# External-leg corrections as an origin of large logarithms

**Based on arXiv:2112.11419 (JHEP 02 (2022) 159),**
**in collaboration with Henning Bahl and Georg Weiglein**

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# Outline

- Introduction (motivation, large logs, external legs)
- Large logs from external-leg corrections I: a toy model
- Large logs from external-leg corrections II: MSSM
- Large logs from external-leg corrections III: N2HDM
- Conclusions

# Introduction

# Motivation

- Numerous reasons to expect BSM physics (e.g. DM, baryon asymmetry, hierarchy problem)

- BSM theories commonly involve **additional scalars**, e.g.
  - ***Extended Higgs sectors*** → bottom-up extensions of the SM (singlet extensions, 2HDM, N2HDM, ...), supersymmetric models (MSSM, NMSSM, ...)
  - **Scalar partners** → SUSY, ...

- To correctly determine the viable parameter space of BSM models, and assess discovery sensitivities of BSM scalars, **precise theory predictions for the production and decay processes of the new scalars are needed**

- Lack of experimental results tends to **favour heavier BSM states**
  *(light states with small couplings to SM also possible, but we won't consider this in this talk)*

# Large logarithms

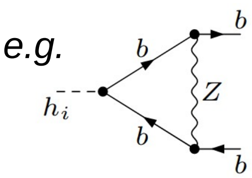
- Calculations in QFT notoriously known to be plagued by (potential) **large logs**, when **widely separated mass scales** are present

- Among the possible types of large logarithms:
  - Logs involving ratio of high and low mass scales, in calculation of quantity/observable at low scale, e.g. $\log(M_{SUSY}/m_t)$ in SUSY Higgs mass calculations
    → Solution: *Resummation of logs via Effective Field Theory*

  - Sudakov logarithms in QCD
    → Solution: *exponentiation, or Soft-Collinear Effective Theory (SCET)*

  - Electroweak Sudakov logarithms (related to exchange of Z, W, h)
    → Solution: *SCET*

*e.g.*




***In this talk: we point out a new type of large, Sudakov-like, logarithms appearing in external-leg corrections involving heavy scalars***

# External-leg corrections

- **LSZ formalism** → to obtain a reliable prediction for an observable, need to ensure correct on-shell properties of external particles → LSZ factor

- External scalar Φ, without mixing: include for each external leg a factor

$$\sqrt{Z_\phi} = \left(1 + \hat{\Sigma}'_{\phi\phi}(p^2 = \mathcal{M}_\phi^2)\right)^{-1/2}$$

$\hat{\Sigma}'_{\phi\phi}$ Derivative of renormalised self-energy w.r.t $p^2$

$\mathcal{M}_\phi$ Complex pole mass

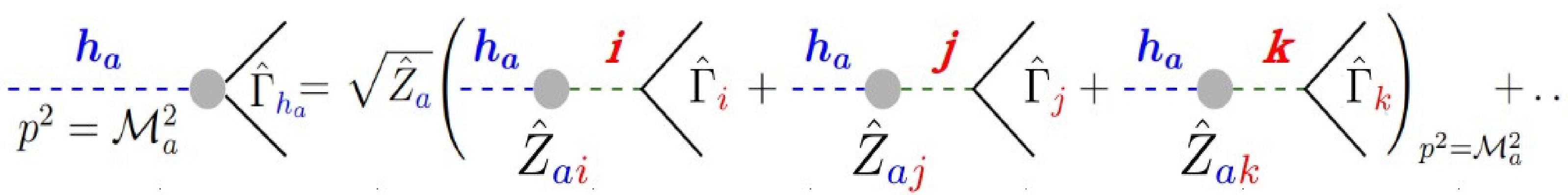
Up to 2L order:

$$\sqrt{Z_\phi} = 1 - \mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\phi\phi}(m^2) - \mathrm{Re}\hat{\Sigma}^{(2)\prime}_{\phi\phi}(m^2) + \left(\mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\phi\phi}(m^2)\right)^2$$
$$-\frac{1}{2}\left(\mathrm{Im}\hat{\Sigma}^{(1)\prime}_{\phi\phi}(m^2)\right)^2 + \mathrm{Im}\hat{\Sigma}^{(1)}_{\phi\phi}(m^2)\cdot\mathrm{Im}\hat{\Sigma}^{(1)\prime\prime}_{\phi\phi}(m^2) + \mathcal{O}(3\mathrm{L})$$

- Case with mixing → we employ the **Z-matrix formalism** [Frank et al. '06, Fuchs and Weiglein '16, '17]

$$\hat{\Gamma}_{h_a^{\mathrm{physical}}} = \sum_j \hat{\mathbf{Z}}_{aj}\hat{\Gamma}_j \qquad \text{with} \qquad \hat{\mathbf{Z}}_{aj} = \sqrt{\hat{Z}_i^a}\,\hat{Z}_{ij}^a$$

*e.g. with 3 scalars i,j,k:*




(more details also in [Bahl, JB, Weiglein '21])

# Large logarithms from external legs I: toy model example

# A simple toy model

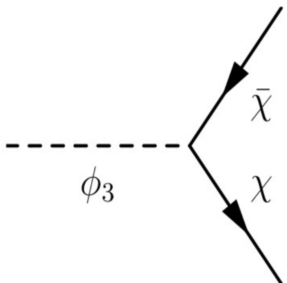
- Three scalars $\Phi_1$, $\Phi_2$, $\Phi_3$, and a Dirac fermion $\chi$
- $Z_2$-symmetry (unbroken): $\Phi_1 \to -\Phi_1$, $\Phi_2 \to -\Phi_2$, $\Phi_3 \to \Phi_3$, $\chi \to \chi$
- Consider a hierarchy where $m_1 \ll m_2, m_3$

$$
\begin{aligned}
\mathcal{L}_{\text{int.}} =& -\frac{1}{2}A_{113}\phi_1^2\phi_3 - \boxed{A_{123}\phi_1\phi_2\phi_3} - \frac{1}{2}A_{223}\phi_2^2\phi_3 - \frac{1}{6}A_{333}\phi_3^3 \\
& -\frac{1}{24}\lambda_{1111}\phi_1^4 - \frac{1}{6}\lambda_{1112}\phi_1^3\phi_2 - \frac{1}{4}\lambda_{1122}\phi_1^2\phi_2^2 - \frac{1}{6}\lambda_{1222}\phi_1\phi_2^3 - \frac{1}{24}\lambda_{2222}\phi_2^4 \\
& -\frac{1}{4}\lambda_{1133}\phi_1^2\phi_3^2 - \frac{1}{2}\lambda_{1233}\phi_1\phi_2\phi_3^2 - \frac{1}{4}\lambda_{2233}\phi_2^2\phi_3^2 - \frac{1}{24}\lambda_{3333}\phi_3^4 \\
& + \boxed{y_3\phi_3\bar{\chi}\chi}
\end{aligned}
$$

- Only $\Phi_3$ can couple to the fermions
- Main focus: trilinear couplings, in particular $\mathbf{A_{123}}$ (light-heavy-heavy coupling)

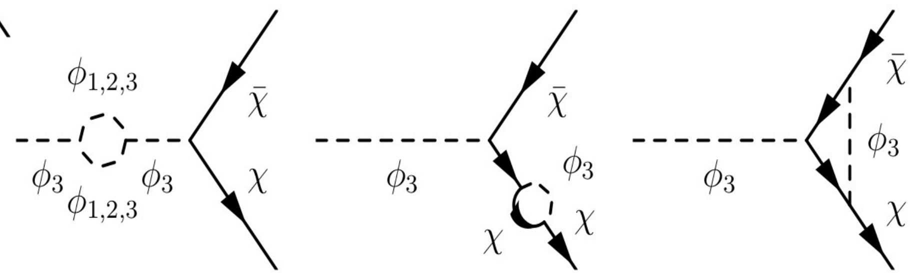
# The $\phi_3 \to \chi\bar{\chi}$ decay process

