Electroweak renormalization based on gauge-invariant vacuum expectation values of non-linear Higgs representations in the SM and beyond

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in collaboration with Heidi Rzehak, arXiv:2203.07236 [hep-ph] + paper in preparation

see also related work with L.Altenkamp, M.Boggia, A.Denner and J.N.Lang,
JHEP 09 (2017) 134; 03 (2018) 110; 04 (2018) 062; 11 (2018) 104

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EW renormalization of vevs and tadpoles

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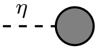
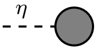
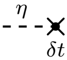
Higgs doublet $\mathbf{\Phi}$ (matrix notation), Higgs field η, and vev parameter v:

$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}\left[(v+\eta)\mathbb{1} + 2\mathrm{i}\boldsymbol{\phi}\right], \qquad \boldsymbol{\phi} = \frac{\phi_j\sigma_j}{2} = \frac{\vec{\phi}\cdot\vec{\sigma}}{2} = \text{Goldstone matrix}$$

Higgs potential: $$V = -\frac{\mu_2^2}{2}\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right] + \frac{\lambda_2}{16}\left(\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right]\right)^2$$

EW corrections: tadpole diagrams unavoidable $\Gamma^\eta =$  $\neq 0$

Very convenient:

cancel tadpoles against counterterm $\Gamma^\eta_{\mathrm{R}} =$  $+$  $\stackrel{!}{=} 0$

Issues with tadpole renormalization:

- On-shell (OS) conditions: no physical effect of tadpoles (just bookkeeping)
- $\overline{\mathrm{MS}}$ conditions: parameters related to masses depend on tadpole ren.!
  - SM: issues with $\overline{\mathrm{MS}}$ masses
  - BSM: issues with $\overline{\mathrm{MS}}$-ren. Higgs mxing angles

Note: tadpole ren. constant $\delta t =$ gauge dependent!

Two commonly used tadpole treatments:

Starting point: bare Lagrangian contains $t_0\eta$ term with

$$t_0 = \tfrac{1}{4} v_0 \left(4\mu_{2,0}^2 - \lambda_{2,0} v_0^2\right)$$

$\hookrightarrow$ Set $t_0 = 0$ in leading order to get the usual free propagators

Question: How to set t_0 in higher orders? $\rightarrow$ part of "tadpole scheme"

1. Parameter-Renormalized Tadpole Scheme (PRTS):

e.g. Böhm/Hollik/Spiesberger '86
Denner '93

- interpret $t_0\eta$ as tadpole counterterm in Lagrangian: $\delta t = t_0$
- technically equivalent:
  δt term generated in ren. transformation of bare parameters $\mu_{2,0}^2$, $\lambda_{2,0}$:

$$\mu_{2,0}^2 \;\rightarrow\; \mu_{2,0}^2 + \frac{3\delta t^{\mathrm{PRTS}}}{2v}, \qquad \lambda_{2,0} \;\rightarrow\; \lambda_{2,0} + \frac{2\delta t^{\mathrm{PRTS}}}{v^3}$$

- expansion of η field about corrected minimum of effective potential
- drawback: δt can enter relations between bare input parameters
  $\hookrightarrow$ gauge-dependent terms $\propto \delta t$ enter relations
  between renormalized parameters and predicted observables

2. Fleischer–Jegerlehner Tadpole Scheme (FJTS):

Fleischer/Jegerlehner '80
Actis et al. '06

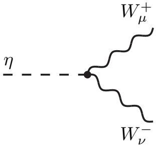
- set $t_0 = 0$ to all orders
- δt generated via field shift $\eta \;\to\; \eta + \Delta v$ with $\Delta v = -\delta t^{\mathrm{FJTS}}/M_{\mathrm{H}}^2$
- field shift has no physical effect (only redistributes tadpole terms)
  $\hookrightarrow$ FJTS equivalent to just including all tadpole loops
- expansion of η field about minimum of lowest-order Higgs potential
- advantage: no δt terms in relations between bare parameters
  $\hookrightarrow$ gauge-independent counterterms and relations between renormalized parameters and observables
- drawback: potentially large corrections in $\overline{\mathrm{MS}}$ schemes

PRTS and FJTS tadpole counterterms:

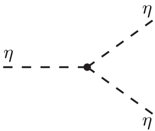
Note: appearance of δt in PRTS and FJTS Feynman rules quite different

see, e.g., Denner, SD, arXiv:1912.06823

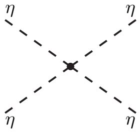
Examples:



$$= \frac{\mathrm{i}eM_\mathrm{W}}{s_\mathrm{W}} g_{\mu\nu} \left[1 + \delta Z_e - \frac{\delta s_\mathrm{W}}{s_\mathrm{W}} + \frac{1}{2}\frac{\delta M_\mathrm{W}^2}{M_\mathrm{W}^2} - \frac{e\delta t^\mathrm{FJTS}}{2s_\mathrm{W} M_\mathrm{H}^2 M_\mathrm{W}} + \frac{1}{2}\delta Z_\eta + \delta Z_W\right]$$



$$= -\frac{3\mathrm{i}e}{2s_\mathrm{W}} \frac{M_\mathrm{H}^2}{M_\mathrm{W}} \left[1 + \delta Z_e - \frac{\delta s_\mathrm{W}}{s_\mathrm{W}} + \frac{\delta M_\mathrm{H}^2}{M_\mathrm{H}^2} + \frac{e(\delta t^\mathrm{PRTS} - \delta t^\mathrm{FJTS})}{2s_\mathrm{W} M_\mathrm{H}^2 M_\mathrm{W}} - \frac{1}{2}\frac{\delta M_\mathrm{W}^2}{M_\mathrm{W}^2} + \frac{3}{2}\delta Z_\eta\right]$$



$$= -\frac{3\mathrm{i}e^2}{4s^2} \frac{M_\mathrm{H}^2}{M_\mathrm{W}^2} \left[1 + 2\delta Ze - 2\frac{\delta s}{s} + \frac{\delta M_\mathrm{H}^2}{M_\mathrm{H}^2} + \frac{e\delta t^\mathrm{PRTS}}{2sM_\mathrm{H}^2 M_\mathrm{W}} - \frac{\delta M_\mathrm{W}^2}{M_\mathrm{W}^2} + 2\delta Z_\eta\right]$$

Gauge-invariant vev renormalization in the SM

Idea:

- as in PRTS: define v_0 from Higgs parameters $\mu^2_{2,0}$, $\lambda_{2,0}$ so that
  $v_0 + \eta_{\mathrm{B}}(x)$ = expansion about corrected minimum of effective potential
  $\hookrightarrow$ no big corrections from tadpoles expected
- use field basis in which $v + h(x)$ is gauge invariant
  $\hookrightarrow$ $\Gamma^h =$ gauge independent according to Nielsen identities
  Kluberg-Stern/Zuber '75;
  Nielsen '75; Gambino/Grassi '00

Solution: GIVS as hybrid scheme

- Non-linear (nl) Higgs field basis:

$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}(v+h)U(\zeta), \qquad h = \text{physical Higgs field (gauge invariant!)}$$

$$U(\zeta) = \exp\left(\frac{2\mathrm{i}\zeta}{v}\right), \qquad \zeta = \frac{\zeta_j\sigma_j}{2} = \text{Goldstone matrix}$$

- loop calculations performed in linear representation,
  but corrections to vev parameter from the nl representation
- generation of δt from 2 parts: $\delta t = \delta t_1^{\mathrm{GIVS}} + \delta t_2^{\mathrm{GIVS}} = -\Gamma^\eta$
  - PRTS part $\delta t_1^{\mathrm{GIVS}} = -\Gamma^h_{\mathrm{nl}} =$ gauge independent
    $\hookrightarrow$ absorbs corrections to the vev into input parameters
  - FJTS part $\delta t_2^{\mathrm{GIVS}} = \Gamma^h_{\mathrm{nl}} - \Gamma^\eta =$ gauge dependent
    $\hookrightarrow$ generates remaining part of $\delta t = -\Gamma^\eta$ from field shift

