

# Invertible Networks

ErUM-Data Community Meeting 1.7.21

Anja Butter

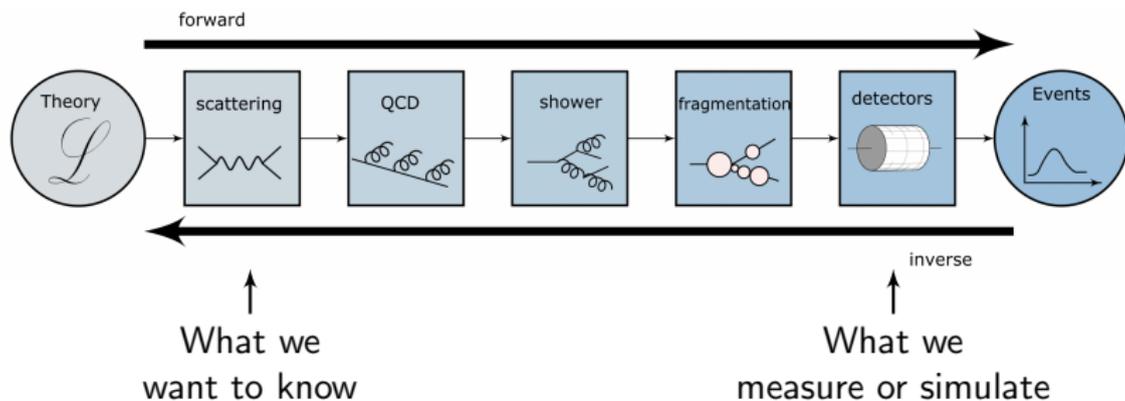
ITP, Universität Heidelberg

arXiv:2006.06685

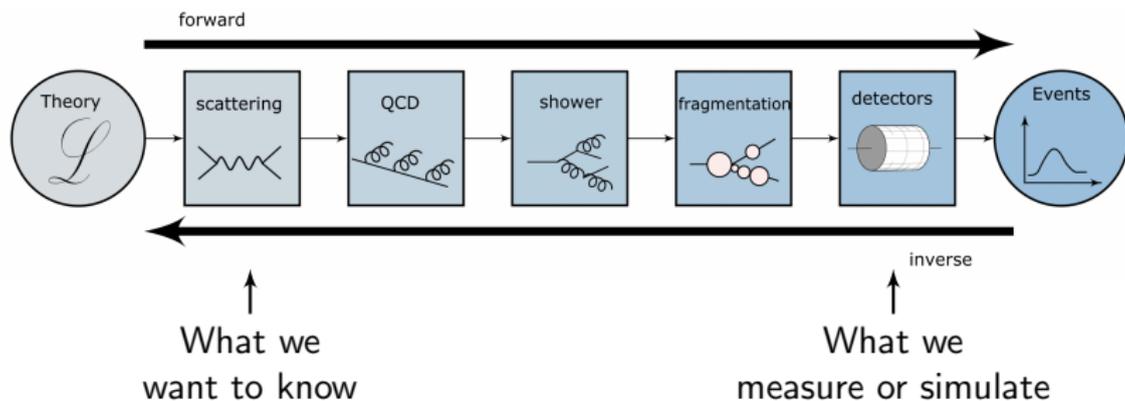
with M. Bellagente, G. Kasieczka, T. Plehn, A. Rousselot,  
R. Winterhalder, L. Ardizzone, U. Köthe



