Synchrotron Radiation Output from the ILC Undulator

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Outline

- The analytic tracking code
- Benchmarking the code with an analytic undulator
- Calculations from measured field maps of single undulators
- Calculations with multiple undulator sections
- Conclusions and future work

Analytic Tracking Code

- Characterise an arbrtrary magnetic field in terms of it's multipole expansion and generalised gradients to produce an analytical description of field as a fuction of the longitudinal coordinate [a]
- Use the analytical expression in differential algebra or Lie algebra code to generate a Taylor or Lie (symplectic) map for the dynamics in the magnet.
- Evaluate the analytical expressions to perform a numerical integration giving a fast particle tracking code to describe the evolution of the canonical coordinates within the magnet.
- The C++ code that has been has been written has a modular structure which facilitates extending the code
- A Synchrotron Radiation Module is being implemented which calculates the synchrotron emission from a particle into an arbitrary observation point
- eg ILC Helical undulator

[a] Venturini and Dragt

Synchrotron Radiation Calculation

Accelerated charges radiate energy. The observed electric field [a] of the emitted radiation is:

$$\vec{E}(t) = \frac{e}{4\pi\epsilon_0 c}\left(\frac{c(1-|\vec{\beta}|^2)(\vec{n}-\vec{\beta})}{|\vec{R}|^2(1-\vec{n}\dot{\vec{\beta}})^3} + \frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{|\vec{R}|(1-\vec{n}\dot{\vec{\beta}})^3}\right)_{RET}$$

$$\frac{d^2U}{d\Omega dt} = \epsilon_0 c E^2 r^2 = \vec{G}(t)^2$$

$$\vec{G}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\vec{G}(t)\exp(\imath\omega t)dt$$

$$\frac{d^2I}{d\Omega d\omega} = |\vec{G}(\omega)|^2 + |\vec{G}(-\omega)|^2$$

[a] Jackson

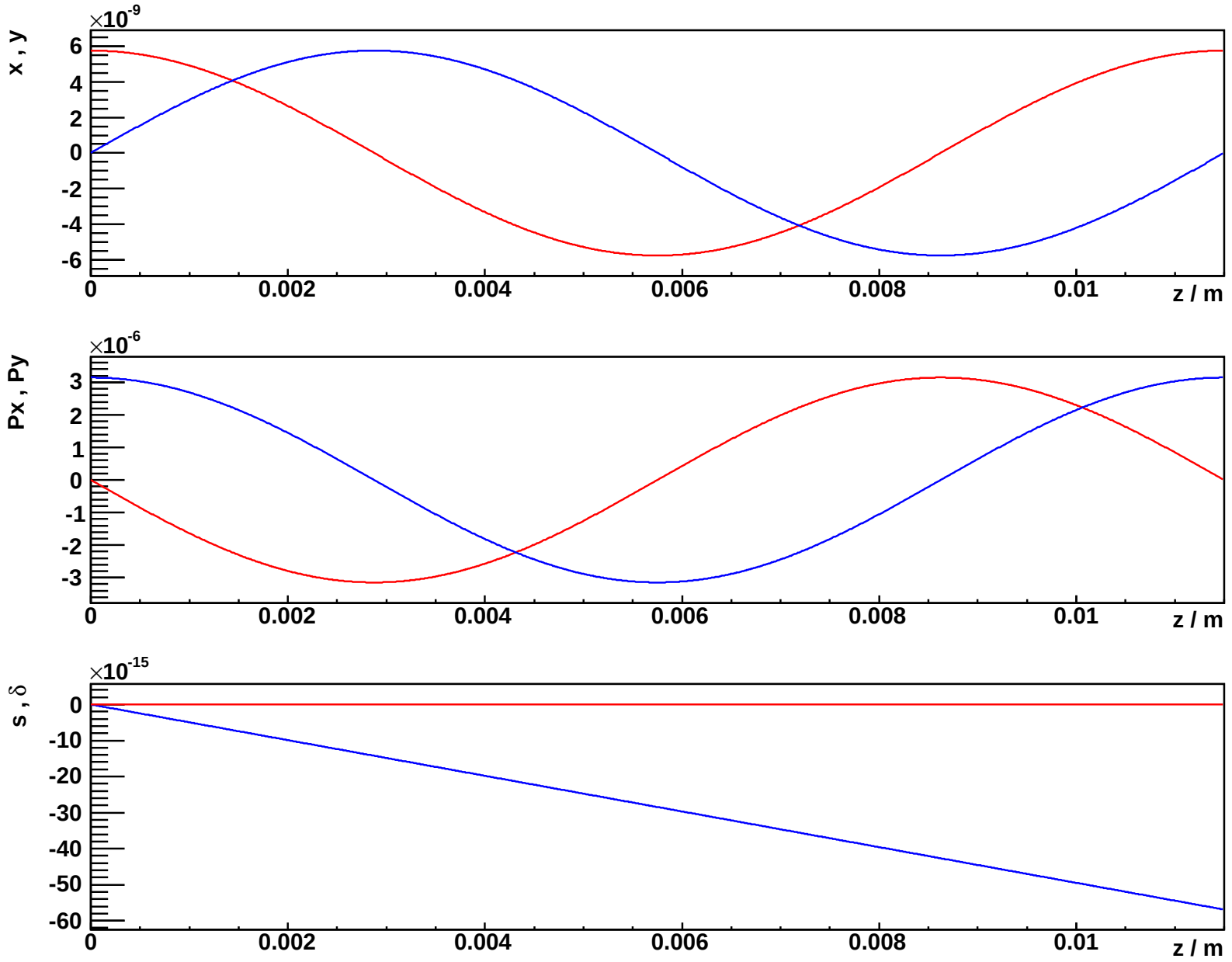
Implementation

At each step of the tracking code, the sychrotron variables, X , $\dot{X}$, $\ddot{X}$ ($X = x, y, z$), are calculated directly from the canonical coordinates and the magnetic field.

These variables are saved to a file and subsequently used to calculate the electric field at an arbitrary observation point.

At the end of the integration the differential intensity is found by performing a DFT on the electric field components.