- Consider the decay of $\Phi_3$ into 2 fermions χ (prototype of scalar → 2 fermions, or fermion → scalar-fermion decays)
- Tree level:




$$\Gamma^{(0)}(\phi_3 \to \chi\bar{\chi}) = \frac{1}{8\pi} m_3 y_3^2 \left(1 - \frac{4m_\chi^2}{m_3^2}\right)^{3/2}$$

- 1L virtual corrections:




- ***Corrections involving $A_{ijk}$*** → no vertex corrections, no mixing contributions

$$\Delta\hat{\Gamma}^{(1)}_{\phi_3\to\bar{\chi}\chi} \supset -\frac{1}{2} k y_3 \text{Re}\bigg[(A_{113})^2 \frac{d}{dp^2} B_0(p^2, m_1^2, m_1^2) + 2(A_{123})^2 \frac{d}{dp^2} B_0(p^2, m_1^2, m_2^2)$$

$(k \equiv 1/(16\pi^2))$

$$+(A_{223})^2 \frac{d}{dp^2} B_0(p^2, m_2^2, m_2^2) + (A_{333})^2 \frac{d}{dp^2} B_0(p^2, m_3^2, m_3^2)\bigg]\bigg|_{p^2=m_3^2} + \cdots,$$

*($B_0$: usual Passarino-Veltmann function)*

# Infrared limits

Derivative of the light-heavy $B_0$ loop function can become **IR divergent** if:

- $\Phi_1$ is light, and $\Phi_2$,$\Phi_3$ are almost mass-degenerate, i.e. $m_1 \to 0$, $m_2 \to m_3$

$$\frac{d}{dp^2} B_0(p^2, m_1^2, m_2^2)\big|_{p^2=m_3^2} = \frac{1}{m_3^2}\left(\frac{1}{2}\ln\frac{m_3^2}{m_1^2} - 1 + \mathcal{O}\left(\epsilon^{1/2}\right)\right)$$

with $\epsilon = m_3^2 - m_2^2$. IR divergence **regulated by $\mathbf{m_1}$**.

- $\Phi_1$ is massless, and $\Phi_2$,$\Phi_3$ are almost mass-degenerate, i.e. $m_1=0$, $m_2 \to m_3$

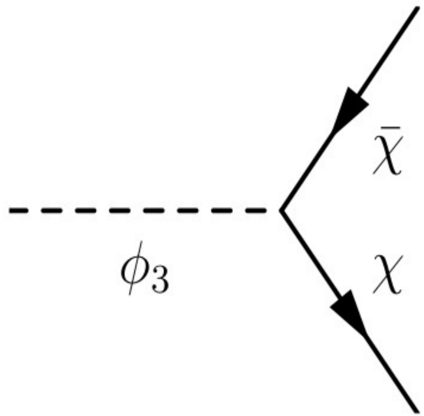
$$\frac{d}{dp^2} B_0(p^2, 0, m_2^2)\big|_{p^2=m_3^2} = \frac{1}{m_3^2}\left(\ln\left(-\frac{m_3^2}{\epsilon}\right) - 1 + \mathcal{O}\left(\epsilon\right)\right)$$

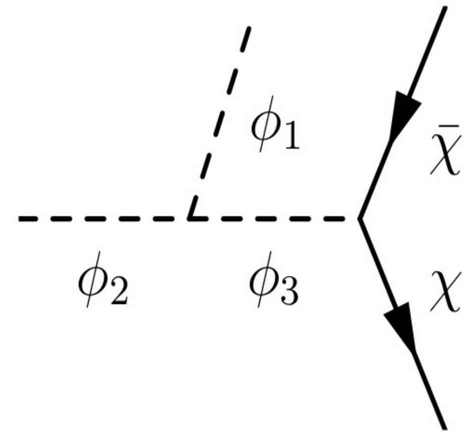
IR divergence **regulated by squared-mass difference** $\epsilon = m_3^2 - m_2^2$

# Curing the IR divergences at 1L – inclusion of real radiation

## In mass scenario where $m_1 \to 0$, $m_2 = m_3$

- Inclusion of real radiation, following Kinoshita-Lee-Nauenberg theorem
  $\to$ **IR divergence interpreted as stemming from lack of inclusiveness of observable**




- $\Phi_1$ radiation not possible from an initial $\Phi_3$ in $\Phi_3 \to \chi\bar{\chi}$ process (would break $Z_2$ symmetry)
  … but KLN theorem requires summing on *energy degenerate states* and $\Phi_2$ can radiate a $\Phi_1$

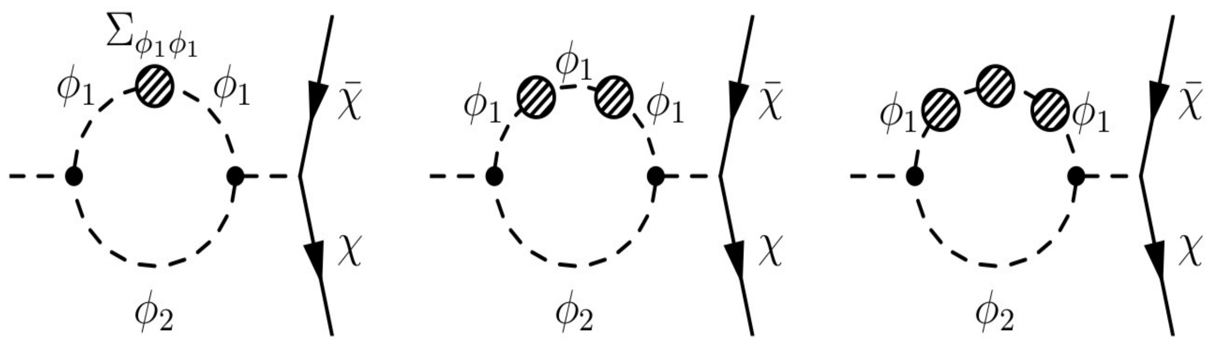
$$\Gamma(\phi_3 \to \chi\bar{\chi}) + \Gamma(\phi_2 \to \phi_1\chi\bar{\chi})|^{\text{soft}} = \text{finite}$$

- $\Gamma(\Phi_2 \to \Phi_1\chi\bar{\chi})|^{\text{soft}}$ contains dependence on energy resolution $E_l$ but this can be removed when including also hard radiation (3-body phase space computed numerically)

# Curing the IR divergences at 1L – resummation

## In mass scenario where $m_1 \to 0$, $m_2=m_3$

- Resummation of $\Phi_1$ contributions (inspired by one of the solutions to Goldstone boson catastrophe [Martin '14], [Elias-Miro, Espinosa, Konstandin '14], [JB, Goodsell '16], [Espinosa, Konstandin '17])




→**IR divergence interpreted as stemming from a bad perturbative expansion**, because in scenarios with large hierarchy, the mass of light scalar $\Phi_1$ receives very significant loop corrections, and thus diagrams with $\Sigma_{\Phi1\Phi1}$ subloop insertions are very large