# Can we invert the simulation chain?

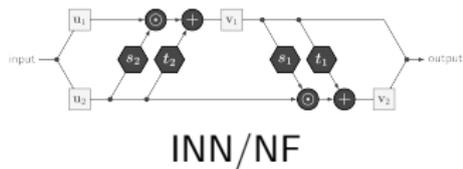
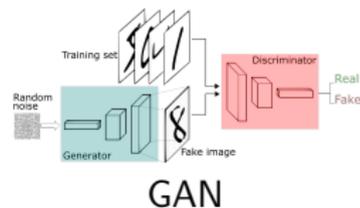
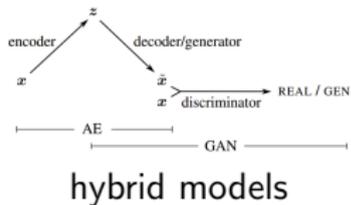
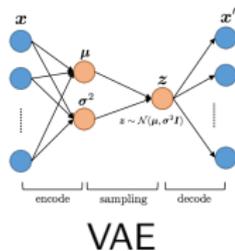


# Can we invert the simulation chain?



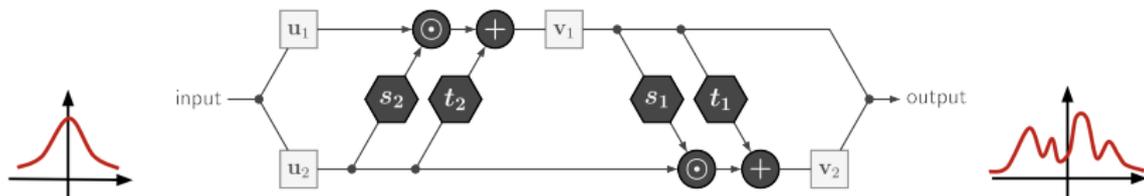
- Get multidimensional probability distribution for each event
- Need generative networks

# Neural network based generative networks



# Invertible networks

[1808.04730] L. Ardizzone, J. Kruse, S. Wirkert, D. Rahner, E. W. Pellegrini, R. S. Klessen, L. Maier-Hein, C. Rother, U. Köthe



- + Arbitrary networks  $s$  and  $t$
- + Fast evaluation in both directions
- + Simple Jacobian  $\rightarrow$  Control over phase space density

# Applications

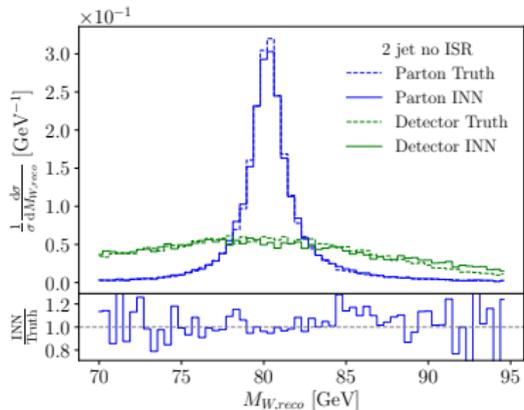
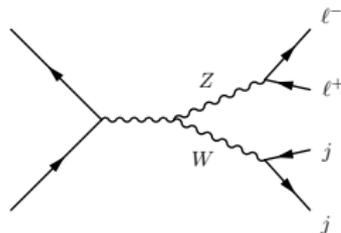
$$r \sim \mathcal{N} \begin{array}{c} \text{EVENT GENERATOR: } g \rightarrow \\ \leftarrow \text{ inversion: } \bar{g} \rightarrow \end{array} x \sim \mathcal{M}(r)$$

$$(x_{\text{parton}}) \begin{array}{c} \text{SHOWER/DETECTOR: } g \rightarrow \\ \leftarrow \text{ inversion: } \bar{g} \rightarrow \end{array} (x_{\text{detector}})$$

$$(x_{\text{model param}}) \begin{array}{c} \text{LHC SIMULATION: } g \rightarrow \\ \leftarrow \text{ inversion: } \bar{g} \rightarrow \end{array} (x_{\text{detector}})$$