Non-linear Higgs representation for the SM

Lee/Zinn-Justin '72; ...
Grosse-Knetter/Kögerler '93
SD/Grosse-Knetter '95

Higgs and Goldstone fields

Relation between fields of the linear and non-linear representations:

$$\eta = \cos\left(\frac{\zeta}{v}\right)(v+h) - v = h - \frac{\zeta^2}{2v}\left(1+\frac{h}{v}\right) + \mathcal{O}(\zeta^4), \quad \zeta \equiv |\vec{\zeta}|$$

$$\vec{\phi} = \frac{1}{\zeta}\sin\left(\frac{\zeta}{v}\right)(v+h)\vec{\zeta} \;= \left(1+\frac{h}{v}\right)\vec{\zeta} \qquad + \mathcal{O}(\zeta^3)$$

Gauge transformations: (θ_a = group parameters, g_n = gauge couplings)

$$\boldsymbol{\Phi} \to \underbrace{S(\boldsymbol{\theta})}_{\exp(\frac{i}{2}g_2\theta_j\sigma_j)}\,\boldsymbol{\Phi}\,\underbrace{S_Y(\theta_Y)}_{\exp(\frac{i}{2}g_1\theta_Y\sigma_3)}, \qquad h \to h, \qquad U(\boldsymbol{\zeta}) \to S(\boldsymbol{\theta})\,U(\boldsymbol{\zeta})\,S_Y(\theta_Y)$$

Higgs potential:

$$V = -\frac{\mu_2^2}{2}\mathrm{tr}\left[\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi}\right] + \frac{\lambda_2}{16}\left(\mathrm{tr}\left[\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi}\right]\right)^2 = -\frac{\mu_2^2}{2}(v+h)^2 + \frac{\lambda_2}{16}(v+h)^4$$

= independent of Goldstone fields

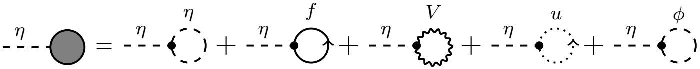
Kinetic Higgs Lagrangian: $\vec{C}^\mu = \left(W_1^\mu, W_2^\mu, Z^\mu/c_\mathrm{W}\right)^\mathrm{T}$

$$\begin{aligned}\mathcal{L}_{\mathrm{H,kin}} = \frac{1}{2}(\partial h)^2 + \frac{(v+h)^2}{2v^2}\Big\{&(\partial_\mu\vec{\zeta})\cdot(\partial^\mu\vec{\zeta}) + \frac{g_2^2 v^2}{4}\,\vec{C}_\mu\cdot\vec{C}^\mu \\ &+ g_1 g_2 B_\mu\left[-W_3^\mu\zeta^2 + (\vec{W}^\mu\cdot\vec{\zeta})\zeta_3\right] - g_2^2 v\,\vec{C}_\mu\cdot(\vec{W}^\mu\times\vec{\zeta}) \\ &- g_2 v\,\vec{C}_\mu\cdot\partial^\mu\vec{\zeta} - g_2(\vec{C}_\mu - 2\vec{W}_\mu)\cdot(\vec{\zeta}\times\partial^\mu\vec{\zeta})\Big\} + \mathcal{O}(\zeta^3)\end{aligned}$$

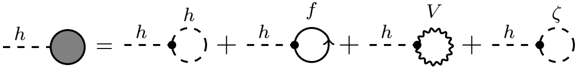
= non-polynomial with arbitrarily high powers in ζ_j

Tadpoles in the linear and non-linear representations:

$\Gamma^{\eta} =$



$\Gamma^{h}_{\mathrm{nl}} =$



(no loops with FP ghosts u)

$=$ gauge independent

$$\Gamma^{\eta} = \Gamma^{h}_{\mathrm{nl}} + M_{\mathrm{H}}^2 \Delta v_{\xi}, \quad \Delta v_{\xi} = \frac{1}{16\pi^2 v} \left\{ \tfrac{1}{2} A_0(\xi_Z M_{\mathrm{Z}}^2) + A_0(\xi_W M_{\mathrm{W}}^2) \right\} = \text{gauge dependent}$$

(ξ_a = gauge parameters, A_0 = 1-point scalar integral)

GIVS tadpole renormalization constants:

$$\left.\begin{aligned} \delta t_1^{\mathrm{GIVS}} &= \delta t_{\mathrm{nl}}^{\mathrm{PRTS}} = -\Gamma^{h}_{\mathrm{nl}} \\ \delta t_2^{\mathrm{GIVS}} &= \Gamma^{h}_{\mathrm{nl}} - \Gamma^{\eta} = -M_{\mathrm{H}}^2 \Delta v_{\xi} \end{aligned}\right\} \quad \delta t = \delta t_1^{\mathrm{GIVS}} + \delta t_2^{\mathrm{GIVS}} = -\Gamma^{\eta}$$

Generation of GIVS tadpole counterterms in Feynman rules:

- PRTS part: $\lambda_{2,0} \to \lambda_{2,0} + \dfrac{2\delta t_{\mathrm{nl}}^{\mathrm{PRTS}}}{v^3}, \quad \mu_{2,0}^2 \to \mu_{2,0}^2 + \dfrac{3\delta t_{\mathrm{nl}}^{\mathrm{PRTS}}}{2v}$
- FJTS part: $\eta_B \to \eta_B + \Delta v_{\xi}$
- or from PRTS/FJTS Feynman rules: $\delta t^{\mathrm{PRTS}} \to \delta t_1^{\mathrm{GIVS}}, \quad \delta t^{\mathrm{FJTS}} \to \delta t_2^{\mathrm{GIVS}}$

Conversion of on-shell to $\overline{\mathrm{MS}}$ masses:

Start from uniqueness of bare mass M_0:

$$M_0 = M^{\mathrm{OS}} + \delta M^{\mathrm{OS}} = \overline{M}(\mu) + \delta\overline{M}(\mu)$$

$\hookrightarrow$ difference of OS and $\overline{\mathrm{MS}}$ masses:

$$\Delta M^{\overline{\mathrm{MS}}-\mathrm{OS}}(\mu) = \overline{M}(\mu) - M^{\mathrm{OS}} = \underbrace{\delta M^{\mathrm{OS}} - \delta\overline{M}(\mu)}_{\text{depends on finite parts of tadpole contributions}}$$

Conversion effects of NLO EW corrections: $(\mu = M^{\mathrm{OS}})$

	M^{OS}[GeV]	$\Delta M_{\mathrm{EW}}^{\overline{\mathrm{MS}}-\mathrm{OS}}$[GeV]		
		FJTS	PRTS	GIVS
W boson	80.379	−2.22	0.82	0.74
Z boson	91.1876	−0.77	1.25	1.14
Higgs boson	125.1	6.34	3.16	2.80
top quark	172.4	10.75 (6%)	0.99	0.54 (0.3%)
bottom quark	4.93	−1.79 (−36%)	0.10	0.13 (3%)
τ lepton	1.77686	−0.93 (−52%)	−0.028	−0.015 (−0.8%)

$\Rightarrow$ GIVS reduces huge EW corrections of FJTS drastically!