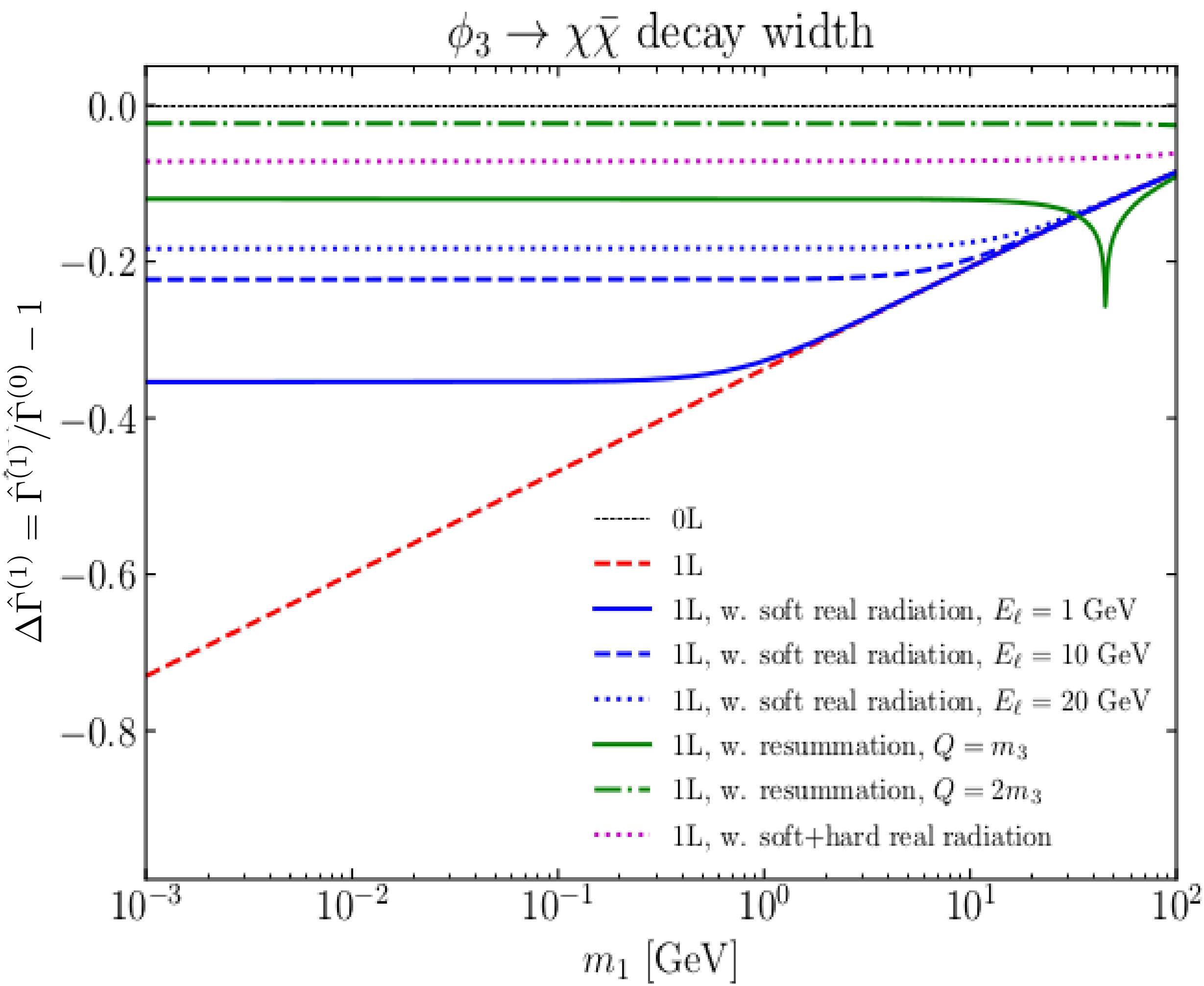
→ resummation produces an effective mass for $\Phi_1$

$$\Delta m_1^2 = \hat{\Sigma}^{(1)}_{\phi_1\phi_1}(p^2=0) = -k\left[\frac{1}{2}\lambda_{1122}A_0(m_2^2) + \frac{1}{2}\lambda_{1133}A_0(m_3^2) + (A_{113})^2 B_0(0,0,m_3^2) + (A_{123})^2 B_0(0,m_2^2,m_3^2)\right]$$
$$= \mathcal{O}(km_3^2)$$

*($A_0$, $B_0$: usual Passarino-Veltmann functions)*

# Curing the IR divergences at 1L – results

## In mass scenario where $m_1 \to 0$, $m_2=m_3$




with
$A_{123}$ =3 TeV
(other $A_{ijk}$=0)
$y_3$=1,
$m_2 = m_3$ = 1 TeV,
$m_\chi$ =200 GeV,
$\lambda_{1122}$ = 0.25,
$\lambda_{1133}$ = 0.4.

*(NB: at 1L, including the width of $\phi_3$ would also cure the IR divergence, but one can devise a model where the width is zero)*

# Remaining large logarithms

- Divergences in IR limit can be cured
  - Resummation (but physical meaning of resummed decay width is ambiguous)
  - Inclusion of (soft) real radiation

- However, if $m_1$ (or $\varepsilon$) is large enough, then $\Phi_3 \to \chi\bar{\chi}$ and $\Phi_2 \to \chi\bar{\chi}\Phi_1$ can be distinguished!

- 1L corrections to $\Phi_3 \to \chi\bar{\chi}$ decay width contain a term of the form

$$\Delta\hat{\Gamma}^{(1)} \supset -\frac{1}{2} k y_3 \boxed{\frac{A_{123}^2}{m_3^2}} \ln \frac{m_3^2}{m_1^2}$$

- Trilinear couplings involving heavy states $\Phi_2$, $\Phi_3$ typically of the order of the heavy mass $\mathbf{A_{123} \sim m_3}$

**→ Large, unsuppressed, logarithm remains in $\Delta\Gamma^{(1)}$!**

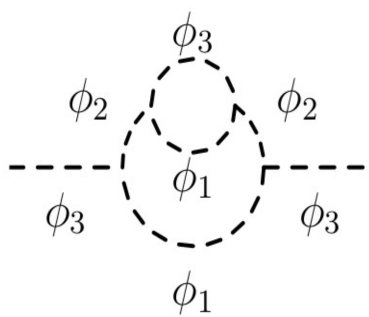
- What happens at 2L?

# External-leg corrections at 2L

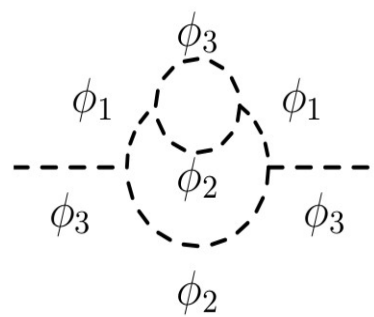
$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi})\Big\{1 - \text{Re}\hat{\Sigma}_{33}^{(1)\prime}(m^2) - \text{Re}\hat{\Sigma}_{33}^{(2)\prime}(m^2) + \left(\text{Re}\hat{\Sigma}_{33}^{(1)\prime}(m^2)\right)^2 - \frac{1}{2}\left(\text{Im}\hat{\Sigma}_{33}^{(1)\prime}(m^2)\right)^2 + \text{Im}\hat{\Sigma}_{33}^{(1)}(m^2)\cdot\text{Im}\hat{\Sigma}_{33}^{(1)\prime\prime}(m^2) + \mathcal{O}(k^3)\Big\}$$

- Genuine 2L O($A_{123}^4$) corrections involve derivatives of 2L self-energy diagrams (with $m_1^2 = \epsilon,\ m_2^2 = m_3^2 \equiv m^2$)

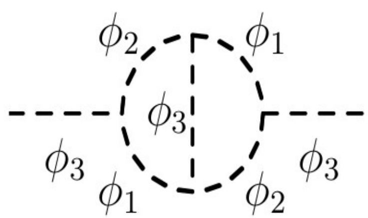
$$\hat{\Sigma}_{33}^{(2,\text{ genuine})\prime}(p^2 = m^2) = k^2 (A_{123})^4 \frac{d}{dp^2}\Big[T_{11234}(p^2, m^2, m^2, \epsilon, m^2, \epsilon) + T_{11234}(p^2, \epsilon, \epsilon, m^2, m^2, m^2) + T_{12345}(p^2, m^2, \epsilon, m^2, \epsilon, m^2)\Big]\Big|_{p^2=m^2}$$



