Positivity bounds on EFTs : the flavored story

Quentin Bonnefoy, 3/06/21, Journal club of the DESY theory group

Positivity bounds on EFTs : the flavored story

Motivation : lots of recent progress in the field

Positivity bounds on EFTs : the flavored story

Motivation : lots of recent progress in the field

First, introduction to **positivity bounds** : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]

Positivity bounds on EFTs : the flavored story

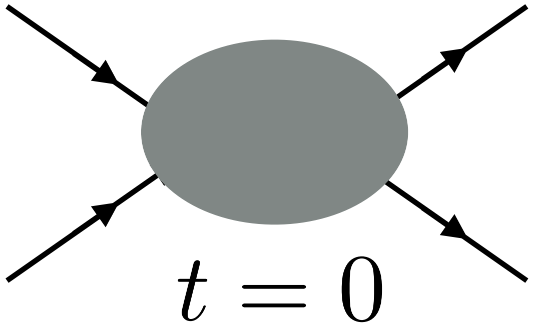
Motivation : lots of recent progress in the field

First, introduction to **positivity bounds** : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]

Ingredients (Lorentz invariance, causality, locality, unitarity) :

- **analyticity**



$$\mathcal{A}(s)$$

Positivity bounds on EFTs : the flavored story

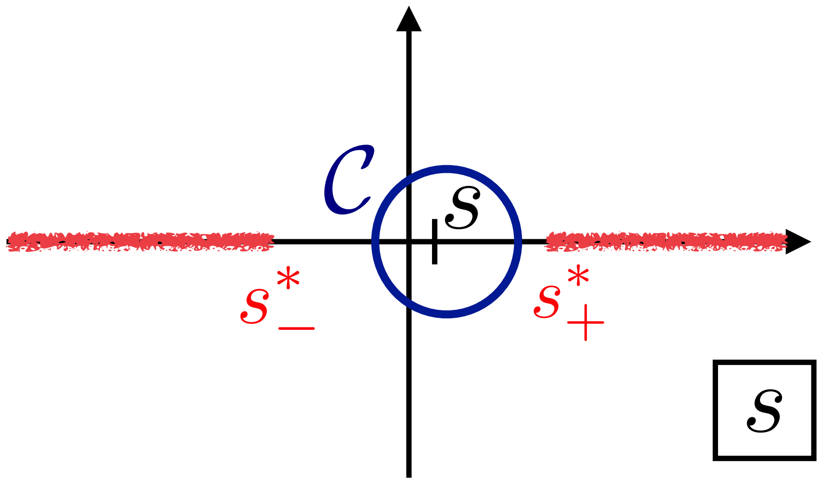
Motivation : lots of recent progress in the field

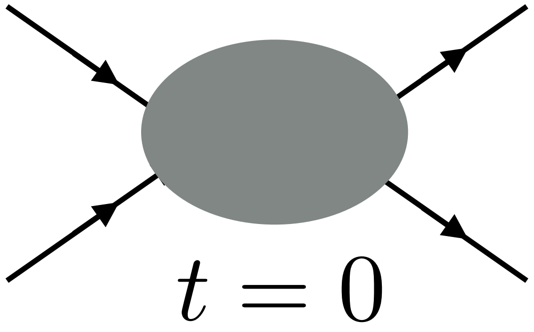
First, introduction to **positivity bounds** : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]

Ingredients (Lorentz invariance, causality, locality, unitarity) :

- **analyticity**





$$\mathcal{A}(s) = \int_{\mathcal{C}} \frac{\mathcal{A}(s')}{2\pi i(s' - s)}$$

Positivity bounds on EFTs : the flavored story

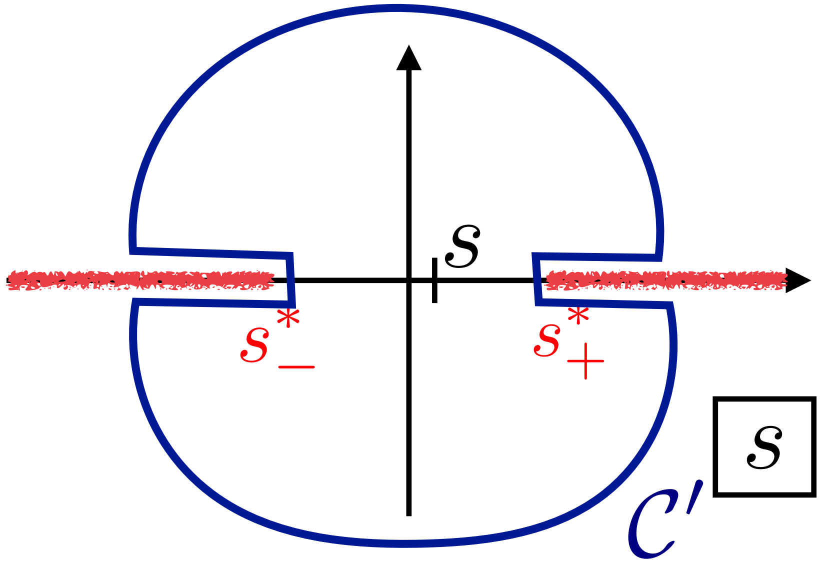
Motivation : lots of recent progress in the field

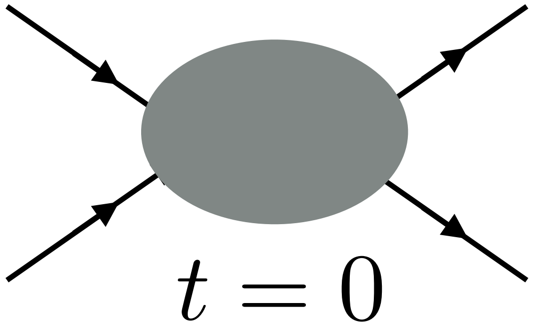
First, introduction to **positivity bounds** : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]

Ingredients (Lorentz invariance, causality, locality, unitarity) :

- **analyticity**





$$\mathcal{A}(s) = \int_{\mathcal{C}'} \frac{\mathcal{A}(s')}{2\pi i(s' - s)}$$

Positivity bounds on EFTs : the flavored story

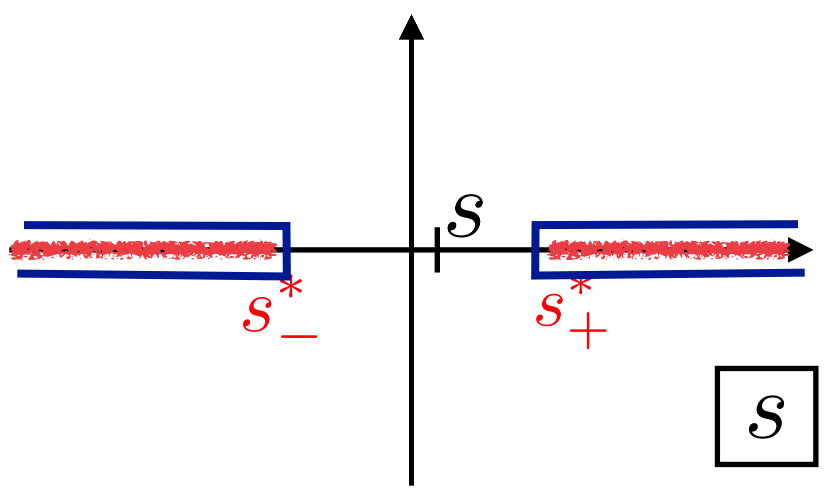
Motivation : lots of recent progress in the field

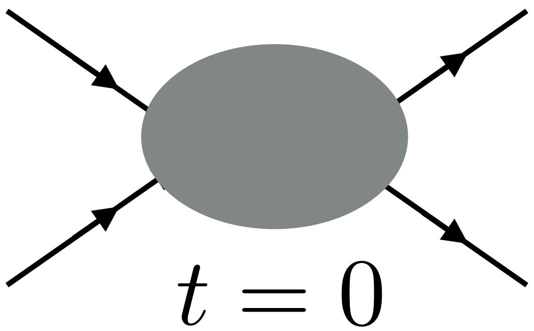
First, introduction to **positivity bounds** : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]

Ingredients (Lorentz invariance, causality, locality, unitarity) :

- **analyticity**





$$\mathcal{A}(s) = \int_{-\infty}^{s_-^*} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)} + \int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)}$$

Positivity bounds on EFTs : the flavored story

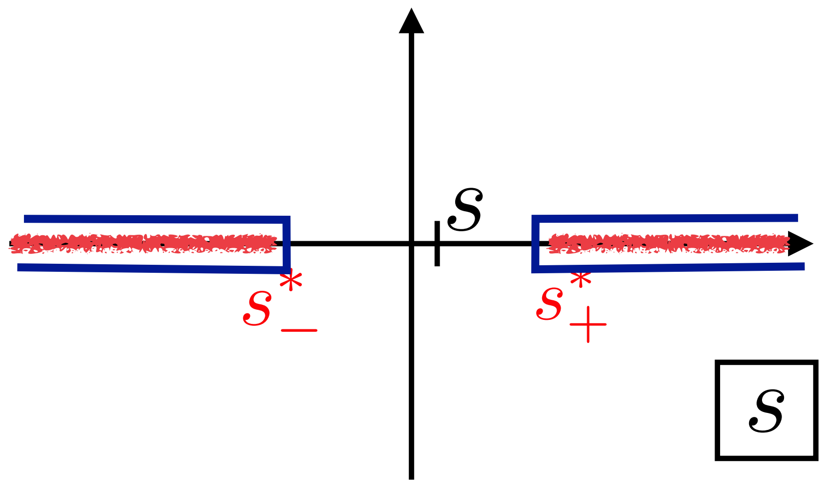
<u>Motivation :</u> lots of recent progress in the field

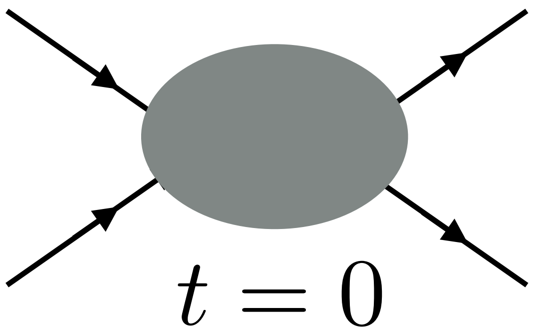
First, introduction to **positivity bounds** : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]