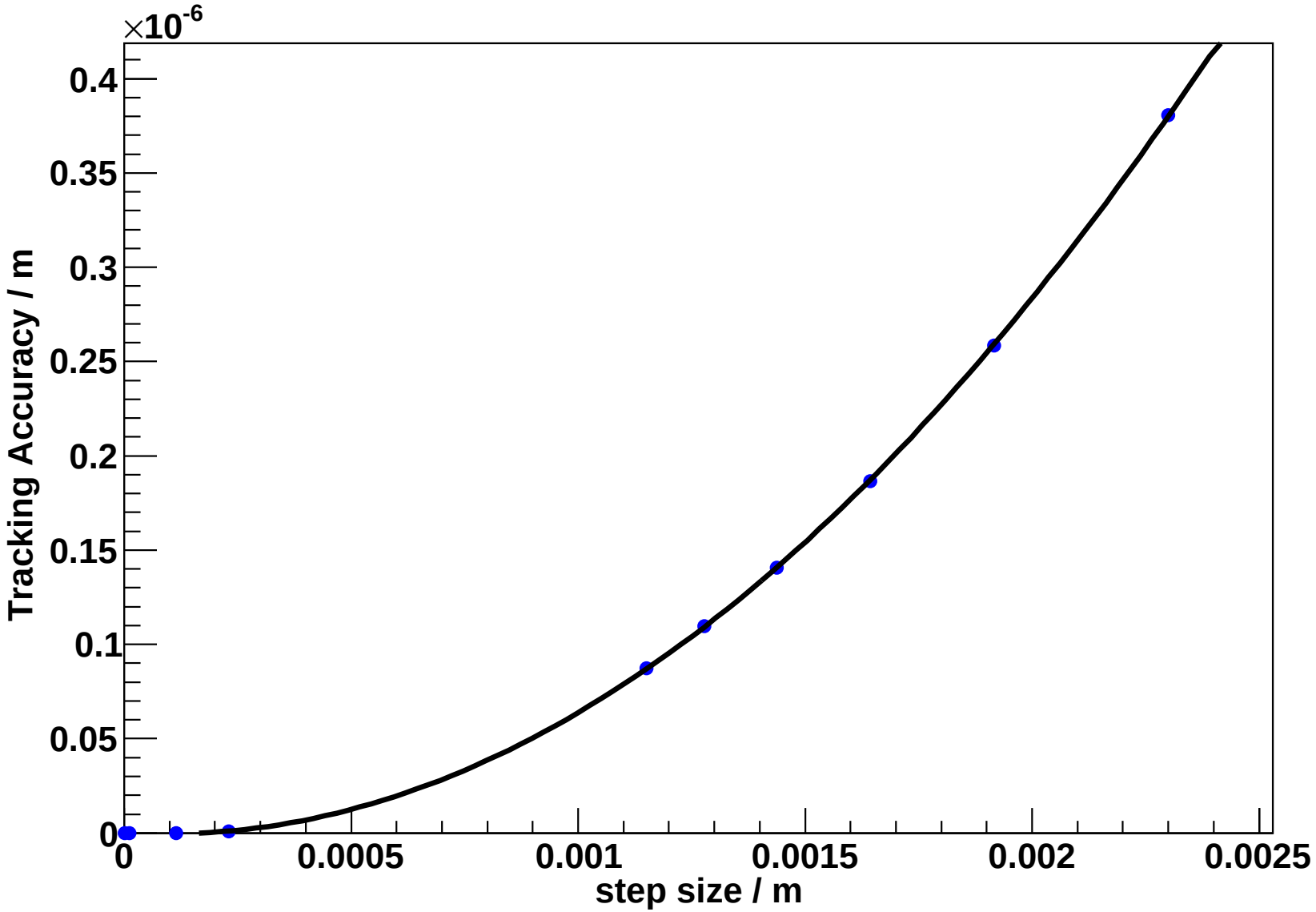
Tracking Particles through an Ideal Undulator



Tracking Accuracy vs StepSize (155 periods)

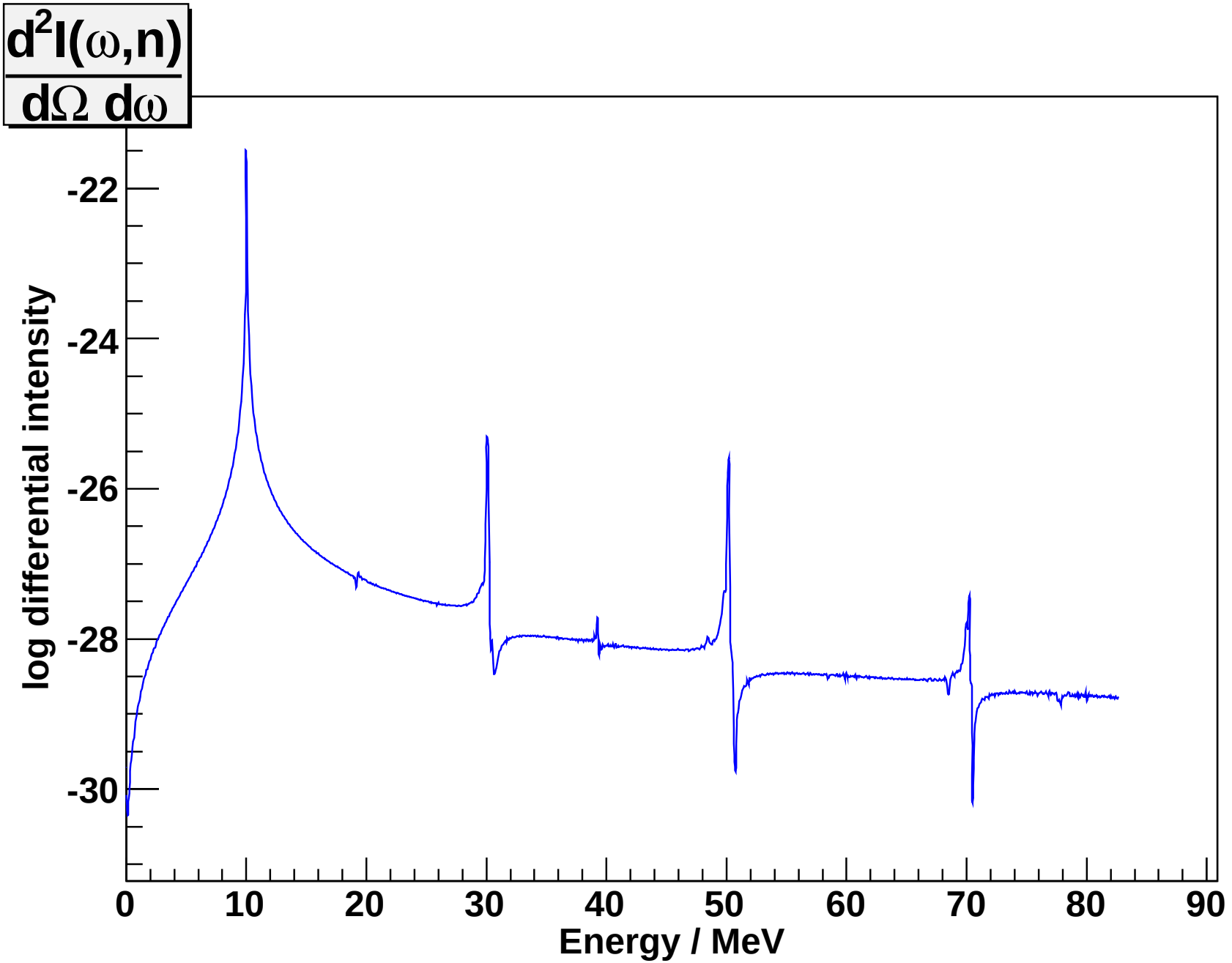
The displacement of the particle can be compared with the analytical solution of the trajectory