$T_{11234}(p^2, m^2, m^2, \epsilon, m^2, \epsilon)$



$T_{11234}(p^2, \epsilon, \epsilon, m^2, m^2, m^2)$



$T_{12345}(p^2, m^2, \epsilon, m^2, \epsilon, m^2)$

with

$$\mathbf{T}_{11234}(p^2, x, y, z, u, v) \equiv \left(\frac{(2\pi\mu)^{2\epsilon_{\text{UV}}}}{i\pi^2}\right)^2 \int\int \frac{d^d q_1 d^d q_2}{(q_1^2 - x)(q_1^2 - y)((q_1 + p)^2 - z)((q_1 - q_2)^2 - u)(q_2^2 - v)}$$

$$\mathbf{T}_{12345}(p^2, x, y, z, u, v) \equiv \left(\frac{(2\pi\mu)^{2\epsilon_{\text{UV}}}}{i\pi^2}\right)^2 \int\int \frac{d^d q_1 d^d q_2}{(q_1^2 - x)((q_1 + p)^2 - y)((q_1 - q_2)^2 - z)(q_2^2 - u)((q_2 + p)^2 - v)}$$

# $\overline{\text{MS}}$ scheme results at 2L

$$\hat{\Sigma}_{33}^{(2,\text{ genuine})\prime}(p^2=m^2) = k^2(A_{123})^4\frac{d}{dp^2}\left[T_{11234}(p^2,m^2,m^2,\epsilon,m^2,\epsilon)+T_{11234}(p^2,\epsilon,\epsilon,m^2,m^2,m^2)+T_{12345}(p^2,m^2,\epsilon,m^2,\epsilon,m^2)\right]\Big|_{p^2=m^2}$$

- Analytical evaluation of derivatives of self-energy integrals at finite $p^2=m^2$ using **differential equations** and special limits from [Martin hep-ph/0307101] (in terms of $\overline{\text{MS}}$ quantities)
- **Expansion in ε** to find IR-dominant terms
- Results cross-checked numerically with TSIL [Martin, Robertson hep-ph/0501132]

$$\frac{d}{dp^2}T_{11234}(p^2,m^2,m^2,\epsilon,m^2,\epsilon)\Big|_{p^2=m^2} = \frac{\pi(2-\overline{\ln}m^2)}{4\sqrt{\epsilon}m^3}+\frac{-6\overline{\ln}\epsilon\overline{\ln}m^2-3\overline{\ln}^2\epsilon+24\overline{\ln}\epsilon+9\overline{\ln}^2m^2-24\overline{\ln}m^2-\pi^2}{24m^4}+\mathcal{O}(\epsilon)$$

$$\frac{d}{dp^2}T_{11234}(p^2,\epsilon,\epsilon,m^2,m^2,m^2)\Big|_{p^2=m^2} = -\frac{\overline{\ln}m^2}{2m^2\epsilon}+\frac{3\pi\overline{\ln}m^2}{8m^3\sqrt{\epsilon}}+\frac{-50+6\pi^2+3\overline{\ln}\epsilon-12\overline{\ln}m^2+18\overline{\ln}\epsilon\overline{\ln}m^2-18\overline{\ln}^2m^2}{36m^4}+\mathcal{O}(\epsilon)$$

$$\frac{d}{dp^2}T_{12345}(p^2,m^2,\epsilon,m^2,\epsilon,m^2)\Big|_{p^2=m^2} = \frac{1}{4m^4}\left[2+\ln\frac{m^2}{\epsilon}+\ln^2\frac{m^2}{\epsilon}\right]-\frac{\pi^2\ln 2-3/2\zeta(3)}{m^4}+\mathcal{O}(\epsilon) \qquad (\overline{\ln}x\equiv\ln x/Q^2)$$

$$\hat{\Gamma}(\phi_3\to\chi\bar{\chi})=\Gamma^{(0)}(\phi_3\to\chi\bar{\chi})\Bigg\{1-\frac{k(A_{123})^2}{m^2}\left[\frac{1}{2}\ln\frac{m^2}{\epsilon}-1\right]+\frac{k^2(A_{123})^4}{m^4}\Bigg[\frac{m^2\overline{\ln}m^2}{2\epsilon}-\frac{m\pi(4+\overline{\ln}m^2)}{8\sqrt{\epsilon}} +\frac{17}{9}-\frac{\pi^2}{8}+\frac{1}{8}\ln^2\frac{m^2}{\epsilon}+\frac{1}{6}\overline{\ln}\epsilon+\frac{1}{12}\overline{\ln}m^2+\pi^2\ln 2-\frac{3}{2}\zeta(3)\Bigg]\Bigg\}$$

→ **unphysically large 1/ε and 1/√ε terms in addition to logε, log²ε**

# Choices of renormalisation schemes at 2L

- Subloop renormalisation: $$\hat{\Sigma}_{33}^{(2,\text{ subloop})}(p^2) = k(A_{123})^2\Bigg[\left(\frac{2\delta^{(1)}A_{123}}{A_{123}}+\delta^{(1)}Z_3\right)B_0(p^2,m_1^2,m_2^2)$$ $$+\delta^{(1)}m_1^2\frac{\partial}{\partial m_1^2}B_0(p^2,m_1^2,m_2^2)+\delta^{(1)}m_2^2\frac{\partial}{\partial m_2^2}B_0(p^2,m_1^2,m_2^2)\Bigg]$$
- **OS renormalisation of scalar masses**:

$$\left.\begin{aligned}\delta^{(1)}m_1^2 &= k(A_{123})^2\text{Re}B_0(m_1^2,m_2^2,m_3^2)\\ \delta^{(1)}m_2^2 &= k(A_{123})^2\text{Re}B_0(m_2^2,m_1^2,m_3^2)\end{aligned}\right\} \Rightarrow \quad \Sigma_{33}^{(2,\text{subloop})\prime} \supset B_0(\epsilon,m^2,m^2)\frac{\partial^2}{\partial p^2\partial x}B_0(p^2,x,m^2)\Big|_{p^2=m^2,x=\epsilon},$$
$$\text{and } B_0(m^2,\epsilon,m^2)\frac{\partial^2}{\partial p^2\partial y}B_0(p^2,\epsilon,y)\Big|_{p^2=y=m^2}$$

→ **cancels with 1/ε and 1/√ε terms in $\overline{\text{MS}}$ decay width result!**

- Different possible choices for renormalisation of $A_{123}$
  - **$\overline{\text{MS}}$** → $\delta^{\text{fin}}A_{123} = 0$
  - **OS** → fix $\delta^{\text{fin}}A_{123}$ by demanding that OS-renormalised loop-corrected amplitude for $\Phi_2 \to \Phi_1\Phi_3$ with momenta on-shell remains equal to its tree-level value
  - **Custom "no-log-sq" scheme**, adjusting $\delta^{\text{fin}}A_{123}$ to cancel the $\log^2$ term in $\Gamma(\Phi_3\to\chi\bar{\chi})$

