Vector and axial-vector coefficient functions for DVCS at NNLO

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Based on 2007.06348, 2106.01437 V. Braun, A.M., S. Moch, J. Schoenleber

Loops and Legs, Ettal, 29 April 2022

DVCS

- Deeply Virtual Compton Scattering, **Müller 94, Ji 96, Radyushkin 96**

$$\gamma^* \ N \longrightarrow \gamma \ N'$$

Goeke et al. 01, Diehl 03, Belitsky, Radyushkin 05, Müller 14

- The DVCS amplitude

$$\mathcal{A}_{\mu\nu}(q,q',p) = i\int d^4x\, e^{-iqx}\langle p'|T\{j_\mu^{\rm em}(x) j_\nu^{\rm em}(0)\}|p\rangle\,.$$

- The leading twist approximation

$$\mathcal{A}_{\mu\nu} = -g_{\mu\nu}^{\perp} V_+ + \epsilon_{\mu\nu}^{\perp} V_- + \ldots$$

(Axial) vector flavor nonsinglet amplitude

$$V_\pm(\xi,t,Q^2) = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{\xi} C_\pm(x/\xi, Q^2/\mu^2)\, F_q^\pm(x,\xi,t,\mu)\,.$$

$C_\pm$ are the coefficient functions, $F_q^\pm$ - the generalized parton distributions (GPD)

$$\xi = -(\Delta,q)/2(P,q), \qquad t=\Delta^2, \qquad \Delta = p'-p, \qquad P=(p+p')/2,$$

$$\langle p'|[\bar q(z_1 n)\Gamma_\pm q(z_2 n)]|p\rangle = 2P_+\int_{-1}^{1} dx\, e^{-iP_+\xi(z_1+z_2)+iP_+x(z_1-z_2)}\, F_q^\pm(x,\xi),$$

where $\Gamma_+=\gamma_+=\gamma\cdot n$, $\Gamma_-=\gamma_+\gamma_5$, $n^2=0$.

LO coefficients functions

$$C_{\pm}^{(0)}(z) = \frac{1}{1-z} \mp \frac{1}{1+z}\,,$$

NLO functions **Ji, Osborne, 98, Belitsky, Müller, 98**

$$C_{\pm}^{(1)}(x) = \frac{C_F}{1-z}\left(-9+\ln^2(1-z)-(2\pm 1)\,\frac{1-z}{1+z}\ln(1-z)\right)$$

DIS $\longleftrightarrow$ DVCS

Amplitudes of both processes are derived from the OPE of two electromagnetic currents:

$$T\{j_\mu^{\rm em}(x)j_\nu^{\rm em}(0)\} = \sum_{N,k} C_{Nk}(x)\partial^k\mathcal{O}_N(0),$$

DIS: Only the operators with $k=0$ are relevant, C_{N0} – moments of the coefficient function.

DVCS: All operators contribute to the amplitude. One need to know C_{Nk} for all k.

In a conformal theory C_{Nk} for $k>0$ are completely determined by C_{N0}

$$T\{j_\mu^{\rm em}(x)j_\nu^{\rm em}(0)\} = \sum_{N} C_N(x,\partial)\mathcal{O}_N(0),$$

Ferrara, Gatto, Grillo, Parisi, 1970

DIS: the coefficient functions and anomalous dimensions are known at NNLO,
Moch, Vermaseren, Vogt, 2004

QCD at the critical point, $\beta(a_*) = -2a_*(\epsilon + \beta_0 a_* + \cdots) = 0$, in $d = 4 - 2\epsilon$ dimensions

