Vector and axial-vector coefficient functions for DVCS at NNLO

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Based on 2007.06348, 2106.01437 V. Braun, A.M., S. Moch, J. Schoenleber

Loops and Legs, Ettal, 29 April 2022
Deeply Virtual Compton Scattering, Müller 94, Ji 96, Radyushkin 96

$\gamma^* N \rightarrow \gamma N'$

Goeke et al. 01, Diehl 03, Belitsky, Radyushkin 05, Müller 14

The DVCS amplitude

$$A_{\mu\nu}(q, q', p) = i \int d^4x \, e^{-iqx} \langle p' | T\{j^\text{em}_\mu(x) j^\text{em}_\nu(0)\} | p \rangle.$$  

The leading twist approximation

$$A_{\mu\nu} = -g^\perp_{\mu\nu} V_+ + \epsilon^\perp_{\mu\nu} V_- + \ldots$$
(Axial) vector flavor nonsinglet amplitude

\[ V_{\pm}(\xi, t, Q^2) = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{\xi} C_{\pm}(x/\xi, Q^2/\mu^2) F_{q}^{\pm}(x, \xi, t, \mu). \]

\( C_{\pm} \) are the coefficient functions, \( F_{q}^{\pm} \) - the generalized parton distributions (GPD)

\[ \xi = -(\Delta, q)/2(P, q), \quad t = \Delta^2, \quad \Delta = p' - p, \quad P = (p + p')/2, \]

\[ \langle p' | [\bar{q}(z_1 n) \Gamma_{\pm} q(z_2 n)] | p \rangle = 2P_+ \int_{-1}^{1} dx \, e^{-iP+\xi(z_1+z_2)+iP+x(z_1-z_2)} F_{q}^{\pm}(x, \xi), \]

where \( \Gamma_+ = \gamma_+ = \gamma \cdot n, \Gamma_- = \gamma_+ \gamma_5, \, n^2 = 0. \)
LO coefficients functions

\[ C_{\pm}^{(0)}(z) = \frac{1}{1 - z} \mp \frac{1}{1 + z}, \]

NLO functions \textit{Ji, Osborne, 98, Belitsky, Müller, 98}

\[ C_{\pm}^{(1)}(x) = \frac{C_F}{1 - z} \left( -9 + \ln^2(1 - z) - (2 \pm 1) \frac{1 - z}{1 + z} \ln(1 - z) \right) \]
**DVCS vs DIS**

**DIS ↔ DVCS**

Amplitudes of both processes are derived from the OPE of two electromagnetic currents:

\[
T\{j_{\mu}^{em}(x)j_{\nu}^{em}(0)\} = \sum_{N,k} C_{Nk}(x) \partial^k \mathcal{O}_N(0),
\]

**DIS:** Only the operators with \( k = 0 \) are relevant, \( C_{N0} \) – moments of the coefficient function.

**DVCS:** All operators contribute to the amplitude. One need to know \( C_{Nk} \) for all \( k \).

In a conformal theory \( C_{Nk} \) for \( k > 0 \) are completely determined by \( C_{N0} \)

\[
T\{j_{\mu}^{em}(x)j_{\nu}^{em}(0)\} = \sum_N C_N(x, \partial) \mathcal{O}_N(0),
\]

**Ferrara, Gatto, Grillo, Parisi, 1970**

**DIS:** the coefficient functions and anomalous dimensions are known at NNLO, **Moch, Vermaseren, Vogt, 2004**
QCD at the critical point, $\beta(a_*) = -2a_*(\epsilon + \beta_0a_* + \cdots) = 0$, in $d = 4 - 2\epsilon$ dimensions

**Conformal OPE** for two conserved vector currents:

$$
T \left\{ j^{\mu}(x)j^{\nu}(0) \right\} = \sum_N \frac{\mu^{\gamma_N}}{(-x^2)^{\tau_N}} \int_0^1 du (u\bar{u})^{j_N-1} \left\{ a_N \left( g^{\mu\nu} - \frac{2x^\mu x^\nu}{x^2} \right) + b_N g^{\mu\nu} + \ldots \right\} O_N^{(x)}(xu),
$$

