Martin Goebel (DESY / Universität Hamburg)
for the Gfitter group*

Advanced Statistics WS 2010
Göttingen
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Gfitter – Global Electroweak Fits in the Standard Model and Beyond

main paper **Eur. Phys. J. C 60, 543 (2009)**

http://cern.ch/Gfitter

*) M. Baak (CERN), M. G. (Univ. Hamburg, DESY), J. Haller (Univ. Göttingen), A. Höcker (CERN),
D. Ludwig (Univ. Hamburg, DESY), K. Mönig (DESY), M. Schott (CERN) J. Stelzer (DESY/Michigan)

A Generic Fitter Project for HEP Model Testing

- SUSY / BSM Fit Working Group of the Helmholtz Alliance
- Goal: provide a state-of-the-art model testing tool for the LHC/ILC era
- modular, object-oriented C++, relying on ROOT, XML, python
- core package for data handling, fitting and statistics tools
  - various fitting tools: Minuit, Genetic Algorithm and Simulated Annealing
  - full statistics analysis: parameter scans, *p*-values, MC analyses, goodness-of-fit tests, …
  - coherent treatment of statistical, systematic errors, and correlations
    - theo. uncertainties included in χ^2 with flat likelihood in allowed ranges
- physics plug-in packages
  - Library for the Standard Model fit to the electroweak precision data → this talk
  - Library for SM extensions via the oblique parameters → this talk
  - Library for the super-symmetric extension of the SM
  - Library for the 2HDM extension of the SM

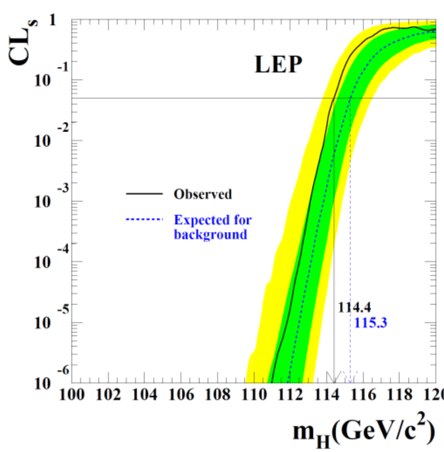
The Global Electroweak Fit

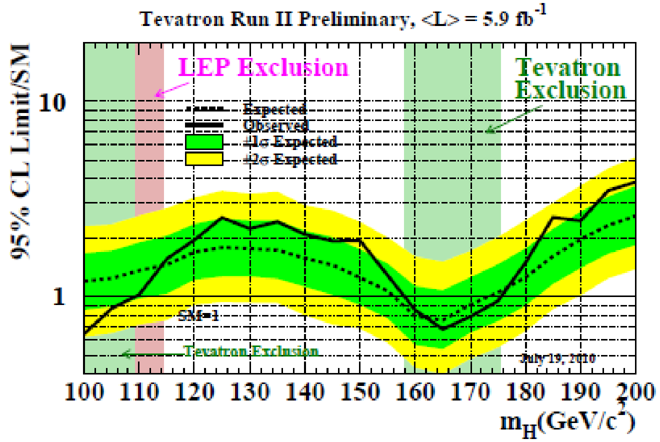
Gfitter SM — A Gfitter Package for the Global Electroweak Fit

- state-of-the art calculations (OMS scheme); in particular:
  - M_W and $\sin^2\theta^f_{eff}$: full two-loop + leading beyond-two-loop correction
    [M. Awramik et al., Phys. Rev D69, 053006 (2004)][M. Awramik et al., JHEP 11, 048 (2006), (M. Awramik et al., Nucl.Phys.B813:174-187 (2009)]
  - radiator functions: N^3LO of the massless QCD Adler function
    [P.A. Baikov et al., Phys. Rev. Lett. 101 (2008) 012022]
- theoretical uncertainties: M_W (δM_W=4-6 GeV), $\sin^2\theta^l_{eff}$ ($\delta\sin^2\theta^l_{eff}$ $=4.7 \cdot 10^{-5}$), truncation of higher QCD orders
  - included in χ^2 with flat likelihood → vary within uncertainties without contribution to χ^2

Experimental Input

- usage of latest experimental results:
  - **Z-pole observables**: LEP/SLD results [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
  - **M_W and Γ_W**: LEP/Tevatron $M_W = 80.399 \pm 0.023$ GeV
    [ADLO, hep-ex/0612034] [CDF, Phys. Lett. 100, 071801 (2008)] [CDF&D0, Phys. Rev. D 70, 092008 (2004)] [CDF&D0, arXiv:0908.1374v1]
  - **m_{top}**: $m_{top} = 173.3 \pm 1.1$ GeV [D0&CDF, arXiv:1007.3178]
  - **$\Delta\alpha_{had}^{(5)}(M_Z^2)$**: including α_S dependency [Davier, Hoecker, Malaescu. Zhang]
  - **$\overline{m}_c$, $\overline{m}_b$**: world averages [PDG, J. Phys. G33,1 (2006)]
- floating fit parameters: M_Z, M_H, m_t, $\Delta\alpha_{had}^{(5)}(M_Z^2)$, $\alpha_S(M_Z^2)$, $\overline{m}_c$, $\overline{m}_b$
- fits are performed in two versions:
  - all data except results from direct Higgs searches
  - all data including results from direct Higgs searches at LEP [ADLO: Phys. Lett. B565, 61 (2003)] and Tevatron [CDF+D0: arXiv: 1007.4587]





Hadronic Contribution to α_{QED}

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)}, \qquad \Delta\alpha(s) = \Delta\alpha_{\mathrm{lep}}(s) + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \Delta\alpha_{\mathrm{top}}(s)$$

- leptonic (top) contribution to running of α precisely known (small)
- new value for hadronic contribution [Davier, Hoecker, Malaescu. Zhang]
  - several improvements: new $\pi\pi$ cross-section data from KLOE, all available multi-hadron data from BABAR, reevaluation of the continuum contributions from perturbative QCD at four loops, ...

$$\Delta\alpha^{(5)}_{had}(M_Z^2) = \left(274.2 \pm 1.0\right)\cdot 10^{-4}$$

- error includes uncertainty of α_S ($0.37\ 10^{-04}$), which is a free fit parameter and has therefore no uncertainty in a certain fit step

⇒ subtract α_S uncertainty from total error

- variation of α_S needs to be included in the central value (Gfitter rescaling mechanism)

$$\Delta\alpha^{(5)}_{had}(M_Z^2) = \left(274.2 \pm 0.97 \pm 0.37 \cdot \left(\frac{\alpha_S(M_Z^2) - 0.1193}{0.0028}\right)\right)\cdot 10^{-4}$$

Goodness-of-Fit

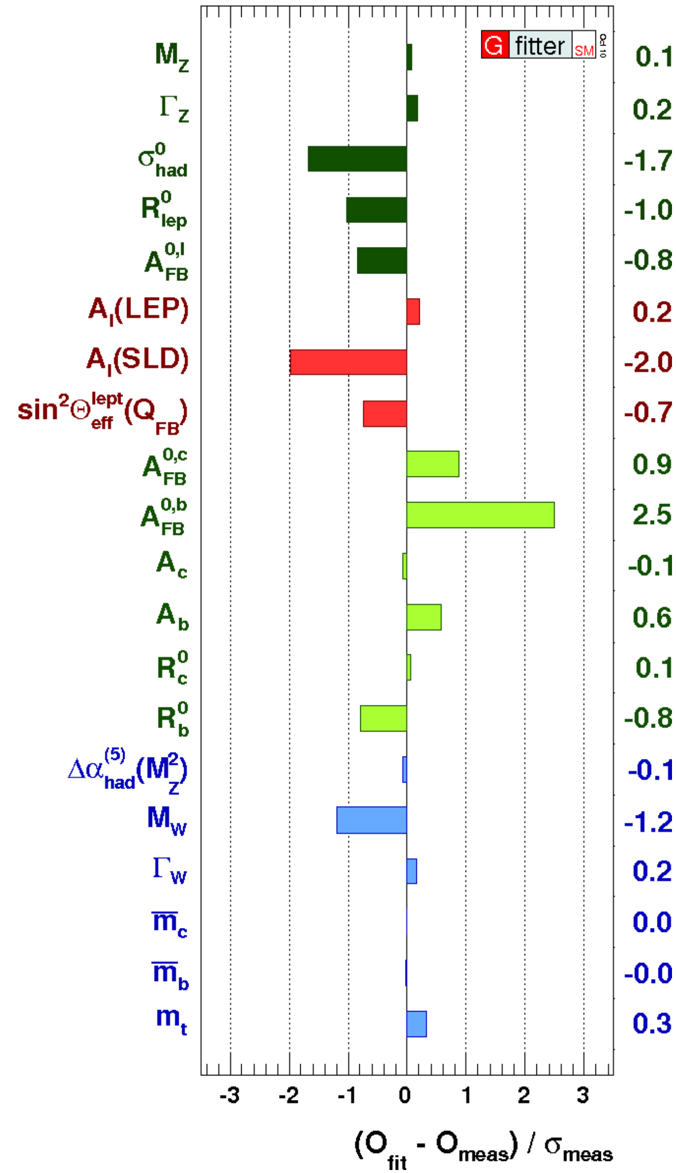
naïve p-value

- w/o direct Higgs searches:
  χ^2_{min}=16.6 → `Prob(`χ^2_{min}`,13)=0.22`
- with direct Higgs searches:
  χ^2_{min}=17.5 → `Prob(`χ^2_{min}`,14)=0.23`

pull-values for the fit with Higgs searches (right figure →)

