

TWO-LOOP MIXED QCD-EW CORRECTIONS TO NEUTRAL CURRENT DRELL-YAN

Loops and Legs in Quantum Field Theory

29/04/2022

Simone Devoto



UNIVERSITÀ
DEGLI STUDI
DI MILANO

In collaboration with:
T. Armadillo, R. Bonciani, N. Rana, A. Vicini

MOTIVATIONS

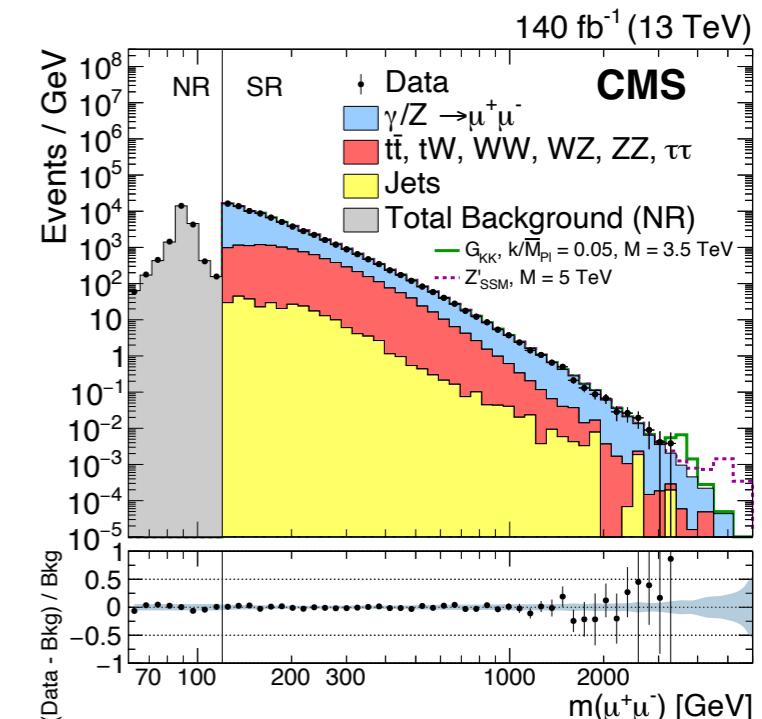
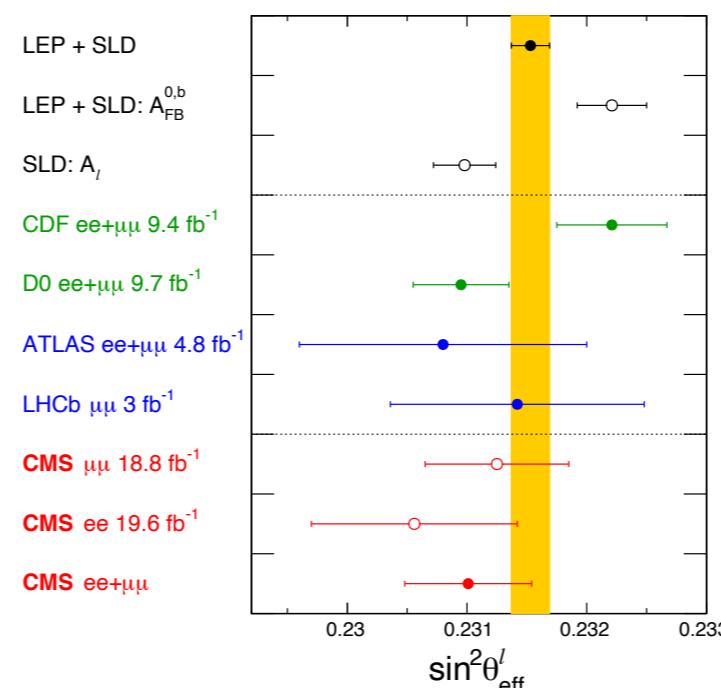
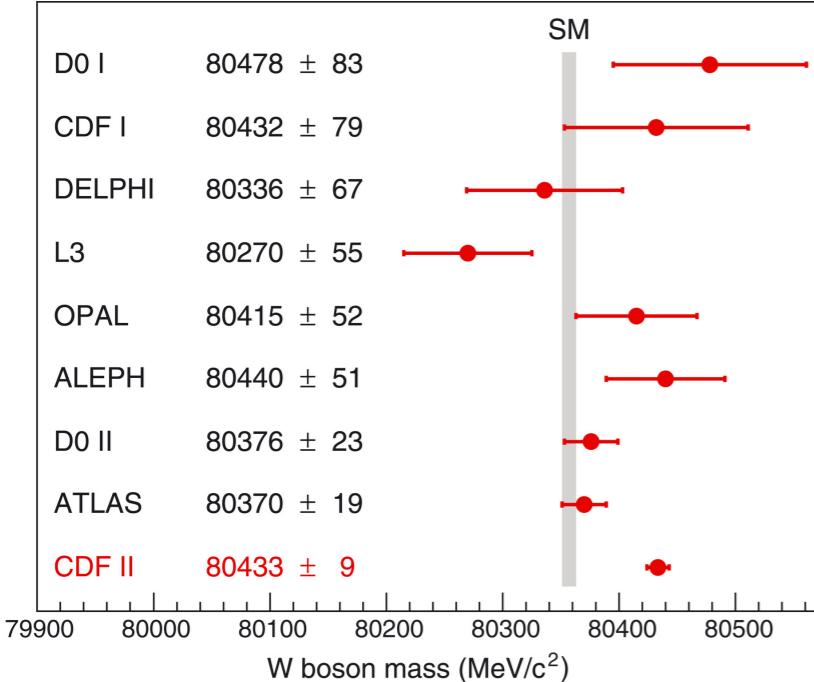
Standard Model Precision Studies

- Extremely precise measurement of W boson mass (10 MeV uncertainties!);
- Determination of $\sin^2 \theta_{\text{eff}}^{\text{lep}}$ is becoming competitive with LEP result: 0.23152(16)

BSM Studies

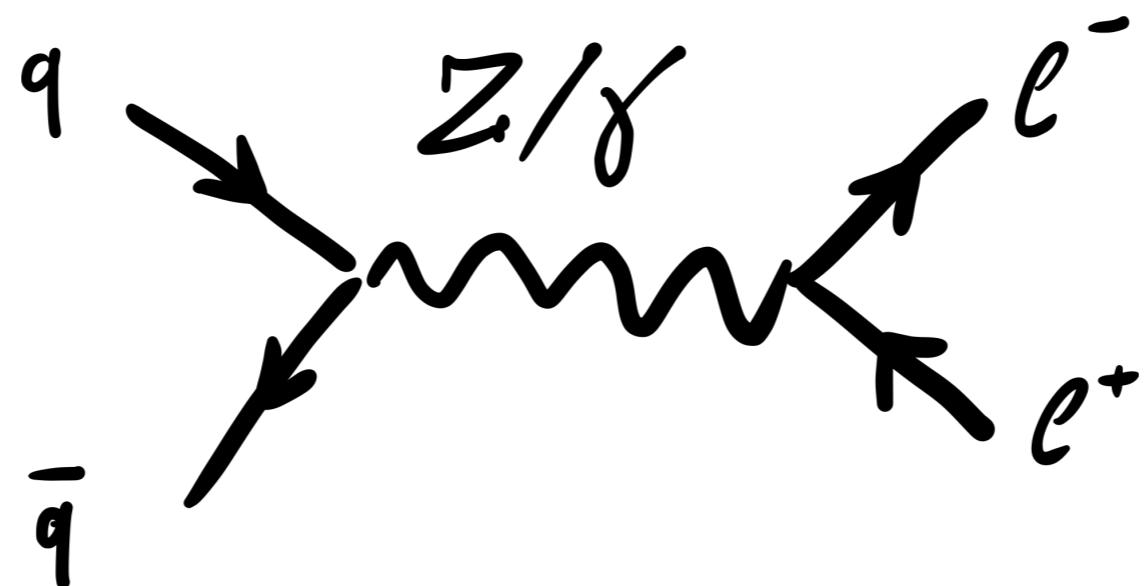
- Precise modelling of SM background crucial for new physics searches;
- Dilepton invariant mass distribution at the $\mathcal{O}(1\%)$ level in the TeV region.

Can we provide SM theoretical predictions matching experimental accuracy?



MIXED QCDxEW CORRECTIONS

$$q(p_1) + \bar{q}(p_2) \rightarrow l^-(p_3) + l^+(p_4) = \sigma^{(0,0)} \quad \text{Drell-Yan (1970)}$$

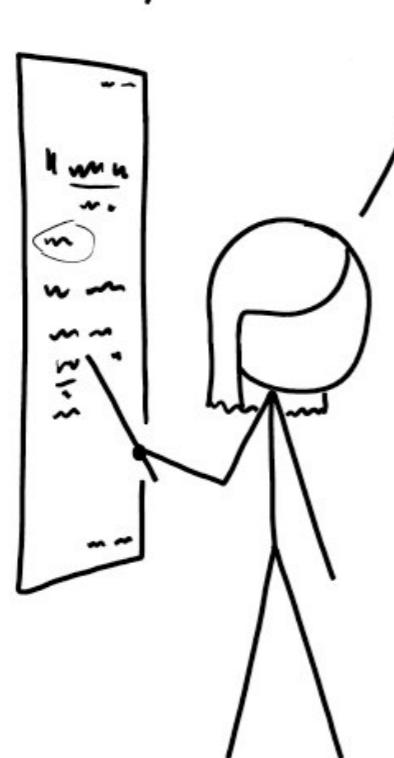


MIXED QCDxEW CORRECTIONS

$$q(p_1) + \bar{q}(p_2) \rightarrow l^-(p_3) + l^+(p_4) = \sigma^{(0,0)} \quad \text{Drell-Yan (1970)}$$

AT THIS POINT, YOU'RE PROBABLY
THINKING, "I LOVE THIS EQUATION
AND WISH IT WOULD NEVER END!"

WELL, GOOD NEWS!



credits: xkcd (2605)

TAYLOR SERIES EXPANSION IS THE WORST.

$$\begin{aligned} &+ \alpha_S \sigma^{(1,0)} &+ \alpha \sigma^{(0,1)} \\ &+ \alpha_S^2 \sigma^{(2,0)} &+ \alpha \alpha_S \sigma^{(1,1)} &+ \alpha^2 \sigma^{(0,2)} \\ &+ \alpha_S^3 \sigma^{(3,0)} &+ \dots \end{aligned}$$

QCD

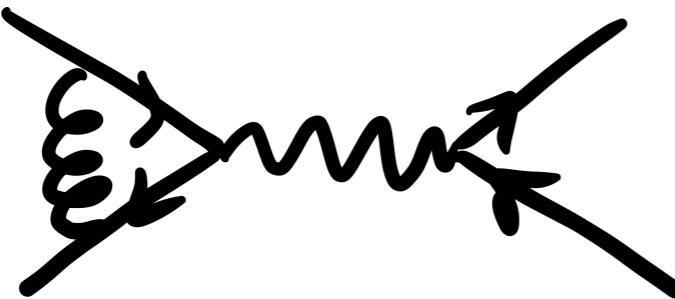
MIXED

EW

MIXED QCDxEW CORRECTIONS

$$q(p_1) + \bar{q}(p_2) \rightarrow l^-(p_3) + l^+(p_4) = \sigma^{(0,0)}$$

QCD CORRECTIONS



$$\begin{aligned} &+ \alpha_S \sigma^{(1,0)} + \alpha \sigma^{(0,1)} \\ &+ \alpha_S^2 \sigma^{(2,0)} + \alpha \alpha_S \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)} \\ &+ \alpha_S^3 \sigma^{(3,0)} + \dots \end{aligned}$$

► Dominant effect;

NLO:

[G.Altarelli, R.Ellis, G.Martinelli *Nucl.Phys.B* 157 (1979)];

NNLO:

[R.Hamberg, T.Matsuura, W.van Nerveen, *Nucl. Phys. B* 359 (1991)];

[C.Anastasiou, L.J.Dixon, K.Melnikov, F.Petriello, *hep-ph:0306192*];

[S.Catani, L.Cieri, G.Ferrera, D.de Florian, M.Grazzini
arXiv:0903.2120];

► Known up to N3LO.

