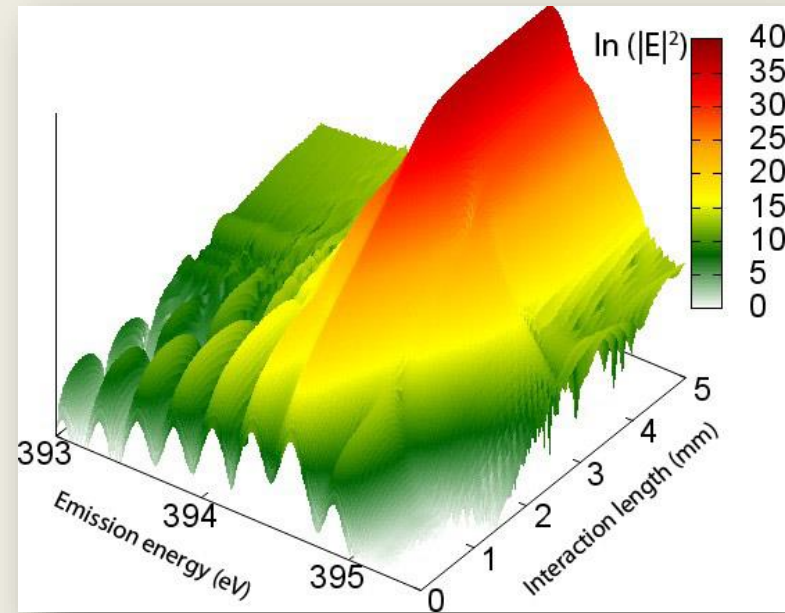




Stimulated X-ray emission in molecules: experience and perspectives

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Workshop on 1D-imaging Soft X-ray Spectroscopy at SQS, The EuroXFEL 21/10/21

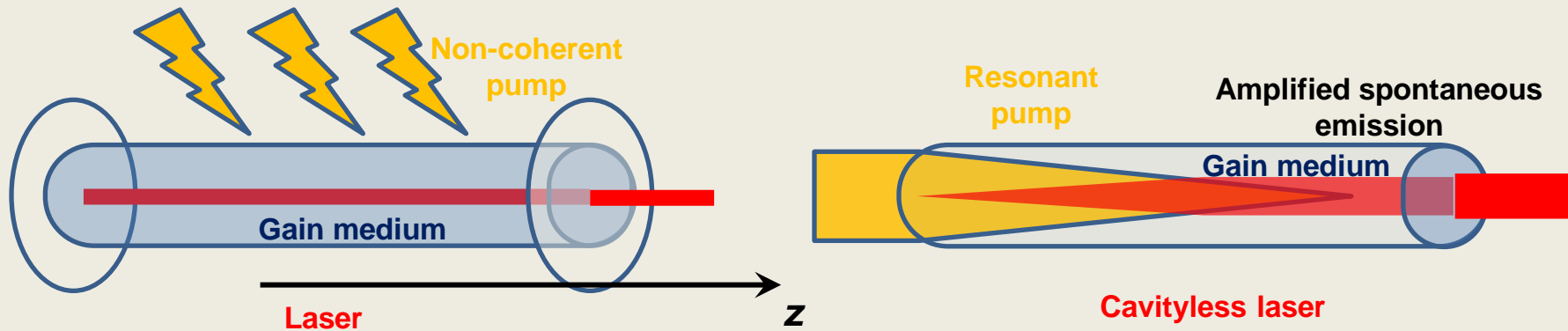
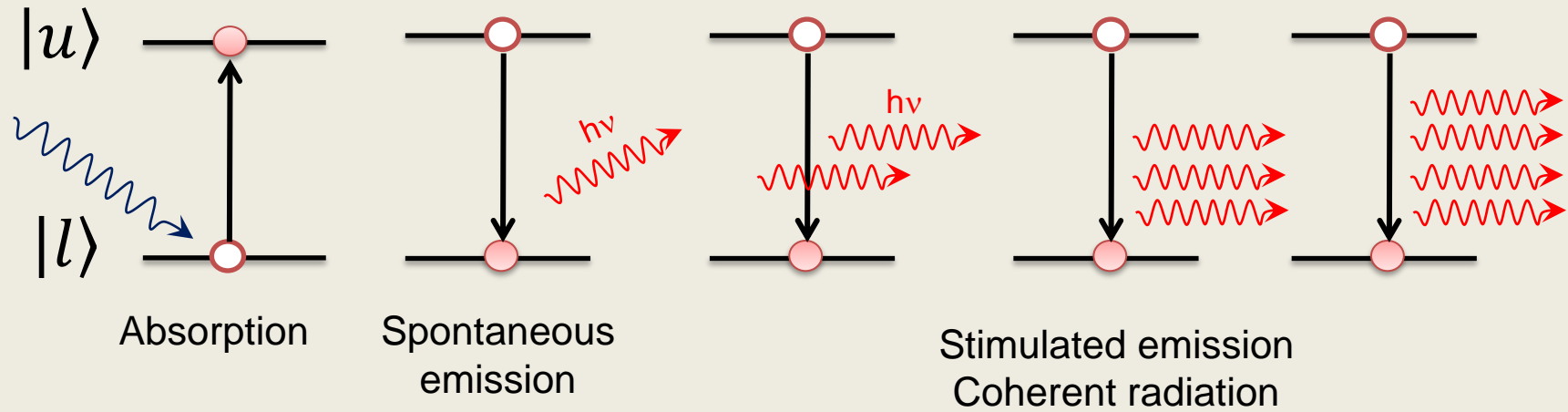


Stimulated X-ray emission in molecules: **theoretical** experience and perspectives **at SQS**

- Basics of stimulated emission, amplified spontaneous X-ray emission in atoms
- Theory of SXE in molecules: new opportunities and challenges
- Stimulated X-ray Raman (RIXS): new dimensions for two X-ray color schemes
- Theory vs experiment

- **New possibilities with 1D imaging Soft X-ray spectroscopy at SQS:**
 - IR+X-ray: molecular alignment for SXE
 - IR+X-ray: IR-pump vibrational dynamics observed in fluorescence
 - Study of electron wave packet in a confinement: shape resonance tunneling time
 - Observation of the pulse slow down in two-color delayed X-ray pulse mode

Stimulated emission and lasing

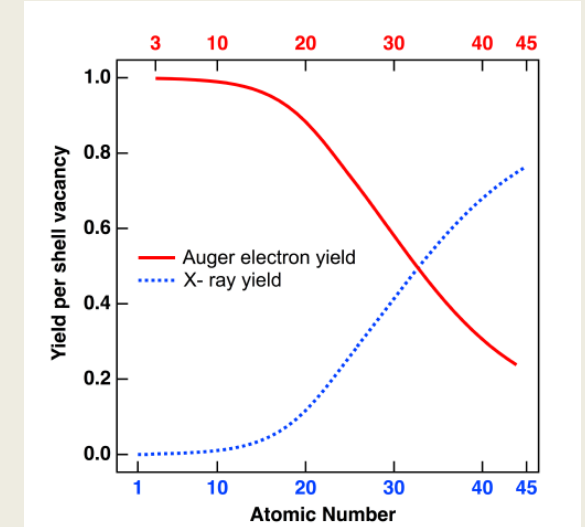
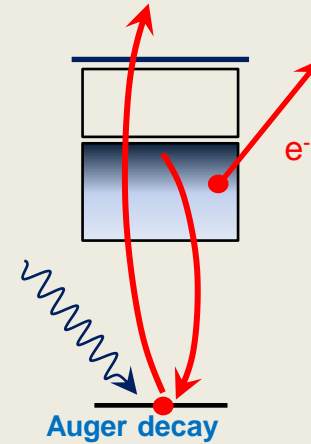


$$\left(\frac{1}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial z} \right) I = gI, \quad g = \sigma_{stim} (N_u - N_l)$$

$$I \approx I_0 \exp(gNz)$$

Towards x-ray nonlinearity: x-ray lasing

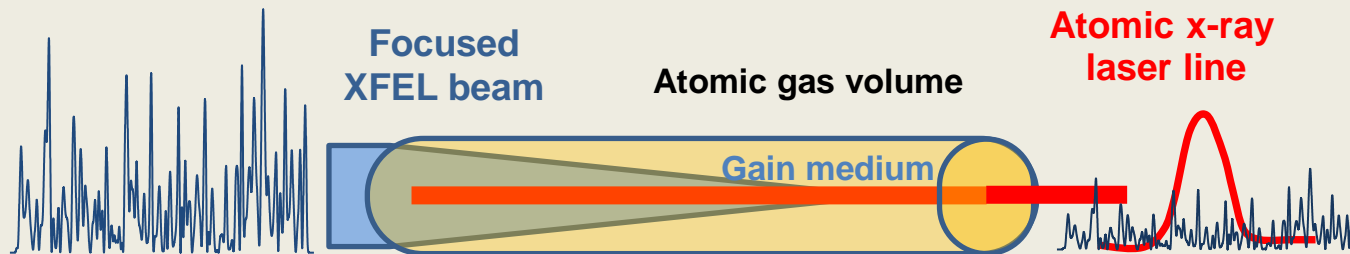
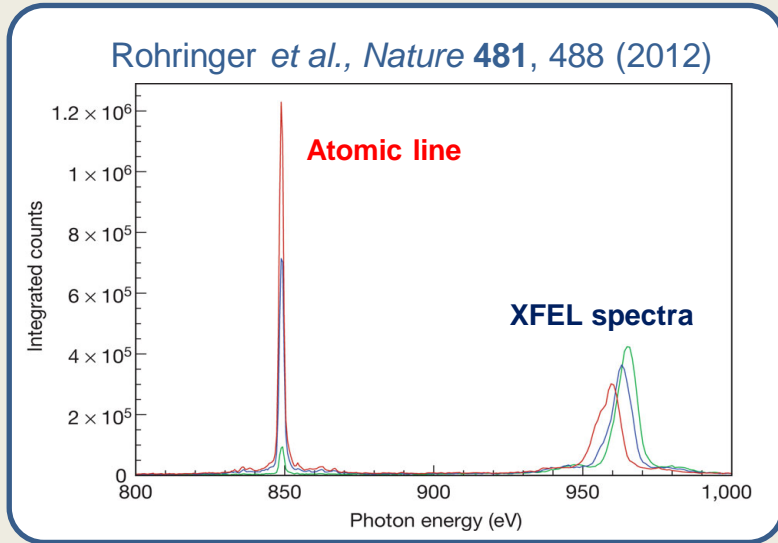
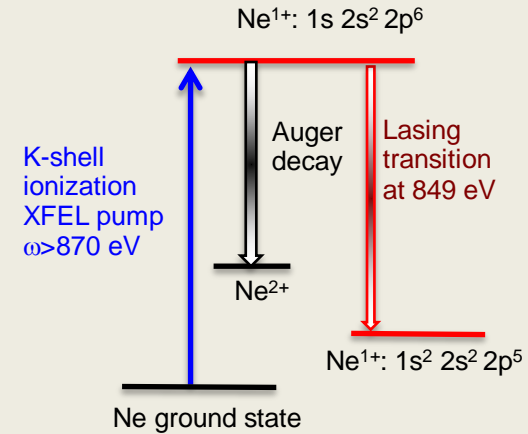
- **X-ray lasing scheme was proposed in 60th**
Duguay & Rentzepis, Appl. Phys. Lett. 10 350 (1967)
- **Strong nonradiative Auger decay channel**
 - Short core-hole lifetime: **1-10 fs**
 - Small transition dipole moments (TFY $\sim 10^{-2} - 10^{-3}$)
- **Create gain media**
 - Pump rate faster than Auger rate!
 - Source: High photon flux and photon energy
Rohringer & London, PRA 80, 013809 (2009)
Sun et al., Phys Rev A 81 (2010) 013812
- **1st demonstration of the XFEL pumped x-laser**
 - 2010: LCLS soft X-ray FEL in operation
 - Gas phase Neon: $K\alpha$ soft x-ray lasing: *Rohringer et al., Nature 481, 488 (2012)*



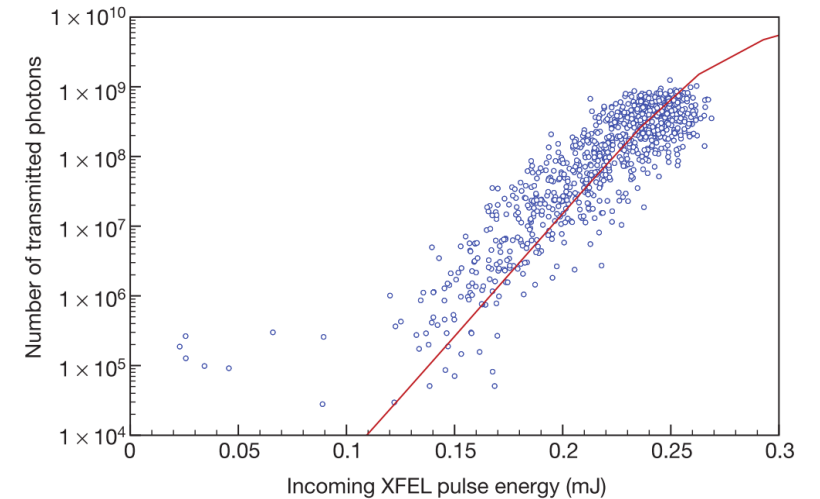
1st experiment on x-ray nonlinear effects: X-ray lasing in Ne gas

- ❑ LCLS pump: $h\nu=960$ eV, 40 fs, focus 2 mkm
- ❑ **Single shot of highest intensity:** $\sim 10^{12}$ photons per XFEL pulse
- ❑ **X-ray emission:** 8×10^9 photons in Ne K- α line
 - ✓ **conversion efficiency** $\sim 4 \times 10^{-3}$

Photo-ionization pumping

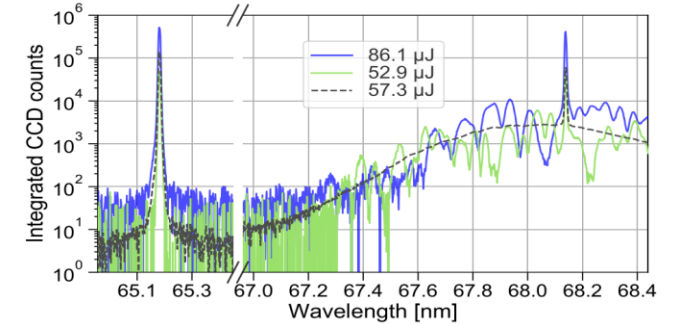
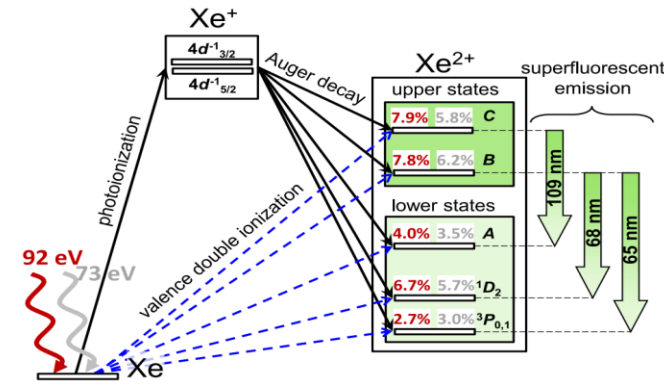
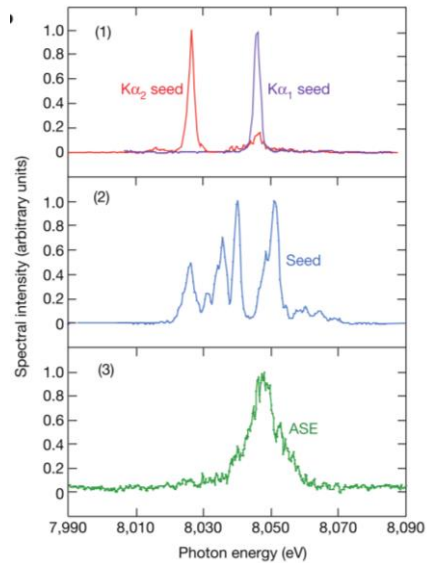
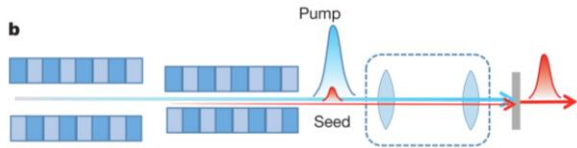
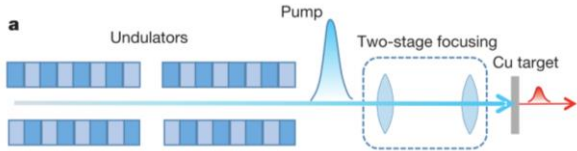


Dependence of the XRL output on pump power.