Conformal OPE for two conserved vector currents:

$$\mathsf{T}\left\{j^\mu(x)j^\nu(0)\right\} = \sum_N \frac{\mu^{\gamma_N}}{(-x^2)^{\tau_N}} \int_0^1 du(u\bar{u})^{j_N-1}\left\{a_N\left(g^{\mu\nu} - \frac{2x^\mu x^\nu}{x^2}\right) + b_N g^{\mu\nu} + \dots\right\}\mathcal{O}_N^{(x)}(xu)\,,$$

where

$$\mathcal{O}_N^{(x)}(y) = x_{\mu_1}\dots x_{\mu_N}\mathcal{O}_N^{\mu_1\dots\mu_N}(y)\,,$$

$$\Delta_N = d - 2 + N + \gamma_N \qquad \text{(scaling dimension)}$$

$$j_N = \frac{1}{2}(\Delta_N + N) \qquad \text{(conformal spin)}$$

$$\tau_N = d - 1 - t_N/2 \qquad t_N = \Delta_N - N \qquad \text{(twist)}$$

The coefficients a_N and b_N are related to the DIS coefficient functions $C_2(N)$ and $C_L(N)$

The coefficient function for DVCS process depends on x/ξ so we can put $\xi = 1$

$$V(\xi = 1) = \int_{-1}^{1} dx\, C(x)\, F(x, \xi = 1)\,. \qquad \text{(definition)}$$

$$V(\xi = 1) = \sum_{N,\text{even}} f_N\, C_1\,(N)\, \frac{\Gamma(d/2-1)\Gamma(2j_N)}{\Gamma(j_N)\Gamma(j_N + d/2 - 1)} \qquad \text{(OPE result, Müller, 95)}$$

and $P_+^N f_N = \langle p'|\mathcal{O}_N|p\rangle$

$$\mathcal{O}_N \longrightarrow P_N(x) \qquad F(x) = \sum_N f_N P_N(x) \qquad \text{(at LO } P_N(x) = (1-x^2)C_N^{3/2}(x))$$

$$\int_{-1}^{1} dx\, C(x)\, P_N(x) = C_1(N) \times \frac{\Gamma(d/2-1)\Gamma(2j_N)}{\Gamma(j_N)\Gamma(j_N + d/2 - 1)}$$

all factors in the r.h.s. of this equation are known.

What is necessary to do

- Construct functions $P_N(x)$, $N = 2, 4, \cdots$
- Restore the coefficient function $C(x)$ from its (P_N) moments

Braun, A.M., Moch, Strohmaier, 1601.05937, 1703.09532

Symmetry generators for the light-ray operator

$$\mathcal{O}(z_1,z_2)=[\bar q(z_1 n)\gamma_+ q(z_2 n)]=\sum_{Nk}\Psi_{Nk}(z_1,z_2)\partial_+^k\mathcal{O}_N$$

$$S_- = -\partial_{z_1}-\partial_{z_2}$$

$$S_0 = z_1\partial_{z_1}+z_2\partial_{z_2}+2+\left(-\epsilon+\frac{1}{2}\mathrm{H}(a_*)\right)$$

$$S_+ = z_1^2\partial_{z_1}+z_2^2\partial_{z_2}+2(z_1+z_2)+\underbrace{(z_1+z_2)\left(-\epsilon+\frac{1}{2}\mathrm{H}(a_*)\right)+(z_1-z_2)\Delta(a_*)}_{\text{Quantum corrections}}$$

RG equation at the critical point

$$\left(\mu\partial_\mu+\mathrm{H}(a_*)\right)\mathcal{O}(z_1,z_2)=0 \qquad\qquad [S_\alpha,\mathrm{H}(a_*)]=0$$

Changing renormalization scheme:

$$\mathcal{O}\mapsto\mathcal{O}'=U\mathcal{O} \qquad \mathrm{H}\mapsto\mathrm{H}'=U\mathrm{H}U^{-1} \qquad \Delta\mapsto\Delta'(=0?)$$

Conformal scheme

In a scheme with $\Delta' = 0$ the generator S_+ depends only H':

$$\mathrm{H}'(z_1 - z_2)^{N-1} = \gamma_N (z_1 - z_2)^{N-1}$$

$$\Psi_{Nk}(z_1, z_2) = \Big(S_+(\mathrm{H}')\Big)^k (z_1 - z_2)^{N-1} = \Big(S_+(\mathrm{H}' \mapsto \gamma_N)\Big)^k (z_1 - z_2)^{N-1}$$

$$P_N(x) = (1-x^2)^{\lambda_N - 1/2} C_{N-1}^{(\lambda_N)}(x), \qquad \lambda_N = \frac{3}{2} - \epsilon + \frac{1}{2}\gamma_N(a_*)$$