N3LO:

[C.Duhr, F.Dulat, B.Mistlberger *arXiv:2007.13313*];

[X.Chen, T.Gehrmann, N.Glover, A.Huss, T.Yang, and H.Zhu
arXiv:2107.09085];

[S.Camarda, L.Cieri, G.Ferrera *arXiv:2103.04974*];

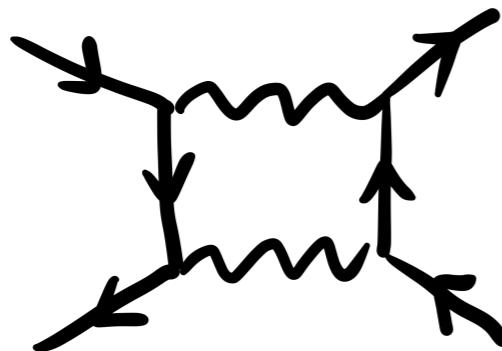
[X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli,
P.Torrielli *arXiv:2203.01565*];

See talk by Xuan Chen!

MIXED QCDxEW CORRECTIONS

$$q(p_1) + \bar{q}(p_2) \rightarrow l^-(p_3) + l^+(p_4) = \sigma^{(0,0)}$$

EW CORRECTIONS



$$+ \alpha_S \sigma^{(1,0)} + \alpha \sigma^{(0,1)}$$

$$+ \alpha_S^2 \sigma^{(2,0)} + \alpha \alpha_S \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)}$$

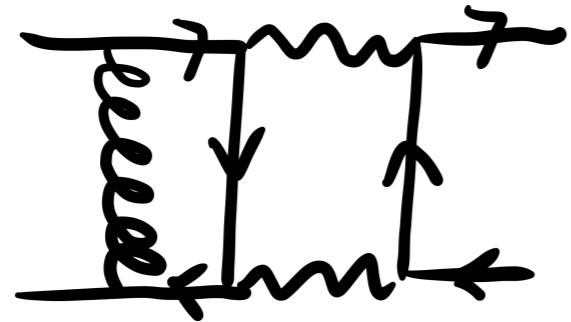
$$+ \alpha_S^3 \sigma^{(3,0)} + \dots$$

- Subdominant with respect to QCD: **physical counting** $\alpha_S \simeq \alpha^2$;
- NLO corrections known;
[U.Baur, O.Brein, W.Hollik, C.Schappacher, D.Wackeroth, *hep-ph:0108274*];
[S.Dittmaier, M.Kramer, *hep-ph:0109062*];
[U.Baur, D.Wackeroth, *hep-ph:0405191*];
- NNLO corrections still missing (available Sudakov high energy approximation).
[B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, *hep-ph:0509157*];

MIXED QCDxEW CORRECTIONS

$$q(p_1) + \bar{q}(p_2) \rightarrow l^-(p_3) + l^+(p_4) = \sigma^{(0,0)}$$

MIXED CORRECTIONS



$$\begin{aligned} &+ \alpha_S \sigma^{(1,0)} + \alpha \sigma^{(0,1)} \\ &+ \alpha_S^2 \sigma^{(2,0)} + \alpha \alpha_S \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)} \\ &+ \alpha_S^3 \sigma^{(3,0)} + \dots \end{aligned}$$

- NLO QCD and NLO EW separately large: what about mixed?
- By physical counting, expected size comparable with N3LO QCD!
- Recently, exact results, showing an effect of $\sim 0.5\%$ with respect to the LO result.

[R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, A.Vicini, arXiv:2106.11953]

[F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, arXiv:2203.11237]

RECENT PROGRESSES IN MIXED CORRECTIONS

► Theoretical Developments

- **2-loop virtual Master Integrals with internal masses** [*U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193*], [*R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581*], [*M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491*], [*M.Long, R.Zhang, W.Ma, Y.Jiang, L.Han,, Z.Li, S.Wang, arXiv:2111.14130*], [*X.Liu, Y.Ma, arXiv:2201.11669*]
- **Altarelli-Parisi splitting functions including QCD-QED effects** [*D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612*]
- **Renormalisation** [*G.Degrassi, A.Vicini, hep-ph/0307122*], [*S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229*], [*S.Dittmaier, arXiv:2101.05154*]

► On-shell Z and W production

- **pole approximation of the NNLO QCD-EW corrections** [*S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016*]
- **analytical total Z production cross section including NNLO QCD-QED corrections** [*D. de Florian, M.Der, I.Fabre, arXiv:1805.12214*]
- **fully differential on-shell Z production including exact NNLO QCD-QED corrections** [*M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428*] [*S.Hasan, U.Schubert, arXiv:2004.14908*]
- **analytical total Z production cross section including NNLO QCD-EW corrections** [*R. Bonciani, F. Buccioni, R.Mondini, A.Vicini, arXiv:1611.00645*], [*R. Bonciani, F. Buccioni, N.Rana, I.Triscari, A.Vicini, arXiv:1911.06200*], [*R. Bonciani, F. Buccioni, N.Rana, A.Vicini, arXiv:2007.06518, arXiv:2111.12694*]
- **fully differential Z and W production including NNLO QCD-EW corrections** [*F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221*], [*A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671*]

► Complete Drell-Yan

- **neutrino-pair production including NNLO QCD-QED corrections** [*L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315*]
- **2-loop amplitudes** [*M.Heller, A.von Manteuffel, R.Schabinger, arXiv:2012.05918*], [*T.Armadillo, R.Bonciani, SD, N.Rana, A.Vicini, arXiv:2201.01754*]
- **NNLO QCD-EW corrections to neutral-current DY including leptonic decay** [*R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, A.Vicini, arXiv:2106.11953*], [*F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, arXiv:2203.11237*] **See talks by Chiara Signorile-Signorile and Luca Buonocore!**
- **NNLO QCD-EW corrections to charged-current DY including leptonic decay (2-loop contributions in pole approximation).** *L.Buonocore*, [*M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539*]

CONTENTS

Process definition: $q(p_1) + \bar{q}(p_2) \rightarrow l^-(p_3)l^+(p_4)$



Feynman Amplitudes



Computation of the interference terms

Reduction to a set of Master Integrals



Evaluation of the Master Integrals



Subtraction of IR and UV divergences



SEASYDE



Numerical grid

Phenomenological application: [R. Bonciani, L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, A. Vicini - PhysRevLett.128.012002] **See talk by Luca Buonocore!**

COMPUTATIONAL FRAMEWORK

[T. Armadillo, R. Bonciani, SD, N.Rana,
A.Vicini, arXiv:2201.01754]

- IR singularities handled by **q_T-subtraction formalism**;

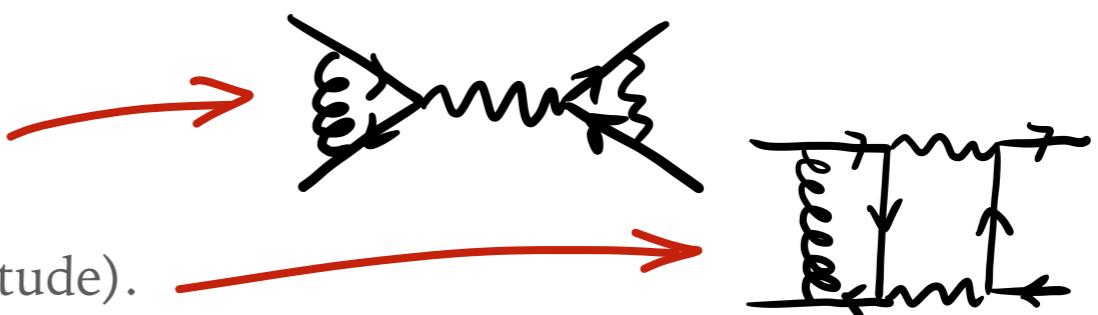
[S. Catani, M. Grazzini (2007)]

[L.Buonocore, M. Grazzini, F.Tramontano (2019)]

$$d\sigma_{(N)NLO}^F = \mathcal{H}_{(N)NLO}^F \otimes d\sigma_{LO}^F + \left[d\sigma_{(N)LO}^{F+jets} - d\sigma_{(N)LO}^{CT} \right]$$

- straightforward implementation of **any other framework** by replacing the **subtraction operator**.

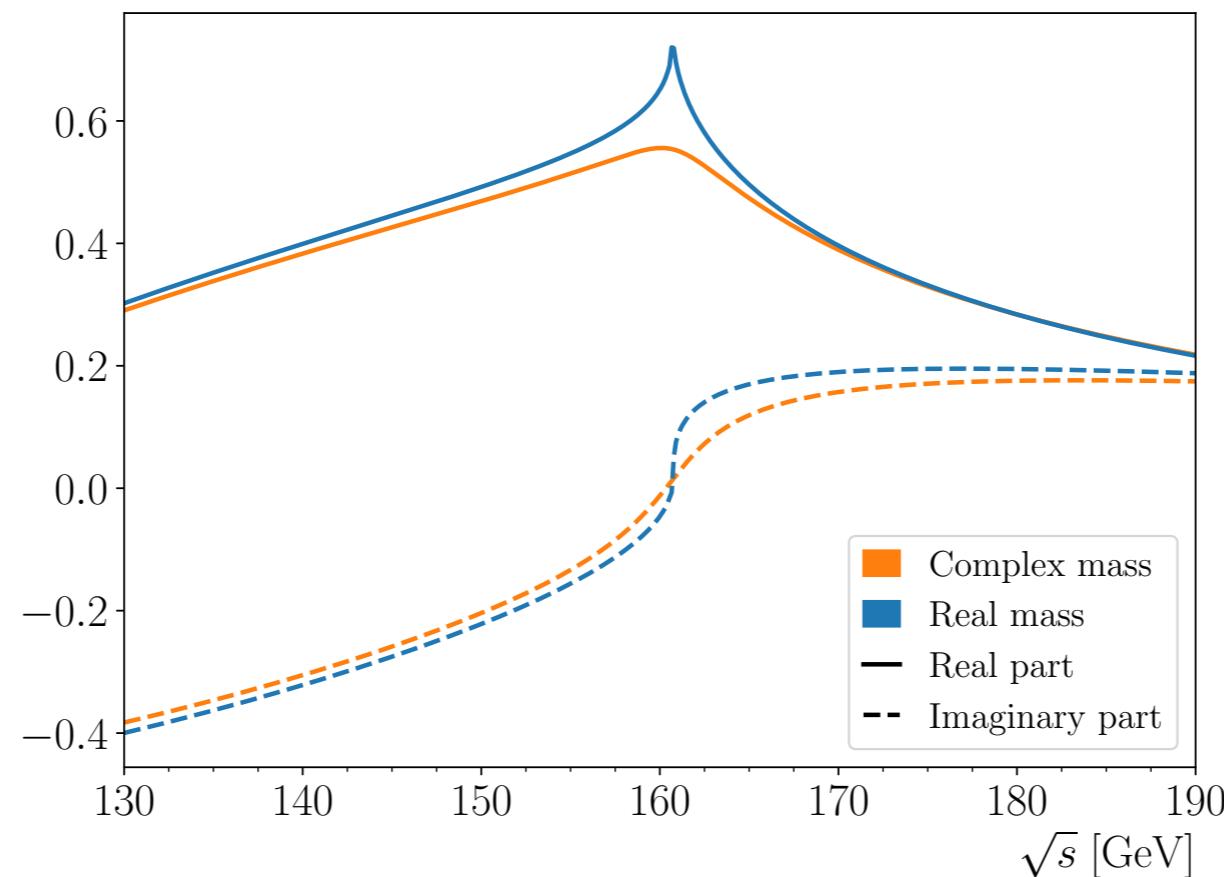
- Final-state collinear singularities regularised by the **lepton mass**;
- small lepton mass limit**: consider the ratio m_l/\sqrt{s} and keep only **logarithmic terms**
 $\sim \log(m_l/\sqrt{s})$;
- leftover lepton mass dependence:
 - factorisable QCD-EW vertex corrections;
 - $\gamma\gamma$ boxes (but cancels in the physical amplitude).



