

Bunch Compression Operation at the European XFEL

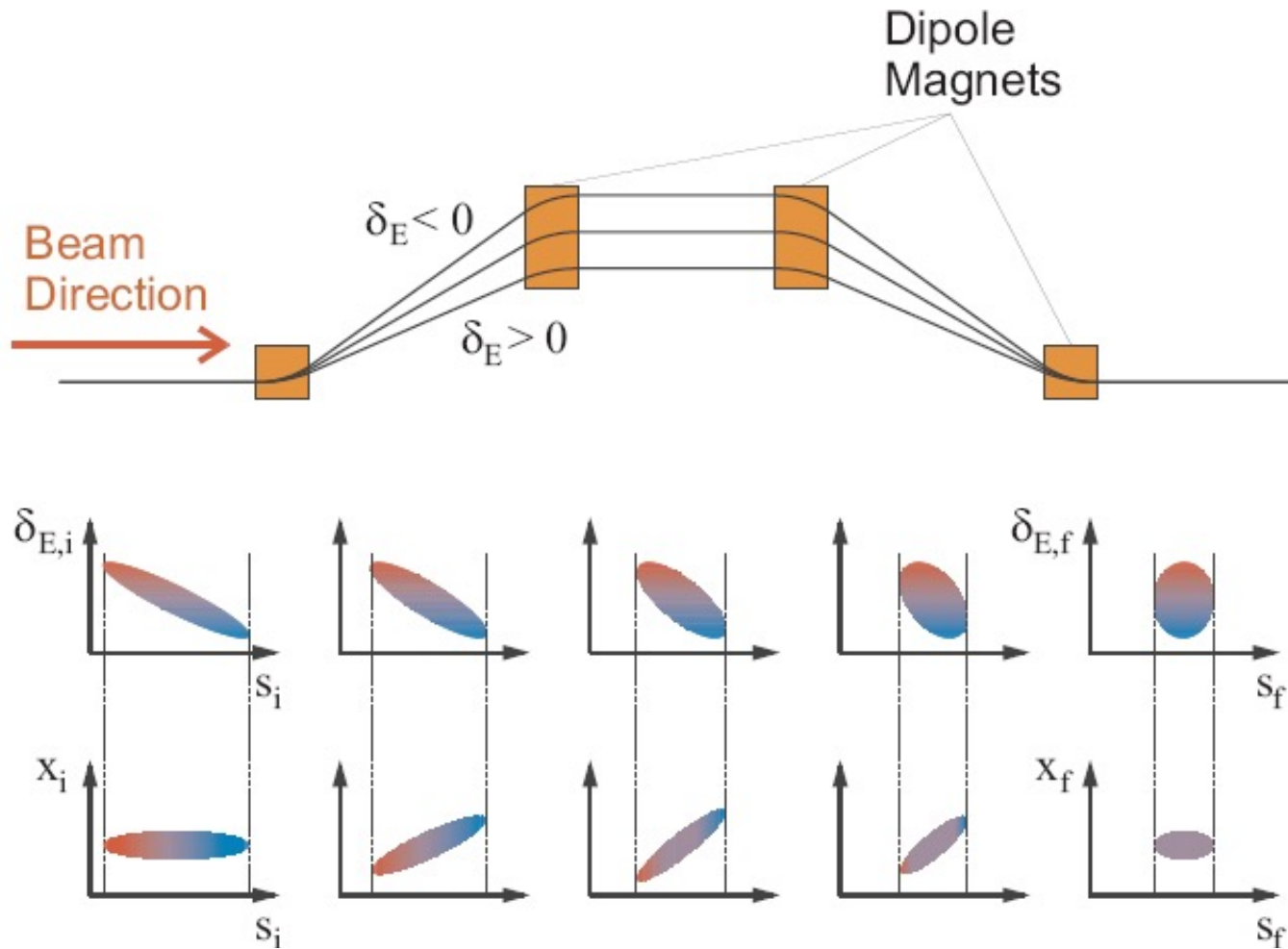
Bolko Beutner

Operator Training
June 2021

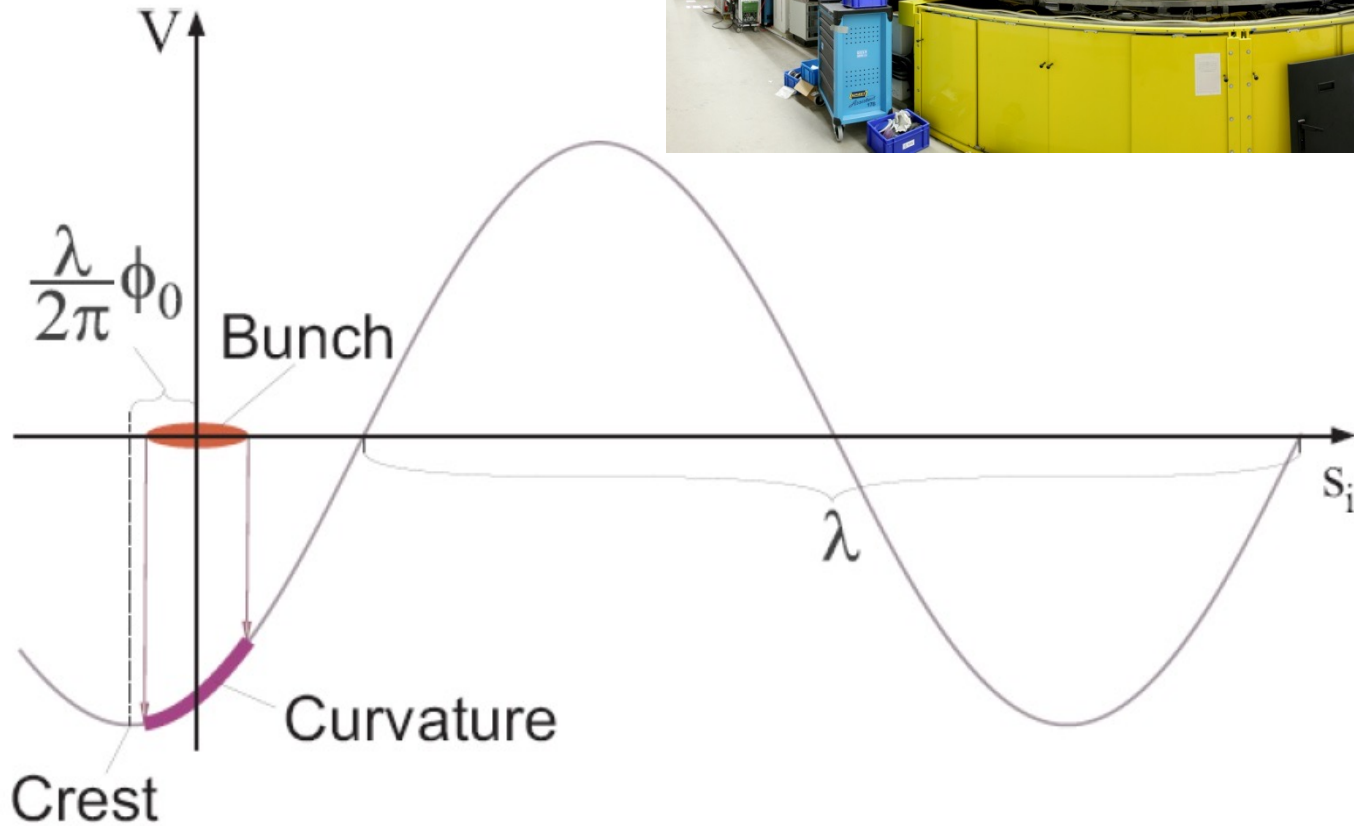
FEL Performance

- FEL performance is determined by the peak current and the emittance – the charge density in 6D.
- Low emittance and high peak current beams are required in the undulators, but not available at feasible electron sources.
- Typically long beams are produced with low emittance and the compressed later.

Basic Principle



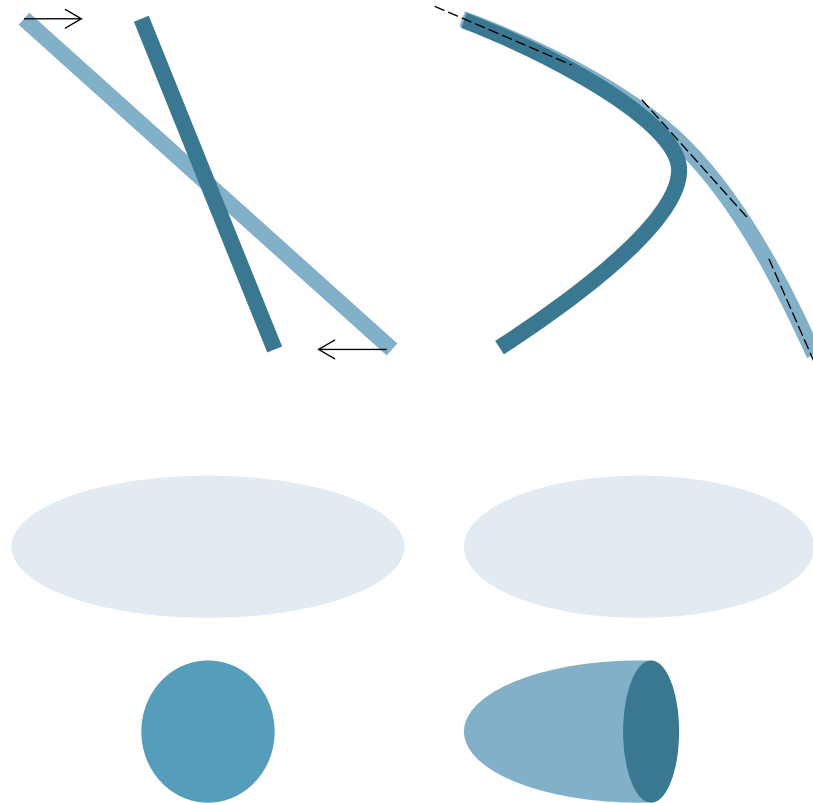
Energy Chirp Generation



“Non-linear” Compression

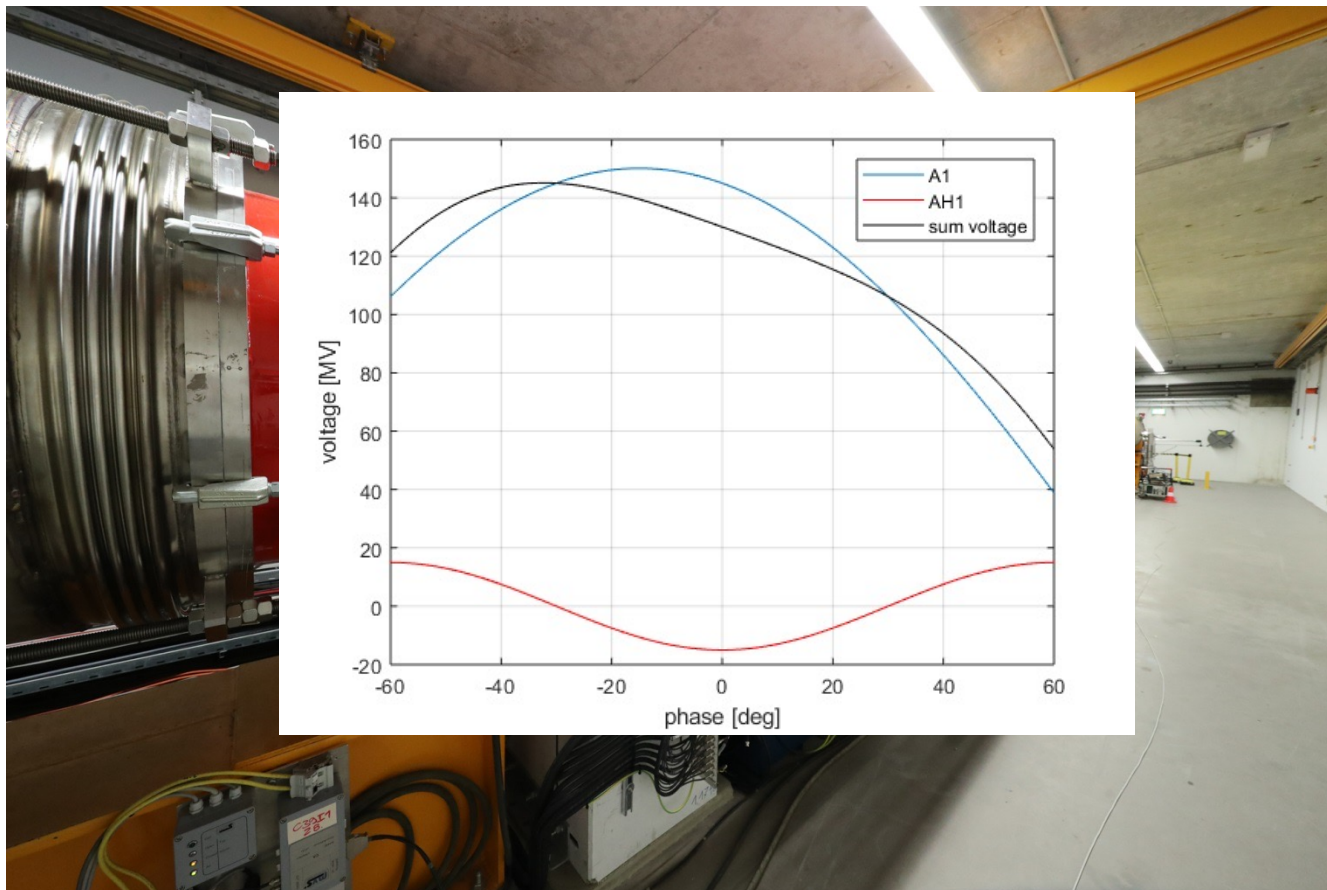
Linear Compression:

