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Automated Choice of the best Renormalization Scheme

Sven Heinemeyer, IFT (CSIC, Madrid)

Kloster Ettal, 04/2022

based on collaboration with *F. v.d. Pahlen* [*arXiv:221m.nnnnn*]

- The problem
- General idea for the solution
- Example for the solution
- Conclusions

1. The Problem

Fact:

The SM cannot be the ultimate theory!

1. gravity is not included
2. the hierarchy problem
3. no unification of the three forces
4. Dark Matter is not included
5. Baryon Asymmetry of the Universe cannot be explained
6. neutrino masses are not included
7. anomalous magnetic moment of the muon shows a $\sim 4\,\sigma$ discrepancy

$\Rightarrow$ Time to get ready for BSM physics

Current at future collider experiments:

LHC (Large Hadron Collider): running

pp collisions at 13 TeV

HL-LHC final high-luminosity phase: approved

HE-LHC new magnets $\Rightarrow$ 27 TeV possible?

ILC (International Linear Collider) decision 2023/24 in Japan

e^+e^- collisions at 250 GeV (final stage 1000 GeV)

CLIC (Compact LInear Collider)

e^+e^- collisions at 380 GeV (final stage 3000 GeV)

FCC-hh (Future Circular Collider)

pp collisions at 100 TeV

FCC-ee/CEPC (Future Circular Collider - CERN/China)

e^+e^- collisions at $\lesssim$ 350/250 GeV

$\Rightarrow$ Higher-order calculations needed for e^+e^- and pp colliders

Automated calculation of BSM production and decay processes

Generic problems for BSM loop calculations:

Problem # 1:

we do not know the values of the BSM parameters

⇒ "normal" in the investigation of BSM models: parameter scans
or at least predictions as a function of the relevant parameters

Problem # 2:

External (BSM) particles should be on-shell particles

⇒ OS renormalization of BSM model required

⇒ known cases that no "good" renormalization scheme exists
for the "full" parameter space

⇒ point-by-point decision on RS?!

Generic problems for BSM loop calculations:

Problem # 2 generalized:

The BSM model has n parameters

One can choose m free parameters to be renormalized with $m < n$

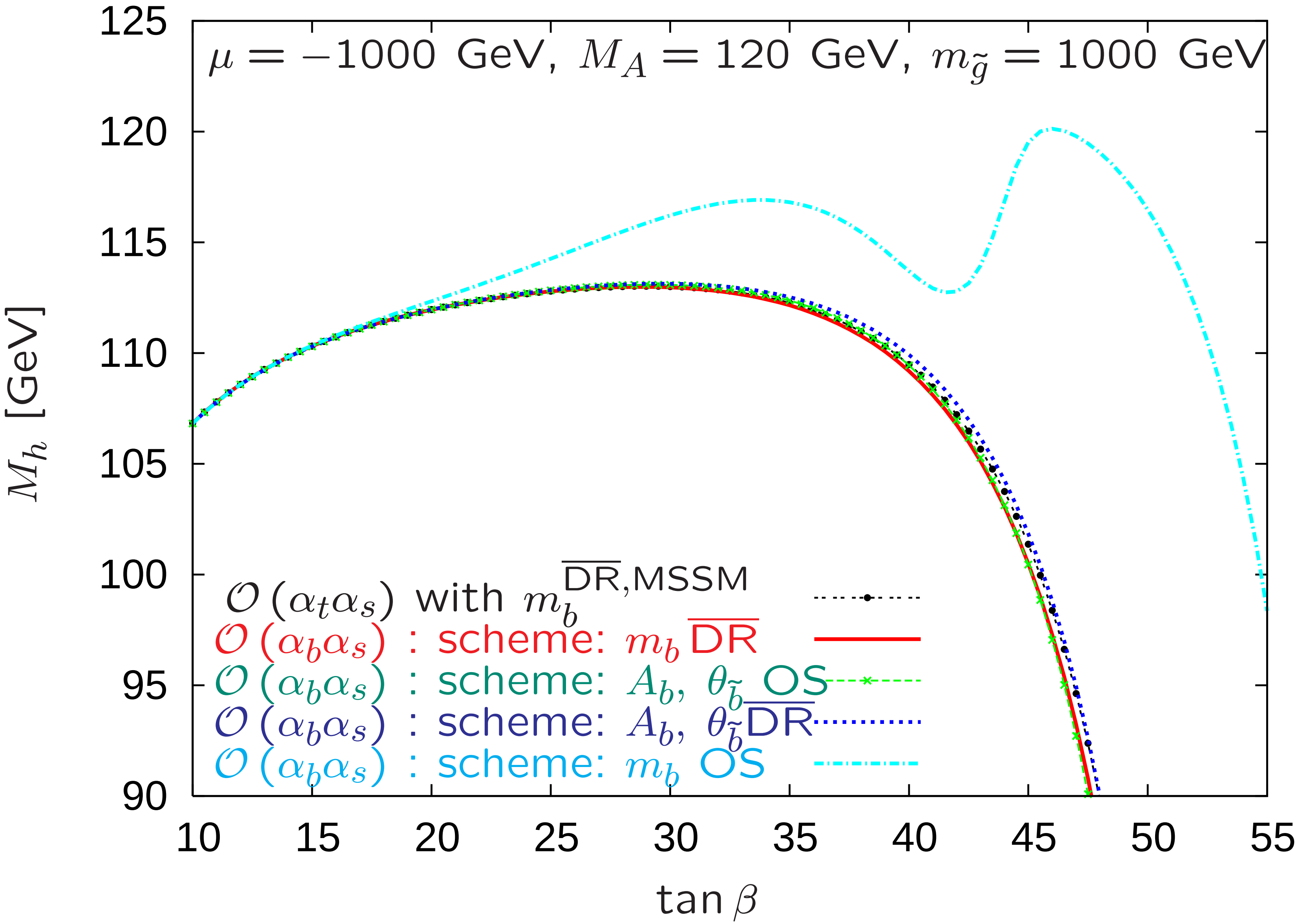
⇒ how to choose the m parameters such that
the higher-order calculation is stable?
(i.e. no unphysically large corrections appear)

⇒ point-by-point decision on RS?!

⇒ some examples of the problem

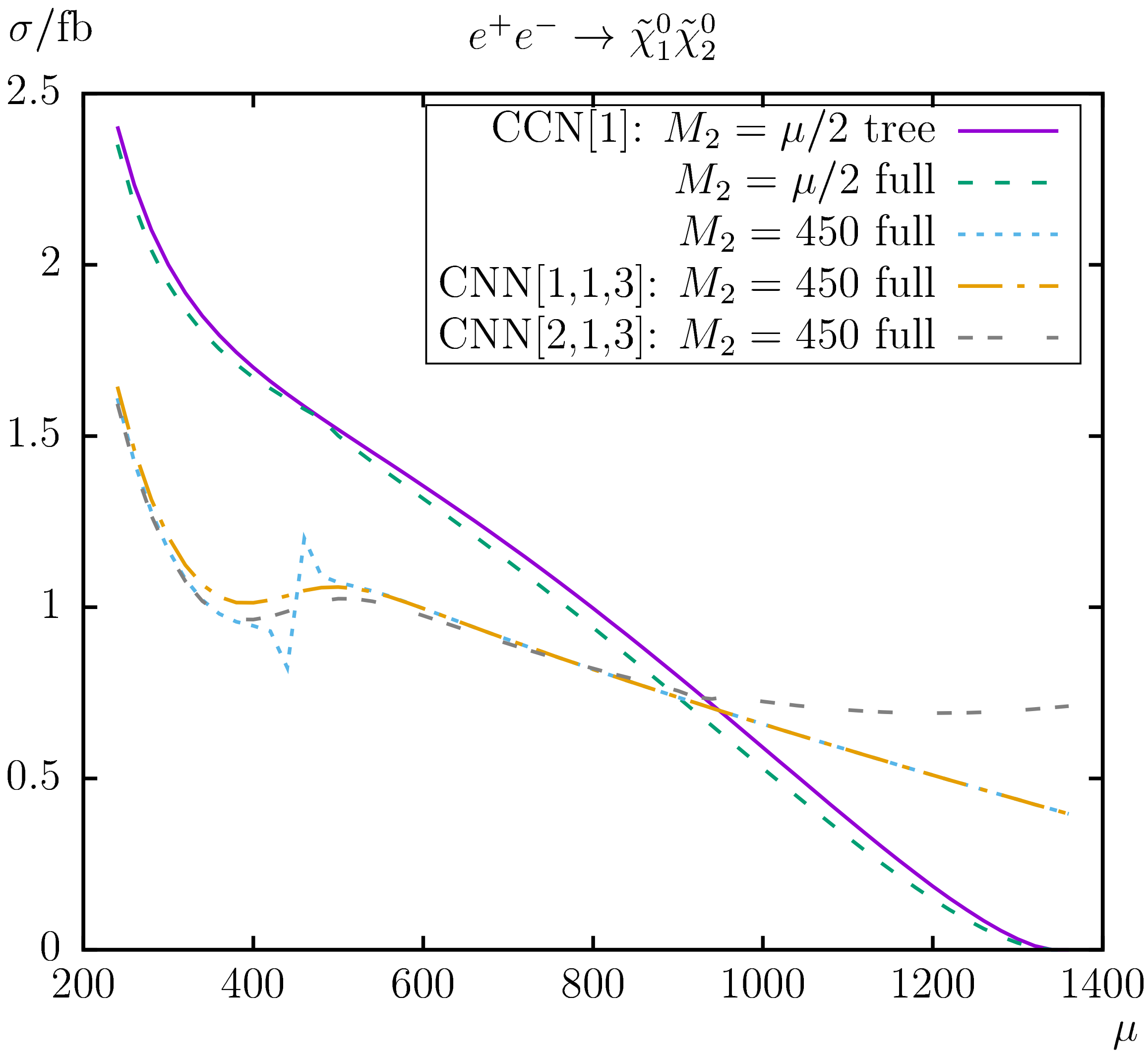
Example I: M_h calculation in the MSSM: [S.H., W. Hollik, H. Rzehak, G. Weiglein '04]

"OS" scheme: $\delta A_b = \frac{1}{m_b}\left[-(A_b - \mu^* \tan\beta)\,\delta m_b + \ldots\right]$



Example II: $\tilde{\chi}^0_1\tilde{\chi}^0_2$ production:

[S.H., C. Schappacher '17]



⇒ Schemes sometimes give good results, sometimes fail completely

2. General idea for the solution

Focus case here:

– BSM model with n particles

– BSM model with m free mass parameters with $m < n$

– we want an RS that renormalizes as many particles as possible OS
(we want calculations for BSM particle production and decay!)

⇒ how to choose the m particles that are renormalized OS?

A good RS fulfills:

– higher-order corrections remain "small"

– external particles fulfill OS requirements

– ...

Interesting to investigate in the future:

→ apply our proposal to other cases in which one RS out of many has to be chosen

Focus case here:

- BSM model with m free mass parameters
  given as $\overline{\text{DR}}$ (or $\overline{\text{MS}}$) parameters
- we want to choose m out of n particles to be renormalized OS
- we can choose out of $N = \begin{pmatrix} n \\ m \end{pmatrix}$ RS

$\Rightarrow$ we give a general recipe

$\Rightarrow$ we give concrete numerical examples

$\Rightarrow$ we are interested in the application of our recipe on other cases. . .

