Muon $g-2$ and physics beyond the SM

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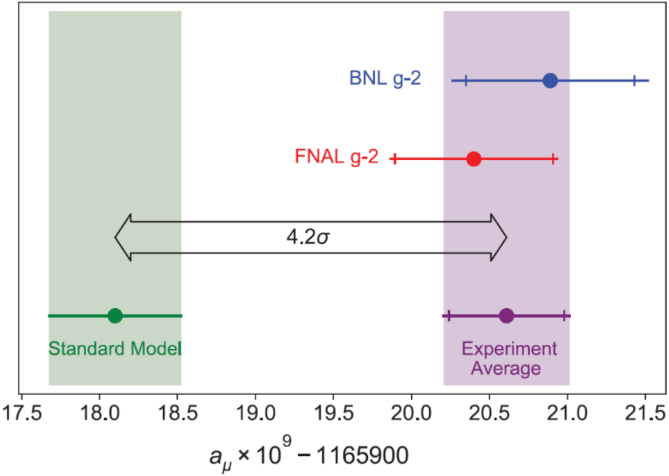
$(g_\mu - 2)/2 = a_\mu$ is among the most precise observables sensitive to all known (and unknown?) interactions

Finally: Fermilab Run 1 versus Theory Initiative SM value



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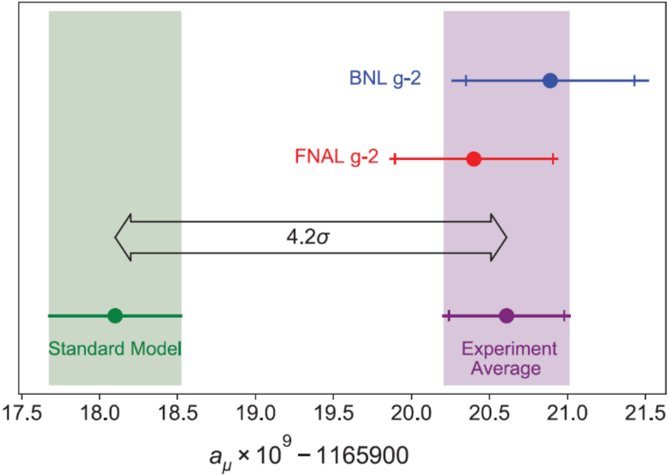


Which models can(not) explain it?

Outline

Technical summary lecture 1

SM prediction too low by $\approx (25 \pm 6) \times 10^{-10}$

$$\mathcal{L}_{\text{eff}} = \frac{Qe}{2}\left(c\bar{\psi}_R\sigma_{\mu\nu}\psi_L + c^*\bar{\psi}_L\sigma_{\mu\nu}\psi_R\right)F^{\mu\nu}$$

$$a_\mu = -2m_\mu \mathrm{Re}(c)$$

$$d_\mu = Qe\,\mathrm{Im}(c)$$

$$a_\mu \sim m_\mu \times (\text{some VEV}) \times (\psi_{L\leftrightarrow R}\text{-flipping param.}) \times \frac{(\text{other couplings})}{M^2_{\text{typical}}}$$

$$\delta m_\mu = \frac{1}{16\pi^2}\left\{ m_\mu \left[|c_L|^2 + |c_R|^2\right] B_1 + m_F\, \mathrm{Re}\left[c_L c_R{}^*\right] B_0\right\}$$

$$a_\mu = \frac{m_\mu}{16\pi^2}\left\{\frac{m_\mu}{12 m_S^2}\left[|c_L|^2 + |c_R|^2\right] F_1^C + \frac{2m_F}{3m_S^2}\mathrm{Re}\left[c_L c_R{}^*\right] F_2^C\right\}$$

Two important general points

discrepancy $\approx 2 \times a_\mu^{\text{SM,weak}}$

but: expect $a_\mu^{\text{NP}} \sim a_\mu^{\text{SM,weak}} \times \left(\frac{M_W}{M_{\text{NP}}}\right)^2 \times$ couplings

loop-induced, CP- and Flavor-conserving, chirality-flipping ψ_R ψ_L

compare: EDMs, $b \to s\gamma$, $B \to \tau\nu$, $\mu \to e\gamma$ EWPO

Questions: Which models can(not) explain it?

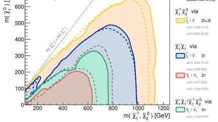
Why is a single number so interesting?

"Why are you happy about a discrepancy?"

$\Rightarrow$ we might make significant progress!

Outlook: concrete models; general relations to flavor, CP, dark matter

Technical summary lecture 2



model without chiral enhancement:

$$C_{\mathrm{BSM}} = \frac{\delta m_\mu}{m_\mu} \sim \frac{|\lambda_L|^2}{16\pi^2}, \quad a_\mu \sim C_{\mathrm{BSM}} \frac{m_\mu^2}{M_\phi^2}$$

LQ S_1 allows coupling to μ_L and μ_R, exemplifies chiral enhancement:

$$C_{\mathrm{BSM}} = \frac{\delta m_\mu}{m_\mu} \sim \frac{\lambda_L \lambda_R m_t}{8\pi^2 m_\mu} \sim 20\lambda_L\lambda_R$$

SUSY preview:

$$C_{\mathrm{BSM}} = \frac{\delta m_\mu}{m_\mu} \sim \frac{yg \times gv_u}{16\pi^2 y v_d} \sim \tan\beta \frac{\alpha}{4\pi}$$

Model-independent relations:

$$d_\mu \approx \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 2\times 10^{-22} e\,\mathrm{cm} \times \tan\phi_\mu,$$

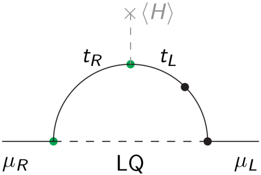
$$d_e \approx \left(\frac{\Delta a_e}{7\times 10^{-14}}\right) 10^{-24} e\,\mathrm{cm} \times \tan\phi_e,$$

$$BR(\mu\to e\gamma) \approx \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2 2\times 10^{-13} \left(\frac{\theta_{\mu e}}{10^{-5}}\right)^2,$$

Ex. from Lec. 2: Analysis of a_μ in the leptoquark model S_1

$$\delta m_\mu = \frac{1}{16\pi^2}\Big\{ \; m_\mu \; \big[|c_L|^2 \ldots\big] \, B_1 + \; m_F \; \mathrm{Re}\,[c_L c_R{}^*] B_0 \Big\}$$

$$a_\mu = \frac{m_\mu}{16\pi^2}\Big\{ \frac{m_\mu}{12 m_S^2} \, \big[|c_L|^2 \ldots\big] \, F_1^C + \frac{2 m_F}{3 m_S^2} \mathrm{Re}\,[c_L c_R{}^*] F_2^C \Big\}$$



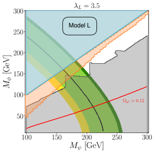
$$C_{\mathrm{BSM}} = \frac{\delta m_\mu}{m_\mu} \sim \frac{\lambda_L \lambda_R m_t}{8\pi^2 m_\mu} \sim 20 \lambda_L \lambda_R$$ huge enhancement, $\mathcal{O}(1)$ possible!

$$a_\mu \sim C_{\mathrm{BSM}} \frac{m_\mu^2}{M_S^2}$$ may explain a_μ for $M_S \gtrsim 2$ TeV