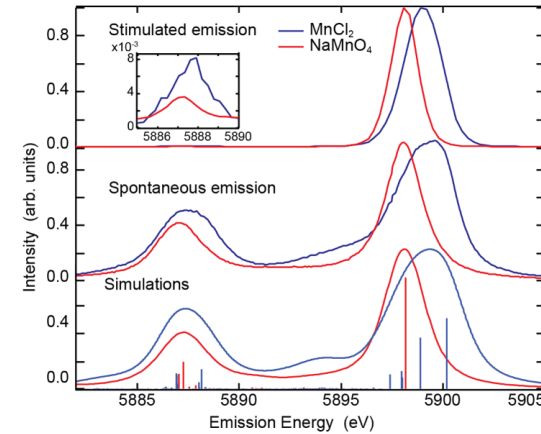
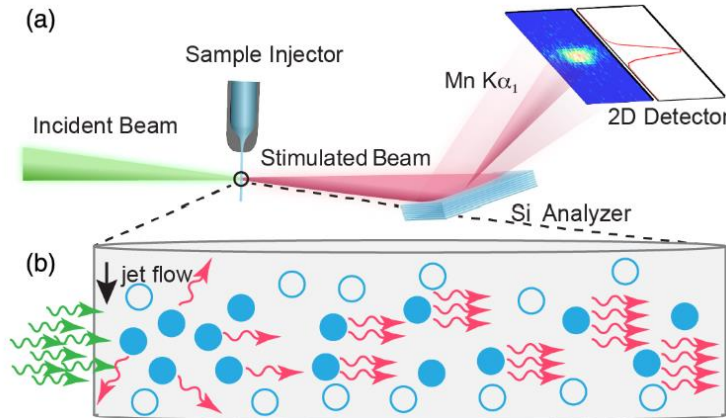


Duguay & Rentzepis, *Appl. Phys. Lett.* **10** 350 (1967)
 Rohringer & London, *PRA* **80**, 013809 (2009)
 Rohringer et al., *Nature* **481**, 488 (2012)

Cu K α line laser: 8 keV



UV superfluorescence Xe



Stimulated X-Ray Emission in TM Complexes

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Cu K α laser, 8 keV: Yoneda, H. et al. *Nature* **524**, 446–449 (2015).

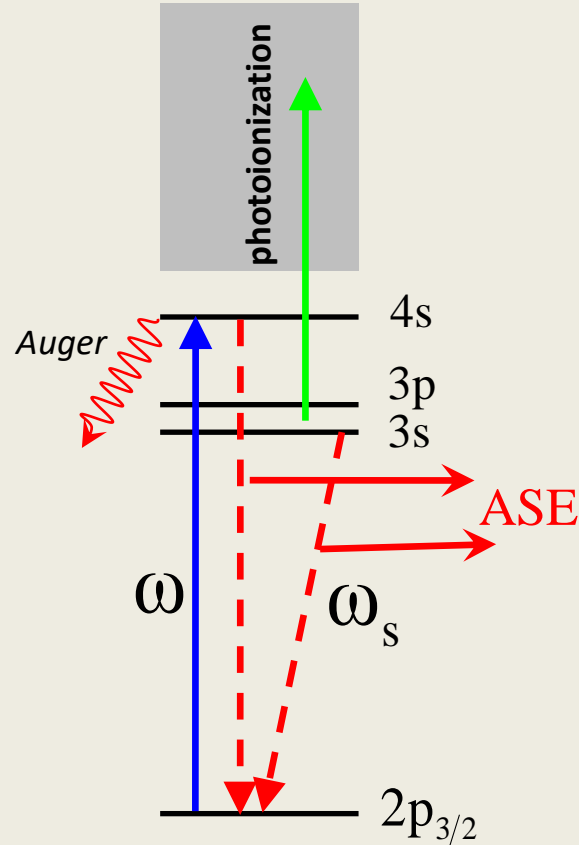
Superfluorescence, FWM in He: J. R. Harries et al., *Phys. Rev. Lett.*, **121**, 263201 (2018)

SRIXS and ASXE Mn K α , K β : Kroll et al., *PRL* **120**, 133203 (2018), *PRL* **125**, 037404 (2020)

Superfluorescence Xe: L. Mercadier et al., *Phys. Rev. Lett.*, **123**, 023201 (2019)

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Theoretical predictions: nonlinear effects for strong X-ray pulse propagating in resonant gas media



- Ar gas medium
- Strong Auger decay and photoionization: an open quantum system is considered

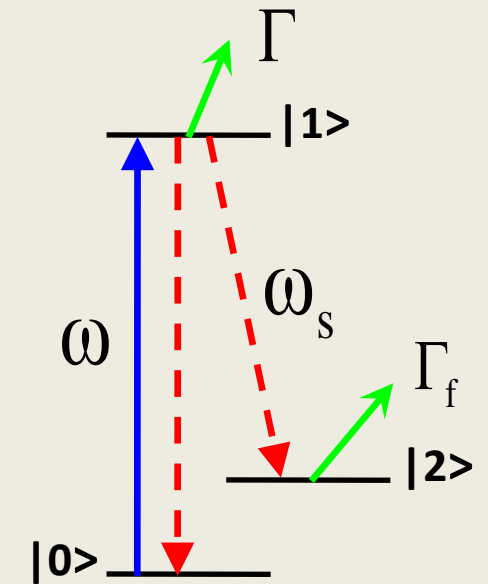
$$\Gamma = \Gamma_{decay} + \gamma_{photoionization}$$

$$\rho_{00} + \rho_{11} + \rho_{22} \approx \rho_{00} e^{-\Gamma\tau} < 1$$

- Scheme of transitions uses

$$|1\rangle = |2p_{3/2}^{-1}4s\rangle$$

$$|2\rangle = |3s^{-1}4s\rangle$$



Coupled density matrix and Maxwell's equations

Density-matrix equations

$$\left[\frac{\partial}{\partial t} + \Gamma_{11} + \gamma_{ph}^{(1)}(t)\right]\rho_{11} = W = 2\Im \sum_{n=0,2} (V_{1n}\rho_{n1})$$

$$\left[\frac{\partial}{\partial t} + \Gamma_{22} + \gamma_{ph}^{(2)}(t)\right]\rho_{22} = 2\Im(V_{21}\rho_{12})$$

$$\left[\frac{\partial}{\partial t} + \Gamma_{00} + \gamma_{ph}^{(0)}(t)\right]\rho_{00} = 2\Im(V_{01}\rho_{10})$$

$$\left[\frac{\partial}{\partial t} + \Gamma_{10} + \gamma_{ph}^{(10)}(t)\right]\rho_{10} = iV_{10}(\rho_{11} - \rho_{00}) - iV_{12}\rho_{20}$$

$$\left[\frac{\partial}{\partial t} + \Gamma_{12} + \gamma_{ph}^{(12)}(t)\right]\rho_{12} = iV_{12}(\rho_{11} - \rho_{22}) - iV_{10}\rho_{02}$$

$$\left[\frac{\partial}{\partial t} + \Gamma_{20} + \gamma_{ph}^{(20)}(t)\right]\rho_{20} = -iV_{21}\rho_{10} + i\rho_{21}V_{10}$$

Maxwell equations

$$\frac{\partial E}{\partial z} + \mu_0 \frac{\partial H}{\partial t} = 0$$

$$\frac{\partial H}{\partial z} + \epsilon_0 \frac{\partial E}{\partial t} = -\frac{\partial P}{\partial t}$$

$$P(t, z) = N\text{Tr}(d\rho) = 2N\Re(d_{10}\rho_{01}e^{i\omega_{10}t} + d_{12}\rho_{21}e^{i\omega_{12}t})$$

Macroscopic nonlinear polarisation

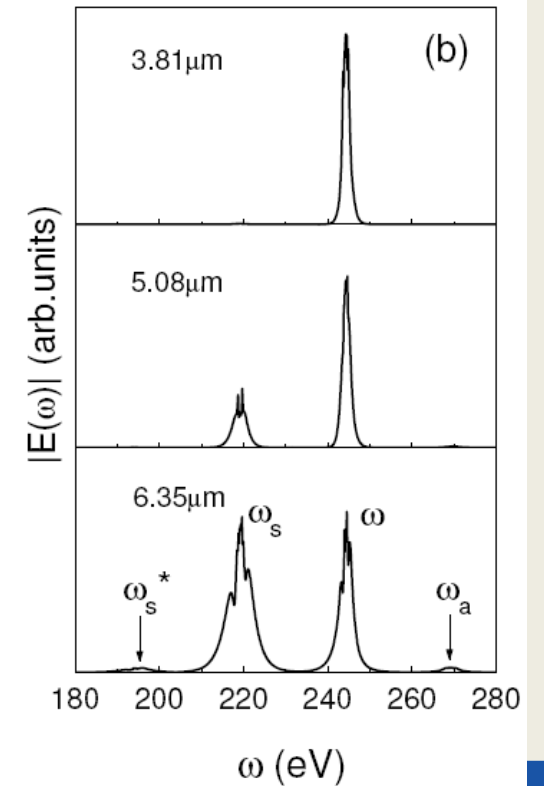
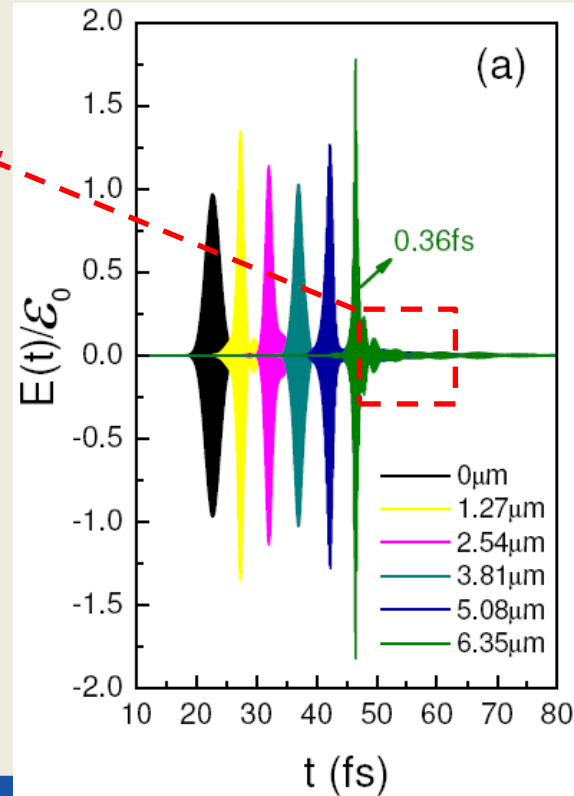
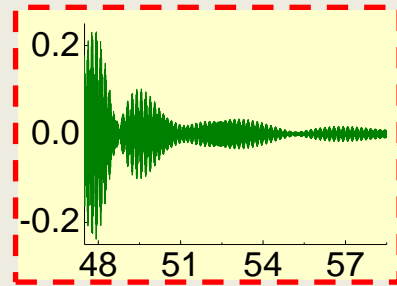
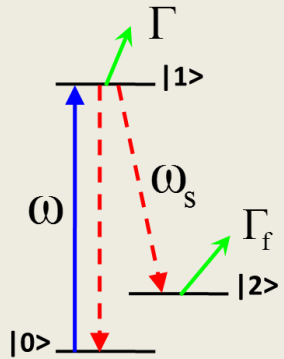
$$V_{mn} = -E(t, z)d_{nm}e^{i\omega_{mn}t}/\hbar$$

$$\gamma_{ph}^{(nm)}(t) = \frac{1}{2}[\gamma_{ph}^{(n)}(t) + \gamma_{ph}^{(m)}(t)], \quad \gamma_{ph}^{(n)}(t) = \sigma_{ph}^{(n)} \frac{I(t)}{\hbar\omega}$$

Numerical solution: Finite-Difference Time Domain method

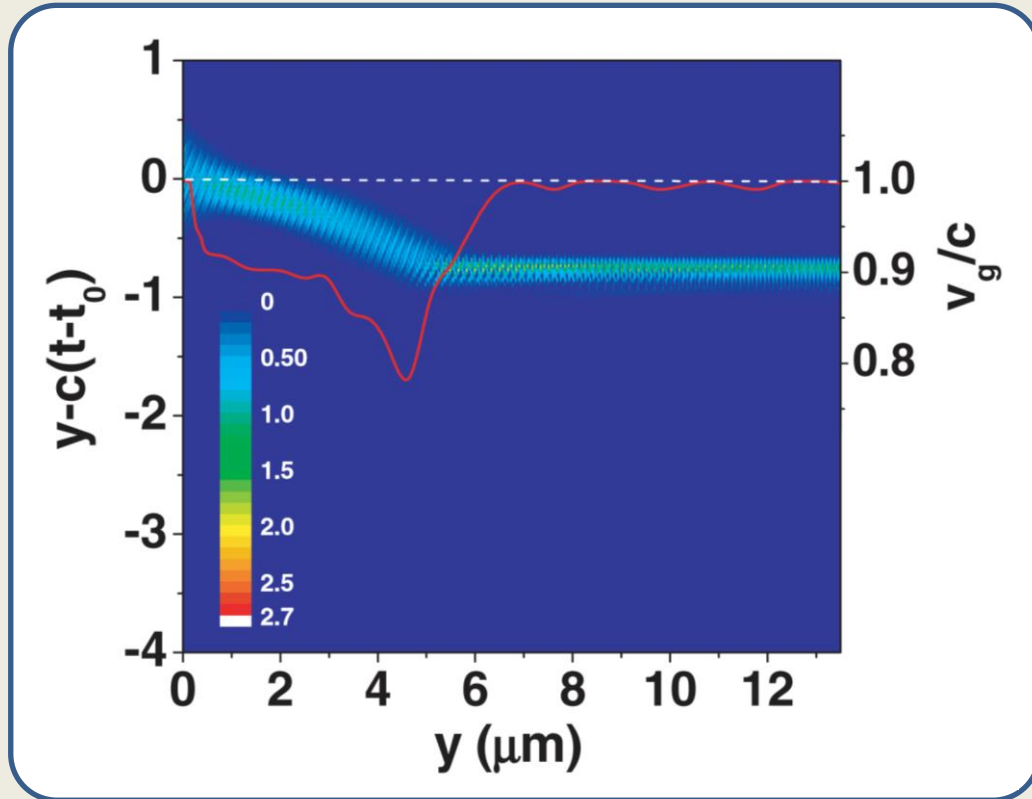
Pulse compression, pulse modulation and four-wave mixing

- Coherent x-ray pulse 2 fs duration, 3π pulse area resonant to Ar transition
- Pulse (Burnham-Chiao) modulations: gain in an inverted medium and the sign-changing modulation of the envelope in the region where the inversion is absent.