Proposal for general solution:

1. We start with m $\overline{\text{DR}}$ parameters, $P_i^{\overline{\text{DR}}}$ from the Lagrangian ($i = 1 \ldots m$)

2. We have N RS_l ($l = 1 \ldots N$).

3. For each RS_l, i.e. each different choice of m particles renormalized OS, we evaluate the corresponding OS parameters

$$P_{i,l}^{\text{OS}} = P_i^{\overline{\text{DR}}} - \delta P_{i,l|\text{fin}}^{\text{OS}} \tag{1}$$

   with the transformation matrix $\mathbf{A}_l^{\overline{\text{DR}}}$.

4. A scheme RS_l is bad
   if one counterterm does not depend on its own parameter
   $\Leftrightarrow$ $|\det \mathbf{A}_l^{\overline{\text{DR}}}|$ is small, or even vanishing.

5. Comparing the various $|\det \mathbf{A}_l^{\overline{\rm DR}}|$ yields RS_L: the best RS (more details later).

6. Inserting $P^{\rm OS}_{i,L}$ into the Lagrangian yields n particle masses out of which m are by definition given as their OS values.
   The remaining OS masses have to be determined calculating $n - m$ finite shifts.

7. The counterterms for the $P^{\rm OS}_{i,L}$ are already known from eq. (1) as $\delta P^{\rm OS}_{i,L}$ and can be inserted as counterterms in a loop calculation.

5. Comparing the various $|\det \mathbf{A}_l^{\overline{\text{DR}}}|$ yields RS$_L$: the best RS (more details later).

6. Inserting $P_{i,L}^{\text{OS}}$ into the Lagrangian yields n particle masses out of which m are by definition given as their OS values.
   The remaining OS masses have to be determined calculating $n - m$ finite shifts.

7. The counterterms for the $P_{i,L}^{\text{OS}}$ are already known from eq. (1) as $\delta P_{i,L}^{\text{OS}}$ and can be inserted as counterterms in a loop calculation.

⇒ so far we have OS counterterms expressed by $\overline{\text{DR}}$ parameters.
One can iterate one step further and express the OS counterterms by OS parameters.

⇒ so let's not decide on RS$_L$ yet, and re-continue at 5.

5. Inserting $P_{i,l}^{\text{os}}$ into the Lagrangian yields n particle masses out of which m are by definition given as their os$_l$ values.
The remaining os$_l$ masses have to be determined calculating $n - m$ finite shifts.

6. RS$_l$ is applied again on the OS$_l$ Lagrangian.

7. This yields now OS counterterms in terms of os$_l$ parameters,

$$\delta P_{i,l}^{\text{OS}}(P_{i,l}^{\text{os}}) \qquad (2)$$

with the transformation matrix $\mathbf{A}_l^{\text{OS}}$.

8. A scheme RS$_l$ is bad
if one counterterm does not depend on its own parameter
$\Leftrightarrow$ ($|\det \mathbf{A}_l^{\overline{\text{DR}}}|$ and/or ($|\det \mathbf{A}_l^{\text{OS}}|$) is small, or even vanishing

9. Comparing the various

$$\min\left\{|\det \mathbf{A}_l^{\overline{\mathsf{DR}}}|, |\det \mathbf{A}_l^{\mathsf{OS}}|\right\} \qquad (3)$$

yields RS_L, the best scheme.

10. The counterterms for the $P_{i,L}^{\mathsf{OS}}$ are already known from eq. (2) as $\delta P_{i,L}^{\mathsf{OS}}$ and can be inserted as counterterms in a loop calculation.

Steps 5-7 could be iterated until convergence is reached,
but we will not do this here.

3. Example for the solution

We will show an example:

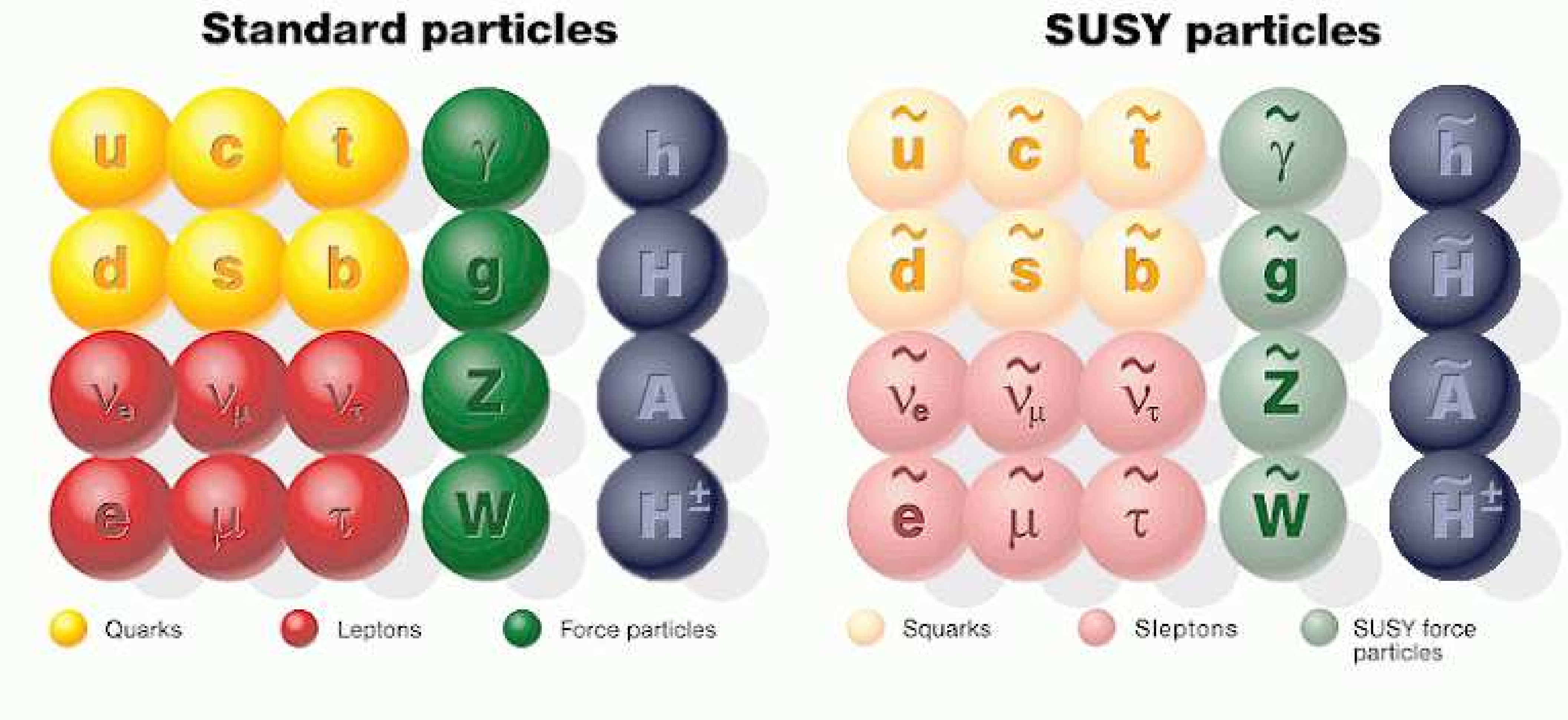
– in a concrete UV-complete model: MSSM
– in a sector with 6 masses , but only 3 mass parameters
– with many RS to choose from
– show how the best RS is chosen
– demonstrate that this choice yields “good results”
– whereas other RS fail compeletely
  while they are good (or even chosen) for other parameter sets

What remains to be shown:

– how this works out for other RS choices
– ...

The MSSM

Superpartners for Standard Model particles



⇒ SUSY partners for the Higgses and gauge bosons

Neutralinos and charginos:

Higgsinos and electroweak gauginos mix

charged:

$$\tilde{W}^+, \tilde{h}_u^+ \to \tilde{\chi}_1^+, \tilde{\chi}_2^+, \qquad \tilde{W}^-, \tilde{h}_d^- \to \tilde{\chi}_1^-, \tilde{\chi}_2^-$$

Diagonalization of the mass matrix:

$$\mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2}\sin\beta\, M_W \\ \sqrt{2}\cos\beta\, M_W & \mu \end{pmatrix} ,$$

$$\mathbf{M}_{\tilde{\chi}^-} = \mathbf{V}^* \, \mathbf{X}^\top \, \mathbf{U}^\dagger = \begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix}$$

$\Rightarrow$ charginos: mass eigenstates

mass matrix given in terms of M_2, μ, $\tan\beta$

neutral:

$$\underbrace{\tilde{\gamma}, \tilde{Z},}_{\tilde{W}^0, \tilde{B}^0} \tilde{h}_u^0, \tilde{h}_d^0 \to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$$

Diagonalization of mass matrix:

$$\mathbf{Y} = \begin{pmatrix} M_1 & 0 & -M_Z\, s_W \cos\beta & M_Z\, s_W \sin\beta \\ 0 & M_2 & M_Z\, c_W \cos\beta & -M_Z\, c_W \sin\beta \\ -M_Z\, s_W \cos\beta & M_Z\, c_W \cos\beta & 0 & -\mu \\ M_Z\, s_W \sin\beta & -M_Z\, c_W \sin\beta & -\mu & 0 \end{pmatrix},$$

$$\mathbf{M}_{\tilde{\chi}^0} = \mathbf{N}^* \, \mathbf{Y} \, \mathbf{N}^\dagger = \mathbf{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0})$$

$\Rightarrow$ neutralinos: mass eigenstates

mass matrix given in terms of M_1, M_2, μ, $\tan\beta$

$\Rightarrow$ only one new parameter

$\Rightarrow$ 6 masses are given in terms of 3 free mass parameters

Some comments on the chargino/neutralino renormalization:

4+2 masses, but only 3 free parameters: M_1, M_2, μ

$\Rightarrow$ OS renormalization for 3 masses:

$$\text{CCN1:} \quad \left(\left[\widetilde{\text{Re}}\hat{\Sigma}_{\tilde{\chi}^-}(p) \right]_{ii} \tilde{\chi}_i^-(p) \right) \bigg|_{p^2 = m^2_{\tilde{\chi}_i^\pm}} = 0 \qquad (i = 1, 2) \; ,$$

$$\left(\left[\widetilde{\text{Re}}\hat{\Sigma}_{\tilde{\chi}^0}(p) \right]_{11} \tilde{\chi}_1^0(p) \right) \bigg|_{p^2 = m^2_{\tilde{\chi}_1^0}} = 0$$

$\Rightarrow$ Scheme can easily be extended to other variants, e.g.