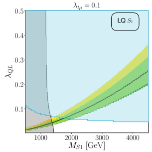
Interpretation and caution:

- additive structure $m_\mu = y_\mu v + \frac{\lambda_L \lambda_R m_t}{8\pi^2}$
- Huge enhancement BUT beware of finetuning ($\Leftrightarrow C_{\mathrm{BSM}} \gg 1$)

Physics summary



- 2-field models:
  - either entirely excluded (for a_μ)
  - Or viable around $M \sim 200$ GeV, DMRD too small



- Single-LQ models:
  - S_1 or R_2: large chiral enhancements
  - Explain a_μ for $M \gtrsim 1.3$ TeV (LHC-limit)
  - finetuning considerations on m_μ, m_e: ultra-large contributions, non-naive scaling implausible

$(SU(3)_C \times SU(2)_L \times U(1)_Y)_{\text{spin}}$	$+\mathbb{Z}_2$	Result for $\Delta a_\mu^{\text{BNL}}$, Δa_μ^{2021}
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Projected LHC 14 TeV exclusion, not confirmed
	Yes	Updated Sec. 4.2
$(\mathbf{1},\mathbf{1},-1)_0 - (\mathbf{1},\mathbf{1},0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{2},-1/2)_0 - (\mathbf{1},\mathbf{1},0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	No	Excluded: LHC searches
	Yes	Updated Sec. 4.2
$(\mathbf{1},\mathbf{2},-1/2)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Excluded: LEP contact interactions
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{1},-1)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{2},-1/2)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: LEP search
$(\mathbf{1},\mathbf{2},-1/2)_0 - (\mathbf{1},\mathbf{3},0)_{1/2}$	No	Excluded: LHC searches
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{2},-1/2)_0 - (\mathbf{1},\mathbf{3},-1)_{1/2}$	No	Excluded: LHC searches + LEP contact interactions
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{3},0)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
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$(\mathbf{1},\mathbf{3},-1)_{1/2} - (\mathbf{1},\mathbf{3},0)_1$	No	Excluded: $\Delta a_\mu < 0$

- Other simple models
  - Often sign wrong⇝excluded
  - Interesting/viable: some LQ, 2HDM, VLL, Z'
- Correlations:
  - a_μ a_e d_μ d_e $\mu \to e\gamma$
  - a_μ tests Re(c^{22}), strong constraints on c^{12} and Im(c^{11})
  - SM (and some other models) naturally predict $c^{12} \approx 0$ and Im(c^{11}) ≈ 0
  - maybe we should prefer BSM with similar properties

Outline

Three obvious relationships

- a_μ versus a_e $\leftarrow$ naive scaling? universal couplings?
- a_μ versus d_μ (and d_e) $\leftarrow$ CP violation?
- a_μ versus $\mu \to e\gamma$ $\leftarrow$ lepton flavour violation?

Of course, further relationships exist as well

Limits on EDMs:	$\|d_e\| < 8.7 \times 10^{-29} e\,\mathrm{cm}$
	$\|d_\mu\| < 1.5 \times 10^{-19} e\,\mathrm{cm}$
Limit on $\mu \to e\gamma$:	$BR(\mu \to e\gamma) < 4.2 \times 10^{-13}$

Efficient formulation, dimension-5 effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{Qe}{2}\left(c^{ij}\bar{\psi}^i_R\sigma_{\mu\nu}\psi^j_L + c^{*ij}\bar{\psi}^i_L\sigma_{\mu\nu}\psi^j_R\right)F^{\mu\nu}$$

$$a_\mu = -2m_\mu \mathrm{Re}(c^{22}) \qquad d_\mu = Qe\,\mathrm{Im}(c^{22})$$

$$a_e = -2m_e \mathrm{Re}(c^{11}) \qquad d_e = Qe\,\mathrm{Im}(c^{11})$$

$$BR(\mu \to e\gamma) = \frac{e^2 m_\mu^3}{\pi\Gamma_\mu}(|c^{21}|^2 + |c^{12}|^2)$$

Relations and estimates

also: [Giudice, Paradisi, Passera 2012] [Crivellin, Hoferichter, Schmidt-Wellenburg 2018]

Can unify description of MDM, EDM, CLFV: generalize $\mathcal{L}_{\text{eff}} \sim c^{ij}$ to leptons i, j with coefficients c^{ij}, $c^{ij} \propto$ VEV $\times$ chir.-flip:

$$d_\mu \approx \left(\frac{\Delta a_\mu}{3\times10^{-9}}\right) 2\times10^{-22} e\,\text{cm} \times \tan\phi_\mu,$$

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$$BR(\mu\to e\gamma) \approx \left(\frac{\Delta a_\mu}{3\times10^{-9}}\right)^2 2\times10^{-13} \left(\frac{\theta_{\mu e}}{10^{-5}}\right)^2,$$

- SM: $\tan\phi \ll 1$, $\theta_{\mu e} \ll 1$.
- Current EDM, MEG limits: $\tan\phi_\mu \lesssim 1000$, ($\tan\phi_e \ll 1$ or $\Delta a_e \ll 10^{-14}$), $\theta_{\mu e} \lesssim 10^{-5}$
- New physics strongly restricted in $\theta_{\mu e}$ and $\tan\phi_e$ but not in $\tan\phi_\mu$ $\rightsquigarrow$ improve!
- Naive scaling $c^{\ell\ell} = m_\ell \times$ const.:

  $\Delta a_e : \Delta a_\mu = m_e^2 : m_\mu^2, \quad d_e : d_\mu = m_e : m_\mu \quad \overset{\text{Exp.}}{\Rightarrow} \quad |d_\mu^{\text{naive sc.}}| \lesssim 10^{-27}$
- New physics possibilities: new flavor structures (LQ, sleptons, 2HDM-Yukawas), new flavor-independent parameters (complex Higgsino mass, gaugino masses)
- Note: neutron EDM and $\mu \to e$ conversion sensitive to non-dipole operators!

  Note 2: naive scaling is different from writing $a_\mu = C_{\text{BSM}} \frac{m_\mu^2}{M_{\text{BSM}}^2} \quad \rightsquigarrow \quad c^{\mu\mu} \sim m_\mu \times C_{\text{BSM}}^{\text{dimensionless BSM-couplings}}$

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SUSY: 3 generations of sleptons

$$c_R \to Y^{ij}, c_L \to g\delta^{ij}$$

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2-field model: lepton number? E.g. three generations of ψ?

$$\lambda_L^{ij} L^i \cdot \psi_d^j \phi, \qquad \text{so far assumed: } \lambda_L^{ij} = \begin{cases} \lambda_L (ij = 22) \\ 0 (ij \neq 22) \end{cases}$$

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LQ model: actually matrix-valued couplings even for just one leptoquark

$$\lambda_{QL}^{ij} Q^i \cdot L^j S_1 + \lambda_{t\mu}^{ij} u^i \ell^j S_1^* \qquad \text{so far assumed: } \lambda_{L,R}^{ij} = \begin{cases} \lambda_{L,R}^{32} \\ 0 (ij \neq 32) \end{cases}$$

In principle there can be non-trivial flavour structure. What happens in that case?
Strong limits on CPV in d_e and on $\mu \to e\gamma$ — need specific patterns!

a_μ versus a_e: Which models lead to naive scaling?

$$\frac{\delta m_\ell^{\rm BSM}}{m_\ell} = C^\ell_{\rm BSM} \qquad a_\ell^{\rm BSM} = \mathcal{O}(C^\ell_{\rm BSM})\frac{m_\ell^2}{M^2_{\rm BSM}} \qquad \frac{a_\mu}{a_e} \approx \frac{m_\mu^2}{m_e^2} \qquad ?$$

SM: gauge interactions are universal $\rightsquigarrow$ naive scaling holds!

