Endpoint factorization and next-to-leading power resummation of “gluon thrust”

M. Beneke (TU München)

Loops & Legs in Quantum Field Theory
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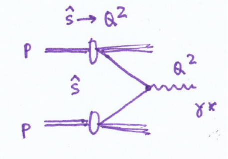
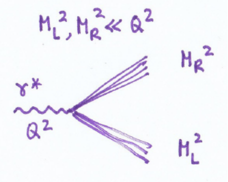
In collaboration with M. Garny, S. Jaskiewicz, J. Strohm, R. Szafron, L. Vernazza, J. Wang, 220m.nnnnn

Resummation for back-to-back jets and threshold kinematics, $\tau \ll 1$



DY , $\tau = 1 - z = 1 - Q^2/\hat{s}$



e^+e^- back-to-back jets /
event shapes (thrust, $\tau = 1 - T$)

$$\frac{d\sigma}{dQ^2} = f_{a/A} \otimes f_{b/B} \times |C^{\rm A0}|^2 \otimes S_{\rm DY}(Q\tau)$$

$$\frac{d\sigma}{d\tau} = |C^{\rm A0}|^2 \times J_c^{(q)} \otimes J_{\bar{c}}^{(\bar{q})} \otimes S_{\rm LP}$$

- Leading power IR logs $\alpha_s^n \left[\frac{\ln^m \tau}{\tau}\right]_+$, $m \leq 2n$ well understood: LL, NLL, NNLL, N3LL
  [Becher, Neubert, Xu, 2006; Becher, Schwartz, 2008], ...
- Where there are logs, there are powers, and powers times logs → next-to-leading power (NLP) resummation
- Structure of NLP Logs $[\alpha_s^n \ln^m \tau]$, $m \leq 2n - 1$ not well known

NLP (overview)

Early precursor work (semi-leptonic B decay)

MB, Campanario, Mannel, Pecjak, 2004; Lee, Stewart, 2004; Bosch, Neubert, Paz, 2004

Now an active field

Larkoski, Neill, Stewart, 2014; Kolodrubetz, Moult, Stewart, 2016; Feige, Kolodrubetz, Moult, Stewart, 2017; MB, Garny, Szafron, Wang, 2017-2019; Moult, Rothen Stewart, Tackmann, Zhu, 2016/17; Boughezal, Liu, Petriello, 2016/17; Moult, Stewart, Vita, Zhu, 2018; Moult, Stewart, Vita, 2019; MB, Broggio, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 2018; MB, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 2019; MB, Broggio, Jaskiewicz, Vernazza, 2018; Broggio, Jaskiewicz, Vernazza, 2021; MB, Bobeth, Szafron, 2017; Alte, König, Neubert, 2018; Moult et al., 2019; Liu, Neubert, 2019; Wang, 2019; Liu, Mecaj, Neubert, Wang, 2020; MB, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 2020; Liu, Neubert, Schnubel, Wang, 2021; Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 2018; Moult, Vita, Yan, 2019; MB, Hager, Szafron, 2021; + ...

also with diagrammatic methods: Bonocore et al., 2016/17; Del Duca et al., 2017; van Beekveld et al., 2019; Bahjat-Abbas et al., 2019; van Beekveld, Vernazza, White, 2021; Ajjath, Mukherjee, Ravindran, 2020; Ajjath et al. 2021; Liu, Penin, 2017/18; Anastasiou, Penin, 2020; Liu, Modi, Penin, 2021; + ...

LBKD theorem, operator bases, renormalization, N-jettiness subtraction, thrust distribution, Drell-Yan and Higgs production near threshold, DIS for $x \to 1$, QED effects in B decays, New Physics decay, Higgs decay through bottom loops, TMD factorization, energy-energy correlation in $N = 4$ SYM, gravitation

- NLP resummation at leading-log (LL) in the diagonal channels for thrust two-jet region [Moult, Stewart, Vita, Zhu, 2018] and DY threshold [MB, Broggio, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 2018]

NLP-LL series $[gg \to H]$ [MB, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 1910.12685]

$$\Delta^{\rm LL}_{\rm NLP}(z,\mu) = \left[\frac{\beta\big(\alpha_s(\mu)\big)}{\alpha_s^2(\mu)}\,\frac{\alpha_s^2(\mu_t)}{\beta\big(\alpha_s(\mu_t)\big)}\right]^2 C_t^2(m_t,\mu_t)$$

$$\times \exp\left[4S^{\rm LL}(\mu_h,\mu) - 4S^{\rm LL}(\mu_s,\mu)\right]\,\frac{-8C_A}{\beta_0}\,\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_s)}\,\theta(1-z)$$