Non-linear Higgs representations in extended Higgs sectors

Singlet Extension of the SM (SESM)

Complex scalar SU(2) doublet Φ & real scalar singlet σ: $v_{1,2}$ = vevs

$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}\left[(v_2+\eta_2)\mathbb{1} + 2\mathrm{i}\phi\right] = \underbrace{\frac{1}{\sqrt{2}}(v_2+h_2)U(\zeta)}_{\text{nl representation as in SM}}, \qquad Y_{\mathrm{W}}(\Phi) = 1$$

$$\sigma = v_1 + \eta_1 = v_1 + h_1$$

Note: h_1, h_2 = gauge invariant

$\hookrightarrow$ Mass basis h, H: $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R(\alpha)\begin{pmatrix} H \\ h \end{pmatrix}, \quad R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

Lagrangian: restriction to real, $\mathbb{Z}_2$-symmetric case!

$$\mathcal{L}_{\mathrm{Higgs}} = \frac{1}{2} tr\left[(D_\mu\mathbf{\Phi})^\dagger(D^\mu\mathbf{\Phi})\right] + \frac{1}{2}(\partial\sigma)^2 - V$$

$$V = -\frac{\mu_2^2}{2}\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right] - \mu_1^2\sigma^2 + \frac{\lambda_2}{16}\left(\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right]\right)^2 + \lambda_1\sigma^4 + \frac{\lambda_{12}}{2}\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right]\sigma^2$$
$$= -\frac{\mu_2^2}{2}(v_2+h_2)^2 - \mu_1^2(v_1+h_1)^2 + \frac{\lambda_2}{16}(v_2+h_2)^4 + \lambda_1(v_1+h_1)^4$$

Tadpoles in the linear and non-linear representations:

$$\Gamma^{h_n}_{\rm nl} = \text{gauge independent}$$

$$\Gamma^{\eta_1} = \Gamma^{h_1}_{\rm nl} + 2\lambda_{12} v_1 v_2 \Delta v_\xi$$

$$\Gamma^{\eta_2} = \Gamma^{h_2}_{\rm nl} + \frac{\lambda_2}{2}\, v_2^2 \underbrace{\Delta v_\xi}_{\text{as in SM}}$$

Transformation of input parameters:

original set: $\{\lambda_1, \lambda_2, \lambda_{12}, \mu_1^2, \mu_2^2, g_1, g_2\}$

$\downarrow$

mass basis: $\{\underbrace{M_{\rm H}, M_{\rm h}, M_{\rm W}, M_{\rm Z}, e}_{\text{renormalized on-shell}}, \underbrace{\lambda_{12}}_{\overline{\rm MS}}, \alpha\}$

Renormalization:

Bojarski et al. '15
Kanemura et al. '15,'17
Denner et al. '17,'18
Altenkamp et al. '18

Two-Higgs-Doublet Model (THDM)

Two complex scalar SU(2) doublets Φ_1, Φ_2: $\quad v_{1,2} = \text{vevs}, \quad v = \sqrt{v_1^2 + v_2^2}$

$$\Phi_n = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{n,2} + \mathrm{i}\phi_{n,1} \\ v_n + \eta_n - \mathrm{i}\phi_{n,3} \end{pmatrix}, \qquad Y_\mathrm{W}(\Phi_n) = 1$$

$$\mathbf{\Phi}_n = \frac{1}{\sqrt{2}} \left[(v_n + \eta_n)\mathbb{1} + 2\mathrm{i}\boldsymbol{\phi}_n \right], \qquad \boldsymbol{\phi}_n = \frac{\phi_{nj}\sigma_j}{2}$$

$$= \underbrace{U(\zeta)}_{\text{Goldstone matrix as in SM}} \mathbf{\Phi}_n^{(\mathrm{u})}, \qquad \mathbf{\Phi}_n^{(\mathrm{u})} = \frac{1}{\sqrt{2}} \underbrace{\left[(v_n + h_n)\mathbb{1} + \mathrm{i} c_n \sigma_j \rho_j \right]}_{h_1,\, h_2,\, \rho_1,\, \rho_2,\, \rho_3 \text{ mix to fields of physical Higgs bosons}}$$

$$\left| \begin{array}{l} c_1 = -v_2/v \equiv -\sin\beta = -s_\beta \\ c_2 = v_1/v \equiv \cos\beta = c_\beta \end{array} \right.$$

Fields corresponding to mass / charge eigenstates:

- CP-even neutral Higgs bosons h, H: $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} H \\ h \end{pmatrix}$ = gauge invariant
- CP-odd neutral Higgs boson A_0: $\quad \rho_3$ = gauge invariant
- Charged Higgs bosons $\mathrm{H}^\pm$: $\quad \rho^\pm = \frac{1}{\sqrt{2}}(\rho_2 \pm \mathrm{i}\rho_1)$ = SU(2) gauge invariant

Lagrangian: restriction to CP-conserving, $\mathbb{Z}_2$-symmetric case!

$$\mathcal{L}_\mathrm{Higgs} = \frac{1}{2} tr\left[(D_\mu \mathbf{\Phi}_1)^\dagger (D^\mu \mathbf{\Phi}_1) \right] + \frac{1}{2} tr\left[(D_\mu \mathbf{\Phi}_2)^\dagger (D^\mu \mathbf{\Phi}_2) \right] - V$$

$$V = \frac{m_{11}^2}{2} \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1\right] + \frac{m_{22}^2}{2} \mathrm{tr}\left[\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2\right] - m_{12}^2 \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2\right] + \frac{\lambda_1}{8} \left(\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1\right]\right)^2 + \frac{\lambda_2}{8} \left(\mathrm{tr}\left[\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2\right]\right)^2$$
$$+ \frac{\lambda_3}{4} \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1\right] \mathrm{tr}\left[\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2\right] + \lambda_4 \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 \Omega_+\right] \mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 \Omega_-\right]$$
$$+ \frac{\lambda_5}{2} \left[\left(\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 \Omega_+\right]\right)^2 + \left(\mathrm{tr}\left[\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 \Omega_-\right]\right)^2 \right], \qquad \Omega_\pm = \frac{1}{2}(1 \pm \sigma_3)$$

Non-polynomial Higgs kinetic Lagrangian more explicitly:

$$
\begin{aligned}
\mathcal{L}_{H,\,kin} = {} & \tfrac{1}{2}(\partial h_n)^2 + \tfrac{1}{2}(\partial\vec{\rho}\,)^2 + \frac{1}{2v^2}\left[(v_n+h_n)^2+\vec{\rho}^{\,2}\right]\Big\{(\partial_\mu\vec{\zeta}\,)\cdot(\partial^\mu\vec{\zeta}\,) + \frac{g_2^2v^2}{4}\,\vec{C}_\mu\cdot\vec{C}^\mu \\
& \qquad + g_1g_2B_\mu\left[-W_3^\mu\zeta^2+(\vec{W}^\mu\cdot\vec{\zeta}\,)\zeta_3\right] - g_2^2v\,\vec{C}_\mu\cdot(\vec{W}^\mu\times\vec{\zeta}\,) \\
& \qquad - g_2v\,\vec{C}_\mu\cdot\partial^\mu\vec{\zeta} - g_2(\vec{C}_\mu-2\vec{W}_\mu)\cdot(\vec{\zeta}\times\partial^\mu\vec{\zeta}\,)\Big\} \\
& + \frac{g_2}{2}\Big\{\vec{C}_\mu - \frac{2}{g_2v}\,\partial_\mu\vec{\zeta} - \frac{2}{v}\,\vec{W}_\mu\times\vec{\zeta} - \frac{2}{g_2v^2}\,(\vec{\zeta}\times\partial_\mu\vec{\zeta}\,) - \frac{2}{v^2}\left[\zeta^2\,\vec{W}_\mu - (\vec{W}_\mu\cdot\vec{\zeta}\,)\,\vec{\zeta}\,\right]\Big\} \\
& \qquad \cdot\left[c_n(\partial^\mu h_n)\vec{\rho} - c_nh_n(\partial^\mu\vec{\rho}\,) + (\vec{\rho}\times\partial^\mu\vec{\rho}\,) + g_1B^\mu\rho_3\vec{\rho}\,\right] \\
& + \frac{g_1^2}{2}\,B^2\left(\rho_1^2+\rho_2^2\right) + g_1B^\mu\left[(\partial_\mu\rho_1)\rho_2-(\partial_\mu\rho_2)\rho_1\right] \\
& + \frac{g_1g_2}{2}\,B_\mu\Big\{c_nh_n\Big[\left(\rho_1C_2^\mu-\rho_2C_1^\mu\right) - \frac{2}{g_2v}\,\left(\rho_1\partial^\mu\zeta_2-\rho_2\partial^\mu\zeta_1\right) \\
& \qquad - \frac{2}{v}\left[(\vec{\rho}\cdot\vec{\zeta}\,)W_3^\mu-(\vec{\rho}\cdot\vec{W}^\mu)\zeta_3\right] - \frac{2}{g_2v^2}\left[(\vec{\rho}\cdot\partial^\mu\vec{\zeta}\,)\zeta_3-(\vec{\rho}\cdot\vec{\zeta}\,)\partial^\mu\zeta_3\right] \\
& \qquad + \frac{2}{v^2}\left[(\vec{W}^\mu\cdot\vec{\zeta}\,)\left(\rho_1\zeta_2-\rho_2\zeta_1\right) - \zeta^2\left(\rho_1W_2^\mu-\rho_2W_1^\mu\right)\right. \\
& \qquad - \vec{\rho}^{\,2}\Big[C_3^\mu - \frac{2}{g_2v}\,\partial^\mu\zeta_3 - \frac{2}{v}\,\left(W_1^\mu\zeta_2-W_2^\mu\zeta_1\right) - \frac{2}{g_2v^2}\,\left(\zeta_1\partial^\mu\zeta_2-\zeta_2\partial^\mu\zeta_1\right) \\
& \qquad - \frac{2}{v^2}\left[\zeta^2\,W_3^\mu - (\vec{W}^\mu\cdot\vec{\zeta}\,)\zeta_3\right]\Big]\Big\} + \mathcal{O}(\zeta^3)
\end{aligned}
$$