With a 2nd order integrator the accuaracy should be of order h^2 for a stepsize h



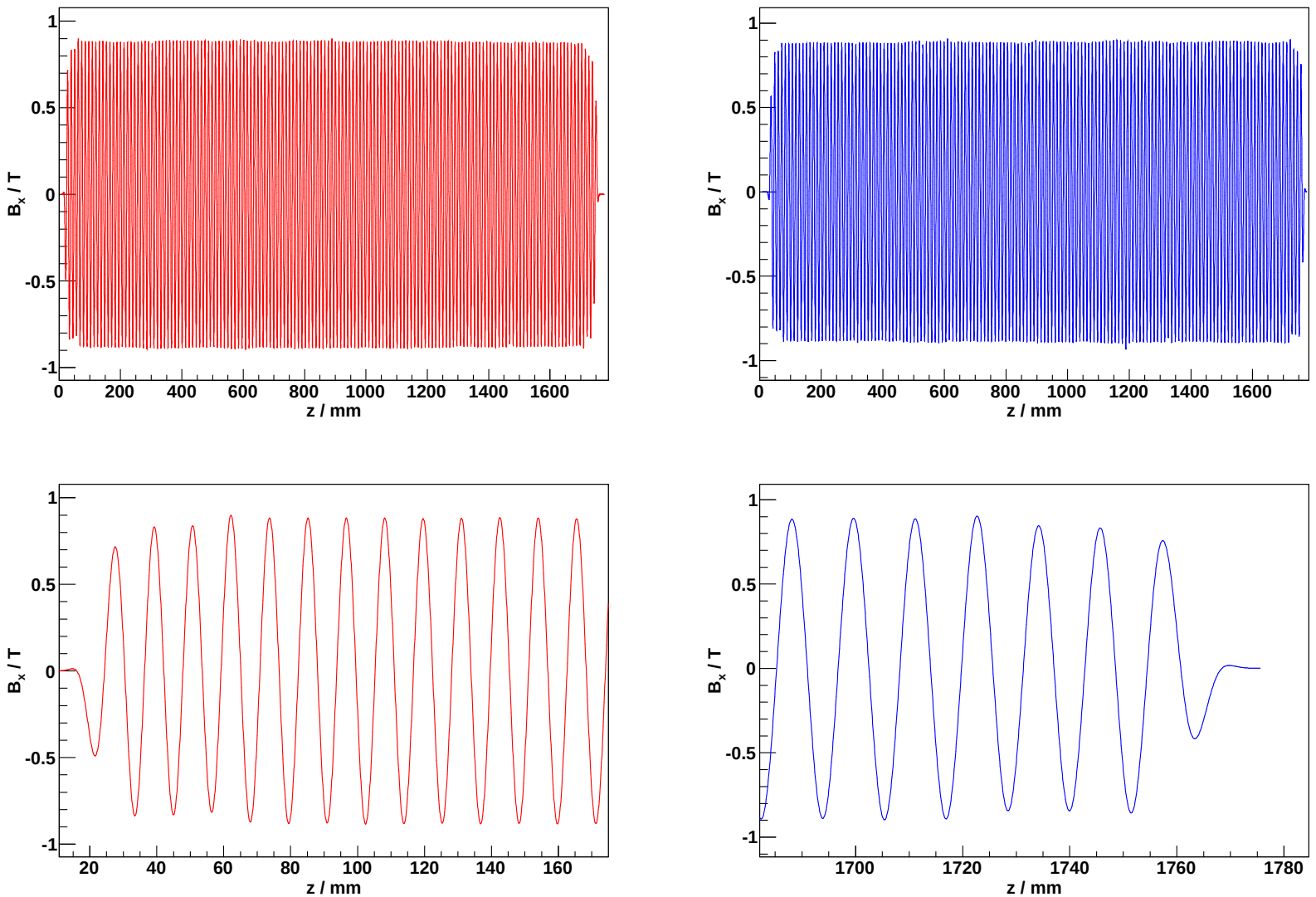
Energy Spectrum

Ideal undulator (155 periods, 2μm aperture)