CCNi ($i = 1, 2, 3, 4$) or CNNijk ($i = 1, 2; j, k = 1, 2, 3, 4$)

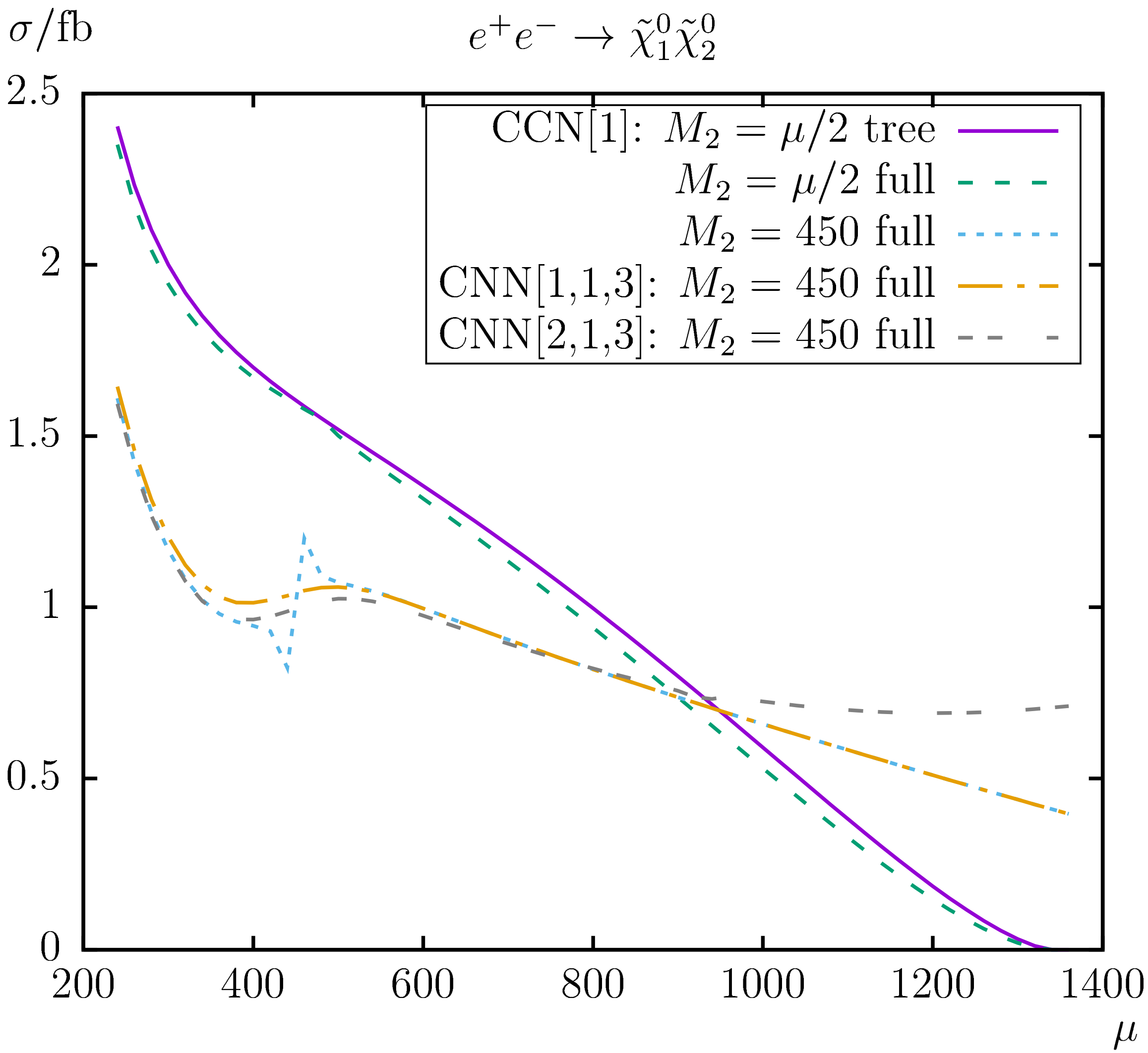
$\Rightarrow$ Scheme requires a shift of three (neutralino) masses to their OS value:

$$\Delta m_{\tilde{\chi}_i^0} = -\frac{1}{2}\text{Re}\Big\{ m_{\tilde{\chi}_i^0} \left(\hat{\Sigma}^L_{\tilde{\chi}_i^0}(m^2_{\tilde{\chi}_i^0}) + \hat{\Sigma}^R_{\tilde{\chi}_i^0}(m^2_{\tilde{\chi}_i^0}) \right) + \hat{\Sigma}^{SL}_{\tilde{\chi}_i^0}(m^2_{\tilde{\chi}_i^0}) + \hat{\Sigma}^{SR}_{\tilde{\chi}_i^0}(m^2_{\tilde{\chi}_i^0}) \Big\}$$

$$m^{\text{OS}}_{\tilde{\chi}_i^0} = m_{\tilde{\chi}_i^0} + \Delta m_{\tilde{\chi}_i^0}$$

Example II: $\tilde{\chi}^0_1\tilde{\chi}^0_2$ production:

[S.H., C. Schappacher '17]



$\Rightarrow$ CCN1 breaks down for $\mu = M_2 = 450$ GeV

How to choose a good RS?

– we can renormalize 3 out of 6 charginos/neutralinos OS
(at best: those ones that appear as external particles . . .)

– each mass should depedent strongly (sufficiently)
on a different mass parameter

– simple example: $M_1 < M_2 < \mu$
$\Rightarrow$ renormalize $\tilde{\chi}_1^\pm$ with $m_{\tilde{\chi}_1^\pm} \sim M_2$
$\tilde{\chi}_2^\pm$ with $m_{\tilde{\chi}_2^\pm} \sim \mu$
$\tilde{\chi}_1^0$ with $m_{\tilde{\chi}_1^0} \sim M_1$
$\Rightarrow$ CCN1

– how to choose in more complicated mass hierarchies?
$\Rightarrow$ application of our $|\det \mathbf{A}|$ based solution

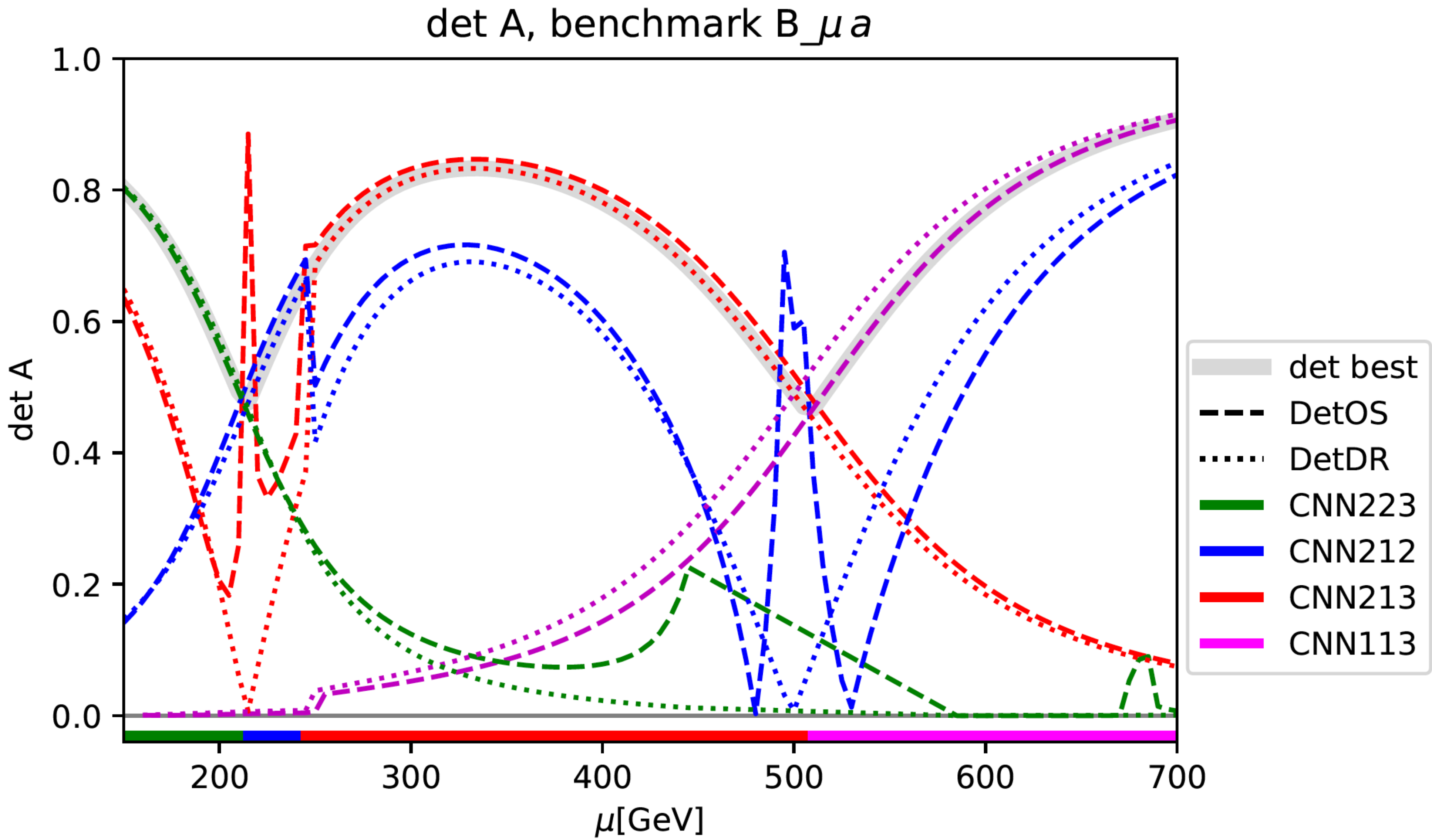
– application in an automated way!

Some numerical examples:

Benchmark	M_1[GeV]	M_2[GeV]	μ[GeV]	$\tan\beta$
B_$M_1\,a$	100-700	200	500	10
B_$M_1\,b$	100-700	500	200	10
B_$M_2\,a$	200	150-700	500	10
B_$\mu\,a$	200	500	150-700	10
B_$M_2\,b$	-200	150-700	500	10
B_$\mu\,b$	-200	500	150-700	10
B_$M_2\,c$	500	150-700	300	10
B_$\mu\,c$	500	300	150-700	10