Ingredients (Lorentz invariance, causality, locality, unitarity) :

- **analyticity**
- **polynomial boundedness** (Froissart bound)





$$\widetilde{\mathcal{A}}(s) = \int_{-\infty}^{s_-^*} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)^3} + \int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)^3}$$

Positivity bounds on EFTs : the flavored story

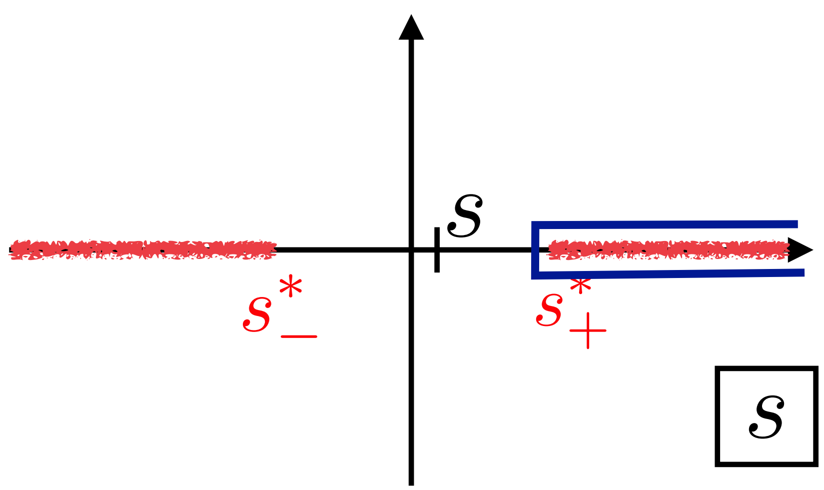
Motivation : lots of recent progress in the field

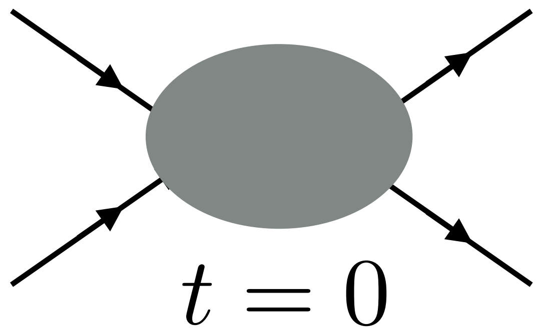
First, introduction to **positivity bounds** : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]

Ingredients (Lorentz invariance, causality, locality, unitarity) :

- **analyticity**
- **polynomial boundedness** (Froissart bound)
- **crossing symmetry**





$$\widetilde{\mathcal{A}}(s) = \int_{4m^2-s_-^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-4m^2+s)^3} + \int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)^3}$$

Positivity bounds on EFTs : the flavored story

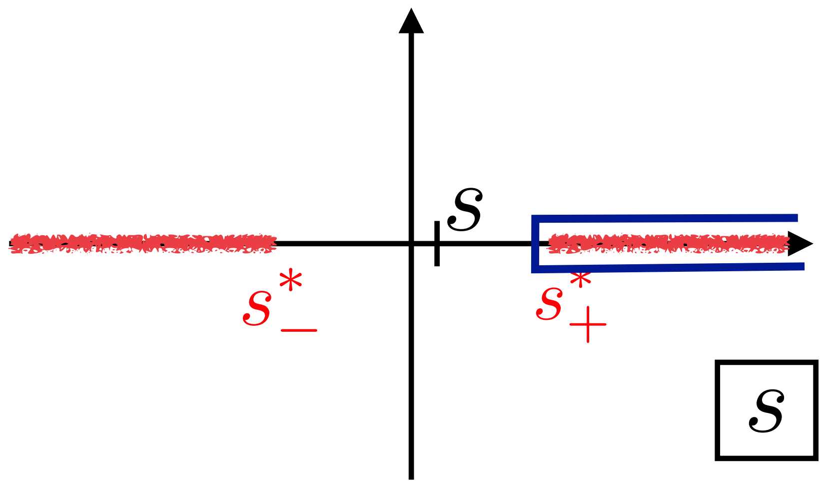
Motivation : lots of recent progress in the field

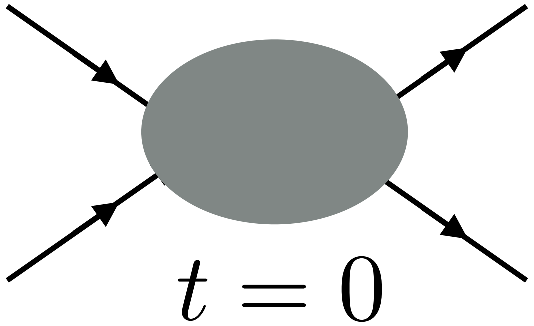
First, introduction to **positivity bounds** : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]

Ingredients (Lorentz invariance, causality, locality, unitarity) :

- **analyticity**
- **polynomial boundedness** (Froissart bound)
- **crossing symmetry**
- **unitarity**





> 0

$$\widetilde{\mathcal{A}}(s) = \int_{4m^2 - s_-^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i (s' - 4m^2 + s)^3} + \int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i (s' - s)^3}$$

Positivity bounds on EFTs : the flavored story

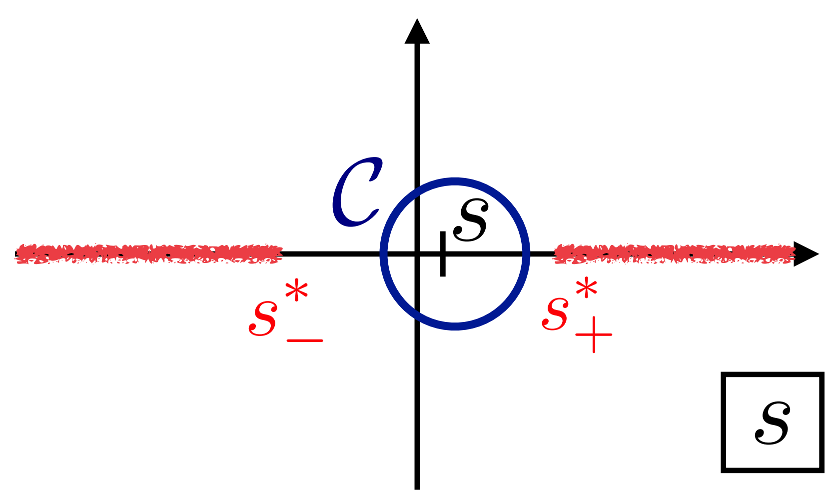
Motivation : lots of recent progress in the field

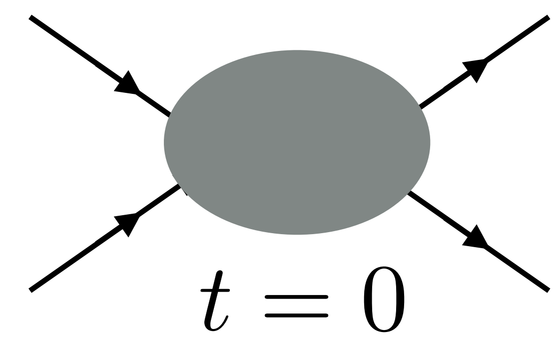
First, introduction to **positivity bounds** : sign constraints on amplitudes

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 06]

Ingredients (Lorentz invariance, causality, locality, unitarity) :

- **analyticity**
- **polynomial boundedness** (Froissart bound)
- **crossing symmetry**
- **unitarity**





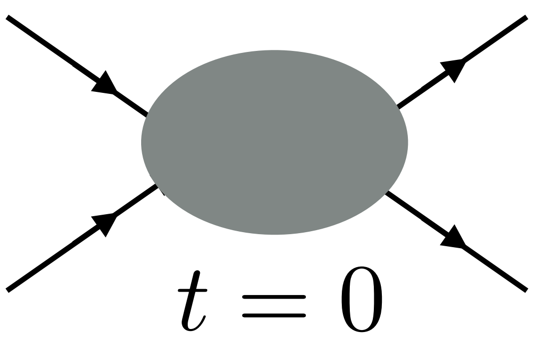
$$\mathcal{A}(s) = \int_{\mathcal{C}} \frac{\mathcal{A}(s')}{2\pi i(s' - s)}$$

At small s : computed in the EFT, **bound on EFT coefficients**

Positivity bounds on EFTs : the flavored story

Motivation : lots of recent progress in the field

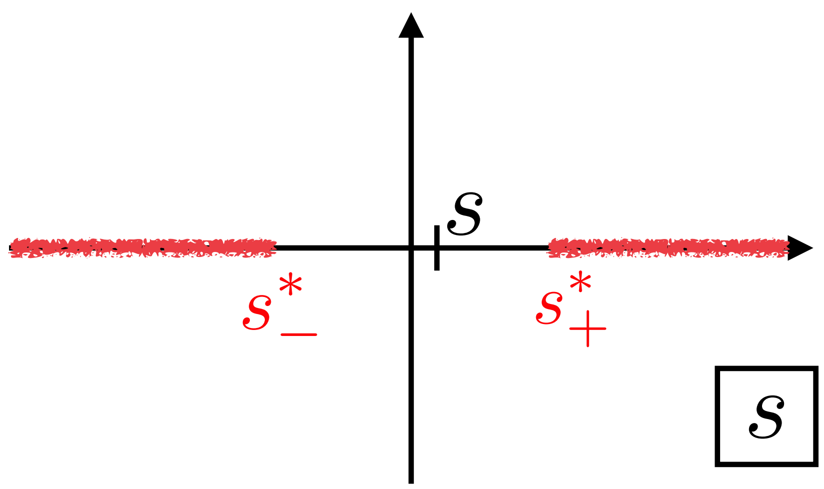
What will not be covered today :
-beyond 2 -> 2