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2-field model:

$$\delta m_\mu = \frac{1}{16\pi^2}\left\{ m_\mu \left[|c_L|^2 + |c_R|^2\right] B_1 + m_F \ldots \right\}$$

$$a_\mu = \frac{m_\mu}{16\pi^2}\left\{ \frac{m_\mu}{12 m_S^2} \left[|c_L|^2 + |c_R|^2\right] F_1^C + \frac{2m_F}{3m_S^2} \ldots \right\}$$

$$C_{\text{BSM}}^{\ell\ell} \sim \frac{|\lambda_L^{\ell\ell}|^2}{16\pi^2}$$

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$$C^{\ell\ell}_{\rm BSM} \sim \frac{|\lambda_L^{\ell\ell}|^2}{16\pi^2} \qquad\qquad \frac{a_\mu}{a_e} \sim \frac{m_\mu^2}{m_e^2}\underbrace{\frac{|\lambda_L^{\mu\mu}|^2}{|\lambda_L^{ee}|^2}}$$

universal couplings plausible

$\rightsquigarrow$naive scaling

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SUSY:
$$\delta m_\mu = \frac{1}{16\pi^2}\left\{ m_\mu \left[|c_L|^2 \ldots\right] B_1 + m_F \, \mathrm{Re}\left[c_L c_R{}^*\right] B_0\right\}$$
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$$C_{\text{BSM}}^{\ell\ell} \sim \frac{y_\ell g \times g v_u}{16\pi^2 y_\ell v_d} \sim \tan\beta \frac{g^2}{16\pi^2}$$

a_μ versus a_e: Which models lead to naive scaling?

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naive scaling

thanks to $c_R \sim y_\ell$

a_μ versus a_e: Which models lead to naive scaling?

$$\frac{\delta m_\ell^{\text{BSM}}}{m_\ell} = C_{\text{BSM}}^\ell \qquad a_\ell^{\text{BSM}} = \mathcal{O}(C_{\text{BSM}}^\ell)\frac{m_\ell^2}{M_{\text{BSM}}^2} \qquad \frac{a_\mu}{a_e} \approx \frac{m_\mu^2}{m_e^2} \qquad ?$$

LQ S_1 (similar to generic models):

$$\delta m_\mu = \frac{1}{16\pi^2}\left\{ m_\mu \left[|c_L|^2 \dots\right] B_1 + m_F \,\mathrm{Re}\left[c_L c_R{}^*\right] B_0 \right\}$$

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$$C^{\ell\ell}_{\rm BSM} \sim \frac{(\lambda_L\lambda_R)^{\ell\ell} m_t}{8\pi^2 m_\ell} \qquad \frac{a_\mu}{a_e} \sim \frac{m_\mu^2}{m_e^2}\underbrace{\frac{m_e}{m_\mu}\frac{(\lambda_L\lambda_R)^{\mu\mu}}{(\lambda_L\lambda_R)^{ee}}}_{\text{depends on } \lambda^{\ell\ell}/m_\ell}$$

a_μ versus a_e: Which models lead to naive scaling?

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Case 1: couplings $(\lambda_L\lambda_R)^{\ell\ell} \sim m_\ell$ like Higgsinos $\Rightarrow$ naive scaling

a_μ versus a_e: Which models lead to naive scaling?

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$$C_{\text{BSM}}^{\ell\ell} \sim \frac{(\lambda_L \lambda_R)^{\ell\ell} m_t}{8\pi^2 m_\ell} \qquad \frac{a_\mu}{a_e} \sim \frac{m_\mu}{m_e}$$

Case 2: couplings $(\lambda_L\lambda_R)^{\text{univ.}}$ flavour-universal $\Rightarrow$ different (=linear) scaling

Case of non-naive (linear) scaling $a_\mu : a_e$ via $(\lambda_L \lambda_R)^{\text{univ.}}$

$$C^{\ell\ell}_{\text{BSM}} \sim \frac{(\lambda_L\lambda_R)^{\text{univ.}} m_t}{8\pi^2 m_\ell} \qquad a_\ell^{\text{BSM}} = \mathcal{O}(C^{\ell}_{\text{BSM}})\frac{m_\ell^2}{M^2_{\text{BSM}}} \qquad \frac{a_\mu}{a_e} \sim \frac{m_\mu}{m_e}$$

Nice, but look what happens in the absolute mass corrections:

$$m_\mu \sim y_\mu v + \frac{(\lambda_L\lambda_R)^{\text{univ.}} m_t}{8\pi^2}$$

$$m_e \sim y_e v + \frac{(\lambda_L\lambda_R)^{\text{univ.}} m_t}{8\pi^2}$$

Case of non-naive (linear) scaling $a_\mu : a_e$ via $(\lambda_L \lambda_R)^{\text{univ.}}$

$$C^{\ell\ell}_{\text{BSM}} \sim \frac{(\lambda_L \lambda_R)^{\text{univ.}} m_t}{8\pi^2 m_\ell} \qquad a^{\text{BSM}}_\ell = \mathcal{O}(C^\ell_{\text{BSM}}) \frac{m_\ell^2}{M^2_{\text{BSM}}} \qquad \frac{a_\mu}{a_e} \sim \frac{m_\mu}{m_e}$$

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- Same additive corrections to the muon and electron mass.
- Plausible if correction small.
- Very implausible if the corrections are bigger than the muon mass! But this is what we need to explain a_μ with $M_{\text{BSM}} \gg 1$ TeV.
- In general, if a_μ is explained,
  the corrections are bigger than the muon mass if $M_{\text{BSM}} \gtrsim 2$ TeV
  the corrections are bigger than the electron mass if $M_{\text{BSM}} \gtrsim 70$ GeV

Bottom line:

- Naive scaling holds in many models without chiral enhancement (e.g. our 2-field model)
- And it holds in many models with chiral enhancement (e.g. SUSY and LQ if couplings $\propto$ lepton mass)
- In models with chiral enhancement also

$$a_\mu : a_e \sim m_\mu : m_e$$

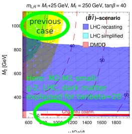
  is plausibly possible.
- However: I regard it as particularly plausible for small $M_{\text{BSM}} \lesssim 70$ GeV, where the contributions to $m_{\mu,e}$ are insignificant.
- I regard it as less plausible in case of models with large masses and huge corrections to m_e (or even to m_μ).

Outline

Survey of many examples. . .