$\mu_t,\,\mu_h \sim m_H,\,\mu_s \sim m_H(1-z)$

$$\begin{aligned}
\Delta^{\rm LL}_{\rm NLP}(z,\mu) \;=\; & -\theta(1-z)\,\Big\{4C_A\frac{\alpha_s}{\pi}\Big[\ln(1-z) - L_\mu\Big] \\
&+ 8C_A^2\left(\frac{\alpha_s}{\pi}\right)^2\Big[\ln^3(1-z) - 3L_\mu\ln^2(1-z) + 2L_\mu^2\ln(1-z)\Big] \\
&+ 8C_A^3\left(\frac{\alpha_s}{\pi}\right)^3\Big[\ln^5(1-z) - 5L_\mu\ln^4(1-z) + 8L_\mu^2\ln^3(1-z) - 4L_\mu^3\ln^2(1-z)\Big] \\
&+ \frac{16}{3}C_A^4\left(\frac{\alpha_s}{\pi}\right)^4\Big[\ln^7(1-z) - 7L_\mu\ln^6(1-z) + 18L_\mu^2\ln^5(1-z) - 20L_\mu^3\ln^4(1-z) \\
&\qquad + 8L_\mu^4\ln^3(1-z)\Big] \\
&+ \frac{8}{3}C_A^5\left(\frac{\alpha_s}{\pi}\right)^5\Big[\ln^9(1-z) - 9L_\mu\ln^8(1-z) + 32L_\mu^2\ln^7(1-z) - 56L_\mu^3\ln^6(1-z) \\
&\qquad + 48L_\mu^4\ln^5(1-z) - 16L_\mu^5\ln^4(1-z)\Big]\Big\} + \mathcal{O}(\alpha_s^6\times(\log)^{11}) \\
&\qquad \left(L_\mu = \ln\frac{\mu}{m_H}\right)
\end{aligned}$$

$\mathcal{O}(\alpha_s^3)$ agrees with expansion of N3LO FO calculation, $\mathcal{O}(\alpha_s^4)$ with [de Florian et al., 2014] based on "physical kernel conjecture" of Vogt.

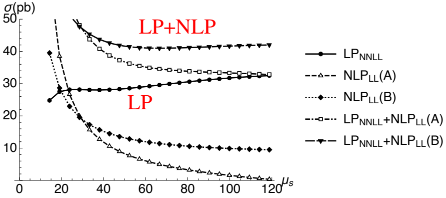
SCET result confirmed by diagrammatic method [Bahjat-Abbas et al., 1905.13710]

NLP numerics for Higgs production [1910.12685]

Fixed-order vs threshold resummation at LP-NNLL + NLP-LL

$$\mu_s^{\rm dyn} = \frac{Q}{\bar{s}_1(\tau)} = 38\,\text{GeV}, \quad \bar{s}_1(\tau) \equiv -e^{\gamma_E} \frac{d\ln\mathcal{L}(y,\mu)}{d\ln y}\bigg|_{y=\tau}$$

[Sterman, Zeng, 2013]



σ (pb)	$\mu_s = \mu_s^{\rm dyn}$	
	$\mu_h^2 = m_H^2$	$\mu_h^2 = -m_H^2$
$\sigma_{\rm LP}^{\rm NNLL}$	24.12	28.04
$\sigma_{\rm LP}^{\rm NNLO}$	28.93	
$\sigma_{\rm LP}^{\rm N^3LO}$	29.24	
$\sigma_{\rm NLP}^{\rm LL}$ (A)	7.18	12.76
$\sigma_{\rm NLP}^{\rm LL}$ (B)	8.82	15.68
$\sigma_{\rm non\,LP}^{\rm NNLO}$	11.90	
$\sigma_{\rm non\,LP}^{\rm N^3LO}$	16.27	
$\sigma_{\rm LP}^{\rm NNLL} + \sigma_{\rm NLP}^{\rm LL}$ (A)	31.30	40.80
$\sigma_{\rm LP}^{\rm NNLL} + \sigma_{\rm NLP}^{\rm LL}$ (B)	32.94	43.72
$\sigma^{\rm NNLO}$	40.82	
$\sigma^{\rm N^3LO}$	45.52	

- Threshold approximation gives a reasonable approximation – once NLP terms are included.
- NLL NLP theory still needs to be understood.