Positivity bounds on EFTs : the flavored story

Motivation : lots of recent progress in the field

What will not be covered today :
-beyond 2 -> 2
-massless particles



Positivity bounds on EFTs : the flavored story

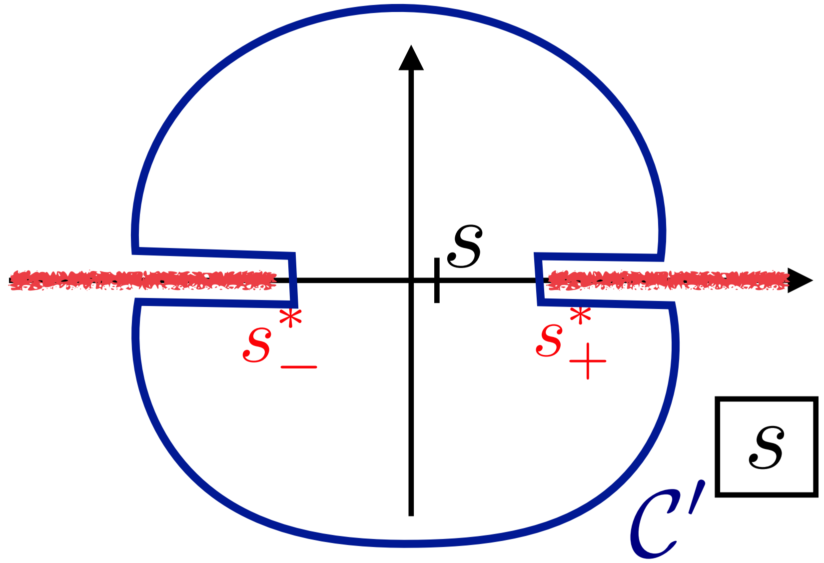
Motivation : lots of recent progress in the field

What will not be covered today :

-beyond 2 -> 2

-massless particles

-less substractions



$$\mathcal{A}(s) = \int_{\mathcal{C}'} \frac{\mathcal{A}(s')}{2\pi i(s'-s)}$$

Positivity bounds on EFTs : the flavored story

Motivation : lots of recent progress in the field

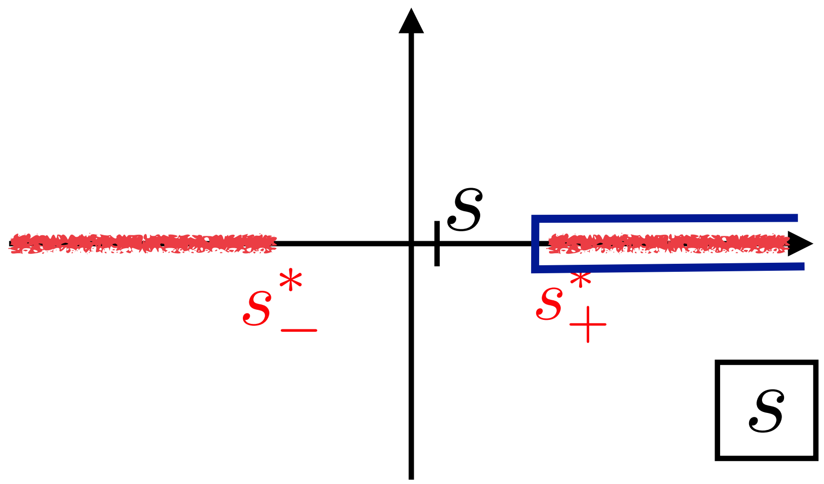
What will not be covered today :

-beyond 2 -> 2

-massless particles

-less substractions

-finite t



$$\widetilde{\mathcal{A}}(s) = \int_{4m^2 - s_-^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i (s' - 4m^2 + s)^3} + \int_{s_+^*}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i (s' - s)^3}$$

Positivity bounds on EFTs : the flavored story

Motivation : lots of recent progress in the field

What will not be covered today :
-beyond 2 -> 2
-massless particles
-less substractions
-finite t
-…

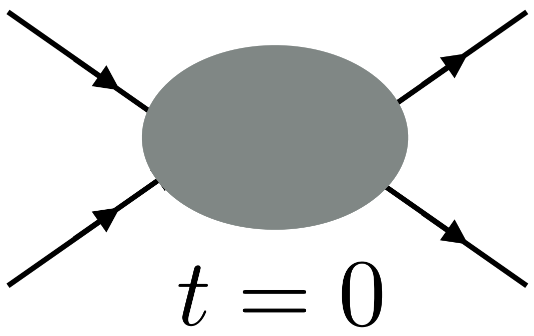
[Arkani-Hamed, Bellazzini, Caron-Huot, Chiang, de Rham, Elias-Miro, Huang, Huang, Li, Martucci, Melville, Nicolis, Rattazzi, Remmen, Riembau, Riva, Rodd, Rodina, Serra, Sgarlata, Tolley, Torre, Trincherini, Van Duong, Wang, Weng, Zhou…]

Check this week's workshop « Positivity and the bootstrap » (CERN)

Positivity bounds on EFTs : the flavored story

Motivation : lots of recent progress in the field

What will not be covered today :
-beyond 2 -> 2
-massless particles
-less substractions
-finite t
-…

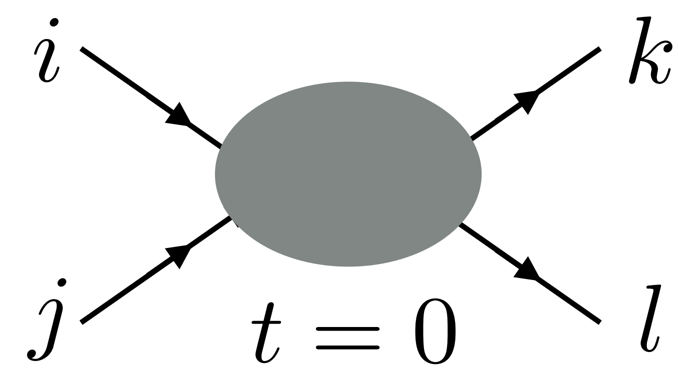


Instead : flavored/inelastic version of the bounds $\mathcal{A}(s)$

Positivity bounds on EFTs : the flavored story

Motivation : lots of recent progress in the field

What will not be covered today :
-beyond 2 -> 2
-massless particles
-less substractions
-finite t
-…

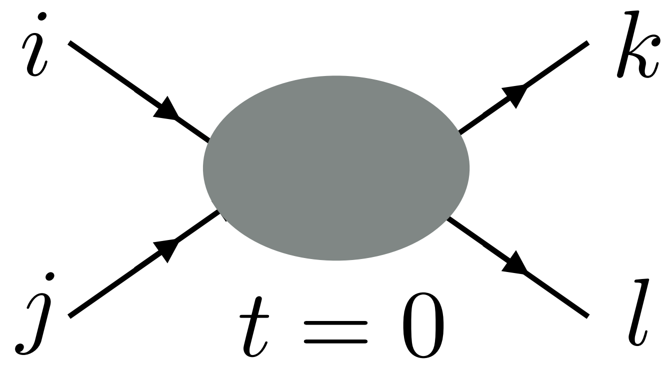


Instead : flavored/inelastic version of the bounds $\mathcal{A}(s) \to \mathcal{A}_{ijkl}(s)$

Positivity bounds on EFTs : the flavored story

Motivation : lots of recent progress in the field

What will not be covered today :
-beyond 2 -> 2
-massless particles
-less substractions
-finite t
-…



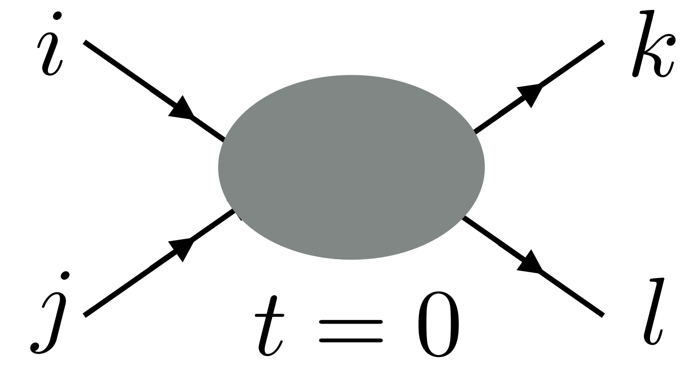
Instead : flavored/inelastic version of the bounds $\mathcal{A}(s) \to \mathcal{A}_{ijkl}(s)$

[*Convex Geometry Perspective to the (Standard Model) Effective Field Theory Space*, Zhang/Zhou, arXiv:2005.03047]
[*Causality, Unitarity and Symmetry in Effective Field Theory*, Trott, arXiv:2011.10058]
([*Symmetries, Sum Rules and Constraints on Effective Field Theories*, Bellazzini/Martucci/Torre, arXiv:1405.2960])

[*Positivity in Multi-Field EFTs*, Li, Yang, Xu, Zhang, Zhou, arXiv:2101.01191]

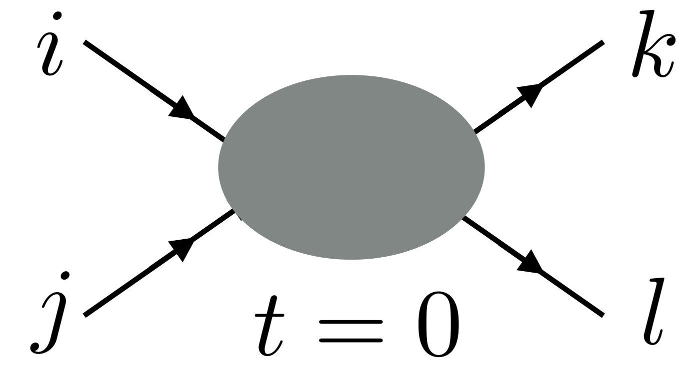
Positivity bounds on EFTs : the flavored story

$\widetilde{\mathcal{A}}_{ijkl}(s)$



Positivity bounds on EFTs : the flavored story

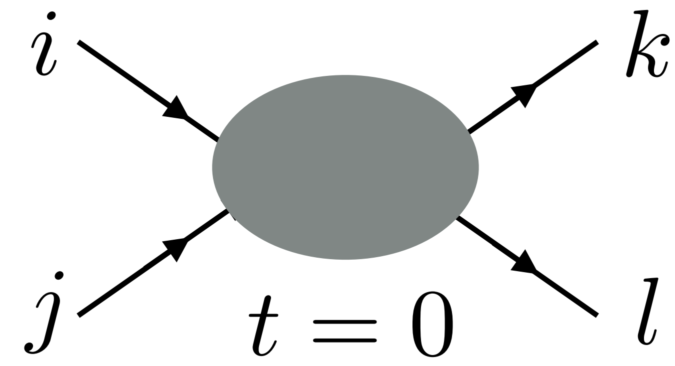
$\widetilde{\mathcal{A}}_{ijkl}(s)$



$$\widetilde{\mathcal{A}}(s) = \int_{4m^2-s^*_-}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-4m^2+s)^3} + \int_{s^*_+}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)^3}$$