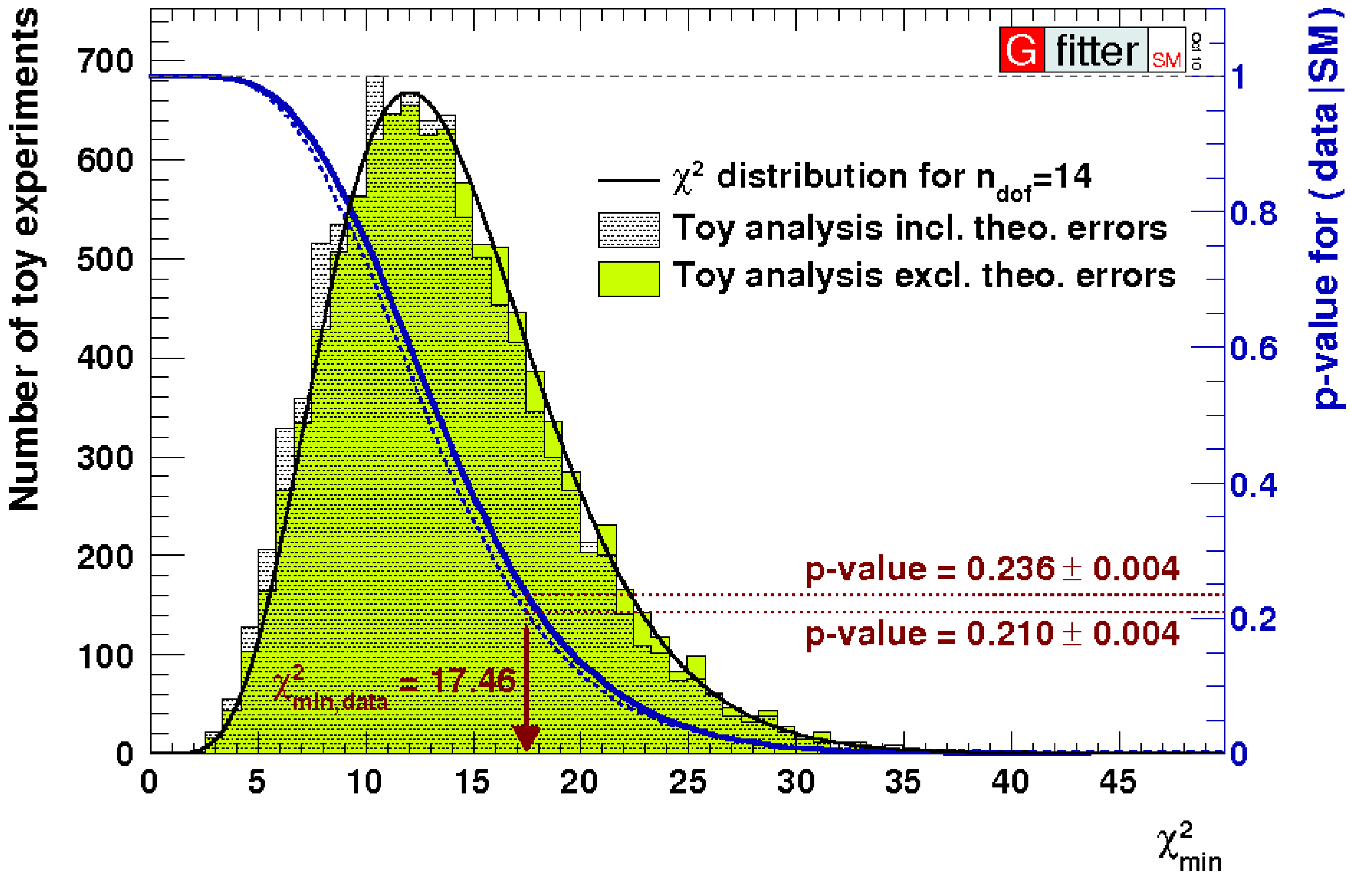
- FB asymmetry of bottom quarks
  → largest contribution to χ^2
- no value exceeds 3σ
- small contributions from M_Z, $\Delta\alpha_{had}^{(5)}$ m_c, and m_b indicate that their input accuracies exceed fit requirements

⇒ no significant requirement for new physics

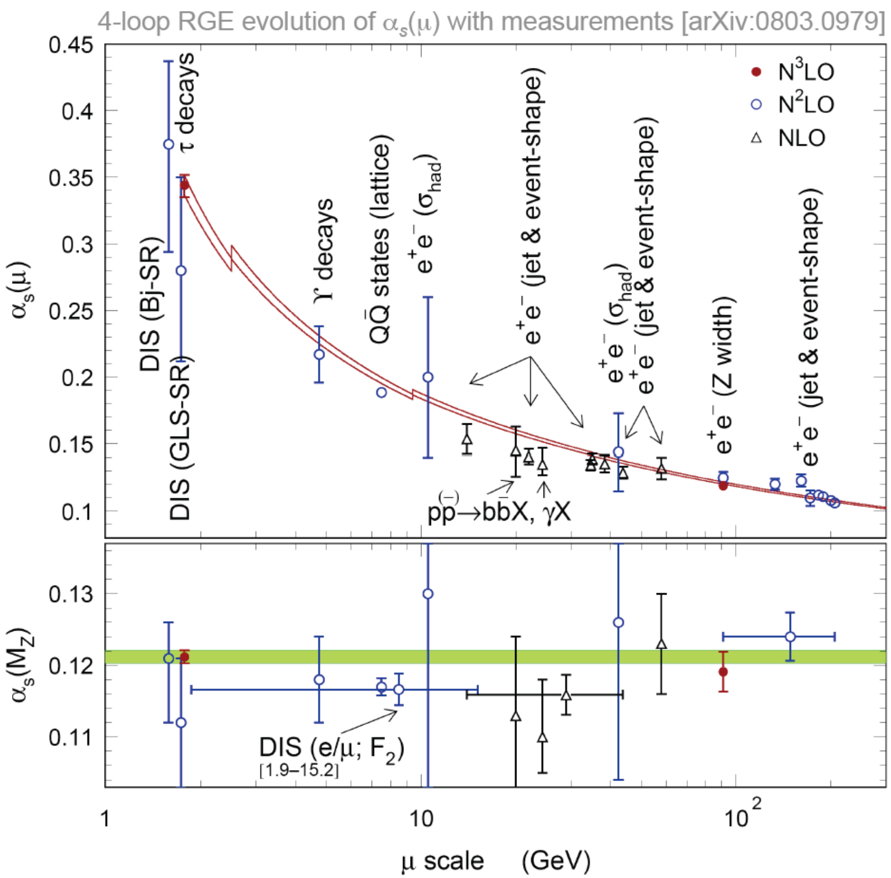


Goodness-of-Fit

- p-value: probability for wrongly rejecting the SM
- p-value: probability for getting a $\chi^2_{min,toy}$ larger than the $\chi^2_{min,data}$ from data
- p-value for fit with Higgs searches $0.24 \pm 0.03 \; -0.02_{theo}$



Determination of Strong Coupling



- R_l observable most sensitive to α_s
- N^3LO (massless Adler function) determination of α_s from complete fit:

$$\alpha_s(M_Z) = 0.1193 \pm 0.0028 \pm 0.0001$$

  - first error experimental
  - second error theoretical
    [incl. variation of renorm. scale from $M_Z/2$ to $2M_Z$ and massless terms of order/beyond $a_s^5(M_Z)$ and massive terms of order/beyond $a_s^4(M_Z)$]
  - excellent agreement with N^3LO result from hadronic τ decays [M. Davier et al., arXiv:0803.0979]

$$\alpha_s(M_Z) = 0.1212 \pm 0.0005_{exp} \pm 0.0008_{theo} \pm 0.0005_{evol}$$

Higgs Mass Constraints (old $\Delta\alpha_{had}$)

w/o direct Higgs searches:

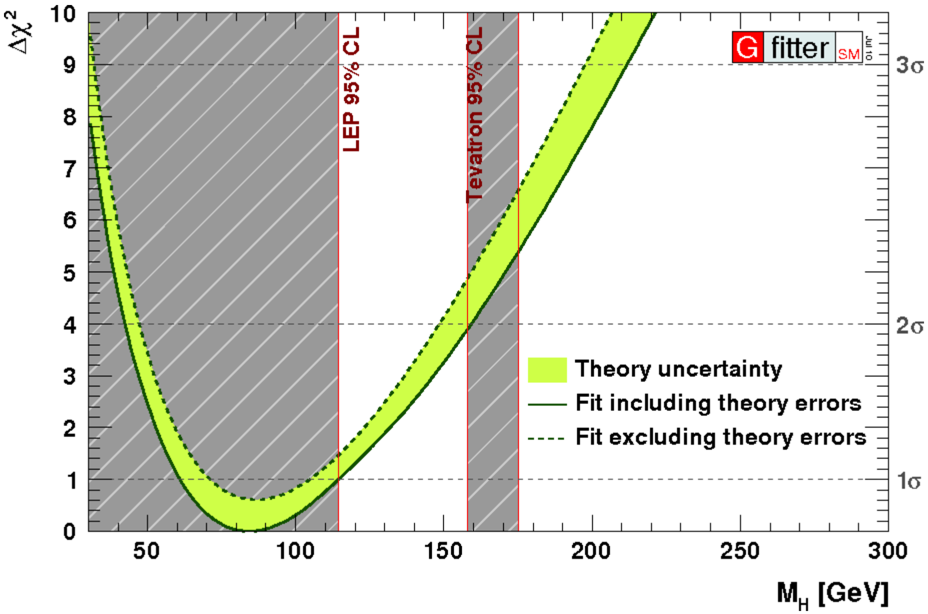
- value at minimum ±1σ:

$$M_H = 83^{+30}_{-23}\ \text{GeV}$$

- 2σ interval: [42, 159] GeV

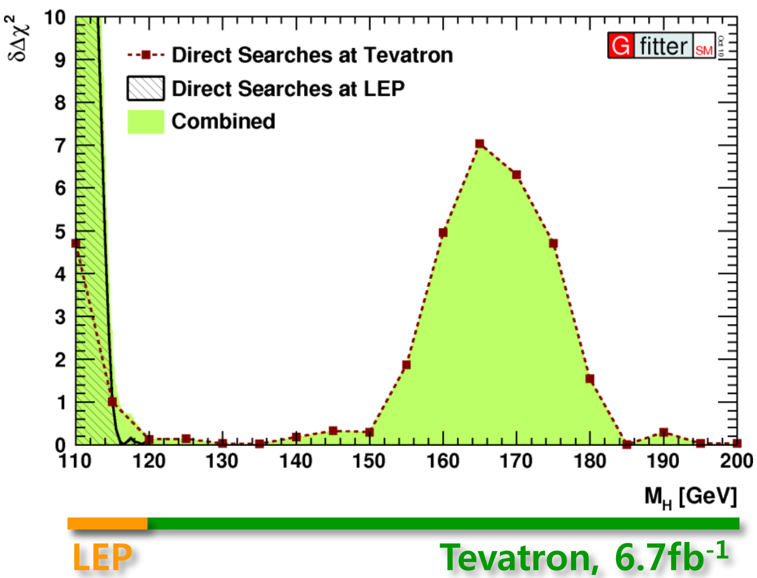
$$\Delta\alpha_{had}^{(5)}(M_Z^2) = (276.8 \pm 2.2)\cdot 10^{-4}$$