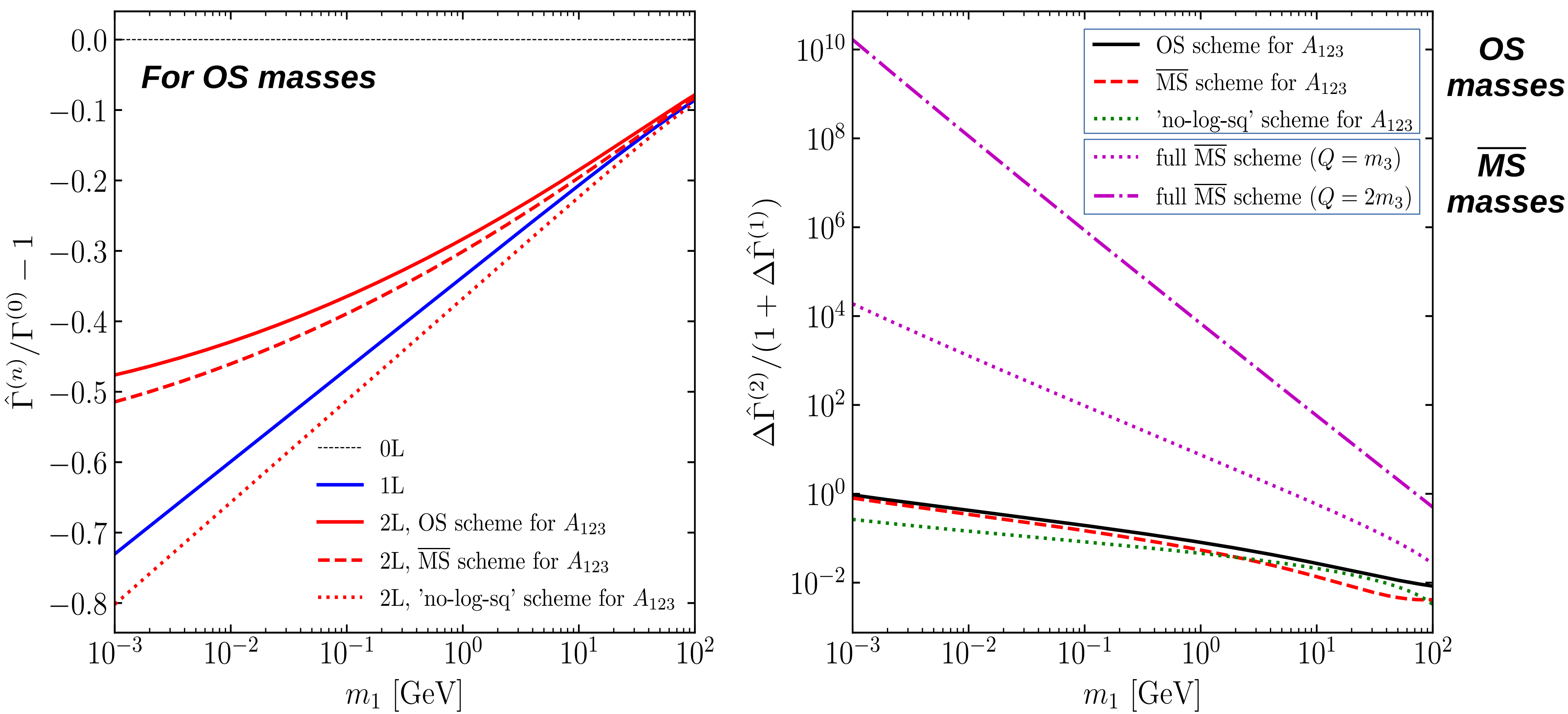
    *NB: this only reshuffles the $\log^2$ into the extraction of $A_{123}$ from a physical observable, e.g. $\Gamma(\Phi_3 \to \Phi_1\Phi_2)$*

- **logε remains at 1L and 2L ($\log^2\varepsilon$ also unless special scheme) !** → full expressions in [Bahl, JB, Weiglein '21]

# Numerical results I

## In mass scenario where $m_1 \to 0$, $m_2 = m_3$

$\phi_3 \to \chi\bar{\chi}$ decay width

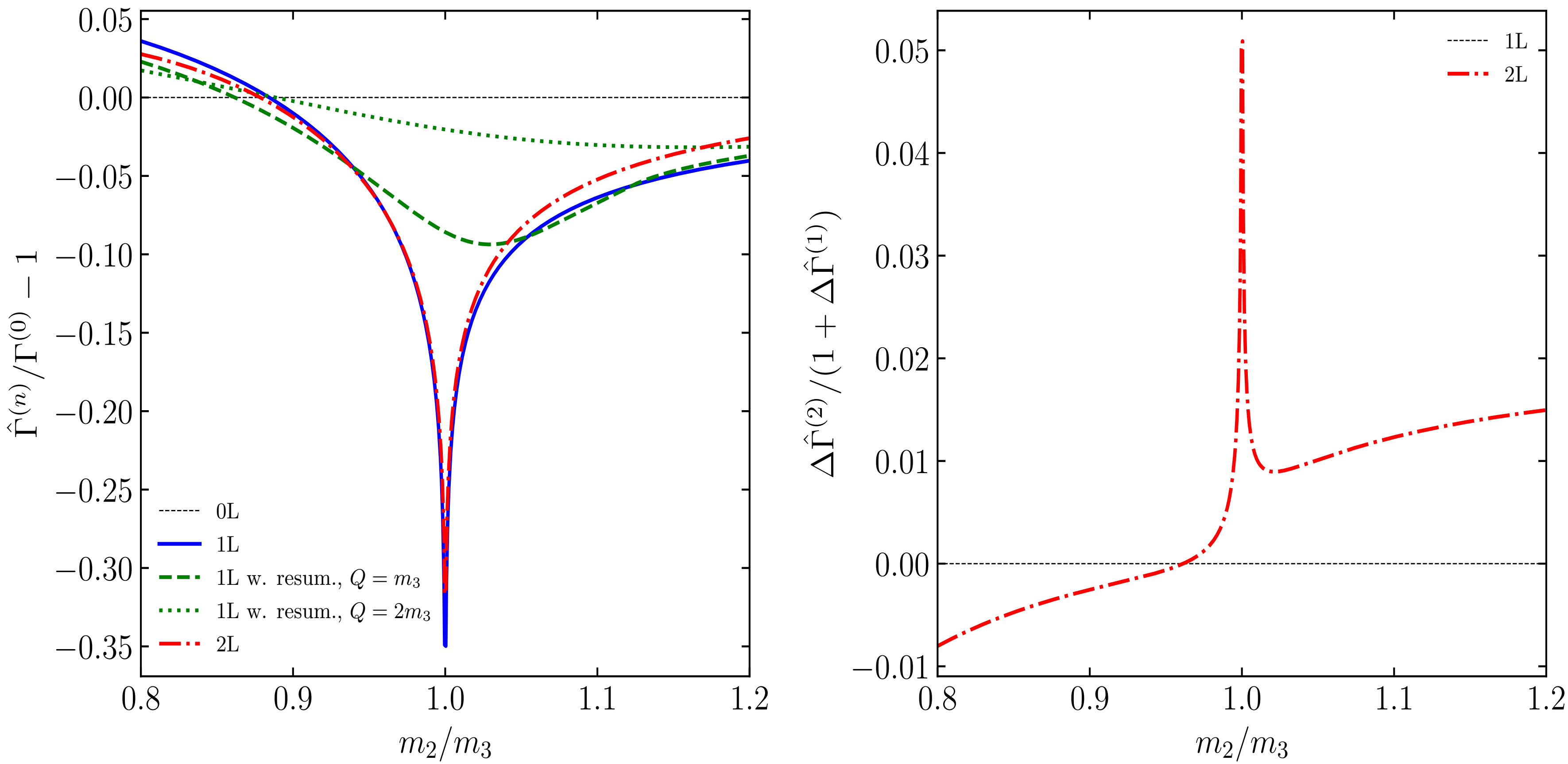



with $m_2 = m_3 = 1$ TeV, $y_3 = 1$, $A_{123} = 3$ TeV (other $A_{ijk} = 0$)