Positivity bounds on EFTs : the flavored story

$\widetilde{\mathcal{A}}_{ijkl}(s)$

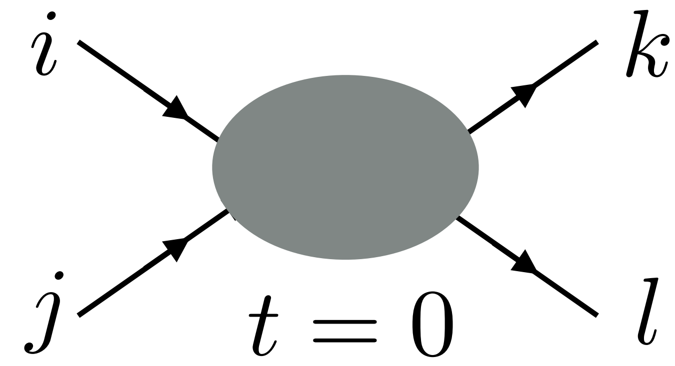


$$\mathcal{A}_{ij\to kl} - \mathcal{A}^*_{kl\to ij} \propto \sum_X \mathcal{A}_{ij\to X}\mathcal{A}^*_{kl\to X}$$

$$\widetilde{\mathcal{A}}(s) = \int_{4m^2-s^*_-}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-4m^2+s)^3} + \int_{s^*_+}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)^3}$$

Positivity bounds on EFTs : the flavored story

$\widetilde{\mathcal{A}}_{ijkl}(s)$



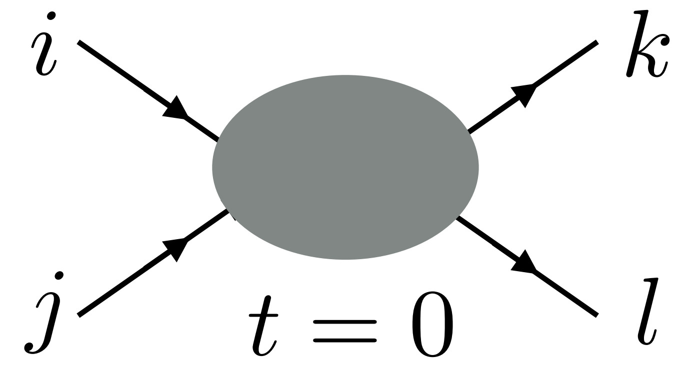
$$\mathcal{A}_{ij\to kl} - \mathcal{A}^*_{kl\to ij} \propto \sum_X \mathcal{A}_{ij\to X}\mathcal{A}^*_{kl\to X}$$

$(j \leftrightarrow l)$

$$\widetilde{\mathcal{A}}(s) = \int_{4m^2-s^*_-}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-4m^2+s)^3} + \int_{s^*_+}^{+\infty} \frac{\mathrm{Im}\mathcal{A}(s')}{2\pi i(s'-s)^3}$$

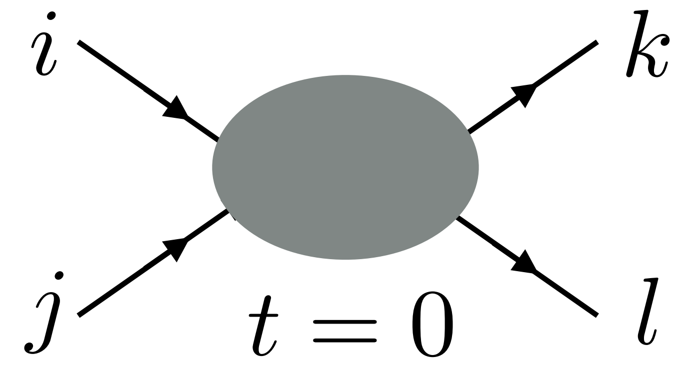
Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



Positivity bounds on EFTs : the flavored story

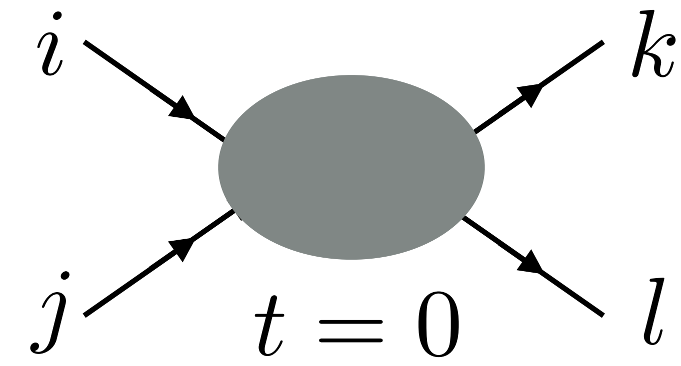
$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



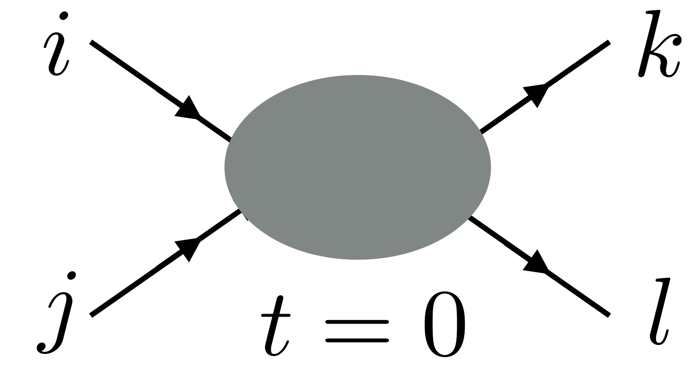
How to get positivity bounds ? <u>Approaches :</u>

- Scatter general elastic states $|in\rangle = |out\rangle = |u^i i\rangle \times |v^j j\rangle$

$$\sum_X (u^T m_X v)(v^\dagger m_X^\dagger u^*) = \sum_X |u^T m_X v|^2$$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? Approaches :

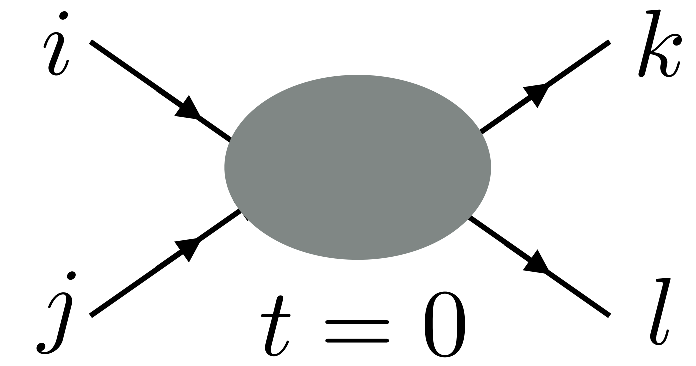
- Scatter general elastic states $|in\rangle = |out\rangle = |u^i i\rangle \times |v^j j\rangle$

$$\sum_X (u^T m_X v)(v^\dagger m_X^\dagger u^*) = \sum_X |u^T m_X v|^2$$

Restricted/necessary set of bounds **[for 4-Fermi ops: Remmen/Rodd 19 -see also Emanuele's JC last year-, QB/Gendy/Grojean 20 + lots of refs therein]**

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? Approaches :

- Scatter general elastic states $|in\rangle = |out\rangle = |u^i i\rangle \times |v^j j\rangle$

$$\sum_X (u^T m_X v)(v^\dagger m_X^\dagger u^*) = \sum_X |u^T m_X v|^2$$

Restricted/necessary set of bounds [for 4-Fermi ops: Remmen/Rodd 19 -see also Emanuele's JC last year-, QB/Gendy/Grojean 20 + lots of refs therein]

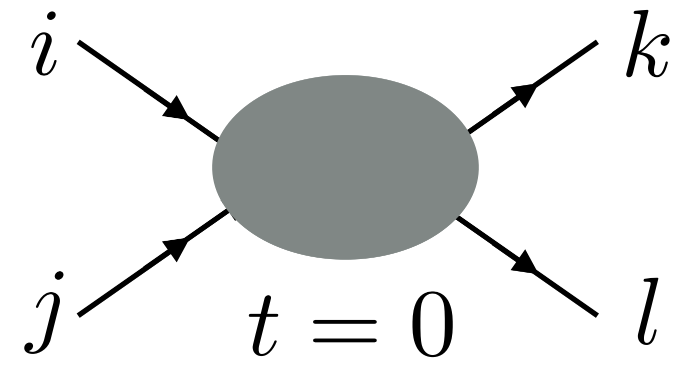
A lesson learnt : **bounds on inelastic transitions**

$$|\tilde{\mathcal{A}}_{ijkl}| + |\tilde{\mathcal{A}}_{ilkj}| \leq \sqrt{\tilde{\mathcal{A}}_{ijij}\tilde{\mathcal{A}}_{klkl}} + \sqrt{\tilde{\mathcal{A}}_{ilil}\tilde{\mathcal{A}}_{kjkj}}$$

From arXiv:2011.10058

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



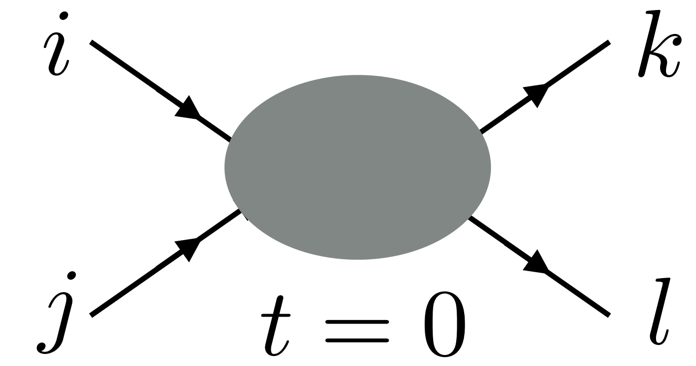
How to get positivity bounds ? Approaches :