[Hagiwara et al., Phys. Lett. B649, 173 (2007)]



with direct Higgs searches:

- direct Higgs searches from LEP and Tevatron
- resulting contribution added to χ^2 during fit



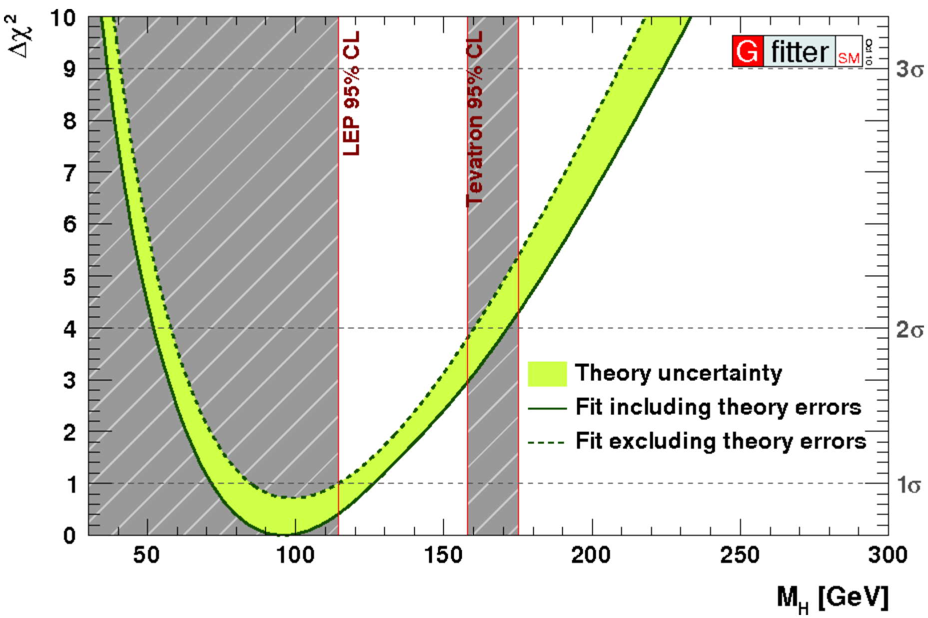
w/o direct Higgs searches:

- value at minimum ±1σ:

$$M_H = 96^{+31}_{-24}\ \text{GeV}$$

- 2σ interval: [52, 172] GeV

$$\Delta\alpha_{had}^{(5)}(M_Z^2) = (274.9 \pm 1.0)\cdot 10^{-4}$$



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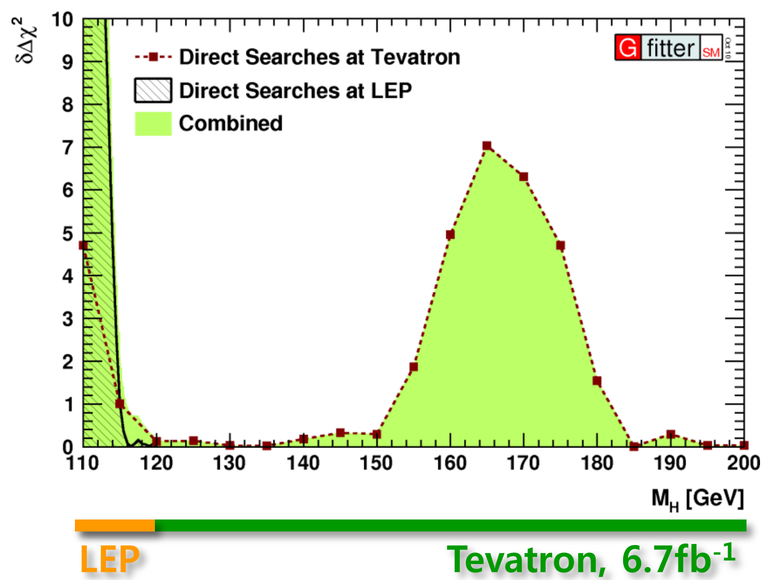

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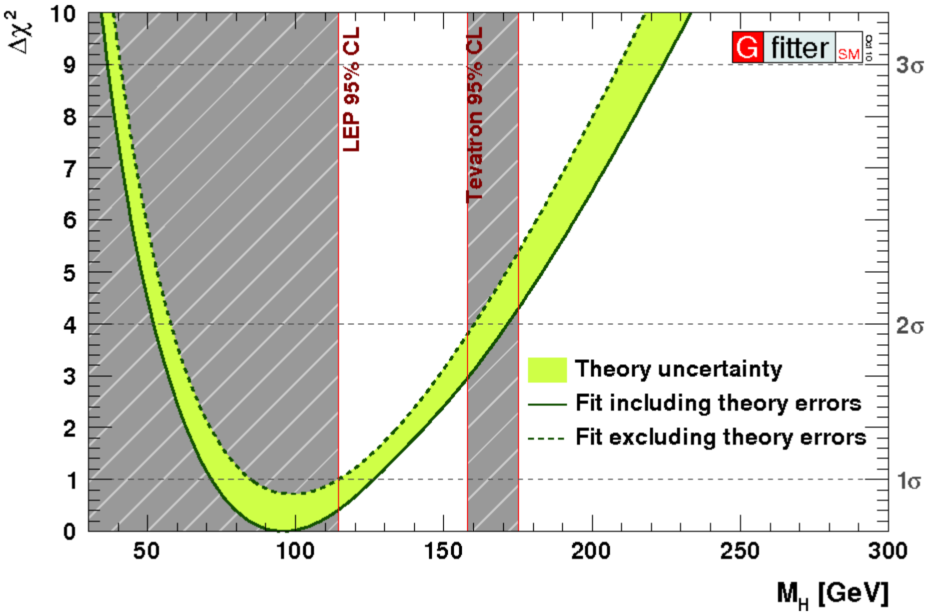
green error band: theory uncertainties directly included in χ^2 ("flat likelihood")



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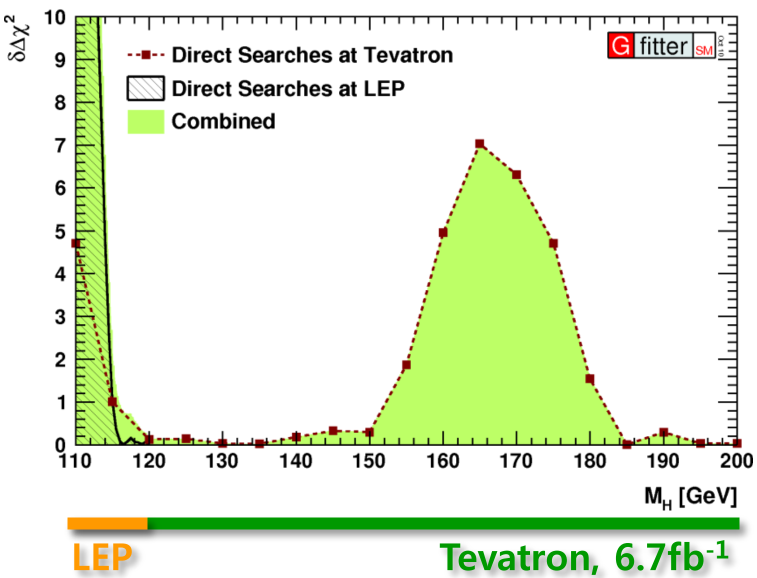

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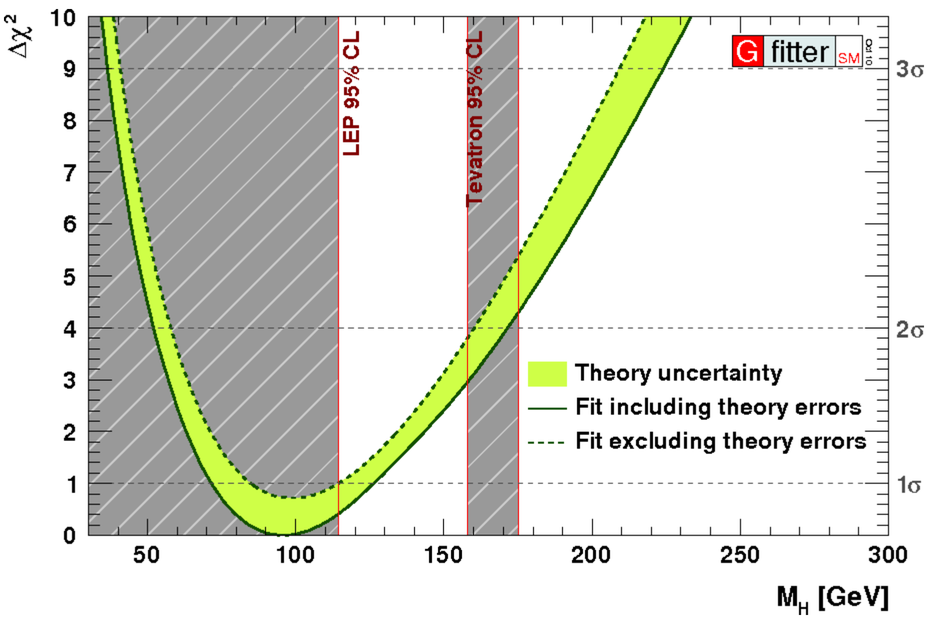
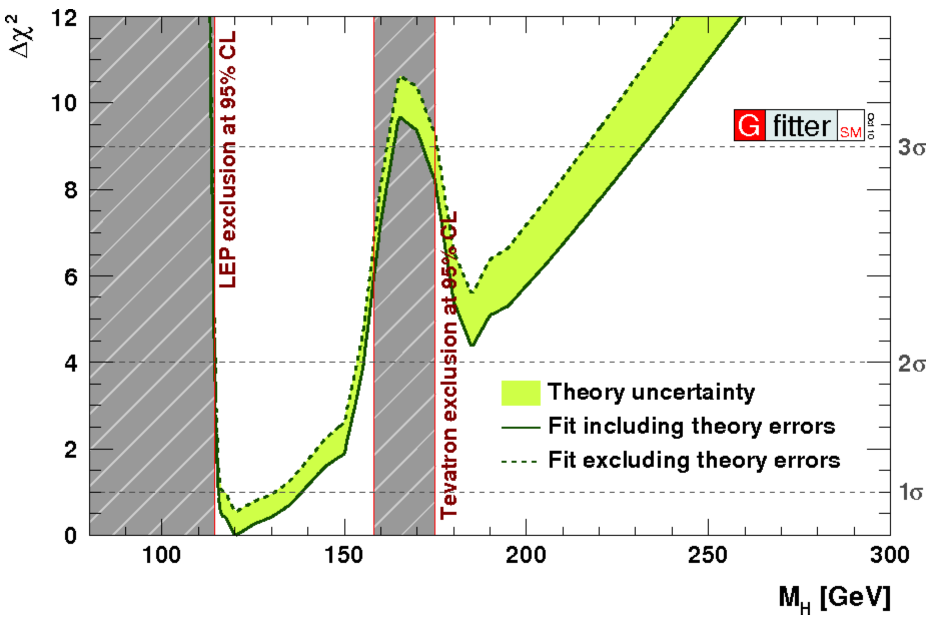


with direct Higgs searches:

- value at minimum ±1σ:

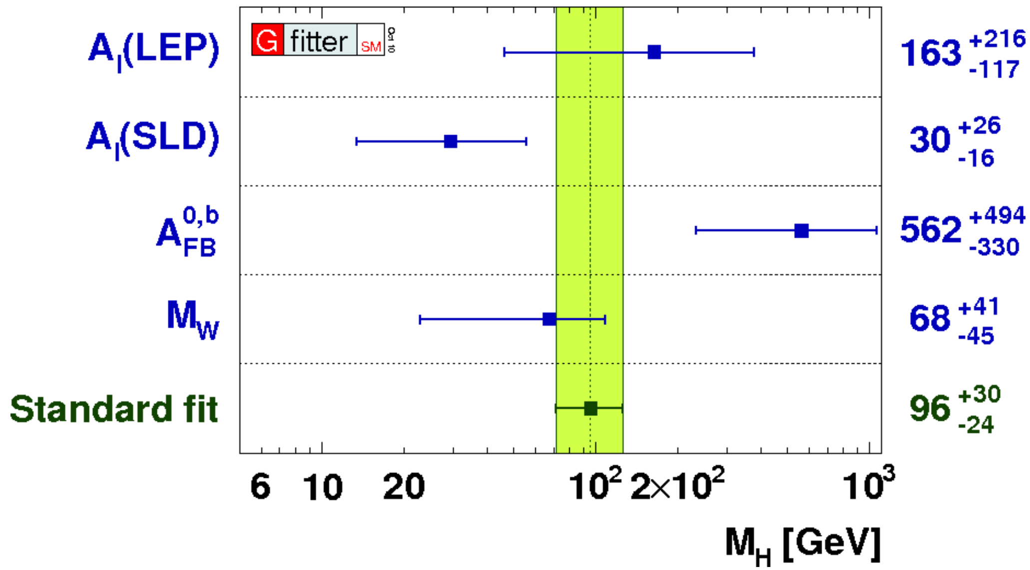
$$M_H = 120.2^{+18.1}_{-4.7}\ \text{GeV}$$

- 2σ interval: [114, 155] GeV



Testing most sensitive observables

- Higgs mass constraints from most sensitive observables
  - known tension between M_W, A_l(SLD), and $A_{FB}^{0,b}$
  - including measurements of floating fit parameters



- compatibility of these measurements:
  - MC toy analysis ("look-elsewhere-effect")
    - compare the χ^2_{min} of the full fit with χ^2_{min} of a fit without the least compatible measurement (here $A_{FB}^{0,b}$) → $\Delta\chi^2_{min}$=7.9
    - Generate toy sample around fitted values and repeat procedure by calculating the $\Delta\chi^2_{min}$ → $\Delta\chi^2_{min}{}^{toy}$-distribution
  - 1.6% (2.4σ) of toys show a result worse than the $\Delta\chi^2_{min}$ of the data

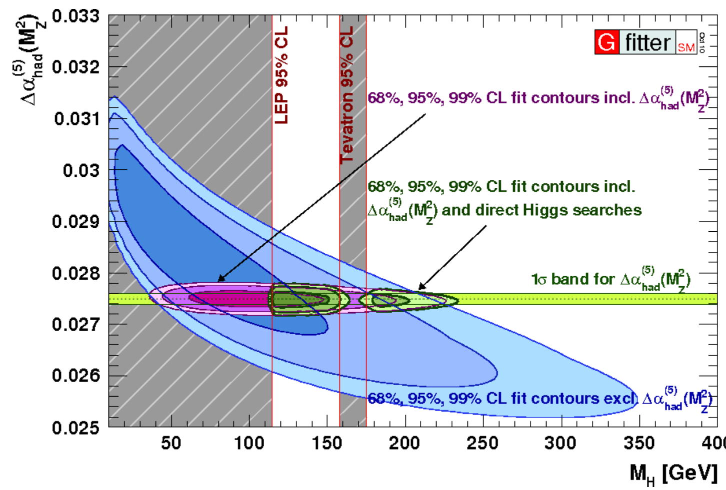
Fit Correlation between Fit Parameters

Parameter	$\ln M_H$	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$	M_Z	$\alpha_s(M_Z^2)$	m_t	$\overline{m}_c$	$\overline{m}_b$
$\ln M_H$	1	−0.17	0.13	0.03	0.32	−0.00	−0.01
$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$		1	−0.01	0.35	0.01	0.00	0.02
M_Z			1	−0.01	−0.01	−0.00	−0.00
$\alpha_s(M_Z^2)$				1	0.03	0.01	0.05
m_t					1	0.00	−0.00
$\overline{m}_c$						1	0.00

due to heavier Higgs

for comparison matrix determined by using old value for hadronic contribution to α_{QED} ➔ lighter Higgs

Parameter	$\ln M_H$	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$	M_Z	$\alpha_s(M_Z^2)$	m_t	$\overline{m}_c$	$\overline{m}_b$
$\ln M_H$	1	−0.395	0.113	0.041	0.309	−0.001	−0.006
$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$		1	−0.006	0.101	−0.007	0.001	0.003
M_Z			1	−0.019	−0.015	−0.000	0.000
$\alpha_s(M_Z^2)$				1	0.021	0.011	0.043
m_t					1	0.000	−0.003
$\overline{m}_c$						1	0.000



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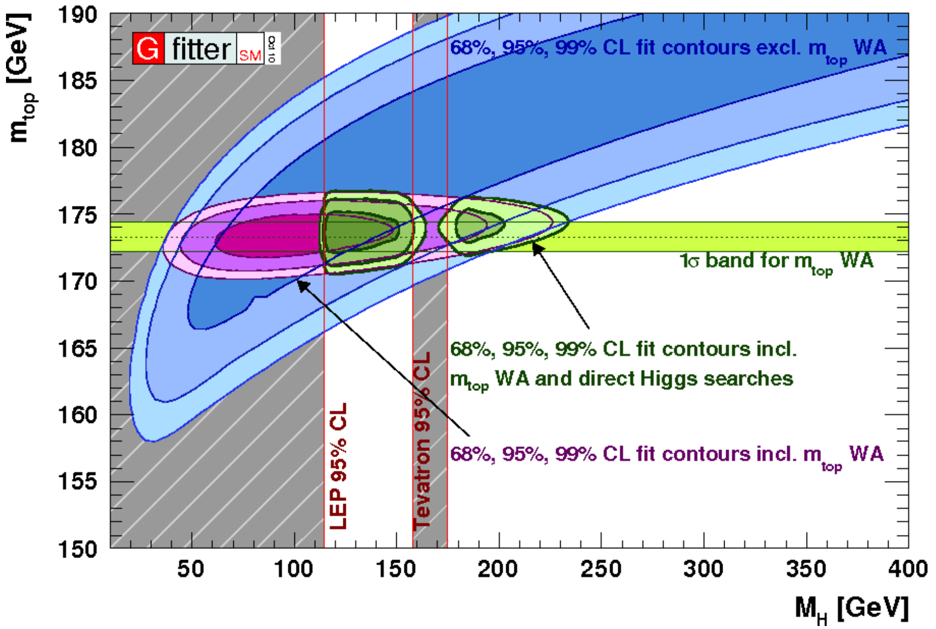
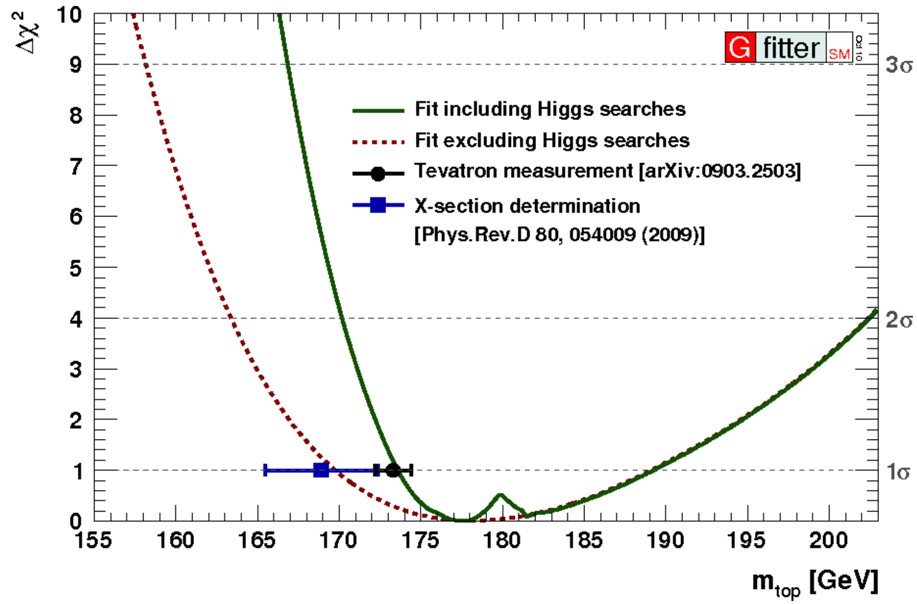
m_t					1	0.00	-0.00
$\overline{m}_c$						1	0.00