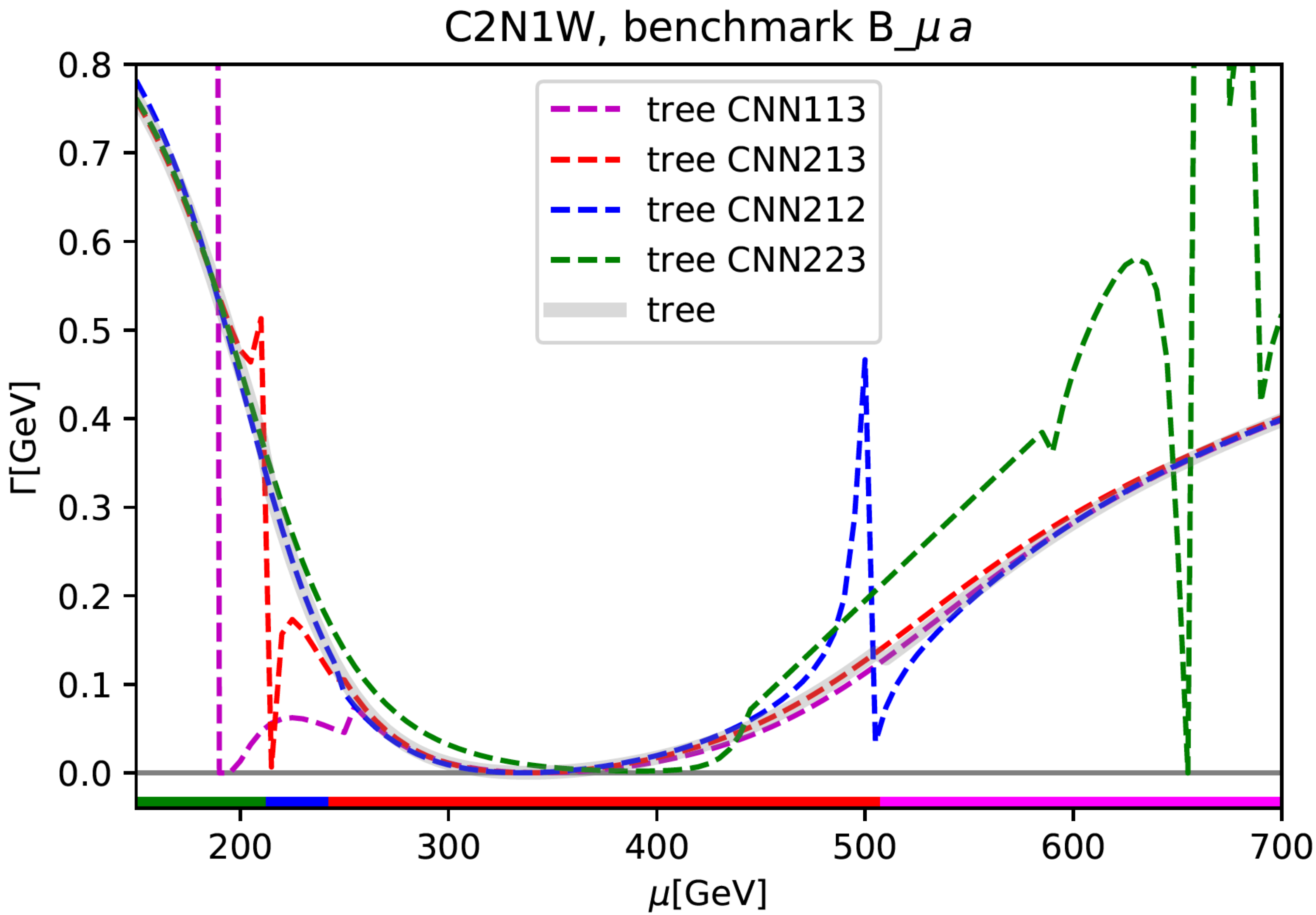
– numerical results as a function of $\overline{\text{DR}}$ input parameter
– tree level predictions for decay widths
  $\Rightarrow$ contain "jumps" already, since OS masses are used
– loop corrections
– full = loop + tree
– size of the various (normalized) transformation matrices $|\det \mathbf{A}_l|$

Example I: $\Gamma(\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm)$ with $\mu^{\overline{\text{DR}}}$ valied: $|\det \mathbf{A}_l|$ [*PRELIMINARY*]



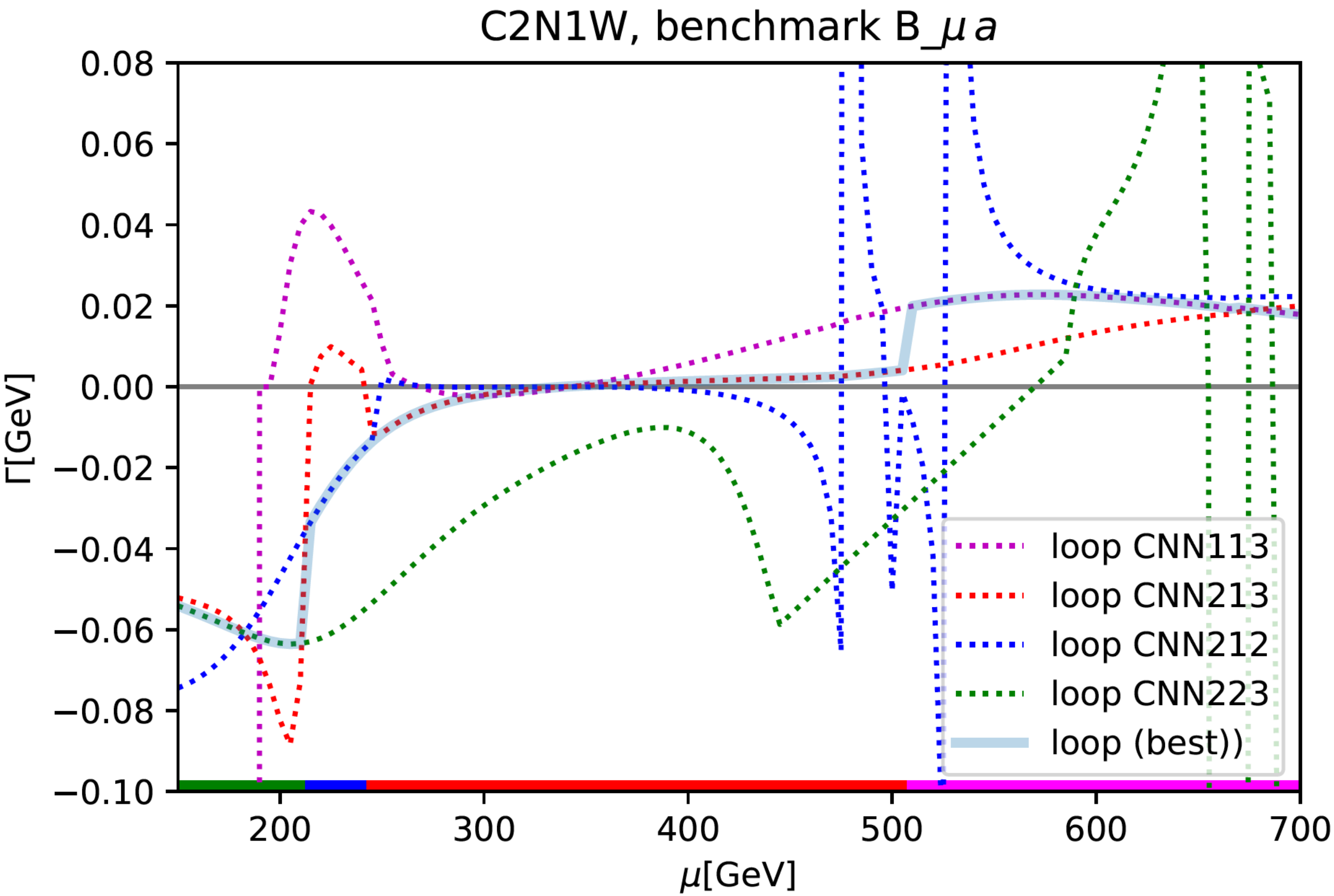
Example I: $\Gamma(\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm)$ with $\mu^{\overline{\rm DR}}$ valied: tree

[PRELIMINARY]



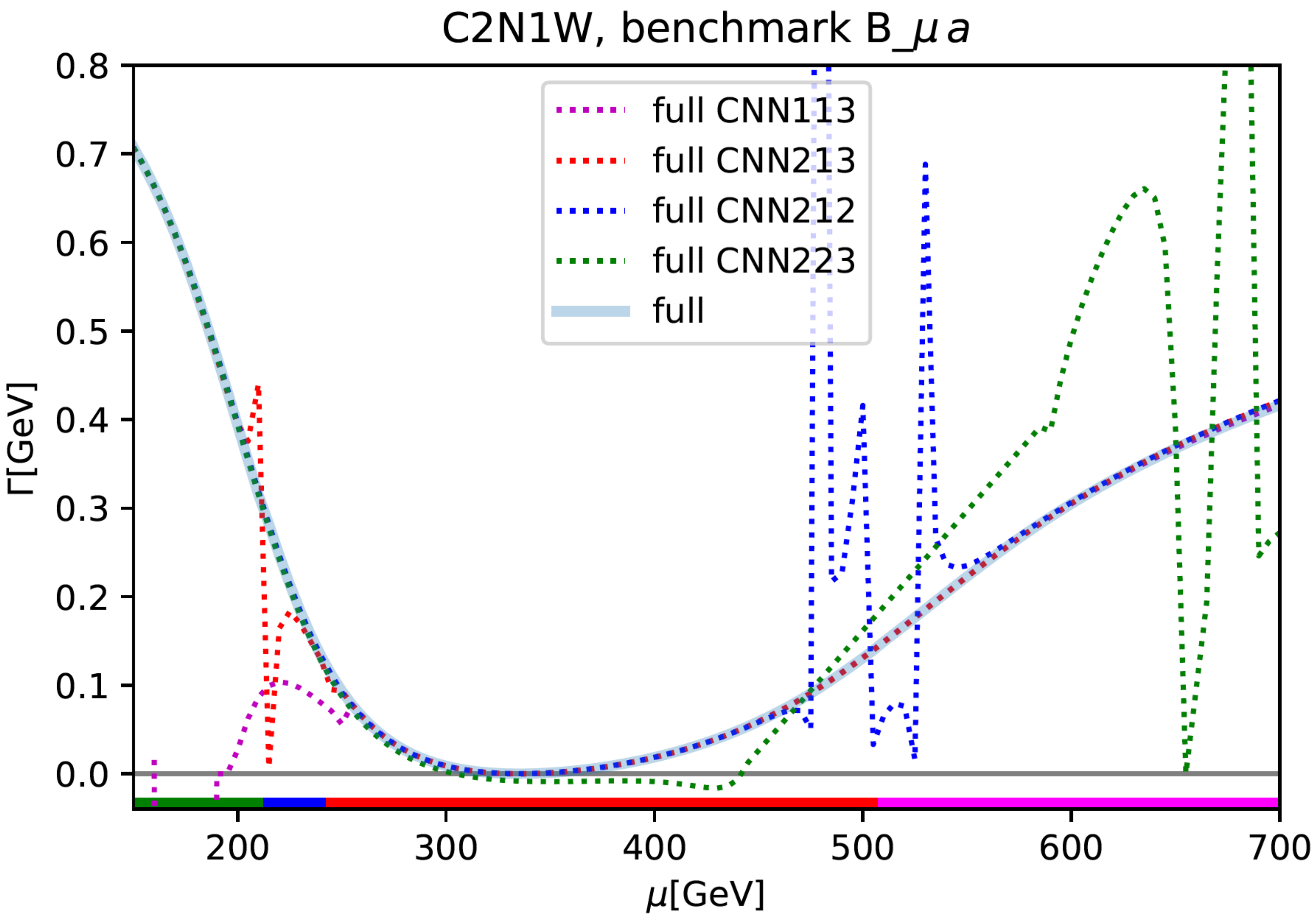
Example I: $\Gamma(\tilde{\chi}_2^{\pm} \to \tilde{\chi}_1^0 W^{\pm})$ with $\mu^{\overline{\rm DR}}$ valied: loop

[PRELIMINARY]