- Related : use symmetries

[*Convex Geometry Perspective to the (Standard Model) Effective Field Theory Space*, Zhang/Zhou, arXiv:2005.03047]
[*Causality, Unitarity and Symmetry in Effective Field Theory*, Trott, arXiv:2011.10058]
([*Symmetries, Sum Rules and Constraints on Effective Field Theories*, Bellazzini/Martucci/Torre, arXiv:1405.2960])

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

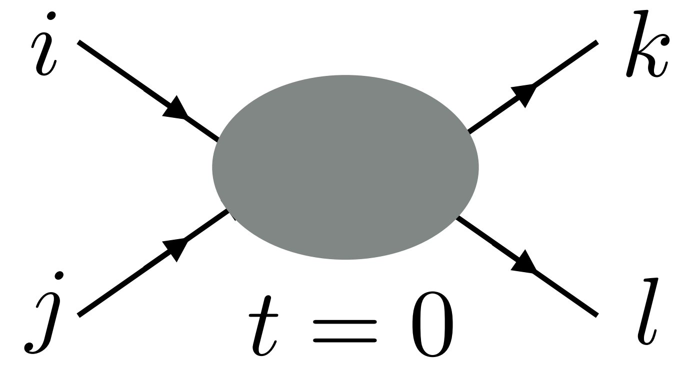
- Related : use symmetries

$$m_{ij\to \text{irrep } I_\alpha} = C_{ij}{}^{\alpha}_{I} m_{I\to I}$$

$$\sum_I C_{ij}{}^{\alpha}_{I} C^{*}_{kl}{}^{\alpha}_{I} |m_{I\to I}|^2$$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- Related : use symmetries

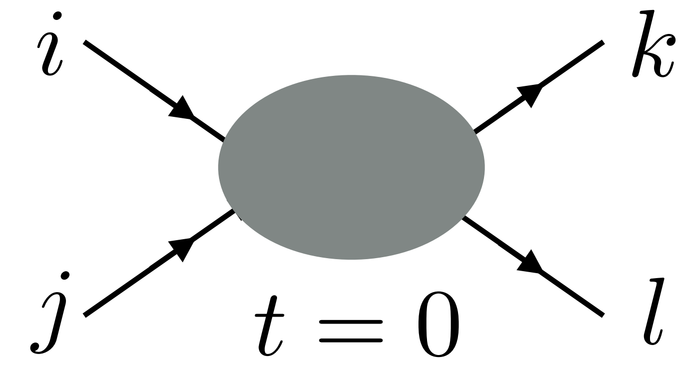
$$m_{ij\to \text{irrep } I_\alpha} = C_{ij}{}^{\alpha}_{I} m_{I\to I}$$

$$\sum_I C_{ij}{}^{\alpha}_{I} C^{*}{}^{\alpha}_{klI} |m_{I\to I}|^2$$

numbers

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- Related : use symmetries

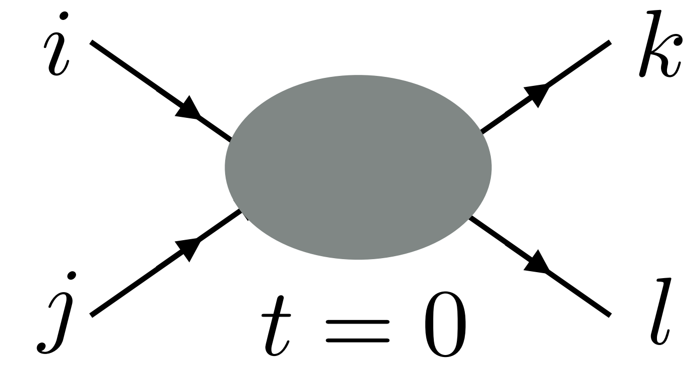
$$m_{ij\to \text{irrep } I_\alpha} = C_{ij\,I}^{\ \alpha} m_{I\to I}$$

$$\sum_I C_{ij\,I}^{\ \alpha} C^{*\ \alpha}_{kl\,I} |m_{I\to I}|^2$$

UV data, but positive

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

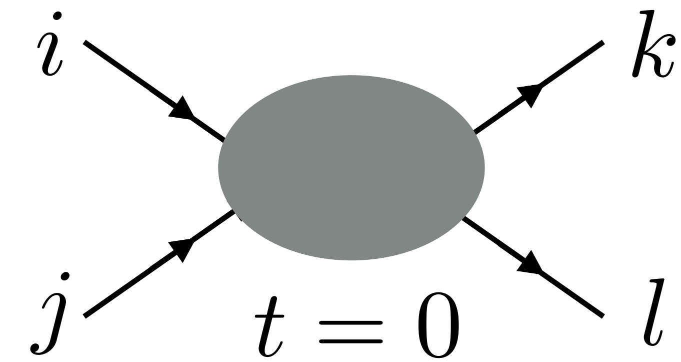
- Related : use symmetries $\quad m_{ij\to \text{irrep } I_\alpha} = C_{ij\,I}^{\ \alpha} m_{I\to I}$

$$\sum_I C_{ij\,I}^{\ \alpha} C^{*\ \alpha}_{kl\,I} |m_{I\to I}|^2$$

The (twice substracted) amplitude belongs to an identified convex cone

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- Related : use symmetries

$$m_{ij\to \text{irrep } I_\alpha} = C_{ij\,I}^{\ \ \alpha} m_{I\to I}$$

$$\sum_I C_{ij\,I}^{\ \ \alpha} C^{*\ \alpha}_{kl\,I} |m_{I\to I}|^2$$

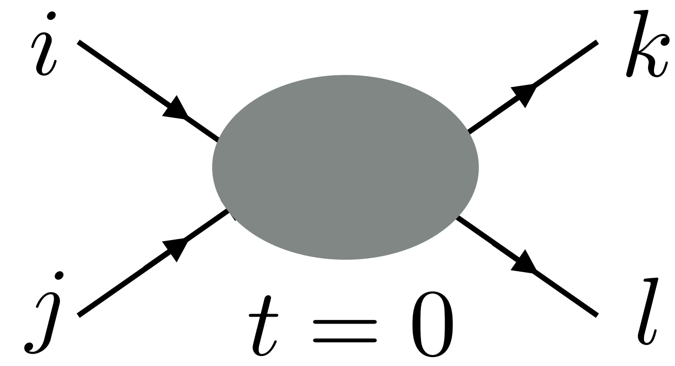
The (twice substracted) amplitude belongs to an identified convex cone

Ex: Higgs scattering

$\mathcal{O}_1^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_2^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_3^{H^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- Related : use symmetries

$$m_{ij\to \text{irrep } I_\alpha} = C_{ij}{}^{\alpha}_{I} m_{I\to I}$$

$$\sum_I C_{ij}{}^{\alpha}_{I} C^{*}_{kl}{}^{\alpha}_{I} |m_{I\to I}|^2$$

The (twice substracted) amplitude belongs to an identified convex cone

Ex: Higgs scattering

$$I = \mathbf{1}, \mathbf{3}$$

$$E_1^{ijkl} = \frac{1}{2}\left[C^{i(j}C^{|k|l)} + (C\gamma_4)^{i(j}(C\gamma_4)^{|k|l)}\right],$$

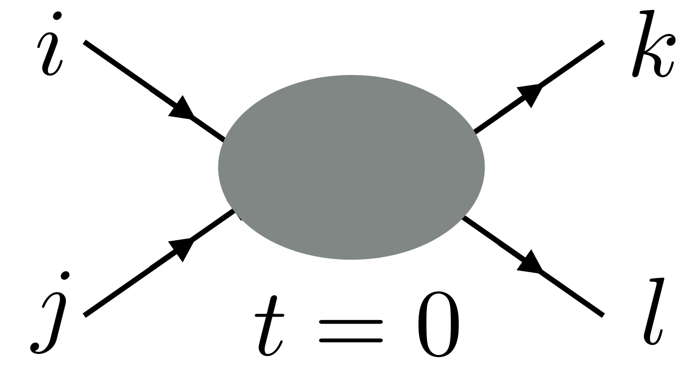
$$E_{1S}^{ijkl} = \mathbf{1}_{4\times4}^{i(j}\mathbf{1}_{4\times4}^{|k|l)}, \; E_{1A} = \gamma_4^{i(j}\gamma_4^{|k|l)},$$

$\mathcal{O}_1^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_2^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_3^{H^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

from arXiv:2005.03047

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X}\, m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- Related : use symmetries

$$m_{ij\to \text{irrep } I_\alpha} = C_{ijI}^{\alpha}\, m_{I\to I}$$

$$\sum_I C_{ijI}^{\alpha} C^{*\,\alpha}_{klI} |m_{I\to I}|^2$$

The (twice substracted) amplitude belongs to an identified convex cone

Ex: Higgs scattering $\quad I = \mathbf{1}, \mathbf{3}$

$$E_1^{ijkl} = \frac{1}{2}\left[C^{i(j}C^{|k|l)} + (C\gamma_4)^{i(j}(C\gamma_4)^{|k|l)}\right],$$

$$E_{1S}^{ijkl} = \mathbf{1}_{4\times4}^{i(j}\mathbf{1}_{4\times4}^{|k|l)}, \quad E_{1A} = \gamma_4^{i(j}\gamma_4^{|k|l)},$$