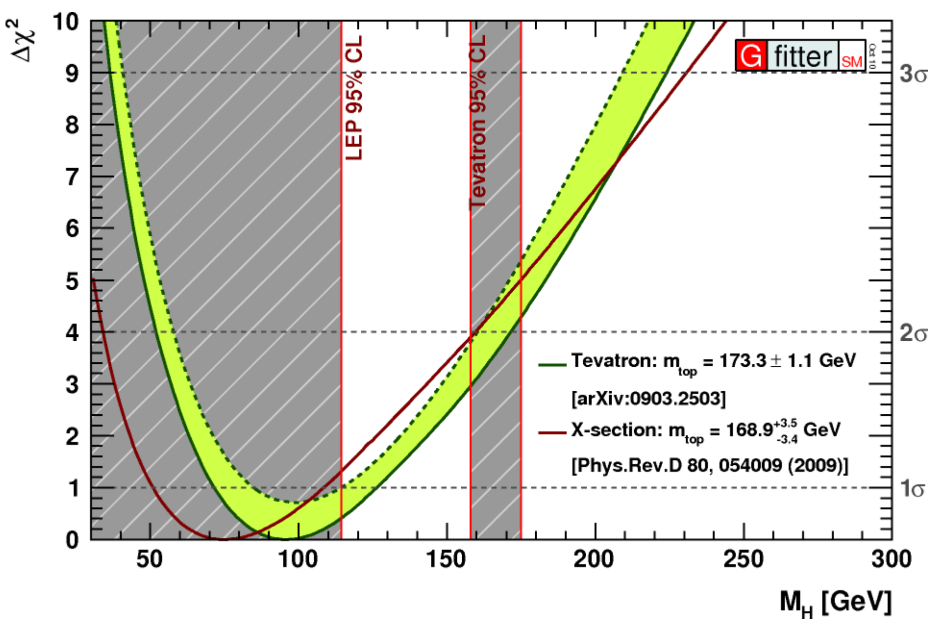


⇒ top mass crucial input parameter for electroweak fit

Top Mass Determination

- theoretical predictions use top pole mass
- unclear definition of top mass at Tevatron: "MC" or pole mass?
  [Hoang &Steward., Nucl.Phys.Proc.Suppl.185:220-226,2008]
  ⇒ additional uncertainty?
- alternative: extract top mass from total top pair cross-section
  [Langenfeld, Moch, Uwer, Phys.Rev.D80:054009,2009]



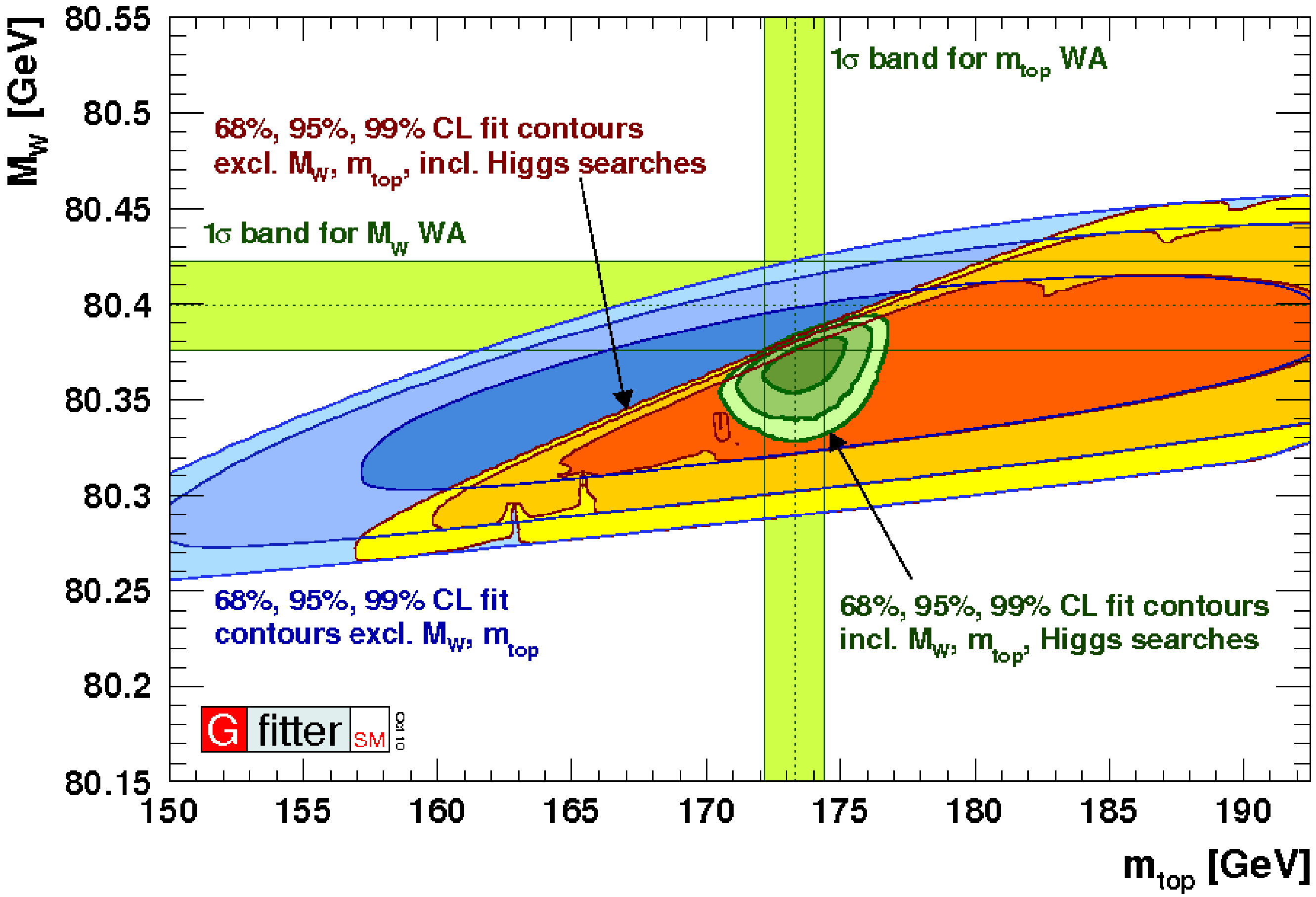


- direct: m_{top}=173.3 ± 1.1 GeV
- X-section: $m_{top}=168.9\ ^{+3.5}_{-3.4}$ GeV

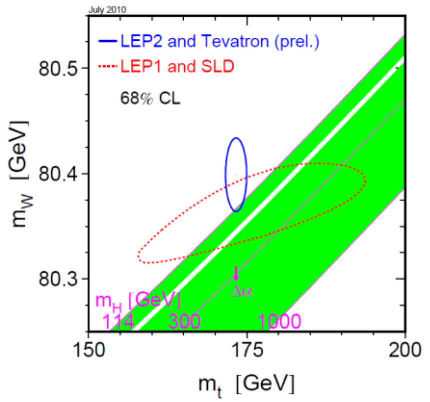
SM Fit:

- w Higgs searches:
  $m_{top}=177.4\ ^{+11.8}_{-3.5}$ GeV
- w/o Higgs searches:
  $m_{top}=178.2\ ^{+10.9}_{-8.8}$ GeV

W and Top Mass



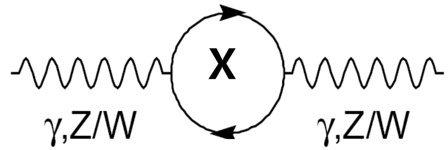
for comparison:



- indirect fit results agree with experimental values
- results from Higgs searches significantly reduce the allowed parameter space
- illustrative probe of SM (if M_H is measured at LHC and/or ILC)

Beyond Standard Model

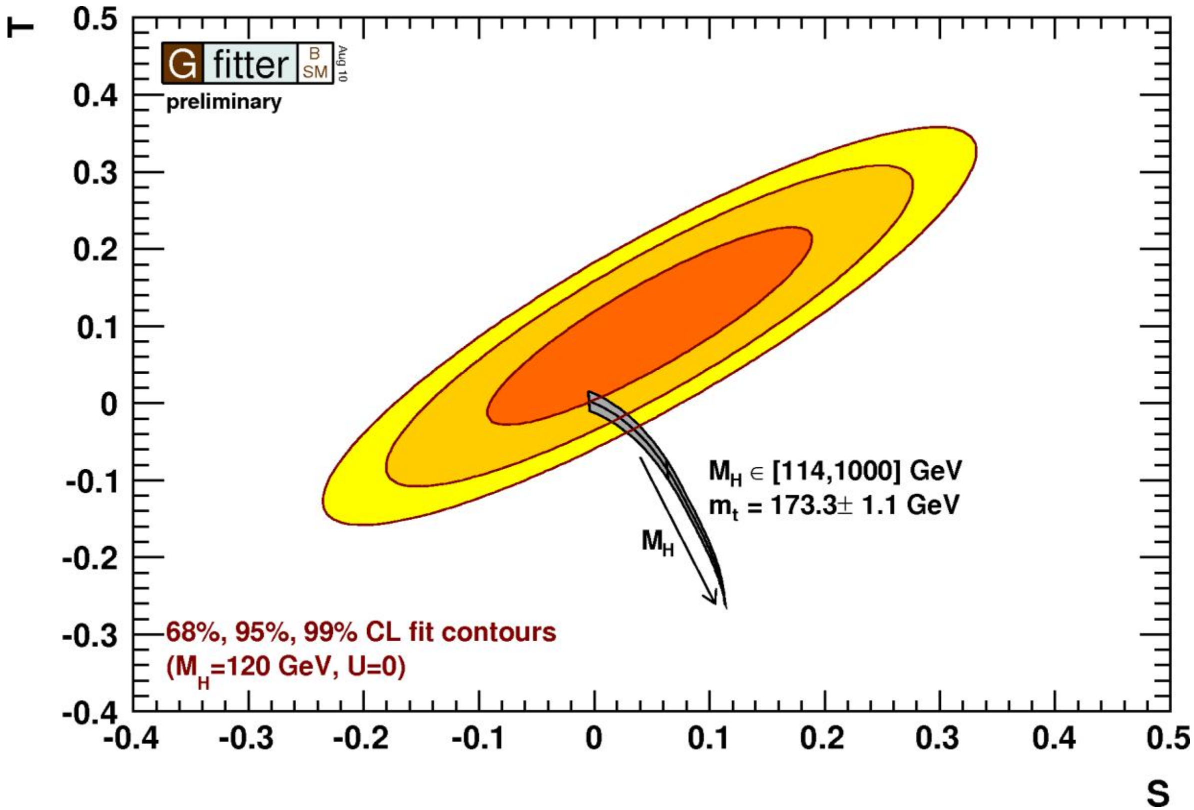
A Gfitter Package for SM Extensions



- oblique electroweak corrections to SM observables (physics beyond SM appear only through vacuum polarizations)
- STU parameters [Peskin and Takeuchi, Phys. Rev. D46, 1 (1991)]
  - $O_{measurement} = O_{SM}(M_{H,ref}, m_{t,ref}) + c_S S + c_T T + c_U U$
    - S=T=U=0 if data are equal to SM_{ref} prediction
    - S : new physics contribution to neutral current processes
    - (S+U) : new physics contribution to charged current processes
      U only sensitive to W mass and width
      usually very small in new physics models (often: U=0)
    - T : difference between neutral and charged current processes (sensitive to isospin violation)
  - also implemented corrections to Zbb couplings
    [Burgess et al., Phys. Lett. B326, 276 (1994)] [Burgess et al., Phys. Rev. D49, 6115 (1994)]