- The pump component change the temporal shape (pulse compression) during the propagation
- Corresponding spectral broadening produce seeding for the Stokes line

Slowdown of XFEL pulse



- High intensity of XFEL pulse results in a change of the refractive index
- Nonlinear refractive index

$$n = n_0 + n_{nl} \approx 1 + n_{nl}$$

- Slowdown of the pulse

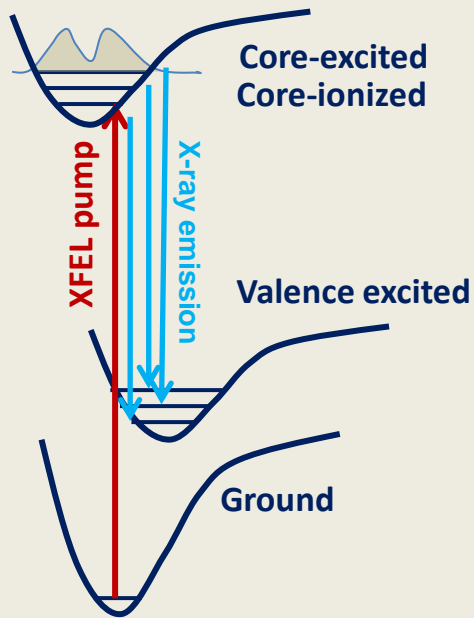
$$v_g = \frac{v}{n - \lambda \frac{dn}{d\lambda}}$$

Well-known in the optical region, but in x-ray n is very close to 1.

- The reported nonlinear change of the refraction index of the order 0.1–1

Stimulated resonant x-ray emission in molecules: Opportunities or difficulties?

What nuclear degrees of freedom can give us?



- **Broader absorption and emission energy ranges:**
 - + various electronic and vibrational transitions, chemical shifts
 - + effective pump on broadened transition
 - weaker emission efficiency
- **Various schemes:**
 - + choosing potentials of the core-excited and final states
 - + variation and control of the nuclear coordinates
 - numerous competing channels
- **X-ray polarization control:**
 - + using the molecular alignment with additional IR field
 - + pump-probe with IR excitation
 - low efficiency due to chaotic orientation of soft matter

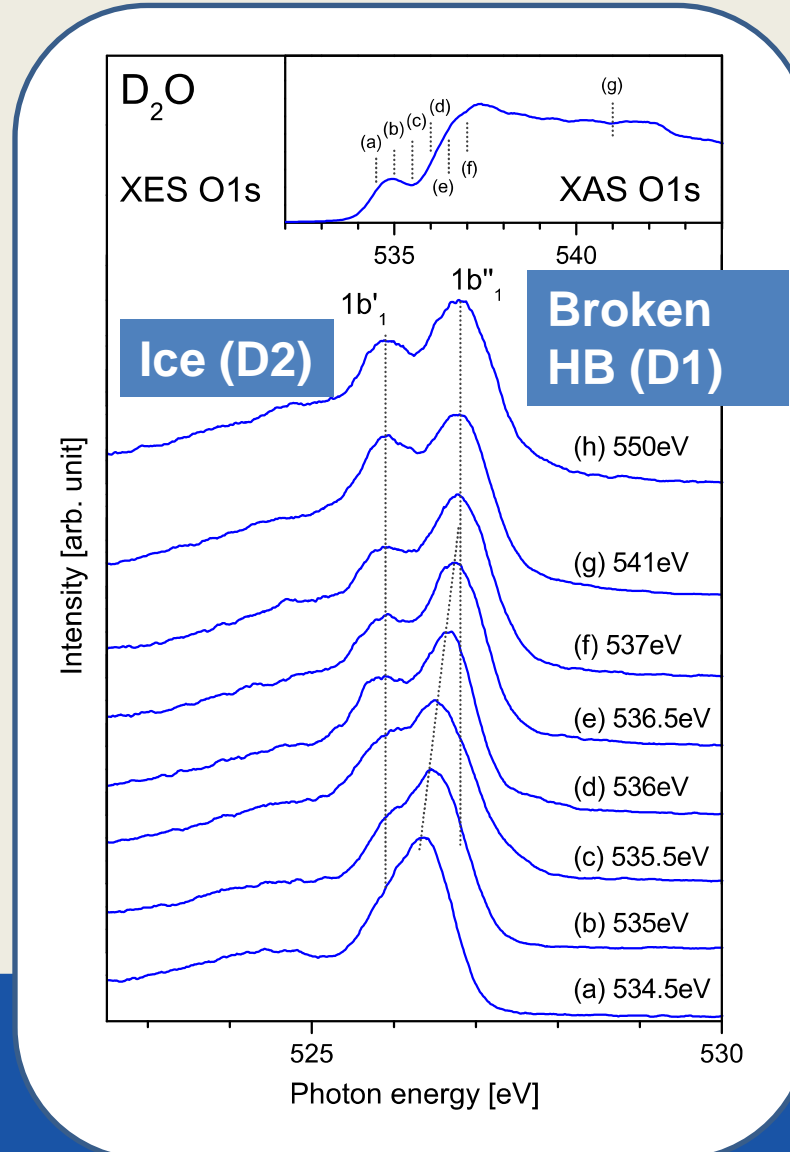
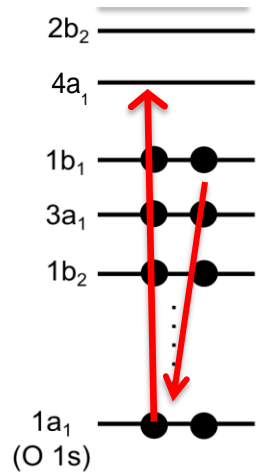
Local structure and nuclear dynamics in RIXS

1b₁ structure splitting in RIXS: structure vs dynamics

Structure

- Nat.Comm. 6, 8998 (2015)
- J.Chem Phys. 148, 144507(2018)

1b₁: first inelastic RIXS channel

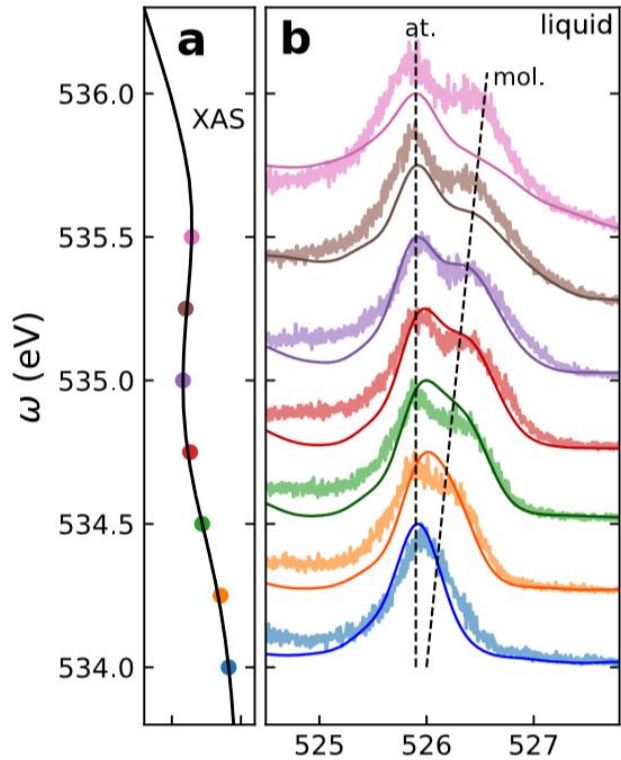


Nuclear dynamics

- PRL, 100, 027801(2008); PRL, 114, 088302(2015); PRB, 79, 144204(2009)
- This mechanism supported by isotope sensitivity of RIXS

Dynamics in real-time XFEL measurements

Pre-edge¹⁾



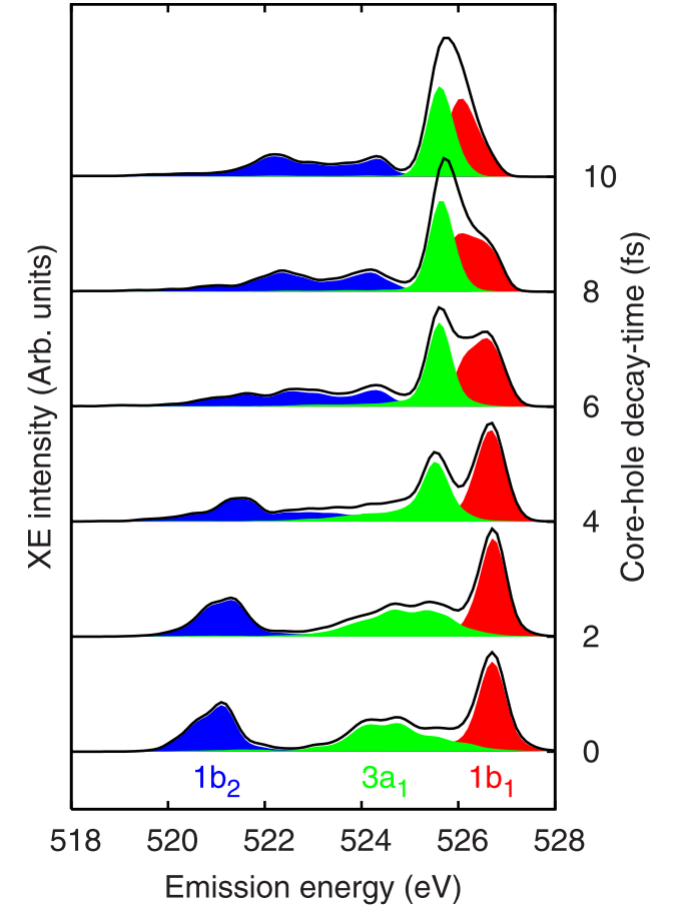
Effective scattering duration time:

$$\tau_s = \frac{1}{\sqrt{\Omega^2 + \Gamma^2}}$$

Structure vs dynamics problem in water

- “Time-dependent” XES modelling with MD²⁾
- Can be measured in XFEL all X-ray pump-probe scheme with **stimulated X-ray emission** via $3a_1$ and $1b_1$ channels
- Different polarization dependence for $3a_1$ and $1b_1$ peaks can be traced in stimulated resonant X-ray emission

Post-edge²⁾

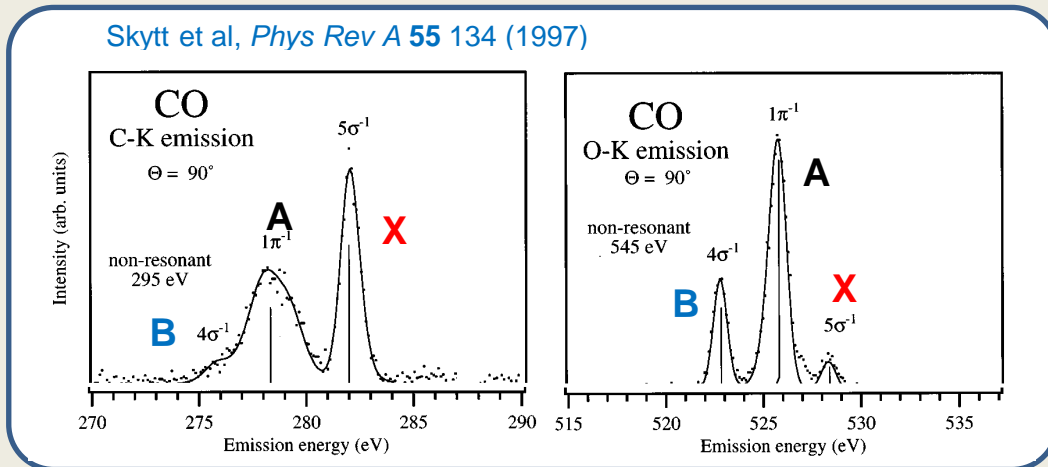


[1] V. Vaz da Cruz, et al, Nat Commun. **10**, 1013 (2019)

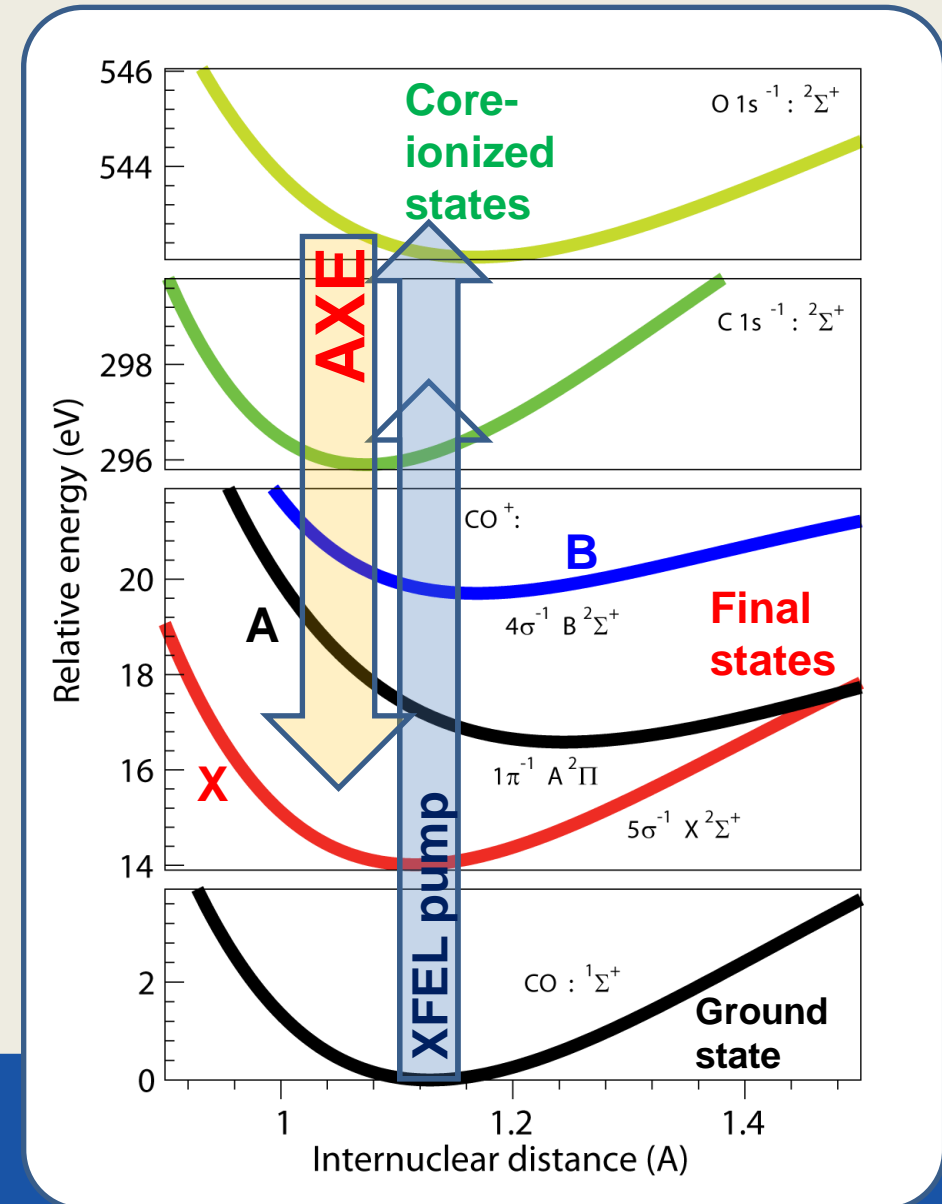
[2] M. Odelius, Phys. Rev. B **79**, 144204 (2009)