... somewhat messy, but straightforward

Higgs potential:

V = function of $h_1, h_2, \rho_3, \rho^{\pm}$, but independent of Goldstone fields ζ_j

Tadpoles in the linear and non-linear representations:

$$\Gamma^{h_n}_{\mathrm{nl}} = \text{gauge independent}$$

$$\Gamma^{\eta_1} = \Gamma^{h_1}_{\mathrm{nl}} + c_\beta \left[c_\beta^2 \lambda_1 + s_\beta^2 \left(\lambda_3 + \lambda_4 + \lambda_5 \right) \right] v^2 \Delta v_\xi$$

$$\Gamma^{\eta_2} = \Gamma^{h_2}_{\mathrm{nl}} + s_\beta \left[s_\beta^2 \lambda_2 + c_\beta^2 \left(\lambda_3 + \lambda_4 + \lambda_5 \right) \right] v^2 \underbrace{\Delta v_\xi}_{\text{as in SM}}$$

Transformation of input parameters:

original set: $\{\lambda_1, \dots, \lambda_5, m_{11}^2, m_{22}^2, m_{12}^2, g_1, g_2\}$

$\downarrow$

mass basis: $\{ \underbrace{M_\mathrm{H}, M_\mathrm{h}, M_{\mathrm{A}_0}, M_{\mathrm{H}^+}, M_\mathrm{W}, M_\mathrm{Z}, e}_{\text{renormalized on-shell}}, \underbrace{\lambda_5}_{\overline{\mathrm{MS}}}, \alpha, \beta \}$

Renormalization:

Santos/Barroso '97; Kanemura et al. '04; Lopez-Val/Sola '09; Degrande '14; Krause et al. '16,'19; Denner et al. '16,'18; Altenkamp '17

Gauge-invariant vev renormalization in extended Higgs sectors

On the role of Higgs mixing angles α, ...

- generic feature of extended Higgs sectors
- typically rescale SM couplings by factors $\cos\alpha$, $\sin\alpha$, ...
  $\hookrightarrow$ phenomenologically well accessible
  $\hookrightarrow$ appropriate input quantities requiring renormalization

Desirable properties for the renormalized mixing angles:

Freitas/Stöckinger '02; Denner/SD/Lang '18

- gauge independence
  $\hookrightarrow$ S-matrix = gauge-independent function of input parameters
- symmetry wrt. mixing degrees of freedom
- process independence
- perturbative stability
  $\hookrightarrow$ higher-order corrections should not get artificially large
- smoothness for degenerate masses or extreme mixing angles
  $\hookrightarrow$ no singularities like $1/(M^2_{\mathrm{H}_1} - M^2_{\mathrm{H}_2})$ or $1/\sin\alpha$, $1/\cos\alpha$, etc.

\+ addendum: decoupling behaviour in large-mass limits
should be respected in transition LO $\to$ NLO $\to$ ...
see, e.g., SD/Schuhmacher/Stahlhofen '21

Types of renormalization schemes for mixing angles

1. $\overline{\mathrm{MS}}$ scheme

   + process independence, simplicity
   + scale dependence as diagnostic tool to check perturbative stability
   - dependence on tadpole scheme:

     FJTS: gauge independence, potentially large corrections
     PRTS: gauge dependence, corrections better behaved
     NEW → GIVS: gauge independence, corrections similar to PRTS

   − prone to problems for extreme parameter scenarios (especially FJTS)

2. Momentum-subtraction schemes

   - procedures based on $\Sigma^{ij}(p^2)$ at some momentum transfer p^2
   + process independence
   − potential gauge dependence (often removed ad hoc)
   − physical meaning and generalizability of ad hoc procedures unclear

3. Process-specific on-shell (OS) conditions: e.g. $\Gamma^{\mathrm{h}\to XY} \stackrel{!}{=} \Gamma^{\mathrm{h}\to XY}_{\mathrm{LO}}$

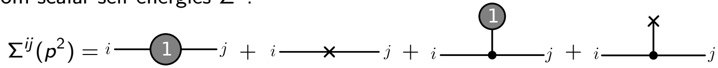
   + gauge independence
   − process dependence
   − "contamination" of $\delta\alpha$ by all types of different corrections
     ↪ perturbative instabilities & "dead corners" in parameter space

Types of renormalization schemes for mixing angles (continued)

4. OS conditions on amplitude/formfactor ratios: e.g. $\frac{\mathcal{M}^{\mathrm{h}\to XY}}{\mathcal{M}^{\mathrm{H}\to XY}} \stackrel{!}{=} \frac{\mathcal{M}_{\mathrm{LO}}^{\mathrm{h}\to XY}}{\mathcal{M}_{\mathrm{LO}}^{\mathrm{H}\to XY}}$
   Denner/SD/Lang '18
   - \+ no dependence on gauge or tadpole scheme
   - • process independence in specific cases (e.g. SESM)
   - \+ great perturbative stability, no "dead corners" in parameter space
5. Symmetry-inspired schemes Kanemura et al. '03; Krause et al. '16; Denner/SD/Lang '18
   - • exploits symmetry relations of UV divergences ("rigid" invariance, background-field gauge invariance)
   - \+ process independence
   - − gauge dependence
   - \+ "dead corners" in parameter space avoidable

$\overline{\mathrm{MS}}$ renormalization of mixing angle α (SESM and THDM)

$\delta\alpha$ from scalar self-energies Σ^{ij}:



($\times$ = tadpole counterterms)

$$\delta\alpha_{\overline{\mathrm{MS}}} = \frac{\Sigma^{Hh}(M_\mathrm{H}^2) + \Sigma^{Hh}(M_\mathrm{h}^2)}{2(M_\mathrm{H}^2 - M_\mathrm{h}^2)}\bigg|_{\mathrm{UV}}$$

(R_ξ gauge used in the following)

$$\delta\alpha_{\overline{\mathrm{MS}},\mathrm{tad}} = \frac{\Sigma^{Hh}_{\mathrm{tad}}}{M_\mathrm{H}^2 - M_\mathrm{h}^2}\bigg|_{\mathrm{UV}} = \begin{cases} \dfrac{e\,(C_{hhH}\,\Delta v_h^{\mathrm{FJTS}} + C_{hHH}\,\Delta v_H^{\mathrm{FJTS}})}{M_\mathrm{H}^2 - M_\mathrm{h}^2}\bigg|_{\mathrm{UV}} & \text{for FJTS} \\ 0 & \text{for PRTS} \\ \dfrac{e\,C^{\mathrm{GIVS}}\,\Delta v_\xi}{M_\mathrm{H}^2 - M_\mathrm{h}^2}\bigg|_{\mathrm{UV}} & \text{for GIVS} \end{cases}$$

with $C_{hhH/hHH}$ = coupling factors of hhH/hHH vertices and

$$C^{\mathrm{GIVS}} = \begin{cases} c_\alpha C_{hhH} + s_\alpha C_{hHH} & \text{for SESM} \\ s_{\beta-\alpha} C_{hhH} + c_{\beta-\alpha} C_{hHH} & \text{for THDM} \end{cases}$$