Non-Linear Compression:



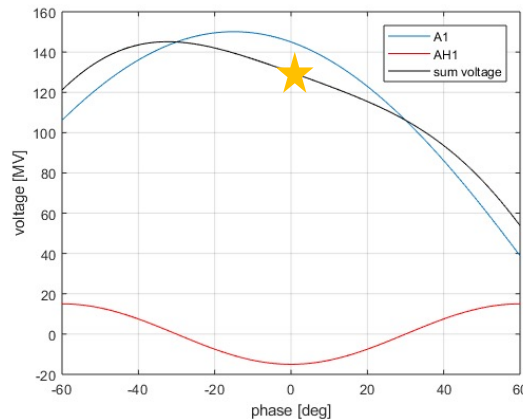
Phase Space Linearisation

- Higher harmonic RF system is used to remove non-linear chirp
- 3rd harmonic (3.9GHz) at XFEL and FLASH



Sum Voltage

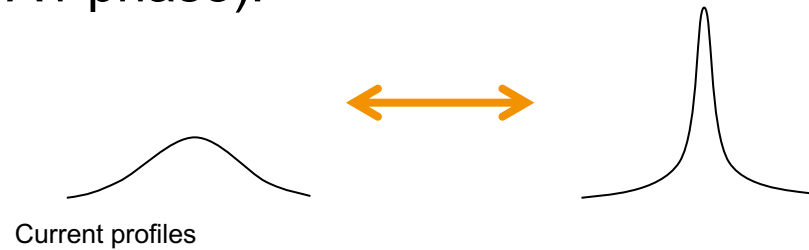
- Setup of RF phase and voltages to get a certain energy chirp is cumbersome
- RF parameters can be directly calculated from the Taylor coefficients (energy, chirp, curvature) at the beam position



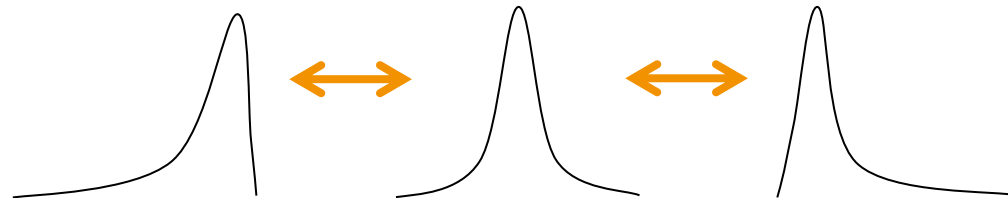
- Tuning one of the Taylor coefficients do not change the others, especially the beam energy is not changed (provided that the on-crest phases are correct)

Sum Voltage Effects

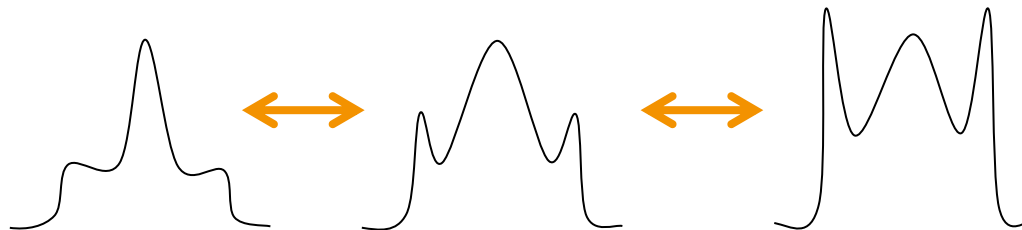
- Chirp (mostly A1 phase):



- Curvature (mostly AH1 voltage):



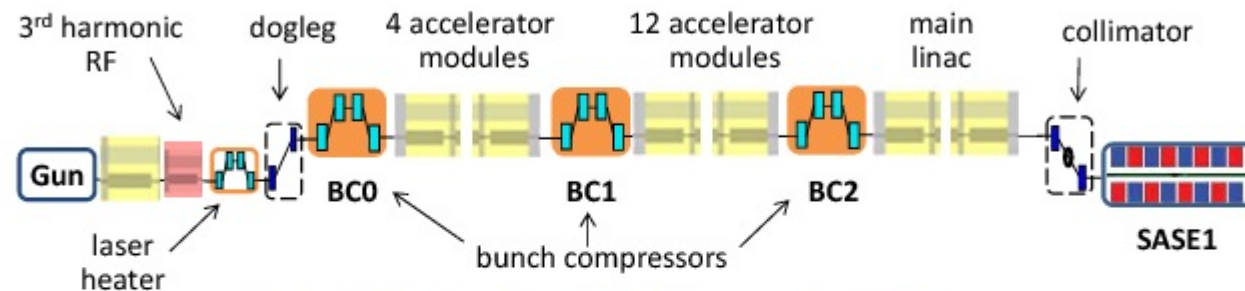
- Third Order "Skewness" (AH1 voltage and phase):



Multi-Stage Compression

- Too much compression at low energies will lead to space-charge dilution of the beam
- Too little compression in the early stage lead to problems with transport of long beams
- Too strong chicanes distort the beam due to synchrotron radiation emission

=> Multi-stage compression



$\sigma_s = 2 \text{ mm}$
 $I_{\text{peak}} = 50 \text{ A}$
 $Q = 1 \text{ nC}$

$R_{56} = 60\text{-}120 \text{ mm}$

$\sigma_s = 1 \text{ mm}$
 $I_{\text{peak}} = 100 \text{ A}$
 $E = 130 \text{ MeV}$

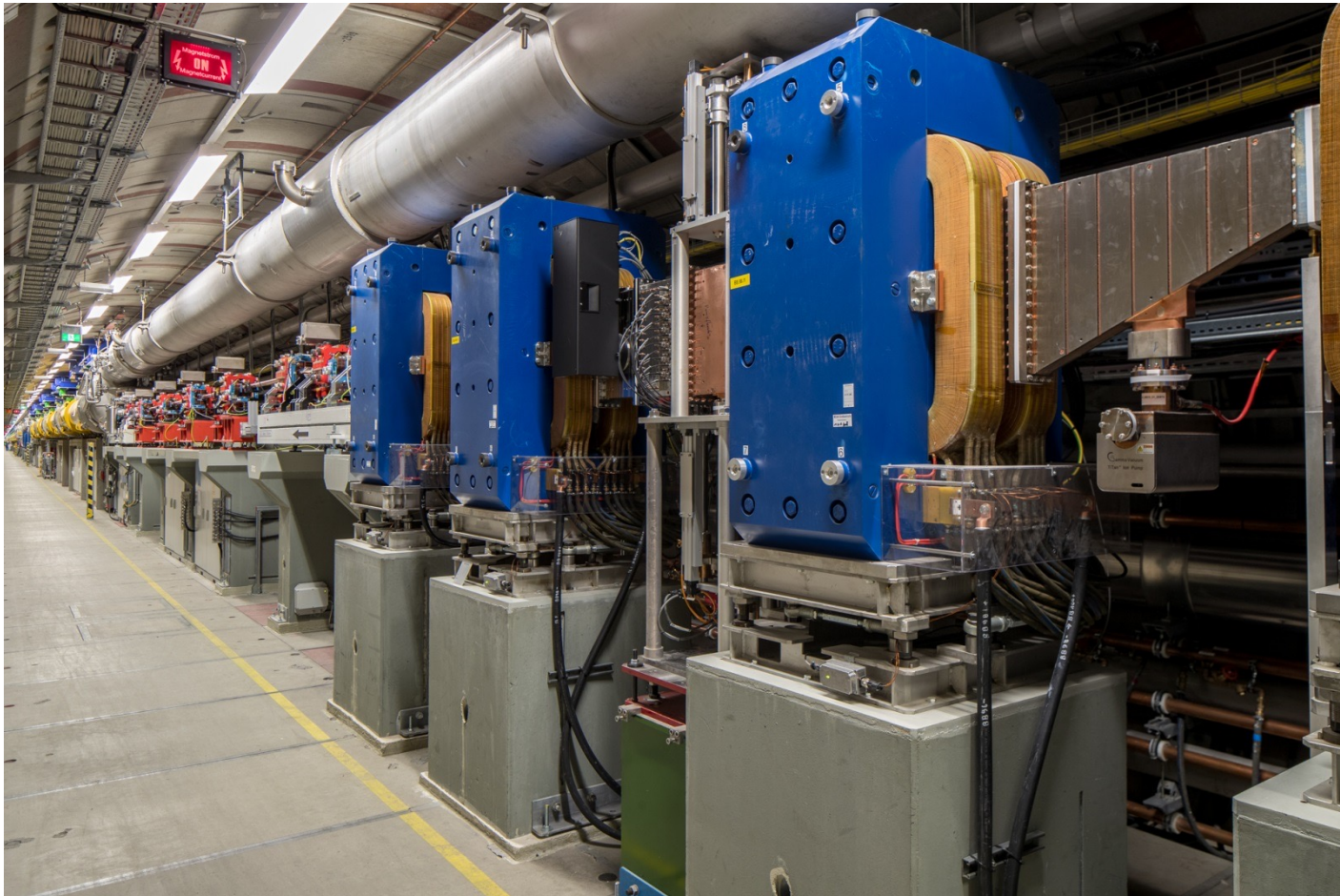
$R_{56} = 60\text{-}120 \text{ mm}$

$\sigma_s = 0.1 \text{ mm}$
 $I_{\text{peak}} = 1 \text{ kA}$
 $E = 600 \text{ MeV}$

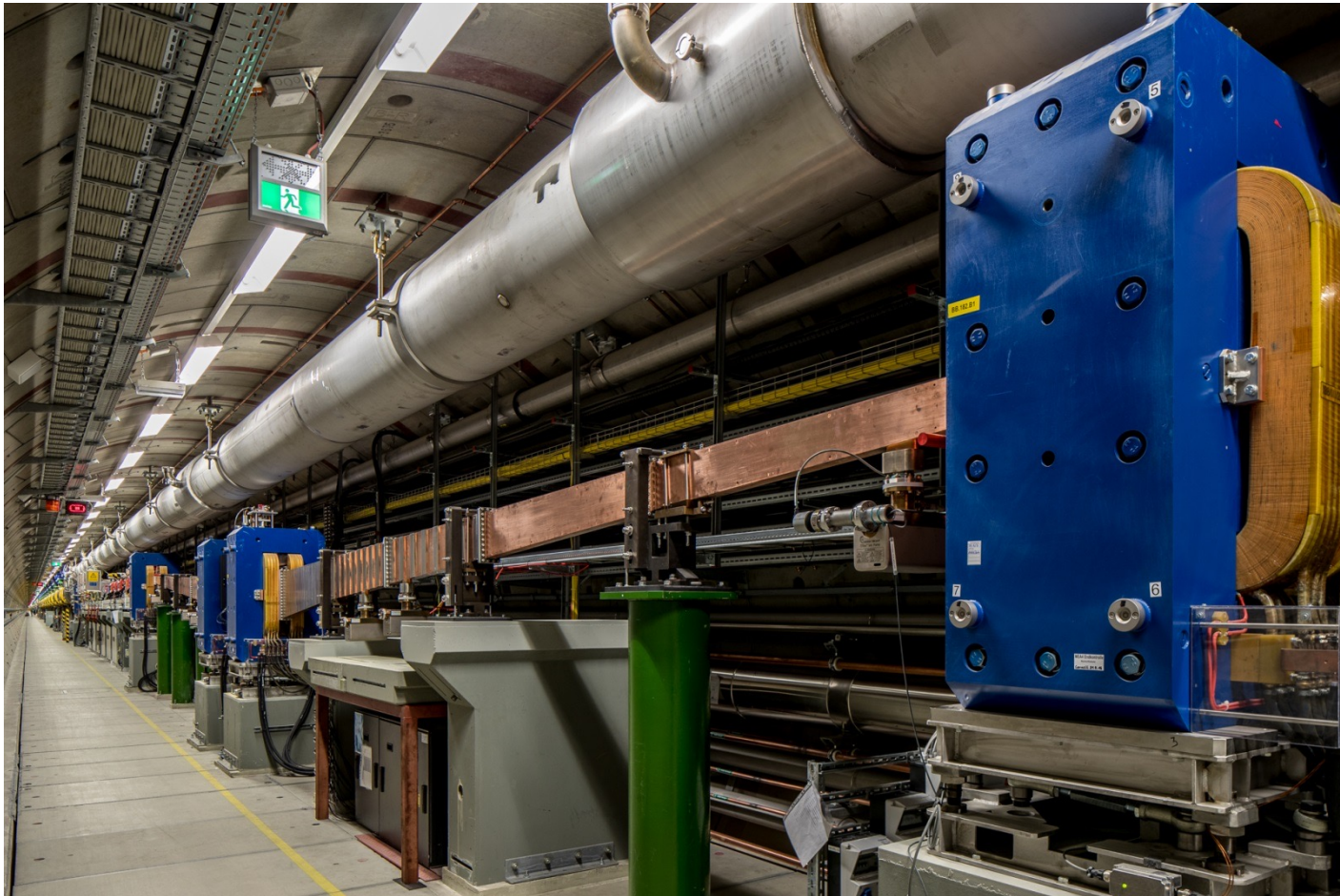
$R_{56} = 30\text{-}100 \text{ mm}$

$\sigma_s = 0.01\text{-}0.02 \text{ mm}$
 $I_{\text{peak}} = 5\text{-}10 \text{ kA}$
 $E = 2400 \text{ MeV}$