where

$$
O_N^{(x)}(y) = x_{\mu_1} \ldots x_{\mu_N} O_N^{\mu_1 \ldots \mu_N}(y),
$$

$$
\Delta_N = d - 2 + N + \gamma_N \quad \text{(scaling dimension)}
$$

$$
\jmath_N = \frac{1}{2}(\Delta_N + N) \quad \text{(conformal spin)}
$$

$$
\tau_N = d - 1 - t_N/2 \quad t_N = \Delta_N - N \quad \text{(twist)}
$$

The coefficients $a_N$ and $b_N$ are related to the DIS coefficient functions $C_2(N)$ and $C_L(N)$.
The coefficient function for DVCS process depends on $x/\xi$ so we can put $\xi = 1$

$$V(\xi = 1) = \int_{-1}^{1} dx \; C(x) \; F(x, \xi = 1). \quad \text{(definition)}$$

$$V(\xi = 1) = \sum_{N, \text{even}} f_N \; C_1(N) \frac{\Gamma(d/2 - 1) \Gamma(2j_N)}{\Gamma(j_N) \Gamma(j_N + d/2 - 1)} \quad \text{(OPE result, Müller, 95)}$$

and $P_N^+ f_N = \langle p' | \mathcal{O}_N | p \rangle$

$$\mathcal{O}_N \rightarrow P_N(x) \quad F(x) = \sum_{N} f_N P_N(x) \quad \text{(at LO} \; P_N(x) = (1 - x^2)C_{N}^{3/2}(x))$$

$$\int_{-1}^{1} dx \; C(x) \; P_N(x) = C_1(N) \times \frac{\Gamma(d/2 - 1) \Gamma(2j_N)}{\Gamma(j_N) \Gamma(j_N + d/2 - 1)}$$

all factors in the r.h.s. of this equation are known.
What is necessary to do

- Construct functions $P_N(x)$, $N = 2, 4, \cdots$

- Restore the coefficient function $C(x)$ from its $(P_N)$ moments
Symmetry generators

Braun, A.M., Moch, Strohmaier, 1601.05937, 1703.09532

Symmetry generators for the light-ray operator

\[ \mathcal{O}(z_1, z_2) = [\bar{q}(z_1 n) \gamma_+ q(z_2 n)] = \sum_{Nk} \Psi_{Nk}(z_1, z_2) \partial_+^k \mathcal{O}_N \]

\[ S_- = -\partial z_1 - \partial z_2 \]

\[ S_0 = z_1 \partial z_1 + z_2 \partial z_2 + 2 + \left( -\epsilon + \frac{1}{2} H(a_*) \right) \]

\[ S_+ = z_1^2 \partial z_1 + z_2^2 \partial z_2 + 2(z_1 + z_2) + (z_1 + z_2) \left( -\epsilon + \frac{1}{2} H(a_*) \right) + (z_1 - z_2) \Delta(a_*) \]

Quantum corrections

RG equation at the critical point

\[ \left( \mu \partial \mu + H(a_*) \right) \mathcal{O}(z_1, z_2) = 0 \]

\[ [S_\alpha, H(a_*)] = 0 \]

Changing renormalization scheme:

\[ \mathcal{O} \mapsto \mathcal{O}' = U \mathcal{O} \]

\[ H \mapsto H' = U H U^{-1} \]

\[ \Delta \mapsto \Delta'(= 0?) \]
Conformal scheme

In a scheme with $\Delta' = 0$ the generator $S_+$ depends only $H'$:

$$H'(z_1 - z_2)^{N-1} = \gamma_N(z_1 - z_2)^{N-1}$$

$$\Psi_{N,k}(z_1, z_2) = (S_+(H'))^k(z_1 - z_2)^{N-1} = (S_+(H' \mapsto \gamma_N))^k(z_1 - z_2)^{N-1}$$

$$P_N(x) = (1 - x^2)^{\lambda_N - 1/2}C^{(\lambda_N)}_{N-1}(x), \quad \lambda_N = \frac{3}{2} - \epsilon + \frac{1}{2} \gamma_N(a_*)$$

$C^\lambda_N$ – Gegenbauer polynomials

$U$: MS scheme $\mapsto$ "conformal scheme": is known with two loop accuracy
How to restore the coefficient function?