Endpoint-divergent convolutions

- At NLP-NLL a convolution in $J \otimes S$ appears, that exists in d dimensions, but does not for $\epsilon \to 0$.

$$\int_0^\Omega d\omega \, \underbrace{(n_+p\,\omega)^{-\epsilon}}_{\text{collinear fn}} \, \underbrace{\frac{1}{\omega^{1+\epsilon}} \frac{1}{(\Omega-\omega)^\epsilon}}_{\text{soft function}}$$

- Do not have a <u>renormalized</u> factorization theorem for the partonic cross section. Have to refactorize the parton distributions for $x \to 1$ as well from NLP.

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For the off-diagonal parton channels [$\to$ soft fermions] there is no LP contribution, and the problem appears already at NLP LL
$\to$ Endpoint factorization

- Up to now only for $B \to \chi_{cJ} K$ [MB, Vernazza, 2008] and in particular $H \to \gamma\gamma$ [Liu, Mecaj, Neubert, Wang, 2019-2020]

Soft-collinear EFT factorization

[Bauer, Fleming, Pirjol, Stewart, 2000; MB, Chapovsky, Diehl, Feldmann, 2002]

Separate fields for collinear and soft modes

$$\text{QCD}[A, \psi] \quad \longrightarrow \quad \text{SCET}[A_c, A_s, \xi_c, q_s]$$

Several collinear directions $n_{i\pm}$ $\rightarrow$ several copies of collinear fields.

$$\mathcal{L}^{(0)}_{\text{SCET}} = \sum_{i=1}^{N} \mathcal{L}^{(0)}_{c_i} + \mathcal{L}_{\text{soft}}$$

$$\mathcal{L}^{(0)}_c(x) = \bar{\xi} \left(in_- D_c + g_s n_- A_s(x_-) + i\not{D}_{\perp c} \frac{1}{in_+ D_c} i\not{D}_{\perp c} \right) \frac{\not{n}_+}{2} \xi + \mathcal{L}^{(0)}_{c,\text{YM}}$$

$$iD_c = i\partial + g_s A_c, \qquad x_-^\mu = \frac{1}{2} n_+ \cdot x \, n_-^\mu$$

- Only $n_- A_s$ appears, with eikonal vertex $ig_s n_-^\mu$, multipole expansion of soft fields around light-cone of collinear modes [BCDF, 2002]
- Soft interaction can be removed by a field redefinition [Bauer, Pirjol, Stewart, 2001]

$$W_c(x) = Y(x) = P\exp\left(ig\int_{-\infty}^{0} ds\, n_- A_s(x+sn_-)\right)$$

$$Y^\dagger in_- D_s\, Y = in_-\partial$$

$$\xi(x) \to Y(x_-)\xi(x) \qquad A_c^\mu(x) \to Y(x_-)A_c^\mu(x)Y^\dagger(x_-)$$

Collinear and soft interactions decoupled at leading power:

$$\mathcal{L}_c^{(0)}(x) = \bar\xi\left(in_- D_c + \cancel{g_s n_- A_s(x_-)} + i\not{D}_{\perp c}\frac{1}{in_+ D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}_{c,\mathrm{YM}}^{(0)}$$

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Next-to-leading power soft-collinear interactions $\mathcal{L}_{c_i} = \mathcal{L}_{c_i}^{(0)} + \mathcal{L}_{c_i}^{(1)} + \mathcal{L}_{c_i}^{(2)} + \ldots$

$$\mathcal{L}_c^{(1)\mathrm{gluon}}(x) = \bar{\xi}\left[\underbrace{x_\perp^\mu n_-^\nu\, W_c\, g_s F_{\mu\nu}^{s}(x_-) W_c^\dagger}_{\mathrm{NLP}}\right]\frac{\not{n}_+}{2}\,\xi + \mathcal{O}(\lambda^2)$$