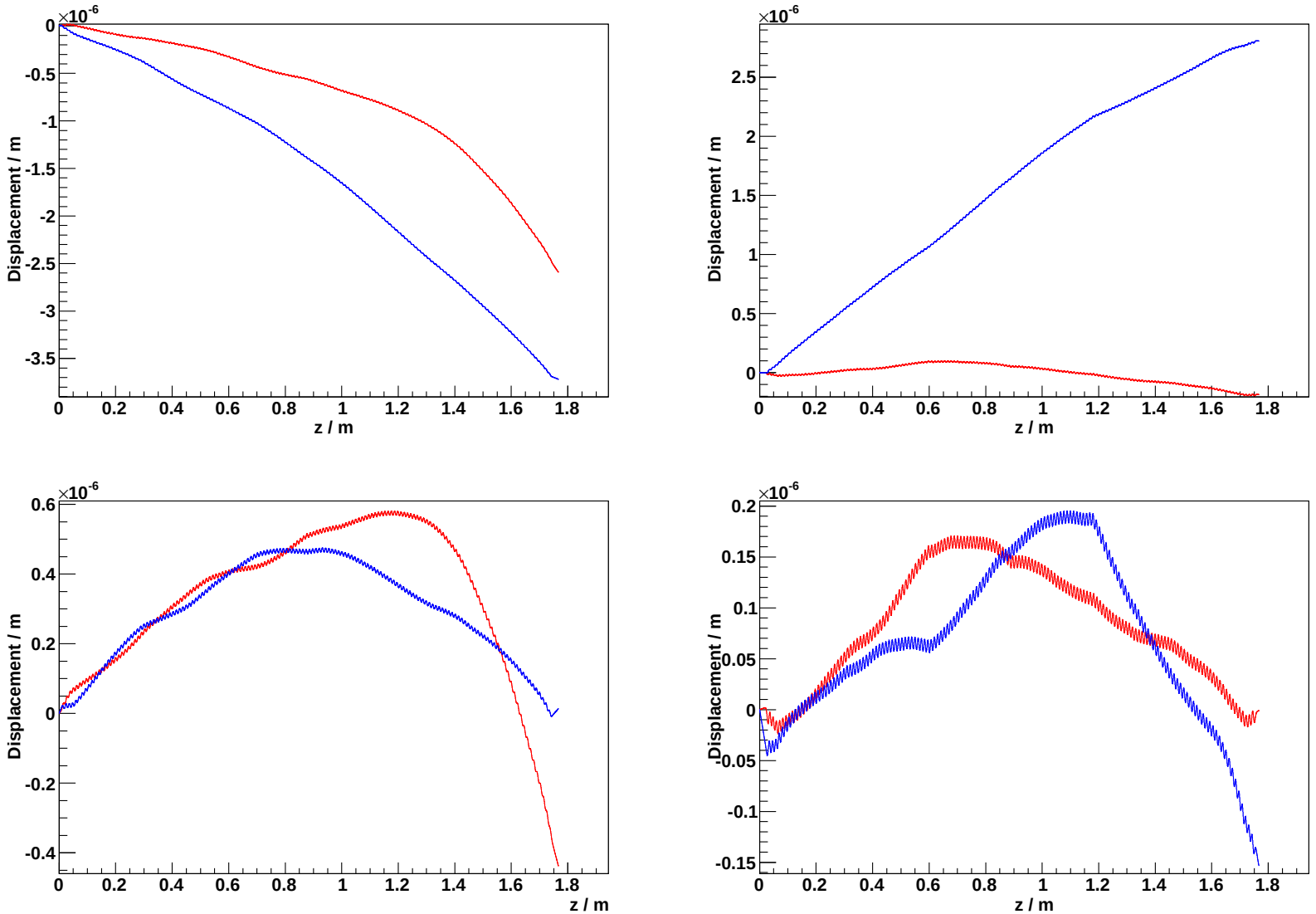
Measured Field Maps

Two different undulator sections



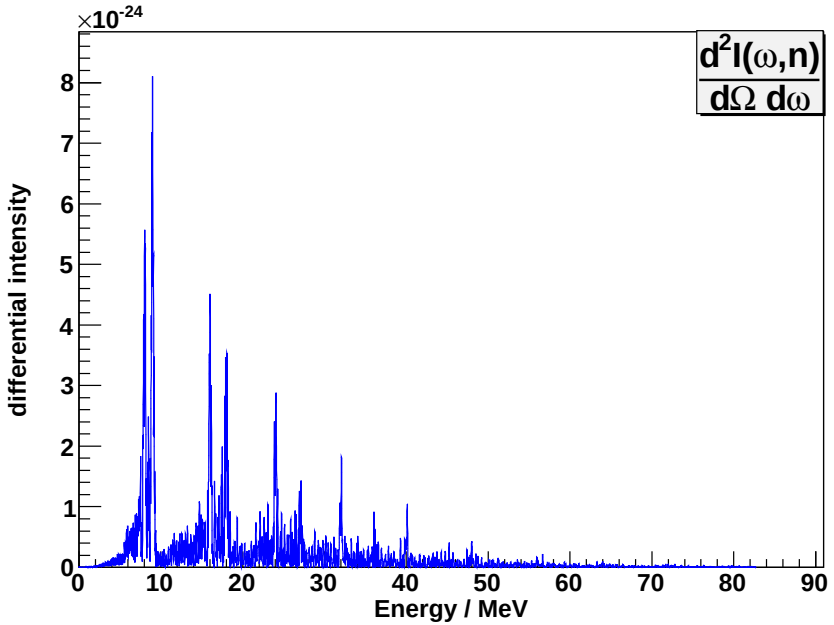
Particle Tracking

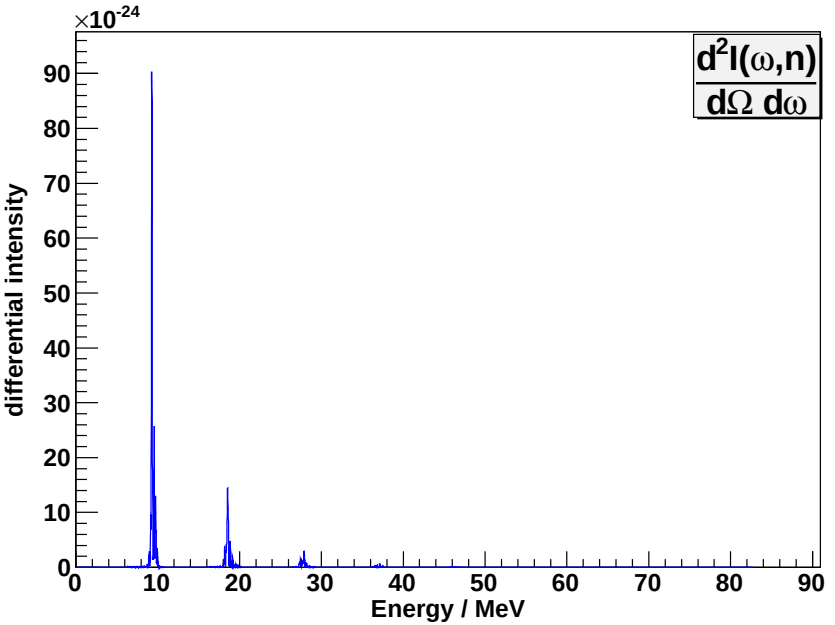
Tracking the x (red) and y (blue) coordinates of an elecron
Kicking the trajectory before the undulator gives a flatter trajectory
left - Undulator 1, right - Undulator 2