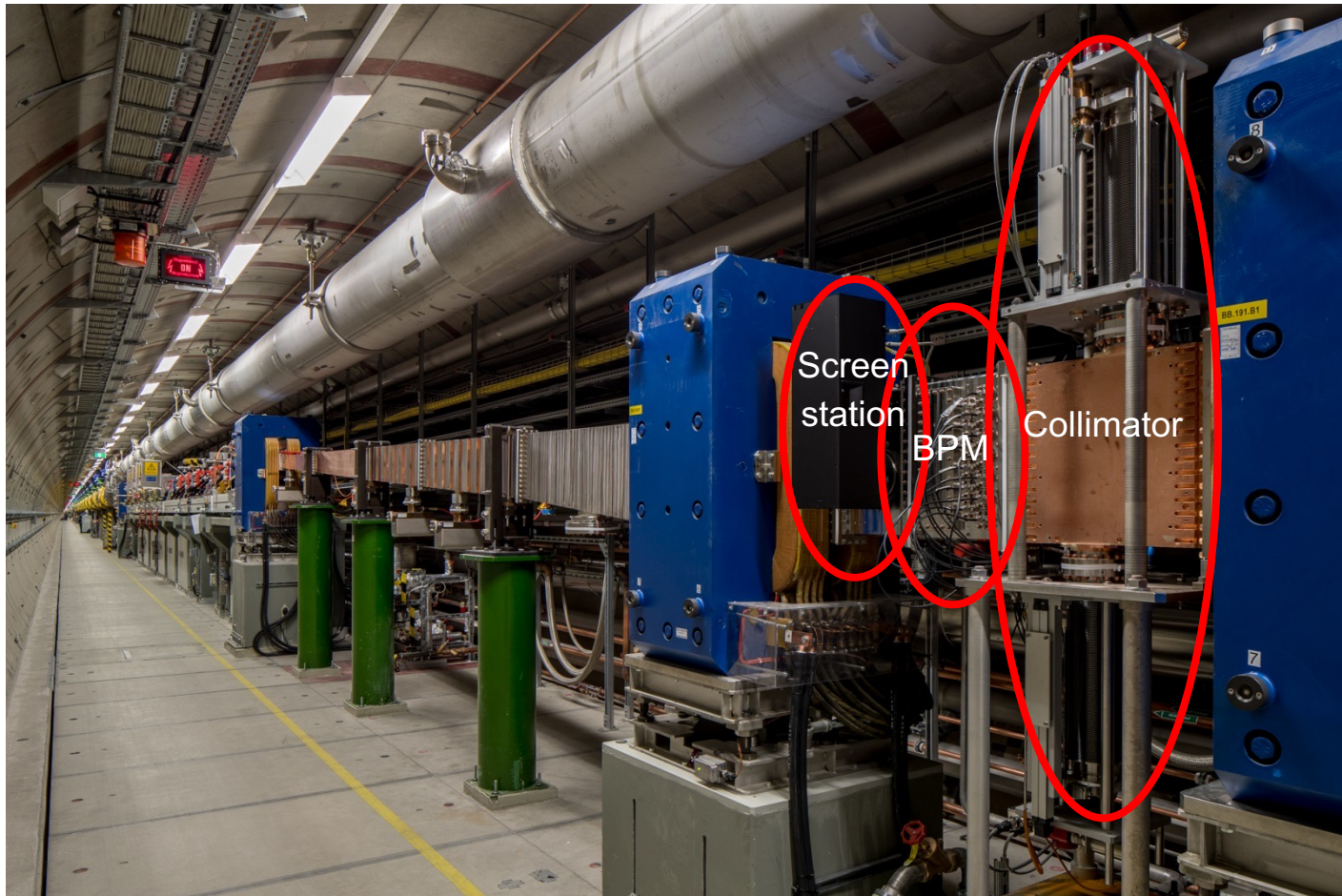
BC0



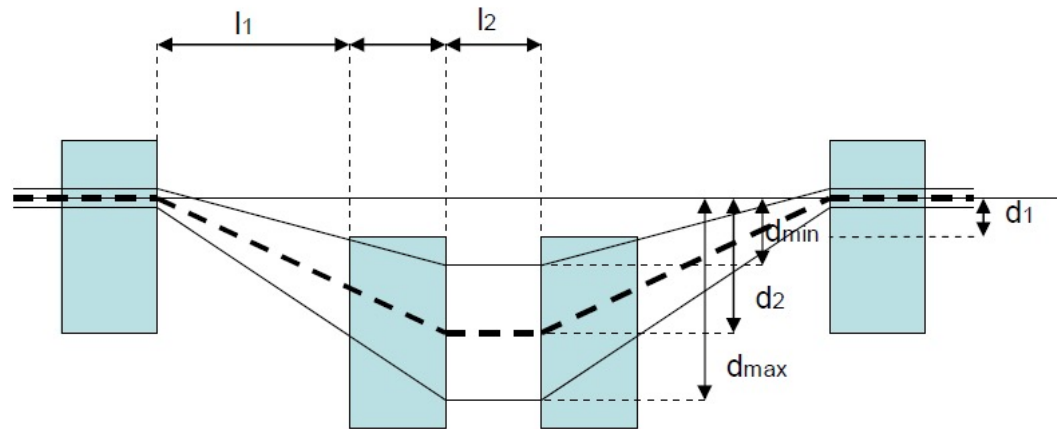
BC1



BC1

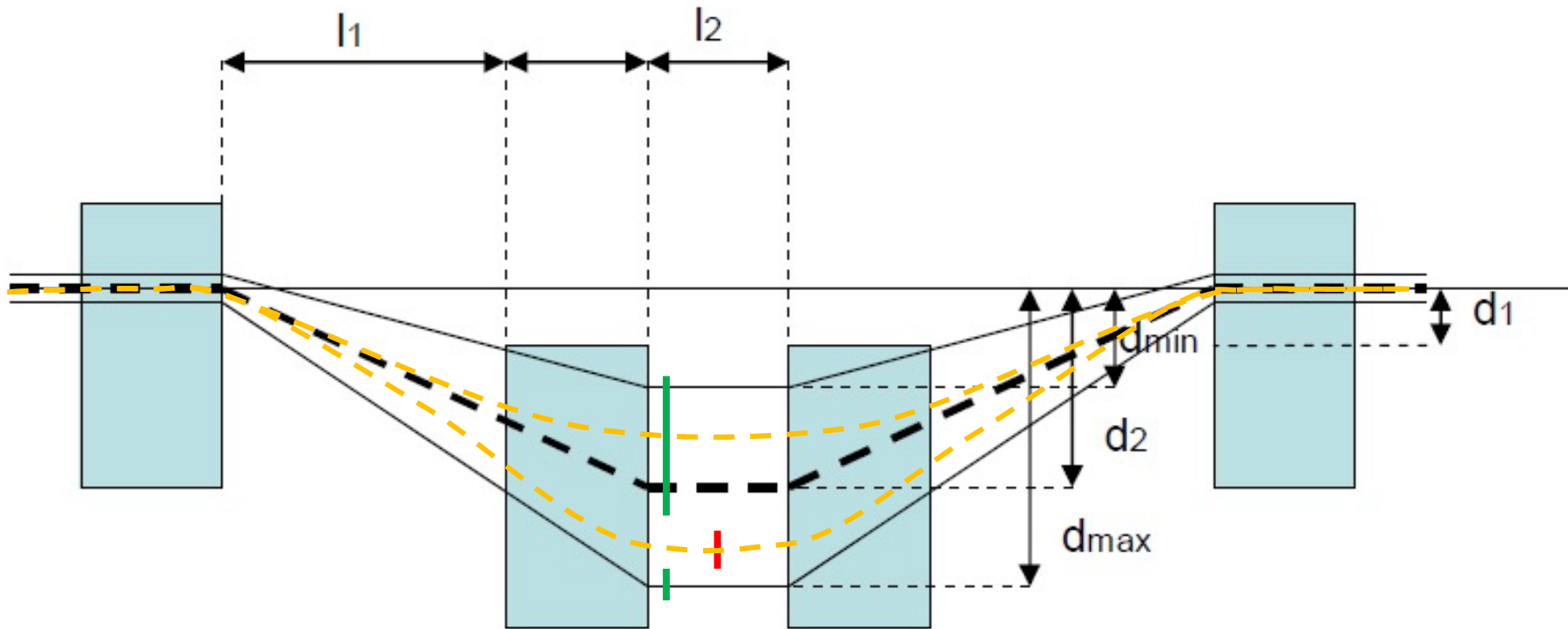


XFEL BC Overview



	R56 range [mm]	Bending Angle [deg]	d_{\min} [mm]	d_{\max} [mm]	Bend offset d_1 [mm]	Bend offset d_2 [mm]	L_1 [mm]	L_2 [mm]
BC0	0, 30-90	0, 5.67 - 9.82	-20	380	100	200	1	1.5
BC1	20-80	1.93 - 3.86	250	650	100	450	8.5	1.5
BC2	10-60	1.36 - 3.34	175	575	100	375	8.5	1.5

Chamber Overview



Screen
Collimator

Not to scale!

BC Collimator Control

The image displays the BC Collimator Control interface. On the left is the 'SUBTRAIN - ALL' overview showing a beamline with various collimators. A red circle highlights a specific collimator. On the right are two 'Motor Expert Panel' windows for 'COLU.192.B1' and 'COLO.192.B1'. The 'COLU.192.B1' panel shows a 'SOLL' value of 60000000 and a 'Wegmessung' of 18836. The 'COLO.192.B1' panel shows a 'SOLL' value of 93110000 and a 'Wegmessung' of 12739. Both panels have 'Start', 'Stopp', 'Reset', and 'Auf 0 setzen' buttons. Below the panels is a status table with checkboxes for motor states and error messages.