COMPUTATIONAL FRAMEWORK

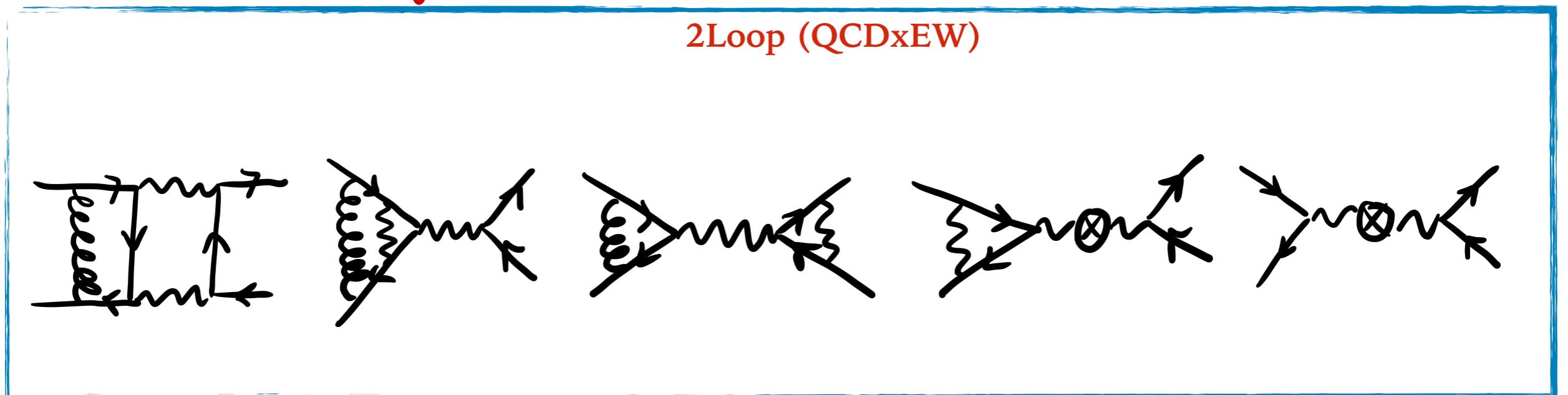
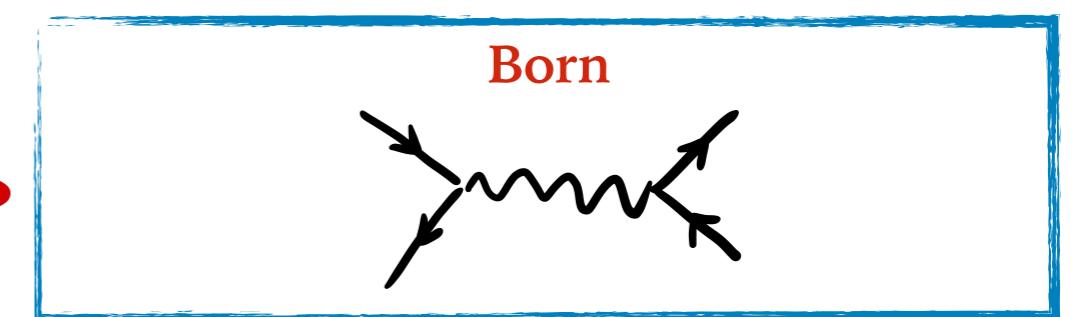
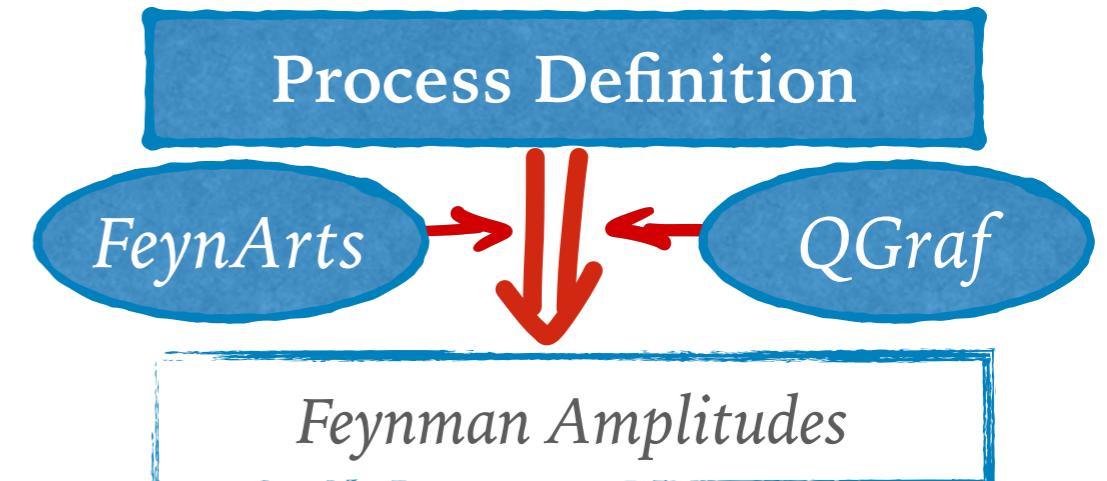
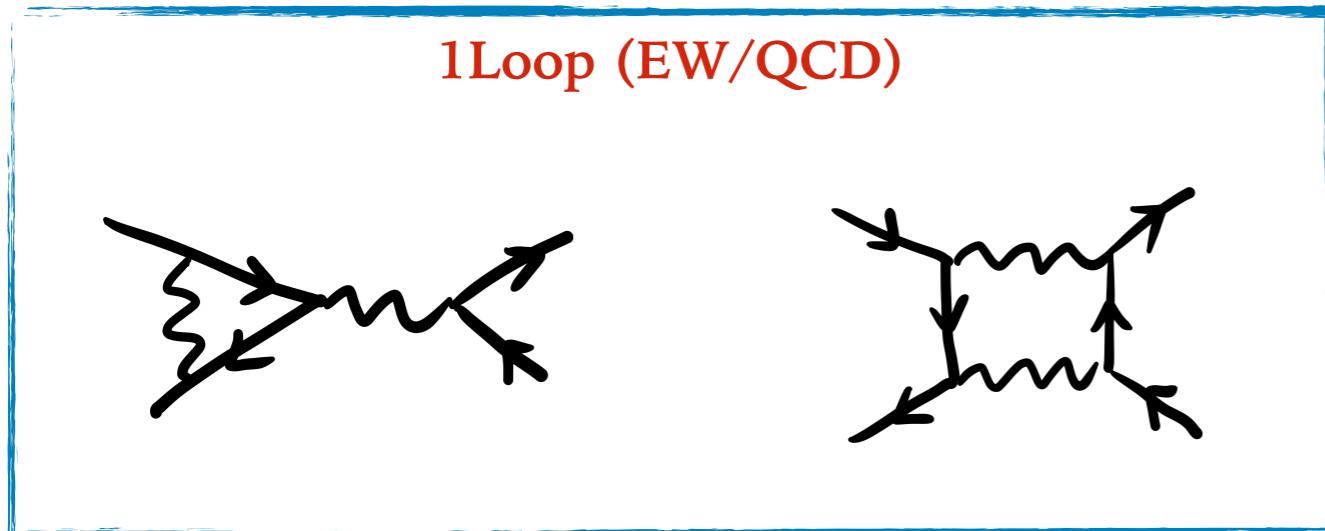
[T. Armadillo, R. Bonciani, SD, N.Rana,
A.Vicini, arXiv:2201.01754]

- When dealing with intermediate unstable particles, such as W and Z, it is useful to perform the calculations in the **complex-mass scheme**;
- We introduce the complex mass $\mu_V^2 = m_V^2 - i\Gamma_V m_V$;
- The complex mass scheme **regularise the behaviour at the resonance**: $\frac{1}{s - \mu_V^2 + i\delta}$;
- the **adimensional kinematical variables** become **complex valued**: $\tilde{s} = \frac{s}{m_V^2} \rightarrow \frac{s}{\mu_V^2}$.



GENERATION OF THE AMPLITUDES

[T. Armadillo, R. Bonciani, SD, N.Rana,
A.Vicini, arXiv:2201.01754]



INTERFERENCE TERMS

[T. Armadillo, R. Bonciani, SD, N.Rana,
A.Vicini, arXiv:2201.01754]

Mathematica/FORM

Two independent In-house routines to automatically perform the Dirac and Lorentz Algebra in dimensional regularisation.

Feynman Amplitudes



Interference terms

- The result can be written as a sum of **tensor integrals** in the form:

$$\int \prod_{i=1}^L dq_i \frac{q_1^{\mu_1} \dots q_1^{\mu_j} \dots q_L^{\mu_1} \dots q_L^{\mu_l}}{P_1 \dots P_t}$$

where:

- $q_i \rightarrow$ loop momentum;
- $L \rightarrow$ number of independent loop momenta;
- $P_i = k_i^2 - m^2 \rightarrow$ inverse propagator, k_i being a linear combination of external and loop momenta.

ISSUE: Handling γ_5 in dimensional regularisation.

Object inherently 4D: how can we use it in arbitrary space-time dimension?

See talk by Dominik Stöckinger!

INTERFERENCE TERMS

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2201.01754]

	ANTICOMMUTATION $\{\gamma_\mu, \gamma_5\} = 0$	CYCLOCITY OF THE TRACE	Feynman Amplitudes
't Hooft and Veltmann <i>Nucl. Phys. B</i> 44 (1972) 189–213	✗	✓	
Kreimer et al. <i>Phys. Lett. B</i> 237 (1990) 59–62	✓	✗	Interference terms

For neutral-current Drell Yan proven that at 2loops the two prescriptions yield:

► **different** scattering amplitudes; ► **same** finite corrections after subtraction.