NLO relations between renormalized angles (from relations of bare parameters)

$$\alpha^{\mathrm{PRTS}}_{\overline{\mathrm{MS}}} = \alpha^{\mathrm{FJTS}}_{\overline{\mathrm{MS}}} - \frac{e}{M_\mathrm{H}^2 - M_\mathrm{h}^2}\left(C_{hhH}\,\frac{\Gamma^h}{M_\mathrm{h}^2} + C_{hHH}\,\frac{\Gamma^H}{M_\mathrm{H}^2}\right)\bigg|_{\mathrm{finite}}$$

$$\alpha^{\mathrm{GIVS}}_{\overline{\mathrm{MS}}} = \alpha^{\mathrm{FJTS}}_{\overline{\mathrm{MS}}} - \frac{e}{M_\mathrm{H}^2 - M_\mathrm{h}^2}\left(C_{hhH}\,\frac{\Gamma^h_{\mathrm{nl}}}{M_\mathrm{h}^2} + C_{hHH}\,\frac{\Gamma^H_{\mathrm{nl}}}{M_\mathrm{H}^2}\right)\bigg|_{\mathrm{finite}} = \text{gauge independent}$$

Change of tadpole scheme $\rightarrow$ finite shift in α

$\overline{\mathrm{MS}}$ renormalization of THDM mixing angle β

$\delta\beta$ from scalar self-energy $\Sigma^{A_0 G_0}$ or $\Sigma^{A_0 Z}$:

$$\delta\beta_{\overline{\mathrm{MS}}} = \left.\frac{\Sigma^{A_0 G_0}(M_\mathrm{A}^2) + \Sigma^{A_0 G_0}(0)}{2M_\mathrm{A}^2}\right|_\mathrm{UV} \qquad (R_\xi \text{ gauge used in the following})$$

$$\delta\beta_{\overline{\mathrm{MS}},\mathrm{tad}} = \left.\frac{\Sigma_\mathrm{tad}^{A_0 G_0}}{M_\mathrm{A}^2}\right|_\mathrm{UV} = \begin{cases} \left.\frac{c_{\beta-\alpha}\Delta v_h^\mathrm{FJTS}(M_\mathrm{A}^2 - M_\mathrm{h}^2) - s_{\beta-\alpha}\Delta v_H^\mathrm{FJTS}(M_\mathrm{A}^2 - M_\mathrm{H}^2)}{vM_\mathrm{A}^2}\right|_\mathrm{UV} & \text{for FJTS} \\ \left.\frac{-c_{\beta-\alpha}\Gamma^h + s_{\beta-\alpha}\Gamma^H}{vM_\mathrm{A}^2}\right|_\mathrm{UV} & \text{for PRTS \& GIVS} \end{cases}$$

NLO relations between renormalized angles: (from relations of bare parameters)

$$\beta_{\overline{\mathrm{MS}}}^\mathrm{GIVS} = \beta_{\overline{\mathrm{MS}}}^\mathrm{FJTS} + \frac{1}{v}\left.\left(c_{\beta-\alpha}\frac{\Gamma_\mathrm{nl}^h}{M_\mathrm{h}^2} - s_{\beta-\alpha}\frac{\Gamma_\mathrm{nl}^H}{M_\mathrm{H}^2}\right)\right|_\mathrm{finite} = \text{gauge independent}$$

$$\beta_{\overline{\mathrm{MS}}}^\mathrm{PRTS} = \beta_{\overline{\mathrm{MS}}}^\mathrm{GIVS} \quad \text{in } R_\xi \text{ gauges}$$

Change of tadpole scheme $\rightarrow$ finite shift in β

Comment on gauge-dependence issue:

- $\beta_{\overline{\mathrm{MS}}}^\mathrm{PRTS}$ gauge dependent outside class of R_ξ gauges
- $\beta_{\overline{\mathrm{MS}}}^\mathrm{GIVS}$ gauge independent (not only in R_ξ gauges)
- $\beta_{\overline{\mathrm{MS}}}^\mathrm{PRTS} \underset{R_\xi}{=} \beta_{\overline{\mathrm{MS}}}^\mathrm{GIVS}$ $\Rightarrow$ PRTS results of R_ξ gauges can be reinterpreted as (gauge-independent) results of GIVS

Conjecture: feature should generalize beyond NLO and to SUSY models
$\hookrightarrow$ chance to put results based on $\beta_{\overline{\mathrm{MS}}}^\mathrm{PRTS}$ on gauge-independent basis

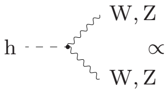
NLO corrections
to h/H$\to$WW/ZZ$\to$4fermions

Light versus heavy Higgs decays $\mathrm{h/H} \to \mathrm{WW/ZZ} \to 4f$

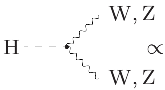
$\hookrightarrow$ SESM and THDM in Monte Carlo program PROPHECY4F

- different resonance patterns:
  - $M_\mathrm{h} = 125\,\mathrm{GeV}$: at least one W/Z off-shell
    $\mathrm{h} \to \mathrm{WW}^*/\mathrm{ZZ}^* \to 4f, \quad \Gamma^{\mathrm{h}\to 4f} \sim 1\,\mathrm{MeV}$
  - $M_\mathrm{H} > 2M_\mathrm{Z}$: on-shell decays $\mathrm{H} \to \mathrm{WW/ZZ}$ possible
    $\mathrm{H} \to \mathrm{WW/ZZ} \to 4f, \quad \Gamma^{\mathrm{H}\to 4f} \sim 100\,\mathrm{MeV}$
- LO prediction suppressed by small mixing factors:

  SESM: $\gamma = \alpha$ THDM: $\gamma = \frac{\pi}{2} + \alpha - \beta$



$\propto \cos\gamma \gtrsim 0.9(0.95)$



$\propto \sin\gamma \lesssim 0.4(0.3)$

LHC result: $\mu = \left.\frac{\Gamma_\mathrm{exp}}{\Gamma_\mathrm{SM}}\right|_{\mathrm{Higgs}\to\mathrm{WW/ZZ}} = 1 \pm 20\%(10\%) \sim \cos^2\gamma$

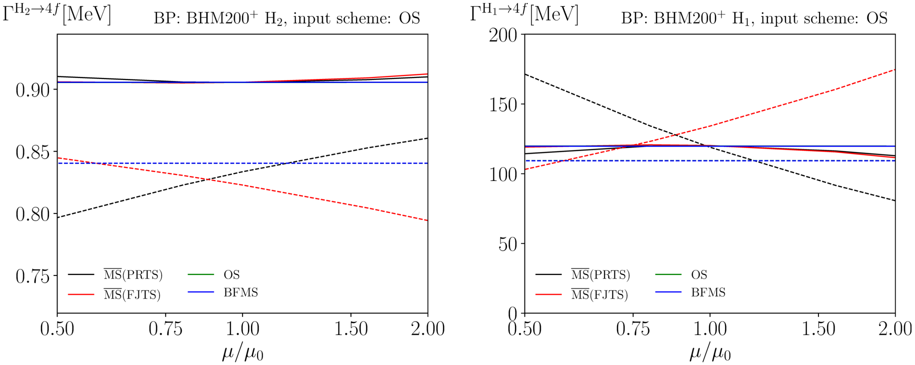
$\Rightarrow$ Potentially large corrections to $\mathrm{H} \to \mathrm{WW/ZZ}$

Perturbatively stable renormalization schemes particularly important!