Stimulated X-ray emission in CO

- Two core-ionization edges: CK (296 eV) and OK (542 eV)
- Lifetime of $O1s^{-1}$ core-hole is twice shorter than $C1s^{-1}$: different nuclear dynamics



- Strongest transitions :
 $C1s^{-1} \rightarrow X$, $O1s^{-1} \rightarrow A$, $O1s^{-1} \rightarrow B$
- Different PECs of the final states: vary the nuclear dynamics and x-ray emission process
- Different transition symmetries



Theoretical framework

Coupled Maxwell and Density Matrix equations

Wave equation for propagation and amplification of the complex field (SVA)

$$\left(\frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) E^+ = -i \frac{2\pi\omega_L}{c} P^+$$

Nonlinear polarization includes all vibrational transitions

$$P^+ = \sum_{i,f} \rho_{if}(\theta) d_{if} \cos(\theta) e^{i(\omega_{if} - \omega_L)t} \quad d_{if} = d_e \langle v_i | v_f \rangle, \quad \theta = \angle(\mathbf{E}, \mathbf{d})$$

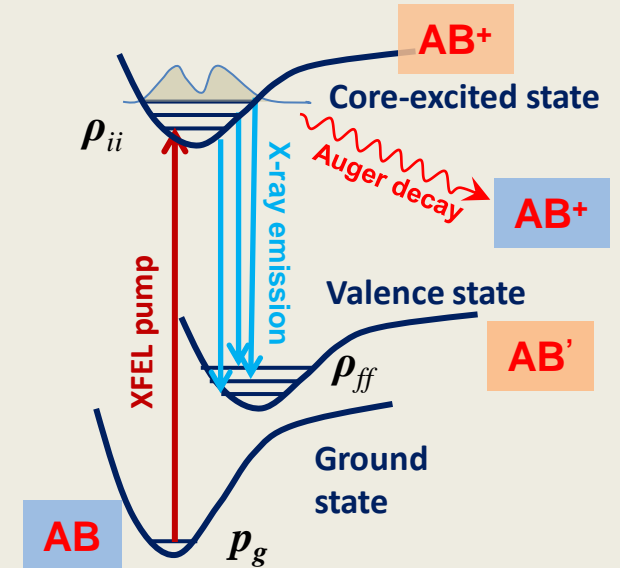
Born-Oppenheimer and Franck-Condon approximations

Reduced density matrix equations

$$\dot{\rho}_{ii}(\theta; z, t) = -\Gamma_i \rho_{ii} + P_{ii} + i \sum_f (\rho_{fi} R_{if}^* - \rho_{if} R_{i'f}^*)$$

$$\dot{\rho}_{if}(\theta; z, t) = -\frac{\Gamma_i}{2} \rho_{if} + i \sum_{f'} \rho_{f'f} R_{if'}^* - i \sum_{i'} \rho_{ii'} R_{i'f}^* + (\rho_{ii} - \rho_{ff}) S(z, t)$$

$$\dot{\rho}_{ff'}(\theta; z, t) = i \sum_i (\rho_{if} R_{if'} - \rho_{fi} R_{if'})$$

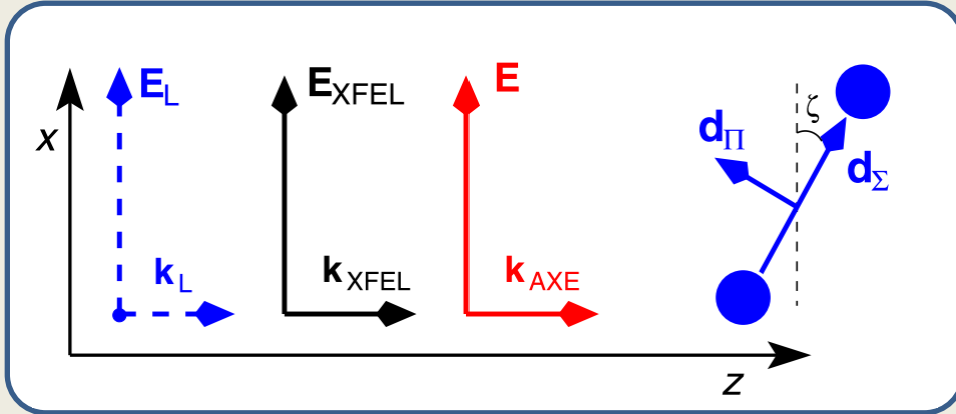


$$j = \{j_e, v_j\}$$

$$R_{if} = E^+ d_{if} \cos(\theta) e^{i(\omega_L - \omega_{if})t}$$

Nonlinear polarization is averaged over molecular orientation ($\cos \theta$) and source term $S(z, t)$

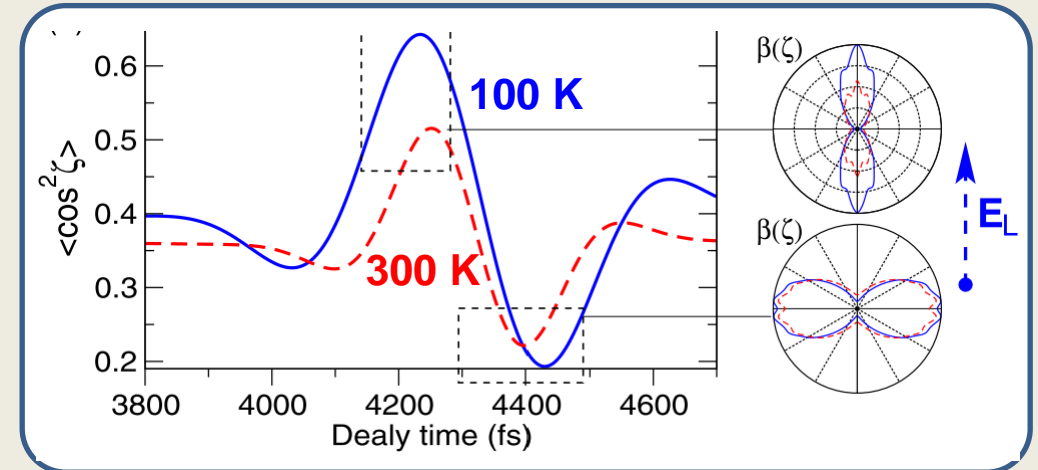
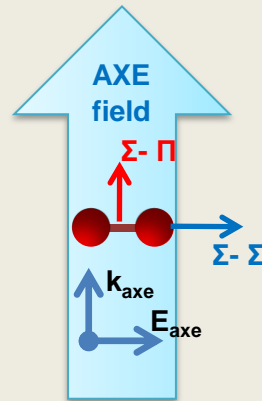
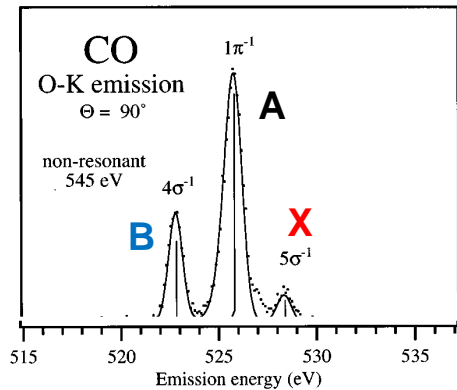
Field-free IR molecular alignment



- Transition symmetry is important :
 $O1s-1 \rightarrow A$ ($\Sigma-\Pi$) dominates for the anti-aligned ensemble
 $O1s-1 \rightarrow B$ ($\Sigma-\Sigma$) dominates for the aligned ensemble

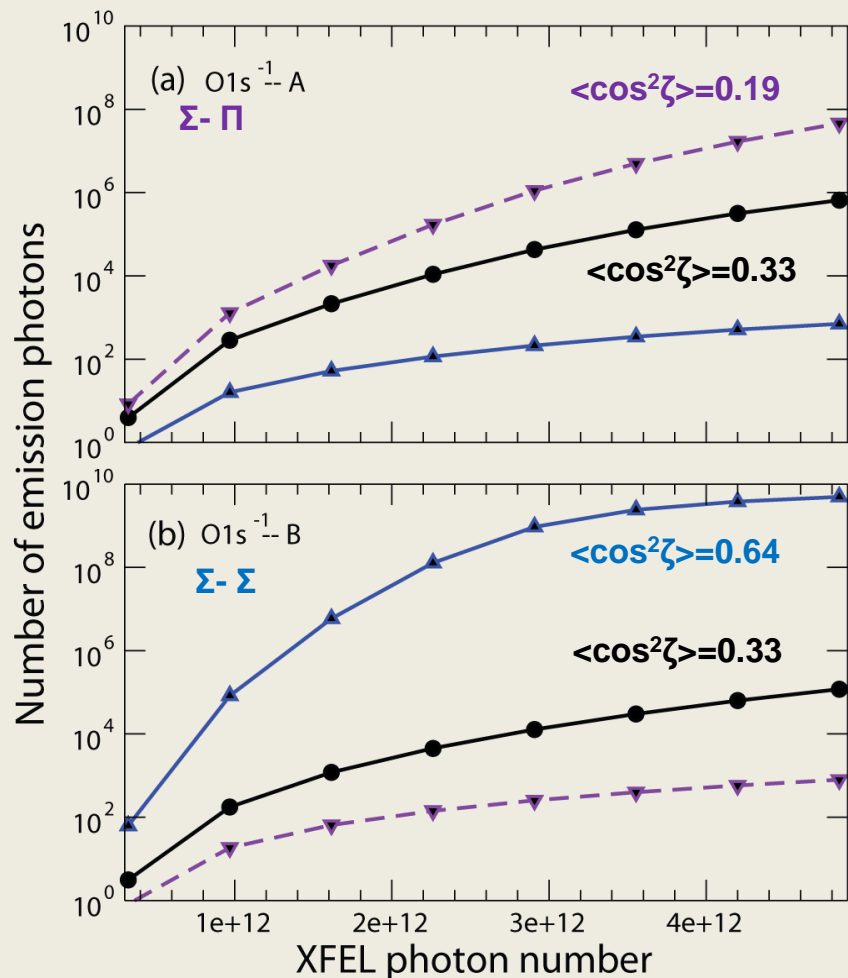
IR-pulse used in simulations:
 800 nm, 100 fs, $1.26e14$ W/cm²

Skytt et al, *Phys Rev A* 55 134 (1997)

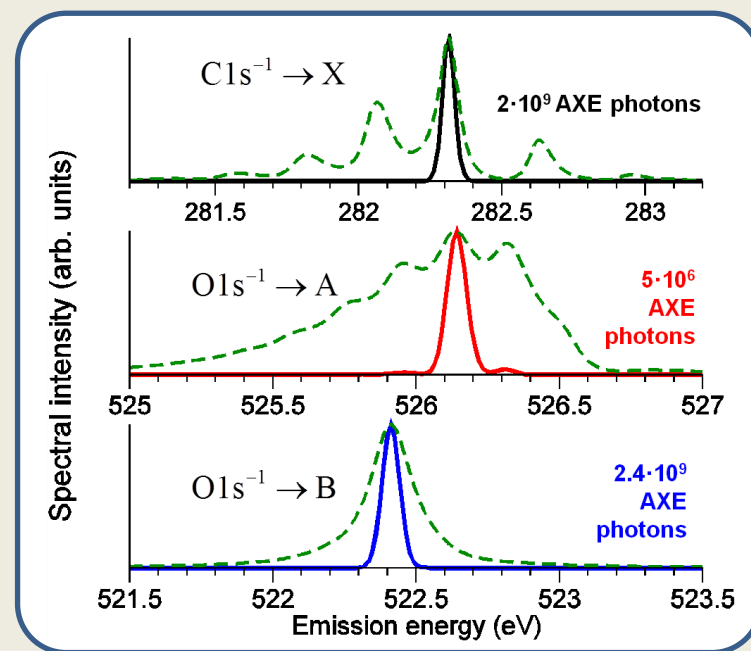
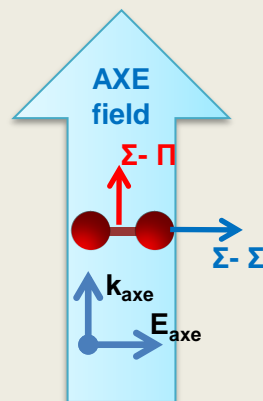


IR laser synchronized with XFEL creates great opportunities for molecular AXE at SQS