# Numerical results II

## In mass scenario where $m_1=0$, $m_2 \sim m_3$

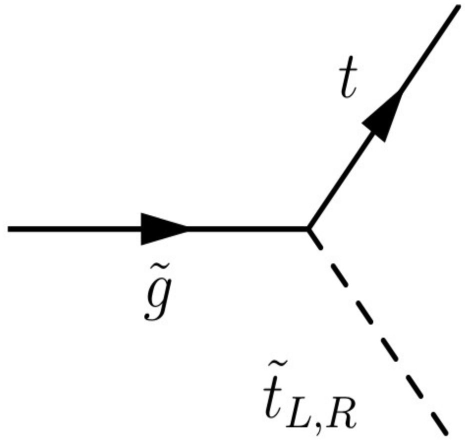
$\phi_3 \to \chi\bar{\chi}$ decay width




with $m_3$ = 500 GeV, $m_\chi$ = 200 GeV, $\lambda_{1122}$ = 1, $\lambda_{1133}$ = 1.2, and $A_{123}$ = 1.5 TeV ($A_{123}$ renormalised $\overline{\text{MS}}$)

# Large logarithms from external legs II: MSSM

# Decay of a gluino in the MSSM




- **Minimal Supersymmetric extension of the Standard Model**
  - Higgs sector (assuming CP conservation):
    2 CP-even states h,H; CP-odd state A; charged Higgs $H^{\pm}$
    (+ would-be Goldstones)
  - Stops – i.e. scalar partners of top quarks

- Consider the **decay of a gluino** (fermionic partner of gluon) **into a top quark and a stop**
- **Stop-Higgs couplings** important for corrections to this decay
  → involve $X_t \equiv A_t - \mu \cot\beta$ or $Y_t \equiv A_t + \mu \tan\beta$
  (with $A_t$ trilinear stop coupling, μ Higgsino mass parameter, and $\tan\beta \equiv v_2/v_1$ ratio of vacuum expectation values of the two Higgs doublets)

- Experimental limits → $M_{SUSY}$ must be large, potentially >> $M_A$ (scale of BSM Higgses)
- Neglect EW gauge couplings and set v~0 (<<$M_{SUSY}$) for simplicity → no stop mixing!

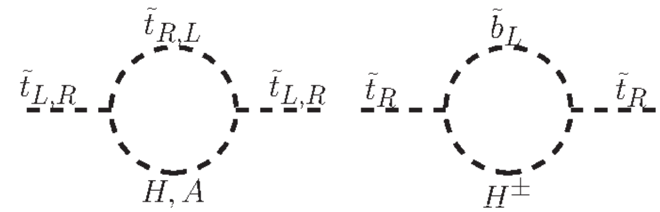
- Typical mass hierarchy: $M_{SUSY} \gg M_A \gg m_h, m_G, m_{G^\pm} \sim 0$

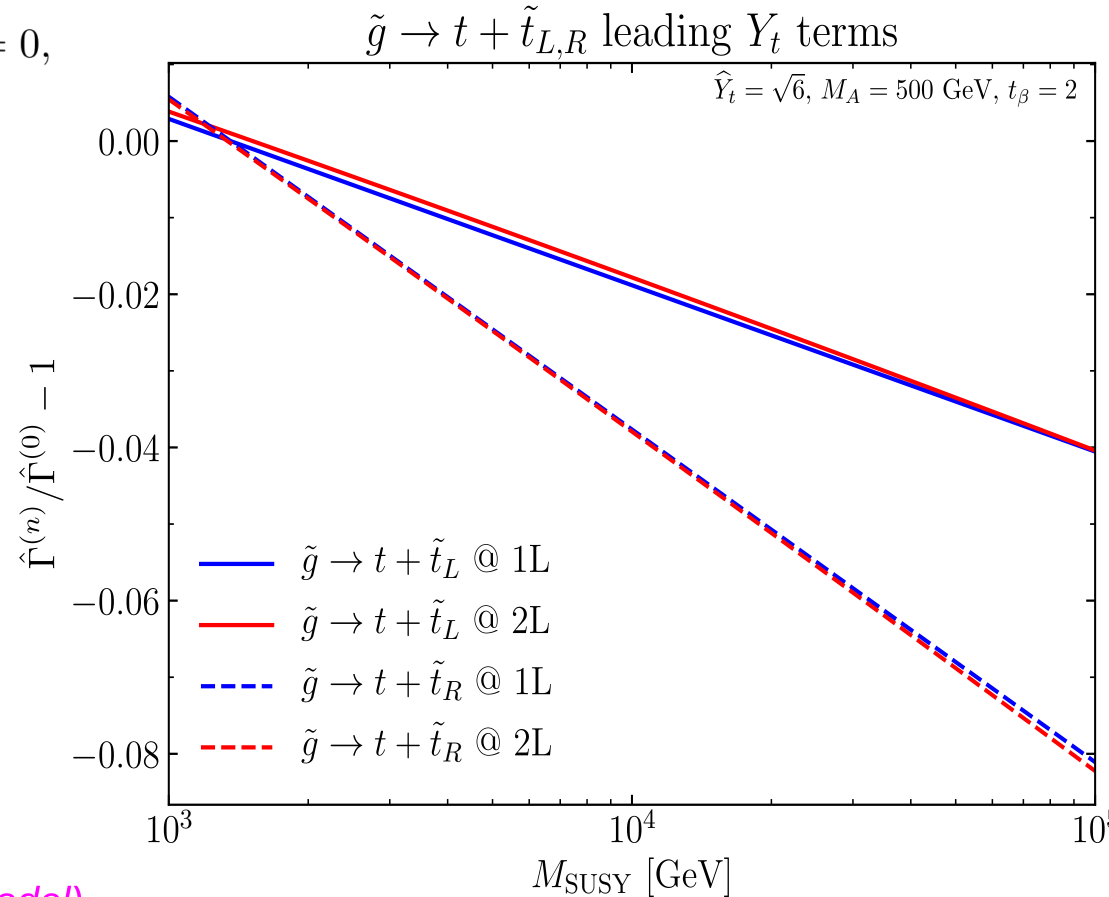
NB: case with v≠0 also considered in [Bahl, JB, Weiglein '21]

# $\tilde{g} \to t\tilde{t}$ decay – $Y_t$ terms

- Terms involving powers of $Y_t \equiv A_t + \mu \tan\beta$
  → **stop—BSM-Higgs couplings**

$$c(H\tilde{t}_L\tilde{t}_L) = c(H\tilde{t}_R\tilde{t}_R) = c(A\tilde{t}_L\tilde{t}_L) = c(A\tilde{t}_R\tilde{t}_R) = 0,$$
$$c(H\tilde{t}_L\tilde{t}_R) = -\frac{1}{\sqrt{2}} h_t c_\beta Y_t,$$
$$c(A\tilde{t}_L\tilde{t}_R) = -c(A\tilde{t}_R\tilde{t}_L) = \frac{1}{\sqrt{2}} h_t c_\beta Y_t,$$
$$c(H^+\tilde{t}_R\tilde{b}_R) = c(H^+\tilde{t}_L\tilde{b}_L) = c(H^+\tilde{t}_L\tilde{b}_R) = 0,$$
$$c(H^+\tilde{t}_R\tilde{b}_L) = -h_t c_\beta Y_t,$$




- Heavy scalars: $\tilde{t}_L$, $\tilde{t}_R$

  $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{SUSY}$
- Light scalars: H, A, $H^\pm$