Status	Konfiguration	Register	Encoder	Modbus	Plots	Expert
<input type="checkbox"/>	Motor fährt/Haltestrom		coil curr A1/A2: 0			
<input type="checkbox"/>	Motor freigegeben		coil curr B1/B2: 0			
<input checked="" type="checkbox"/>	Bereit für neue Verfahrbewegung					
<input type="checkbox"/>	Fehler: 0					
<input type="checkbox"/>	Positiven Endschalter erreicht					
<input type="checkbox"/>	Negativen Endschalter erreicht					
<input type="checkbox"/>	Virtuelles Limit erreicht					

- Set "Sollwert" and the press "start"
- Move collimators to in direction 0 to remove them
- COLU is down "unten", COLO is up "oben"

Dipole Setup

/svn/global/Magnet/Chicane/ChicaneControlMain.xml XFEL.MAGNETS/CHICANE//

European XFEL **XFEL Chicane Server** Print

LH		BC0		BC1		BC2	
Energy	130.00 MeV H	Energy	130.00 MeV H	Energy	700.00 MeV H	Energy	2418.74 MeV H
Angle	5.70 deg H	Angle	8.19 deg H	Angle	3.09 deg H	Angle	2.61 deg H
Design:	5.7000	Design:	-6.8509	Design:	-2.8628	Design:	-2.3857
R ₅₆	4.34 mm H	R ₅₆	54.80 mm H	R ₅₆	51.44 mm H	R ₅₆	36.67 mm H
h _{BC}	29.9 mm H	h _{BC}	215.5 mm H	h _{BC}	485.8 mm H	h _{BC}	410.2 mm H
dt	7.19 ps H	dt	90.16 ps H	dt	85.61 ps H	dt	61.06 ps H
Gen. field	0.1741 T H	Gen. field	0.0000 T H	Gen. field	0.2255 T H	Gen. field	0.6582 T H

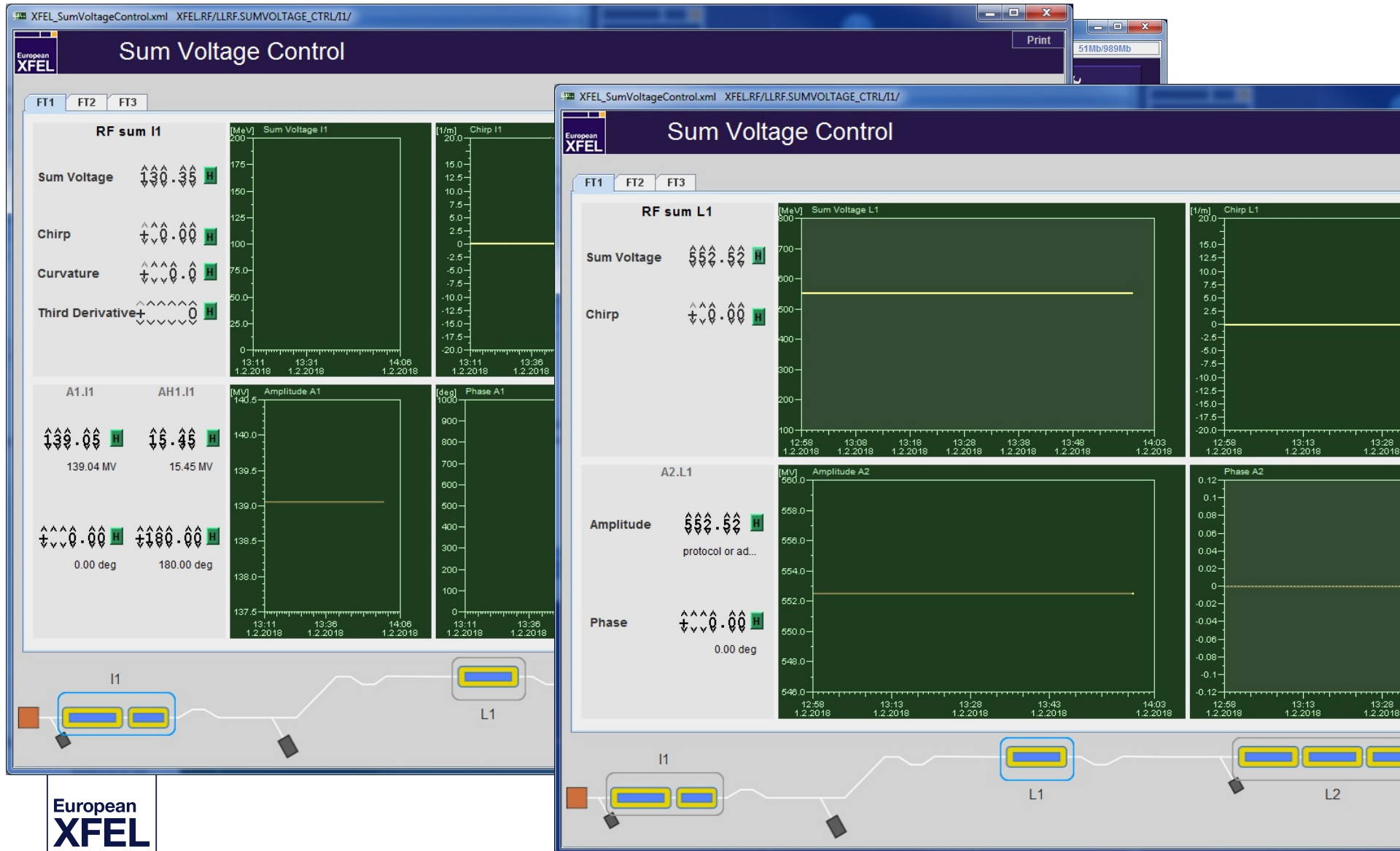
Power supply circuit		Power supply circuit		Power supply circuit		Power supply circuit	
Class	Dipole	Class	Dipole	Class	Dipole	Class	Dipole
PS Circuit	BL.1.11	PS Circuit	BB.1.11	PS Circuit	BB.1.B1	PS Circuit	BB.1.B2
<input checked="" type="checkbox"/> PS On/Off		<input checked="" type="checkbox"/> PS On/Off		<input checked="" type="checkbox"/> PS On/Off		<input checked="" type="checkbox"/> PS On/Off	
<input type="checkbox"/> Circuit write protection		<input type="checkbox"/> Circuit write protection		<input type="checkbox"/> Circuit write protection		<input type="checkbox"/> Circuit write protection	
<input checked="" type="checkbox"/> Switched on		<input checked="" type="checkbox"/> Switched on		<input checked="" type="checkbox"/> Switched on		<input checked="" type="checkbox"/> Switched on	
<input type="checkbox"/> No fault		<input type="checkbox"/> No fault		<input type="checkbox"/> No fault		<input type="checkbox"/> No fault	
<input type="checkbox"/> Idle		<input type="checkbox"/> Idle		<input type="checkbox"/> Idle		<input type="checkbox"/> Idle	
Magnet 1:	BL.48I.11	Magnet 1:	BB.96.11	Magnet 1:	BB.182.B1	Magnet 1:	BB.393.B2
Magnet 2:	BL.48II.11	Magnet 2:	BB.98.11	Magnet 2:	BB.191.B1	Magnet 2:	BB.402.B2
Magnet 3:	BL.50I.11	Magnet 3:	BB.101.11	Magnet 3:	BB.193.B1	Magnet 3:	BB.404.B2
Magnet 4:	BL.50II.11	Magnet 4:	BB.101.11	Magnet 4:	BB.202.B1	Magnet 4:	BB.413.B2

Static values		B0		B1		B2	
h _{BC} according to longlist		h _{BC} according to longlist	-180 mm	h _{BC} according to longlist	-450 mm	h _{BC} according to longlist	-375 mm
h _{BC} OTR mid position		h _{BC} OTR mid position	-216 mm	h _{BC} OTR mid position	-486 mm	h _{BC} OTR mid position	-411 mm

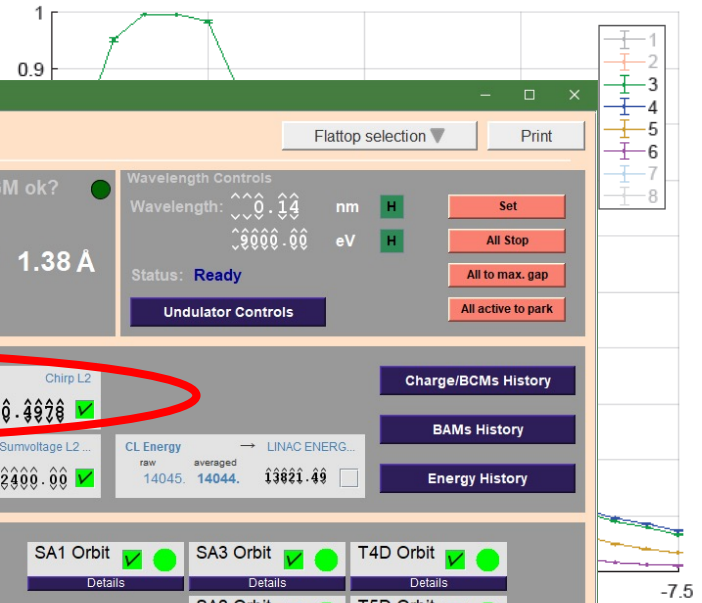
These Values are the design parameters, which do not necessarily reflect the current configuration!

Always use the current values defined by the run coordination (will be discussed later)

RF Setup and Tuning



BCM



XFEL SASE2 Tuning Panel - Flattop: 1

Beam allowed / Shutter: Beam OFF

Bunch frequency: 2.25MHz

SASE2 bunches: 10

SASE Gain @ XGM.2595.T6: 1348.6 μ J

XGM ok?

Wavelength Controls: Wavelength: 0.14 nm

Status: Ready

Longitudinal FB Loops:

Toroid TORA.25.I1	Laser Attenuat...	BC0 BCM.2	A1 Phase	BC1 BCM.1	A2 Phase	BC2 BCM.1	Chirp L2
raw: 0.2529, averaged: 0.2514	raw: 0.0997, averaged: 0.0993	raw: 0.1213, averaged: 0.1213	raw: 0.0997, averaged: 0.0993	raw: 0.1213, averaged: 0.1213	raw: 0.5067, averaged: 0.4947	raw: 0.1213, averaged: 0.1213	raw: 0.0997, averaged: 0.0993

Orbit FB Loops: INJ Orbit, L1 Orbit, L2 Orbit, L3 Orbit, TL Orbit, SA1 Orbit, SA3 Orbit, T4D Orbit, SA2 Orbit, T5D Orbit

Final Energies: LLRF Energy Gain, Beam Energy Meas., I1/LH, L1/B1, L2/B2, L3/CL, T3, T5, T5...

RF Control: GUN RF, RF sum I1, RF sum L1, RF sum L2, Final Energy L3

SASE2 Undulator Launch: CFX.2154.T1, CEX.2196.T1, CFY.2168.T1, CNY.2196.T1



Compression Setup Procedure I

■ Chicane Magnet Setup

- Make sure BC dipoles are set correctly and cycled and the reference energy of the dipoles are correct (130MeV for BC0, 700MeV for BC1, and 2400MeV for BC2)

	BC0 angle [deg]	BC1 angle [deg]	BC2 angle [deg]	BC0 R ₅₆ [mm]	BC1 R ₅₆ [mm]	BC2 R ₅₆ [mm]
250pC	-7.83	-3.05	-2.36	-50	-50	-30
100pC	-7.34	-3.05	-2.36	-43.8	-50	-30

- Set the correct optics for this bending angle is set in the machine

250pC	Inj76/XX Injector (a)symmetric FODO 76/XX	BC2 2019
100pC	Inj76/XX Injector (a)symmetric FODO 76/XX BC0 = -7.34	BC2 2019

Compression Setup Procedure II

■ RF Setup

- Deactivate the compression feedbacks
- phase scan to set on-(anti-)crest to A1,AH1,L1,L2 to 0(180)deg
- Set the design configuration by using the Sum Voltage Knobs to **(250pC/100pC)**:

	sum voltage	chirp	curvature	third derivative
I1	130.0	-9.1 / -9.8	270 / 265	25966 / 20000
L1	569.8	-9.1 / -9.4	-	-
L2	1698.7	-12 / -8	-	-

The RC or an expert might suggest new values here. Alternatively parameters from a previous good SASE delivery run might be used.

- scale the cold magnets between A1 and AH1 to the correct energy

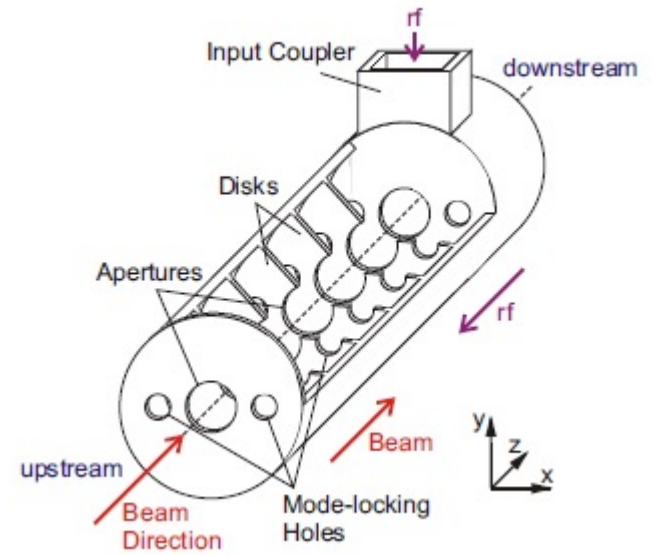
Compression Setup Procedure III

- confirm that beam energy is set to 130, 700, 2400MeV at I1, B1, and B2
- Check with B1D and B2D or at least with the BPMS energy server if in hurry.
- Correct by using the sum voltage knob in the corresponding linac (L1 or L2).
- The I1T energy server should read 130 which typically corresponds to about 128MeV in the LH.
- When set up the above values subtract up to 150 from the 'curvature' parameter in I1 do this in steps of max 10.
- close compression feedbacks by transferring the **current** compression monitor reading to the target value (arrow button) and then set the activation checkbox.