SESM: $\mathrm{h}, \mathrm{H} \to 4f$ at NLO

Altenkamp et al. '17, Denner et al. '18

$\mathrm{h} \to 4f$: $(M_\mathrm{H} = 200\,\mathrm{GeV})$ $\mathrm{H} \to 4f$: $(M_\mathrm{H} = 200\,\mathrm{GeV})$



Transition from LO to NLO:

- $\overline{\mathrm{MS}}$ schemes: drastic reduction of ren. scale dependence
- comparison of schemes: drastic reduction of ren. scheme dependence, i.e. good agreement of all scheme after conversion of input
- overall uncertainty of NLO prediction $\lesssim 0.5\%$ as in SM

New: $\overline{\mathrm{MS}}$ renormalization of α with GIVS SD/Rzehak '22

$\mathrm{h} \to 4f$ in the SESM:

Ren. scheme	tadpoles	BHM400 ($M_\mathrm{H} = 400\,\mathrm{GeV}$) LO	BHM400 NLO	BHM600 ($M_\mathrm{H} = 600\,\mathrm{GeV}$) LO	BHM600 NLO
OS		0.85548(3)	$0.92178(6)^{+0.0\%}_{-0.0\%}$	0.87309(3)	$0.94078(7)^{+0.0\%}_{-0.0\%}$
$\overline{\mathrm{MS}}$	FJTS	$0.85349(3)^{-2.1\%}_{+1.6\%}$	$0.92166(7)^{+0.1\%}_{+0.3\%}$	$0.87608(3)^{-1.5\%}_{+1.2\%}$	$0.94106(7)^{-0.0\%}_{+0.3\%}$
$\overline{\mathrm{MS}}$	PRTS	$0.85209(3)^{+0.5\%}_{-0.5\%}$	$0.92159(7)^{+0.0\%}_{-0.0\%}$	$0.87067(3)^{+0.1\%}_{-0.1\%}$	$0.94060(7)^{+0.0\%}_{-0.0\%}$
$\overline{\mathrm{MS}}$	GIVS	$0.85239(3)^{+0.5\%}_{-0.5\%}$	$0.92160(7)^{+0.0\%}_{-0.0\%}$	$0.87087(3)^{+0.1\%}_{-0.1\%}$	$0.94061(7)^{+0.0\%}_{-0.0\%}$

$\mathrm{H} \to 4f$ in the SESM:

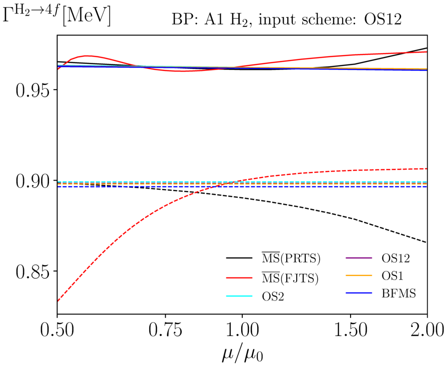
Ren. scheme	tadpoles	BHM400 ($M_\mathrm{H} = 400\,\mathrm{GeV}$) LO	BHM400 NLO	BHM600 ($M_\mathrm{H} = 600\,\mathrm{GeV}$) LO	BHM600 NLO
OS		1533.42(4)	$1643.86(8)^{-0.0\%}_{+0.0\%}$	4295.9(1)	$4532.4(2)^{-0.0\%}_{+0.0\%}$
$\overline{\mathrm{MS}}$	FJTS	$1582.44(4)^{+27.6\%}_{-21.7\%}$	$1646.83(8)^{-1.5\%}_{-3.6\%}$	$4007.1(1)^{+32.5\%}_{-24.8\%}$	$4509.4(3)^{-0.3\%}_{-6.0\%}$
$\overline{\mathrm{MS}}$	PRTS	$1617.26(4)^{-6.2\%}_{+6.3\%}$	$1648.62(8)^{-0.6\%}_{+0.6\%}$	$4530.1(1)^{-2.3\%}_{+2.1\%}$	$4546.0(2)^{-0.4\%}_{+0.6\%}$
$\overline{\mathrm{MS}}$	GIVS	$1609.86(4)^{-6.6\%}_{+6.7\%}$	$1648.26(8)^{-0.7\%}_{+0.6\%}$	$4511.4(1)^{-2.6\%}_{+2.4\%}$	$4545.1(2)^{-0.4\%}_{+0.6\%}$

- GIVS and PRTS $\overline{\mathrm{MS}}$ results almost identical
- all $\overline{\mathrm{MS}}$ results perturbatively stable
- some degradation in precision for $\mathrm{H} \to 4f$ in FJTS $\overline{\mathrm{MS}}$ for large M_H

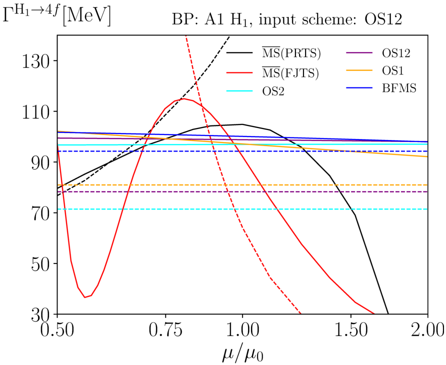
THDM: $\mathrm{h}/\mathrm{H} \to 4f$ at NLO

Altenkamp et al. '17, Denner et al. '18

$\mathrm{h} \to 4f$:



$\mathrm{H} \to 4f$:



Transition from LO to NLO:

- $\overline{\mathrm{MS}}$ schemes: useful results for $\mathrm{h} \to 4f$ in "moderate scenarios"
  But: perturbative instability in extreme scenarios and for $\mathrm{H} \to 4f$
- OS & BFM schemes: very good agreement after conversion of input
- NLO uncertainty estimate should include ren. scheme dependence (including well-behaved schemes)

New: $\overline{\mathrm{MS}}$ renormalization of α with GIVS SD/Rzehak '22

$\mathrm{h} \to 4f$:

Ren. scheme	tadpoles	A1 LO	A1 NLO	A2 LO	A2 NLO
OS12 (α, β)		0.89832(3)	$0.96194(7)^{-0.1\%}_{+0.1\%}$	0.87110(3)	$0.92947(7)^{-0.2\%}_{+0.1\%}$
$\overline{\mathrm{MS}}(\alpha, \beta)$	FJTS	$0.89996(3)^{+0.7\%}_{-7.4\%}$	$0.96283(7)^{+0.8\%}_{-0.2\%}$	$0.88508(3)^{+2.2\%}_{-10.0\%}$	$0.93604(7)^{+3.1\%}_{-11.0\%}$
$\overline{\mathrm{MS}}(\alpha, \beta)$	PRTS	$0.89035(3)^{-2.8\%}_{+0.9\%}$	$0.96103(7)^{+1.2\%}_{+0.4\%}$	$0.86130(3)^{-6.1\%}_{+2.3\%}$	$0.92784(7)^{+1.3\%}_{+1.3\%}$
$\overline{\mathrm{MS}}(\alpha, \beta)$	GIVS	$0.89082(3)^{-2.7\%}_{+0.9\%}$	$0.96106(7)^{+1.2\%}_{+0.5\%}$	$0.86249(3)^{-5.8\%}_{+2.3\%}$	$0.92808(7)^{+1.3\%}_{+1.3\%}$
$\overline{\mathrm{MS}}(\lambda_3, \beta)$	FJTS	$0.89246(3)^{-15.1\%}_{+1.6\%}$	$0.96108(7)^{+17.3\%}_{+1.9\%}$	$0.85590(3)^{-29.8\%}_{+5.5\%}$	$0.92723(7)^{+18.3\%}_{+2.8\%}$
$\overline{\mathrm{MS}}(\lambda_3, \beta)$	PRTS/GIVS	$0.89156(3)^{-8.4\%}_{+1.7\%}$	$0.96111(7)^{+3.8\%}_{+2.1\%}$	$0.85841(3)^{-12.7\%}_{+5.0\%}$	$0.92729(7)^{+4.6\%}_{+2.6\%}$