  $M_A \neq 0$ but << $M_{SUSY}$

  → e.g. $M_A$ = 500 GeV

*(Same as $m_1 \neq 0$, $m_2 = m_3$ in toy model)*

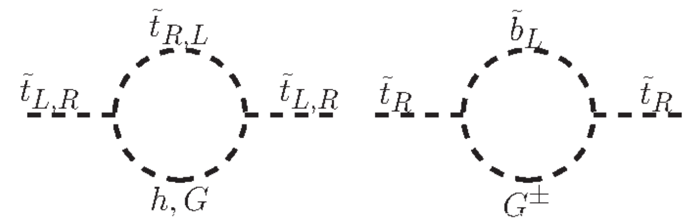
# $\tilde{g} \to t\tilde{t}$ decay – $X_t$ terms (at v=0)

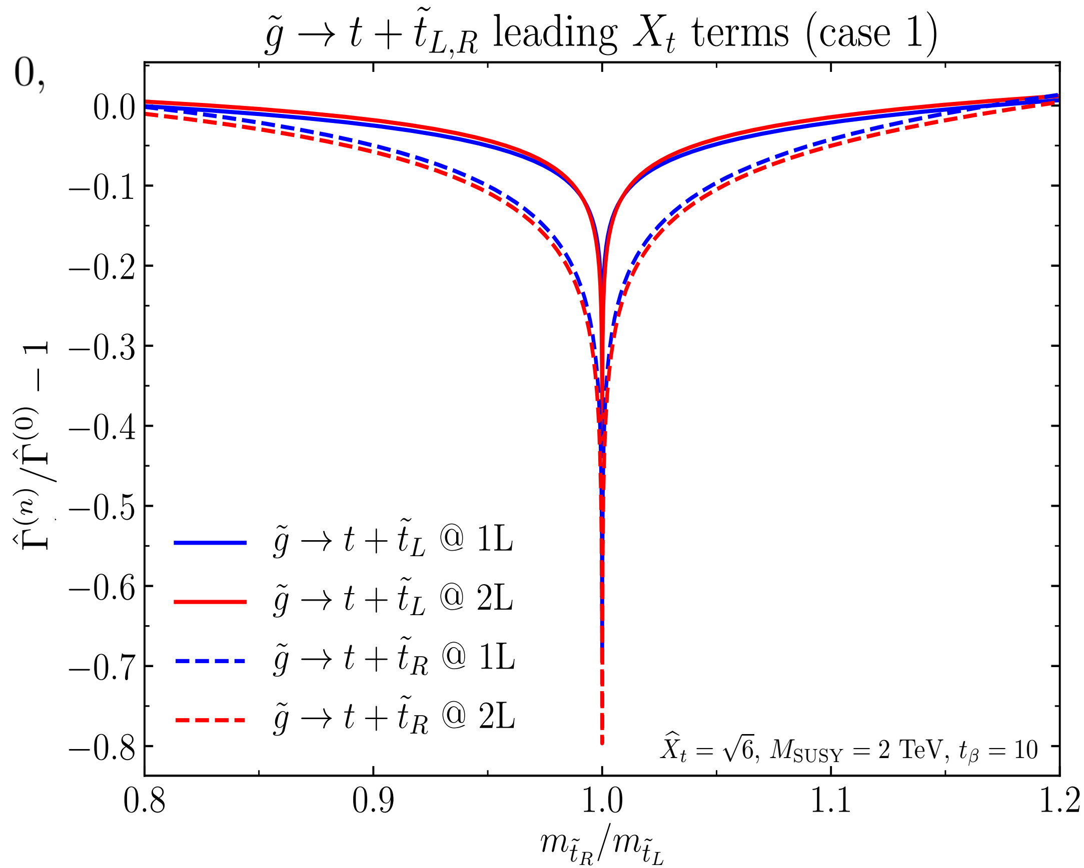
- Terms involving powers of $X_t \equiv A_t - \mu \cot\beta$
  → **stop—Higgs + Goldstone couplings**

$$c(h\tilde{t}_L\tilde{t}_L) = c(h\tilde{t}_R\tilde{t}_R) = c(G\tilde{t}_L\tilde{t}_L) = c(G\tilde{t}_R\tilde{t}_R) = 0,$$
$$c(h\tilde{t}_L\tilde{t}_R) = \frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(G\tilde{t}_L\tilde{t}_R) = -c(G\tilde{t}_R\tilde{t}_L) = \frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(G^+\tilde{t}_R\tilde{b}_R) = c(G^+\tilde{t}_L\tilde{b}_L) = c(G^+\tilde{t}_L\tilde{b}_R) = 0,$$
$$c(G^+\tilde{t}_R\tilde{b}_L) = -h_t s_\beta X_t.$$

- Heavy scalars: $\tilde{t}_L$, $\tilde{t}_R$
  $m_{\tilde{t}L} \neq m_{\tilde{t}R} \sim M_{SUSY}$
- Light scalars: h, G, $G^\pm$
  - $m_h = 0$ in gaugeless limit
  - $m_G = m_{G\pm} = 0$

*(Same as $m_1$=0, $m_2$~$m_3$ in toy model)*



# Large logarithms from external legs III: N2HDM

# Decay of a heavy Higgs boson in the N2HDM

- Extend SM scalar sector by an **additional Higgs doublet** (→2HDM) plus a **real singlet $\Phi_S$**

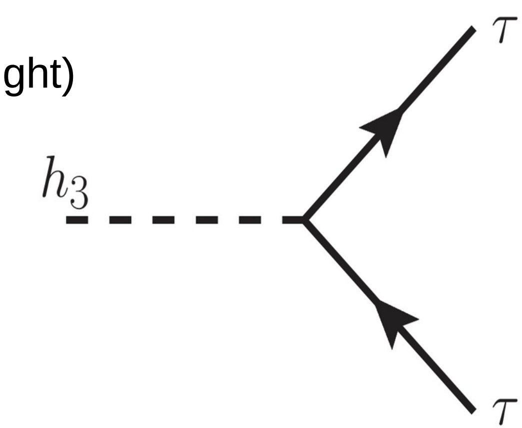
$$V^{(0)} = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \text{h.c.}\right) + \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{1}{2}\lambda_5\left((\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right)$$
$$+ \frac{1}{2}m_S^2\Phi_S^2 + \boxed{\frac{1}{6}a_S\Phi_S^3} + \frac{1}{24}\lambda_S|\Phi_S|^4 + \boxed{\frac{1}{2}a_{1S}|\Phi_1|^2\Phi_S + \frac{1}{2}a_{2S}|\Phi_2|^2\Phi_S} + \frac{1}{6}\lambda_{1S}|\Phi_1|^2\Phi_S^2 + \frac{1}{2}\lambda_{2S}|\Phi_2|^2\Phi_S^2\,.$$

- $Z_3$ symmetry often imposed to forbid trilinear couplings in Lagrangian, *but not in our case*
- For convenience, define $X_a \equiv \frac{1}{4}(a_{1S} - a_{2S})\,, Y_a \equiv \frac{1}{4}a_{1S}s_\beta^2 + a_{2S}c_\beta^2\,, Z_a \equiv \frac{a_S}{4} - Y_a$

- ***Physical spectrum*** (assuming CP-conservation):
  3 CP-even states, $h_1$,$h_2$,$h_3$; 1 CP-odd state A; 1 charged Higgs boson $H^\pm$; (G, $G^\pm$ would-be Goldstones)