Energy Spectrum

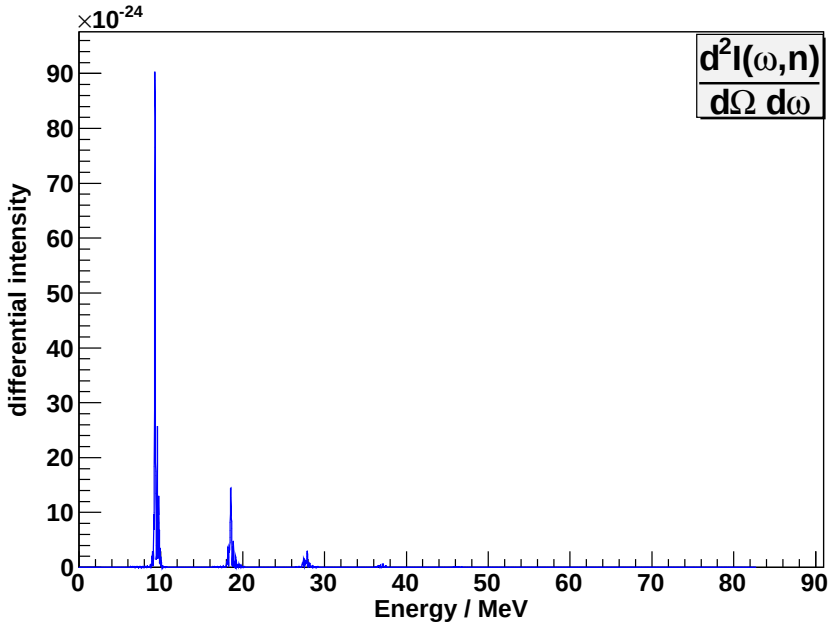
Observed in a $2\mu m$ aperture at the end of the undulator
Undulator 1 without (left) and with (right) trajectory correction

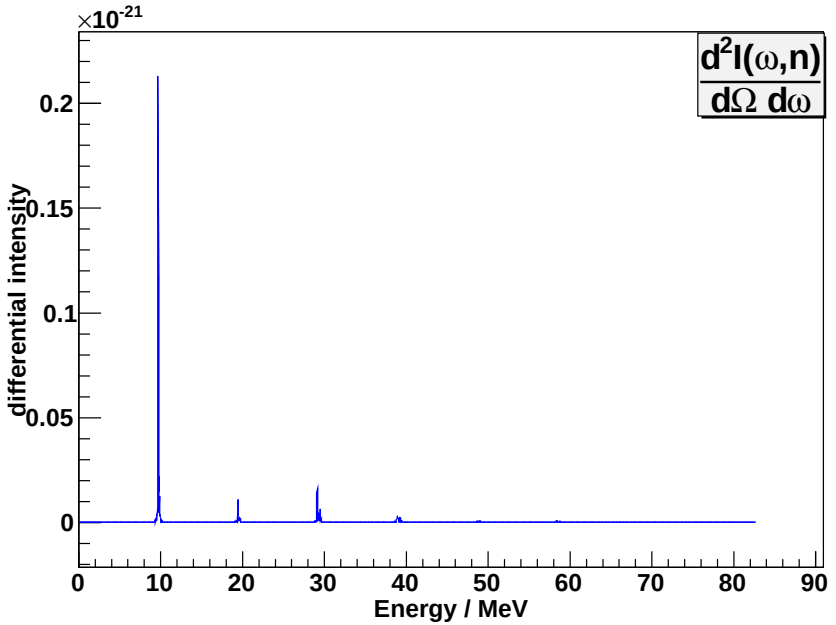




Energy Spectrum

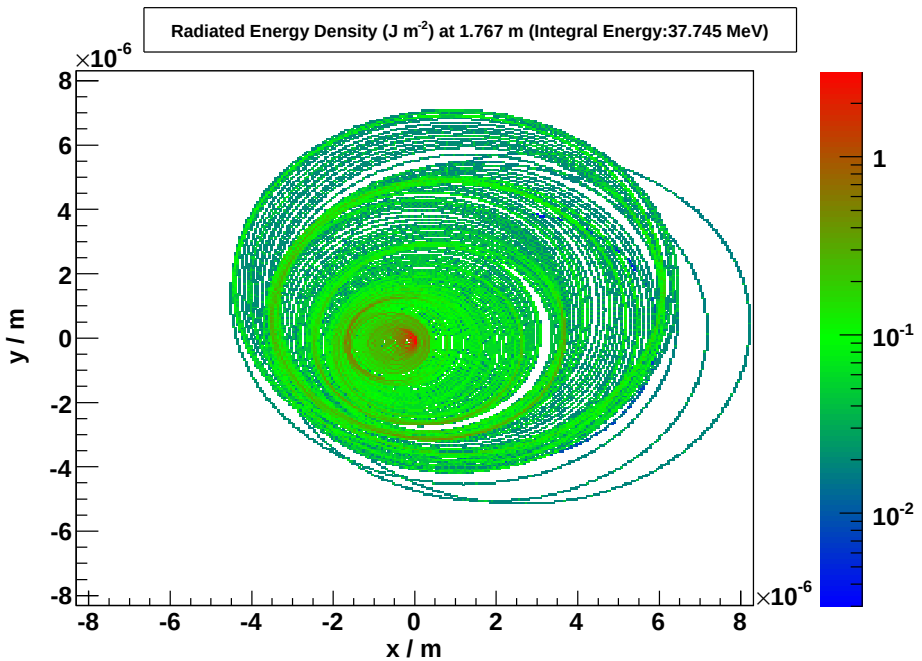
Observed in a $2\mu m$ aperture at the end of the undulator
Undulator 1 (left) and Undulator 2 (right) with trajectory correction