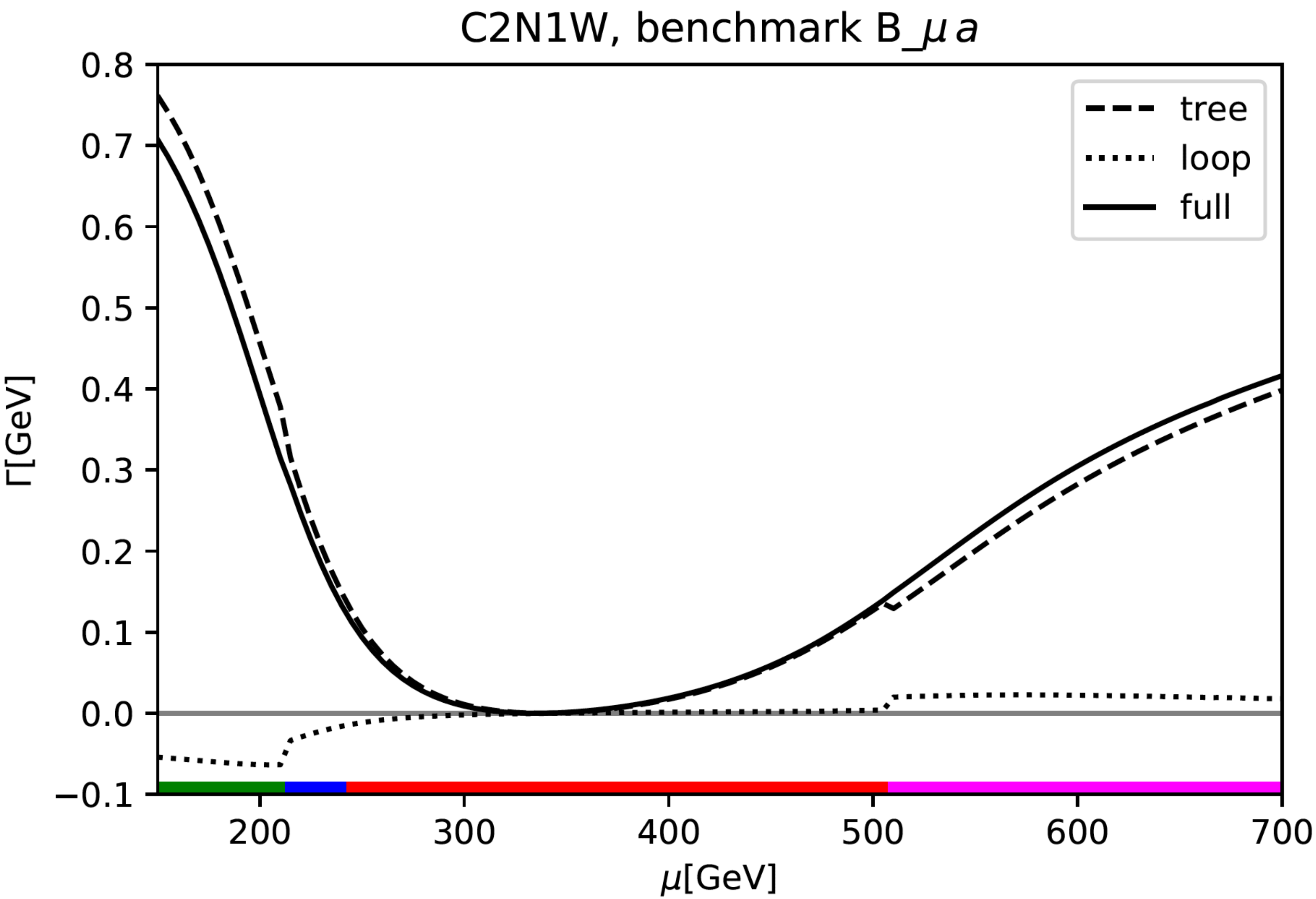
Example I: $\Gamma(\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm)$ with $\mu^{\overline{\rm DR}}$ valied: full

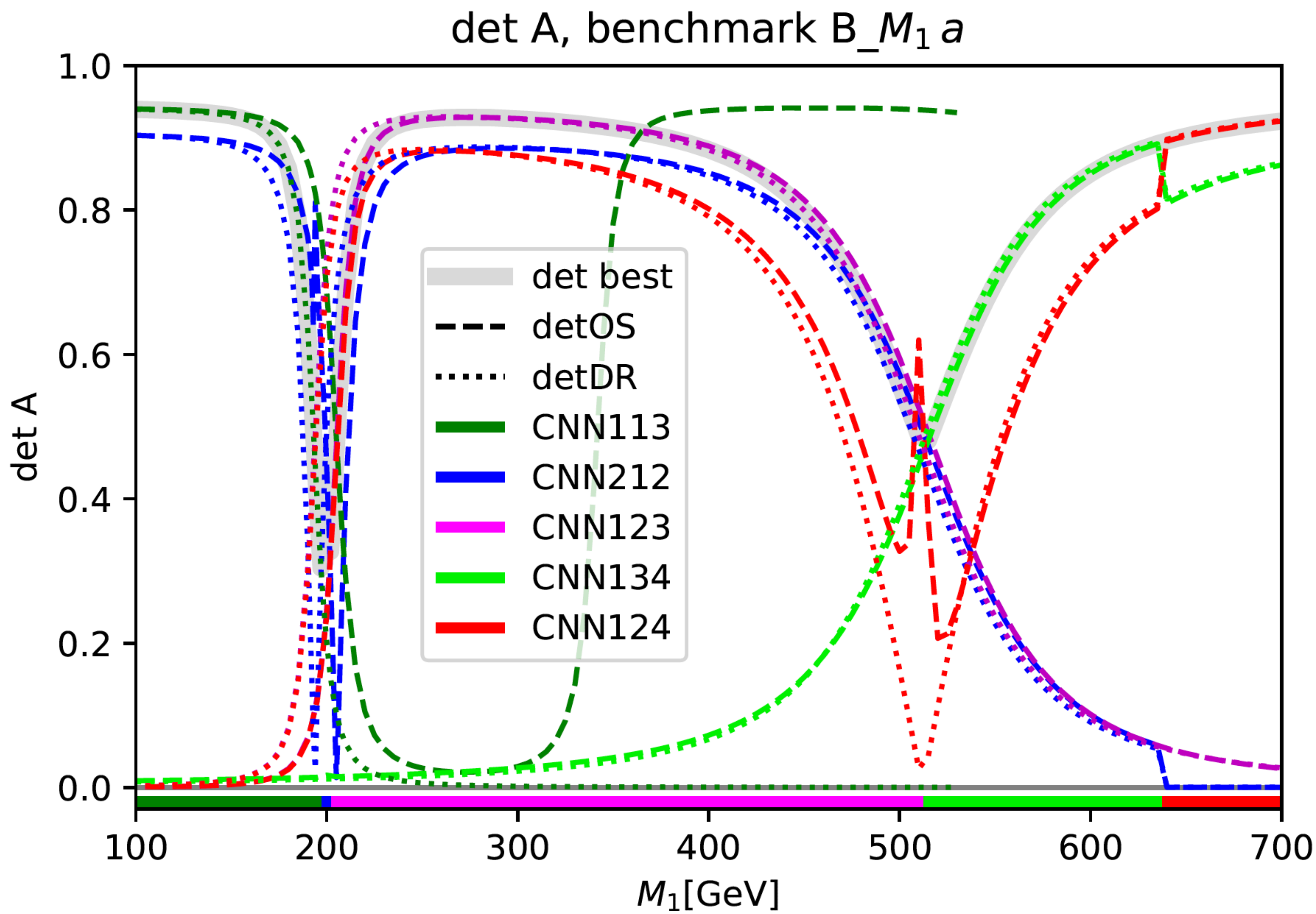
[*PRELIMINARY*]



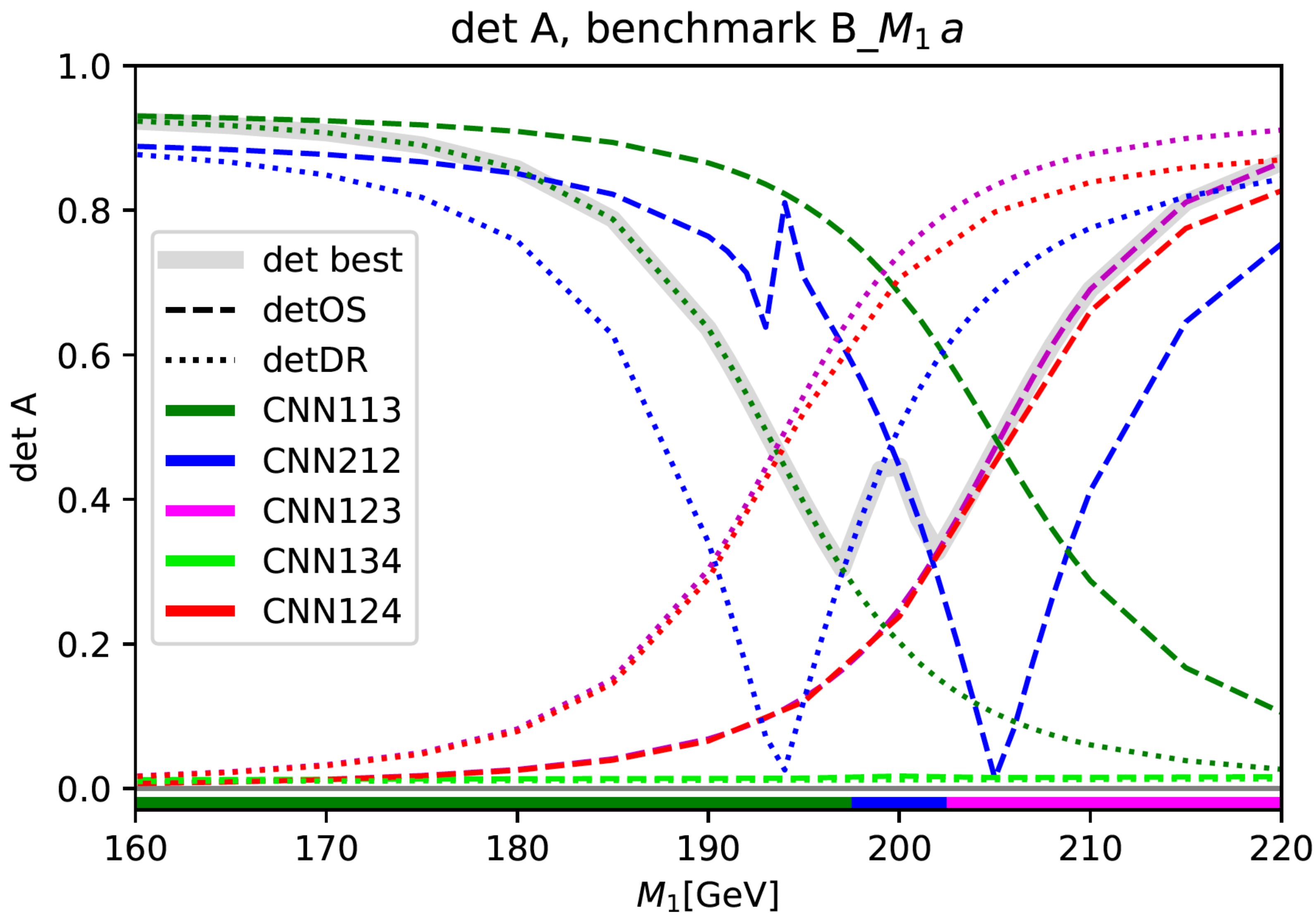
Example I: $\Gamma(\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm)$ with $\mu^{\overline{\mathrm{DR}}}$ valied: best