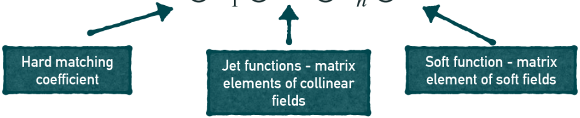
$$\mathcal{L}_c^{(1)\mathrm{quark}}(x) = \underbrace{\bar{q}(x_-)\, W_c^\dagger\, i\not{D}_{\perp c}\,\xi}_{\text{starts at NLP}} + \mathrm{h.c.} + \mathcal{O}(\lambda^2)$$

SCET is the effective QFT that reproduces QCD (or another “full theory”) in the collinear and/or soft regions to any order.
Operator definitions of collinear and soft contributions.

$$p^\mu = n_+ \cdot p \, \frac{n_-^\mu}{2} + p_\perp^\mu + n_- \cdot p \, \frac{n_+^\mu}{2}$$

- Strict expansion in $\lambda \sim p_\perp / n_+ p \equiv \sqrt{1-x}, \sqrt{1-z}, \sqrt{\tau}$. NLP is $\mathcal{O}(\lambda^2)$
- Factorize into **single scale** (“homogeneous”) objects, which have **gauge-invariant operator definitions: hard, jet/collinear and soft functions**

$$\sigma \sim H \otimes J_1 \otimes \ldots \otimes J_n \otimes S$$

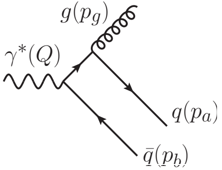


- IR logs in QCD are UV logs in SCET. **Resummation is an operator renormalization / mixing + renormalization group problem**

NLP endpoint factorization and resummation for “gluon thrust”

[MB, Garny, Jaskiewicz, Strohm, Szafron, Vernazza, Wang, 220m.nnnnn]

$$e^+e^- \to \gamma^* \to [g]_c + [q\bar{q}]_{\bar{c}}$$



Double-log resummation with d-dim factorization and consistency [Moult, Stewart, Vita, Zhu, 2019; MB, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 2020]

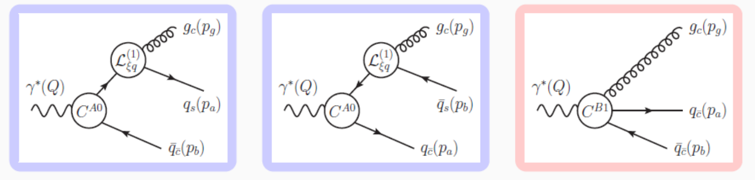
Gluon-thrust as off-diagonal channel in e^+e^-

"Direct" term (B-type) and time-ordered product soft-quark term (A-type)

$$\bar{\psi}\gamma_\perp^\mu\psi(0) = \int dt\,d\bar{t}\,\tilde{C}^{\mathrm{A0}}(t,\bar{t}) \times \bar{\chi}_c(tn_+)\gamma_\perp^\mu\chi_{\bar{c}}(\bar{t}n_-) + (c \leftrightarrow \bar{c})$$
$$+ \sum_{i=1,2}\int dt\,d\bar{t}_1\,d\bar{t}_2\,\tilde{C}_i^{\mathrm{B1}}(t,\bar{t}_1,\bar{t}_2)\,\bar{\chi}_{\bar{c}}(\bar{t}_1 n_-)\Gamma_i^{\mu\nu}\mathcal{A}_{c\perp\nu}(tn_+)\chi_{\bar{c}}(\bar{t}_2 n_-) + \ldots$$

Soft-quark interaction

$$\mathcal{L}_{\xi q}(x) = \bar{q}_s(x_-)\not{\mathcal{A}}_{c\perp}(x)\chi_c(x) + \text{h.c.}$$



B-type direct (anti-) collinear emission

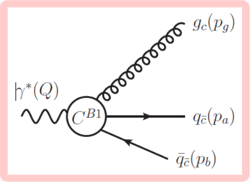
"Direct" B-type term expressed in hard, (anti-) collinear and soft function

$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\text{B}-\text{type}} \sim \int_0^1 dr dr' \, C^{B1}(r) C^{B1}(r')^* \times \mathcal{J}_{\bar{c}}^{q\bar{q}}\left(r, r'\right) \otimes \mathcal{J}_c^{(g)} \otimes S^{(g)}}$$

Endpoint divergence

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{dM_L^2\, dM_R^2}|_{\text{B}-\text{type}} \propto \int_0^1 dr \left[\frac{1}{r^{1+\epsilon}} + \frac{1}{(1-r)^{1+\epsilon}}\right]$$

from quark ($r \to 0$) or antiquark ($r \to 1$) becoming soft.



