$.

Motivation: why identified hadrons?

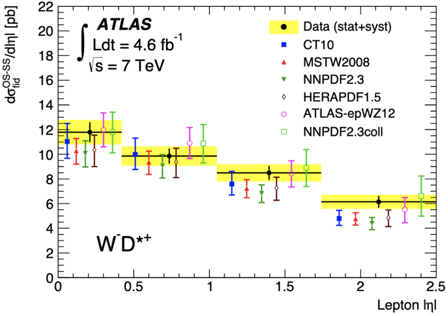
Fit of light fragmentation functions e.g. BDSS21 [De Florian et al. 2015], JAF20 [Moffat et al. 2021] and NNFF1.0 [Bertone, Carrazza, et al. 2017; Bertone, Hartland, et al. 2018]



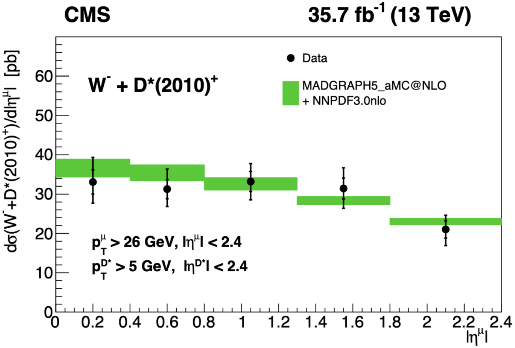
[PDG 2020]

Motivation: why identified hadrons?

Heavy quarks produced in association with vector bosons e.g. $W + c$, with the c-quark fragmenting into the D-meson detected $\rightarrow$ A. Kardos' talk



[ATLAS 1402.6263]



[CMS 1811.10021]

This talk

- **Extend the antenna subtraction formalism to include identified hadrons.**
- First progress in this direction: photon fragmentation → R. Schürmann's talk
- We focus on an identify hadron H in fully exclusive e^+e^- annihilation:

$$e^+ + e^- \to H(\,+\text{jets}\,) + X$$

  Possible presence of additional jets, which may contain the identified hadron.
- A step towards hadron fragmentation in ep or pp collisions.

Inclusion of FFs in the sector-improved residue subtraction scheme → M. Czakon's talk
First application: B-hadron production in $t\bar{t}$ events [Czakon et al. 2021]

Antenna subtraction

Perturbative expansion of the short-distance cross section $\mathrm{d}\hat{\sigma}$:

$$\mathrm{d}\hat{\sigma} = \mathrm{d}\hat{\sigma}^{\mathrm{LO}} + \left(\frac{\alpha_s}{2\pi}\right) \mathrm{d}\hat{\sigma}^{\mathrm{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\hat{\sigma}^{\mathrm{NNLO}} \,.$$

Required subtraction scheme to deal with infrared divergences in the intermediate steps of the calculation. In the antenna subtraction scheme [Gehrmann-De Ridder, Gehrmann, and Glover 2005]:

$$\mathrm{d}\hat{\sigma}^{\mathrm{LO}} = \int_n \left[\mathrm{d}\hat{\sigma}^{\mathrm{B}}\right]$$

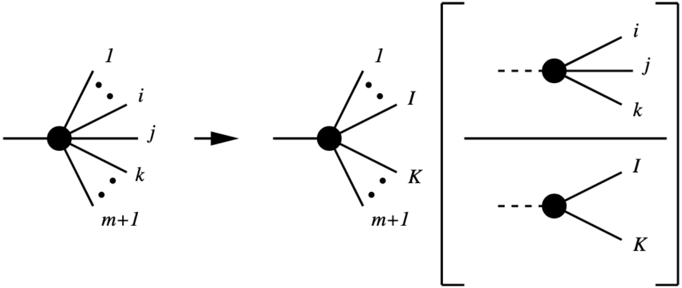
$$\mathrm{d}\hat{\sigma}^{\mathrm{NLO}} = \int_{n+1} \left[\mathrm{d}\hat{\sigma}^{\mathrm{R}} - \mathrm{d}\hat{\sigma}^{\mathrm{S}}\right] + \int_n \left[\mathrm{d}\hat{\sigma}^{\mathrm{V}} - \mathrm{d}\hat{\sigma}^{\mathrm{T}} - \mathrm{d}\hat{\sigma}^{\mathrm{MF}}\right]$$

$$\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}} = \int_{n+2} \left[\mathrm{d}\hat{\sigma}^{\mathrm{RR}} - \mathrm{d}\hat{\sigma}^{\mathrm{S}}\right] + \int_{n+1} \left[\mathrm{d}\hat{\sigma}^{\mathrm{RV}} - \mathrm{d}\hat{\sigma}^{\mathrm{T}} - \mathrm{d}\hat{\sigma}^{\mathrm{MF,RV}}\right] + \int_n \left[\mathrm{d}\hat{\sigma}^{\mathrm{VV}} - \mathrm{d}\hat{\sigma}^{\mathrm{U}} - \mathrm{d}\hat{\sigma}^{\mathrm{MF,VV}}\right]$$