[M. Heller, A. von Manteuffel, R. M. Schabinger and H. Spiesberger, arXiv:hep-ph/2012.05918]

Our procedure:

1. Use anticommutation relation, bring all γ_5 at the end of the Dirac trace;
2. Use $\gamma_5^2 = 1$, end up with zero or one γ_5 in each Dirac trace;
3. Replace the (single) leftover γ_5 with the relation: $\gamma_5 = \frac{i}{4!} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma$.

INTERFERENCE TERMS

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2201.01754]

Mathematica/FORM

*Independent in-house routines to automatically write the tensor integrals in terms of **Integral families of Scalar integrals**, better suited for the application of reduction algorithms*

Feynman Amplitudes



Interference terms

Tensor Integral

$$\int \prod_{i=1}^L d^n q_i \frac{q_1^{\mu_1} \dots q_1^{\mu_j} \dots q_L^{\mu_1} \dots q_L^{\mu_1}}{P_1 \dots P_t}$$



Scalar Integral

$$\int \prod_{i=1}^L d^n q_i \frac{1}{P_1^{\alpha_1} \dots P_t^{\alpha_t} P_{t+1}^{\alpha_{t+1}} \dots P_N^{\alpha_N}}$$

- A set of inverse propagators $\{P_1^{\alpha_1}, \dots, P_N^{\alpha_N}\}$ defines an **integral family**;
- A scalar integral in an integral family is **uniquely identified** by the (positive or negative) powers of the exponents of the inverse propagators:

$$\int \prod_{i=1}^L d^n q_i \frac{1}{P_1^{\alpha_1} \dots P_t^{\alpha_t} P_{t+1}^{\alpha_{t+1}} \dots P_N^{\alpha_N}} = \text{Family1}[\alpha_1, \dots, \alpha_t, \alpha_{t+1}, \dots, \alpha_N]$$

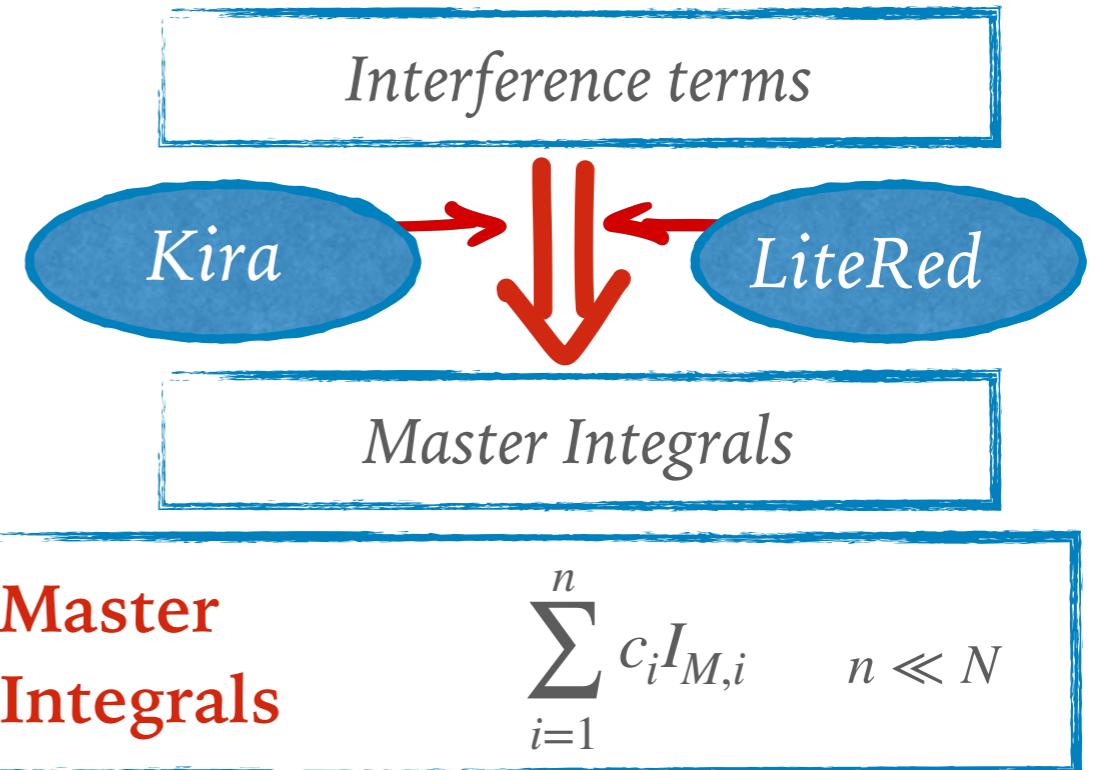
REDUCTION TO MASTER INTEGRALS

[T. Armadillo, R. Bonciani, SD, N.Rana,
A.Vicini, arXiv:2201.01754]

Kira - arXiv:hep-ph/1705.05610

LiteRed - arXiv:hep-ph/1310.1145

Reduction programs implementing Laporta algorithm.



Scalar
Integrals

$$\sum_{i=1}^N \hat{c}_i I_{S,i}$$

Master
Integrals

$$\sum_{i=1}^n c_i I_{M,i} \quad n \ll N$$

- Expressions written as a sum of **scalar integrals** with the respective coefficient;
- All the scalar integrals are not independent: linear relations between them are provided by **integration by parts (IBP) identities**;
- We can reduce the large set of scalar integrals to a smaller set of **master integrals (MI)**;
- Kira applies **Laporta algorithm** to apply IBP identities to a set of integrals in order to find the linear relations between them.

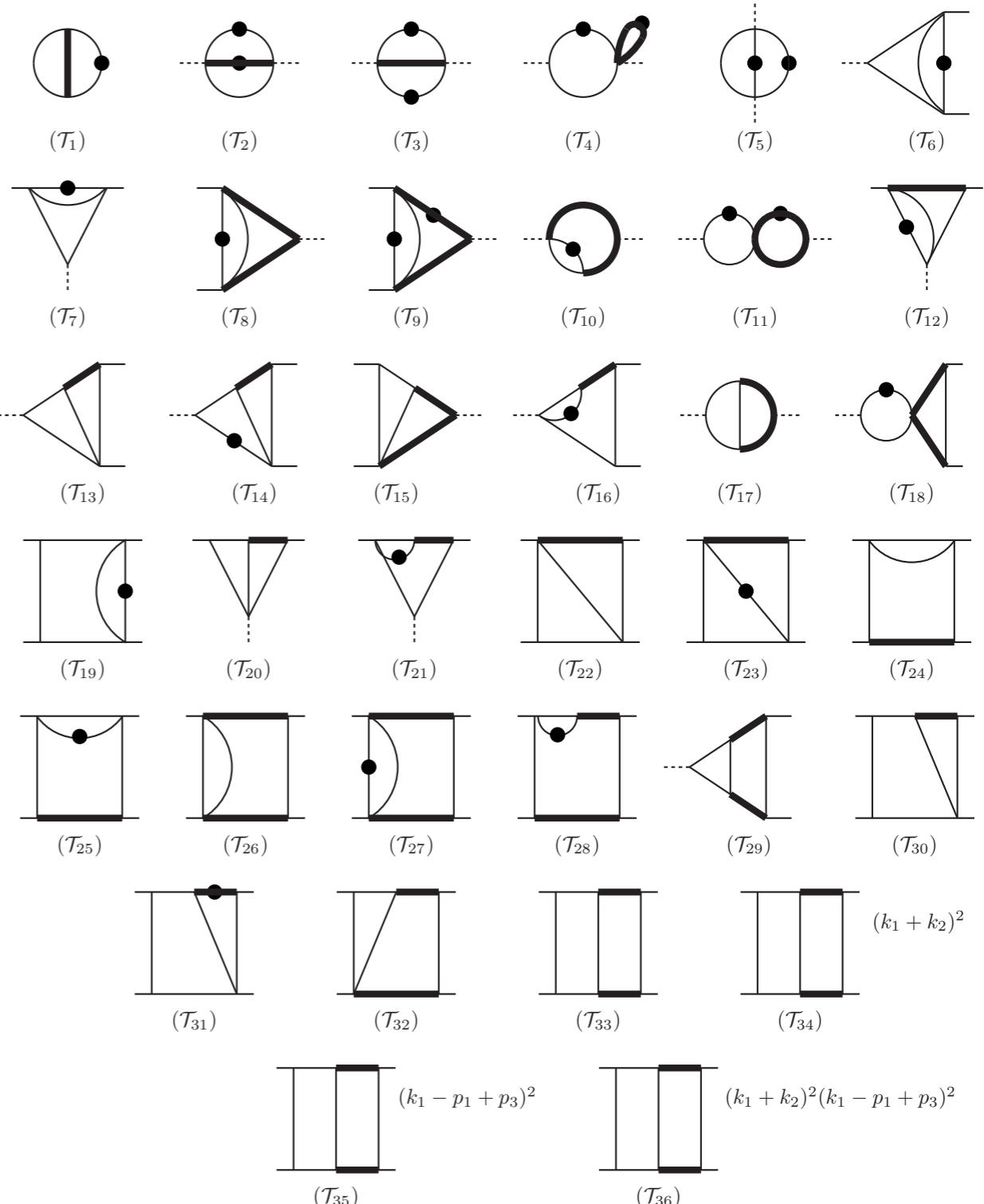
BASIS OF MASTER INTEGRALS

Our basis of MIs is composed by:

- MIs relevant for the **QCD-QED corrections**, with massive final state;
 $[R.Bonciani, A.Ferroglia, T.Gehrmann, D.Maitre, C.Studerus,$
 $arXiv:0806.2301, 0906.3671]$
- MIs with 1 or 2 internal mass relevant for the **EW form factor**;
 $[U.Aglietti, R.Bonciani, hep-ph/0304028, hep-ph/040119]$
- 31 MIs with 1 mass and 36 MIs with 2 masses including boxes, relevant for the **QCD-EW corrections to the full Drell-Yan**.