A-type soft quark emission

Time-ordered product A-type term expressed in hard, (anti-) collinear and soft function

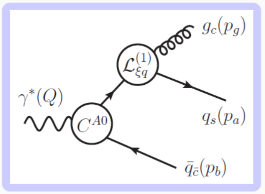
$$\boxed{\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2\,dM_L^2}\Big|_{\text{A-type}} \sim \int_0^\infty d\omega\, d\omega' \left|C^{A0}\right|^2 \times \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes \mathcal{J}_c\left(\omega,\omega'\right) \otimes S_{\text{NLP}}\left(\omega,\omega'\right)}$$

Endpoint divergence

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{dM_L^2\,dM_R^2}\Big|_{\text{A-type}} \propto 2\int_{M_R^2/Q}^{\infty} d\omega\,\frac{1}{\omega^{1+\epsilon}}$$

from soft quark or antiquark becoming energetic, $\omega \to \infty$.

$\hookrightarrow$ Breakdown of standard factorization



NLP soft quark function

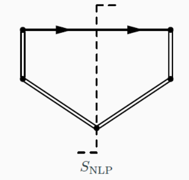
- Soft functions in leading-power are matrix elements of soft Wilson lines Y only.
- At NLP, there are soft functions with Lagrangian insertions, which introduce the additional convolutions.

$$g_s^2 \int \frac{dx_-}{2\pi}\frac{dx'_-}{2\pi}\, e^{-i(x_-\omega - x'_-\omega')}\, \langle 0|\,\overline{T}\left\{\left[Y^\dagger_{n_+}(0) Y_{n_-}(0)\right]_{cb'} \left[Y^\dagger_{n_-} q_s\right]_{\alpha' a'}(x'_-)\right\}$$

$$\times\, \mathcal{P}_s(l_+, l_-)\, T\left\{\left[\bar{q}_s Y_{n_-}\right]_{\alpha a}(x_-)\left[Y^\dagger_{n_-}(0)\, Y_{n_+}(0)\right]_{bc}\right\}|0\rangle$$

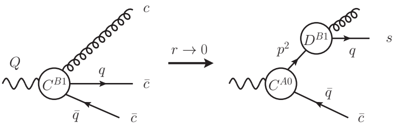
$$= \left(\frac{\not{n}_+}{2}\right)_{\alpha'\alpha}\left\{\delta_{a'a}\delta_{bb'}\, S_{\rm NLP}(l_+, l_-, \omega, \omega') + \delta_{ba}\delta_{a'b'}\, \widehat{S}_{\rm NLP}(l_+, l_-, \omega, \omega')\right\} + \ldots$$

$$\mathcal{P}_s(l_+, l_-) \equiv \sum_{X_s}\int d\mathrm{PS}_{X_s}\, \delta(l_+ - n_+ p^R_{X_s})\delta(l_- - n_- p^L_{X_s})\, |X_s\rangle\langle X_s$$



- Similarly, generalized collinear / jet functions.

Factorization of the B1 hard matching coefficient



$$[\![C_1^{\mathrm{B1}}(Q^2, r)]\!] = C^{\mathrm{A0}}(Q^2) \times \frac{D^{\mathrm{B1}}(rQ^2)}{r}$$

- When the small-r (or $r \to 1$) region is relevant, the B1 coefficient is a two-scale object, which refactorizes, since the intermediate state develops an on-shell pole.

- D^{B1} appears as a universal coefficient that renormalizes soft quark emission [MB, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 2020]. Its double logarithms are proportional to the change of colour charge of the collinear particles.

$$\langle g_c^a(p_c) q_{\overline{sc}}(p_{\overline{sc}})| \int d^4x\, T\{\bar{\chi}_c(0), \mathcal{L}_{\xi q}(x)\}\, |0\rangle = g_s \bar{u}(p_{\overline{sc}}) t^a \not{\epsilon}_{c\perp}(p_c) \frac{in_+p_c}{p^2} \frac{\not{n}_-}{2} D^{\mathrm{B1}}(p^2)$$

