On the way to four-loop anomalous dimensions

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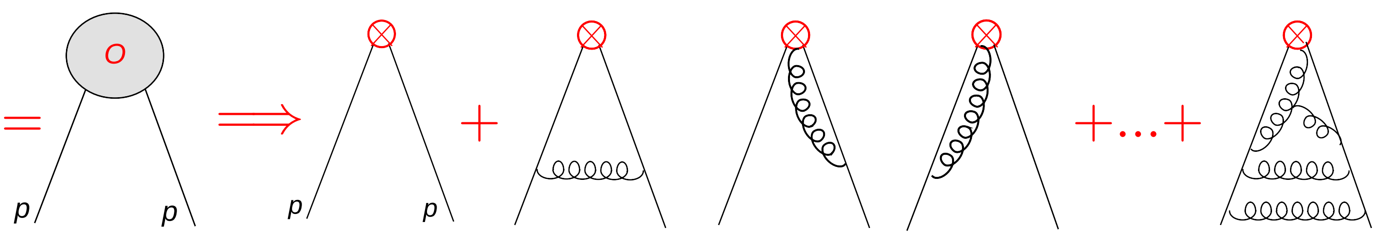
Motivation

- Self-consistent NNNLO analysis of **DIS**: 3-loop *Wilson Coefficients* and 4-loop *anomalous dimensions*
  KNOWN:
  Wilson Coeff. $\sim$ up to 3-loops for *all* values of spin Moch, Vermaseren, Vogt'05
  Anom. dim. $\sim$ also up to 3-loops for *all* values of spin Moch, Vermaseren, Vogt'04 (also some polarized results Vogt, Moch, M.R., Vermaseren'08)
- The higher loops calculations: to *decrease theoretical errors* in analysis of **DIS** ($\mapsto$ PDF)
- This project is a part of an approved project A.1 (*'Massless propagators, vertices and anomalous dimesions'*) of SFB TR 9 (princ. ivest. Kühn)
  The aim - to create a **ZFOUR** - FORM based program using MINCER, MATAD and EXP to compute a variety of 4-loop anom. dim. for evaluation of higher moments ($N > 4$) of **DIS**
  $N > 4$ - not possible with BAICER at the moment Baikov'96-...
- Results - immediately useful for project B3 (*'Parton Distribution Functions'*) of SFB TR 9 (princ. invest. Blümlein, Jansen)
- Useful to check the *KLOV* conjecture on leading transcedentality terms in AdS/CFT Kotikov, Lipatov, Onishchenko, Velizhanin '04

Known ways to calculate anom. dim.: Usual way

- suggest O is an operator (e.g. composite twist-2 non-singlet operator $O^{\alpha,\{\mu_1,\cdots,\mu_N\}} = \overline{\psi}\lambda^\alpha\gamma^{\{\mu_1}D^{\mu_2}\ldots D^{\mu_N\}}\psi$)
- $O_{bare} = Z_O O_{ren}$, $\gamma_O \equiv \frac{\mu}{Z_O}\frac{\partial Z_O}{\partial\mu} \Rightarrow$
  $Z_O = 1 - \frac{g^2}{2\varepsilon}\gamma_O^0 + \mathcal{O}(g^4)$
- for anomalous dimension γ_O :
  matrix element between parton states with momenta p:

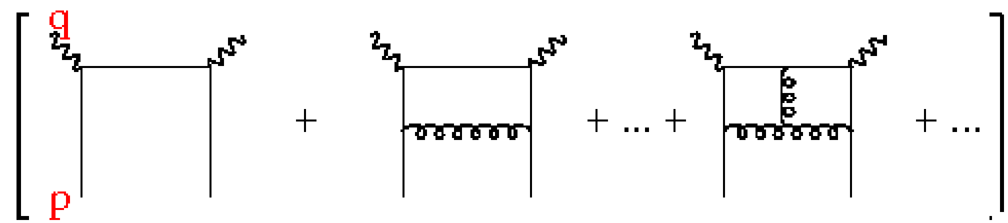
$< p|O|p >=$ 

- in MS ($\overline{MS}$) need only poles in ε, ($\varepsilon:\ dim = 4 - 2\varepsilon$),
  i.e. only terms $\sim \left(\frac{1}{\varepsilon}\right)^n, n > 1$. In general we have to calculate pole parts of n-loop diagrams with 2 legs to get n-loop anomalous dimensions γ_O.