$[R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert,$
 $arXiv:1604.08581]$

2 masses MIs



BASIS OF MASTER INTEGRALS

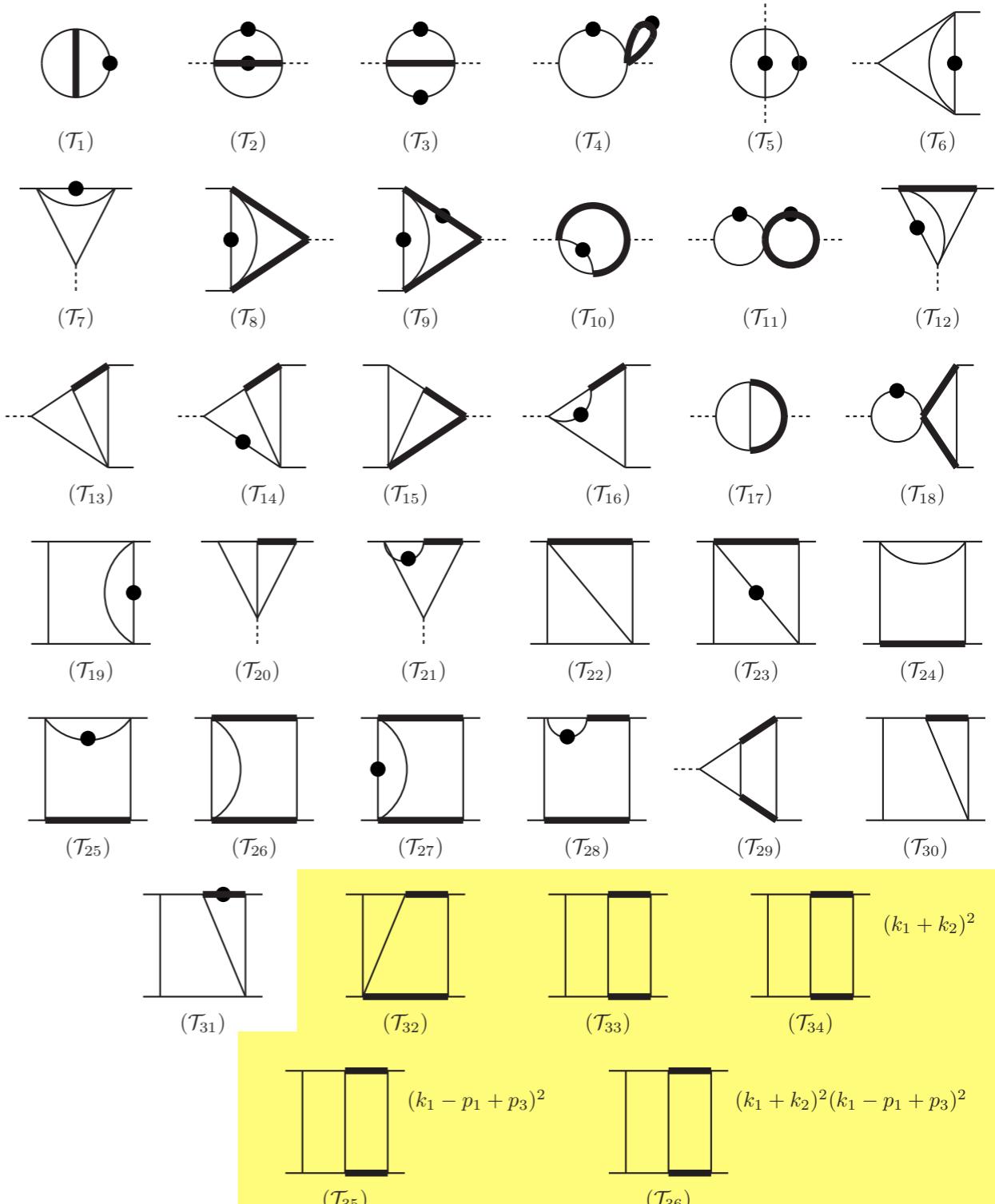
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 $[R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581]$

- 5 box integrals are in **Chen-Goncharov** representation;
- closed form found but not public
 $[M.Heller, A.von Manteuffel, R.M. Schabinger, arXiv:1907.0049]$
- difficult numerical evaluation requires alternative strategy.

Semi-analytical approach!

2 masses MIs



SEMI-ANALYTICAL APPROACH?

Numerical Result

The result of the master integral is provided as a numerical **grid**.

Analytical Result

The result of the master integral can be expressed in closed form as a combination of elementary and special functions, whose **power expansion is known**.

SEMI-ANALYTICAL APPROACH?

Numerical Result

The result of the master integral is provided as a numerical **grid**.

Analytical Result

The result of the master integral can be expressed in closed form as a combination of elementary and special functions, whose **power expansion is known**.

Semi-Analytical Result

The result of the master integral can be expanded as a power series at every point of its domain, but without any additional functional relations.

We solve the master integral with **series expansion!**

SOLVING MASTER INTEGRALS BY SERIES EXPANSION

- The Master Integrals satisfy a **system of differential equations**;
- The system can be solved by **series expansion**.

A SIMPLE EXAMPLE

$$\begin{cases} f'(x) + \frac{1}{x^2 - 4x + 5} f(x) = \frac{1}{x+2} \\ f(0) = 1 \end{cases}$$

$$\begin{cases} rc_0 = 0 \\ \frac{1}{5}c_0 + c_1(r+1) = 0 \\ \frac{4}{25}c_0 + \frac{1}{5}c_1 + c_2(2+r) = 0 \\ \frac{11}{125}c_0 + \frac{4}{25}c_1 + \frac{1}{5}c_2 + c_3(3+r) = 0 \\ \dots \end{cases}$$

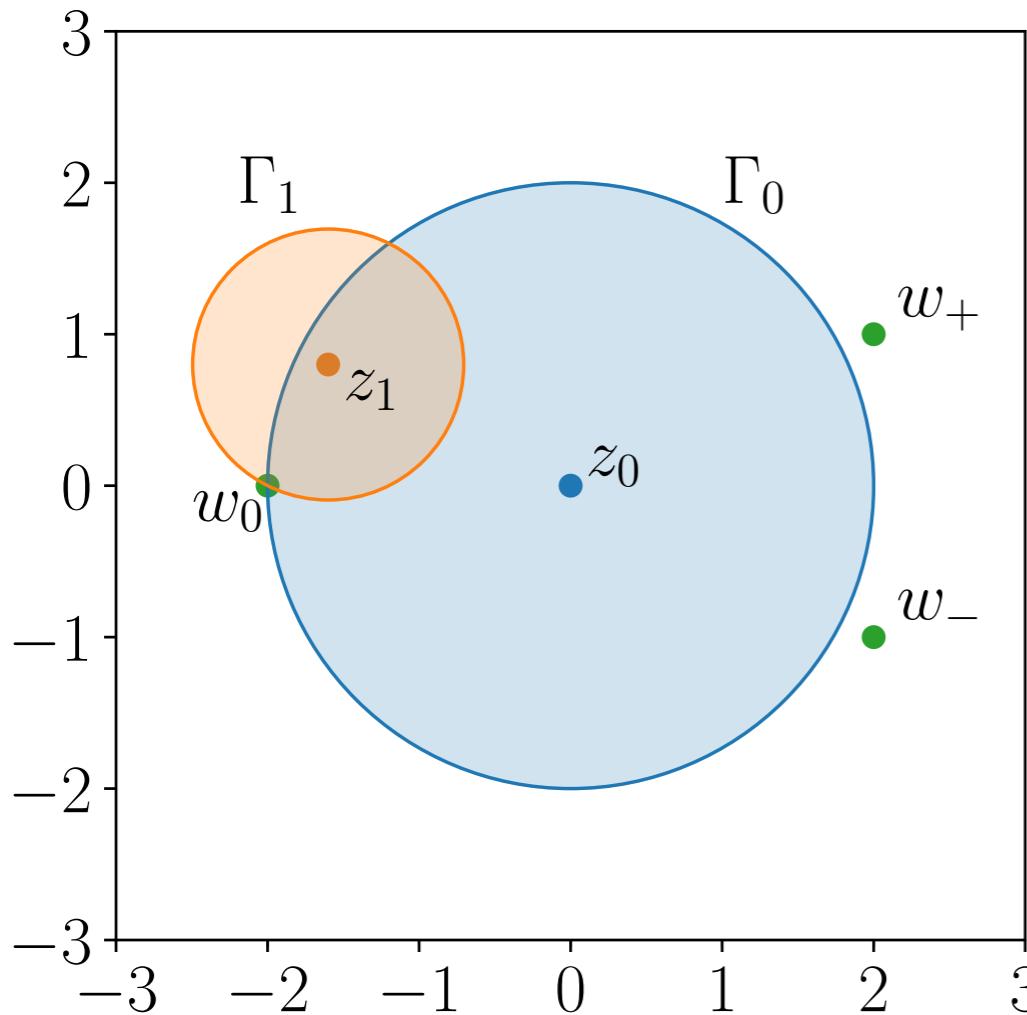
$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f_{hom}(x) = 5 - x - \frac{3}{10}x^2 + \frac{11}{150}x^3 + \dots$$

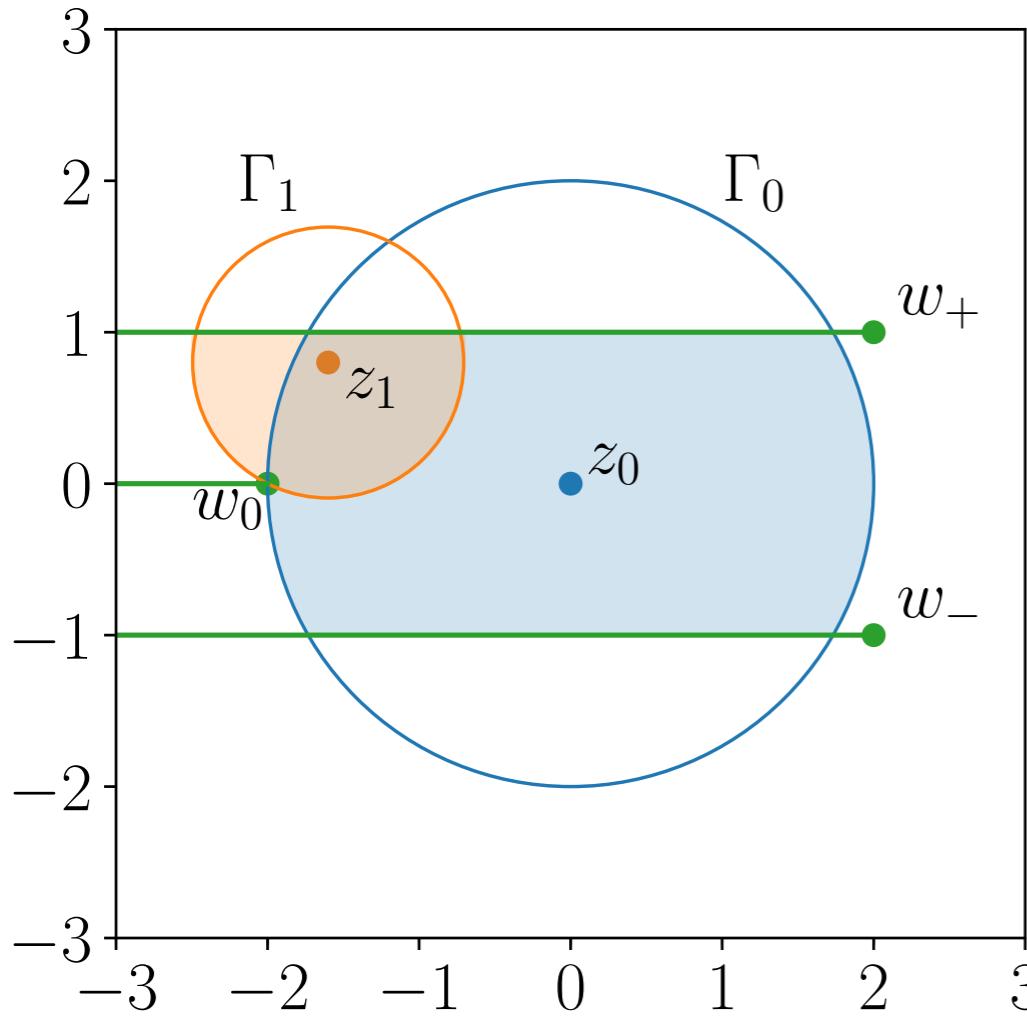
$$\begin{aligned} f_{part}(x) &= f_{hom}(x) \int_0^x dx' \frac{1}{(x'+2)} f_{hom}^{-1}(x') \\ &= \frac{1}{2}x - \frac{7}{40}x^2 + \frac{2}{75}x^3 + \dots \end{aligned}$$

Method implemented in the MATHEMATICA package DIFFEXP for real kinematic variables [F.Moriello, arXiv:1907.13234], [M.Hidding, arXiv:2006.05510]