- The same coefficient appears in the endpoint factorization theorem for $H \to gg$ through light-quark loops [Liu, Neubert, Schnubel, Wang, 2022]

Renormalization of the soft-quark emission coefficient

$$[\![C_1^{\mathrm{B1}}(Q^2, r)]\!] = C^{\mathrm{A0}}(Q^2) \times \frac{D^{\mathrm{B1}}(rQ^2)}{r}$$

Obtain $D^{\mathrm{B1}}(p^2)$ $[p^2 = rQ^2]$ from the limit $r \to 0$ of the full NLP B1 operator and its RGE:

$$D^{\mathrm{B1}}(p^2) = 1 + \frac{\alpha_s}{4\pi}\,(C_F - C_A)\left(\frac{2}{\epsilon^2} - 1 - \frac{\pi^2}{6}\right)\left(\frac{\mu^2}{-p^2 - i\varepsilon}\right)^{\epsilon} + \mathcal{O}(\alpha_s^2).$$

$$\frac{d}{d\ln\mu} D^{B1}(p^2) = \int_0^\infty d\hat{p}^2\, \gamma_D(\hat{p}^2, p^2) D^{B1}(\hat{p}^2)\,,$$

with

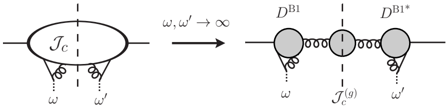
$$\gamma_D(\hat{p}^2, p^2) = \frac{\alpha_s(C_F - C_A)}{\pi}\,\delta(\hat{p}^2 - p^2)\ln\left(\frac{\mu^2}{-p^2 - i\varepsilon}\right) + \frac{\alpha_s}{\pi}\left(\frac{C_A}{2} - C_F\right) p^2 \left[\frac{\theta(\hat{p}^2 - p^2)}{\hat{p}^2(\hat{p}^2 - p^2)} + \frac{\theta(p^2 - \hat{p}^2)}{p^2(p^2 - \hat{p}^2)}\right]_+$$

Endpoint factorization relations

- In d dimensions the $1/\epsilon$ poles from the divergent convolution integrals cancels. The integrands match in the asymptotic limits $\omega, \omega' \to \infty$ (A-type) and $r, r' \to 0(1)$.

- This allows a rearrangement between the terms that makes them separately finite, provided two refactorization conditions hold for the soft and jet functions:

$$\text{(I)} \qquad \mathcal{J}_c\left(p^2, \omega, \omega'\right) = \mathcal{J}_c^{(g)}(p^2)\,\frac{D^{\mathrm{B1}}(\omega Q)}{\omega}\,\frac{D^{\mathrm{B1}*}(\omega' Q)}{\omega'} + \mathcal{O}\left(\frac{1}{\omega^{(\prime)}}\right),$$



$$\text{(II)} \qquad Q\,\widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R)\,\widetilde{S}_{\mathrm{NLP}}\left(s_R, s_L, \omega, \omega'\right)\Big|_{\omega^{(\prime)}\to\infty}$$

$$= \widetilde{\mathcal{J}}_{\bar{c}}^{q\bar{q}(8)}\left(s_R, r, r'\right)\widetilde{S}^{(g)}(s_R, s_L)\Big|_{r^{(\prime)}=\omega^{(\prime)}/Q\to 0}.$$

Rearrangement

- Add the scaleless integral

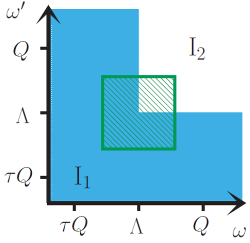
$$0 = \frac{2C_F}{Q} f(\epsilon)\,|C^{A0}(Q^2)|^2 \widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}(s_R)\widetilde{\mathcal{J}}_{c}^{(g)}(s_L) \\ \times \int_0^\infty d\omega d\omega' \, \frac{D^{\mathrm{B1}}(\omega Q)}{\omega} \frac{D^{\mathrm{B1}*}(\omega' Q)}{\omega'} \left[\!\left[\widetilde{S}_{\mathrm{NLP}}(s_R, s_L, \omega, \omega')\right]\!\right]$$

- Split

$$0 = I_1 + I_2$$

by introducing a factorization parameter Λ, and subtract I_1 from the B-type term and I_2 from the A-type term, which makes both endpoint-finite as $d \to 4$.