- GIVS and PRTS $\overline{\mathrm{MS}}(\alpha, \beta)$ results almost identical (conicide for λ_3, β input)
- GIVS and PRTS $\overline{\mathrm{MS}}$ results perturbatively trustworthy
- FJTS $\overline{\mathrm{MS}}$ results results perturbatively unstable

$\mathrm{H} \to 4f$: Denner, S.D., Lang '18

$\overline{\mathrm{MS}}$ predictions generally not trustworthy

But: OS or symmetry-inspired schemes perturbatively very well behaved

Conclusions

Renormalization of vev parameters v and tadpoles

- important ingredient in EW renormalization
- impact on predictions in $\overline{\mathrm{MS}}$ schemes (but not in OS schemes)
- most common tadpole schemes:
  - PRTS: correction to v absorbed in parameter relations
    $\hookrightarrow$ typically moderate corrections, but gauge dependence
  - FJTS: field expansion about bare vev v_0
    $\hookrightarrow$ potentially large corrections, but gauge independence
- new proposal: GIVS = Gauge-Invariant Vacuum expectation value Scheme
  - fields expanded about corrected vev v
    $\hookrightarrow$ good perturbative stability as PRTS
  - gauge independence

Applications:

- SM: drastic reduction of EW corrections in conversion $M^{\mathrm{OS}} \to M^{\overline{\mathrm{MS}}}$

  e.g.: $$\Delta m_{\mathrm{t}}^{\overline{\mathrm{MS}}-\mathrm{OS}}|_{\mathrm{EW}} = 11\,\mathrm{GeV}(\mathrm{FJTS}) \to 0.5\,\mathrm{GeV}(\mathrm{GIVS})$$
  $$\Delta m_{\mathrm{b}}^{\overline{\mathrm{MS}}-\mathrm{OS}}|_{\mathrm{EW}} = -1.8\,\mathrm{GeV}(\mathrm{FJTS}) \to 0.1\,\mathrm{GeV}(\mathrm{GIVS})$$
- BSM: $\overline{\mathrm{MS}}$ Higgs mixing angles in singlet Higgs extension of the SM and THDM
  - NLO results of GIVS and PRTS very similar and more stable than for FJTS
  - THDM: $\beta_{\overline{\mathrm{MS}}}^{\mathrm{PRTS}} = \beta_{\overline{\mathrm{MS}}}^{\mathrm{GIVS}}$ in R_ξ gauges
    $\hookrightarrow$ gauge-independent reinterpretation of results with $\beta_{\overline{\mathrm{MS}}}^{\mathrm{PRTS}}$

Backup slides

Renormalization schemes for the SESM and THDM

OS schemes

OS renormalization schemes for the SESM and THDM Denner, S.D., Lang '18

Idea: Renormalization condition on ratio of S-matrix elements

$$\frac{\mathcal{M}^{H_1\to XY}}{\mathcal{M}^{H_2\to XY}} \stackrel{!}{=} \frac{\mathcal{M}_0^{H_1\to XY}}{\mathcal{M}_0^{H_2\to XY}} = \rho(\alpha) = \text{function of } \alpha \text{ only}$$

- $X, Y \stackrel{!}{=}$ neutral, otherwise problem with IR divergences
- vertex corrections $\delta^{H_i XY}$ to $\delta\alpha$ might be avoidable for clever choice of X, Y

SESM: extension by fermionic singlet ψ with Yukawa coupling $y_\psi \to 0$

$$\mathcal{L}_\psi = \mathrm{i}\bar{\psi}\partial\!\!\!/\psi - y_\psi \bar{\psi}\psi \underbrace{(v_1 + H_1 c_\alpha - H_2 s_\alpha)}_{=\sigma} = \text{ gauge invariant}$$

- ratio $\rho(\alpha) = -c_\alpha/s_\alpha$ for $XY = \bar{\psi}\psi$
- vertex corrections $\delta^{H_i\bar{\psi}\psi} \to 0$ for $y_\psi \to 0$
- $\delta\alpha = \frac{1}{2}(\delta Z^H_{11} - \delta Z^H_{22})c_\alpha s_\alpha + \frac{1}{2}(\delta Z^H_{12} c_\alpha^2 - \delta Z^H_{21} s_\alpha^2)$

  $=$ gauge independent, symmetric in $\mathrm{H}_1/\mathrm{H}_2$, perturbatively stable

THDM: refinements required (no gauge-inv. coupling to fermion singlet)
$\hookrightarrow$ use right-handed neutrinos, $\delta\beta$, vertex corrections contribute, ...

OS renormalization schemes for the THDM Denner, S.D., Lang '18

Problem:

no Higgs singlet $\rightarrow$ no gauge-invariant $h\bar{\psi}\psi$ operator with singlet ψ

Solution:

Add two right-handed singlet neutrinos $\nu_{1/2,\mathrm{R}}$ of THDM types 1/2

$$\mathcal{L}_{\nu_\mathrm{R}} = \mathrm{i}\bar{\nu}_{1\mathrm{R}}\partial\!\!\!/\nu_{1\mathrm{R}} + \mathrm{i}\bar{\nu}_{2\mathrm{R}}\partial\!\!\!/\nu_{2\mathrm{R}} - \left[y_{\nu_1}\bar{L}_{1\mathrm{L}}\tilde{\Phi}_1\nu_{1\mathrm{R}} + y_{\nu_2}\bar{L}_{2\mathrm{L}}\tilde{\Phi}_2\nu_{2\mathrm{R}} + \mathrm{h.c.}\right]$$

Renormalization conditions for α, β from appropriate ratios

$$\frac{\mathcal{M}^{H_1\to\nu_1\bar{\nu}_1}}{\mathcal{M}^{H_2\to\nu_1\bar{\nu}_1}} \stackrel{!}{=} \frac{\mathcal{M}_0^{H_1\to\nu_1\bar{\nu}_1}}{\mathcal{M}_0^{H_2\to\nu_1\bar{\nu}_1}} = -\frac{c_\alpha}{s_\alpha}, \qquad \frac{\mathcal{M}^{A_0\to\nu_1\bar{\nu}_1}}{\mathcal{M}^{H_1\to\nu_1\bar{\nu}_1}} \stackrel{!}{=} \frac{\mathcal{M}_0^{A_0\to\nu_1\bar{\nu}_1}}{\mathcal{M}_0^{H_1\to\nu_1\bar{\nu}_1}} \propto \frac{s_\beta}{c_\alpha}$$

deliver gauge-independent renormalization constants: ("OS1" scheme)

$$\delta\alpha = (\delta_{H_1\nu_1\bar{\nu}_1} - \delta_{H_2\nu_1\bar{\nu}_1})c_\alpha s_\alpha + \tfrac{1}{2}(\delta Z^H_{11} - \delta Z^H_{22})c_\alpha s_\alpha + \tfrac{1}{2}(\delta Z^H_{12}c^2_\alpha - \delta Z^H_{21}s^2_\alpha),$$

$$\delta\beta = \frac{s_\beta}{c_\beta}\left[\delta_{H_1\nu_1\bar{\nu}_1} - \delta_{A_0\nu_1\bar{\nu}_1} - \tfrac{1}{2}(\delta Z_{A_0A_0} - \delta Z^H_{11}) - \tfrac{1}{2}\frac{s_\alpha}{c_\alpha}\delta Z^H_{21}\right] + \tfrac{1}{2}\delta Z_{G_0A_0} - \frac{s_\beta}{c_\beta}\frac{s_\alpha}{c_\alpha}\delta\alpha$$

Comments:

- vertex corrections $\delta_{S\nu_i\bar{\nu}_i}$ unavoidable in spite of $y_{\nu_i} \to 0$
- but: singular factors $1/c_\alpha, 1/s_\beta$ in $\delta\beta$ can be avoided