Amplified spontaneous X-ray emission: various lasing channels

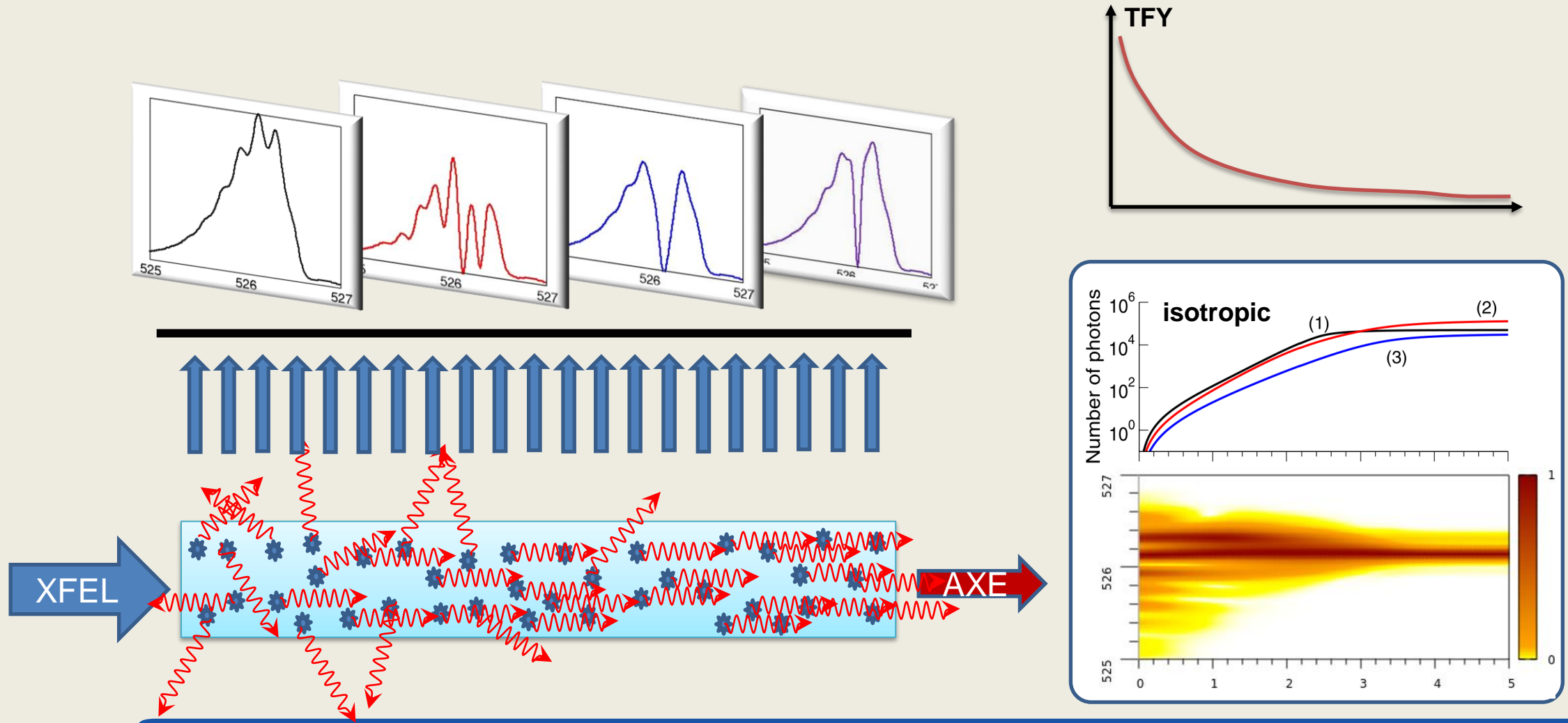


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 $O1s^{-1} \rightarrow A$ ($\Sigma - \Pi$) dominates for the anti-aligned ensemble
 $O1s^{-1} \rightarrow B$ ($\Sigma - \Sigma$) dominates for the aligned ensemble



IR laser synchronized with XFEL creates great opportunities for molecular AXE at SQS

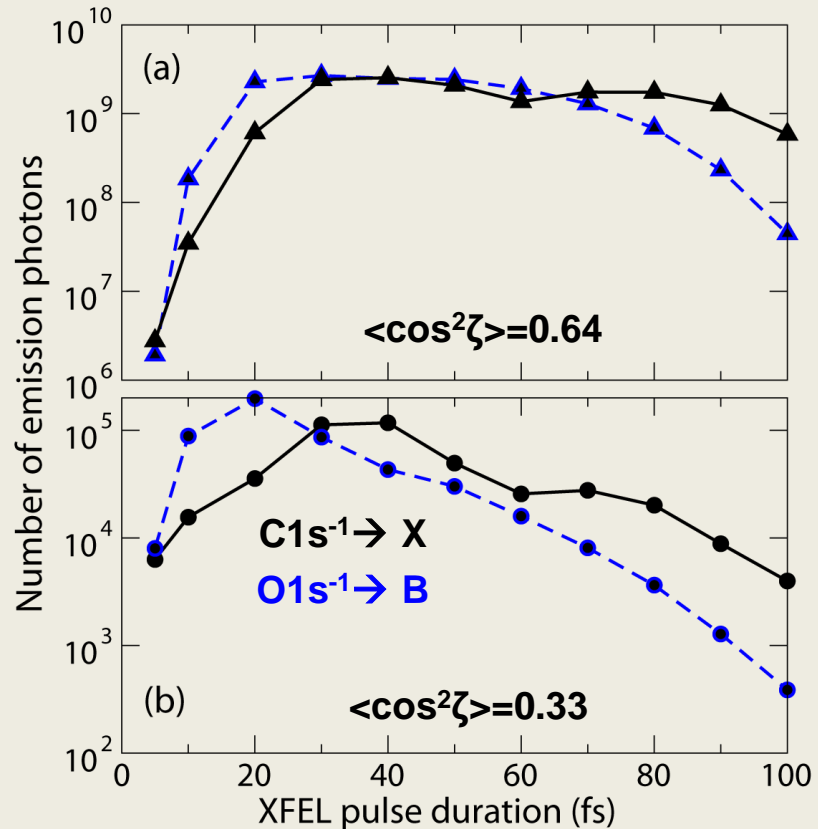
1D-imaging of the nonlinear propagation in fluorescence



Suppression of TFY and change of XE spectra of 1D IXS are the fingerprints of the SXE

Pump pulse duration

XFEL photon flux=const

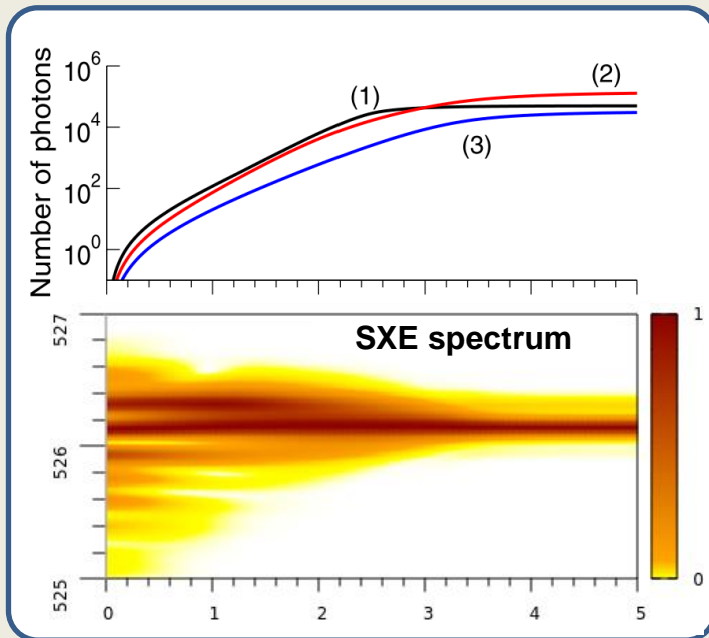


- Pump pulse duration allows to optimize lasing conditions for according to the core-hole lifetime
- Same photon flux, different pulse length
 - Shorter pulse is more efficient for shorter lifetime
 - Optimal conditions are broader for the aligned ensemble
- The higher pumping rate (shorter pump pulse) beats the stronger Auger decay and results in a larger AXE efficiency

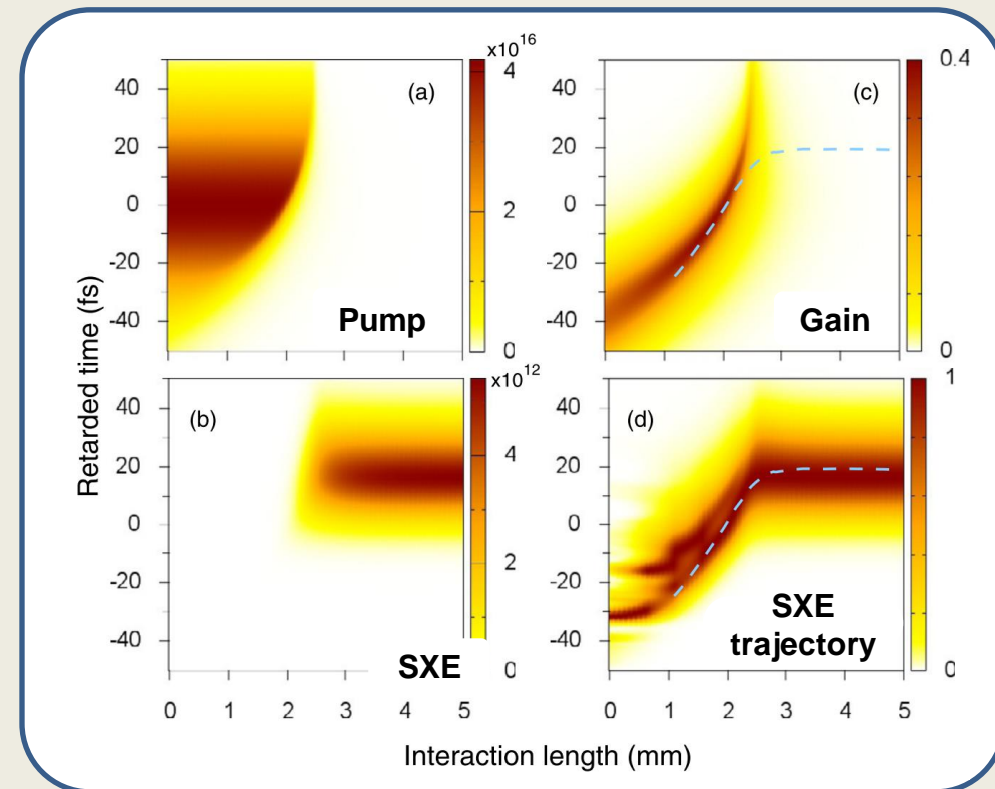
Dynamics of amplified x-ray emission in CO

□ Spectral dynamics

- Low intensity: broadband emission
- Narrow line in the linear gain region
- Broadening and shift of the spectral line due to strong field (nonlinear) effects above the saturation ($z > 3$ mm)



- Group velocity of pulse propagating in high gain resonance medium is slowdown¹⁾
- Gain “catch-up” effect²⁾ (not observed without pump absorption)
- **Strong absorption** results in a shift of the XFEL front -> population inversion shifts to later times as well



Nonlinear effects and pulse delay by about 40 fs

¹⁾Casperson & Yariv, PRL **29**, 293 (1971)

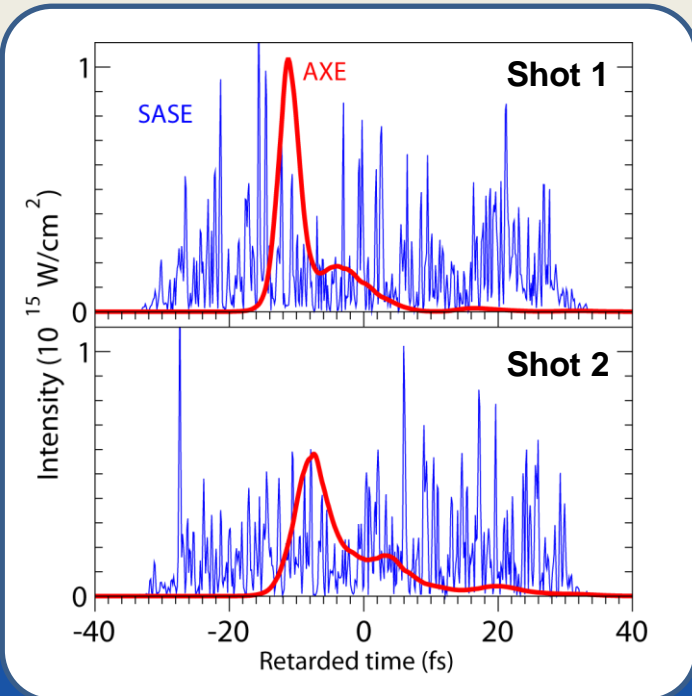
²⁾Miao et al PRL **109** 233905 (2012)

SASE pump pulse: N₂ showcase

- Broad-band excitation (~ 8 eV)
- Core-ionization: due to the off-resonant excitation SASE show no drawback as compared to Gaussian profile
- **AXE conversion:** broadband **SASE pulse** to a narrow band (0.1 eV) short (~5-10 fs) **AXE pulse**
- Shot to shot results are slightly different: averaging over large number of SASE pulses

□ Spectral dynamics

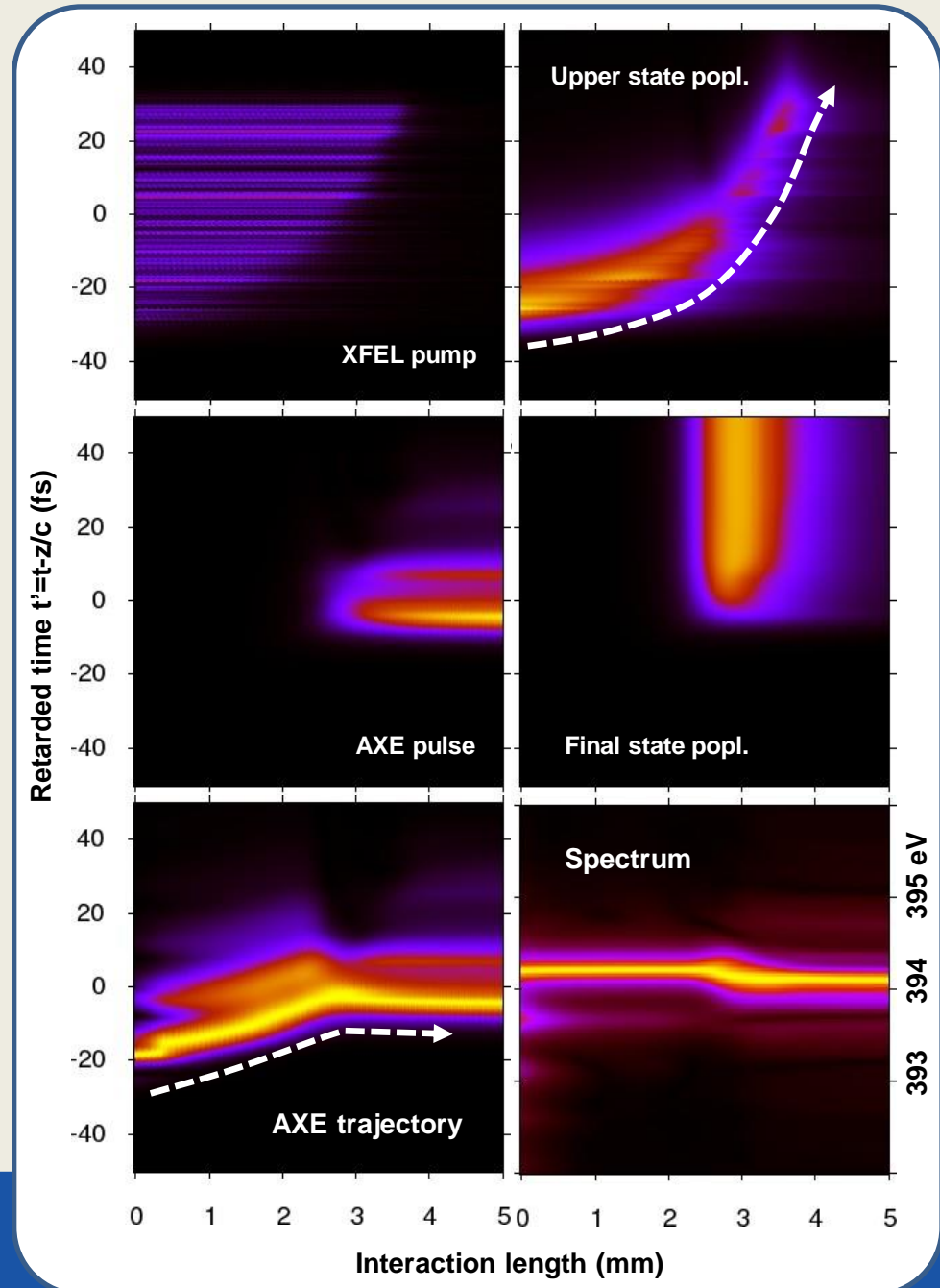
- Low intensity: broadband emission, narrow line in the linear gain region
- Broadening and shift of the spectral line due to strong field (nonlinear) effects above the saturation ($z > 3$ mm)