# Inverting detector effects

- $pp \rightarrow ZW \rightarrow (\ell\ell)(jj)$
- Train: parton  $\rightarrow$  detector
- Evaluate: parton  $\leftarrow$  detector



multi-dimensional ✓ bin independent ✓ only for deterministic process

# Including the probabilistic aspect

$$(X_{part}) \xleftarrow{\text{PYTHIA,DELPHES:}g \rightarrow} (X_{det})$$

$\leftarrow \text{inversion:} \bar{g}$

$$\text{with } \mathcal{L} = \mathcal{L}_{part} + \mathcal{L}_{det}$$

## Including the probabilistic aspect

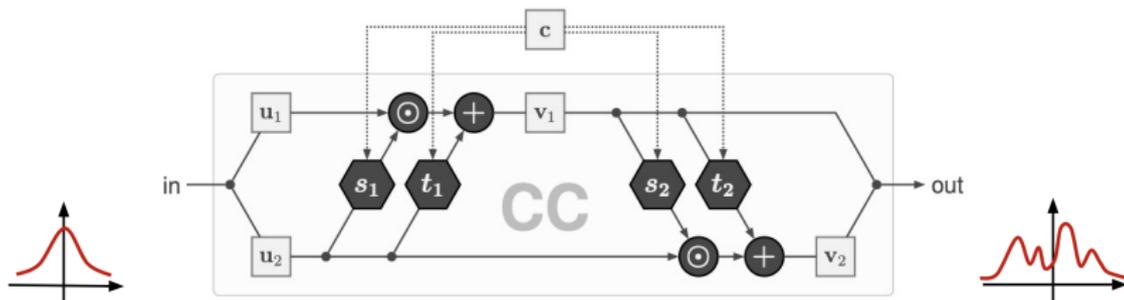
$$\begin{array}{ccc} \begin{pmatrix} x_{part} \\ r_{part} \end{pmatrix} & \begin{array}{c} \xrightarrow{\text{PYTHIA,DELPHES:}g\rightarrow} \\ \xleftarrow{\text{inversion:}\bar{g}} \end{array} & \begin{pmatrix} x_{det} \\ r_{det} \end{pmatrix} \\ \text{with } \mathcal{L} = \mathcal{L}_{part} + \mathcal{L}_{det} + \mathcal{L}_r & & \end{array}$$

Arbitrary choice of  $\mathcal{L}_{part}$ ,  $\mathcal{L}_{det}$ ,  $\mathcal{L}_r$  determines (un)supervised training

# Conditional INN

Rephrasing the problem

Given detector level information [ $\rightarrow$  condition  $c$ ]  
 $\rightarrow$  What is the probability density at parton level?



# How to train the network

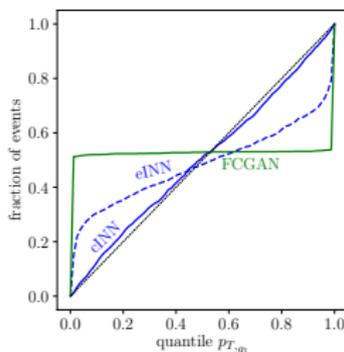
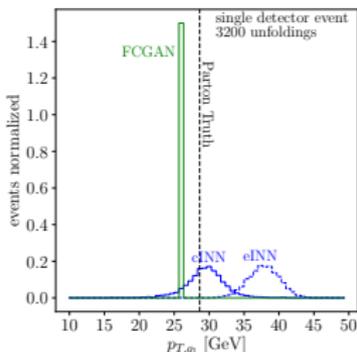
→ Training: Maximize posterior over model parameters

$$\begin{aligned} L &= - \langle \log p(\theta|y) \rangle_{y \sim \text{data}} \\ &= - \langle \log p(y|\theta) \rangle_{y \sim \text{data}} - \log p(\theta) + \text{const.} \quad \leftarrow \text{Bayes} \\ &= - \left\langle \log p(g_\theta(y)) + \log \left| \frac{\partial g_\theta(y)}{\partial y} \right| \right\rangle_{y \sim \text{data}} - \log p(\theta) \quad \leftarrow \text{change of var} \\ &= \left\langle \underbrace{0.5 \|g(y)\|_2^2}_{\text{Gaussian latent variable } g(y) \rightarrow \mathcal{N}} - \underbrace{\log \left| \frac{\partial g_\theta(y)}{\partial y} \right|}_{\text{Jacobian}} \right\rangle_{y \sim \text{data}} - \underbrace{\log p(\theta)}_{\text{Regularization } \|\theta\|^2} \end{aligned}$$