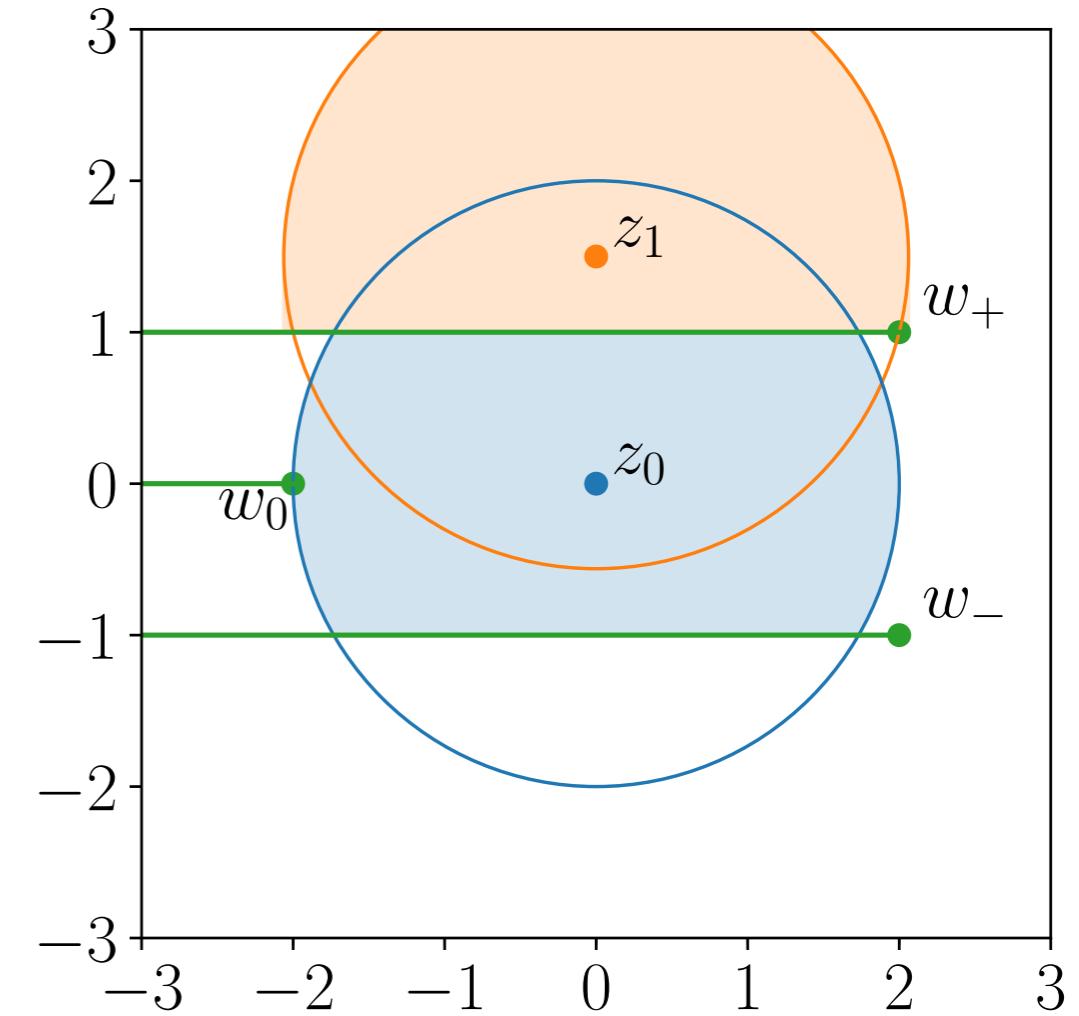
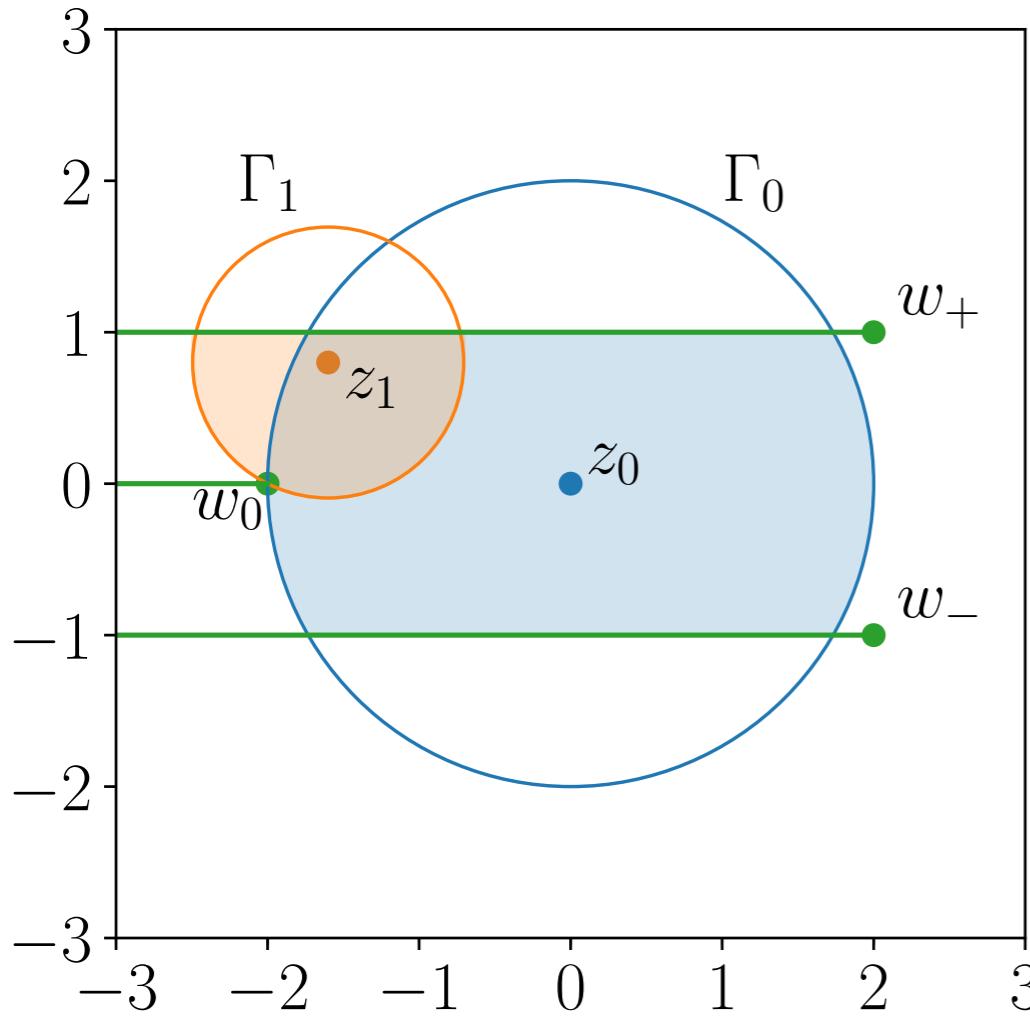
- We implemented in the MATHEMATICA package **SEASYDE** (Series Expansion Approach for SYstems of Differential Equations) the same method, generalising it to **arbitrary complexed-value masses** → **complex plane!**
- The radius of convergence of the series is limited by the presence of **poles**;
- Transport of boundary conditions need to consider **branch-cuts**.