- Group velocity of pulse propagating in high gain resonance medium is slowdown¹⁾
- Gain “catch-up” effect²⁾ (not observed without pump absorption)
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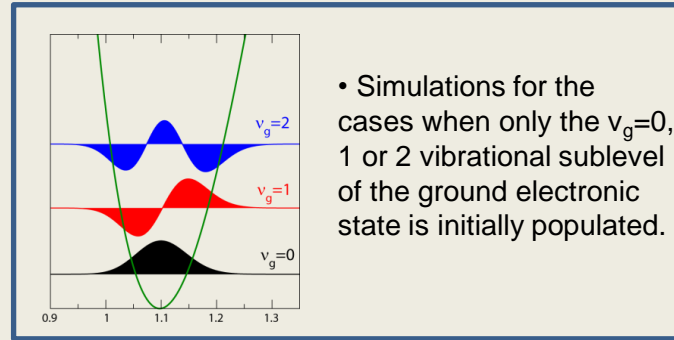
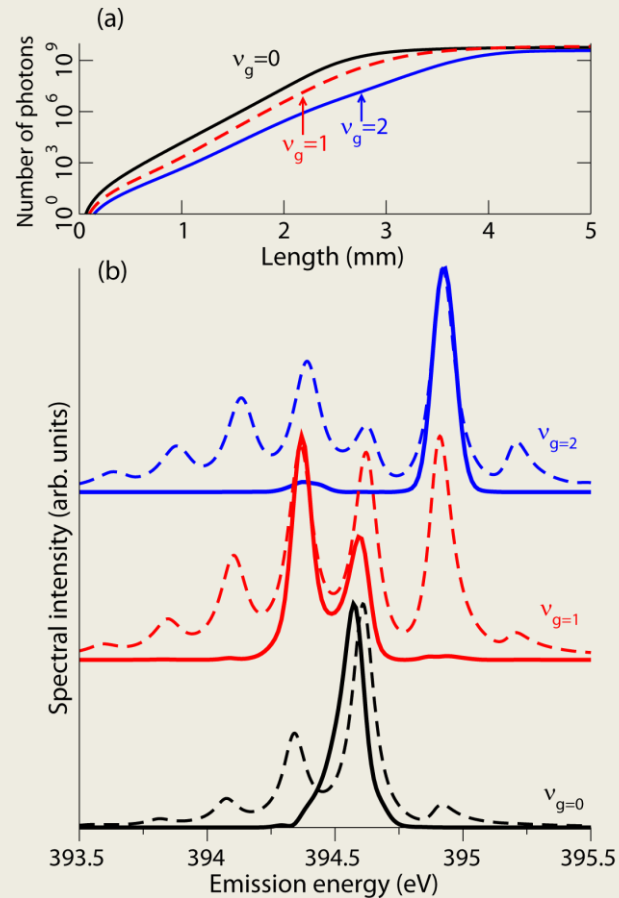
¹⁾Casperson & Yariv, PRL **29**, 293 (1971)

²⁾Miao et al PRL **109** 233905 (2012)



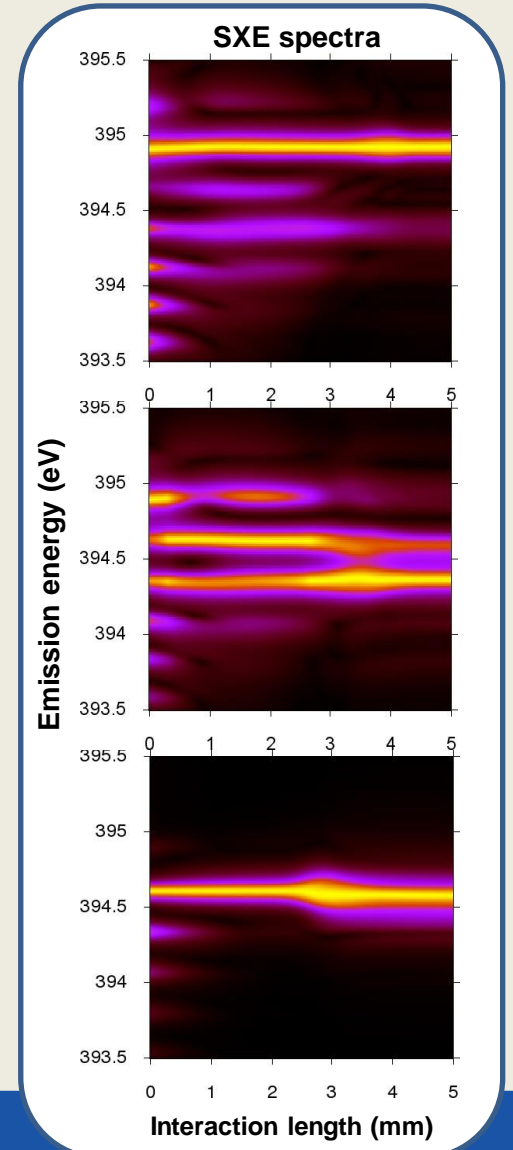
Stimulated X-ray emission from IR-excited molecules

□ By fixing the initial vibrational quantum state (e.g. IR Raman), a shift of the SXE to the maximum of the fluorescence spectrum can be achieved (N_2 showcase).



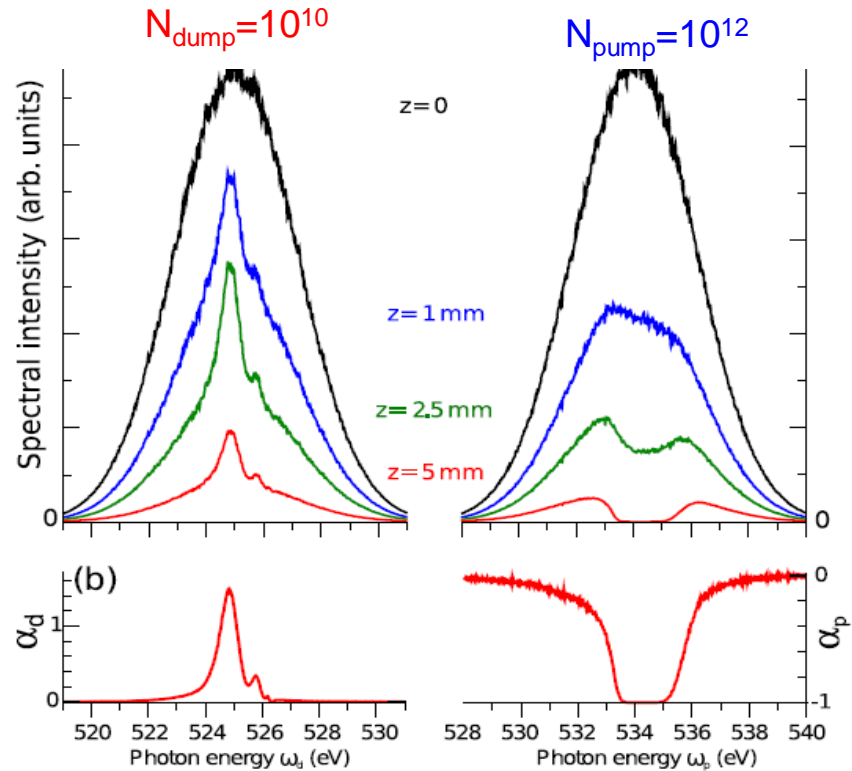
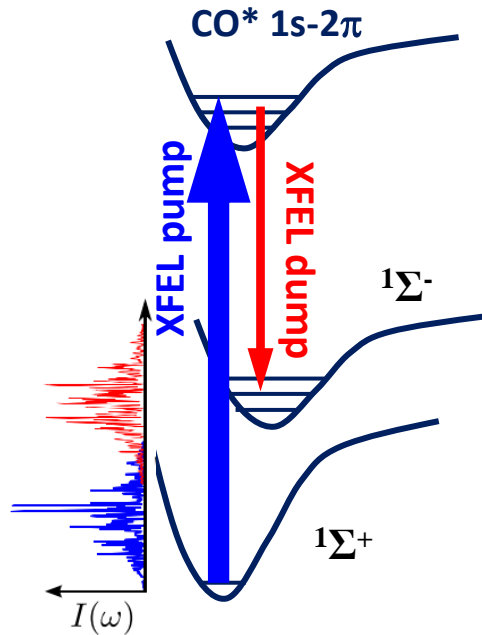
- Simulations for the cases when only the $v_g=0$, 1 or 2 vibrational sublevel of the ground electronic state is initially populated.

- Difference in the gain is due to the different Franck-Condon factors
- The propagation spectral dynamics is more complex in the cases of the broad vibrational band excitation
- Any coherent superposition of the vibrational states which differs sufficiently from the $v_g=0$ results in a change of the AXE spectrum.



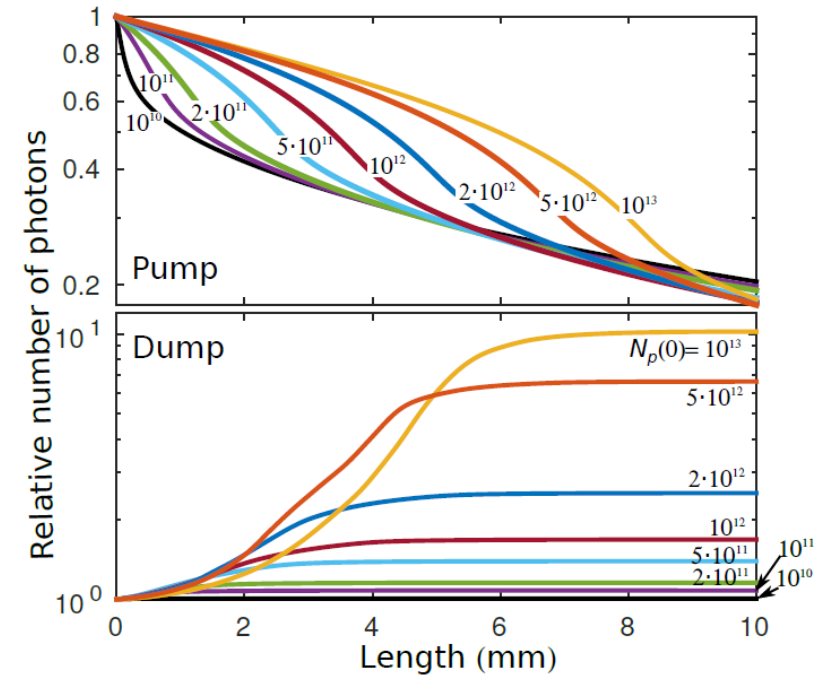
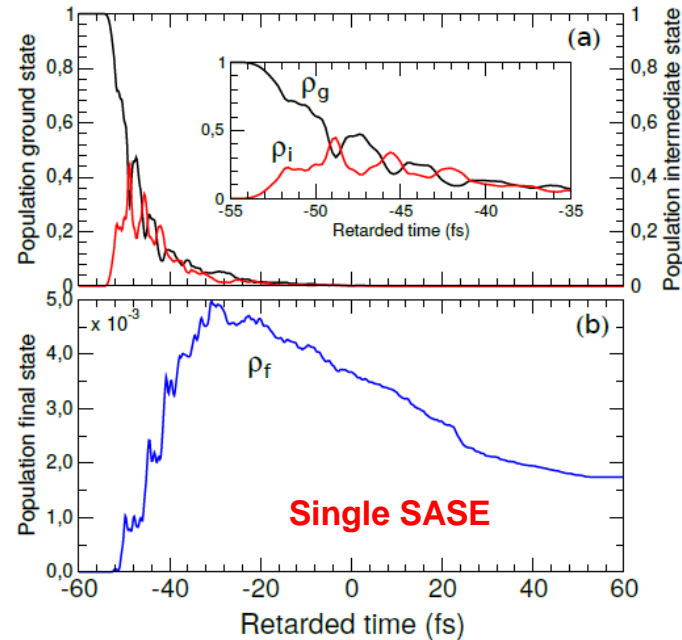
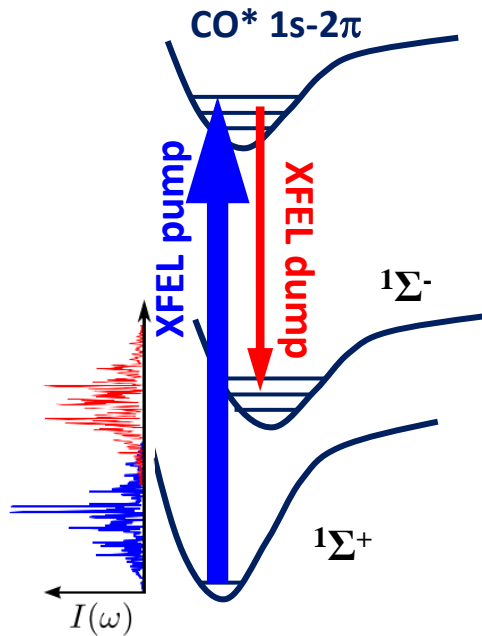
Stimulated RIXS in two-color regime

- High flux pump pulse: 10^{12} photons per target (3 mkm focus, 100 fs)
- Population inversion and amplified stimulated emission conditions
- Gross features in x-ray amplified emission
- Feasibility study for the experiment (LCLS, AMO beam line at 2014)



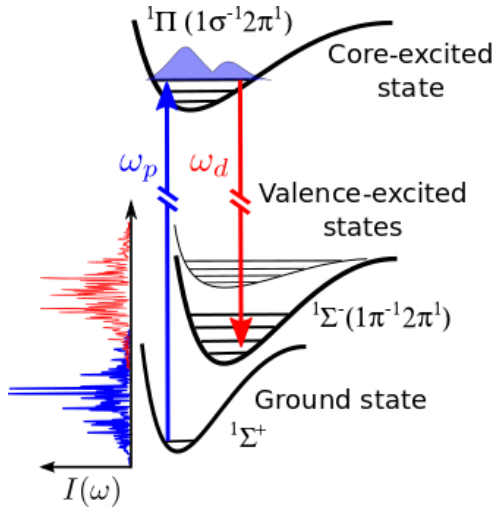
Stimulated RIXS in strong pump regime

- Population inversion and amplified stimulated emission conditions
- Dipole moment (dump) < dipole moment (pump)
- Gross features in x-ray amplified emission
- Feasibility study for the experiment (LCLS, AMO beam line at 2014)