# cINN result for calibration

$$\begin{array}{c}
 g(x_p, f(x_d)) \rightarrow \\
 \leftarrow \text{unfolding: } \bar{g}(r, f(x_d)) \rightarrow r \\
 x_p
 \end{array}$$

$$\text{Minimizing } L = \langle 0.5 \|\bar{g}(x_p, f(x_d))\|_2^2 - \log |J| \rangle_{x_p \sim P_p, x_d \sim P_d} - \log p(\theta)$$



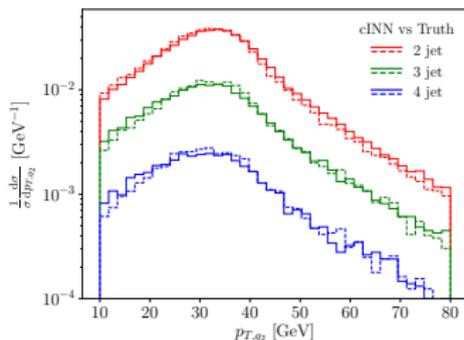
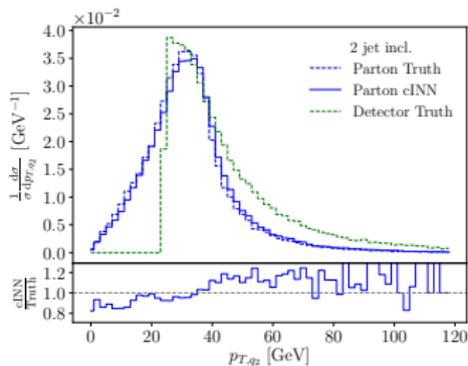
multi-dimensional ✓ bin independent ✓ statistically well defined ✓

# Inverting the full event

$pp > WZ > q\bar{q}l^+l^- + \text{ISR}$

Train on inclusive dataset

Evaluate  
exclusive 2/3/4 jet channels

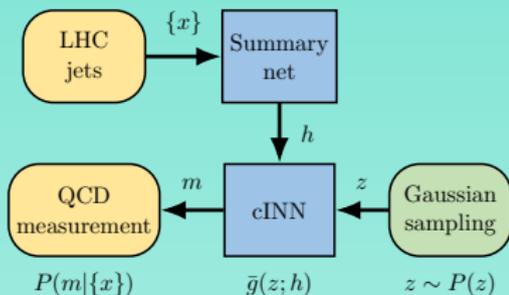


# Going beyond unfolding

## Same principle for inference

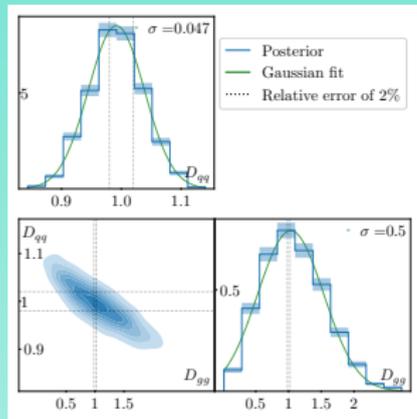
Measure parton shower parameters

Inference



## Infer splitting kernels

$$P_{qq}(z, y) = C_F \left[ D_{qq} \frac{2z(1-y)}{1-z(1-y)} + F_{qq}(1-z) + C_{qq}yz(1-z) \right]$$



# Invertible networks

- Fast evaluation and tractable Jacobian
- Trainable on samples as well as densities
- Invertible networks for deterministic mapping
- **cINN** guaranties correct calibration in probabilistic mapping
- Simultaneous unfolding of different exclusive channels