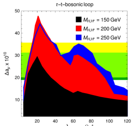
SUSY: MSSM, MRSSM

- MSugra. . . many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns



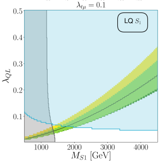
Two-Higgs doublet model

- Type I, II, Y, Type X(lepton-specific), flavour-aligned



Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to μ_L and μ_R



Simple models (one or two new fields)

- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_\mu - L_\tau$)

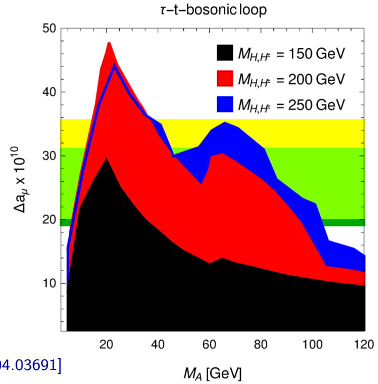
Model	Spin	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Result
1	0	$(\mathbf{1},\mathbf{1},1)$	Ruled out: $\Delta a_\mu < 0$
2	0	$(\mathbf{1},\mathbf{1},2)$	Ruled out: $\Delta a_\mu < 0$
3	0	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	Special cases viable
4	0	$(\mathbf{1},\mathbf{3},-1)$	Ruled out: $\Delta a_\mu < 0$
5	0	$(\bar{\mathbf{3}},\mathbf{1},1/3)$	Updated Sec. 3.2.
6	0	$(\bar{\mathbf{3}},\mathbf{1},4/3)$	Ruled out: $\Delta a_\mu < 0$
7	0	$(\bar{\mathbf{3}},\mathbf{3},1/3)$	Ruled out: $\Delta a_\mu < 0$
8	0	$(\mathbf{3},\mathbf{2},7/6)$	Updated Sec. 3.2.
9	0	$(\mathbf{3},\mathbf{2},2/3)$	Ruled out: $\Delta a_\mu < 0$
10	1/2	$(\mathbf{1},\mathbf{1},0)$	Ruled out: $\Delta a_\mu < 0$
11	1/2	$(\mathbf{1},\mathbf{1},-1)$	Ruled out: $\Delta a_\mu < 0$ or too small (disputed)
12	1/2	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$	Ruled out: $\Delta a_\mu < 0$ or too small (disputed)
13	1/2	$(\mathbf{1},\mathbf{2},-\frac{3}{2})$	Ruled out: $\Delta a_\mu < 0$
14	1/2	$(\mathbf{1},\mathbf{3},0)$	Ruled out: $\Delta a_\mu < 0$
15	1/2	$(\mathbf{1},\mathbf{3},-1)$	Ruled out: $\Delta a_\mu < 0$
16	1	$(\mathbf{1},\mathbf{1},0)$	Special cases viable
17	1	$(\mathbf{1},\mathbf{2},-3/2)$	Ruled out: UV compl. $M_{Z'}$ limit

[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]

Two-Higgs doublet model: $M_A < 100$ GeV

- Aligned 2-Higgs doublet model, rich new Higgs/Yukawa sectors

  [Type X extensively studied by E.J. Chun et al, Aligned (incl. full 2-loop) by Cherchiglia et al]



[2104.03691]

Details on Yukawa couplings:

Type X/lepton-specific: $Y_\ell \propto \tan\beta$ Type II: $Y_{\ell,d} \propto \tan\beta$ Aligned: $Y_\ell \propto \zeta_\ell$

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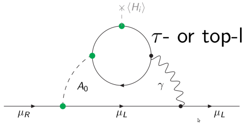
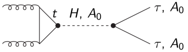
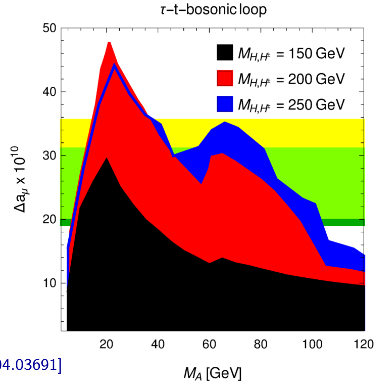


LHC constraints:



Also: τ-dec., $Z \to \tau\tau$, EWPO



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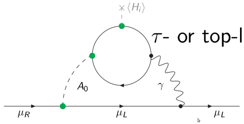

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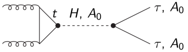

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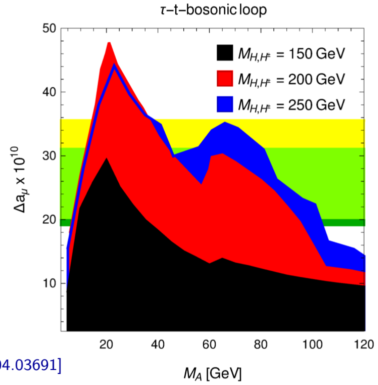

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- under pressure, testable at LHC, lepton colliders, B-physics

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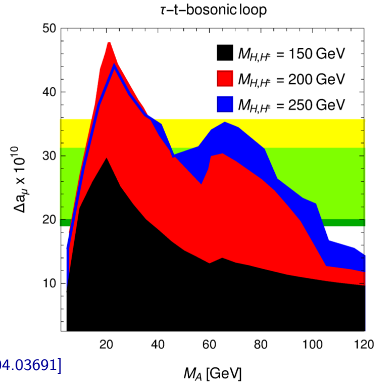
Further constraints

- τ-, Z-decays, LEP
- b-decays, LHC

$\Rightarrow$ maximum Yukawa couplings

- lepton Yukawa $<\sim 100$
- quark Yukawas $<\sim 0.5$
- (for $M_A = 20 \ldots 100$ GeV, else even stronger)



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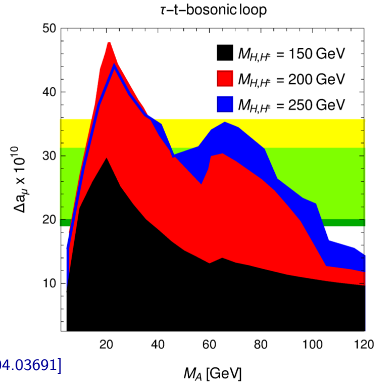

[2104.03691]

Minimal Supersymmetric Standard Model

- fundamental new QFT symmetry
- predicts Higgs potential/mass
- dark matter candidate
- chirality flip enhancement $\rightsquigarrow g-2$
- viable (LHC)?

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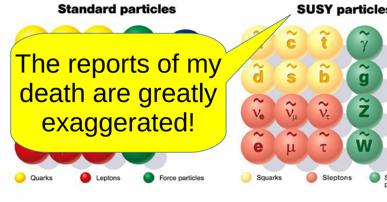

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Superpartners and SUSY Higgs sector $\rightsquigarrow \tan\beta = \frac{v_u}{v_d}$, Higgsino mass μ

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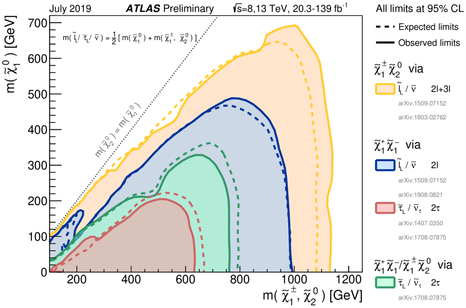

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- fundamental new QFT symmetry
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- dark matter candidate
- chirality flip enhancement $\rightsquigarrow g-2$
- viable (LHC)?