Fit to Oblique Parameters

- derived from fit to electroweak observables (see global SM fit)
  - SM_{ref}: M_H=120 GeV, m_t=173.3 GeV
- comparison with SM prediction of ST parameters



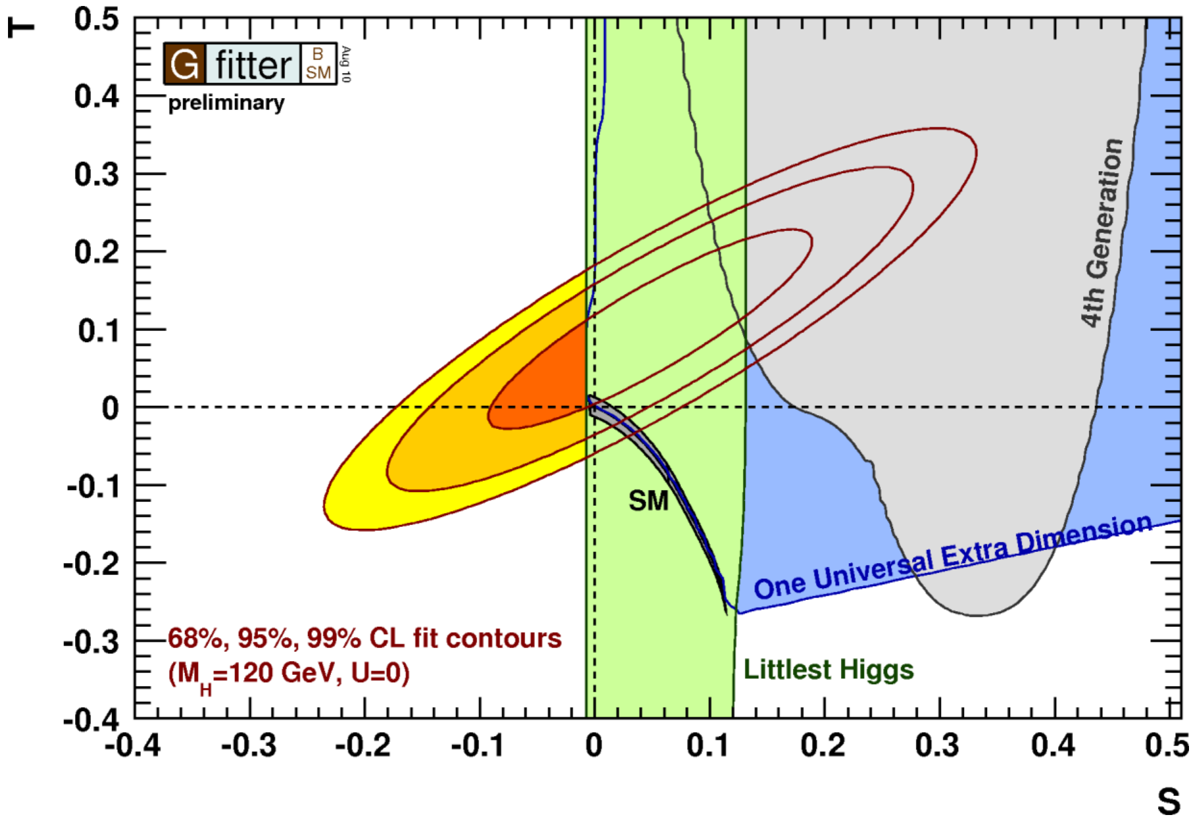
Fit Results:

$S = 0.02 \pm 0.11$
$T = 0.05 \pm 0.12$
$U = 0.07 \pm 0.12$

Correlation:

	S	T	U
S	1	0.879	-0.469
T		1	-0.716
U			1

- derived from fit to electroweak observables (see global SM fit)
  - SM_{ref}: M_H=120 GeV, m_t=173.3 GeV
- comparison with new physics prediction



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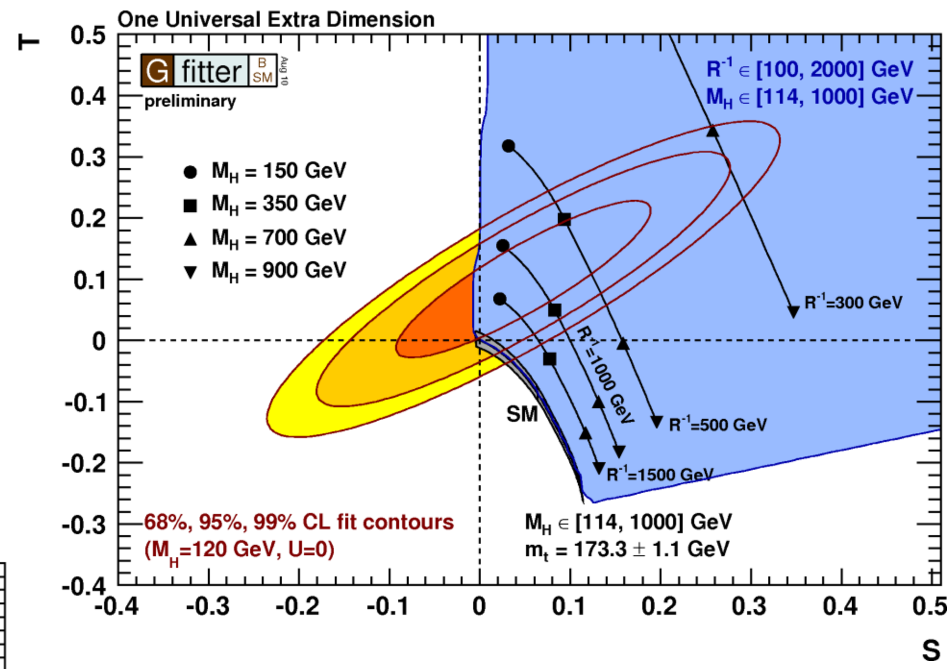
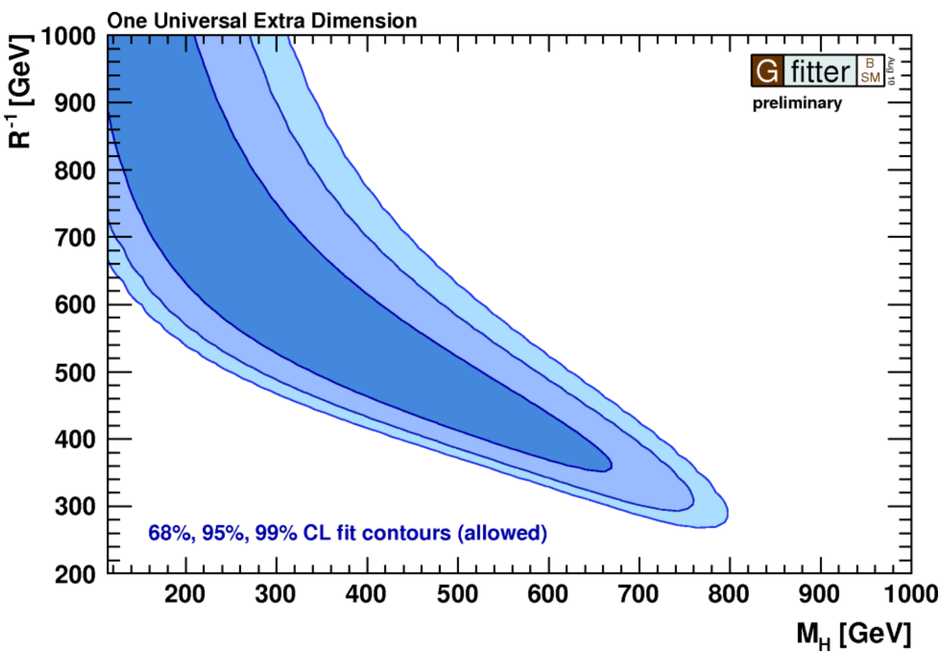
One Universal Extra Dimension

- all SM particles propagate in extra Dimension
- conservation of Kaluza-Klein (KK) parity → similar phenomenology as SUSY
- lightest KK state stable → Dark Matter candidate