Outline

Outline

Real subtraction term $\mathrm{d}\hat{\sigma}^{\mathrm{S}}$



$$M_{n+1}(\text{original momenta}) \to M_n(\text{mapped momenta}) \times X(\text{original momenta})$$

Real subtraction term $\mathrm{d}\hat{\sigma}^{\mathrm{S}}$

New ingredient: unresolved parton between the identified parton k_p and another hard parton.

$$M(k_1, \ldots, k_p, k_j, k_k, \ldots, k_{1+m}) \to M(k_1, \ldots, K_p, K, \ldots, k_{1+m}) \times X^{0,\mathrm{id}.p}(k_p, k_j, k_k)$$

with the mapping defined as

$$z = \frac{s_{pj} + s_{pk}}{s_{pj} + s_{pk} + s_{jk}}\,, \quad K_p = k_p/z\,, \quad K = k_j + k_k - (1-z)\frac{k_p}{z}$$

Phase space factorizes as

$$\begin{aligned} \mathrm{d}\Phi_{m+1}(k_1, \ldots, k_p, k_j, k_k, \ldots, k_{m+1}; \mathcal{Q}) &= \mathrm{d}\Phi_m(k_1, \ldots, K_p, K, \ldots, k_{m+1}; \mathcal{Q}) \\ &\times \frac{q^2}{2\pi}\mathrm{d}\Phi_2(k_j, k_k; q - k_h)\, z^{1-2\epsilon}\, \mathrm{d}z \end{aligned}$$

with $q = k_j + k_k + k_h$. Factor $z^{1-2\epsilon}$ from identified particle phase space.

Virtual subtraction term $\mathrm{d}\hat{\sigma}^{\mathrm{T}}$ and $\mathrm{d}\hat{\sigma}^{\mathrm{MF}}$

Integration of the fragmentation X_{30} antenna

$$\mathcal{X}_{30}^{\mathrm{id}.p}(z) = \frac{1}{C(\epsilon)} \int \mathrm{d}\Phi_2 \frac{q^2}{2\pi}\, z^{1-2\epsilon}\, X_{30}^{\mathrm{id}.p}\,, \quad C(\epsilon) = (4\pi e^{-\gamma_E})^{\epsilon}/(8\pi^2)$$

over the two-particle phase space with kinematics

$$q(q^2) + (-k_p) \to k_j + k_k$$

with $s_{jk} = (q-k_p)^2 = q^2(1-z)$. After integration, it can be directly combined with the one-loop mass factorisation kernels

$$\mathcal{X}_{30}^{\mathrm{id}.p}(z) - \mu_a^{-2\epsilon}\Gamma^{(1)}(z)$$

which subtracts all the explicit poles of the virtual matrix element.

Outline

Real-real subtraction term $\mathrm{d}\hat{\sigma}^{\mathrm{S}}$

- **Single unresolved limit**:
  as in the NLO case
- **Double unresolved limit**:
  New ingredient: tree-level four-parton fragmentation antenna function $X^{0,\mathrm{id}.p}_{pjkl}$

$$M(k_1,\dots,k_p,k_j,k_k,k_l,\dots,k_{1+m}) \to M(k_1,\dots,K_p,K,\dots,k_{1+m}) \times \left[X^{0,\mathrm{id}.p}_{pjkl} - X^{0,\mathrm{id}.p}_{pjk} X^{0,\mathrm{id}.P}_{PKl} - X^{0}_{jkl} X^{0,\mathrm{id}.p}_{pJL}\right]$$

with the NNLO mapping defined as the generalized version of the NLO mapping

$$z = \frac{s_{pj}+s_{pk}+s_{pl}}{s_{pj}+s_{pk}+s_{jk}+s_{pl}+s_{jl}+s_{kl}}\,, \quad K_p = k_p/z\,, \quad K = k_j+k_k+k_l-(1-z)\frac{k_p}{z}$$

It turns into a NLO phase space mapping in its single unresolved limits, as required in order to cancel the single unresolved limits of the $X^{40,\mathrm{id}.p}$ antenna function.

- ...