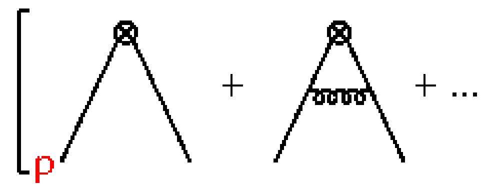
Known ways to calculate anom. dim.: Indirect method

Method of projection Gorishnii, Larin, Tkachev ‘83; Gorishnii, Larin ‘87

- Operator product expansion $\longrightarrow$ for Green functions with external partons



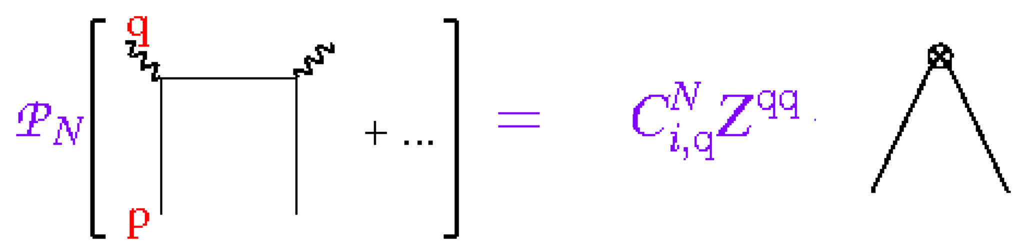
$$= \sum_N \left(\frac{1}{2x}\right)^N \left\{ C^N_{i,q} Z^{qq} \right.$$



$$\left. \vphantom{C^N_{i,q}} \right\}$$

With projection $\mathcal{P}_N \equiv \left[\frac{q^{\{\mu_1} \ldots q^{\mu_N\}}}{N!} \frac{\partial^N}{\partial p^{\mu_1} \cdots \partial p^{\mu_N}} \right] \Bigg|_{p=0}$

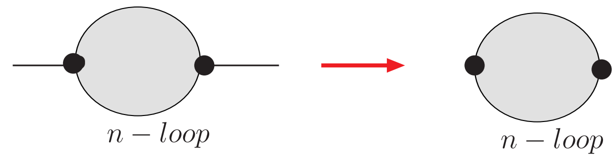
  - only tree level operator matrix elements survive



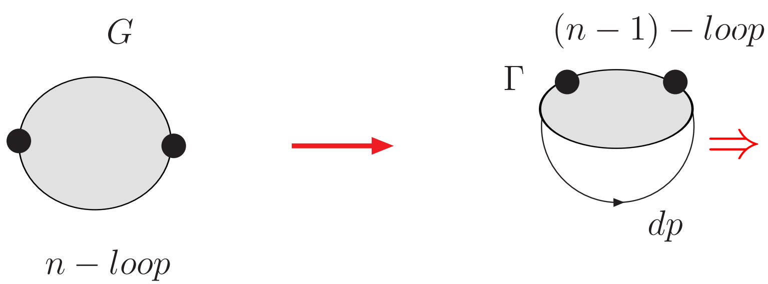
- Extracting from Z^{qq}: $\gamma(\alpha_s, N)$ anomalous dimensions are known to 3-loops ($\mathcal{O}(\alpha_s{}^3)$) analytic in N
  Moch, Vogt,Vermaseren’04

Reduction of four-loop problem to three-loop one. Use of R-operation

- for n-loop anom. dim. γ_O is enough to calculate only UV part of n-loop tadpole (graph with amputated legs)



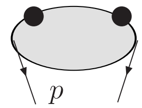
- IR is not relevant $\Rightarrow$ put mass M on every internal line
  Chetyrkin, Larin, Tkachev'80; Misiak'95

- 

  $\Rightarrow G = \int \Gamma \, dp$, UV $\Leftrightarrow$ limit $p_E^2 \to \infty$

  van Ritbergen, Vermaseren, Larin'97; Chetyrkin, Misiak, Münz'98
  Given Γ has no **divergencies** and **subdiv.**, perform *Asymptotic Expansion* (**AsE**) in $p_E^2 >> M^2$. Final integration over dp gives UV-poles for G graph. Problem is reduced by 1 loop.