$\hookrightarrow$ Renormalized factorization formula



NLP renormalized factorization

$$\frac{1}{\sigma_0}\frac{d\widetilde{\sigma}}{ds_R ds_L}\bigg|_{\text{A}-\text{type}} = \frac{2C_F}{Q} f(\epsilon)\,|C^{\text{A0}}(Q^2)|^2\,\widetilde{\mathcal{J}}^{(\bar q)}_{\bar c}(s_R)\int_0^\infty d\omega d\omega'$$
$$\times\Big\{\widetilde{\mathcal{J}}_c(s_L,\omega,\omega')\,\widetilde{S}_{\text{NLP}}(s_R,s_L,\omega,\omega')$$
$$-\theta(\omega-\Lambda)\theta(\omega'-\Lambda)\,[\![\widetilde{\mathcal{J}}_c(s_L,\omega,\omega')]\!][\![\widetilde{S}_{\text{NLP}}(s_R,s_L,\omega,\omega')]\!]$$
$$+\,\widetilde{\widetilde{\mathcal{J}}}_c(s_L,\omega,\omega')\,\widetilde{\widetilde{S}}_{\text{NLP}}(s_R,s_L,\omega,\omega')\Big\}$$

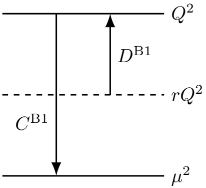
$$\frac{1}{\sigma_0}\frac{d\widetilde{\sigma}}{ds_R ds_L}\bigg|^{\text{B}-\text{type}}_{i=i'=1} = \frac{2C_F}{Q^2} f(\epsilon)\,\widetilde{\mathcal{J}}^{(g)}_c(s_L)\,\widetilde{S}^{(g)}(s_R,s_L)\int_0^\infty dr dr'$$
$$\times\Big[\theta(1-r)\theta(1-r')\,C_1^{\text{B1}*}(Q^2,r')C_1^{\text{B1}}(Q^2,r)\,\widetilde{\mathcal{J}}^{q\bar q(8)}_{\bar c}(s_R,r,r')$$
$$-[1-\theta(r-\Lambda/Q)\theta(r'-\Lambda/Q)]$$
$$\times\,[\![C_1^{\text{B1}*}(Q^2,r')]\!]_0\,[\![C_1^{\text{B1}}(Q^2,r)]\!]_0\,[\![\widetilde{\mathcal{J}}^{q\bar q(8)}_{\bar c}(s_R,r,r')]\!]_0\Big]$$

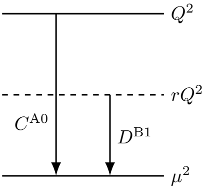
- Λ dependence cancels between the two terms. Each separately independent of dim reg μ
- Valid to any log accuracy. At LL only the subtraction terms contribute do to the extra log from large ω / small r

Example of an asymptotic RGE equation

$$\frac{d}{d\ln\mu}\left[\!\left[C_1^{\mathrm{B1}}(Q^2,r,\mu^2)\right]\!\right]_0 = \Bigg[C_F\gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{-Q^2}{\mu^2}$$
$$-(C_F-C_A)\gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{-rQ^2}{\mu^2}+\gamma_{\mathrm{A0}}(\alpha_s)\Bigg]\left[\!\left[C_1^{\mathrm{B1}}(Q^2,r,\mu^2)\right]\!\right]_0$$
$$+Q\int_0^\infty d\hat{r}\,\frac{\hat{r}}{r}\,\hat{\gamma}_D(\hat{r}Q,rQ)\left[\!\left[C_1^{\mathrm{B1}}(Q^2,\hat{r},\mu^2)\right]\!\right]_0$$

- Two scale object, hard scale Q^2 and the “dynamical” scale rQ^2.
- Either resum the $\ln r$ terms in the initial condition (left), or use two initial scales.