Real-virtual subtraction term $\mathrm{d}\hat{\sigma}^{\mathrm{T}}$ and $\mathrm{d}\hat{\sigma}^{\mathrm{MF,RV}}$

- **Explicit infrared poles**:
  subtracted by product of $\mathcal{X}_{30}^{\mathrm{id}.p}(z)$ with tree-level $(m+1)$-parton matrix element.
- **Single unresolved limit**:

$$
\begin{aligned}
&M_{m+1}^{1}(k_1,\ldots,k_p,k_j,k_k,\ldots,k_{1+m})\\
&\to \Big[X^{0,\mathrm{id}.p}(k_p,k_j,k_k)\,M_m^{1}(k_1,\ldots,K_p,K,\ldots,k_{m+1})\\
&\qquad + X^{1,\mathrm{id}.p}(k_p,k_j,k_k)\,M_m^{0}(k_1,\ldots,K_p,K,\ldots,k_{m+1})\Big]
\end{aligned}
$$

  New ingredient: one-loop three-parton fragmentation antenna function $X^{1,\mathrm{id}.p}$.
- ...

to be combined with mass factorisation contribution containing one-loop $\Gamma^{(1)}(z)$ kernels.

Virtual-virtual subtraction term $\mathrm{d}\hat{\sigma}^{\mathrm{U}}$ and $\mathrm{d}\hat{\sigma}^{\mathrm{MF,VV}}$

The integrated antenna functions

$$\mathcal{X}_{40}^{\mathrm{id}.p}(z) = \frac{1}{[C(\epsilon)]^2} \int \mathrm{d}\Phi_3 \frac{q^2}{2\pi} z^{1-2\epsilon} X_{40}^{\mathrm{id}.p}$$

$$\mathcal{X}_{31}^{\mathrm{id}.p}(z) = \frac{1}{C(\epsilon)} \int \mathrm{d}\Phi_2 \frac{q^2}{2\pi} z^{1-2\epsilon} X_{31}^{\mathrm{id}.p}$$

combine with the NNLO mass factorization terms (containing two-loop $\Gamma^{(2)}(z)$ kernels or products of $\Gamma^{(1)}(z)$ kernel), in order to cancel the explicit poles of the double-virtual matrix elements.

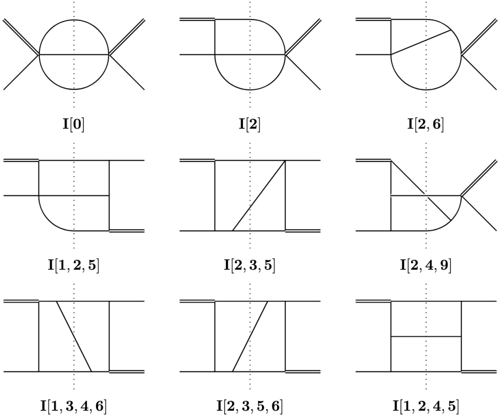
Outline

Integration of real-real antenna function $X_{40}^{\mathrm{id}.p}$

- Three-particle phase space with $2 \to 3$ kinematics

$$q(q^2) + (-k_p) \to k_1 + k_2 + k_3$$

  and invariants q^2 and $s = (q - k_p)^2 = q^2(1 - z)$.
- Well-known technique:
  - write phase space in terms of cut propagators
  - run reduction with REDUZE2 [Manteuffel and Studerus 2012]
  - evaluate master integral with differential equation method [Gehrmann and Remiddi 2000] in the canonical form [Henn 2015; Henn 2013].
  - solution in terms of harmonic polylogarithms (HPLs) [Remiddi and Vermaseren 2000]
  - boundary condition by internal consistency or explicit evaluation at $z = 1$
- After the master integrals have been inserted, expansion of the $(1 - z)^{-\epsilon}$ and $(1 - z)^{-2\epsilon}$ factors in terms of distributions.



9 master integrals

Deep relationship between space-like and time-like kinematics.

$$q(q^2) + (-k_p) \to k_1 + k_2 + k_3 \,, \quad q^2 > 0 \,, \quad z = \frac{2\,k_p \cdot q}{q^2}$$

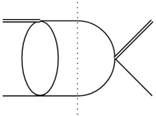
$$q(-Q^2) + k_p \to k_1 + k_2 + k_3 \,, \quad Q^2 > 0 \,, \quad x = \frac{Q^2}{2\,k_p \cdot q}$$