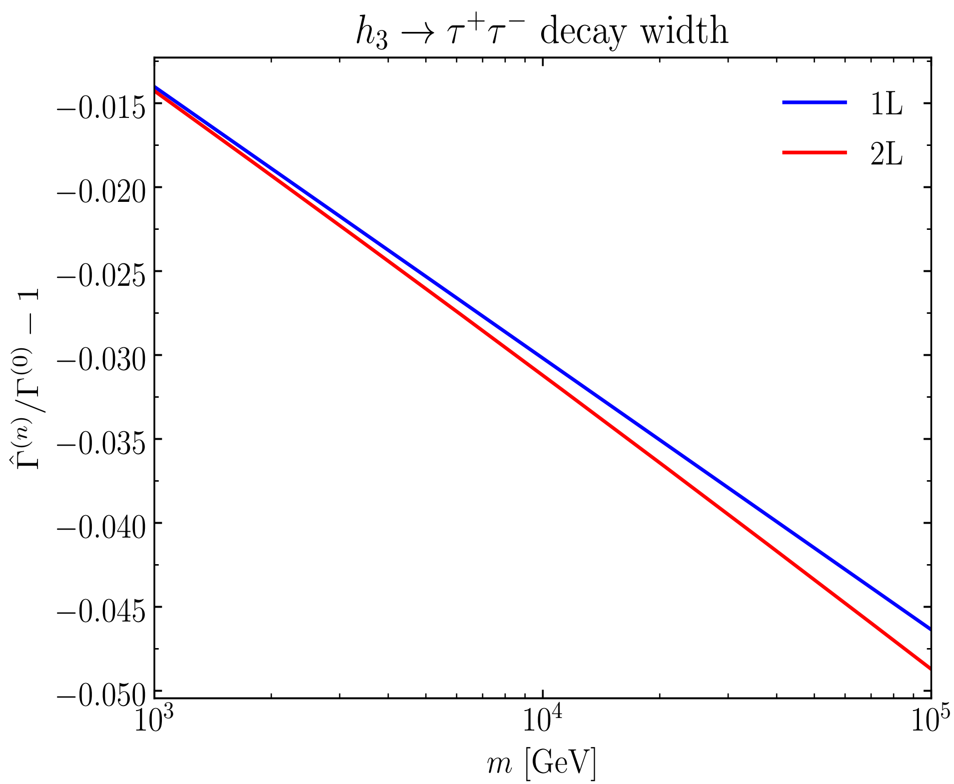
- Consider a scenario with mass hierarchy $m_{h_1} \sim m_{h_2} \sim m_G \sim m_{G^\pm} \sim \sqrt{\epsilon}$ (light) and $m_{h_3} = m_A = m_{H^\pm} = m$ (heavy)

- Investigate trilinear-enhanced contributions to **$h_3 \to \tau^+\tau^-$ decay** process ($h_3$ being doublet-like), involving **$X_a$**

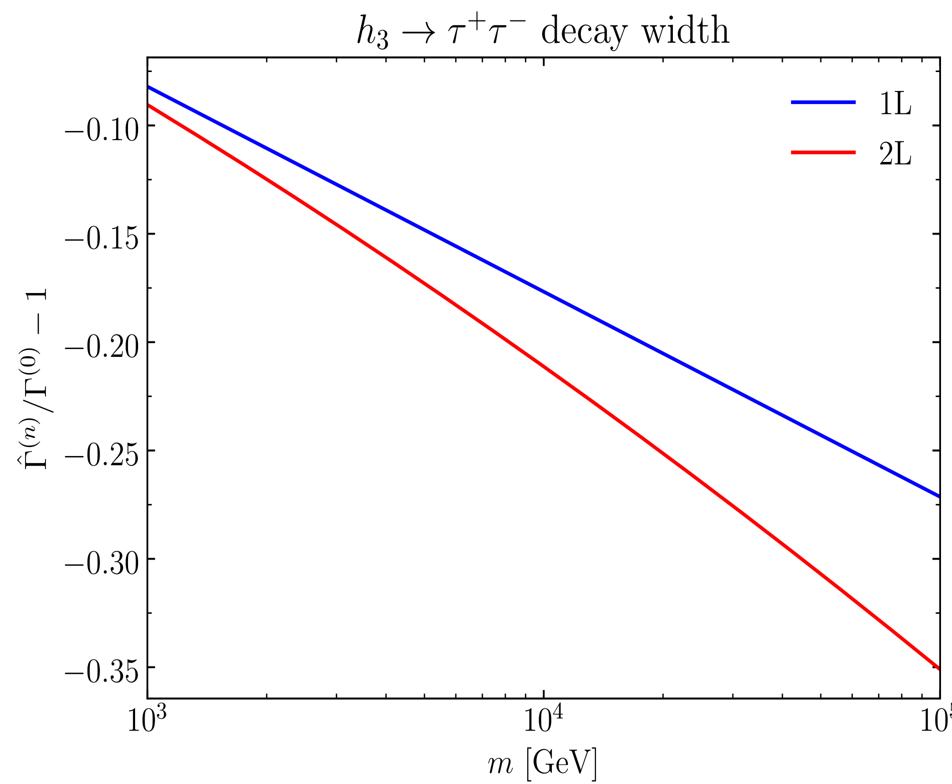


# $h_3 \to \tau^+\tau^-$ decay – trilinear-coupling enhanced $X_a$ terms

**Set $\varepsilon=(50\text{ GeV})^2$, $X_a=3m$, vary m between 1 and 100 TeV** *(Same as $m_1\neq 0$, $m_2=m_3$ in toy model)*




tanβ=1.4, sin$\alpha_3$=0.99




tanβ=1.26, sin$\alpha_3$=0.94

- Effects can be significant! (enhanced by deviation from alignment and by multiplicity of diagrams)
- 2L corrections always well smaller than 1L ones

# Summary

**Precise theory predictions are of paramount importance to properly assess BSM discovery sensitivities, and to constrain parameter space of BSM models**

- We pointed out the existence of a **new type of large Sudakov-like logarithms, in external-leg corrections of heavy scalars**, in presence of mass hierarchy
- Can be further **enhanced by large trilinear couplings**
- At 1L, we showed how these logs are related to singularities in IR limit, and we discussed how to address these divergences
- **Computed large logs at 2L** (derivatives of self-energies with non-zero masses and at finite $p^2$)
- Showed the importance of OS renormalisation of masses
- In MSSM and N2HDM examples: large effects at 1L; size of 2L effects well below that of 1L ones → SCET resummation doesn't seem compulsory

# Thank you for your attention

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# $\tilde{g} \to t\tilde{t}$ decay – $X_t$ terms (at $v \neq 0$)

- $v \neq 0 \to$ stop mixing
- Heavy scalars: $\tilde{t}_1$, $\tilde{t}_2$
  - Assume $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{SUSY}$
  - $m_{\tilde{t}2}^2 - m_{\tilde{t}1}^2 = 2\, m_t\, X_t$

$$c(h\tilde{t}_1\tilde{t}_1) = -c(h\tilde{t}_2\tilde{t}_2) = \frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(h\tilde{t}_1\tilde{t}_2) = c(h\tilde{t}_2\tilde{t}_1) = 0,$$
$$c(G\tilde{t}_1\tilde{t}_1) = c(G\tilde{t}_2\tilde{t}_2) = 0,$$
$$c(G\tilde{t}_1\tilde{t}_2) = -c(G\tilde{t}_2\tilde{t}_1) = \frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(G^+\tilde{t}_1\tilde{b}_1) = c(G^+\tilde{t}_2\tilde{b}_1) = -\frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(G^+\tilde{t}_1\tilde{b}_2) = c(G^+\tilde{t}_2\tilde{b}_2) = 0.$$

- Light scalars:
  - $m_h \neq 0$ but << $M_{SUSY}$
  - Set $m_h \sim m_G \sim m_{G\pm} \sim m_{IR}$ (IR regulator mass)

*(Same as $m_1$~0, $m_2$~$m_3$ in toy model)*

***IR divergence cured by real radiation***