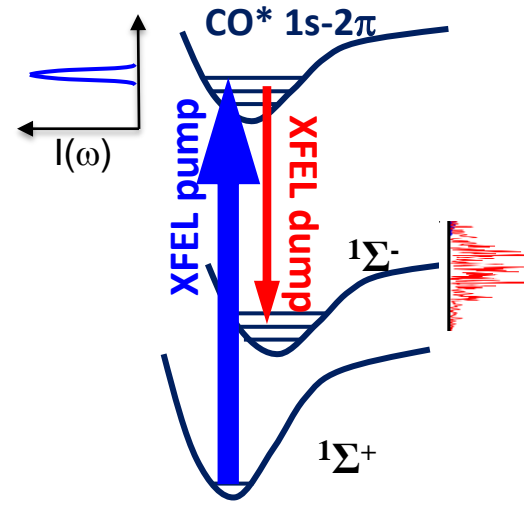


Two 2-color SRIXS schemes

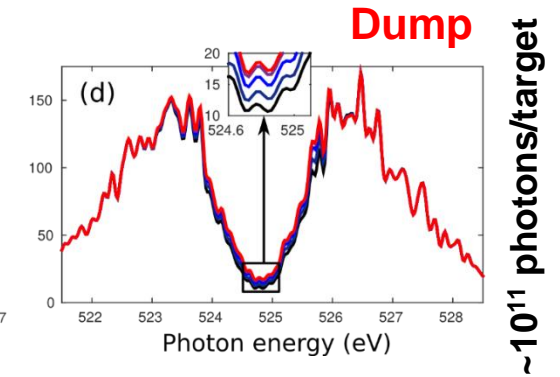
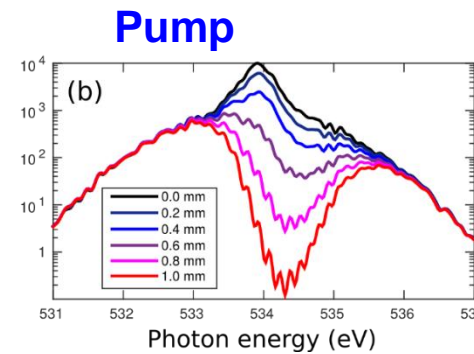
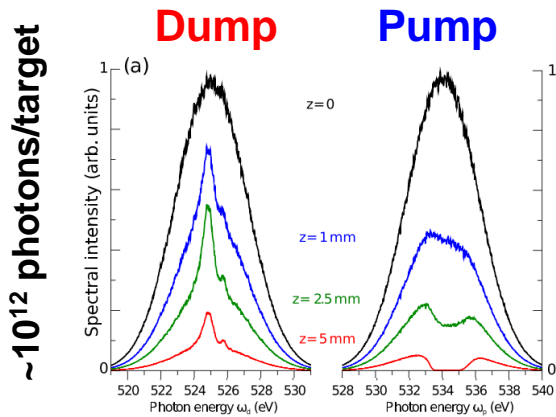
- SASE pump – SASE dump



- Self-seeded pump – SASE dump



Theory predictions

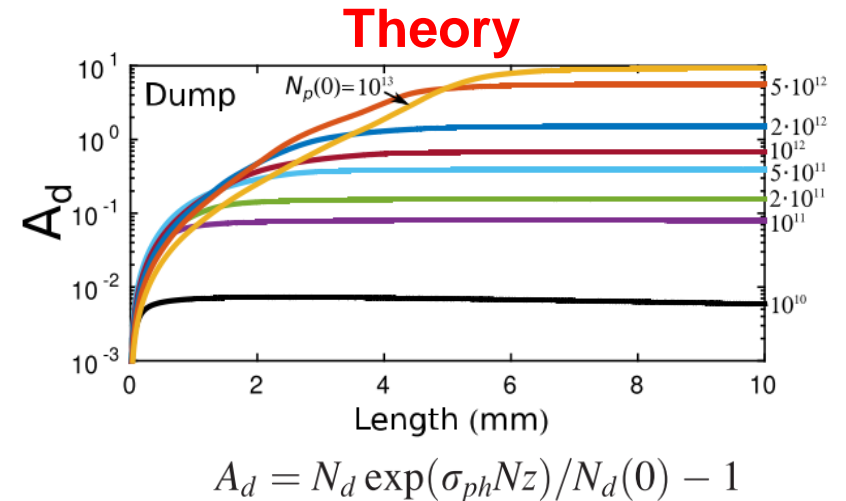
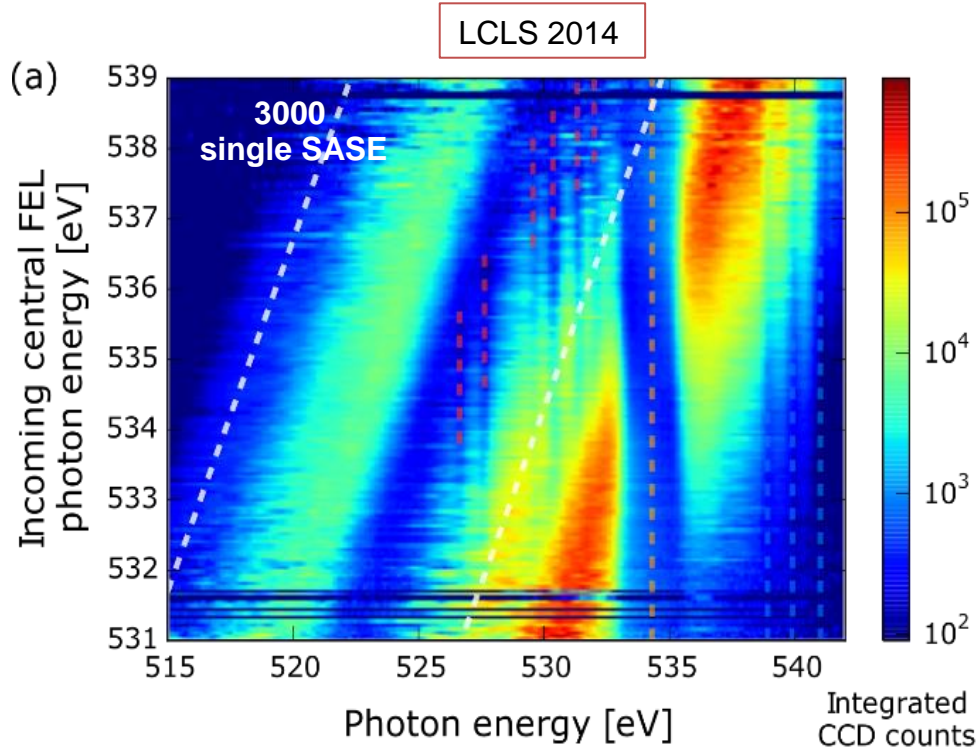


LCLS Feb 2014
AMO beamline

Experiments

LCLS Jul 2015
AMO beamline

Critical assessment: pump intensity



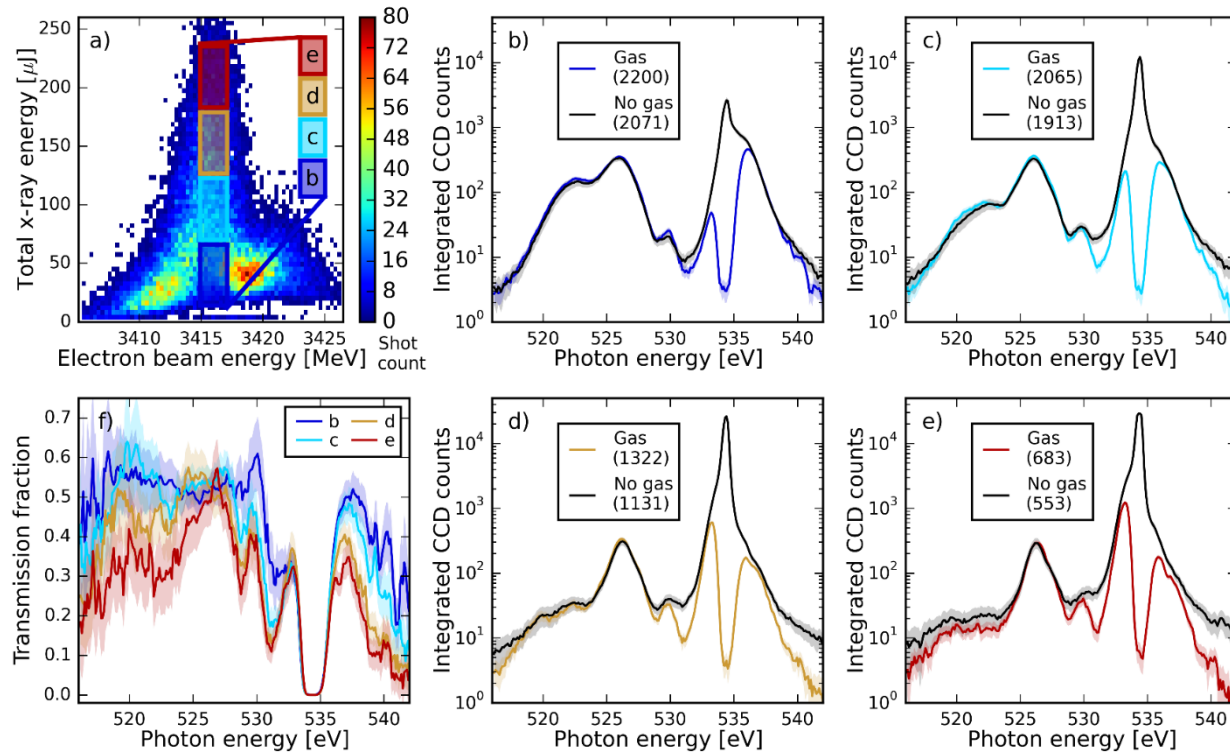
- Stimulated RIXS using 2-color SASE/SS scheme: **no evidence of SRIXS**
- **Main requirement:** high spectral intensity of the pump overlapping with resonance

- **Intensities were on border of attainability!**
- Degradation of transmission and focusing x-ray optics, non-collinearity of pump/dump in the gas cell...
- Solution: pushing up XFEL intensities, using molecules alignment

Advanced analysis of the experimental data

- New measuring protocol for **weak signal measurement**
- Gas/no-gas average spectra comparison for different pressure; Partial averaging of restricted ensembles (e-beam/pulse energy)

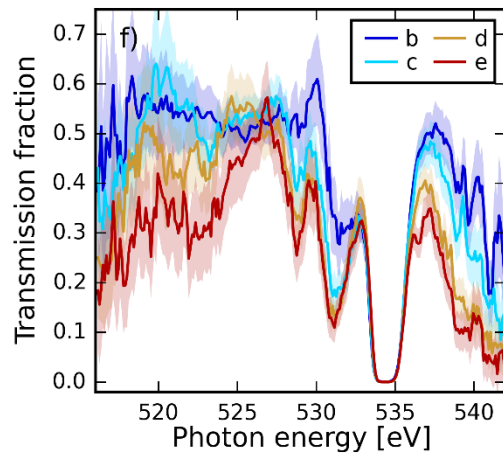
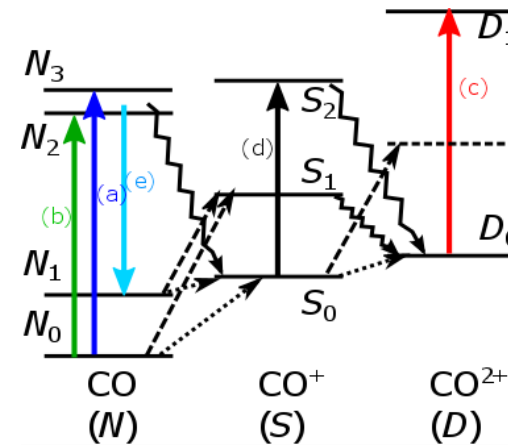
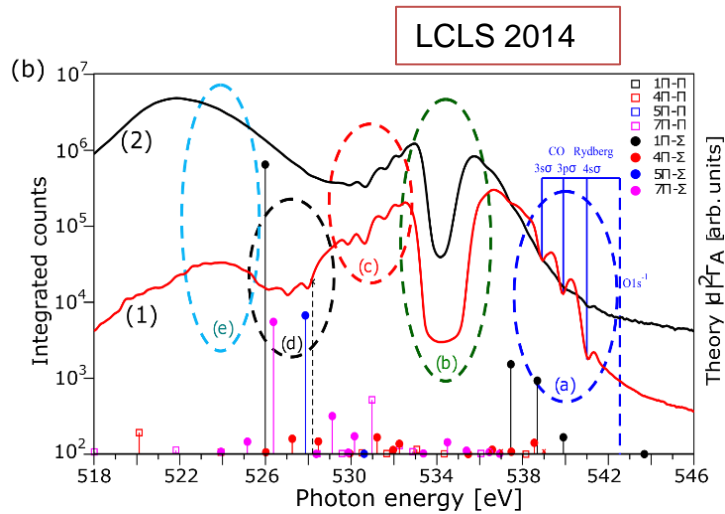
LCLS 2015
AMO, LAMP



- High x-ray beam energies are only around certain e-beam energy interval
- High fluctuation of the total x-pulse energy
- The average spectrum is different for different regions on the correlation plot
- **Fluctuations complicate the detection of a small SRIXS signal**

Despite fluctuations, averaged spectra are well reproducible
Relative differences $> 0.05\%$ observable !!!