- Investigated in the literature e.g. [Stratmann and Vogelsang 1997; Almasy, Moch, and Vogt 2012]
- **Same set of master integrals** appearing in the integration of the antenna functions with initial-final kinematics.
- After analytic continuation of the HPLs $z \to 1/x$, with the package HPL [Maitre 2006; Maitre 2012], we recover the initial-final master integrals.
- Simple "recipe" to relate double-real antenna functions with space-like and time-like kinematics:

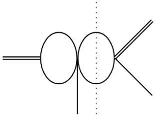
$$x \to 1/z \,, \quad Q^2 \to -q^2$$

Integration of real-real antenna function $X_{31}^{\text{id}.p}$

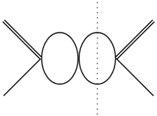
- Integrating one-loop matrix elements over two-particle phase space with $2 \to 2$ kinematics.
- Same chain as before, with one loop momentum plus two cut propagators.
- We find 6 master integrals. Explicit evaluation of four of them, the remaining two found with differential equation with boundary condition evaluated at $z = 1$.



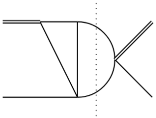
$\mathbf{V[1,3]}$



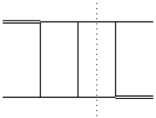
$\mathbf{V[1,4]}$



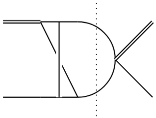
$\mathbf{V[2,4]}$



$\mathbf{V[1,3,4]}$



$\mathbf{V[1,2,3,4,5]}$



$\mathbf{C[1,2,3,4]}$

Again, same master integrals appearing in the integration of initial-final antenna functons. Does the recipe

$$x \to 1/z\,, \quad Q^2 \to -q^2$$

work at the real-virtual level?

- ✓ Integrals related to one-loop bubble, by "keeping track of $(-1)^{-\epsilon}$ factors"
- ✗ Integrals related to one-loop box: analytic continuation required at the integrand level $\to$ it prevents a simple relationship between master integrals which can be propagated to the antenna level

Outline

One-particle inclusive spectrum in e^+e^- annihilation

Integrated antenna functions can be related to known analytical results for one-particle inclusive cross sections in in e^+e^- annihilation:

$$\frac{\mathrm{d}\sigma^H}{\mathrm{d}x} = \sum_{i=q,\bar{q},g} \int_x^1 \frac{\mathrm{d}z}{z} D_i^H\left(\frac{x}{z}\right) \frac{\mathrm{d}\sigma_i^H}{\mathrm{d}z} = \sum_j \sigma_j^{(0)} \int_x^1 \frac{\mathrm{d}z}{z} D_j^H\left(\frac{x}{z}\right) \mathbb{C}_j(z)$$

with $x = 2E_h/\sqrt{s}$ the energy fraction. The coefficient functions $\mathbb{C}_j$ are the mass-renormalised and UV-renormalised version of the parton fragmentation functions $\hat{\mathcal{F}}_j$:

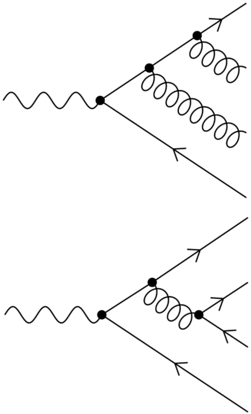
$$\hat{\mathcal{F}}_j = \hat{\mathcal{F}}_j^{(0)} + \left(\frac{\hat{\alpha}_s}{4\pi}\right) S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \hat{\mathcal{F}}_j^{(1)} + \left(\frac{\hat{\alpha}_s}{4\pi}\right)^2 S_\epsilon^2 \left(\frac{\mu^2}{Q^2}\right)^{2\epsilon} \hat{\mathcal{F}}_j^{(2)} + \mathcal{O}(\hat{\alpha}_s^3)\,.$$

The leading order $\hat{\mathcal{F}}_j^{(0)}$ are just zeros or δ-functions. $\hat{\mathcal{F}}_j^{(1)}$ and $\hat{\mathcal{F}}_j^{(2)}$ can be directly compared to a linear combination of integrated antenna functions and virtual form factors.