[PRELIMINARY]



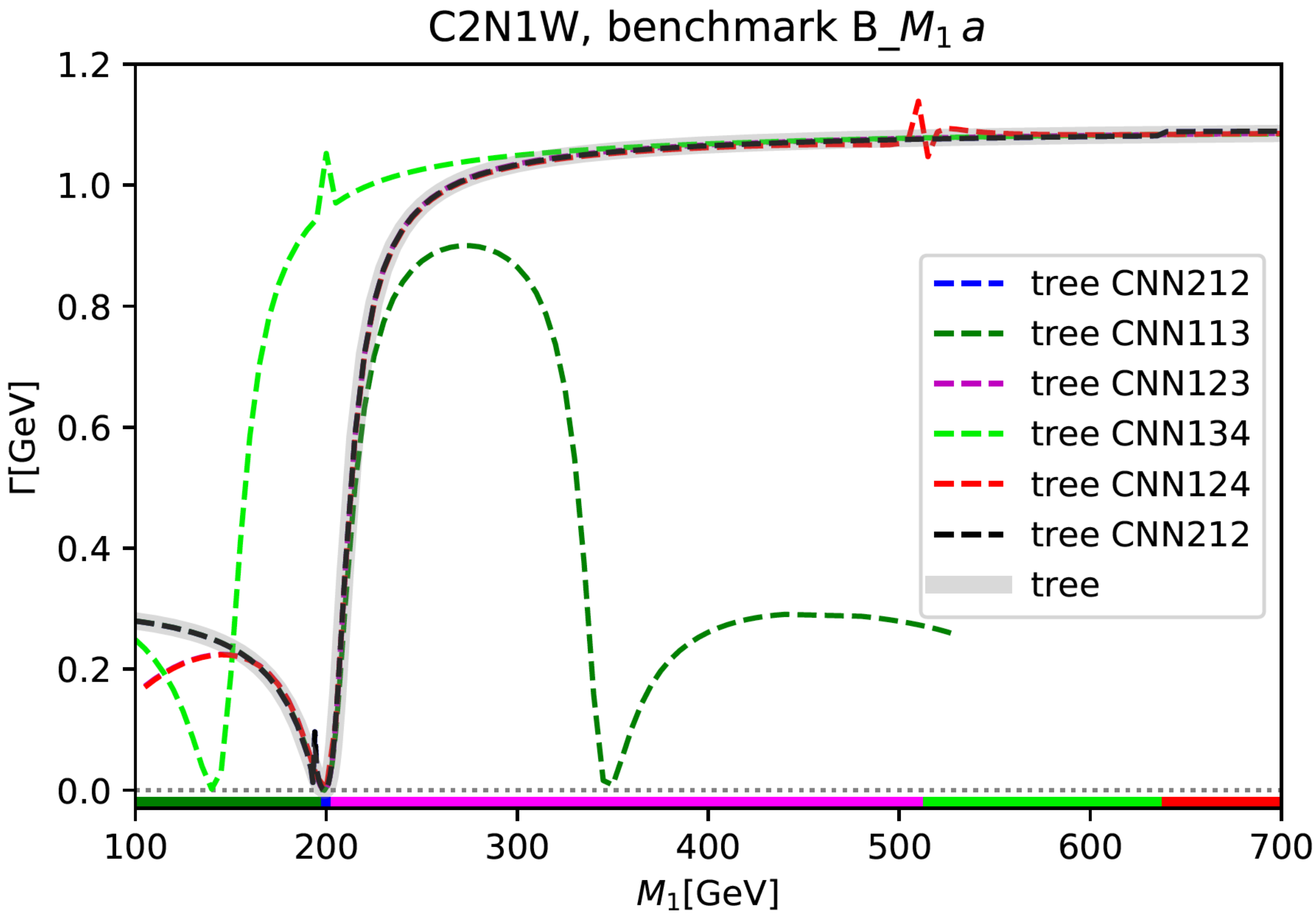

det A, benchmark B_M_1 a
1.0
0.8
0.6
0.4
0.2
0.0
det A
det best
detOS
detDR
CNN113
CNN212
CNN123
CNN134
CNN124
100
200
300
400
500
600
700
M_1[GeV]

Example II: $\Gamma(\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm)$ with $M_1^{\overline{\rm DR}}$ valied: $|\det \mathbf{A}_l|$ (zoom) [*PRELIMINARY*]



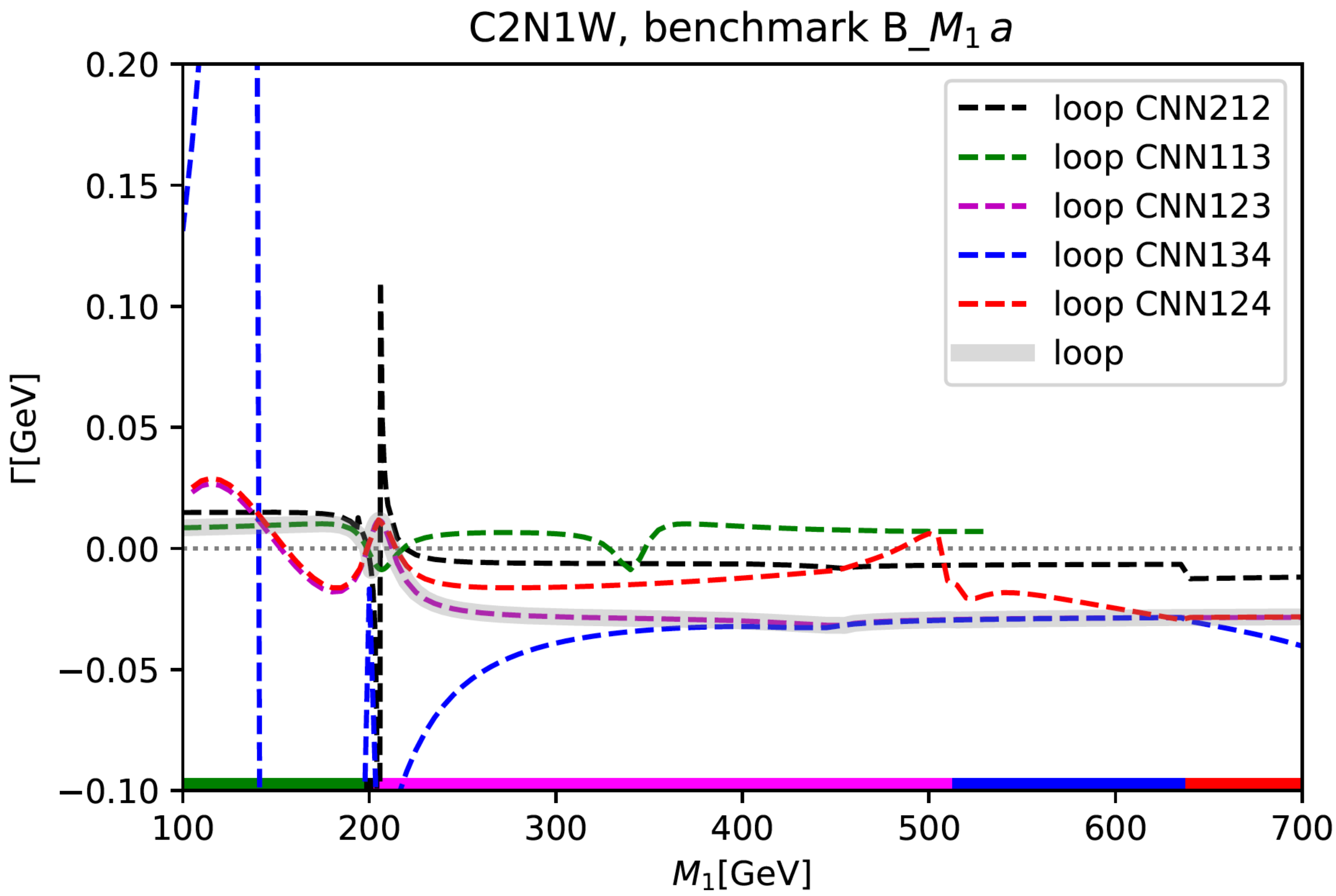
Example II: $\Gamma(\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm)$ with $M_1^{\overline{\rm DR}}$ valied: tree

[*PRELIMINARY*]



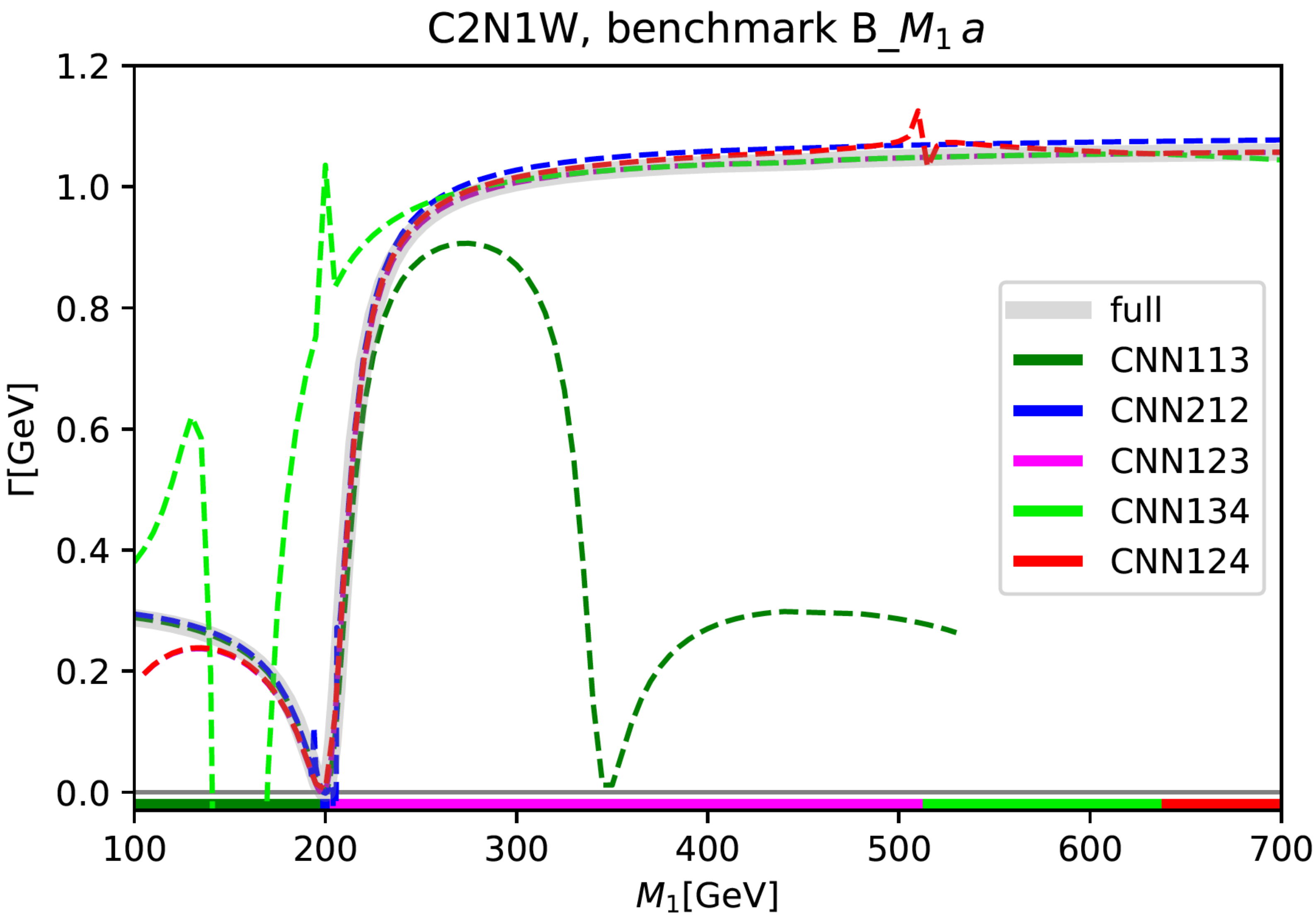
Example II: $\Gamma(\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm)$ with $M_1^{\overline{\rm DR}}$ valied: loop