Remarks on dark matter:

- Bino-LSP $\approx$requires chargino- or slepton/stau-coannihilation
- Higgsino- or Wino-LSP produce underabundant DM (unless masses $\gtrsim 1$ TeV)

Superpartners and SUSY Higgs sector $\rightsquigarrow \tan\beta = \frac{v_u}{v_d}$, Higgsino mass μ

Analysis: a_μ in the MSSM

Typical SUSY contributions are chirally enhanced — Two interesting cases:

Wino–Higgsino–smuon or Bino–smuonL–smuonR(+heavy Higgsino)

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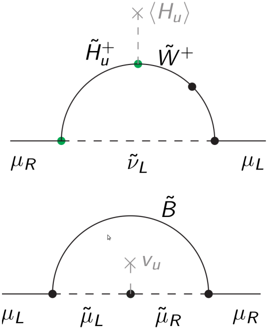
Higgsino-coupling	$c_R \sim y$
Bino/Wino-coupling	$c_L \sim g_{1,2}$



$\propto \mu$ for $\mu \to \infty$

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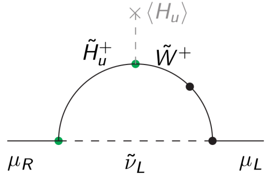
Wino–Higgsino–smuon   or   Bino–smuonL–smuonR(+heavy Higgsino)

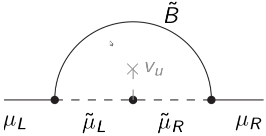
$$\begin{aligned} \text{Higgsino-coupling} \quad & c_R \sim y \\ \text{Bino/Wino-coupling} \quad & c_L \sim g_{1,2} \end{aligned}$$

$$C_{\text{BSM}} = \frac{\delta m_\mu}{m_\mu} \sim \frac{yg \times gv_u}{16\pi^2 y v_d} \sim \tan\beta \frac{g^2}{16\pi^2}$$

$$a_\mu(WHL) \approx 21 \times 10^{-10} \left(\frac{500\text{ GeV}}{M_{\text{SUSY}}}\right)^2 \frac{\tan\beta}{40}$$

$$a_\mu(BLR) \approx 2.4 \times 10^{-10} \left(\frac{500\text{ GeV}}{M_{\text{SUSY}}}\right)^2 \frac{\tan\beta}{40} \frac{\mu}{500\text{ GeV}}$$





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MSSM can explain $g-2$ and dark matter

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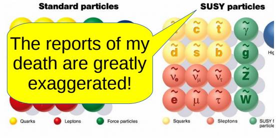
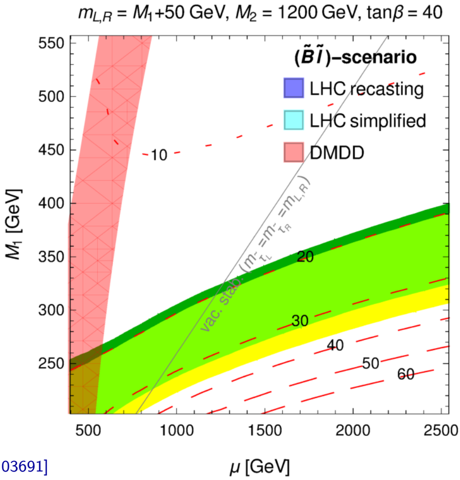
- Bino-LSP, close-by sleptons
- DM explained by stau/slepton-coannihilation
- explains $g-2$ in large region (expands for $\tan\beta \neq 40$) (both WHL and BLR important)
- this automatically evades (current) LHC limits





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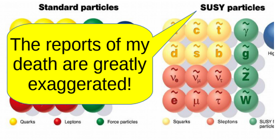
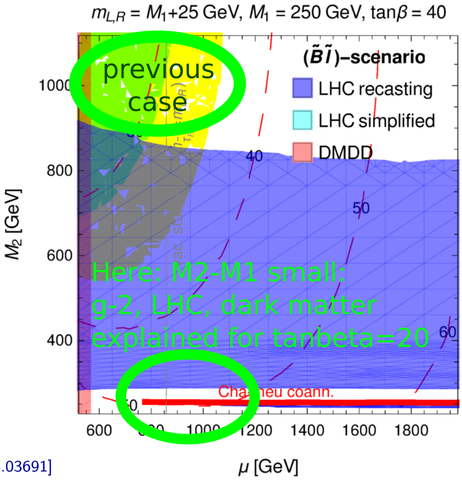
- Still Bino-LSP and close-by sleptons
- Now lower M_w: strong LHC limits
- DM also explained by Wino-coannihilation
- again evades (current) LHC limits





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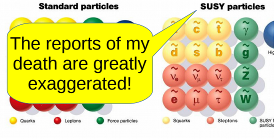
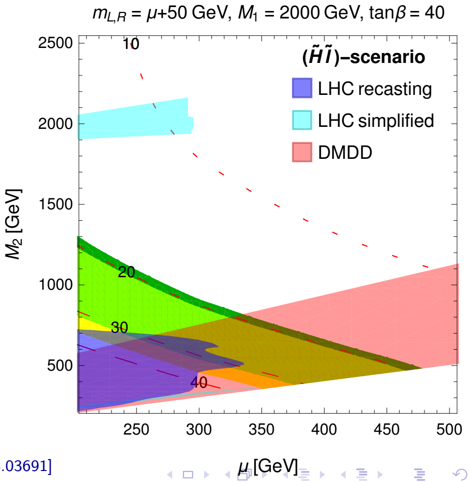
- Higgsino-LSP and $\approx$ light sleptons
- DMRD too small
- significant LHC limits on M_2
- $\Rightarrow$ attractive, generic scenario





[2104.03691]

Brief MSSM highlights — promising scenarios

- Bino-LSP:
  - DM explained via slepton-coannihilation (heavy M_2, $\mu \gtrsim 1$ TeV ok)

    super-large μ often motivated in high-scale models

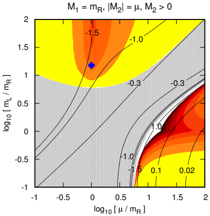
    typically $M_1 < M_2/2$ — how to arrange that?
  - DM explained via Wino-coannihilation (sleptons close-by)

    how to arrange Bino, Wino, sleptons to have similar masses?
- Higgsino-LSP (and Wino-LSP is similar)
  - fine if we accept some other DM candidate
  - sleptons reasonably light to evade LHC

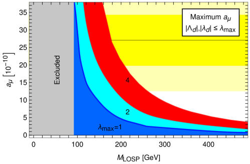
    such scenarios appear e.g. in GMSB, Bhattacharyya, Yanagida, Yokozaki '18
- Cannot explain a_μ: mSUGRA/CMSSM,...
- There are other possibilities, e.g.
  radiative m_μ (zero Yukawa [Crivellin,Nierste,Westhoff], $\tan\beta \to \infty$ [Bach,Park,DS,Stöckinger-Kim]),

  many ("flavourful") VEVs [Altmannshofer,Gadam,Gori,Hamer]

Further SUSY models: SUSY is more than MSSM!