$\mathcal{O}_1^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$\mathcal{O}_2^{H^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$\mathcal{O}_3^{H^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

$$c_1^{H^4} + c_2^{H^4} + c_3^{H^4} > 0$$

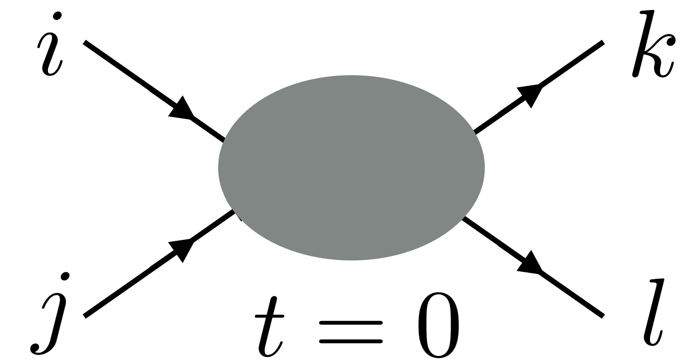
$$c_1^{H^4} + c_2^{H^4} > 0$$

$$c_2^{H^4} > 0$$

from arXiv:1908.09845

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- Related : use symmetries

$$m_{ij\to\text{irrep } I_\alpha} = C_{ij\,I}^{\;\;\alpha} m_{I\to I}$$

$$\sum_I C_{ij\,I}^{\;\;\alpha} C^{*\;\alpha}_{kl\,I} |m_{I\to I}|^2$$

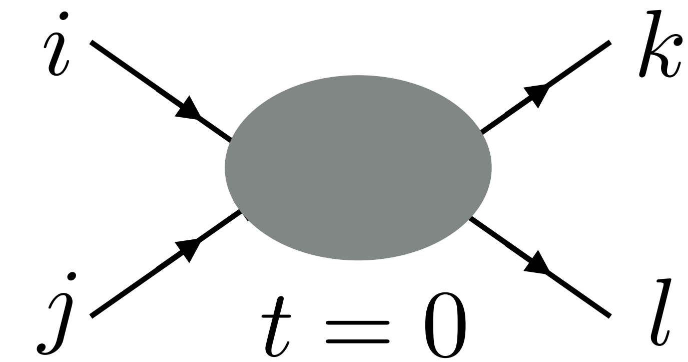
The (twice substracted) amplitude belongs to an identified convex cone

Ex: Higgs scattering

A lesson learnt : **inverse problem**

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- Related : use symmetries

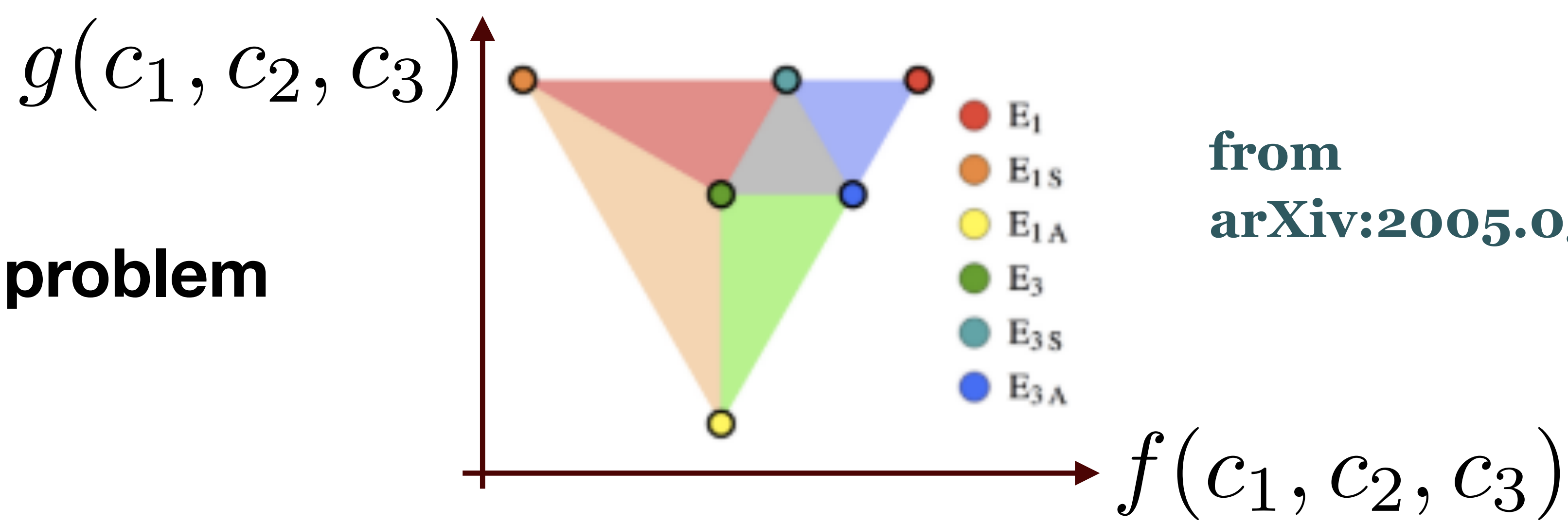
$$m_{ij\to \text{irrep } I_\alpha} = C_{ij\,I}^{\;\;\alpha} m_{I\to I}$$

$$\sum_I C_{ij\,I}^{\;\;\alpha} C^{*\,\alpha}_{kl\,I} |m_{I\to I}|^2$$

The (twice substracted) amplitude belongs to an identified convex cone

Ex: Higgs scattering

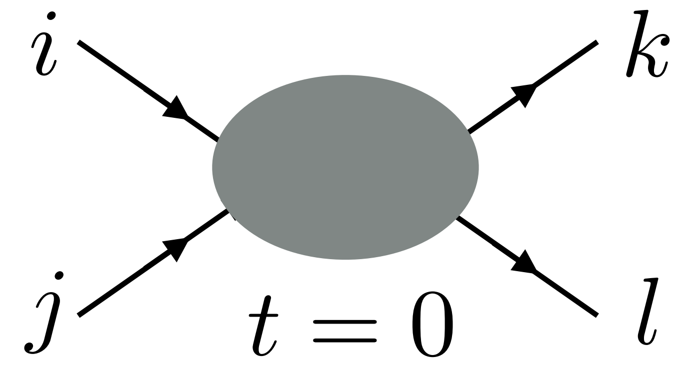
A lesson learnt : **inverse problem**



from arXiv:2005.03047

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



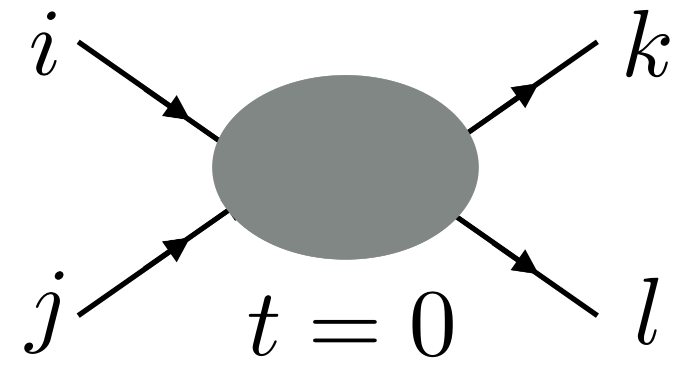
How to get positivity bounds ? <u>Approaches :</u>

- Related : use symmetries $\quad m_{ij\to \text{irrep } I_\alpha} = C_{ij}{}^{\alpha}_{I} m_{I\to I}$

With degenerate irreps : $\quad \sum_I C_{ij}{}^{\alpha}_{I_\xi} C^{*}_{kl}{}^{\alpha}_{I_{\xi'}} \cancel{|m_{I\to I}|^2}\, m_{I_\xi} m^*_{I_{\xi'}}$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- Related : use symmetries $\qquad m_{ij\to \text{irrep } I_\alpha} = C_{ij}{}^{\alpha}_{I} m_{I\to I}$

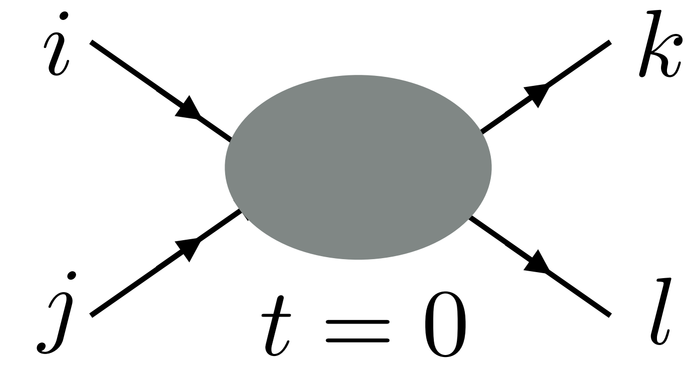
With degenerate irreps : $\qquad \sum_I C_{ij}{}^{\alpha}_{I_\xi} C^{*}_{kl}{}^{\alpha}_{I_{\xi'}} \cancel{|m_{I\to I}|^2}\, m_{I_\xi} m^*_{I_{\xi'}}$

$$\rightarrow \sum_I |m_{I_1}|^2 \left(C_{ij}{}^{\alpha}_{I_\xi} C^{*}_{ij}{}^{\alpha}_{I_\xi} \frac{m_{I_\xi} m^*_{I_{\xi'}}}{|m_{I_1}|^2} \right)$$

unfixed parameters with Cauchy-Schwarz identities

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? Approaches :

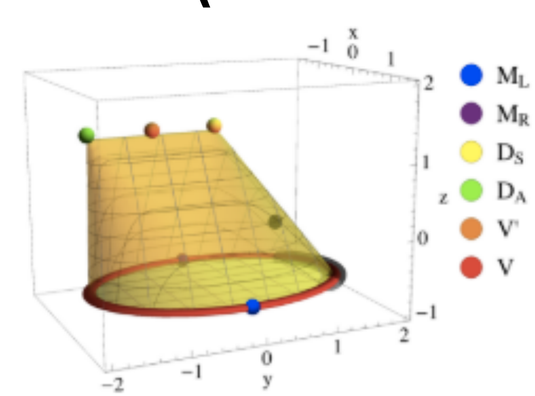
- Related : use symmetries $\quad m_{ij\to \text{irrep } I_\alpha} = C_{ij\,I}^{\,\alpha} m_{I\to I}$