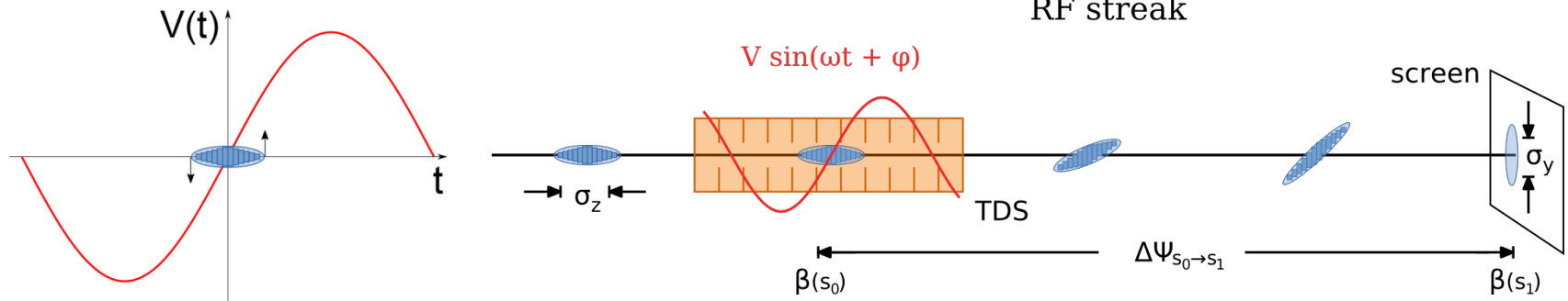
SASE Tuning

- open longitudinal feedback loops
- use chirps first L1, I1, and L2 in that order
 - I1 is the most sensitive tune in the 0.1 or 0.01 range
 - L1 should be tuned in with 0.1 steps
 - L2 is the least sensitive knob step sizes of 0.1 to 1 can be used.
Make sure that the L2 phase does not exceed 30deg!
- If chirp tuning is not successful adjust curvature in I1. Steps of 1 or 10.
- The I1 third derivative knob can be used in steps of 100 or 1000. Smaller steps have no real impact!
- After tuning is finished e.g. after reaching a new SASE optimum, close the longitudinal feedbacks **after transferring the BCM signals to the target values using the arrow button.**

Transverse Deflecting Structure (TDS)



RF streak



TDS Measurements

- TDS Monitor (Main Taskbar-> Beam Dynamics->TDS Monitor)

The screenshot shows the 'TDS Monitor' window with the following configuration:

- TDS section: I1
- camera section: OTRC.55.I1
- 1st TDS phase range [deg]: 111, 95
- 2nd TDS phase range [deg]: -69, -85
- kicker operation: disable
- bunch number: 1
- store raw images: off

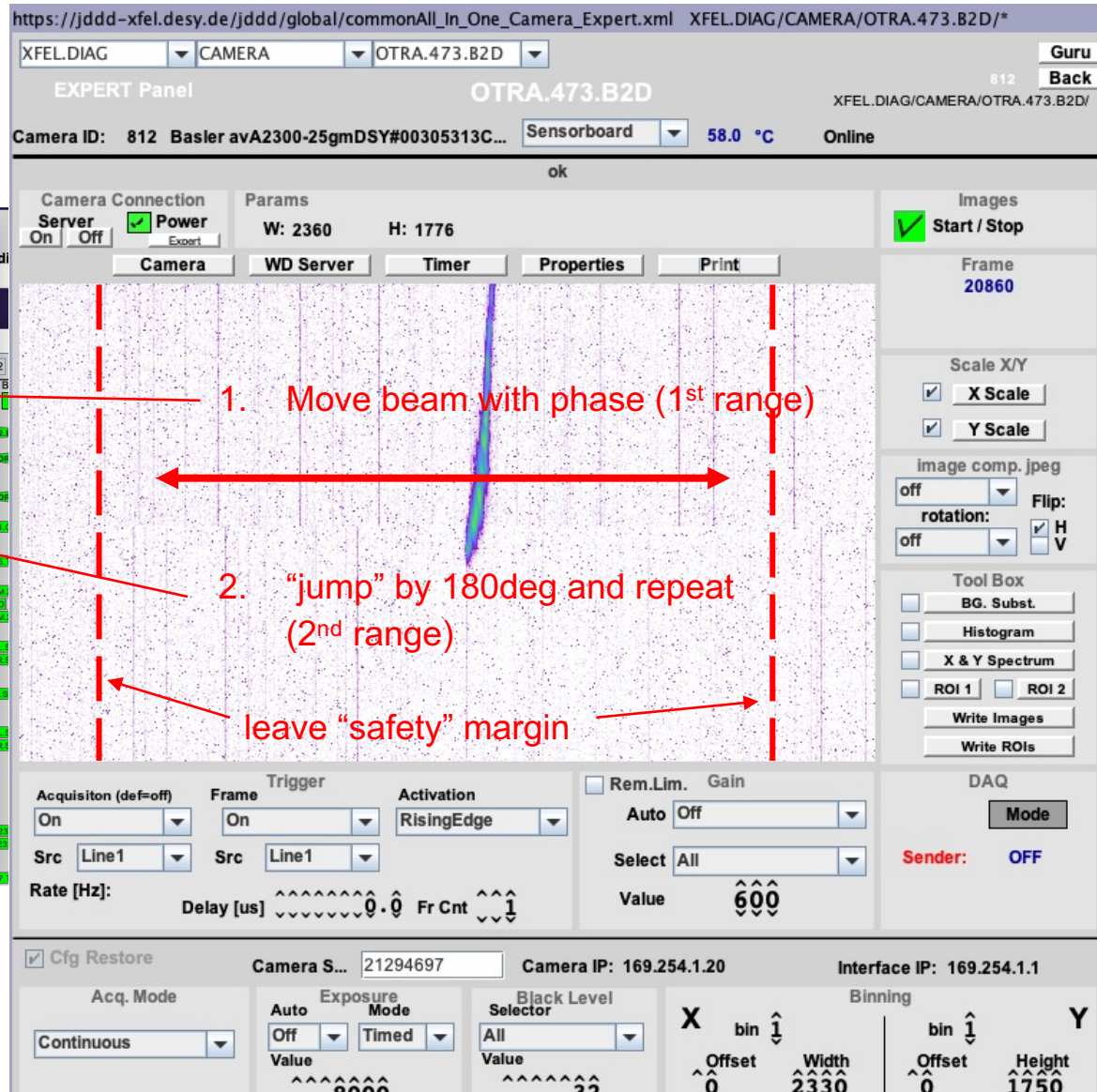
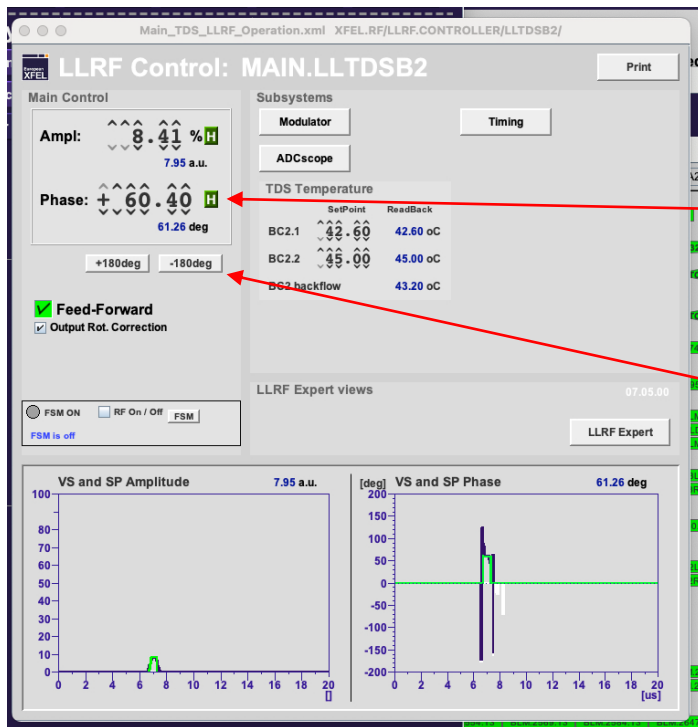
Instructions for configuration:

1. Select TDS station
2. Select Camera
3. Find phase ranges manually
4. (optional) configure Kicker
5. Start Measurement

TDS Phase Range Setup

Example:

B2 in spectrometer B2D
in I1 the streak direction is horizontal



Setup B2D Measurements

Injector 1 (62 m)

● BB.62.I1D	-0.279 deg
-30.000 deg.	0.000 A
● CBB.62.I1D	0.000 mrad
0.000	1.800
● QI.60.I1	0.000 mrad/m
470.128	-71.310
-1901.500	470.128
● QI.61.I1	0.000 mrad/m
207.003	0.000
-119.380	207.003
● QI.63.I1D	0.000 mrad/m
1065.100	1065.100
1065.100	1065.100
● QI.64.I1D	0.000 mrad/m
0.000	0.000
0.000	0.000

BC 1 (229 m)

● BB.229.B1D	-0.052 deg
-12.000 deg.	0.000 A
● CBB.229.B1D	-0.000 mrad
0.000	-0.240
● QD.231.B1D	-4.384 mrad...
-710.100	-710.100
● QD.232.B1D	4.384 mrad...
10.100	10.100

DC 2 (467 m)

● BB.467.B2D	0.010 deg
-12.000 deg.	0.000 A
● QF.469.B2D	-5.022 mrad...
-1170.310	-1170.310
● QE.471.B2D	2.265 mrad...
320.530	320.530
● QF.472.B2D	-5.022 mrad...
-1170.310	-1170.310
● QF.476.B2D	5.022 mrad...
1665.990	1665.990
● QF.477.B2D	5.022 mrad...
409.909	409.909

TLD (1980 m)

● BL.1939.TL	0.014 mrad
0.540 mrad	0.000 A
0.014 mrad	
● BL.1964.TL	0.014 mrad
0.470 mrad	0.000 A
0.014 mrad	

Horizontal Orbit

Persistence Absolute Orbit Difference Orbit Difference Orbit (filtered)

Vertical Orbit

Persistence Absolute Orbit Difference Orbit Difference Orbit (filtered)

Select Beamline ▼ Selected: ALL Reference Orbit Set as ref at: 19:23.52 24.05.2021 Set as reference (ALL)