- In general Γ does have **divergencies** and **subdiv.**. Procedure is not allowed.

- Solution: make Γ 

finite!!!
  $\Gamma \to \Gamma' = \mathrm{R}\circ\Gamma$, R-renormalization operator for graphs (R-operation).
  $\mathrm{R} = 1 - \mathrm{T}$- convention, T- some operator that extracts the divergence.

$$
\begin{aligned}
UV(G) &= UV\int \Gamma dp = UV\int (1-\mathrm{T})\circ\Gamma dp + UV\int \mathrm{T}\circ\Gamma dp \\
&= UV\int \mathbf{AsE}_{(p_E^2>>M^2)}\left\{(1-\mathrm{T})\circ\Gamma\right\} dp + UV\int \mathrm{T}\circ\Gamma dp
\end{aligned}
$$

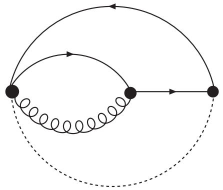
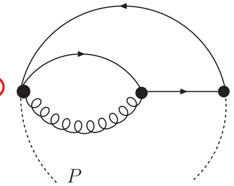
- formal T-operator, *FOREST* formula : Zimmermann'69
  FOREST - **any set of disjoint or nested subgraphs**

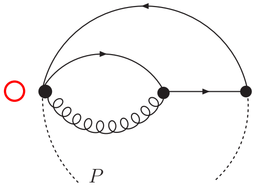
$$
\mathrm{R}\circ\Gamma = \sum_{U_i\in\mathcal{F}(\Gamma)}\left[\prod_{\gamma_i j\in U_i}(-T_{\gamma_{ij}}\circ)\Gamma\right] = 1 + \sum_{U_i\in\mathcal{F}(\Gamma)/\{\}}\left[\prod_{\gamma_j\in U_i}(-T_{\gamma_{ij}}\circ)\Gamma\right]
$$

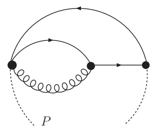
  U_i - forests, γ_{ij} - elements of forests, $\mathcal{F}(\Gamma)$-set of all possible forest, $\{\}$-empty forest *(explanations on an example, next slide)*

- **AsE** and/or T effectively reduce problem by at least 1 loop.

Example with non-trivial three-loop topology

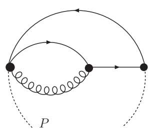
$UV:$  $\Longrightarrow \int dp\,\mathbf{AsE}_{(p_E^2>>M^2)}(1-\mathrm{T})\circ$ 

$+\int dp\,\mathrm{T}\circ$ 

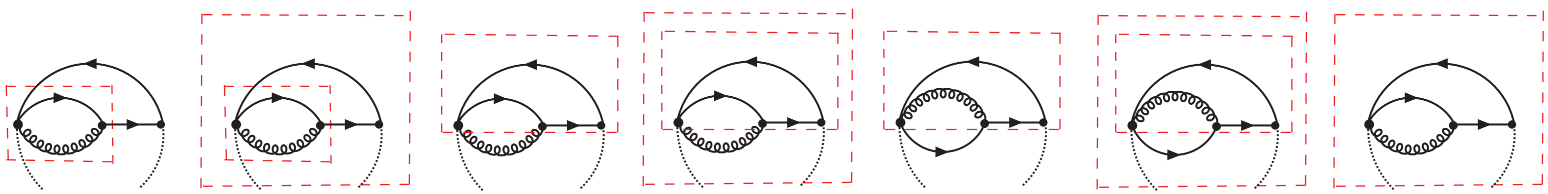
- $\mathbf{AsE}_{(p_E^2>>M^2)}\circ$ 

- usual task for bundle EXP-MATAD

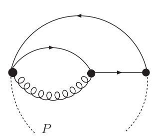
  Chetyrkin, Steinhauser, Seidensticker

- $\mathrm{T}\circ$ 

- *FOREST* formula, construct all possible forests for $\Gamma \Rightarrow$

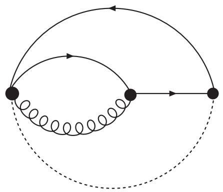
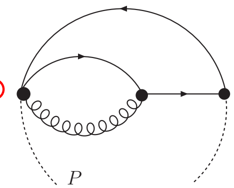
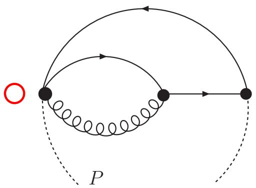


Forest (topologically) can be builded by special feature of EXP. *Changing scale control* it is **possible** to do R-operation by MATAD!!!