With degenerate irreps : $\quad \sum_I C_{ij\,I_\xi}^{\,\alpha} C^{*\,\alpha}_{kl\,I_{\xi'}} \cancel{|m_{I\to I}|^2}\, m_{I_\xi} m^*_{I_{\xi'}}$

$$\rightarrow \sum_I |m_{I_1}|^2 \left(C_{ij\,I_\xi}^{\,\alpha} C^{*\,\alpha}_{ij\,I_\xi} \frac{m_{I_\xi} m^*_{I_{\xi'}}}{|m_{I_1}|^2} \right)$$

unfixed parameters with Cauchy-Schwarz identities

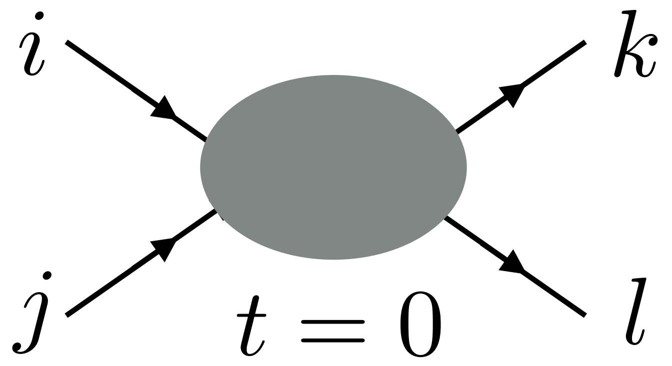
Non-polyhedral cone : ex, fermions



from arXiv:2005.03047

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



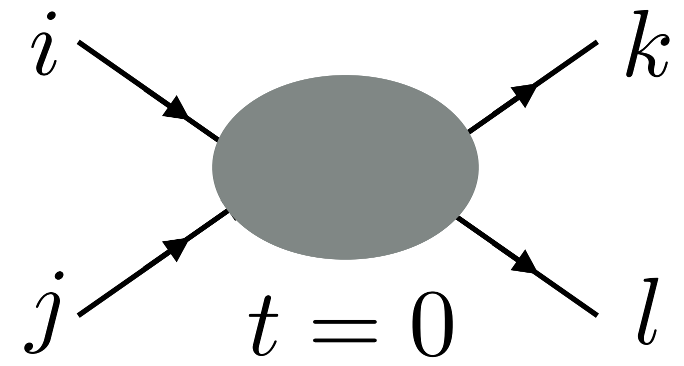
How to get positivity bounds ? Approaches :

- characterize the most general cone

[*Positivity in Multi-Field EFTs*, Li, Yang, Xu, Zhang, Zhou, arXiv:2101.01191]

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$

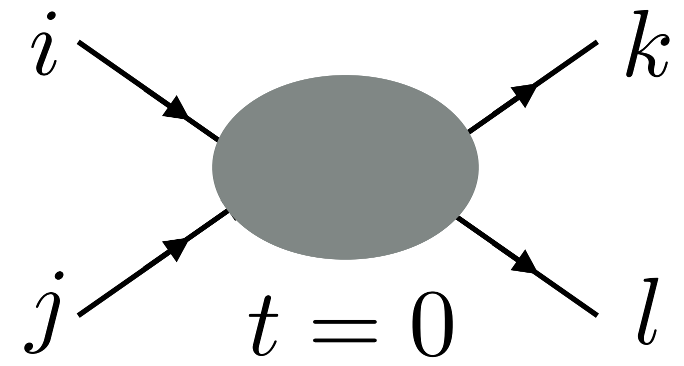


How to get positivity bounds ? <u>Approaches :</u>

- characterize the most general cone

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

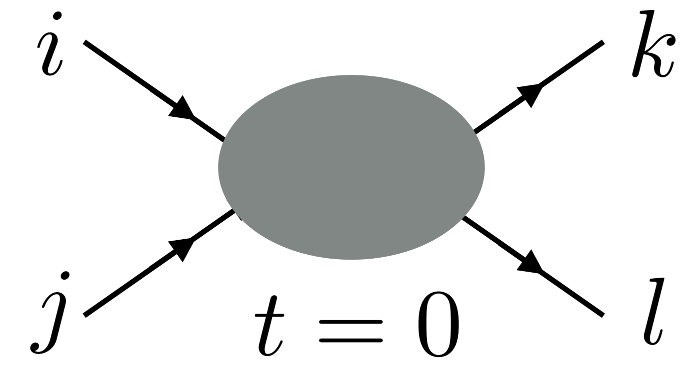
- characterize the most general cone

For a theory of (real) scalars,
Bose + crossing symmetry

$$\tilde{\mathcal{A}}_{ijkl} = \tilde{\mathcal{A}}_{ilkj} = \tilde{\mathcal{A}}_{klij} = \tilde{\mathcal{A}}_{jilk}$$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? Approaches :

- characterize the most general cone

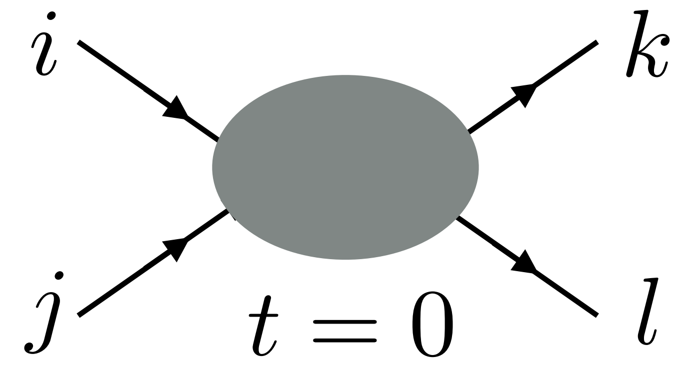
For a theory of (real) scalars,
Bose + crossing symmetry

$$\tilde{\mathcal{A}}_{ijkl} = \tilde{\mathcal{A}}_{ilkj} = \tilde{\mathcal{A}}_{klij} = \tilde{\mathcal{A}}_{jilk}$$

A cone is the dual of its dual $C^* = \left\{ Q \text{ such that } Q.\tilde{A} \geq 0 \right\}$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- characterize the most general cone

For a theory of (real) scalars,
Bose + crossing symmetry

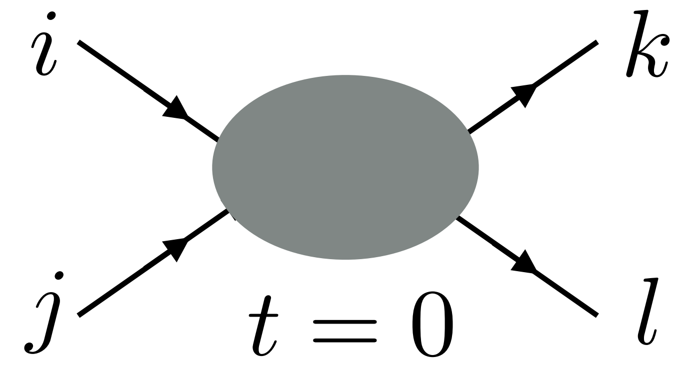
$$\tilde{\mathcal{A}}_{ijkl} = \tilde{\mathcal{A}}_{ilkj} = \tilde{\mathcal{A}}_{klij} = \tilde{\mathcal{A}}_{jilk}$$

A cone is the dual of its dual $C^* = \left\{ Q \text{ such that } Q.\tilde{A} \geq 0 \right\}$

The (useful part of the) dual cone is simply $\mathrm{PSD}_{n^2\times n^2} \cap \overleftrightarrow{S}_{n^2\times n^2}$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? Approaches :

- characterize the most general cone

For a theory of (real) scalars, Bose + crossing symmetry $\tilde{\mathcal{A}}_{ijkl} = \tilde{\mathcal{A}}_{ilkj} = \tilde{\mathcal{A}}_{klij} = \tilde{\mathcal{A}}_{jilk}$

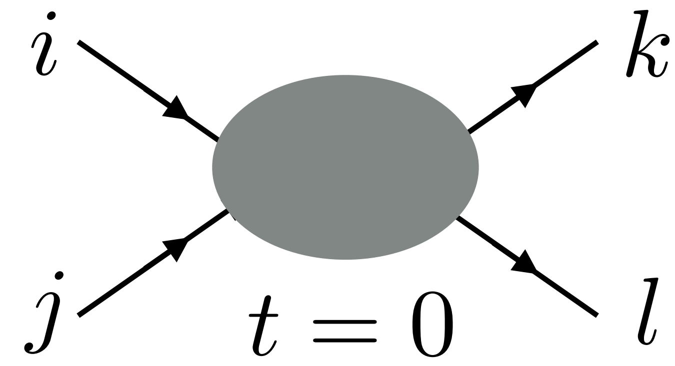
A cone is the dual of its dual $C^* = \left\{ Q \text{ such that } Q.\tilde{A} \geq 0 \right\}$

The (useful part of the) dual cone is simply $\mathrm{PSD}_{n^2\times n^2} \cap \overleftrightarrow{S}_{n^2\times n^2}$

Known as a **spectrahedron**, well studied. Existence of SDP algorithms, allows to find extremal rays of the dual cone, and facet inequalities

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- characterize the most general cone

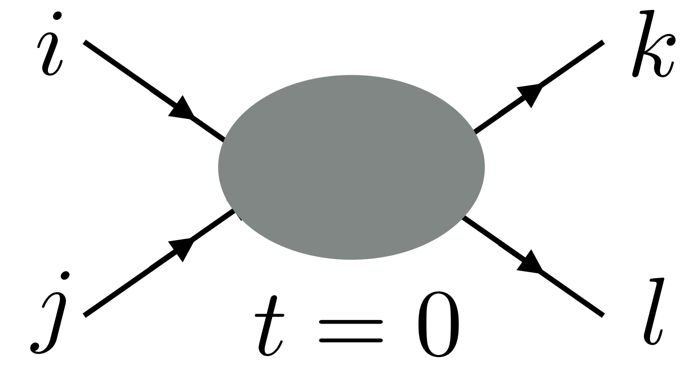
Ex: n=2 scalars with $\mathbb{Z}_2$

$$Q = \begin{pmatrix} Q_{1111} & Q_{1122} & 0 & 0 \\ Q_{1122} & Q_{2222} & 0 & 0 \\ 0 & 0 & Q_{1212} & Q_{1122} \\ 0 & 0 & Q_{1122} & Q_{1212} \end{pmatrix} \begin{matrix} 11 \\ 22 \\ 12 \\ 21 \end{matrix} \quad \text{with } Q_{1122}^2 \leq \min\left(Q_{1111}Q_{2222}, Q_{1212}^2\right)$$

(and the other ones positive)

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- characterize the most general cone