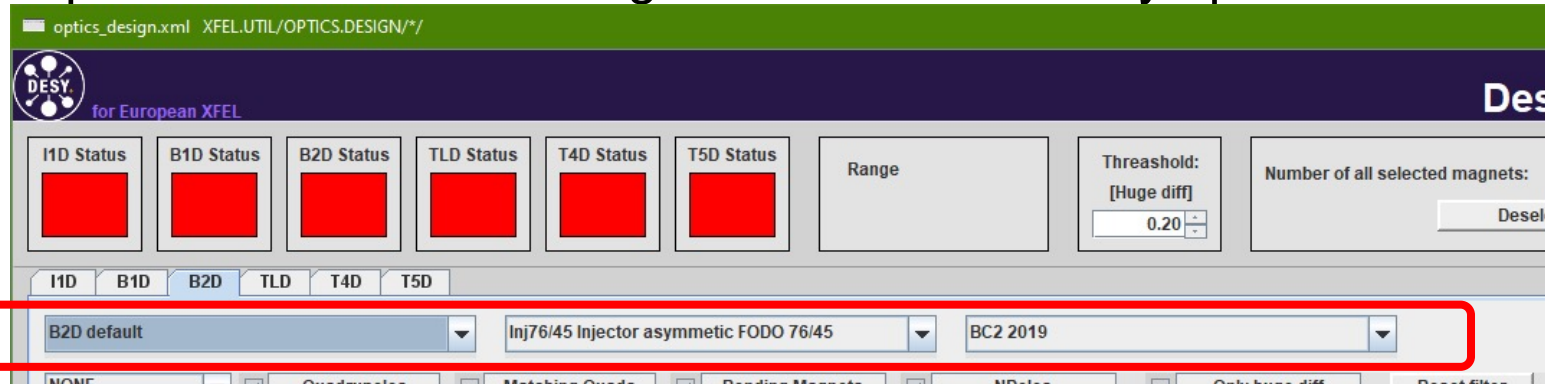
Beam OFF

Bunch Destination (User Mode): Preset ▼

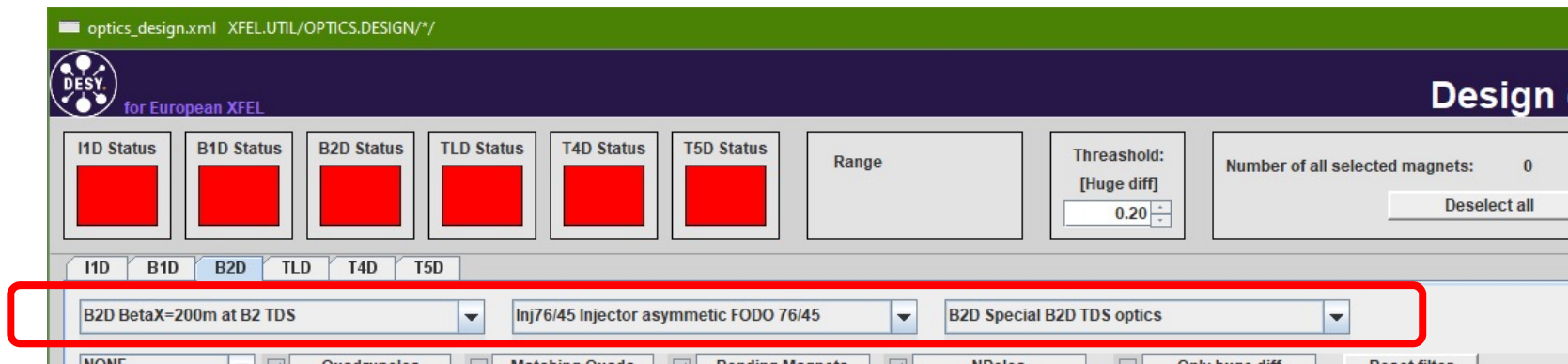
Send to xfellog... Help

Setup B2D Measurements

- After Dump switch was used change from SASE delivery optics



to High-res TDS optics – beta at TDS is increased to 200m

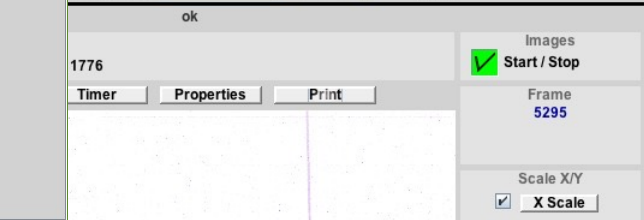
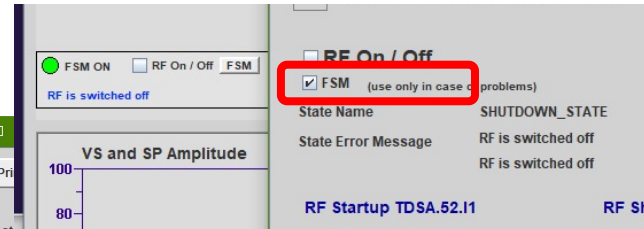
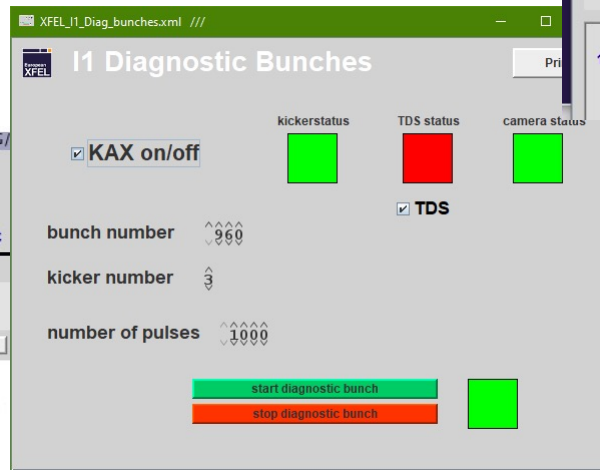
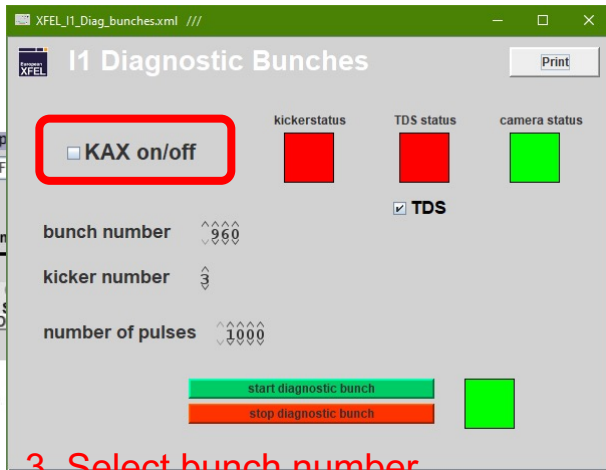


set and cycle to selected design – reset when done with measurements

Off-Axis TDS Operation

■ (Main Taskbar-> Beam Dynamics->Diag Bunches Inj/BC2)

1. Activate Kicker (e.g. KAX on/off) => kickerstatus
2. Activate TDS FSM (required for TDS status)



3. Select bunch number
("pure" bunch number)

4. Select kicker:

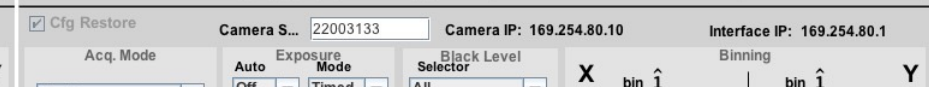
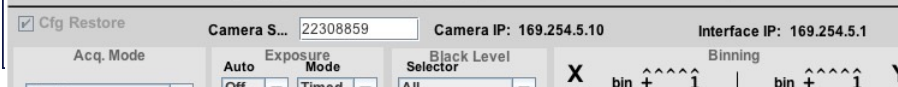
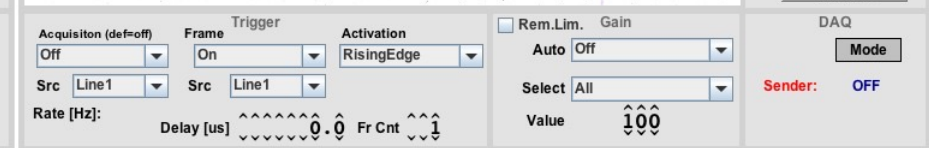
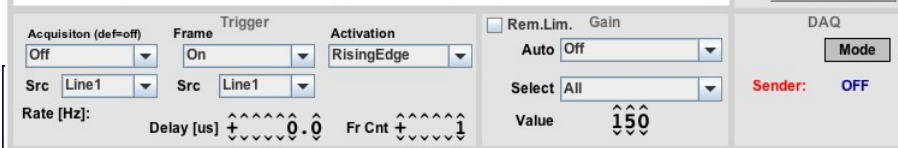
- 1-4 for OTRC.55-59.11
- 1 for 1st and 2nd for kicker pair in B2

5. Request number of shots

6. Start measurement

7. Stop when done

(will stop after the requested number)



TDS Operation I1

RF system is switched on using the “FSM RF On/Off”-button

The screenshot displays the control interface for the XFEL TDS System Injector, divided into several functional panels:

- Modulator Switch ON/OFF:** Contains checkboxes for PS 24V Control, Modulator Controller ON/..., Transformer Bias PS, Filament ON/OFF, Solenoid Contactor ON/OFF, Solenoid PS ON/OFF, and Main Modulator ON/OFF. It also features buttons for 'Connect Modulator', 'SWITCH Driver ON', and 'SWITCH Driver OFF'.
- ETHERNET CONNECTION STATUS:** Shows 'Modulator Control: ok' and 'PLC: ok'.
- ION PUMP:** Displays 'Voltage 3668.0 V' and 'Press... 2.09E-10 m...'. A 'Properties' button is present.
- SOLENOID:** Shows 'Voltage 0.0 V' and 'Current 0.0 A'. A 'Ready' indicator and 'Properties' button are included.
- MODULATOR CHARGE VOLTAGE:** Shows a temperature of 34.1 °C and a '590 V work value'.
- FILAMENT:** Displays 'Primary Voltage 1.6 V', 'Primary Current 0.00 A', 'Filament Current 0.17 A', and 'Filament Ready' status. A 'Properties' button is available.
- KLYSTRON HV PULSE:** Includes 'Pulse Voltage' and 'Pulse Current' controls.
- EXPERT SETTING:** Features a 'Setup' button.
- FAST INTERLOCKS:** Shows 'Status' and 'Lock' indicators with a 'Clear' button and 'Fast Interlocks Registers'.
- HV PULSE TRIGGER NUMBER:** Displays 'Trigger Pulse 10334365' and 'HV Pulse 5219992'.
- LLRF Control: MAIN.LLTDISI1:** A sub-window showing 'Ampl: 0.85 %' and 'Phase: +103.00 deg'. It includes 'Feed-Forward' and 'Output Rot. Correction' checkboxes. A red box highlights the 'FSM ON' and 'RF On / Off' buttons, with a note 'RF is switched off'.
- VS and SP Amplitude and Phase Graphs:** Two plots showing amplitude (0.01 a.u.) and phase (0.00 deg) over time.

At the bottom left, another red box highlights the 'FSM ON' and 'RF On / Off' buttons, with the text 'RF is switched off' below it.



TDS Operation B2

Keep

Manu

Set

“Fe

Set

Pre

Pre

Wa

rea

fron

Inc

115

Set

Inc

BC2 TDS Operation Overview Print

https://jddd-xfel.desy.de/jddd/user/meykopff/bc2_tds_conditioning_overview.xml XFEL.RF/LLRF.CONTROLLER/LLTDSB2/

Main Control

Ampl: 85.41% 84.32 a.u.

Phase: -126.00 -126.3 deg

Feed-Forward

Output Rot. Correction

OVC:

FSM ON RF On / Off FSM

FSM is off

STATE: Trigger

State: Trig ON

Remote Mode

Charge Voltage: 1100.00

Charge Voltage: 1100.00

Pulse Voltage: 222.82

Pulse Current: 206.65

Experts:

RF: Joerg Herrmann (9)3782
Ingo Peperkorn (9)3374

LLRF: Hoffmann, Matthias 1670

Beam Dynamics: Beutner, Bolko 2757

TDS Power

actual = 84.32%

target > 80.00%

VS and SP Amplitude 84.32 a.u.

VS and SP Phase -126.3 deg

B2 Special Bunch ML:

bunch number: 57

kicker number: 1000

number of pulses: 1000

TDS

B2 Diagnostic Kicker Control:

EXPERTS

Legacy Timing Control:

TDS

Vacuum Monitoring Graph:

Y-axis: [mbar]

X-axis: Time [23.5.2021]

Legend: BC2.46TDS.01/P [mbar], BC2.45TDS.01/P [mbar], BC2.46TWG.01/P [mbar], BC2.43TDS.01/P [mbar]

while monitoring vacuum

TDS Measurements Analysis

ttfinfo.desy.de/XFELelog/show.jsp?dir=/2021/06/12.02_M&pos=2021-02-12T11:53:58

Status: Shut Down Accelerator:
 SASE1: SASE2: SASE3:
 News: XFEL emergencies: also inform 94500 or 94600

12.02.2021 11:53 xfeloper@xfelbkr3.desy.de TDS Longitudinal Profile Measurement
 Longitudinal measurements at OTRA.473.B2D in B2
 File: /Users/xfeloper/data/tds_longitudinal_profile/2021-02-12T115356.mat
 Bunch Charge: 253.65pC

Results Summary:

	Gaussian	RMS
bunch length [fs]	36.166	34.7608
long. resolution [fs]	7.2305	9.2274
intrinsic spot size [μm]	21.1123	27.7707
correlation C/S	0.0046126	0.035471

calibration scan:

	1st slope	2nd slope
min phase [deg]	63	-117
max phase [deg]	62	-116
measurement phase [deg]	63.2454	-116.6923

Longitudinal calibration:

	1st slope	2nd slope	average
long calibration [m/ps]	-0.0030599	0.0027531	0.0029065

TDS Summary:

power SP	72.25
modulator charge voltage	1179.9725
amplitude input	176.7111
amplitude output	140.4444

RF Summary:

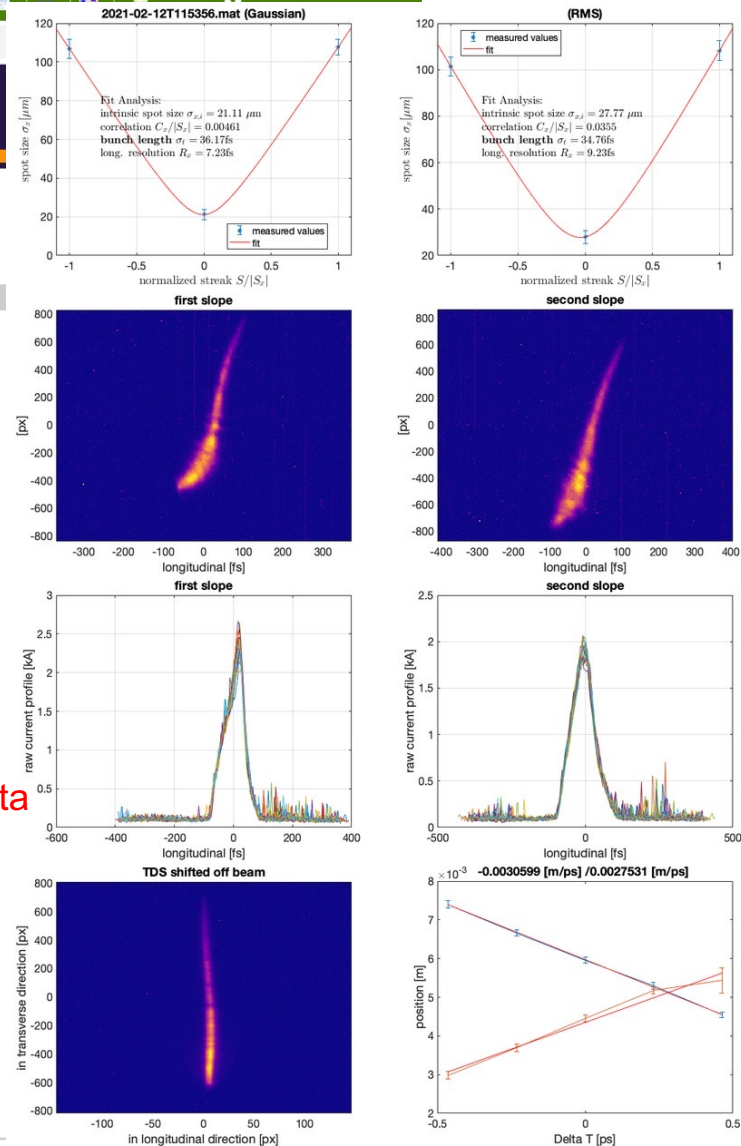
	sum voltage	chirp	curvature	third derivative
T1	129.48	-9.1	119.9999	26000
L1	589.64	-9.4	-	-
L2	1778.0928	-12	-	-

	voltage [MV]	phase [deg]
A1	143.4168	11.1956
AH1	18.6768	164.0471
A2	640.0323	22.8881
A3	647.1862	31.5722
A4	787.1184	31.5722
A5	652.7059	31.5722

Results are directly send to elog
 No summary screen in TDSMonitor GUI!