C_N^λ – Gegenbauer polynomials

U: MS scheme $\mapsto$ "conformal scheme": is known with two loop accuracy

CF in the conformal scheme $C'(x) = \int dx' C(x') U^{-1}(x', x)$

$$\int dx C'(x) P_N(x) = C_1^{\mathsf{DIS}}(N)/U_N \times \frac{\Gamma(d/2-1)\Gamma(2j_N)}{\Gamma(j_N)\Gamma(j_N+d/2-1)} = 1 + O(a)$$

Ansatz for $C'(x)$:

$$C'(x) = \int dx' C_0(x') K(x', x),$$

where K is $SL(2)$ invariant operator,

$$K P_N = K(N) P_N,$$

$C_0(x)$ is the leading order coefficient function, $C_0(x) = 1/(1-x) - 1/(1+x)$.

$$K(N) \times B(\lambda_N + 1/2, \lambda_N - 1/2) = C_1^{\mathsf{DIS}}(N)/U_N \times \frac{\Gamma(d/2-1)\Gamma(2j_N)}{\Gamma(j_N)\Gamma(j_N+d/2-1)}$$

$$K(N) = 1 + aK_1(N) + a^2 K_2(N) + \ldots \qquad K(N) \simeq K(-N-1) \text{ (reciprocity)}$$

K is completely determined by its eigenvalues!.

$$
\begin{aligned}
K(N) = {} & 1 \\
& + a_s 2C_F \left\{ \left(\bar{\gamma}_N^{(1)} + \frac{3}{2} \right)^2 + \frac{5}{2} \frac{1}{N(N+1)} - \frac{9}{2} \right\} \\
& + a_s^2 \frac{C_F}{N_c} \Bigg(16 S_1 \Big(2S_{1,-2} - S_{-3} \Big) - 12 S_{-2}^2 - 8S_{-4} + 16 S_1 S_3 + 4\Big(2S_{1,3} - S_4 \Big) - \frac{20 S_3}{N(N+1)} \\
& + \frac{32 \left(S_{-3} - 2S_{1,-2} \right)}{N(N+1)} + \left(\frac{44}{N^2(N+1)^2} + \frac{24}{(N-2)(N+3)} + \frac{52}{N(N+1)} + 8 \right) S_{-2} \\
& + \frac{32}{3} S_1^2 + \left(-\frac{8}{N^3(N+1)^3} - \frac{8}{N^2(N+1)^2} - \frac{86}{3N(N+1)} + \frac{52}{9} \right) S_1 \\
& + \frac{20}{3N^2(N+1)^2} - \frac{59}{9N(N+1)} + \frac{18}{(N-2)(N+3)} - \frac{35}{4} + \left(\frac{50}{N(N+1)} + 54 \right) \zeta_3 \\
& - \frac{\pi^4}{9} - 36 \zeta_3 S_1 - \frac{2\pi^2 S_1}{N(N+1)} + \pi^2 \left(\frac{4}{3N^2(N+1)^2} + \frac{2}{3N(N+1)} - \frac{10}{9} \right) \Bigg) + \dots
\end{aligned}
$$

The invariant kernel in the coordinate representation has the form:

$$[Kf](z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, \boldsymbol{k}\,(\tau)\, f(z_{12}^\alpha, z_{21}^\beta)\,,$$

$z_{12}^\alpha = z_1(1-\alpha) + z_2\alpha$ and $\tau = \alpha\beta/\bar{\alpha}\bar{\beta}$

The eigenvalues

$$K(N) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, \boldsymbol{k}(\tau)(1-\alpha-\beta)^{N-1}$$

Some examples

$$1 \mapsto \frac{1}{N(N+1)}, \qquad -\ln(\bar{\tau}) \mapsto \frac{1}{N^2(N+1)^2} \qquad \frac{1}{2}\,\mathrm{Li}_2(\tau) \mapsto \frac{(-1)^N(S_{-2}(N) + \zeta_2/2)}{N(N+1)}$$

$$K(N) \mapsto \boldsymbol{k}(\tau)$$

Coordinate representation $\mapsto$ ~~momentum fraction~~ $\mapsto$ CF

$$C'(x) = \int dx' C^{(0)}(x') K(x', x) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \left(\frac{\boldsymbol{k}(\tau)}{\bar{\alpha}(1-x) + \beta(1+x)} - (x \leftrightarrow -x) \right) .$$