[PRELIMINARY]



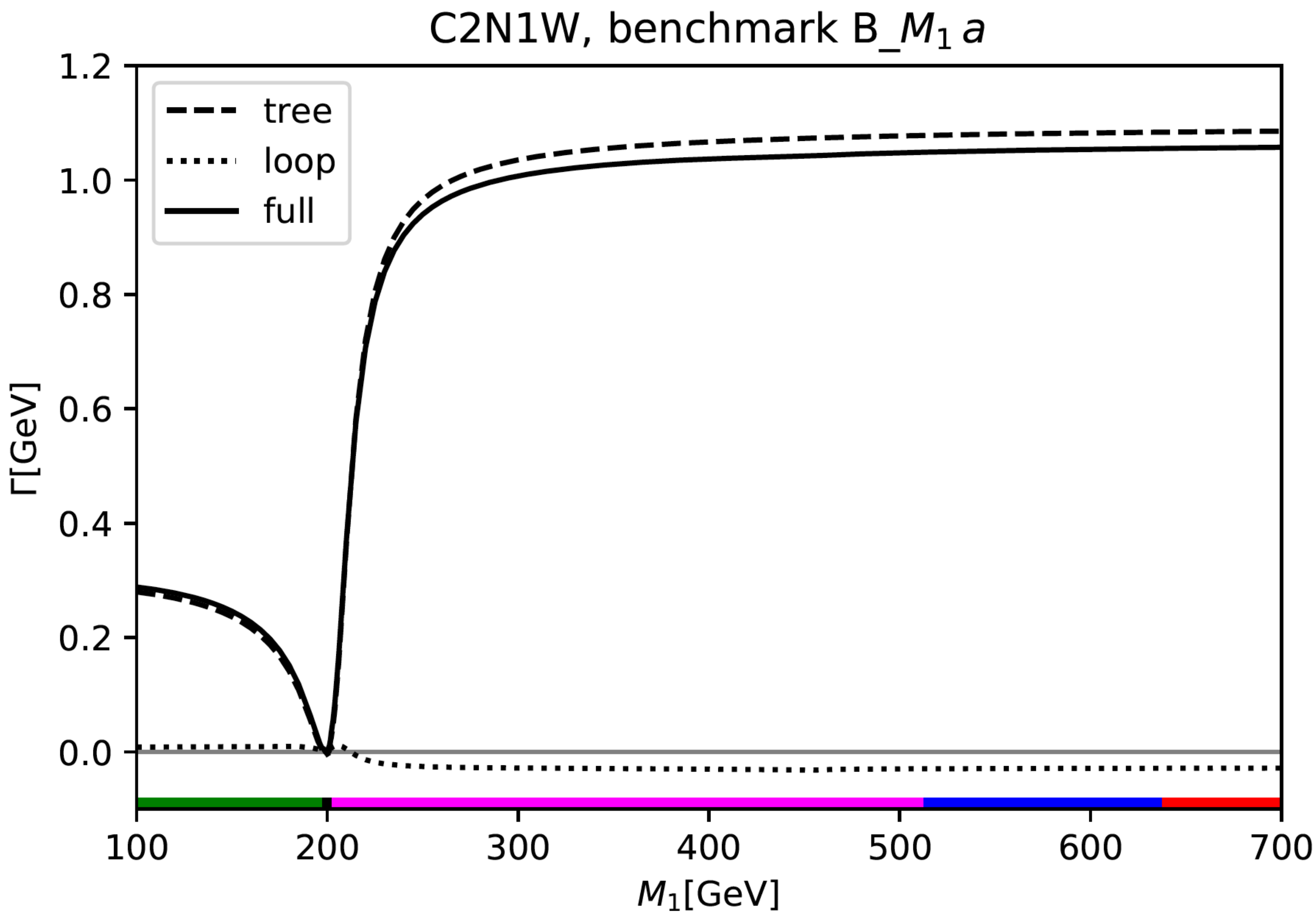
Example II: $\Gamma(\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm)$ with $M_1^{\overline{\text{DR}}}$ valied: full

[*PRELIMINARY*]



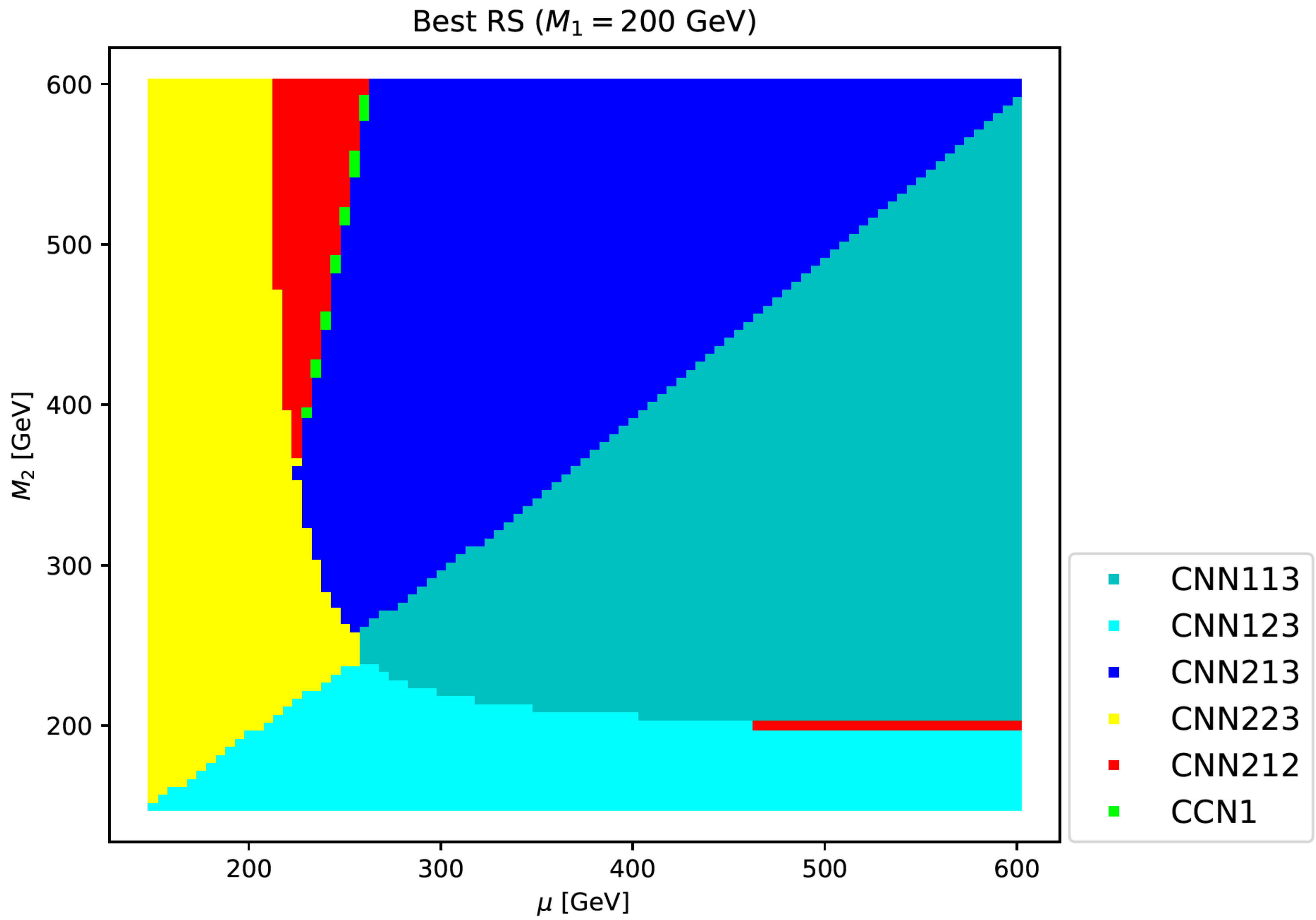
Example II: $\Gamma(\tilde{\chi}_2^\pm \to \tilde{\chi}_1^0 W^\pm)$ with $M_1^{\overline{\rm DR}}$ valied: best

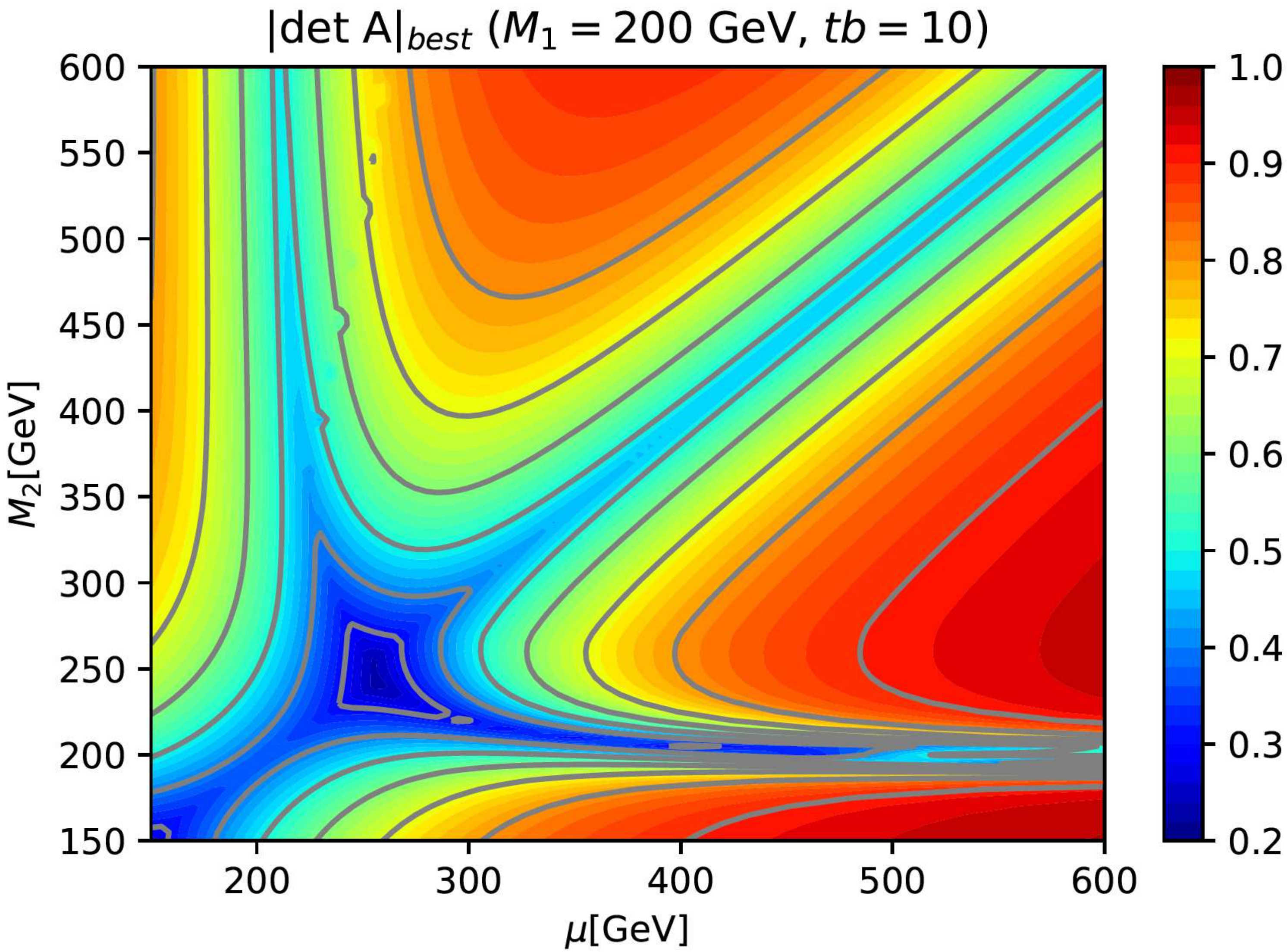
[PRELIMINARY]