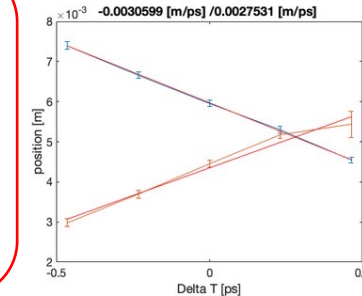
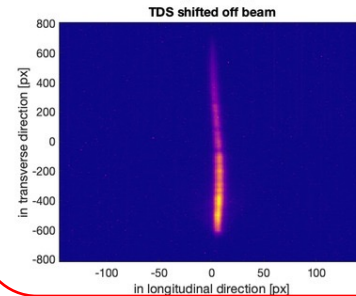
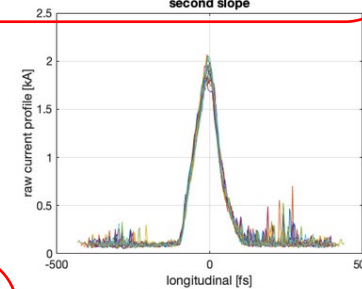
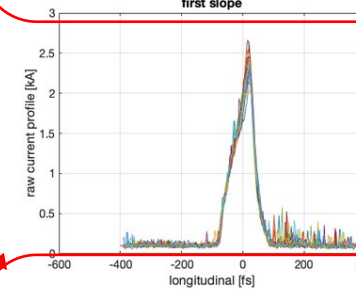
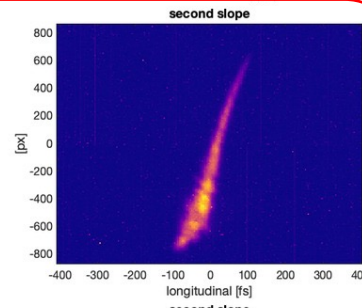
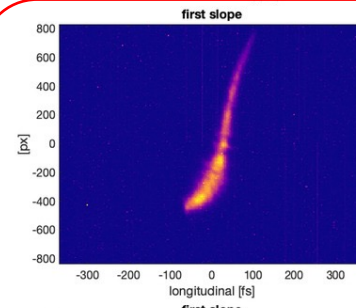
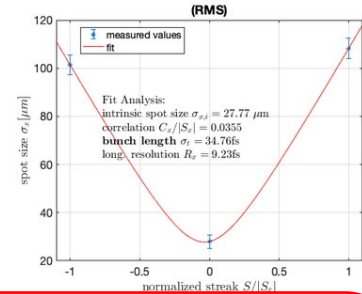
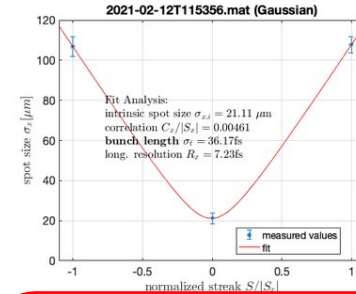
Additional data are send to elog
 (e.g. bunch charge, TDS LLRF)

Caution: Sum voltage and LLRF data
 are only stored for the first Flattop
 even in off-axis operation!

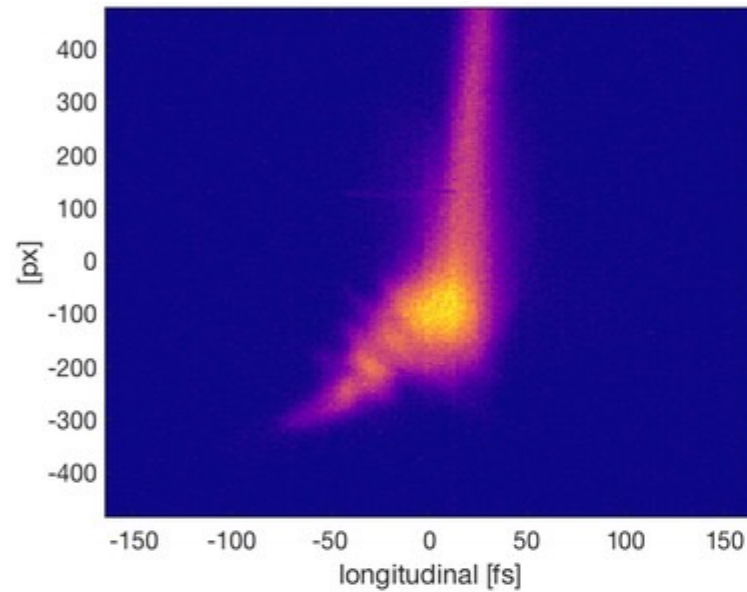


Is the compression properly setup?

- Bunch length at B2 should be between 30-40fs for good SASE (250pC)
- Longitudinal PS should look similar to the example
- If unstreaked beam is “tilted” consider checking dispersion between injector and linac
- A strong asymmetry between the spot-size of both “zero-crossings” is another indication for beam tilts



Thank You for Your Attention!



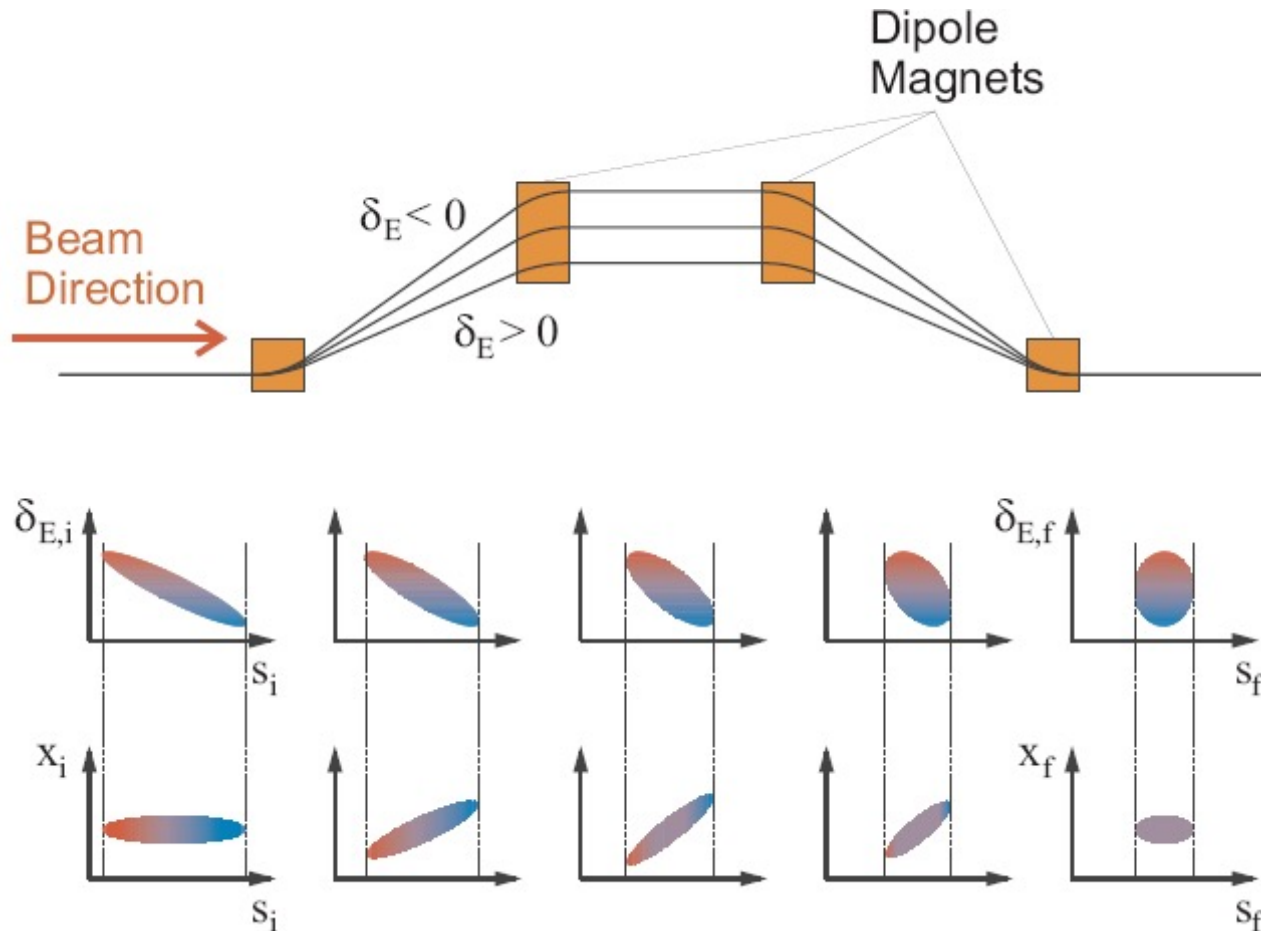
Questions?

Literature:

- Igor Zagorodnov and Martin Dohlus, “Semianalytical modeling of multistage bunch compression with collective effects”
Phys. Rev. ST Accel. Beams 14, 014403 – Published 13 January 2011
- ICFA Beam Dynamics Newsletter No. 38, (http://icfa-usa.jlab.org/archive/newsletter/icfa_bd_nl_38.pdf)
- Various PhD theses:
Frank Stulle (<http://www-library.desy.de/cgi-bin/showprep.pl?desy-thesis-04-041>),
BB (<http://www-library.desy.de/cgi-bin/showprep.pl?desy-thesis-07-040>), ...

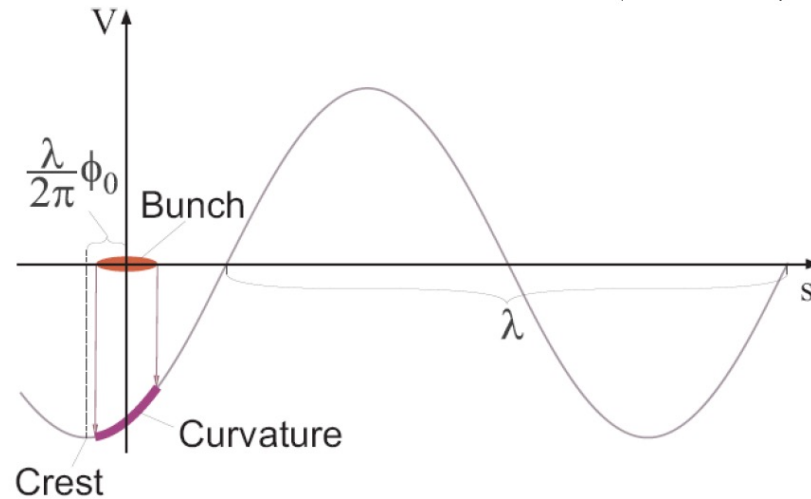
In-house Experts: Martin Dohlus, Igor Zagorodnov, Torsten Limberg, BB

Bunch Compression – with Formulas



$$\Delta s = R_{56} \cdot \delta + T_{566} \cdot \delta^2 + U_{5666} \cdot \delta^3 + \dots \quad \text{with} \quad \delta = \frac{E - E_0}{E_0}$$

Energy Chirp Generation



$$E(s) = eV \cos\left(\varphi + \frac{2\pi}{\lambda}s\right) + E_0$$

$$= E_0 + eV \left(\cos(\varphi) - \frac{2\pi}{\lambda} \sin(\varphi)s - \frac{2\pi^2}{\lambda} \cos(\varphi)s^2 + \dots \right)$$

$$\delta = \frac{E(s) - E(0)}{E(0)}$$

$$= -\frac{2\pi eV}{E\lambda} \sin(\varphi)s - \frac{2\pi^2}{E\lambda^2} \cos(\varphi)s^2 + \dots$$

$$\approx As + Bs^2$$

$$A = -\frac{2\pi eV}{E\lambda} \sin(\varphi)$$

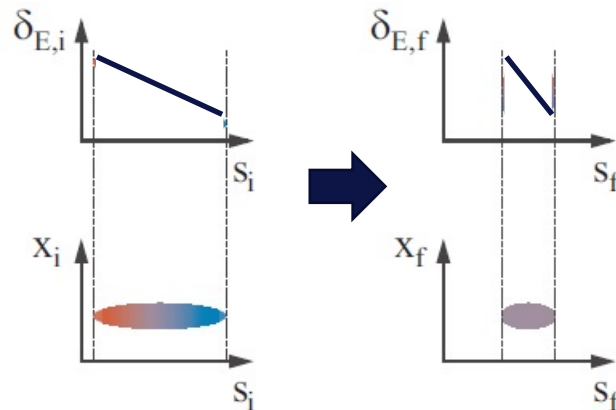
Linear Compression

$$\delta(s_{\text{initial}}) = A s_{\text{initial}}$$

$$s_{\text{final}} = s_{\text{initial}} + R_{56} \delta$$

$$s_{\text{final}} = (1 + AR_{56}) s_{\text{initial}}$$

$$\begin{aligned} \sigma_{s_{\text{final}}} &= \sqrt{(1 + AR_{56})^2 \sigma_{s_{\text{initial}}}^2} \\ &= |1 + AR_{56}| \sigma_{s_{\text{initial}}} = \frac{\sigma_{s_{\text{initial}}}}{C} \end{aligned}$$



Full compression at $(1 + AR_{56}) = 0$, however $C = 1/0$!