CF in the conformal scheme $C'(x) = \int dx' C(x') U^{-1}(x', x)$

\[
\int dx C''(x) P_N(x) = C_{1}^{\text{DIS}}(N)/U_N \times \frac{\Gamma(d/2 - 1)\Gamma(2j_N)}{\Gamma(j_N)\Gamma(j_N + d/2 - 1)} = 1 + O(a)
\]

Ansatz for $C'(x)$:

\[
C'(x) = \int dx' C_0(x') K(x', x),
\]

where $K$ is $SL(2)$ invariant operator,

\[
KP_N = K(N)P_N,
\]

$C_0(x)$ is the leading order coefficient function, $C_0(x) = 1/(1 - x) - 1/(1 + x)$.

\[
K(N) \times B(\lambda_N + 1/2, \lambda_N - 1/2) = C_{1}^{\text{DIS}}(N)/U_N \times \frac{\Gamma(d/2 - 1)\Gamma(2j_N)}{\Gamma(j_N)\Gamma(j_N + d/2 - 1)}
\]

\[
K(N) = 1 + aK_1(N) + a^2K_2(N) + \ldots \quad K(N) \simeq K(-N - 1) \text{ (reciprocity)}
\]

$K$ is completely determined by its eigenvalues!
\[ K(N) = 1 \]

\[ + a_s 2 C_F \left\{ \left( \frac{\tilde{\gamma}_N^{(1)}}{2} + \frac{3}{2} \right)^2 + \frac{5}{2} \frac{1}{N(N + 1)} - \frac{9}{2} \right\} \]

\[ + a_s^2 \frac{C_F}{N_c} \left[ 16 S_1 (2 S_1 - 2 - S_3) - 12 S_{-2}^2 - 8 S_{-4} + 16 S_1 S_3 + 4 (2 S_{1,3} - S_4) - \frac{20 S_3}{N(N + 1)} \right] \]

\[ + \frac{32}{N(N + 1)} \left( S_{-3} - 2 S_{1, -2} \right) + \left( \frac{44}{N^2(N + 1)^2} + \frac{24}{(N - 2)(N + 3)} + \frac{52}{N(N + 1)} + 8 \right) S_{-2} \]

\[ + \frac{32}{3} S_1^2 + \left( -\frac{8}{N^3(N + 1)^3} - \frac{8}{N^2(N + 1)^2} - \frac{86}{3 N(N + 1)} + \frac{52}{9} \right) S_1 \]

\[ + \frac{20}{3 N^2(N + 1)^2} - \frac{59}{9 N(N + 1)} + \frac{18}{(N - 2)(N + 3)} - \frac{35}{4} + \left( \frac{50}{N(N + 1)} + 54 \right) \zeta_3 \]

\[ - \frac{\pi^4}{9} - 36 \xi_3 S_1 - \frac{2 \pi^2 S_1}{N(N + 1)} + \pi^2 \left( \frac{4}{3 N^2(N + 1)^2} + \frac{2}{3 N(N + 1)} - \frac{10}{9} \right) \right) + \ldots \]
The invariant kernel in the coordinate representation has the form:

\[
[Kf](z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \; k(\tau) f(z_{12}^\alpha, z_{21}^\beta),
\]

\[z_{12}^\alpha = z_1(1 - \alpha) + z_2 \alpha \text{ and } \tau = \alpha \beta / \bar{\alpha} \bar{\beta}\]

The eigenvalues

\[K(N) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \; k(\tau)(1 - \alpha - \beta)^{N-1}\]

Some examples

\[1 \mapsto \frac{1}{N(N+1)}, \quad -\ln(\bar{\tau}) \mapsto \frac{1}{N^2(N+1)^2}, \quad \frac{1}{2} \operatorname{Li}_2(\tau) \mapsto \frac{(-1)^N(S_{-2}(N) + \zeta_2/2)}{N(N+1)}\]

\[K(N) \mapsto k(\tau)\]
Coordinate representation $\leftrightarrow$ momentum fraction $\leftrightarrow$ CF

$$C'(x) = \int dx' C^{(0)}(x') K(x', x) = \int_0^1 d\alpha \int_0^{\tilde{\alpha}} d\beta \left( \frac{\kappa(\tau)}{\tilde{\alpha}(1 - x) + \beta(1 + x)} - (x \leftrightarrow -x) \right).$$