Ex: n=2 scalars with $\mathbb{Z}_2$

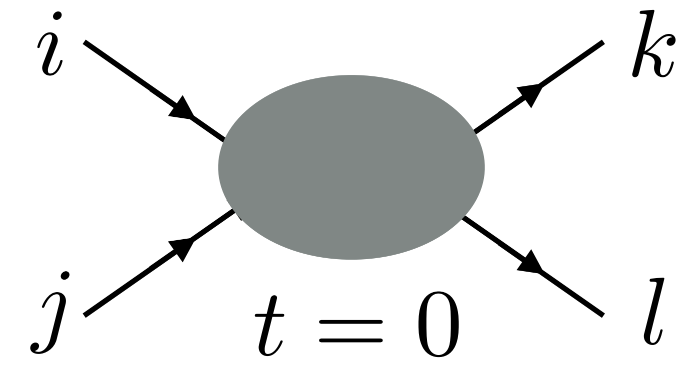
$$Q = \begin{pmatrix} Q_{1111} & Q_{1122} & 0 & 0 \\ Q_{1122} & Q_{2222} & 0 & 0 \\ 0 & 0 & Q_{1212} & Q_{1122} \\ 0 & 0 & Q_{1122} & Q_{1212} \end{pmatrix} \begin{matrix} 11 \\ 22 \\ 12 \\ 21 \end{matrix} \text{with } Q^2_{1122} \leq \min\left(Q_{1111}Q_{2222}, Q^2_{1212}\right)$$

(and the other ones positive)

$$Q = a\begin{pmatrix} 1 & r & 0 & 0 \\ r & r^2 & 0 & 0 \\ 0 & 0 & |r| & r \\ 0 & 0 & r & |r| \end{pmatrix} + b\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? <u>Approaches :</u>

- characterize the most general cone

Ex: n=2 scalars with $\mathbb{Z}_2$

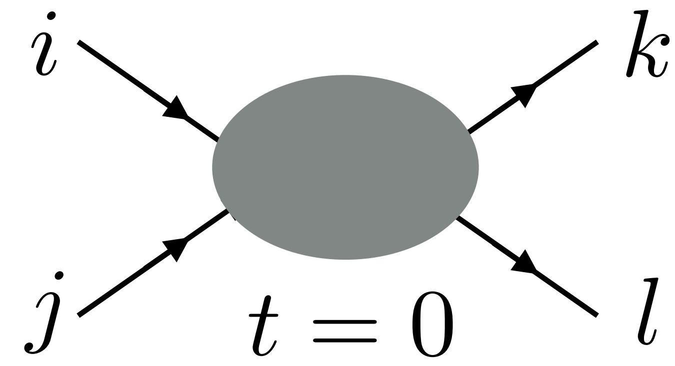
$$Q = \begin{pmatrix} Q_{1111} & Q_{1122} & 0 & 0 \\ Q_{1122} & Q_{2222} & 0 & 0 \\ 0 & 0 & Q_{1212} & Q_{1122} \\ 0 & 0 & Q_{1122} & Q_{1212} \end{pmatrix} \begin{matrix} 11 \\ 22 \\ 12 \\ 21 \end{matrix} \text{with } Q^2_{1122} \leq \min\left(Q_{1111}Q_{2222}, Q^2_{1212}\right)$$

(and the other ones positive)

$$Q = a \begin{pmatrix} 1 & r & 0 & 0 \\ r & r^2 & 0 & 0 \\ 0 & 0 & |r| & r \\ 0 & 0 & r & |r| \end{pmatrix} + b \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



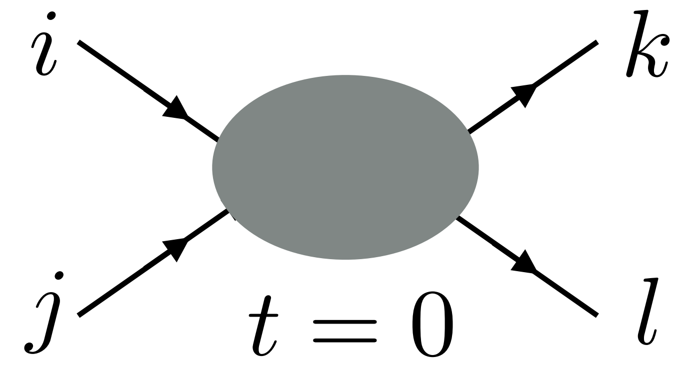
How to get positivity bounds ? <u>Approaches :</u>

- characterize the most general cone

Ex: n=2 scalars with $\mathbb{Z}_2$ $\quad \mathcal{L} \supset \frac{1}{\Lambda^4} C_{ijkl} O_{ijkl}, \quad O_{ijkl} = \partial_\mu \phi_i \partial^\mu \phi_j \partial_\nu \phi_k \partial^\nu \phi_l$

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$



How to get positivity bounds ? Approaches :

- characterize the most general cone

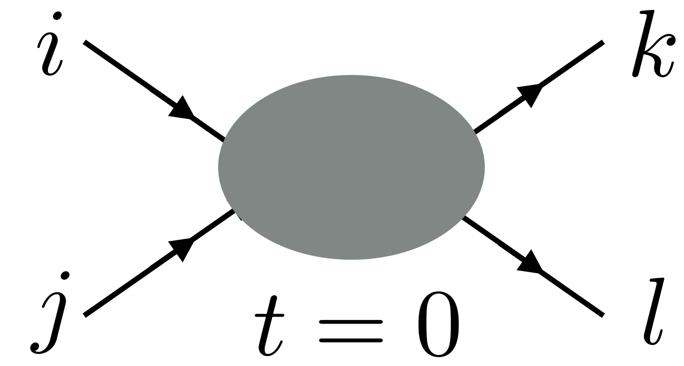
Ex: n=2 scalars with $\mathbb{Z}_2$ $\quad \mathcal{L} \supset \frac{1}{\Lambda^4} C_{ijkl} O_{ijkl}, \quad O_{ijkl} = \partial_\mu \phi_i \partial^\mu \phi_j \partial_\nu \phi_k \partial^\nu \phi_l$

$$C_{1111} \geq 0,\ C_{2222} \geq 0,\ C_{1212} \geq 0$$
$$4\sqrt{C_{1111}C_{2222}} \geq \pm(2C_{1122} + C_{1212}) - C_{1212}$$

From arXiv:2101.01191

Positivity bounds on EFTs : the flavored story

$$\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$$

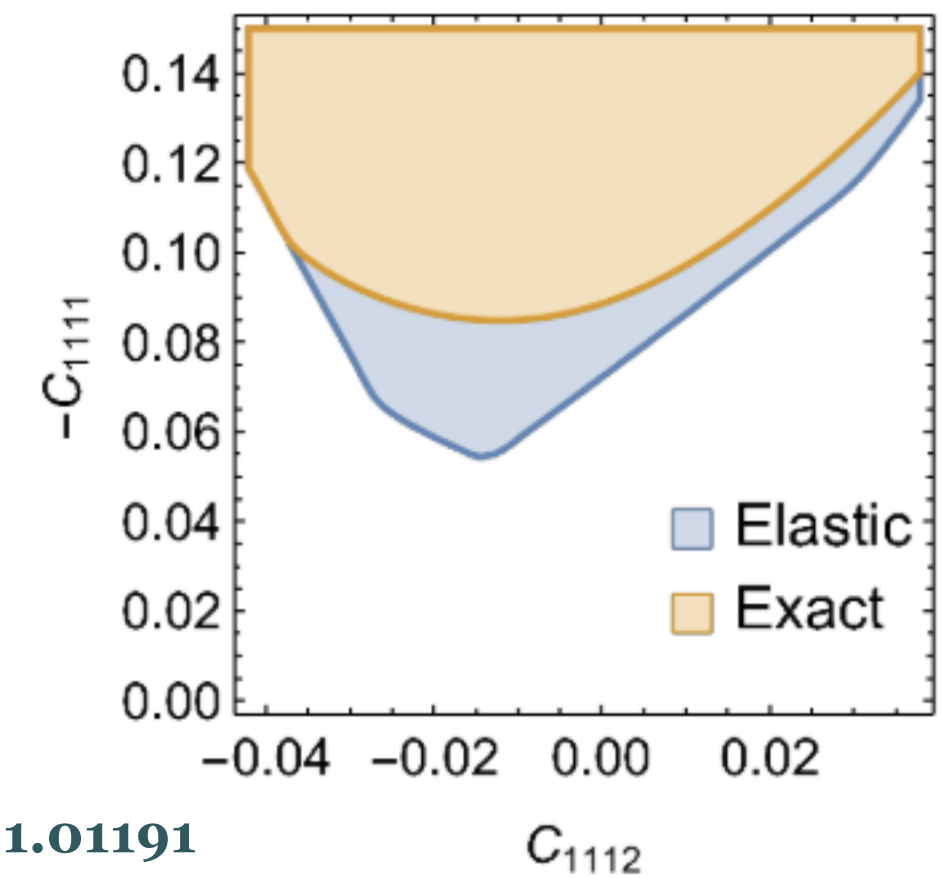


How to get positivity bounds ? Approaches :

- characterize the most general cone

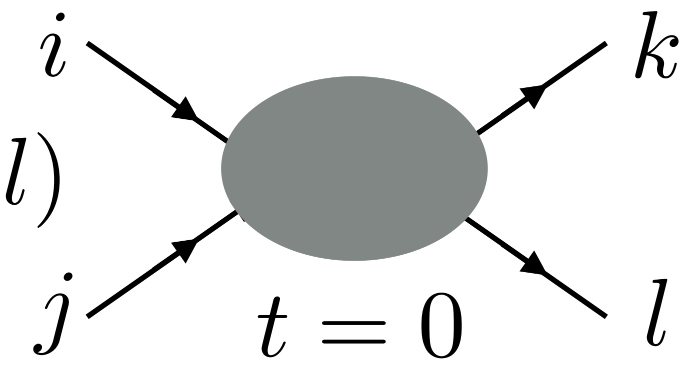
A lesson learnt : **best bounds**

(two RH fermions)



From arXiv:2101.01191

Positivity bounds on EFTs : the flavored story



Summary : $\widetilde{\mathcal{A}}_{ijkl}(s) = \sum_X m_{ij\to X} m^*_{kl\to X} + (j \leftrightarrow l)$

-dispersion relations relate IR amplitues and UV properties

-at dim-8 : **positivity bounds**

-with flavours : inelastic amplitudes described by **convex geometry**

-powerful results on **spectrahedra** solve (at least numerically) the problem

-if extremal amplitude : constraints on the UV spectra (e.g. symmetry irreps) - relation with the **inverse problem**