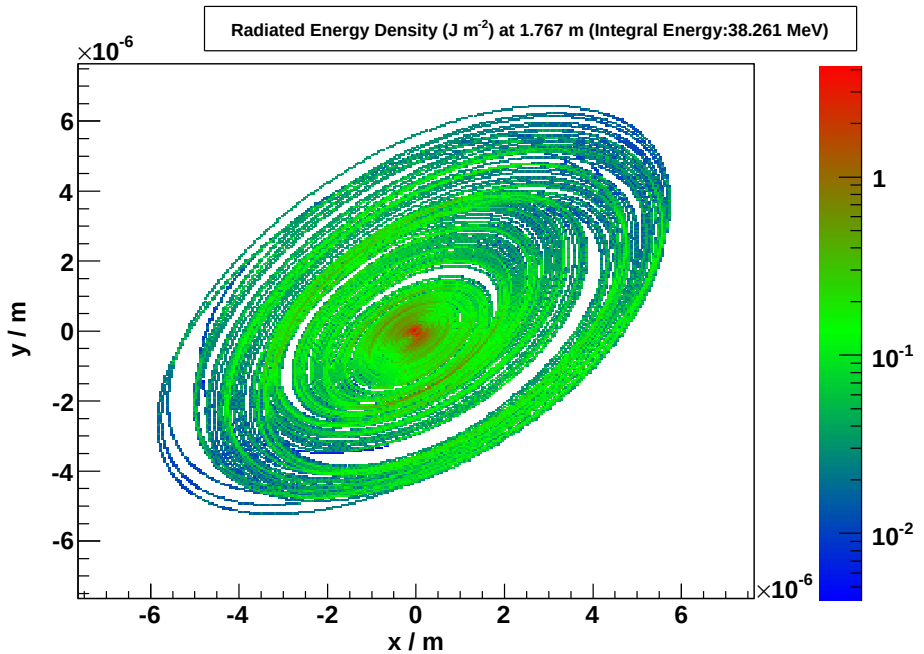




Power Radiated

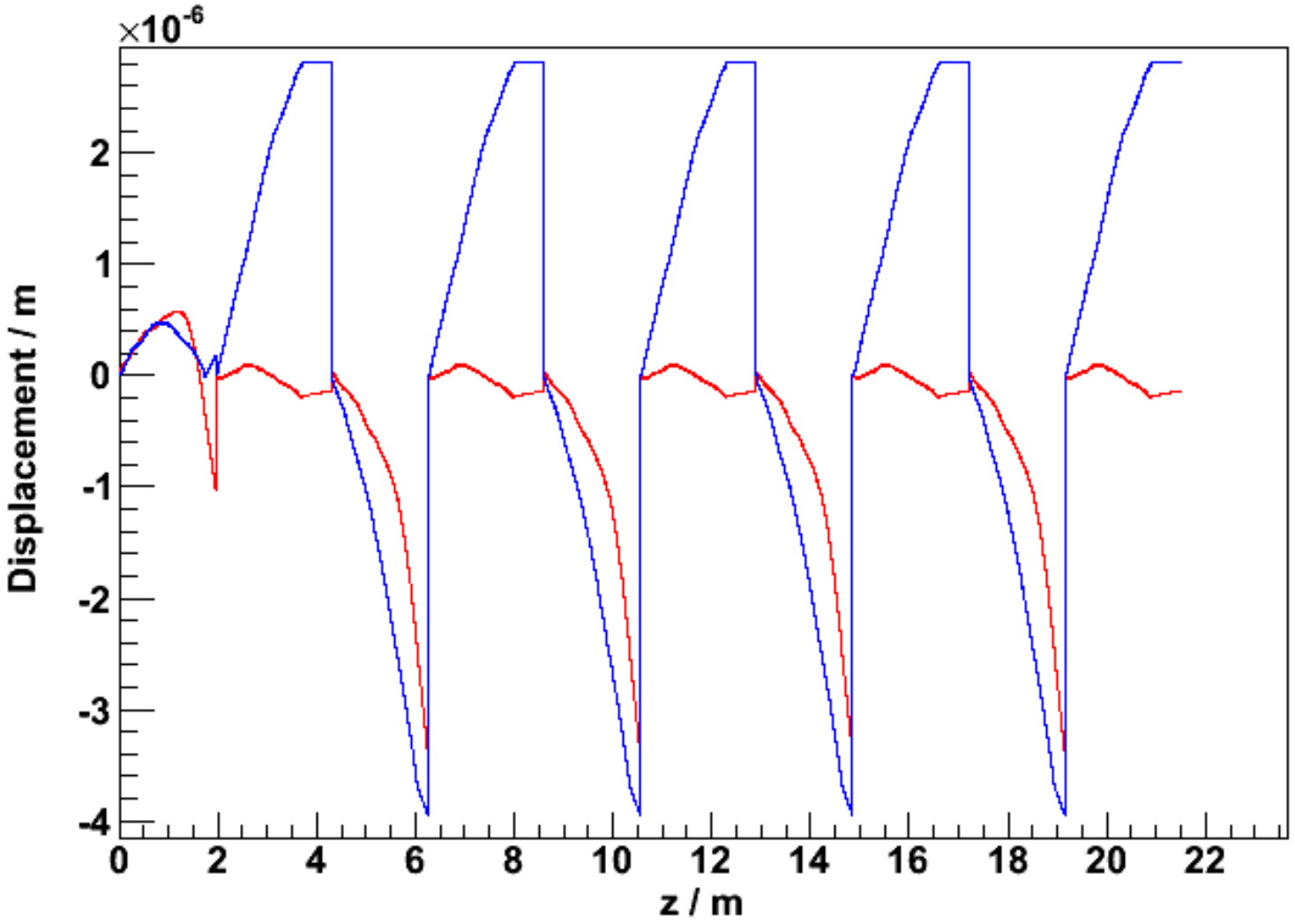
Power incident at the end of undulator 1 (left) and undulator 2 (right)
38.281 MeV is radiated, $\sim$ 0.02 % of the electron energy