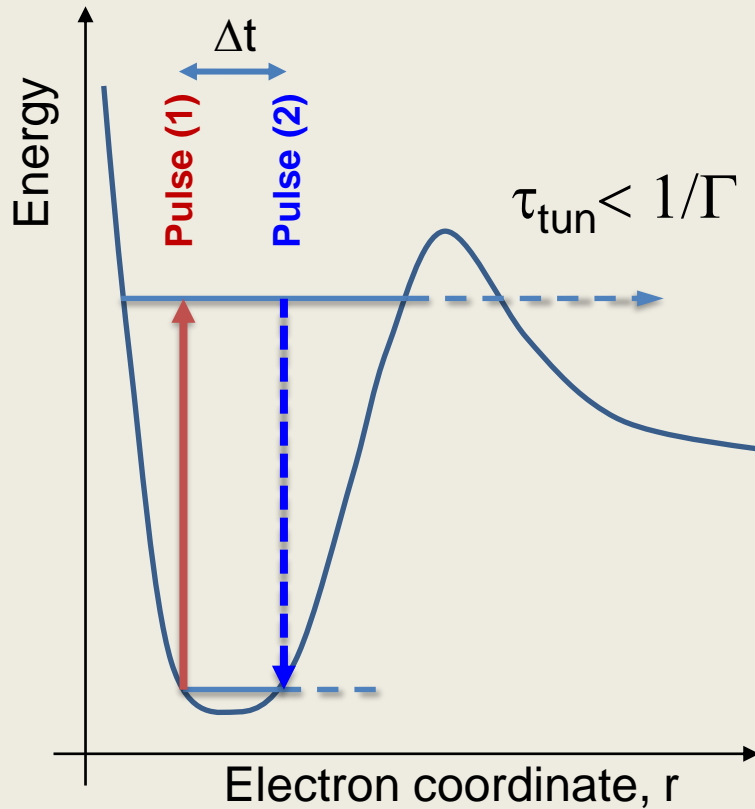
Absorption features of transiently created ions



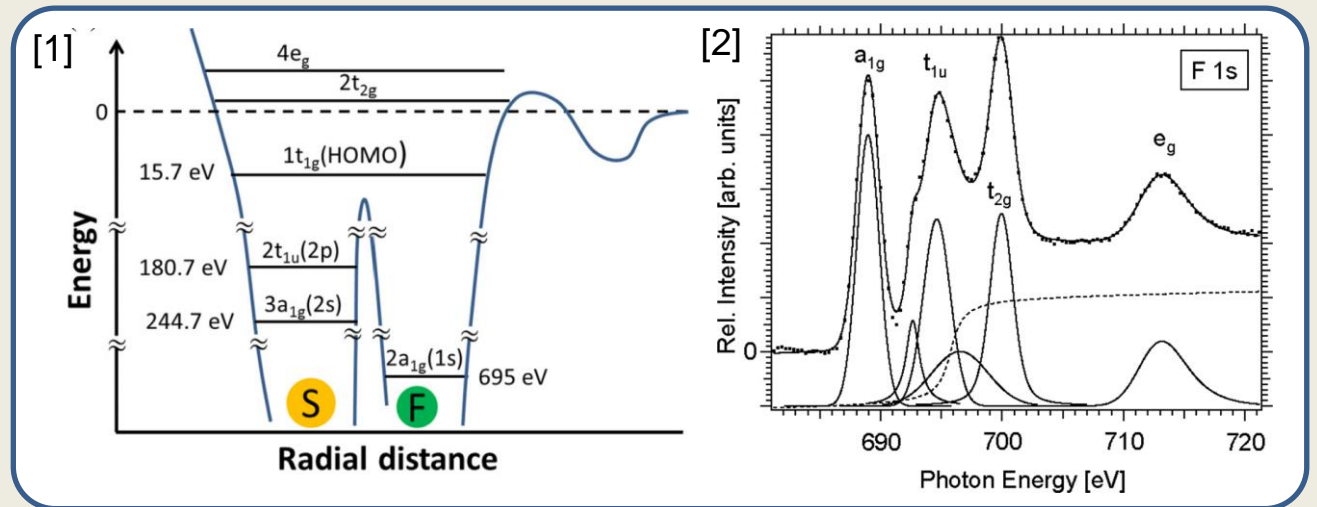
- (a) high energy 539-541 eV: Rydberg states absorption by neutral CO
- (b) $1s-\pi^*$ resonance at 534 eV
- (c) middle energy 529-533 eV: Tentatively assigned to absorption lines in CO⁺⁺
- (d) low energy 525-528 eV : O1s excitation of single-charge CO⁺ (confirmed by XES)
- (e) expected SRIXS region

- 1D imaging: different channels will be visible via fluorescence measurement at different propagation length, giving complementary information for SRIXS

Electron tunnelling time at the shape resonances



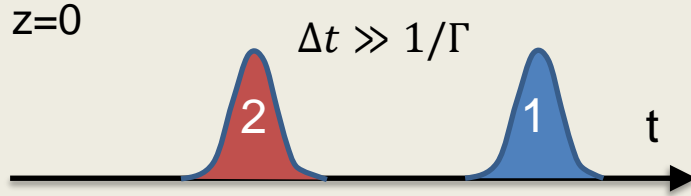
- SF_6 molecule or SF_6 cluster (for example)
- No pulse (2) or $\Delta t > \tau_{\text{tun}}$: we will see X-ray fluorescence during $1/\Gamma$
- $\Delta t < \tau_{\text{tun}}$: fluorescence is absent as core-hole is filled by SXE
- Direct measurements of the tunnelling time and electron dynamics for various molecular systems and cluster size
- Dynamical features along the medium as pulse intensities are changed



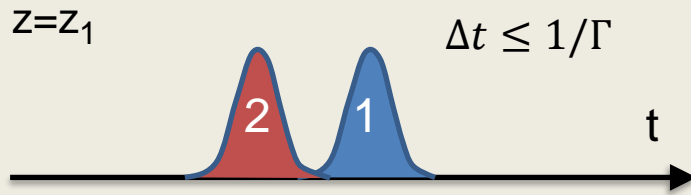
- 1) Nguyen, et al., Phys Rev A 93, 063419 (2016)
- 2) Grunewald, et al., Zeitschrift für Physikalische Chemie 234, 1371 (2020)

Pulse slowdown on resonant X-ray transition

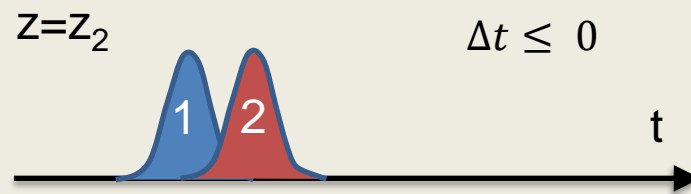
Time \longleftrightarrow Distance



Fluorescence

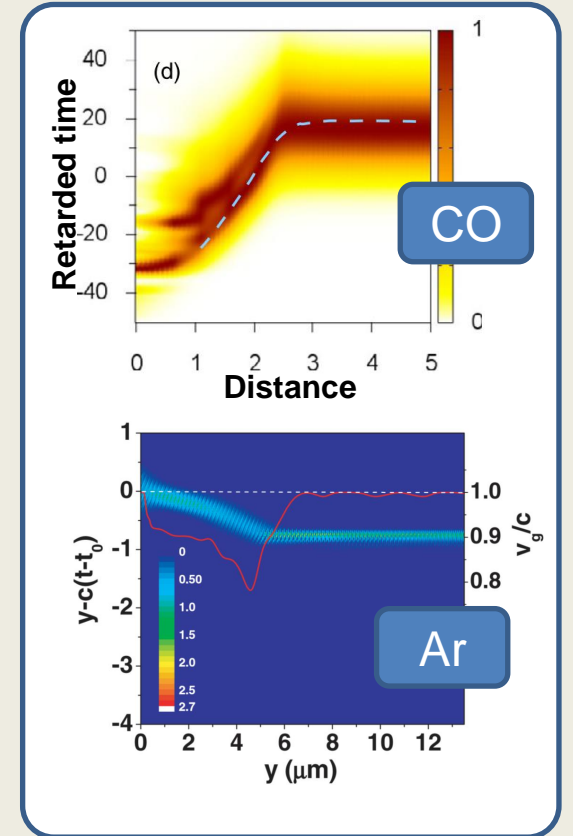
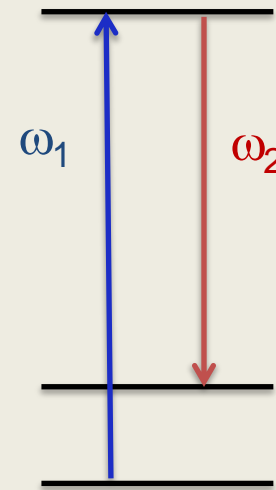


Fluorescence



Fluorescence

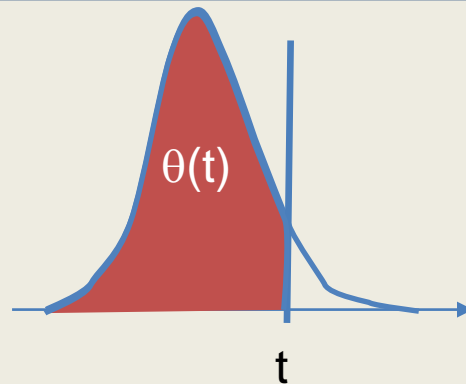
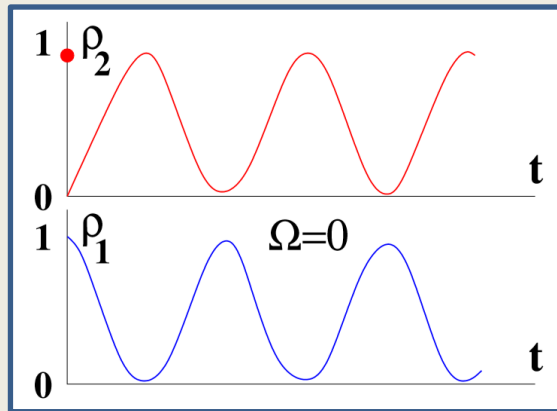
Ne (GS, $1s^{-1}3p^1$, $2p^{-1}3^{-1}$)
 CO (GS, $1\sigma^{-1}2\pi^1$, $1\pi^{-1}2\pi^1$)
 etc...



Delay of several tens fs

1D-imaging spectrometer allows to collect spatial information on nonlinear propagation which can be converted to temporal description of slowdown of the X-ray pulses

Pulse: $m\pi$ -pulses and area theorem



- Another important case: resonant interaction $\Omega=0$

$$\dot{a}_1 = i \frac{G(t)}{2} a_2, \quad \dot{a}_2 = i \frac{G(t)}{2} a_1$$

- Can be decoupled using $a_{\pm} = a_1 \pm a_2$

$$\dot{a}_+ = i \frac{G(t)}{2} a_+, \quad \dot{a}_- = i \frac{G(t)}{2} a_-$$

- Solution is

$$a_{\pm}(t) = A_{\pm} e^{\pm i\theta(t)/2}, \quad \theta(t) = \int_{-\infty}^t G(t_1) dt_1 = d_{12} \int_{-\infty}^t E(t_1) dt_1$$

- Taking into account initial condition ($a_1=1, a_2=0$): $A_+=A_-=1$

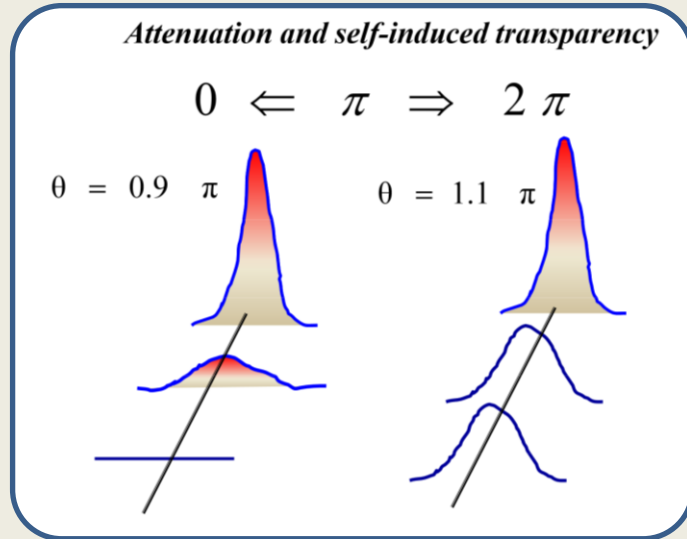
$$\rho_1(t) = |a_1(t)|^2 = \cos^2 \frac{\theta(t)}{2}, \quad \rho_2(t) = |a_2(t)|^2 = \sin^2 \frac{\theta(t)}{2}$$

$$\rho_1(\infty) = \cos^2 \frac{\theta}{2}, \quad \rho_2(\infty) = \sin^2 \frac{\theta}{2}$$



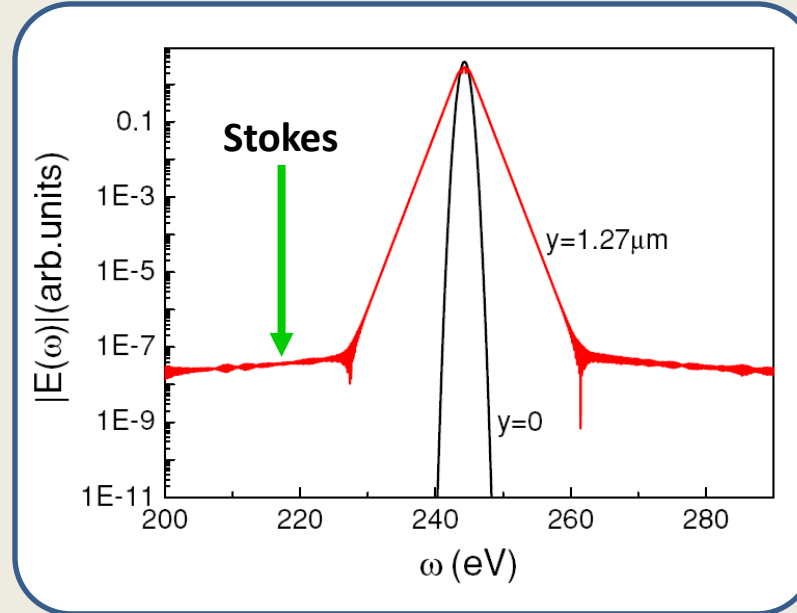
- ✓ Area theorem (Mc Call-Hahn): the system is inverted when pulse area is $(2n+1)\pi$ (unstable)
- ✓ Self induced transparency: for $2n\pi$ pulse the system stays in the ground state, no absorption!
- ✓ Control over population transfer using pulse area

Pulse compression and seeding for the Stokes line

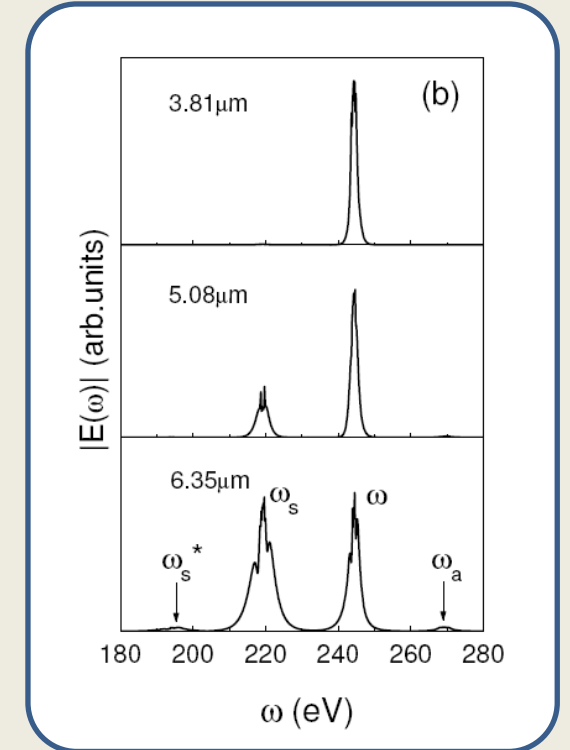


- For pulses with $\theta < \pi$: $\theta \rightarrow 0$
- For pulses with $\theta > \pi$: $\theta \rightarrow 2\pi$
- Energy conservation low

$$E_{3\pi}^2 \tau_{3\pi} = E_{2\pi}^2 \tau_{2\pi}$$



- Burnham-Chiao modulation of the XFEL pulse results in the broadening while the envelope of the BC oscillations forms the spectral tail
- This spectral tail gives the seed radiation for Stokes and four-wave mixing fields





Summary

- Stimulated X-ray emission (SXE) and related processes in molecules
 - Amplifies X-ray emission (single X-ray pulse)
 - Stimulated RIXS schemes (two-color X-ray)

- Application of 1D-imaging X-ray spectrometer at SQS
 - IR+X-ray: molecular alignment for an efficient AXE
 - IR+X-ray: pumped vibrational dynamics observed with X-ray fluorescence
 - SXE near shape resonances: study of the electron wave packet dynamics in a confinement
 - Slow down of resonant X-ray pulse measured in two X-ray pulse mode
 - Pulse broadening and compressing, self-induced transparency, four-wave mixing, ...

Acknowledgements: Swedish Research Council and Russian Science Foundation