Example III: μ–M_2 plane: selected RS

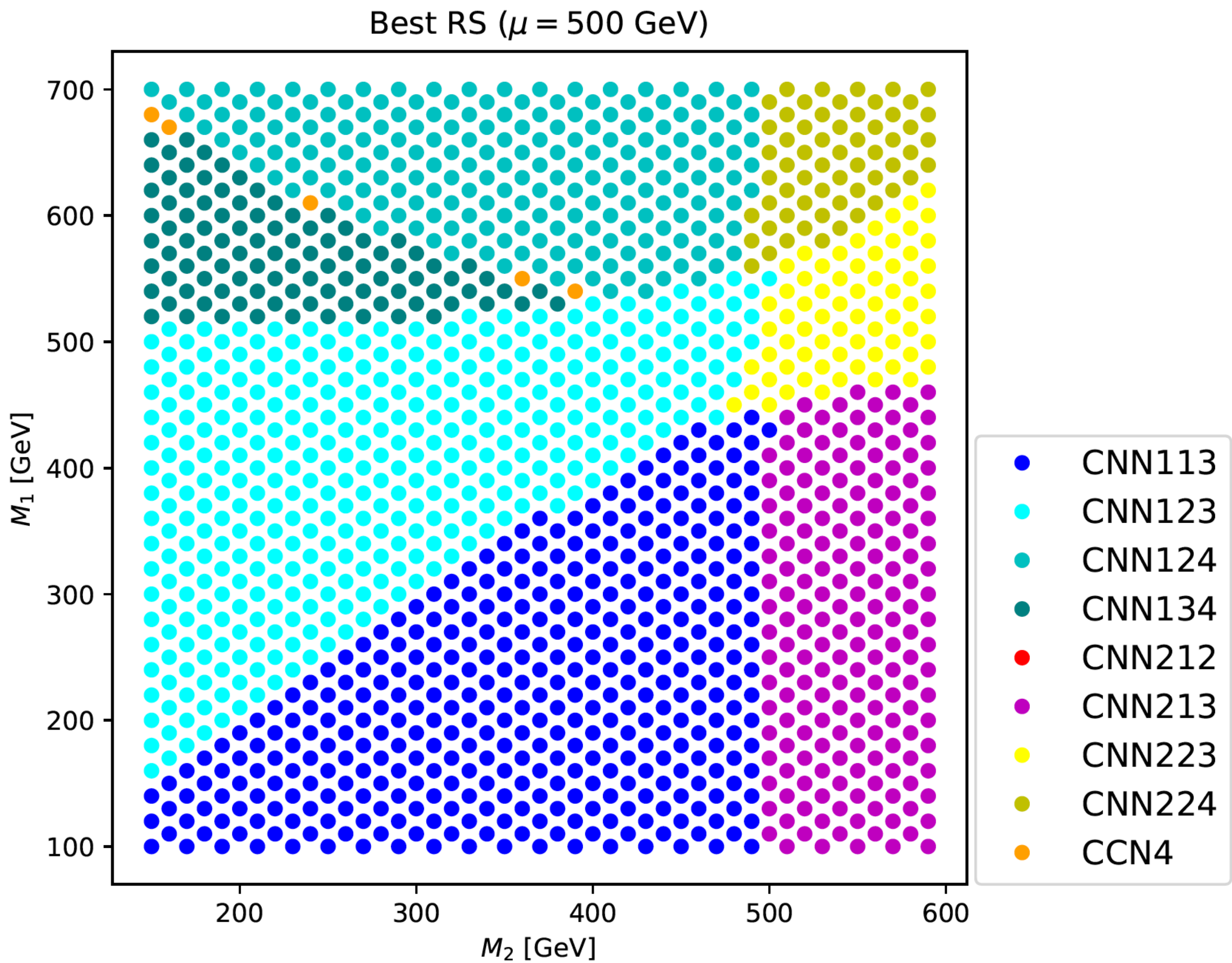
[PRELIMINARY]

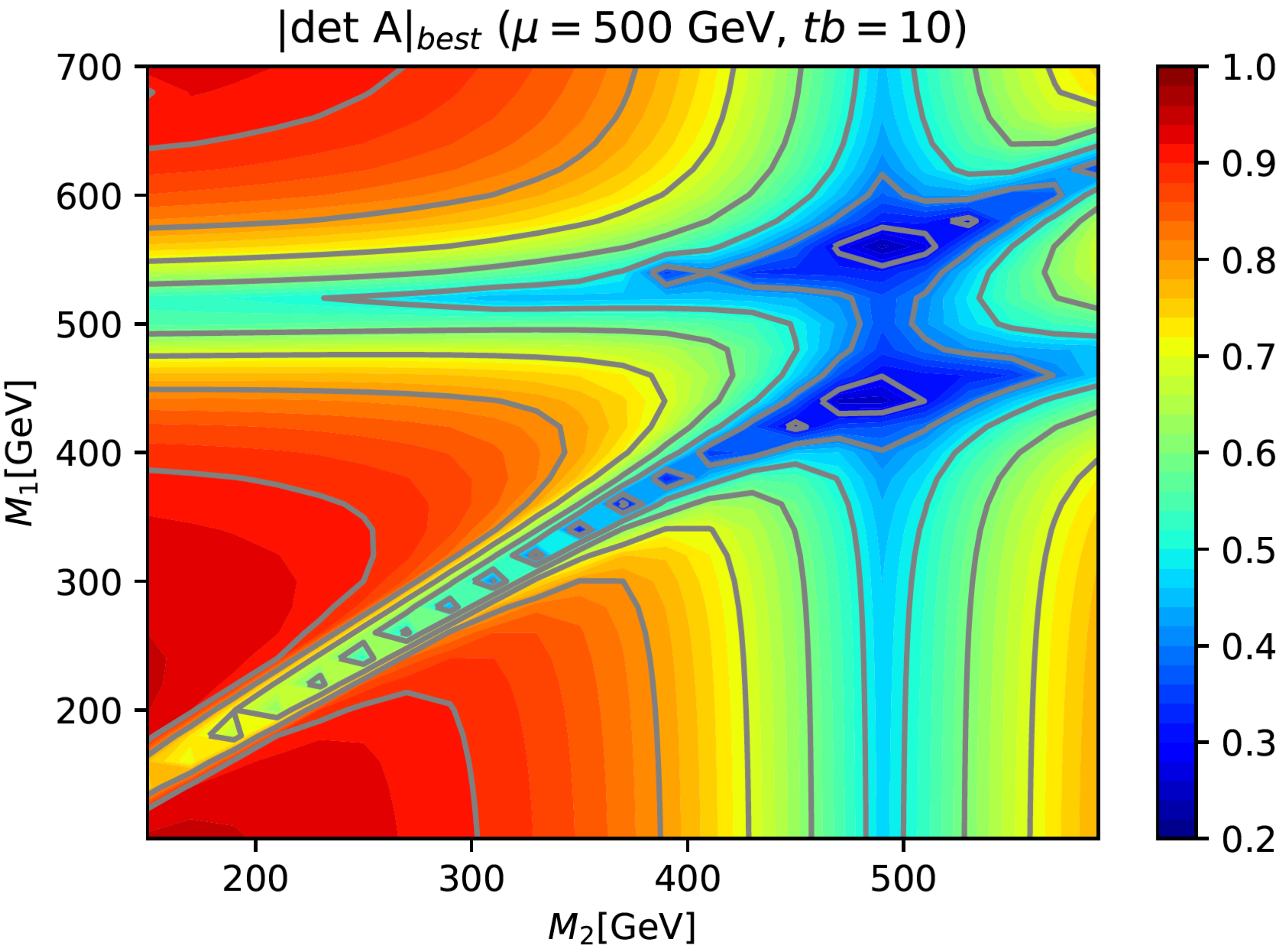



|det A|$_{best}$ (M_1 = 200 GeV, tb = 10)
600
550
500
450
400
350
300
250
200
150
M_2[GeV]
200
300
400
500
600
μ[GeV]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2

Example IV: M_2–M_1 plane: selected RS

[*PRELIMINARY*]




|det A|$_{best}$ (μ = 500 GeV, tb = 10)
700
600
500
400
300
200
M_1[GeV]
200
300
400
500
M_2[GeV]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2

4. Conclusions

- BSM predictions for pp or e^+e- colliders require loop corrections
- Often many different RS are possible
  $\Rightarrow$ how to choose a good (the best?) one?
  $\Rightarrow$ choice depends strongly on the parameter point
- Focus case: BSM model with m free mass parameters ($\overline{\text{DR}}$ or $\overline{\text{MS}}$)
  $\Rightarrow$ we want to choose m out of n particles to be renormalized OS
- General recipe: use the transformation matrix $|\det \mathbf{A}_l|$ that connects (physical) counterterms with the underlying parameter
  RS$_l$ is bad $\Leftrightarrow$ $|\det \mathbf{A}_l|$ is small
  i.e. one counterterm does not depend on its own parameter
- Concrete example: chargino/neutralino sector of the MSSM
  6 physical masses, but only 3 mass parameters: CCN$_i$, CNN$_{ijk}$
- RS selection varies strongly over the paremeter space
  Selected RS gives stable results over the whole parameter range
  while non-selected RS's give unphysical results $\Rightarrow$ it works :-)
- Interested in the application of our recipe on other cases. . .


Further Questions?