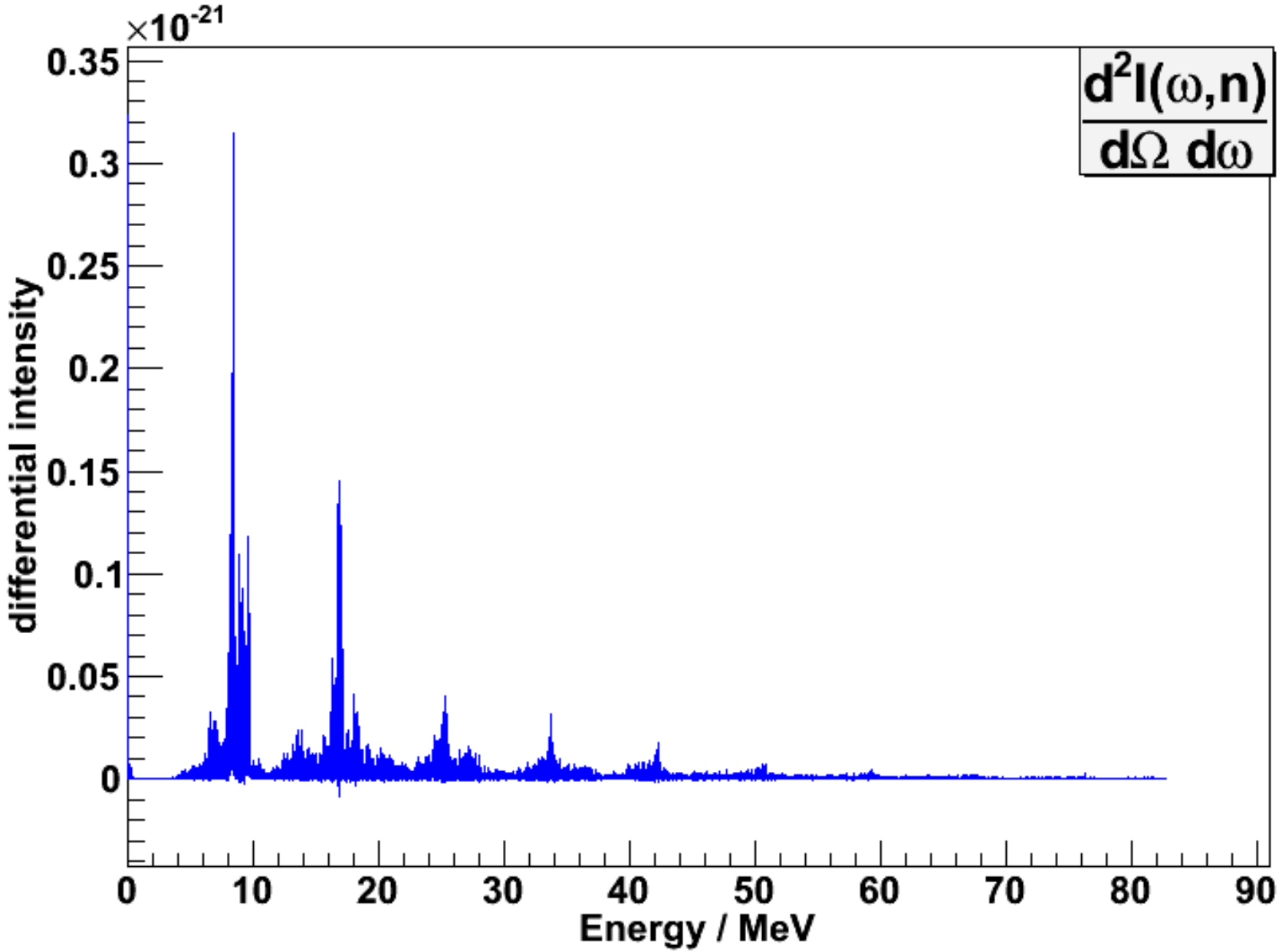
10 Undulator Sections

Particle Tracking through 10 Undulator sections Drift spaces included between undulator sections Particles are refocused every two undulator sections



Energy Spectrum

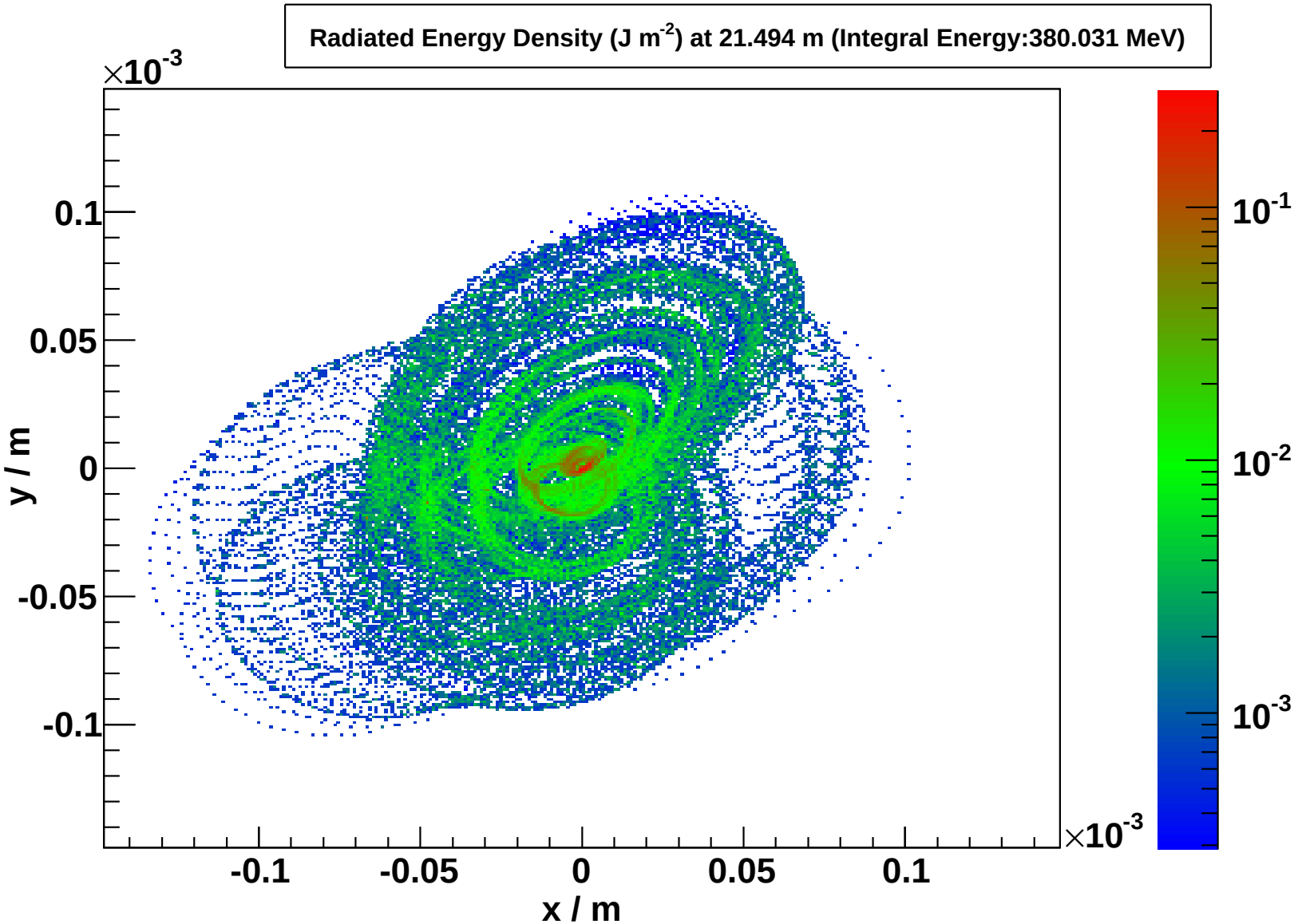
Measured in a 2μm aperture at the end of the final undulator