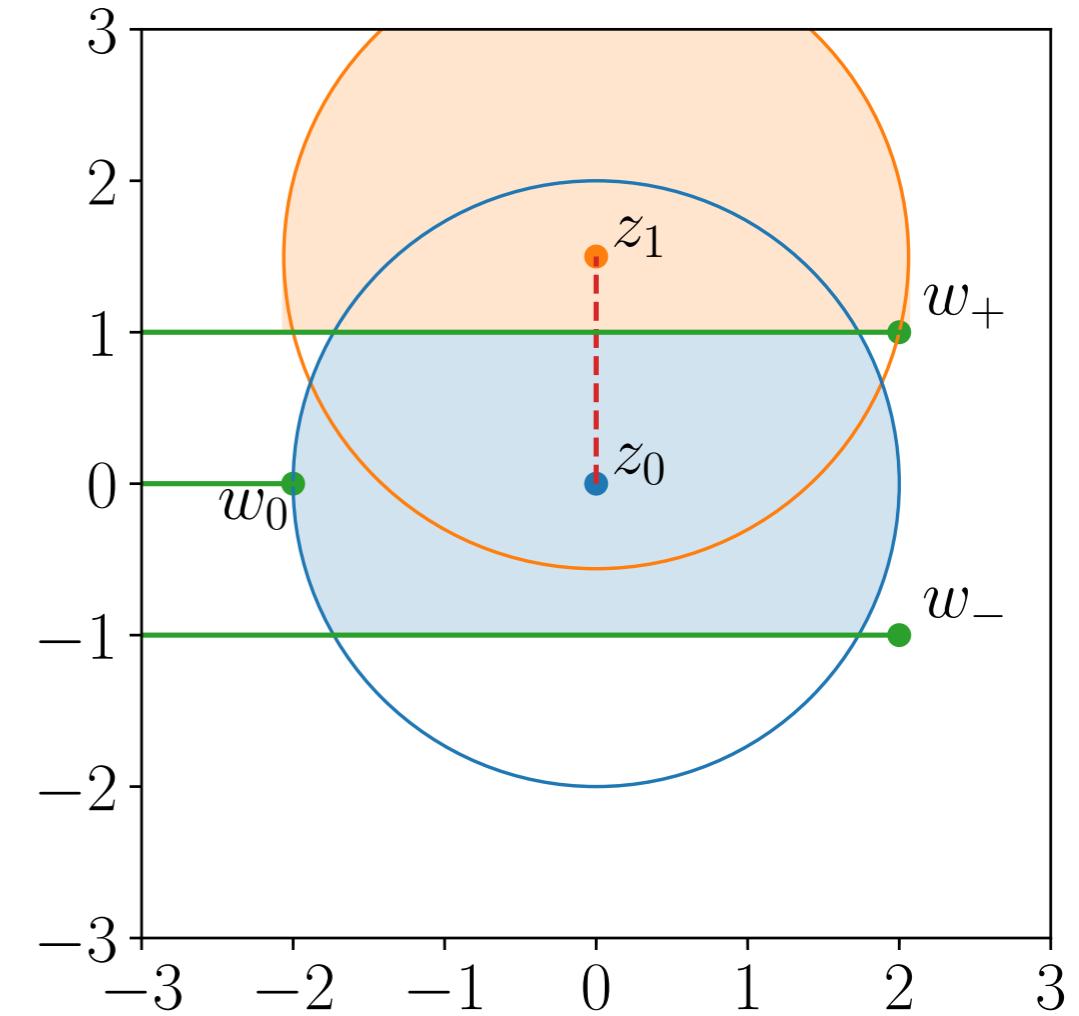
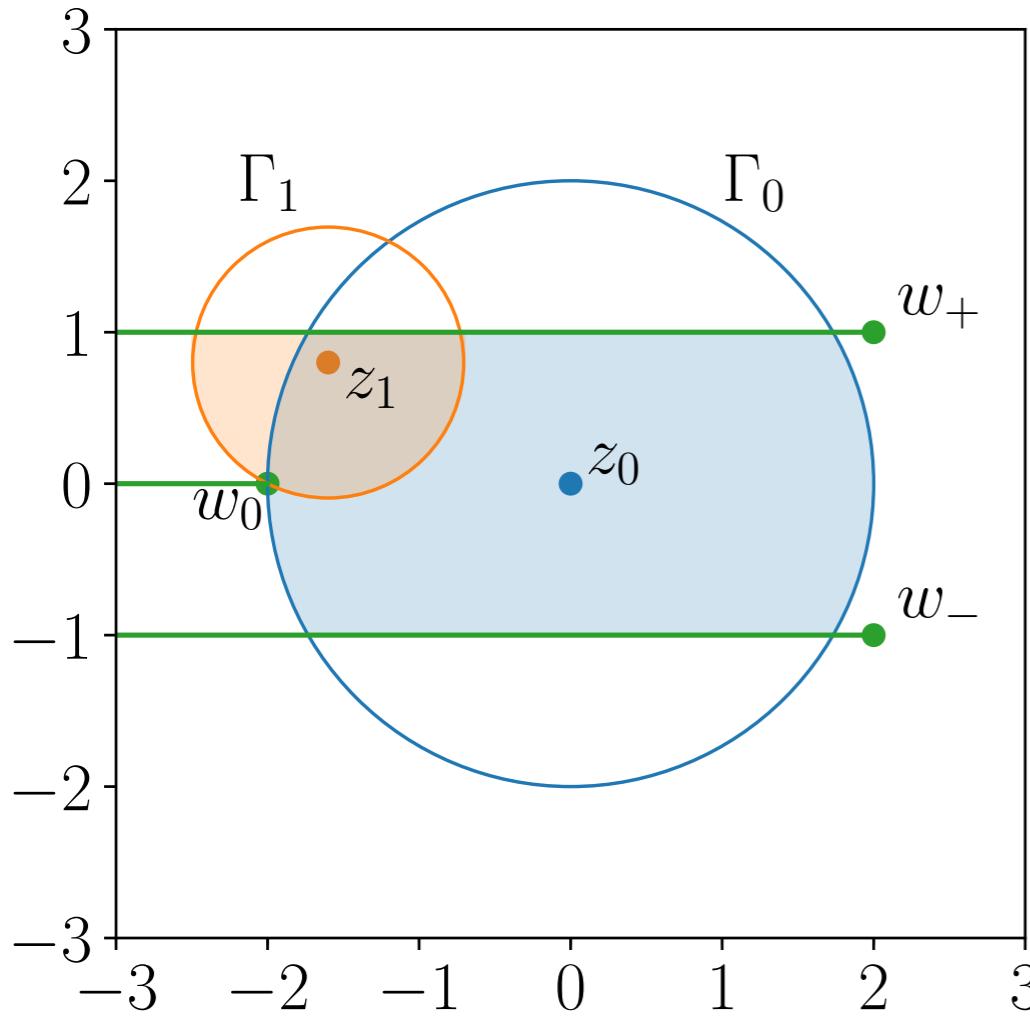
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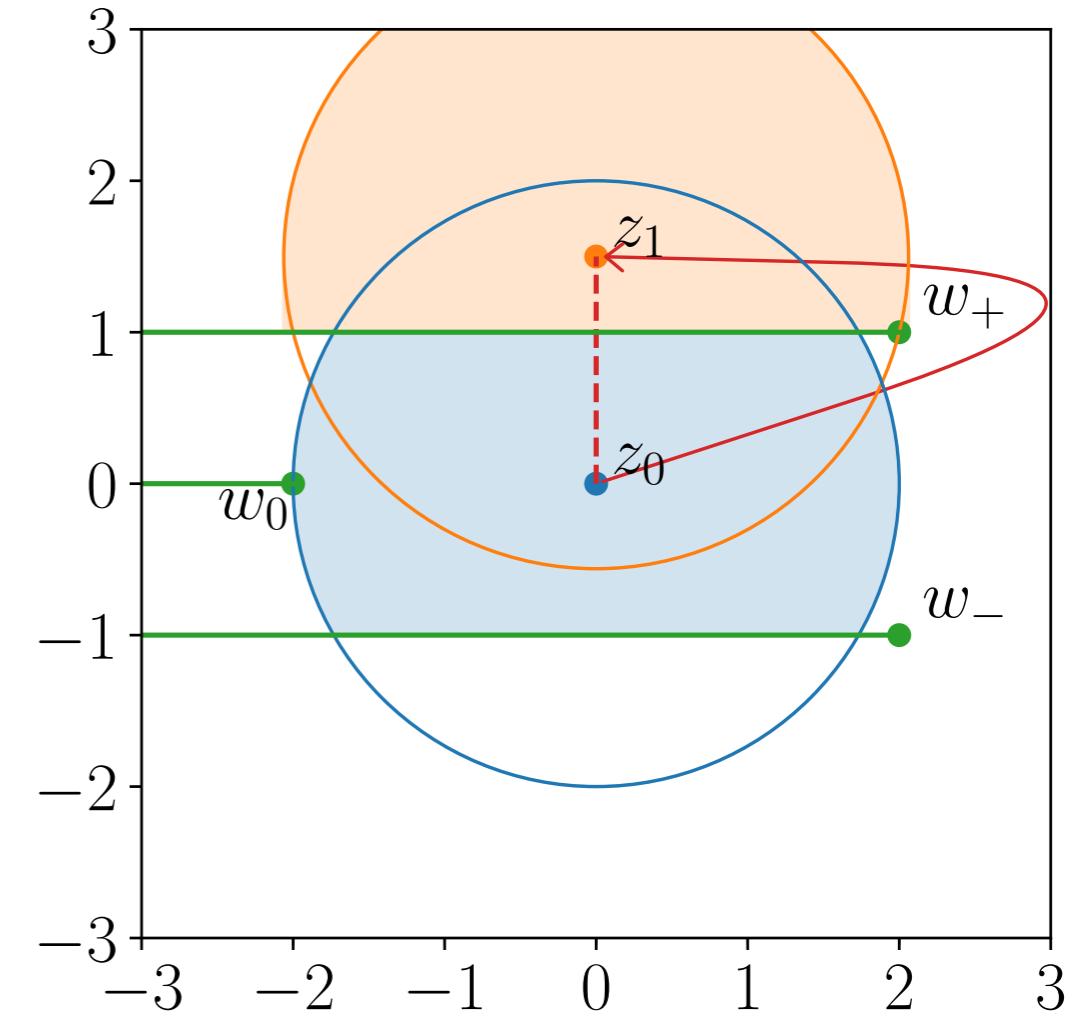
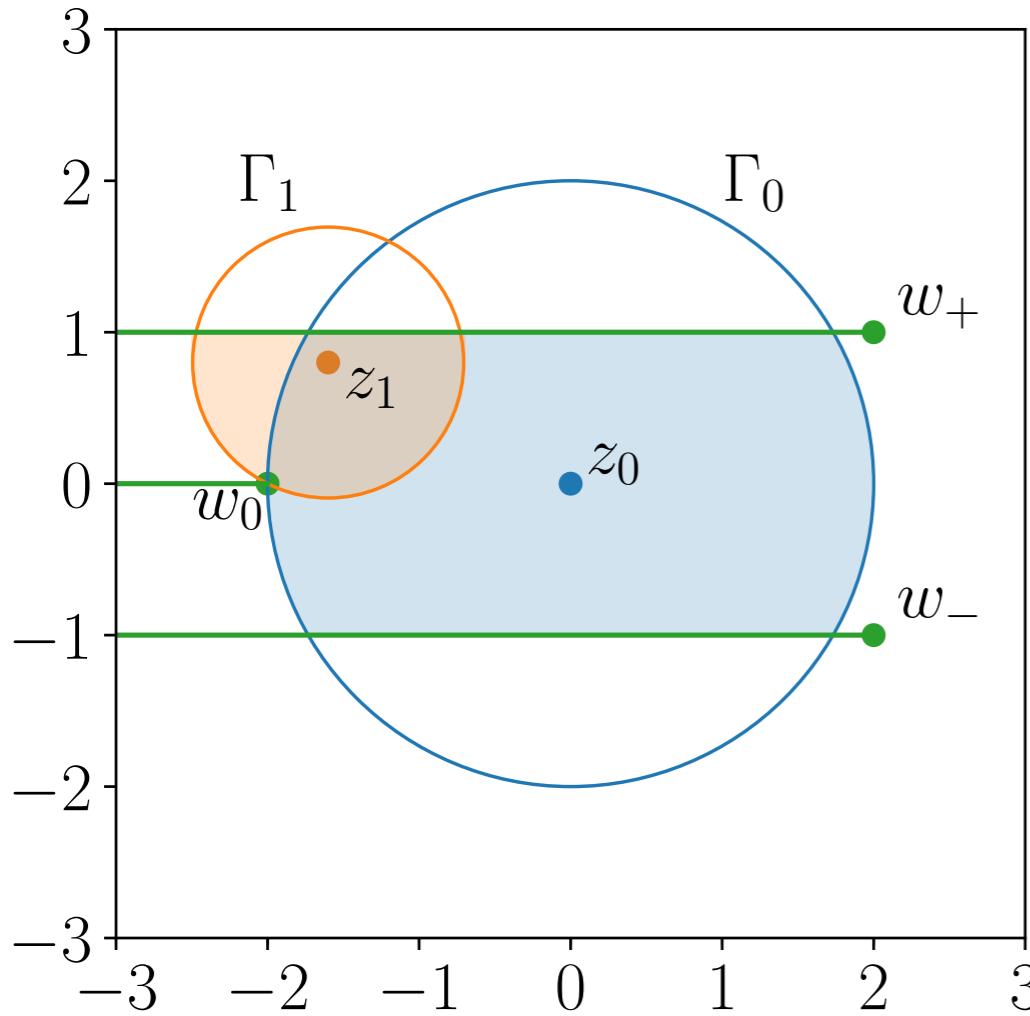
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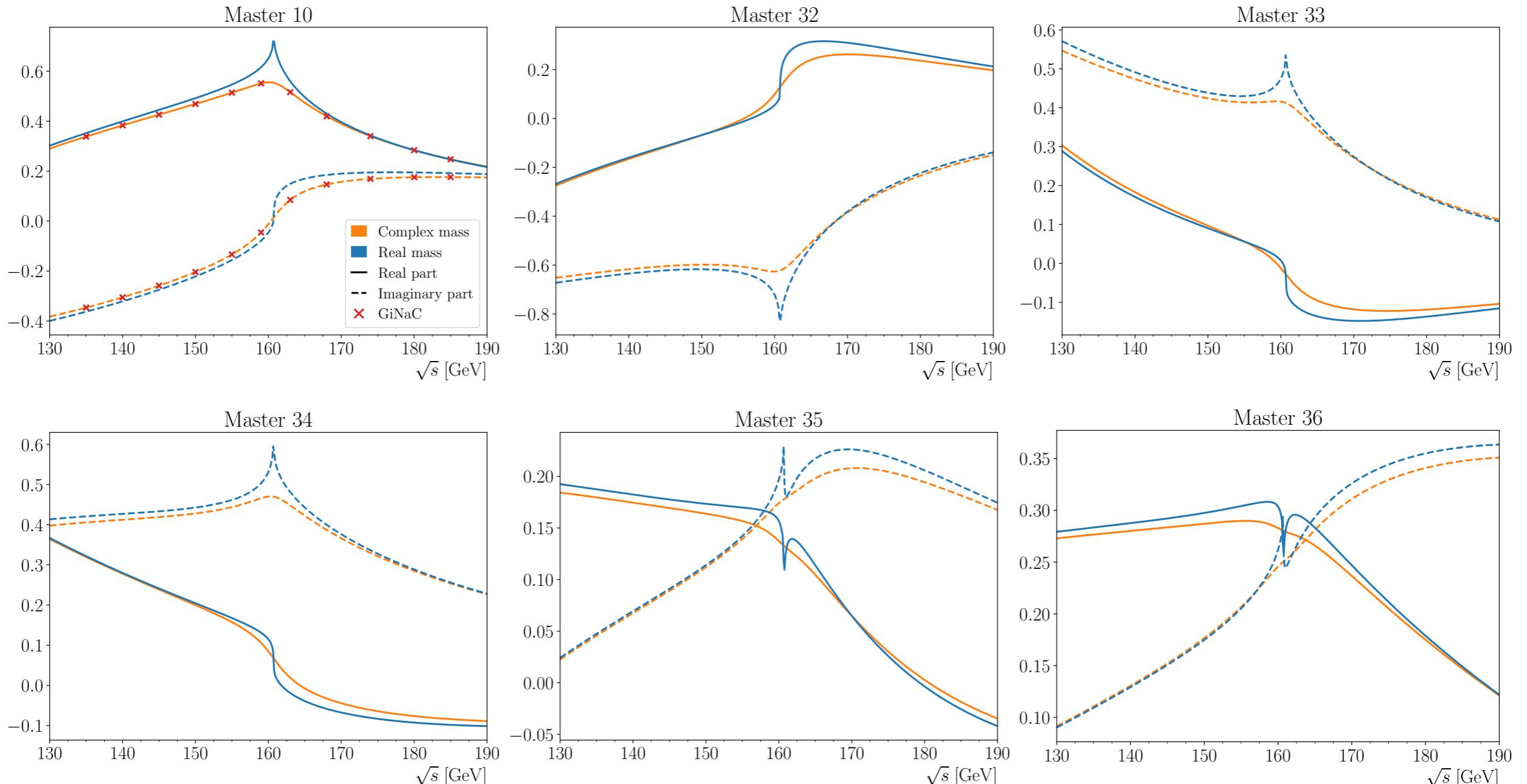
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NUMERICAL GRIDS (MASTERS)