- $\mathbf{AsE}_{(p_E^2>>M^2)}\mathrm{T}\circ$ 

- idea is to make an *MIXED FOREST* with EXP,

then again change usual MATAD for R-operation.

$UV:$  $\Longrightarrow \int dp\,\mathbf{AsE}_{(p_E^2>>M^2)}(1-\mathrm{T})\circ$  $+ \int dp\,\mathrm{T}\circ$ 

- My example $\Rightarrow \int dp\mathbf{AsE}\circ + \int dp\mathbf{AsE}(-\mathrm{T})\circ + + \int dp\mathrm{T}\circ$:

$$+\left(\frac{1}{\varepsilon}\right)^3(-14+8\xi)+\left(\frac{1}{\varepsilon}\right)^2\left(-\frac{98}{3}+\frac{80}{3}\xi\right)$$
$$+\left(\frac{1}{\varepsilon}\right)\left(-\frac{10}{3}-81S_2+\frac{164}{3}\xi-39\zeta_2+12\zeta_2\xi\right)$$
$$+\left(\frac{1}{\varepsilon}\right)^3(+12-8\xi)+\left(\frac{1}{\varepsilon}\right)^2\left(+\frac{93}{2}-\frac{51}{2}\xi\right)$$
$$+\left(\frac{1}{\varepsilon}\right)\left(+\frac{5069}{36}-175S_2-\frac{3625}{54}\xi+\frac{160}{3}\xi S_2+36\zeta_2-12\zeta_2\xi\right)$$
$$+\left(\frac{1}{\varepsilon}\right)^3(-12+8\xi)+\left(\frac{1}{\varepsilon}\right)^2\left(-\frac{93}{2}+\frac{51}{2}\xi\right)$$
$$+\left(\frac{1}{\varepsilon}\right)\left(-\frac{5177}{36}+94S_2+\frac{3625}{54}\xi-\frac{160}{3}\xi S_2-18\zeta_2+12\zeta_2\xi\right),$$

ξ-gauge parameter.

To compare with known result for the tadpole G:

$$+\left(\frac{1}{\varepsilon}\right)^3(-14+8\xi)+\left(\frac{1}{\varepsilon}\right)^2\left(-\frac{98}{3}+\frac{80}{3}\xi\right)$$
$$+\left(\frac{1}{\varepsilon}\right)\left(-\frac{19}{3}-162S_2+\frac{164}{3}\xi-21\zeta_2+12\zeta_2\xi\right)+const.\,term$$

Outline

Using this technique

- obtain with QGRAF whole set of needed 4-loop diags to calculate γ_O
- calculate UV-poles of this set using R-operation:
  - Pearl program to create an proper input for EXP for parts with just AsE, with FOREST and mixed FOREST,
  - to run EXP and MATAD with all pieces.
  - In MATAD one must take over scales control to do R-operation.
- Extract 4-loop γ_O anomalous dimension

- R-operation : number if diags is increased by many times.
- Nevertheless: since practically problem is now at least 1 loop less, CPU time for one diagram is decreased
- No problem to send big number $\mathcal{O}(10\,000)$ of diags to computer cluster and to use parallel versions of FORM (no licenses needed!!!): TFORM, ParFORM Vermaseren, Tentyukov, Vollinga

Summary

- results on 4-loop anomalous dimensions are experimentally important: **DIS** analysis
- to get 4-loop anomalous dimensions is enough to calculate UV part of 4-loop tadpoles
- Using the *Asymptotic Expansion* and the R-operation it is possible to reduce initial task by at least 1 loop
- At the moment we have program that computes some 3-loop diagrams with scalar, spinor and gluon propagators.
  This is a prototype of future **ZFOUR** program
- Far future **- ZFIVE** program for 5-loop anom. dim. (should be theoretically possible)