Power Radiated

Power radiated into a plane at the end of the final undulator

382.8 MeV is radiated, $\sim$ 0.2 % of the electron energy



Bunch Tracking

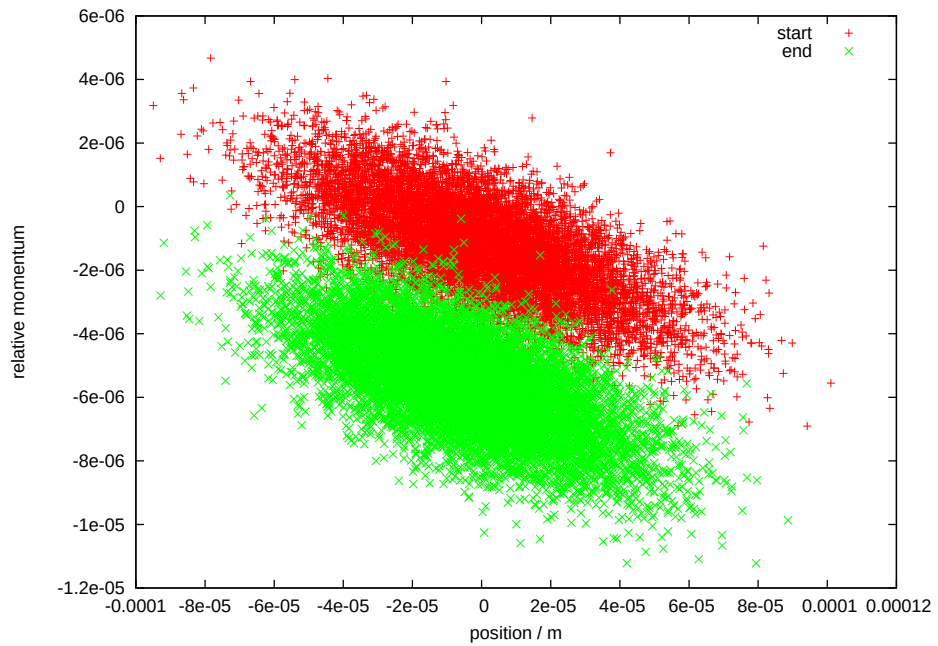
10,000 particles tracked through one undulator section

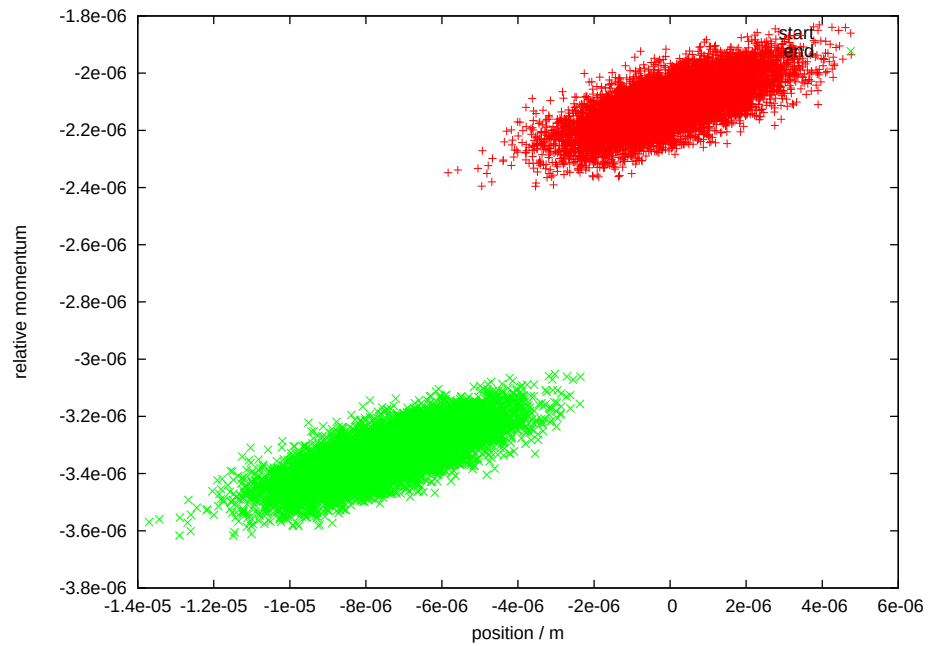
$\alpha_x = -1.0$, $\alpha_y = 1.0$

$\beta_x = \beta_y = 24.0m$

$\epsilon_{nx} = 8.4\mu m$, $\epsilon_{ny} = 24nm$

$\sigma E = 0.05\%$





Tracking x, p_x (left) and y, p_y (right) for the initial beam coordinates (red) and the final beam coordinates (green)

Conclusion and Future Work

- Benchmarking is underway with analytical solutions and SPECTRA calculations (D. Scott)
- Analytic tracking code has been heavily optimised for speed
- Typical run times (2.66 GHz processor, 4Mb RAM)
  - single undulator section (155 periods) $\sim$ 1 minute
  - 10 undulator sections $<$ 10 minutes
- Trajectory corrections greatly improve the quality of the energy spectrum
  - But the spectrum produced by a bunch must be optimised, not a single particle
- Aim to introduce parallel capability to the code
  - This will allow the spectrum from $\sim$ 10,000 particles to be calculated in a reasonable time
- Energy losses willbe included in the code
  - In theory the code is already set up to do this
  - Needs benchmarking