(**HyperInt** package by E. Panzer)

CF at the critical point

$$C_*^{(2)}(x) \;=\; \beta_0 C_F\, C_*^{(2\beta)}(x) + C_F^2 C_*^{(2P)}(x) + \frac{C_F}{N_c} C_*^{(2A)}(x)\,.$$

$$\begin{aligned}
C_*^{(2P)}(x) = {} & \frac{1}{\omega}\Big(6\mathrm{H}_{0000} - \mathrm{H}_{1000} - 2\mathrm{H}_{200} - \mathrm{H}_{1100} - \mathrm{H}_{120} - \mathrm{H}_{210} + \mathrm{H}_{1110}\Big) \\
& - \frac{1}{\bar\omega}\,\mathrm{H}_{000} - \left(\frac{4}{\omega} - \frac{2}{\bar\omega}\right)\mathrm{H}_{100} + \frac{1}{\bar\omega}\,\mathrm{H}_{20} + \frac{2}{\omega}\,\mathrm{H}_{110} \\
& - \left(\frac{13}{2\bar\omega} + \frac{19}{3\omega}\right)\mathrm{H}_{00} + \left(\frac{3}{\bar\omega} + \frac{11}{3\omega}\right)\mathrm{H}_{10} + \frac{1}{\omega}\zeta_2\Big(\mathrm{H}_{11} - \mathrm{H}_{2} - \mathrm{H}_{10} - 4\mathrm{H}_{00}\Big) \\
& + \left(\frac{1}{\bar\omega}\left(\frac{223}{12} + 5\zeta_2 - 2\zeta_3\right) + \frac{1}{\omega}\left(3\zeta_2 + 16\zeta_3 - \frac{32}{9}\right)\right)\mathrm{H}_0 \\
& + \frac{1}{48\omega}\Big(701 + 128\zeta_2 + 936\zeta_3 + 72\zeta_2^2\Big) - (\omega \leftrightarrow \bar\omega)
\end{aligned}$$

$\omega = \frac{1-x}{2}$ and $\mathrm{H}_n = \mathrm{H}_n(\omega)$ HPL functions

Two loop CF

$$C^{(2)}(x) = C_*^{(2)}(x) + \beta_0 C^{(1,1)}(x)\,,$$

$$C^{(1,1)}(x) = -C_F \frac{1}{2\omega}\left\{18 - \frac{\pi^2}{4} - \left(5 - \frac{4}{\bar\omega} + \frac{\pi^2}{6}\right)\ln\omega - \frac{3}{2}\frac{\omega}{\bar\omega}\ln^2\omega + \frac{1}{3}\ln^3\omega\right\} - (\omega \leftrightarrow \bar\omega)$$

$C^{(11)}$ comes from one loop diagrams.

The leading double-logarithmic asymptotic of the CF at $\omega \to 0$:

$$C(x, a_s) \simeq \frac{1}{2\omega}\left(1 + C_F a_s \ln^2\omega + \frac{1}{2}(C_F a_s)^2 \ln^4\omega + \mathcal{O}(a_s^3)\right),$$

suggesting that the series exponentiates. (disagrees with **Altinoluk, Pire, Szymanowski and Wallon, 2012**)

Axial-vector CF:

agrees with **Jing Gao, Tobias Huber, Yao Ji, Yu-Ming Wang, Phys.Rev.Lett. 128 (2022)**
B. Melic, B. Nizic, K. Passek 2002 (β_0 contribution)