[T. Armadillo, R. Bonciani, SD, N.Rana,
A.Vicini, in preparation]

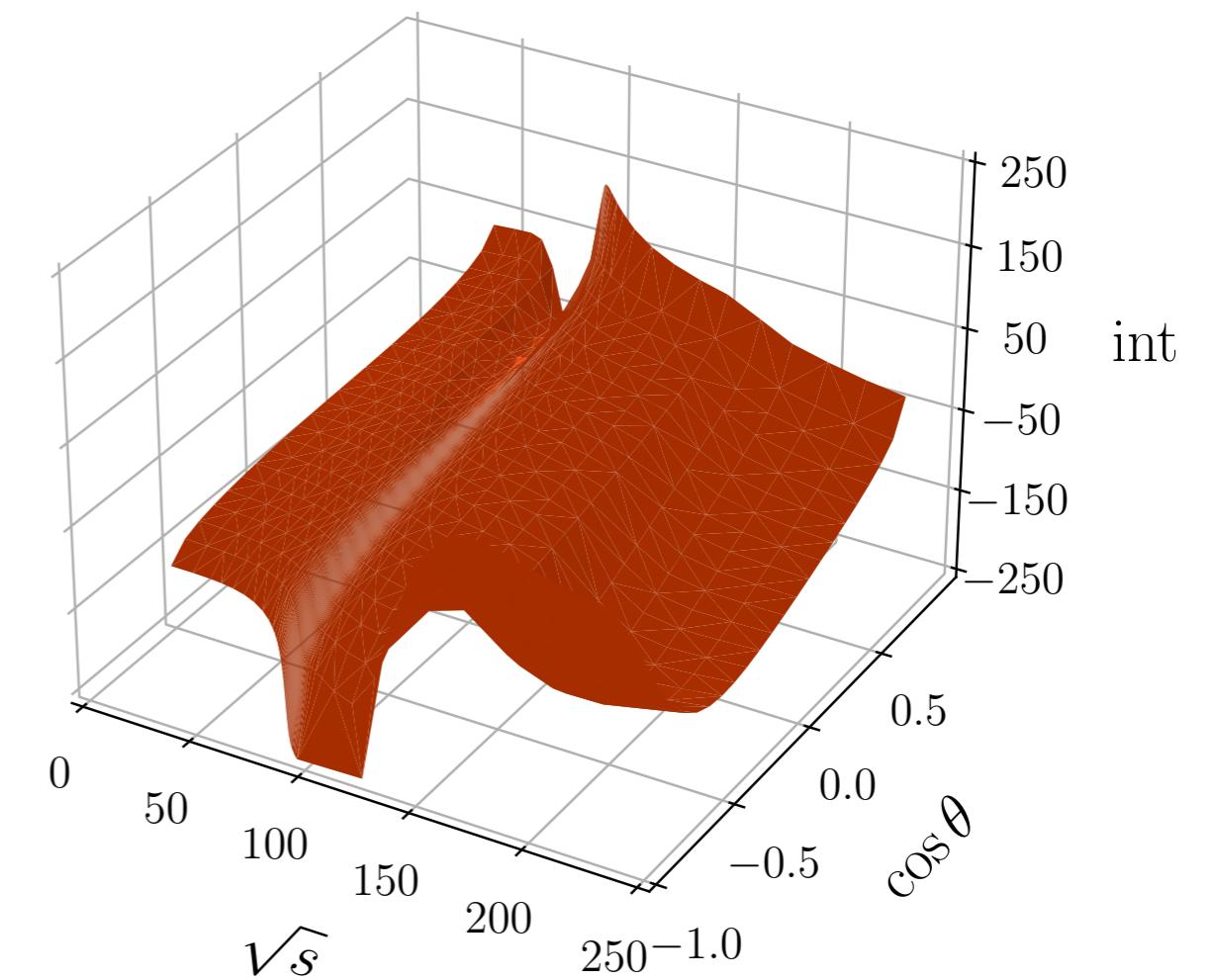
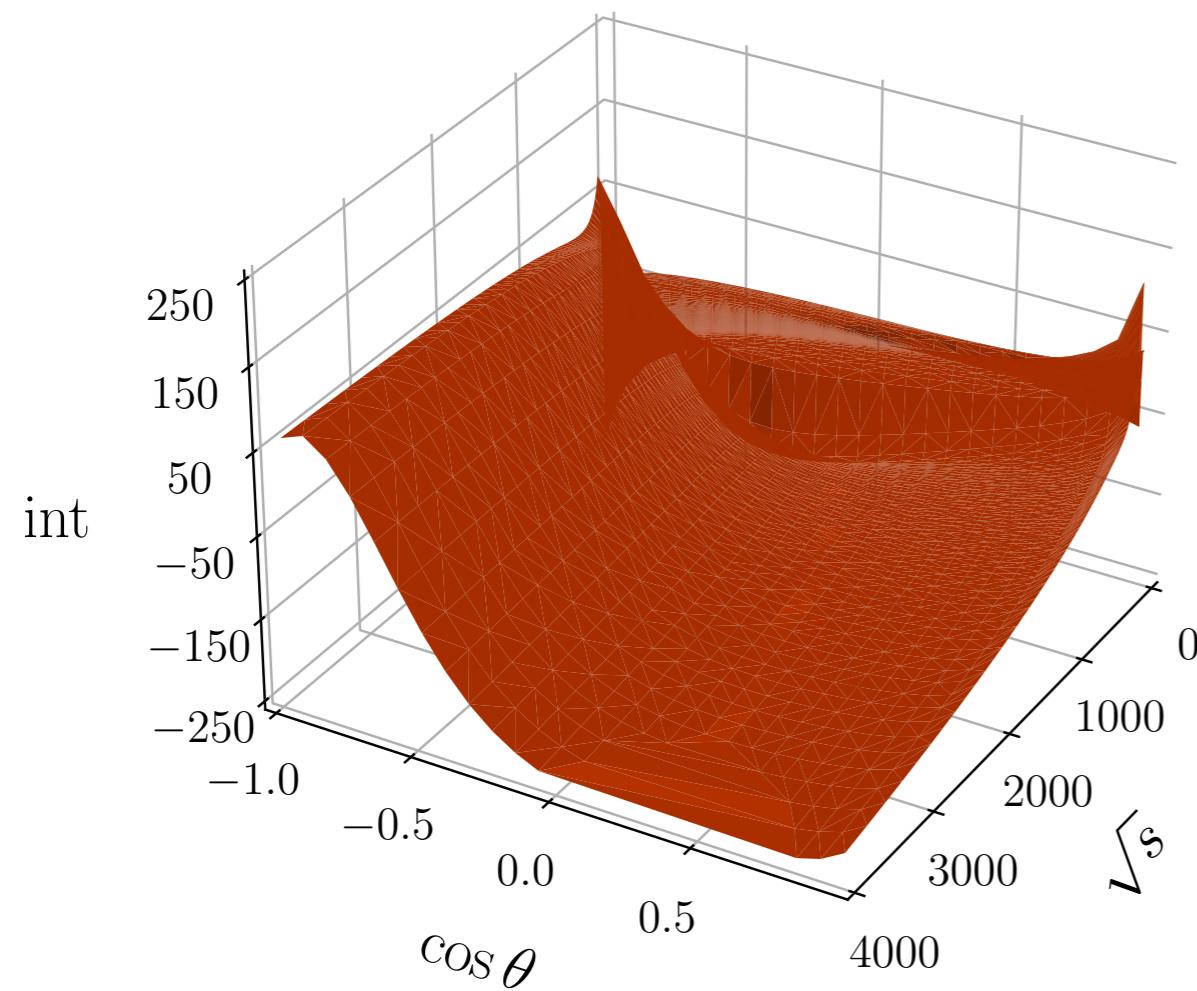
- 31 masters provide **cross checks** with analytic results, 5 are a **prediction**;
- solution can be computed with **arbitrary number of significant digits**;
- **complex mass scheme** smoothens behaviour at threshold.



NUMERICAL GRIDS (HARD FUNCTION)

[T. Armadillo, R. Bonciani, SD, N.Rana,
A.Vicini, arXiv:2201.01754]

- After subtracting IR and UV divergences, we obtain the **hard function**;
- **Publicly available** as a MATHEMATICA notebook;
- Checks on the Master Integrals performed with FIESTA, PYSECDEC, DIFFEXP;
- Production of the grid (3250 points) required $\mathcal{O}(12h)$ on a 32-cores machine;
- Interpolation of the grid requires **negligible time**.



SUMMARY & OUTLOOK

- We computed the **2-loop virtual corrections for neutral-current Drell-Yan**:
 - in the **small lepton mass limit** (keeping collinear logarithms);
 - in the **complex mass scheme**;
- For the evaluation of master integrals with 2 internal masses we wrote **SEASYDE**, a package that generalises the series expansion method implemented by DIFFEXP to **complex-valued kinematic variables**.

SUMMARY & OUTLOOK

- Public release of **SEASYDE**;
- More phenomenological studies;
- NNLO QCD-EW corrections for **charged current Drell-Yan**;
- Towards NNLO-EW?

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THANKS!