Linear Compression with Uncorrelated Energy Spread

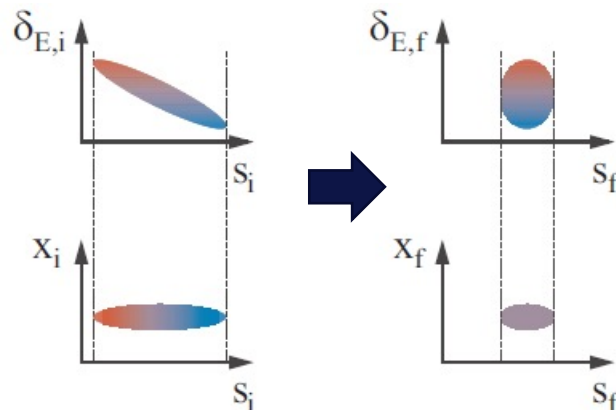
each particle i has an individual “random” energy offset

$$\delta(s_{\text{initial}}, i) = A s_{\text{initial}} + \delta^i$$

$$s_{\text{final}} = s_{\text{initial}} + R_{56} \delta$$

$$s_{\text{final}} = (1 + AR_{56}) s_{\text{initial}} + R_{56} \delta^i$$

$$\sigma_{s_{\text{final}}} = \sqrt{(1 + AR_{56})^2 \sigma_{s_{\text{initial}}}^2 + R_{56}^2 \sigma_{\delta^i}^2}$$



Typically the “full” compression is avoided...

$$\text{if } (1 + AR_{56}) = 0$$

$$\sigma_{s_{\text{final}}}^{\text{min}} \approx |R_{56}| \sigma_{\delta^i}$$

Simple Compression Setup



$$A = -\frac{E_1 k_1}{E} \sin(\varphi)$$

$$\sigma_{\text{final}} = |1 + AR_{56}| \sigma_{\text{initial}}$$

$$E = E_0 + E_1 \cos(\varphi)$$

$$\Rightarrow \varphi = \arctan \left(-\frac{E}{E - E_0} \frac{\left(\frac{\sigma_{\text{final}}}{\sigma_{\text{initial}}} - 1 \right)}{k_1 R_{56}} \right)$$

$$E_1 = \frac{E - E_0}{\cos(\varphi)}$$

Is this Ok?

Non-linear Compression

$E_0, k_1, 0 \text{ deg}$

E_1, k_1, phi

total energy E

$$s_{\text{final}} = s_{\text{initial}} + R_{56}\delta + T_{566}\delta^2 + \dots$$

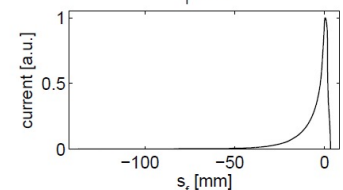
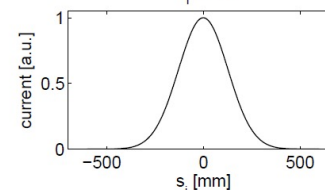
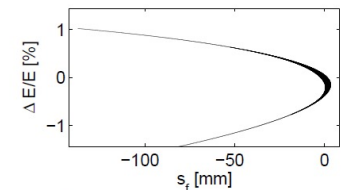
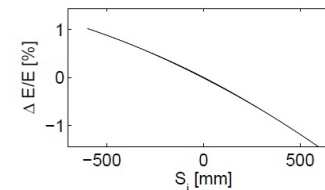
$$\delta = A s_{\text{initial}} + B s_{\text{initial}}^2 + \dots$$

$$\delta(s_{\text{initial}}) = \frac{-E_1 \sin(\varphi) k_1 s_{\text{initial}} - E_1 \cos(\varphi) \frac{k_1^2}{2} s_{\text{initial}}^2}{E_0 + E_1 \cos(\varphi)}$$

$$s_{\text{final}} = \frac{R_{56}}{E} \left(-E_1 \sin(\varphi) k_1 s_{\text{initial}} - E_1 \cos(\varphi) \frac{k_1^2}{2} s_{\text{initial}}^2 \right) + \frac{T_{566}}{E^2} E_1^2 \sin^2(\varphi) k_1^2 s_{\text{initial}}^2 + s_{\text{initial}}$$

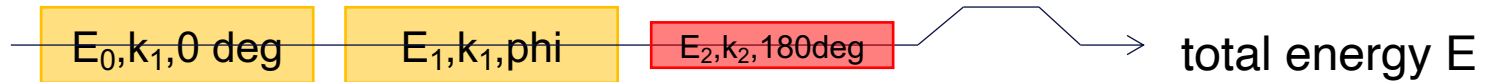
Can we tune the T_{566} -term to compensate the E_1 quadratic-term?

No since $T_{566} \approx -\frac{3}{2} R_{56}$



This is NOT OK!

Linarised Compression



$$s_{\text{final}} = s_{\text{initial}} + R_{56}\delta + T_{566}\delta^2 + \dots$$

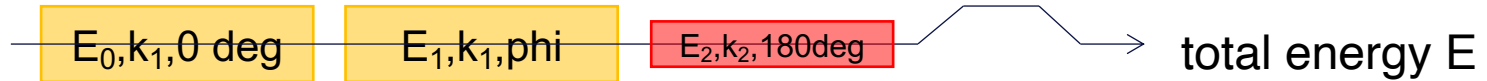
$$\delta = As_{\text{initial}} + Bs_{\text{initial}}^2 + \dots$$

$$\delta(s_{\text{initial}}) = \frac{-E_1 \sin(\varphi)k_1 s_{\text{initial}} - E_1 \cos(\varphi) \frac{k_1^2}{2} s_{\text{initial}}^2 + E_2 \frac{k_2^2}{2} s_{\text{initial}}^2}{E_0 + E_1 \cos(\varphi) - E_2}$$

$$s_{\text{final}} = \frac{R_{56}}{E} \left(-E_1 \sin(\varphi)k_1 s_{\text{initial}} - E_1 \cos(\varphi) \frac{k_1^2}{2} s_{\text{initial}}^2 + E_2 \frac{k_2^2}{2} s_{\text{initial}}^2 \right) + \frac{T_{566}}{E^2} E_1^2 \sin^2(\varphi) k_1^2 s_{\text{initial}}^2 + s_{\text{initial}}$$

$$E_2 = \left(E_1 \cos(\varphi) + 3 \frac{E_1^2}{E} \sin^2(\varphi) \right) \frac{k_1^2}{k_2^2}$$

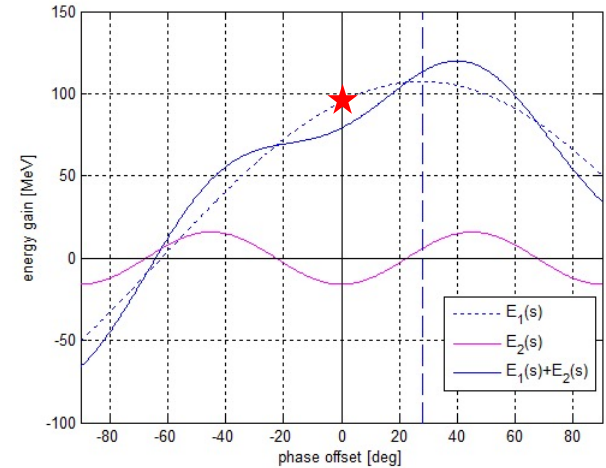
Energy Compensation



$$E_2 = \left(E_1 \cos(\varphi) + 3 \frac{E_1^2}{E} \sin^2(\varphi) \right) \frac{k_1^2}{k_2^2}$$

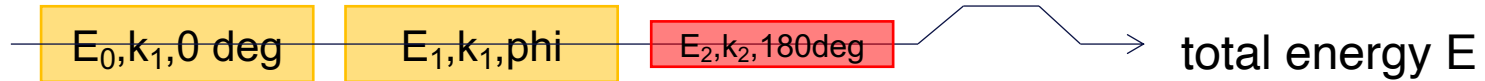
$$E_1 \sin(\varphi) = E_1' \sin(\varphi')$$

$$E_1 \cos(\varphi) + E_2 = E_1' \cos(\varphi')$$



Compensation of energy loss is required while the slope must be maintained.

Energy Compensation



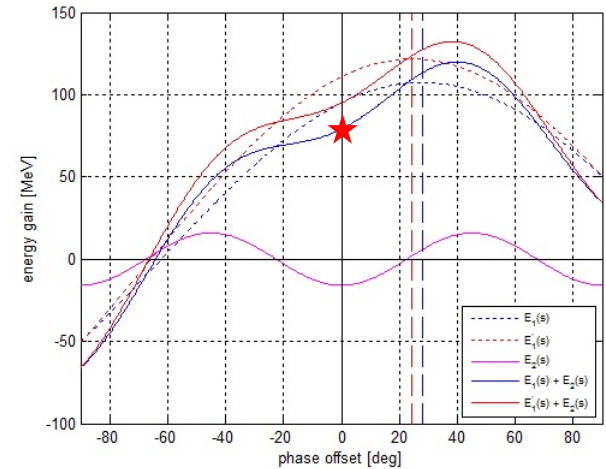
$$E_2 = \left(E_1 \cos(\varphi) + 3 \frac{E_1^2}{E} \sin^2(\varphi) \right) \frac{k_1^2}{k_2^2}$$

$$E_1 \sin(\varphi) = E_1' \sin(\varphi')$$

$$E_1 \cos(\varphi) + E_2 = E_1' \cos(\varphi')$$

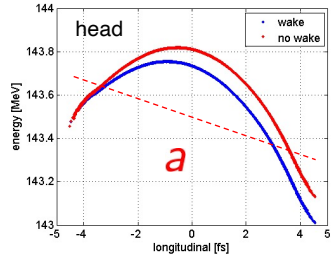
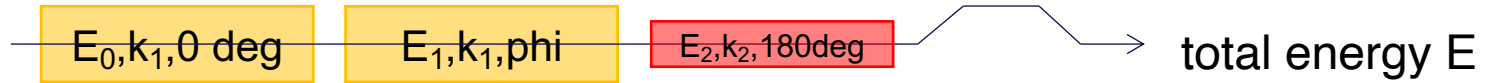
$$\Rightarrow \varphi' = \arctan \left(\frac{E_1 \sin(\varphi)}{E_1 \cos(\varphi) + E_2} \right)$$

$$E_1' = E_1 \frac{\sin(\varphi)}{\sin(\varphi')}$$



Compensation of energy loss is required while the slope must be maintained.

(Almost) General Solution for Single Stage BC



\bar{E}_0

$$\varphi = \arctan \left(-\frac{E}{E - E_0} \frac{\left(\frac{\sigma_{\text{final}}}{\sigma_{\text{initial}}} - 1 - aR_{56} \right)}{k_1 R_{56}} \right)$$

$$E_1 = \frac{E - E_0}{\cos(\varphi)}$$

$$E_2 = \left(E_1 \cos(\varphi) + \bar{E}_0 + 3 \frac{E_1^2}{E} \sin^2(\varphi) \right) \frac{k_1^2}{k_2^2}$$

$$\varphi' = \arctan \left(\frac{E_1 \sin(\varphi)}{E_1 \cos(\varphi) + E_2} \right)$$

$$E_1' = E_1 \frac{\sin(\varphi)}{\sin(\varphi')}$$

If x-band is not operated “anti-on-crest” an additional linear compression term plays a role.

The End