Numerics

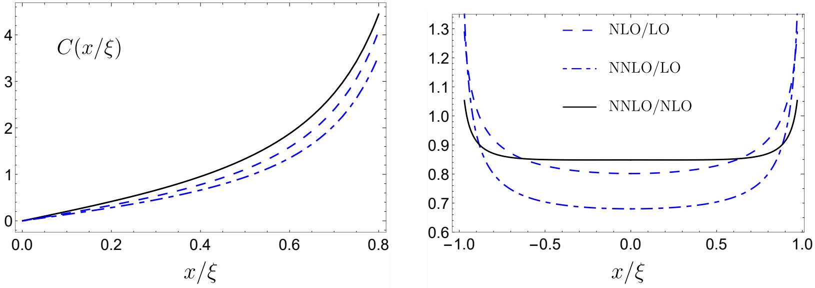


Figure: The DVCS CF $C(x/\xi)$ at $\mu = Q = 2$ GeV in the ERBL region $x < \xi$. The LO (tree-level), NLO (one-loop) and NNLO (two-loop) CFs are shown by the black solid, blue dashed and blue dash-dotted curves on the left panel, respectively. The right panel shows the ratios NLO/LO (dashed), NNLO/LO (dash-dotted) and NNLO/NLO (solid).

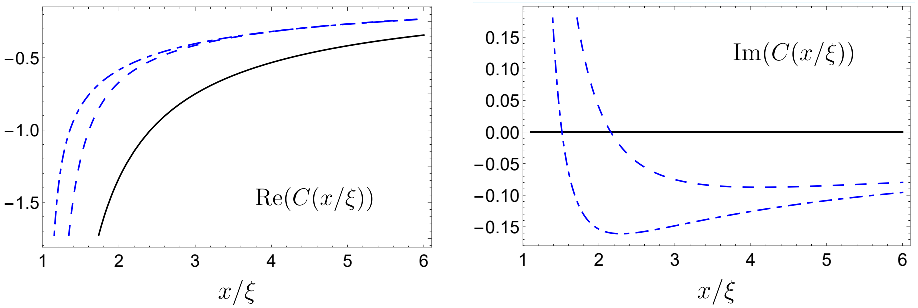


Figure: The DVCS CF $C(x/\xi)$ at $\mu = Q = 2$ GeV analytically continued into the DGLAP region $x > \xi$: real part on the left and imaginary part on the right panel. The LO (tree-level), NLO (one-loop) and NNLO (two-loop) CFs are shown by the black solid, blue dashed and blue dash-dotted curves. Note, that imaginary part of the LO CF contains a local term $\sim \delta(x - \xi)$ (not shown).

Numerics

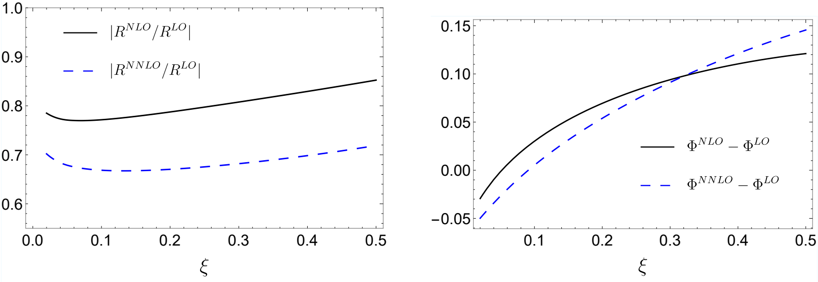
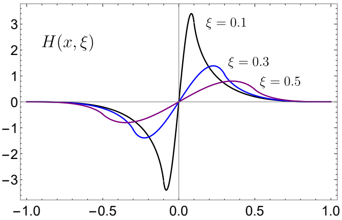


Figure: Higher-order QCD corrections to the Compton form factor $\mathcal{H}(\xi)$.The ratios of the Compton form factor calculated to the NNLO and NLO accuracy with respect to the tree-level are shown for the absolute value and the phase of $\mathcal{H}(\xi) = Re^{i\Phi}$ on the left and the right panels, respectively.



Conclusions

- Using an approach based on conformal symmetry we have calculated two loop DVCS CFs in $\overline{\text{MS}}$ scheme for the flavor-nonsinglet vector and axial-vector operators
- Numerical estimates show that the two loop corrections to the Compton Form Factors are relatively large