(*HyperInt* package by E. Panzer)
Two loop CF

CF at the critical point

\[ C^{(2)}_*(x) = \beta_0 C_F C^{(2\beta)}_*(x) + C_F^2 C^{(2P)}_*(x) + \frac{C_F}{N_c} C^{(2A)}_*(x). \]

\[ C^{(2P)}_*(x) = \frac{1}{\omega} \left( 6H_{0000} - H_{1000} - 2H_{200} - H_{1100} - H_{120} - H_{210} + H_{1110} \right) \]

\[- \frac{1}{\bar{\omega}} H_{000} - \left( \frac{4}{\omega} - \frac{2}{\bar{\omega}} \right) H_{100} + \frac{1}{\bar{\omega}} H_{20} + \frac{2}{\omega} H_{110} \]

\[ - \left( \frac{13}{2\bar{\omega}} + \frac{19}{3\omega} \right) H_{00} + \left( \frac{3}{\bar{\omega}} + \frac{11}{3\omega} \right) H_{10} + \frac{1}{\omega} \zeta_2 \left( H_{11} - H_2 - H_{10} - 4H_{00} \right) \]

\[ + \left( \frac{1}{\bar{\omega}} \left( \frac{223}{12} + 5\zeta_2 - 2\zeta_3 \right) + \frac{1}{\omega} \left( 3\zeta_2 + 16\zeta_3 - \frac{32}{9} \right) \right) H_0 \]

\[ + \frac{1}{48\omega} \left( 701 + 128\zeta_2 + 936\zeta_3 + 72\zeta_2^2 \right) - (\omega \leftrightarrow \bar{\omega}) \]

\[ \omega = \frac{1-x}{2} \text{ and } H_n = H_n(\omega) \text{ HPL functions} \]
Two loop CF

\[
C^{(2)}(x) = C_\star^{(2)}(x) + \beta_0 C^{(1,1)}(x),
\]

\[
C^{(1,1)}(x) = -C_F \frac{1}{2\omega} \left\{ 18 - \frac{\pi^2}{4} - \left( 5 - \frac{4}{\bar{\omega}} + \frac{\pi^2}{6} \right) \ln \omega - \frac{3}{2} \frac{\omega}{\bar{\omega}} \ln^2 \omega + \frac{1}{3} \ln^3 \omega \right\} - (\omega \leftrightarrow \bar{\omega})
\]

\(C^{(1)}\) comes from one loop diagrams.

The leading double-logarithmic asymptotic of the CF at \(\omega \to 0\):

\[
C(x, a_s) \simeq \frac{1}{2\omega} \left( 1 + C_F a_s \ln^2 \omega + \frac{1}{2} (C_F a_s)^2 \ln^4 \omega + \mathcal{O}(a_s^3) \right),
\]

suggesting that the series exponentiates. (disagrees with Altinoluk, Pire, Szymanowski and Wallon, 2012)

Axial-vector CF:

agrees with Jing Gao, Tobias Huber, Yao Ji, Yu-Ming Wang, Phys.Rev.Lett. 128 (2022)
B. Melic, B. Nizic, K. Passek 2002 (\(\beta_0\) contribution)
Figure: The DVCS CF $C(x/\xi)$ at $\mu = Q = 2$ GeV in the ERBL region $x < \xi$. The LO (tree-level), NLO (one-loop) and NNLO (two-loop) CFs are shown by the black solid, blue dashed and blue dash-dotted curves on the left panel, respectively. The right panel shows the ratios NLO/LO (dashed), NNLO/LO (dash-dotted) and NNLO/NLO (solid).
Figure: The DVCS CF $C(x/\xi)$ at $\mu = Q = 2$ GeV analytically continued into the DGLAP region $x > \xi$: real part on the left and imaginary part on the right panel. The LO (tree-level), NLO (one-loop) and NNLO (two-loop) CFs are shown by the black solid, blue dashed and blue dash-dotted curves. Note, that imaginary part of the LO CF contains a local term $\sim \delta(x - \xi)$ (not shown).
Figure: Higher-order QCD corrections to the Compton form factor $\mathcal{H}(\xi)$. The ratios of the Compton form factor calculated to the NNLO and NLO accuracy with respect to the tree-level are shown for the absolute value and the phase of $\mathcal{H}(\xi) = Re^{i\Phi}$ on the left and the right panels, respectively.
Conclusions

- Using an approach based on conformal symmetry we have calculated two loop DVCS CFs in MS scheme for the flavor-nonsinglet vector and axial-vector operators.

- Numerical estimates show that the two loop corrections to the Compton Form Factors are relatively large.