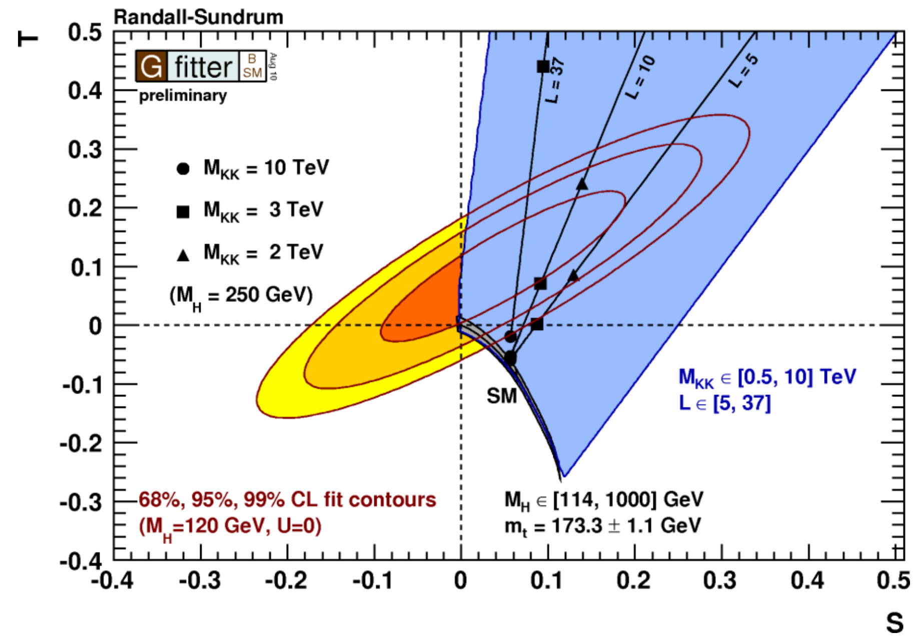


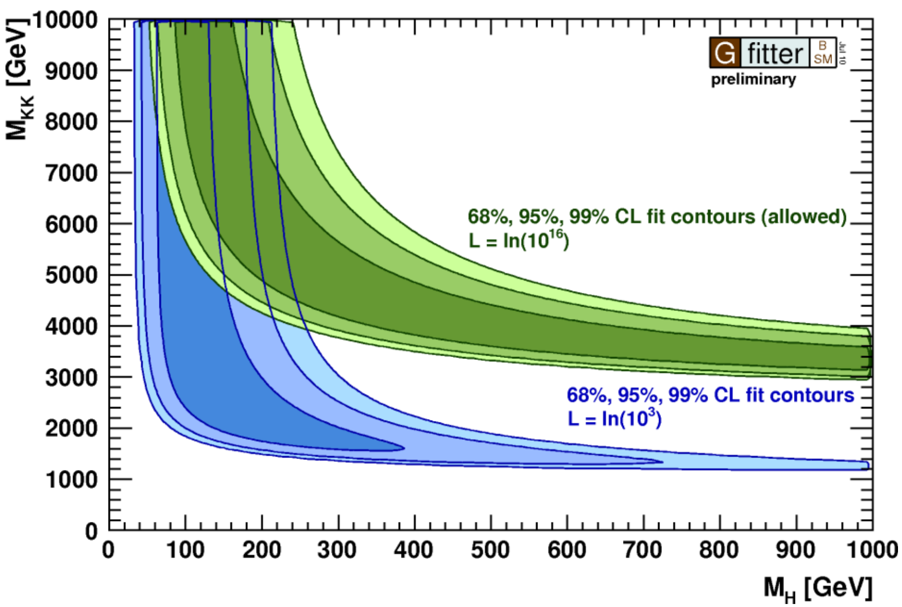


- parameters of UED model
  - R^{-1} compactification scale (size of extra dimension) $m_{KK} \cong n/R$
  - oblique parameters depend on M_H
- oblique parameters replaced by corrections from UED model
  [Gogoladze et al., Phys.Rev. D 74, 093012 (2006)]
  [Appelquist et al., Phys.Rev. D67 (2003) 055002]

Warped Extra Dimensions (Randall-Sundrum)

- introducing one extra dimension (ED) for solving the hierarchy problem
- RS model characterized by one warped ED confined by two three-branes
- one brane contains SM particles
- extension: SM particles allowed to propagate in bulk region
- observation: heavy Kaluza-Klein modes



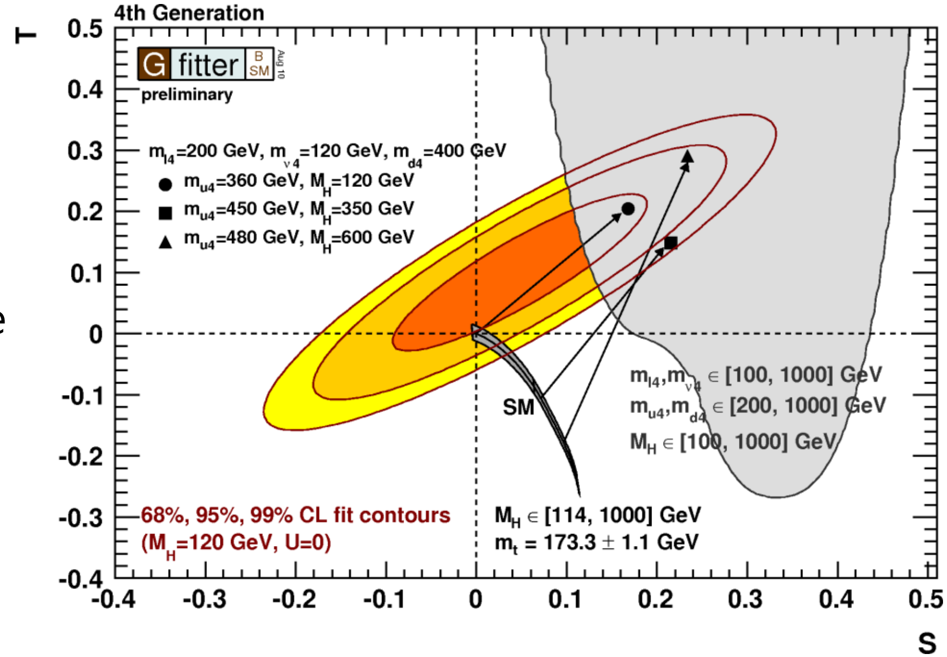


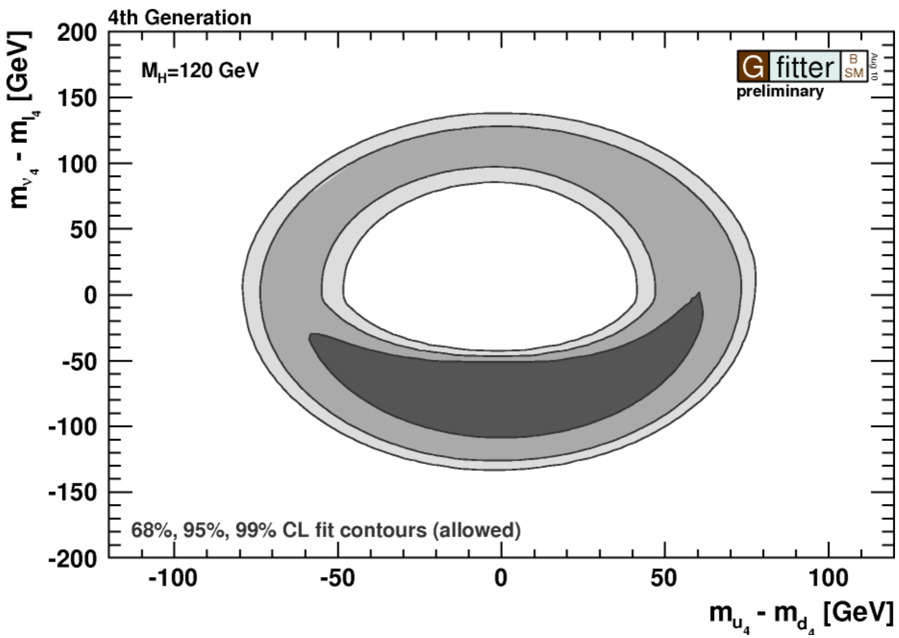
- parameters of RS model
  - M_{KK}: KK state
  - L: inverse warp factor, function of compactification radius
- oblique parameters replaced by corrections from RS model

[S. Casagrande et al., JHEP10(2008)094]

Fourth Generation

- motivation for fourth generation:
  - predicted by some GUT theories
  - can play an important role in electroweak symmetry breaking
- number of light neutrino (**$m_\nu < M_Z/2$**) due to measurement of Z width





- mixing between first 3 generations and new generation neglected
- oblique parameters mostly sensitive to mass difference of new generation
  [Phys.Rev.D64, 053004 (2001)]

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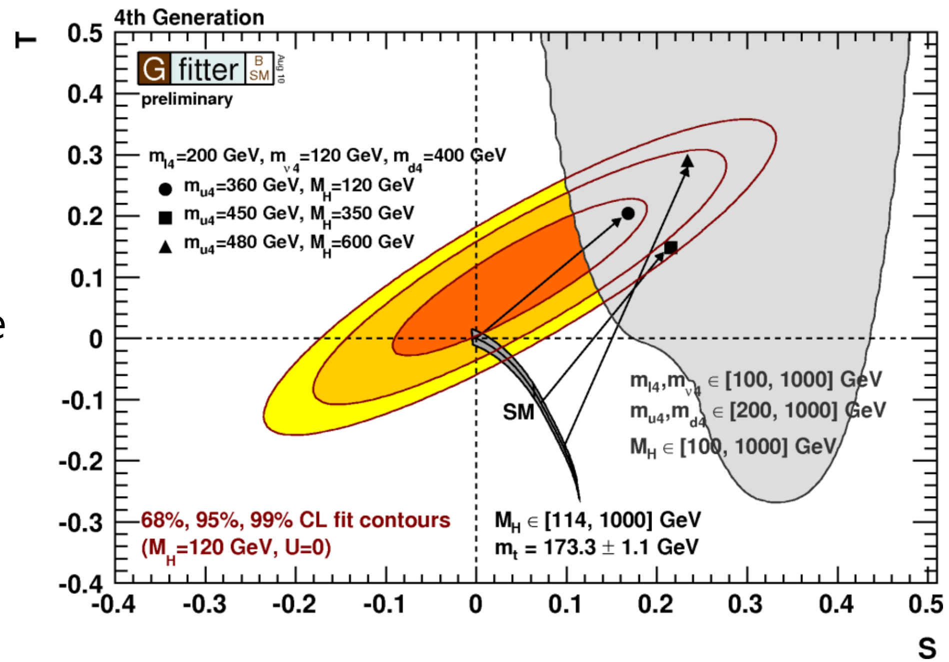