γ/Z boson decay

First derived in [Rijken and Van Neerven 1996; Rijken and Van Neerven 1997] and independently in [Mitov, Moch, and Vogt 2006]. Quark function usually written in terms of non-singlet $\hat{\mathcal{F}}^{\mathrm{NS}}$ and pure-singlet $\hat{\mathcal{F}}^{\mathrm{PS}}$ components, related to:

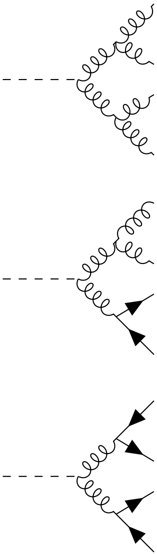
$$
\begin{aligned}
\hat{\mathcal{F}}^{\mathrm{NS}}|_N &= 2\mathcal{A}_{40}^{\mathrm{id}.q} + 8\mathcal{A}_{31}^{\mathrm{id}.q} + \delta(1-z)V_q^{(2)}|_N \\
\hat{\mathcal{F}}^{\mathrm{NS}}|_{N_F} &= 2\mathcal{B}_{40}^{\mathrm{id}.q} + \delta(1-z)V_q^{(2)}|_{N_F} \\
\hat{\mathcal{F}}^{\mathrm{NS}}|_{1/N} &= -\tilde{\mathcal{A}}_{40}^{\mathrm{id}.q} - 8\tilde{\mathcal{A}}_{31}^{\mathrm{id}.q} - 4\mathcal{C}_{40}^{\mathrm{id}.\bar{q}} \\
&\quad - 2\mathcal{C}_{40}^{\mathrm{id}.q_1} - 2\mathcal{C}_{40}^{\mathrm{id}.q_3} + \delta(1-z)V_q^{(2)}|_{1/N} \\
\hat{\mathcal{F}}^{\mathrm{PS}} &= \mathcal{B}_{40}^{\mathrm{id}.q'} \\
\hat{\mathcal{F}}_g|_N &= 4\mathcal{A}_{40}^{\mathrm{id}.g} + 8\mathcal{A}_{31}^{\mathrm{id}.g} \\
\hat{\mathcal{F}}_g|_{1/N} &= -2\tilde{\mathcal{A}}_{40}^{\mathrm{id}.g} - 8\tilde{\mathcal{A}}_{31}^{\mathrm{id}.g}
\end{aligned}
$$



Higgs decay in the heavy-top limit

First obtained in [Almasy, Moch, and Vogt 2012]. Quark $\hat{\mathcal{F}}_q$ and gluon function $\hat{\mathcal{F}}_g$ related to:

$$
\begin{aligned}
\hat{\mathcal{F}}_g|_{N^2} &= \mathcal{F}_{40}^{\mathrm{id}.g} + 4\mathcal{F}_{31}^{\mathrm{id}.g} + 4\delta(1-z)V_g^{(2)}|_{N^2} \\
\hat{\mathcal{F}}_g|_{NN_F} &= 2\mathcal{G}_{40}^{\mathrm{id}.g} + 4\mathcal{G}_{31}^{\mathrm{id}.g} + 4\hat{\mathcal{F}}_{31}^{\mathrm{id}.g} + 4\delta(1-z)V_g^{(2)}|_{NN_F} \\
\hat{\mathcal{F}}_g|_{N_F/N} &= -\tilde{\mathcal{G}}_{40}^{\mathrm{id}.g} + 4\tilde{\mathcal{G}}_{31}^{\mathrm{id}.g} + 4\delta(1-z)V_g^{(2)}|_{N_F/N} \\
\hat{\mathcal{F}}_g|_{N_F^2} &= 4\hat{\mathcal{G}}_{31}^{\mathrm{id}.g} + 4\delta(1-z)V_g^{(2)}|_{N_F^2} \\
\hat{\mathcal{F}}_q|_{N} &= -2\mathcal{G}_{40}^{\mathrm{id}.q} - 8\mathcal{G}_{31}^{\mathrm{id}.q} \\
\hat{\mathcal{F}}_q|_{1/N} &= \tilde{\mathcal{G}}_{40}^{\mathrm{id}.q} + 8\tilde{\mathcal{G}}_{31}^{\mathrm{id}.q} - 4\tilde{\mathcal{J}}_{40}^{\mathrm{id}.q} \\
\hat{\mathcal{F}}_q|_{N_F} &= -2\mathcal{H}_{40}^{\mathrm{id}.q} - 8\hat{\mathcal{G}}_{31}^{\mathrm{id}.q}
\end{aligned}
$$



Outline

Conclusions and outlook

- We have extended the antenna subtraction method to include hadron fragmentation processes up to NNLO in QCD in e^+e^+ collisions.
- Local subtraction scheme allows more **exclusive calculations** e.g. hadrons inside jets, any kind of pre-processing before hadron detection.
- We have derived integrated fragmentation antenna functions in the final-final kinematics, and compared them to known inclusive calculations.
- Final-final fragmentation antenna functions as a step towards fragmentation in pp collisions. Several ingredients in the initial-final kinematics from photon fragmentation.
- Stay tuned for some first phenomenological application!