Largest MSSM (tan $\beta \to \infty$)



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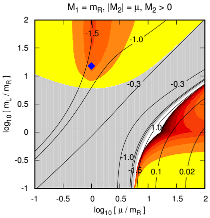

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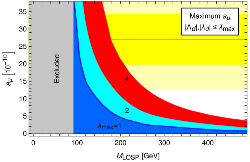


Largest MRSSM

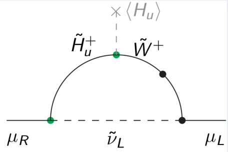
a_μ in SUSY with $\tan\beta \to \infty$ ($m_\mu^{\text{tree}} = y_\mu v_d = 0$) [Bach,Park,DS,Stöckinger-Kim '15]

First: standard SUSY,

$\tan\beta = v_u / v_d \sim 50$

$$a_\mu^{\text{SUSY}} \approx y_\mu v_u \times \text{loop}$$

$$m_\mu^{\text{pole}} \approx y_\mu v_d$$



Can explain Δa_μ if $M_{\text{SUSY}, \tilde{\mu}, \chi} \lesssim 500$ GeV

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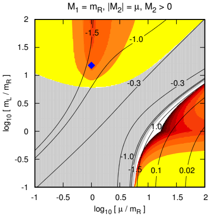

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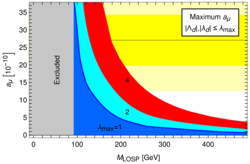

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$$a_\mu^{\text{SUSY}} \approx y_\mu v_u \times \text{loop}$$

$$m_\mu^{\text{pole}} \approx y_\mu v_u \times \text{loop}$$

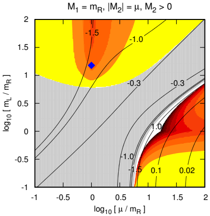
Results: a_μ explained even if $M_{\text{LSP}} > 1$ TeV $\rightsquigarrow$ largest a_μ^{SUSY}

tests: 1TeV chargino searches, Higgs-physics/couplings,...

Similar idea: decouple v_{u3}, v_{d3}, v_{u12}, v_{d12} allows $\tan\beta^\mu_{\text{eff}} \sim 500$

[Altmannshofer et al'21]

Further SUSY models: SUSY is more than MSSM!



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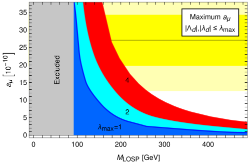
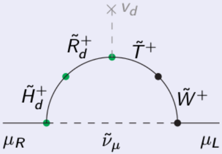


Largest MRSSM

a_μ in SUSY with continuous R-symmetry [Kotlarski,DS,Stöckinger-Kim '19]

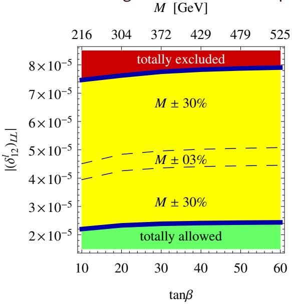


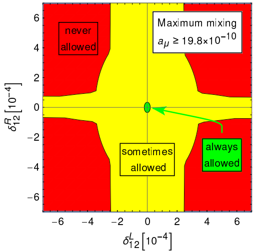
No Majorana gaugino masses!

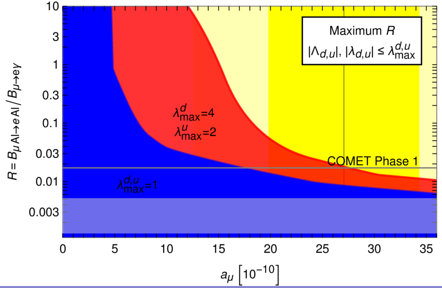
Results: no tan β-enhancement! a_μ explained for $M_{\text{SUSY}} \sim 100$GeV, compressed spectra; testable by LHC/ILC, $\mu \to e/\mu \to e\gamma$

Connection to CP and flavor (example)

illustration how $g-2$ forces us into special parameter regions





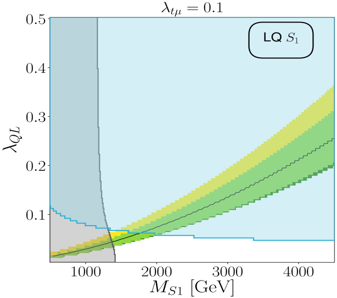


- given $g-2$, derive upper limits on LFV parameters from $\mu \to e\gamma$

  MSSM:
  [Kersten,Park,DS,Velasco-Sevilla '14]
  MRSSM:
  [Kotlarski,DS,Stöckinger-Kim'19]

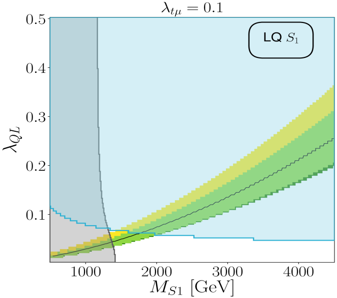
- MRSSM: large $g-2$ enforces special parameter space with restricted $\mu \to e/\mu \to e\gamma$

Leptoquarks and other chirally enhanced models



[2104.03691]

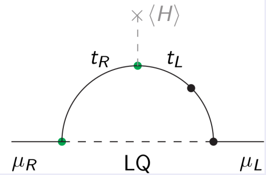
Leptoquarks and other chirally enhanced models



[2104.03691]

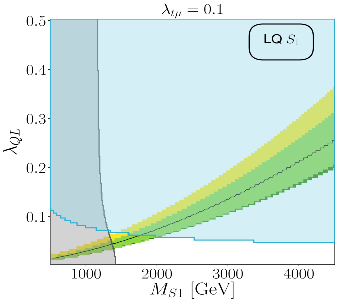
a_μ from LQ (or VLL) $\mathcal{L}_{S_1} = -\left(\lambda_{QL} Q_3 \cdot L_2 S_1 + \lambda_{t\mu} t \mu S_1^*\right)$

Specific LQ that works:



- Chiral enhancement $\sim y_{\text{top}}, y_{\text{VLL}}$ versus y_μ
- LHC: lower mass limits
- Flavour constraints $\rightsquigarrow$ assume only couplings to muons
- Viable window above LHC (without m_μ-finetuning)

Leptoquarks and other chirally enhanced models



[2104.03691]

Comments, extensions

- Need specific flavour pattern!
- Several specific LQ types work
- Example Greljo,Stangl,Thomsen'21: Gauged $U(1)_{B-3L_\mu}$ (e.g. sub-GeV Z') "Muo"quarks S_1 and S_3 explain a_μ, $R(K)$
- Example Spin-1-LQ Ban,Jho,Kwon,Park,Park,Tseng'21: Specific type: U1 with couplings μ–b, s can explain a_μ, $R(K)$ and $R(D)$

a_μ from LQ (or VLL) $\mathcal{L}_{S_1} = -\left(\lambda_{QL} Q_3 \cdot L_2 S_1 + \lambda_{t\mu} t \mu S_1^*\right)$

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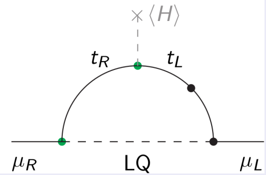

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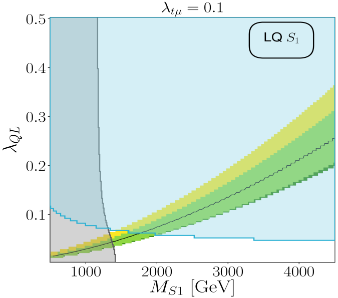
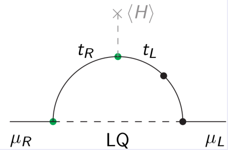