"OS12" renormalization schemes for the THDM Denner, S.D., Lang '18

Modification:

replace matrix elements $\mathcal{M}^{S\to\nu_j\bar{\nu}_j}$ by appropriate formfactors $F^{S\to\nu_j\bar{\nu}_j}$

$$\mathcal{M}^{H_i\to\nu_j\bar{\nu}_j} = [\bar{u}_\nu v_\nu]_{H_i} F^{H_i\to\nu_j\bar{\nu}_j}, \quad \mathcal{M}^{A_0\to\nu_j\bar{\nu}_j} = [\bar{u}_\nu \mathrm{i}\gamma_5 v_\nu]_{A_0} F^{A_0\to\nu_j\bar{\nu}_j}$$

Renormalization condition for β alone:

$$0 \stackrel{!}{=} \frac{F^{A_0\to\nu_1\bar{\nu}_1} c_\beta}{c_\alpha F^{H_1\to\nu_1\bar{\nu}_1} - s_\alpha F^{H_2\to\nu_1\bar{\nu}_1}} + \frac{F^{A_0\to\nu_2\bar{\nu}_2} s_\beta}{s_\alpha F^{H_1\to\nu_2\bar{\nu}_2} + c_\alpha F^{H_2\to\nu_2\bar{\nu}_2}}$$

Renormalization constant $\delta\beta$:

$$\delta\beta = \tfrac{1}{2} c_\beta s_\beta \left[(c_\alpha^2 - s_\alpha^2)(\delta Z_{11}^H - \delta Z_{22}^H) - 2c_\alpha s_\alpha(\delta Z_{12}^H + \delta Z_{21}^H)\right] + \tfrac{1}{2}\delta Z_{G_0 A_0}$$
$$+ c_\beta s_\beta \left(\delta_{A_0\nu_2\bar{\nu}_2} + c_\alpha^2 \delta_{H_1\nu_1\bar{\nu}_1} + s_\alpha^2 \delta_{H_2\nu_1\bar{\nu}_1} - \delta_{A_0\nu_1\bar{\nu}_1} - s_\alpha^2 \delta_{H_1\nu_2\bar{\nu}_2} - c_\alpha^2 \delta_{H_2\nu_2\bar{\nu}_2}\right)$$

Comments:

- $\delta\alpha$ fixed similar as above
- both $\delta\alpha, \delta\beta$ gauge invariant and perturbatively stable without "dead corners"

Renormalization schemes
for the SESM and THDM

symmetry-inspired schemes

Mixing of physical states and "rigid invariance"

Idea: UV divergences can be removed via renormalization in unbroken phase of theory 't Hooft '71; Lee, Zinn-Justin '72–'74

$\hookrightarrow$ field renormalization matrix $(Z^H)^{1/2}$ can be taken diagonal in "η basis":

$$(Z^H)^{1/2}\big|_{\mathrm{UV}} = R^{\mathrm{T}}(\alpha+\delta\alpha)\begin{pmatrix} 1+\frac{1}{2}\delta Z_1^\eta & 0 \\ 0 & 1+\frac{1}{2}\delta Z_2^\eta \end{pmatrix} R(\alpha)\Big|_{\mathrm{UV}}$$

$\Rightarrow$ Relations among UV divergences in δZ_{ij}^H and $\delta\alpha$:

$$\begin{aligned} \delta Z_{11}^H\big|_{\mathrm{UV}} &= c_\alpha^2 \delta Z_1^\eta\big|_{\mathrm{UV}} + s_\alpha^2 \delta Z_2^\eta\big|_{\mathrm{UV}}, \\ \delta Z_{22}^H\big|_{\mathrm{UV}} &= s_\alpha^2 \delta Z_1^\eta\big|_{\mathrm{UV}} + c_\alpha^2 \delta Z_2^\eta\big|_{\mathrm{UV}}, \\ \delta Z_{12}^H\big|_{\mathrm{UV}} + \delta Z_{21}^H\big|_{\mathrm{UV}} &= 2c_\alpha s_\alpha(\delta Z_2^\eta - \delta Z_1^\eta)\big|_{\mathrm{UV}}, \\ \delta Z_{12}^H\big|_{\mathrm{UV}} - \delta Z_{21}^H\big|_{\mathrm{UV}} &= 4\delta\alpha\big|_{\mathrm{UV}} \end{aligned}$$

Kanemura et al. '03
Krause et al. '16
Denner et al. '18

$\Rightarrow$ $\delta\alpha$ can be defined via symmetry relation

$$\delta\alpha = \frac{1}{4}\left(\delta Z_{12}^H - \delta Z_{21}^H\right) = \frac{\Sigma_{12}^H(M_{\mathrm{H}_2}^2) + \Sigma_{12}^H(M_{\mathrm{H}_1}^2)}{2(M_{\mathrm{H}_1}^2 - M_{\mathrm{H}_2}^2)}$$

Note: $\frac{1}{2}\delta Z_{12}^H - \delta\alpha = \delta\alpha + \frac{1}{2}\delta Z_{21}^H = \frac{1}{4}\left(\delta Z_{12}^H + \delta Z_{21}^H\right) =$ regular for $M_{\mathrm{H}_1} \to M_{\mathrm{H}_2}$

Mixing of physical and unphysical states and background-field invariance

Problem: Gauge-fixing terms break rigid invariance.
$\hookrightarrow$ modification of method necessary for mixing with Goldstone fields

Solution: quantization via Background-Field Method (BFM) Abbott '81, ...

BFM – basic features and EW higher orders: Denner, S.D., Weiglein '94

- fields split into "quantum" and "background" parts: $\phi \;\to\; \phi + \hat{\phi}$
  - ϕ: gauge fixed, appear in loops in diagrams
  - $\hat{\phi}$: sources of gauge-invariant effective action, on trees in diagrams
- vertex functions obey "classical" (ghost-free) Ward identities
  $\hookrightarrow$ many desirable properties of vertex functions
- Ward identities can keep their forms after renormalization
  $\hookrightarrow$ simple relations between renormalization constants,
  e.g. electric charge ren. constant $Z_e = Z_{\hat{\gamma}\hat{\gamma}}^{-1/2}$ as in QED
  $\Rightarrow$ use analogous relations to fix $\delta\alpha$, $\delta\beta$, ... ($\delta\alpha$ as from rigid invariance)

Application to the SESM and THDM Denner, S.D., Lang '18

Relations involving $\delta\beta$ in the THDM:

$$\delta Z_1^{\hat{\eta}} = -2\delta Z_e - \frac{c_{\rm W}^2}{s_{\rm W}^2}\frac{\delta c_{\rm W}^2}{c_{\rm W}^2} + \frac{\delta M_{\rm W}^2}{M_{\rm W}^2} + 2\frac{\delta c_\beta}{c_\beta} + \text{tadpoles},$$

$$\delta Z_2^{\hat{\eta}} = -2\delta Z_e - \frac{c_{\rm W}^2}{s_{\rm W}^2}\frac{\delta c_{\rm W}^2}{c_{\rm W}^2} + \frac{\delta M_{\rm W}^2}{M_{\rm W}^2} + 2\frac{\delta s_\beta}{s_\beta} + \text{tadpoles}$$

$\Rightarrow$ with above relations for $\delta Z_i^{\hat{\eta}}$:

$$\delta\beta = \tfrac{1}{2}c_\beta s_\beta\left[(s_\alpha^2 - c_\alpha^2)(\delta Z_{11}^{\hat{H}} - \delta Z_{22}^{\hat{H}}) + 2c_\alpha s_\alpha(\delta Z_{12}^{\hat{H}} + \delta Z_{21}^{\hat{H}})\right] + \text{tadpoles}$$

(similar results obtained by Krause et al. '16)

Comments on BFM schemes (BFMS):

- $\delta\alpha$, $\delta\beta$ depend on choice of symmetry relations and on gauge
  But: S-matrix depends on α, β in a gauge-independent way
- process independence
- absence of singularities for mass degeneracy or $s_\alpha, s_\beta, \cdots \to 0$