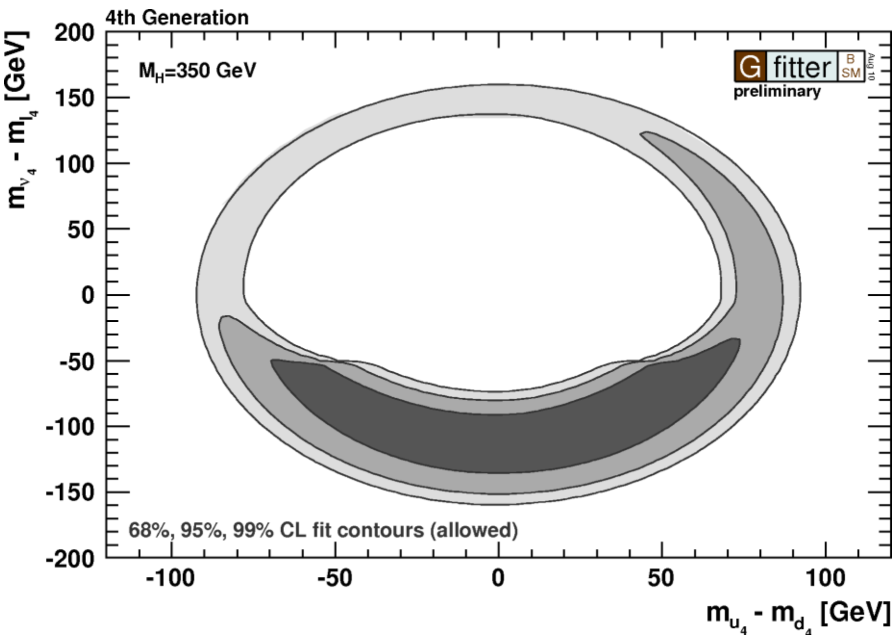

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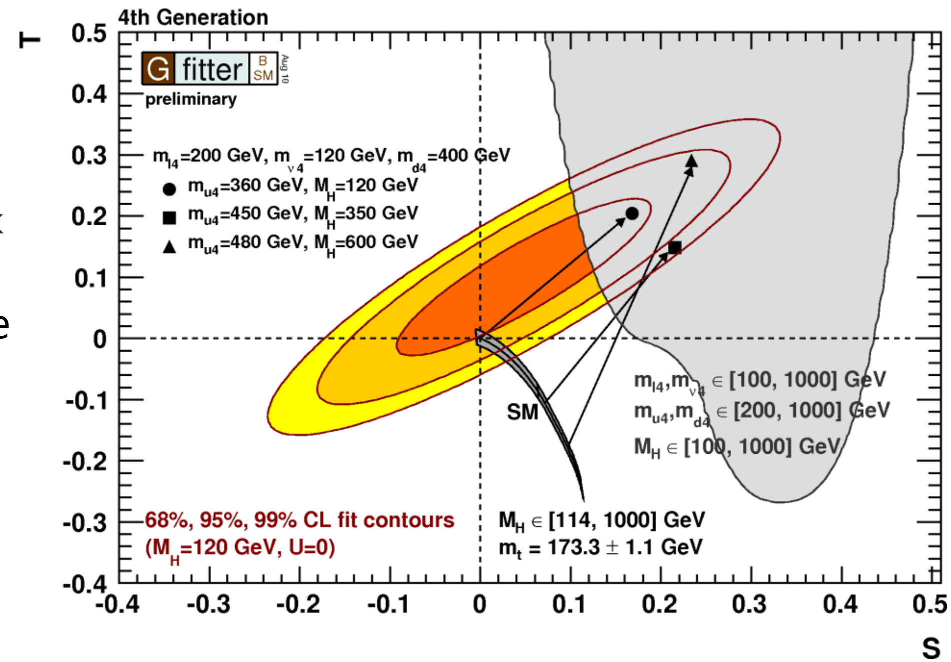
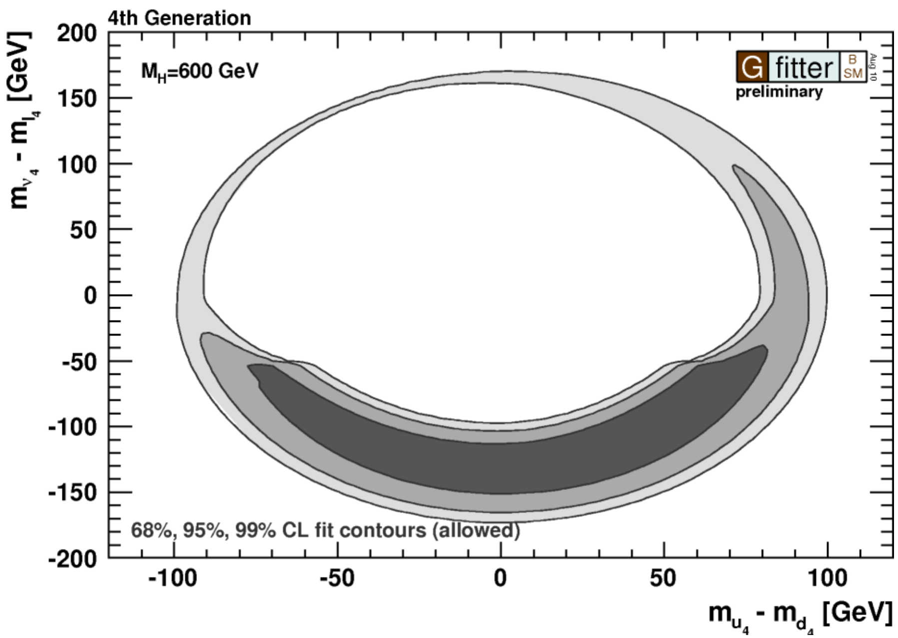
- number of light neutrino (**m_ν<M_Z/2**) due to measurement of Z width



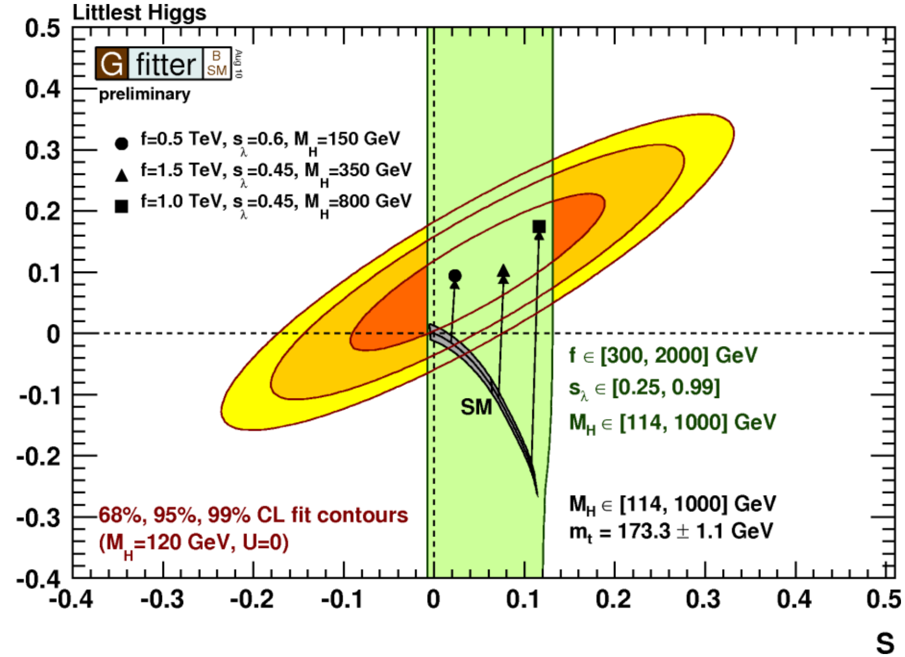


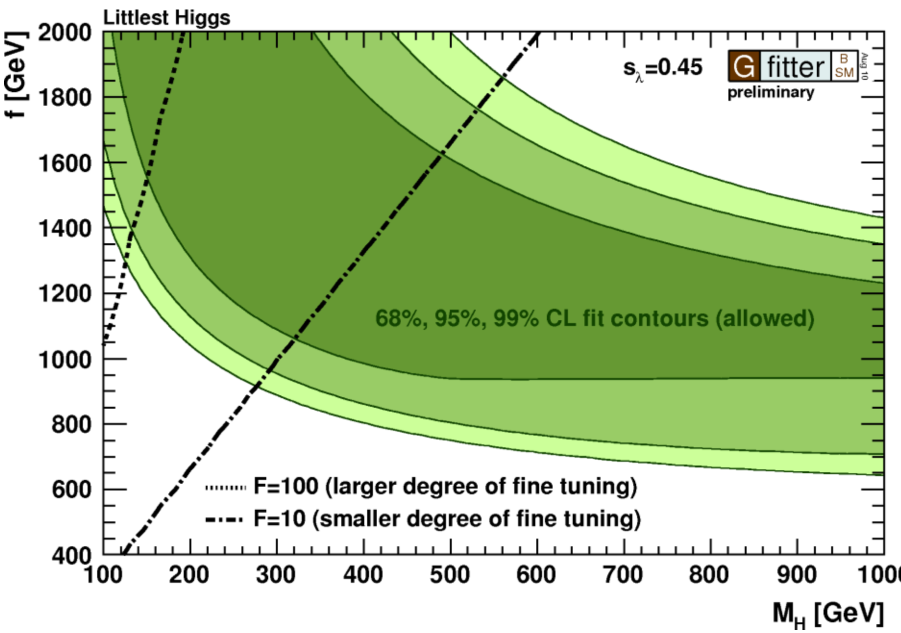
- mixing between first 3 generations and new generation neglected
- oblique parameters mostly sensitive to mass difference of new generation

[Phys.Rev.D64, 053004 (2001)]

Littlest Higgs with T-Parity

- Higgs pseudo-Nambu-Goldstone boson
- new fermions and new gauge bosons
  - two new top states (T-odd m_{T-} and T-even m_{T+})
  - LH solves hierarchy problem (new particles cancel SM loops)
- T-parity
  - provide dark matter candidate
  - forbids tree-level contribution from heavy gauge bosons to SM observables





- parameters of LH model
  - f symmetry breaking scale (scale of new particles)
  - $s_\lambda \cong m_{T-} / m_{T+}$ ratio of masses in top sector
  - order one-coefficient δ_c (exact value depends on detail of UV physics)
    - treated as theory uncertainty in fit (Rfit) δ_c=-5…5
- oblique parameters replaced by corrections from LH model

[Hubisz et al., JHEP 0601:135 (2006)]

Conclusion

Gfitter Package

- flexible, generic C++ program including the statistical framework
- not shown SUSY, 2HDM
- results on http://cern.ch/Gfitter

Global SM Fit

- using state-of-the art predictions for the electroweak observables
- Toy Analysis of p-value: $p = 0.23 \pm 0.01\ {-0.02}$
- small Higgs masses are preferred from SM Fit
- N^3LO determination of $\alpha_s(M_Z) = 0.1193 \pm 0.0028 \pm 0.0001$

Oblique Corrections

- SM extension allow heavy Higgs Boson
- more models implemented (2HDM, TechniColor, Inert Doublet Model, ADD, …)

Interpretation of Direct Higgs Searches

- direct Higgs searches from LEP and Tevatron
  - using one-sided CL_{s+b}
    - sensitive to too few Higgs-like events
  - we are interested in any kind of deviation from “s+b” hypothesis
    - also too many Higgs-like events
    - transform one-sided CL_{s+b} into 2-sided $CL_{s+b}^{2\text{-sided}}$
  - compute contribution to χ^2 assuming symmetric PDF: $\delta\chi^2 = \mathrm{Erf}^{-1}(1 - CL_{s+b}^{2\text{-sided}})$
- alternative: use of test statistics -2lnQ
  - similar behavior, but deeper minimum
  - ⇒ slightly stronger constraint