[2104.03691]

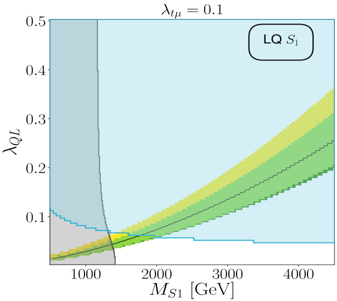
a_μ from vector-like leptons Dermisek,Raval'13

$$\mathcal{L} \ni -\lambda_L \bar{L}_L e_R H - M_L \bar{L}_L L_R - \bar{\lambda} H^\dagger \bar{E}_L L_R - M_E \bar{E}_L E_R - \lambda_R \bar{l}_L E_R H$$



- Similar to LQ: $\lambda_L \lambda_R y_t \longrightarrow \lambda_L \lambda_R \bar{\lambda}$
- Interesting: additional contributions to m_μ^{tree}
- $\mathcal{L}_{\text{eff}} \sim (h+v)^3 \bar{\mu}_L \mu_R$: if large $\rightsquigarrow$ factor $3^2 = 9$ in $R_{h\to\mu\mu}$!
- illustrates role of a_μ vs m_μ vs $h \to \mu\mu$

Leptoquarks and other chirally enhanced models



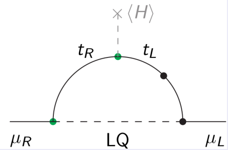
[2104.03691]

Other similar models

- Many generic 3-field extensions have chiral enhancements [Kowalska,Sessolo'17–'21][Calibbi et al'18–'21][Athron et al'21]
- can explain a_μ (and contain large δm_μ)
- Need at least 3 new fields for a_μ, LHC, dark matter
- Need at least 4 new fields for a_μ, LHC, dark matter and B-physics [Arcadi,Calibbi,Fedele,Mescia]

a_μ from vector-like leptons Dermisek,Raval'13

$$\mathcal{L} \ni -\lambda_L \bar{L}_L e_R H - M_L \bar{L}_L L_R - \bar{\lambda} H^\dagger \bar{E}_L L_R - M_E \bar{E}_L E_R - \lambda_R \bar{l}_L E_R H$$



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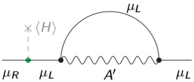
Light/dark sectors — compatible with large a_μ?

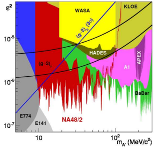
Very light, weakly interacting new particles

Light/dark sectors — compatible with large a_μ?

Very light, weakly interacting new particles

- "dark photon" NO $\mathcal{L} = -\frac{\epsilon}{2\cos\theta_W} F^{\mu\nu} B_{\mu\nu}$ $\quad a_\mu \sim \frac{\alpha}{2\pi}\epsilon^2$





[NA48: 1504.00607]
excludes minimal dark photon for a_μ

- "dark Z_d" Better $\quad a_\mu \sim \frac{\alpha}{2\pi}(\epsilon + \sim \delta' m_{Z_d}/m_Z)^2$

Additional mass mixing δ, may assume invisible decays into dark sector, can evade limits (still nontrivial)

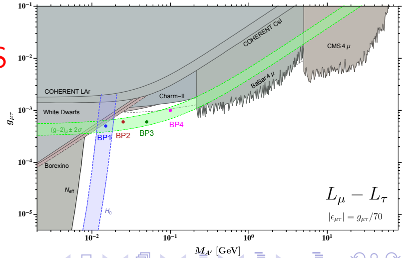
Davoudiasl,Lee,Marciano. . . Cadeddu,Cargioli,Dordei,Giunti,Picciau

- Z' with quantum number $L_\mu - L_\tau$ YES
  [Ma,Roy,Roy'01,Heeck,Rodejohann'11. . .]
  (plot from [Amaral,Cerdeno,Cheek,Foldenauer'21])

  Evades collider constraints,

  subject to low-E constraints,

  viable window 10 . . . 100 MeV



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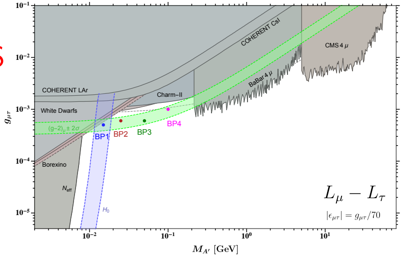
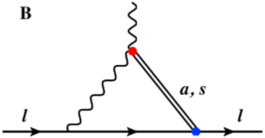
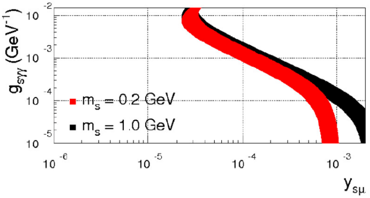


- "ALPs" YES however: UV completions may change the picture [Buen-Abad,Fan,Reece,Sun'21]

$$\mathcal{L} = \frac{1}{4} g_{s\gamma\gamma} s F^{\mu\nu} F_{\mu\nu} + y_s s \bar{\mu}\mu \qquad a_\mu^{\rm BZ} \sim \frac{m_\mu}{4\pi^2} g_{s\gamma\gamma} y_s \ln(\Lambda/m_s)$$



O



[Marciano, Masiero, Paradisi, Passera '16]

Outline

Summary of main points

discrepancy $\approx 2 \times a_\mu^{\mathrm{SM,weak}}$

but: expect $a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{SM,weak}} \times \left(\frac{M_W}{M_{\mathrm{NP}}}\right)^2 \times$ couplings

a_μ is loop-induced, CP- and flavor-conserving and chirality-flipping

rather light, neutral (?) particles $\rightsquigarrow$ Connection to dark matter?

Chirality flip enhancement $\rightsquigarrow$ Window to muon mass generation? EWSB/generations?

Which models can still accommodate large deviation?

Many (but not all) models!

but always: experimental constraints!

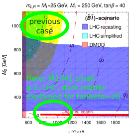
Outlook:

- $g-2$ + LHC, DM $\rightsquigarrow$ constraints on BSM physics, great potential for future
- often chirality flips/new flavor structures/light particles $\rightsquigarrow$ tests: Higgs couplings, B-physics, CLFV, EDM, light-particle searches, e^+e^-/muon collider

Survey of many examples. . .