Example of a leading-log solution

$$
\begin{aligned}
\left[\!\left[C_1^{\mathrm{B1}}(Q^2, r, \mu^2) \right]\!\right]_0 &= \exp\left[2C_F S(\mu_h, \mu) - 2(C_F - C_A) S(\mu_{h\Lambda}, \mu)\right] \\
&\times \left(\frac{-Q^2}{\mu_h^2}\right)^{-C_F A_{\gamma_{\mathrm{cusp}}}(\mu_h, \mu)} \left(\frac{-rQ^2}{\mu_{h\Lambda}^2}\right)^{(C_F - C_A) A_{\gamma_{\mathrm{cusp}}}(\mu_{h\Lambda}, \mu)} \\
&\times \left[\!\left[C_1^{\mathrm{B1}}(Q^2, r, \mu_h^2, \mu_{h\Lambda}^2) \right]\!\right]_0 ,
\end{aligned}
$$

$$
\begin{aligned}
S(\nu, \mu) &= -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \, \frac{\gamma_{\mathrm{cusp}}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} d\alpha' \, \frac{1}{\beta(\alpha')}, \\
A(\nu, \mu) &= -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \, \frac{\gamma_{\mathrm{cusp}}(\alpha)}{\beta(\alpha)}.
\end{aligned}
$$

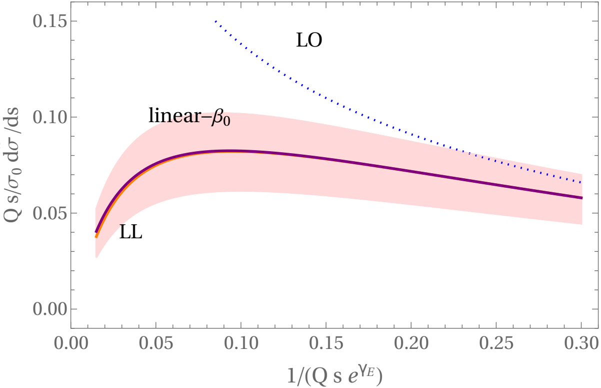
NLP-LL resummed gluon trust

$$
\frac{1}{\sigma_0}\frac{\widetilde{d\sigma}}{ds_R ds_L}|_{\rm LL} = \frac{1}{Q s_R}\frac{\alpha_s(\mu_c)C_F}{\pi}\ \exp\left[4C_F S\left(\mu_h,\mu_{\bar c}\right) + 4C_A S\left(\mu_s,\mu_c\right)\right]
$$
$$
\times\left(\frac{Q^2}{\mu_h^2}\right)^{-2C_F A(\mu_h,\mu_{\bar c})}\left(\frac{1}{s_L s_R e^{2\gamma_E}\mu_s^2}\right)^{-2C_A A(\mu_s,\mu_c)}
$$
$$
\times\int_\sigma^Q\frac{d\omega}{\omega}\ \exp\left[4\left(C_F - C_A\right)S\left(\mu_{s\Lambda},\mu_{h\Lambda}\right)\right]\left(\frac{\omega}{s_R e^{\gamma_E}\mu_{s\Lambda}^2}\right)^{-2(C_F-C_A)A(\mu_{s\Lambda},\mu_{h\Lambda})}
$$
$$
\times\ \left(s_R e^{\gamma_E} Q\right)^{2C_F A(\mu_{h\Lambda},\mu_{\bar c})+2C_A A(\mu_c,\mu_{h\Lambda})}
$$

Previous double-logarithmic result [Moult, Stewart, Vita, Zhu, 2019; MB, Garny, Jaskiewicz, Szafron, Vernaz-za, Wang, 2020] recovered

$$
\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}|_{\rm DL} = \frac{C_F}{C_F - C_A}\frac{1}{\ln(1/\tau)}\,e^{-\frac{\alpha_s C_A}{\pi}\ln^2\tau}\left\{1 - e^{-\frac{\alpha_s}{\pi}(C_F-C_A)\ln^2\tau}\right\}
$$

Effect of resummation



Shown in Laplace space. Roughly $\tau \leftrightarrow 1/(sQe^{\gamma_E})$

Summary

- Renormalized, endpoint-divergence-free NLP factorization theorem for gluon thrust.
- First example for a $\mathrm{SCET_I}$ process at cross section level.
- Rearrangement bears many similarities with the NLP factorization of $H \to \gamma\gamma$ decay through light-quark loops [Liu, Mecaj, Neubert, Wang, 2019-2020]
- Resummation can now be done with standard RGE methods, similar to LP, but with some twists. Completed leading log, formula should work to higher logirithmic accruacy.
- Identified a universal soft-quark emission function D^{B1}.
- Soft-gluon emission ant NLP is different from soft-quark emission. Complete NLP must include the "diagonal" channels.