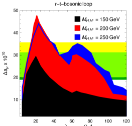
SUSY: MSSM, MRSSM

- MSugra. . . many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns



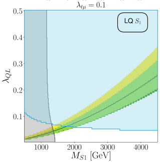
Two-Higgs doublet model

- Type I, II, Y, Type X(lepton-specific), flavour-aligned



Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to μ_L and μ_R



Simple models (one or two new fields)

- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_\mu - L_\tau$)

Model	Spin	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Result
1	0	$(\mathbf{1}, \mathbf{1}, 1)$	Ruled out: $\Delta a_\mu < 0$
2	0	$(\mathbf{1}, \mathbf{1}, 2)$	Ruled out: $\Delta a_\mu < 0$
3	0	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	Special cases viable
4	0	$(\mathbf{1}, \mathbf{3}, -1)$	Ruled out: $\Delta a_\mu < 0$
5	0	$(\mathbf{3}, \mathbf{1}, 1/3)$	Updated Sec. 3.2.
6	0	$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	Ruled out: $\Delta a_\mu < 0$
7	0	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	Ruled out: $\Delta a_\mu < 0$
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9	0	$(\mathbf{3}, \mathbf{2}, 2/3)$	Ruled out: $\Delta a_\mu < 0$
10	1/2	$(\mathbf{1}, \mathbf{1}, 0)$	Ruled out: $\Delta a_\mu < 0$
11	1/2	$(\mathbf{1}, \mathbf{1}, -1)$	Ruled out: $\Delta a_\mu < 0$ or too small (disputed)
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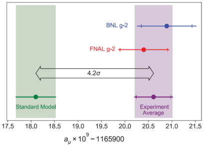
16	1	$(\mathbf{1}, \mathbf{1}, 0)$	Special cases viable
17	1	$(\mathbf{1}, \mathbf{2}, -3/2)$	Ruled out: UV compl. $M_{Z'}$ limit

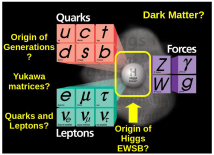
[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]

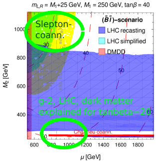
Conclusions

- SM prediction for $g-2$:
  - All known particles relevant (and all QFT tricks)
  - Theory Initiative: worldwide (ongoing!) effort, agreed & conservative value
  - Next week: next TI workshop at KEK

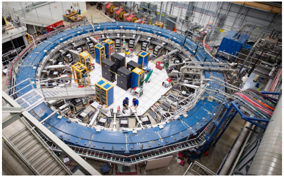


- BSM contributions to $g-2$:
  - large effect needed
  - Connections to deep questions
  - many models . . . and constraints
  - Exp. tests:
    Higgs couplings, B-physics, CLFV,
    EDM, light-particle searches, e^+e^-/muon collider



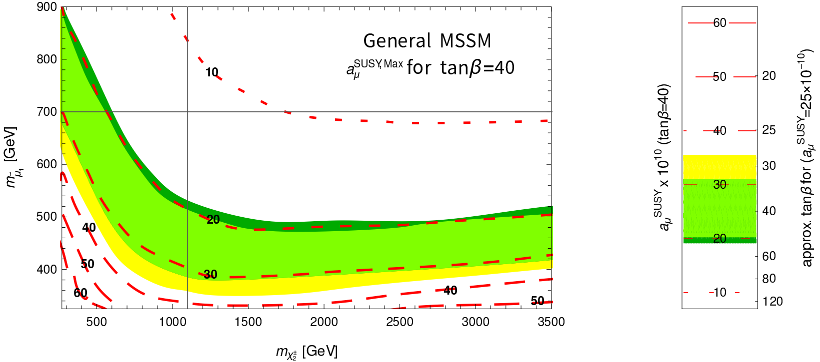


- Fermilab $g-2$ experiment
  - 20 years after BNL. . . deviation confirmed!
  - stat. dominated! Only 6% data used!
  - Best possible starting point . . .
    . . . promising future



Full MSSM overview in 7 plots

[Peter Athron, Csaba Balasz, Douglas Jacob, Wojciech Kotlarski, DS, Hyejung Stöckinger-Kim, 2104.03691]



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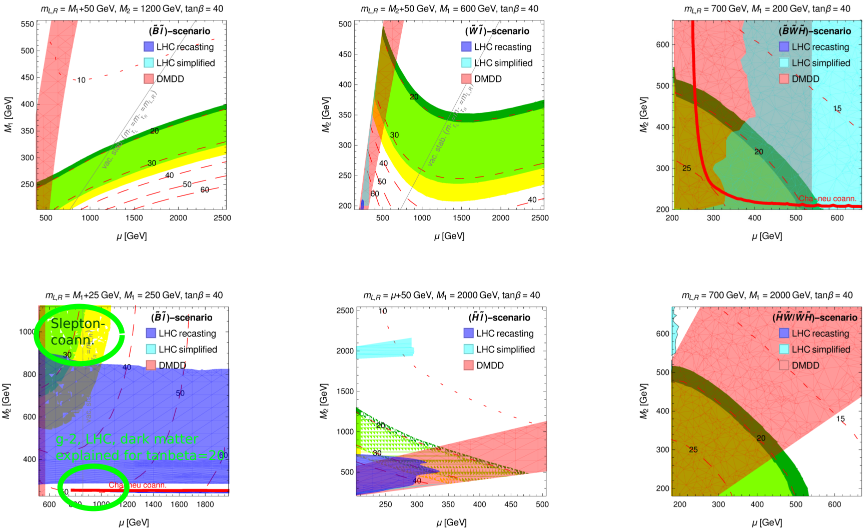

Summary: Bino-LSP: a_μ and DM. Wino-/Higgsino-LSP: a_μ. Both cha<slepton: ≈disfavoured.

DM+LHC⇒mass patterns! Coannihilation regions help! Specific cases excluded, e.g. Constrained MSSM

One-field, two-field models (renormalizable, spin $0, 1/2$)

$\times \langle H \rangle$

t_R t_L

F

$\times \langle H \rangle$

μ_R LQ μ_L

μ_R μ_L S μ_L

- many models: excluded
- very special models: chiral enhancement specific leptoquarks, specific 2HDM versions
- however, no dark matter

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- even more models: excluded
- no chirality flip
- few models: either a_μ^{BNL} or dark matter

$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Projected LHC 14 TeV exclusion, not confirmed
	Yes	Updated Sec. 4.2
$(\mathbf{1},\mathbf{1},-1)_0 - (\mathbf{1},\mathbf{1},0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{2},-1/2)_0 - (\mathbf{1},\mathbf{1},0)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	No	Excluded: LHC searches
	Yes	Updated Sec. 4.2
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Excluded: LEP contact interactions
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{1},-1)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{2},-1/2)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: LEP search
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Excluded: LHC searches
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Excluded: LHC searches + LEP contact interactions
	Yes	Viable with under abundant DM
$(\mathbf{1},\mathbf{3},0)_0 - (\mathbf{1},\mathbf{2},-1/2)_{1/2}$	Both	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{1},0)_0 - (\mathbf{1},\mathbf{1},-1)_{1/2}$	No	Excluded: LHC searches
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$(\mathbf{1},\mathbf{1},-1)_{1/2} - (\mathbf{1},\mathbf{1},0)_1$	No	Excluded: $\Delta a_\mu < 0$
$(\mathbf{1},\mathbf{2},-1/2)_{1/2} - (\mathbf{1},\mathbf{1},0)_1$	No	Excluded: $\Delta a_\mu < 0$
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$(\mathbf{1},\mathbf{1},0)_{1/2} - (\mathbf{1},\mathbf{1},1)_1$	No	Excluded: LHC searches + LEP contact interactions
$(\mathbf{1},\mathbf{2},-1/2)_{1/2} - (\mathbf{1},\mathbf{1},-1)_1$	No	Excluded: LHC searches + LEP contact interactions

Three-field models

$\times \langle H \rangle$

F_R F_L

μ_R S μ_L

- many models: viable, large chirality enhancements
- can explain a_μ^{BNL} and LHC and dark matter