Threshold resummation for top-quark production

(and other coloured particles)

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(Based on M.Beneke, P.Falgari, CS, arXiv:0907.1443 [hep-ph], arXiv:1007.5414 [hep-ph]

M.Beneke, M.Czakon, P.Falgari, A.Mitov, CS arXiv:0911.5166 [hep-ph]

M.Beneke, P.Falgari, S. Klein, CS, in progress)

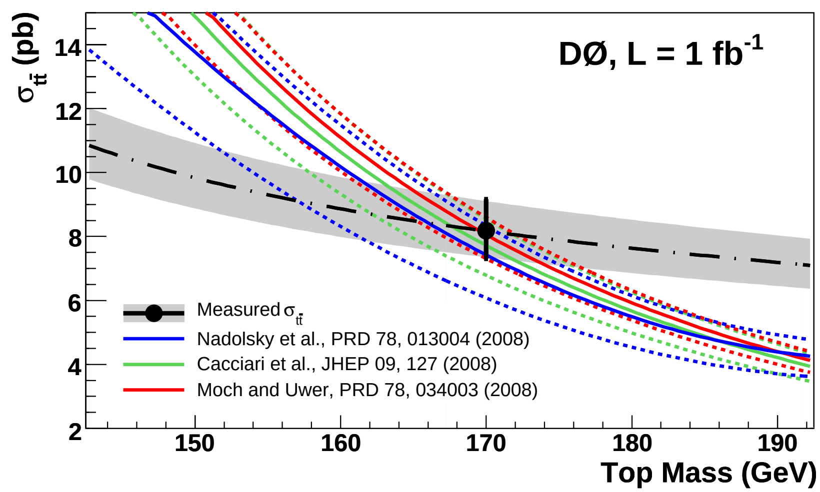
Pair production of heavy coloured particles at Tevatron/LHC

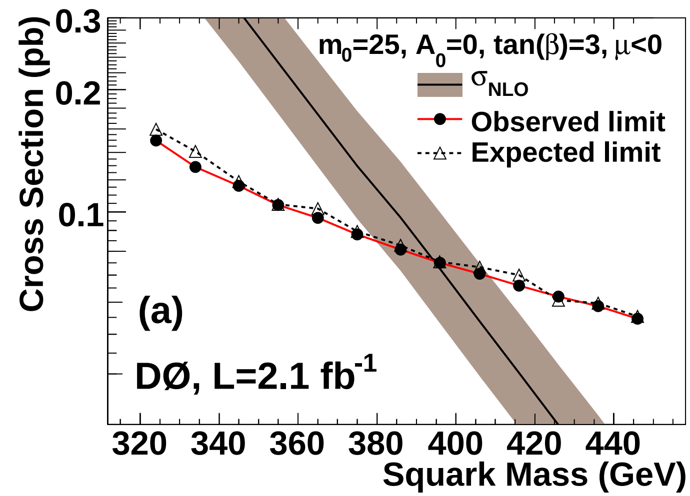
$$NN' \to HH' + X$$

- N, N': pp, $p\bar{p}$; HH': top-quark, squark, gluino... pairs

Precise knowledge of total cross sections:

- top-quarks: sensitivity on mass, constraining gluon PDFs
- new particles: Exclusion bounds, model discrimination,...





Experimental knowledge of $t\bar{t}$ cross section:

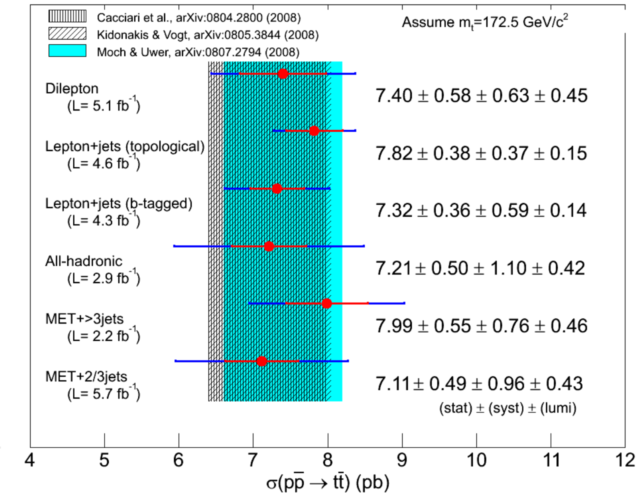
Tevatron: $\Delta\sigma_{t\bar{t}} = 6.8\%$;

LHC Goal: $\Delta\sigma_{t\bar{t}} \approx 5\%$

Theory status:

NLO + higher-order soft gluons

$\Rightarrow \Delta\sigma_{t\bar{t}} \approx 10\%$



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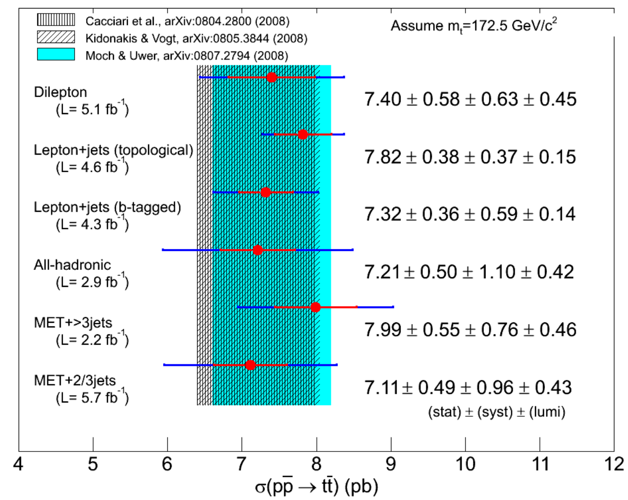

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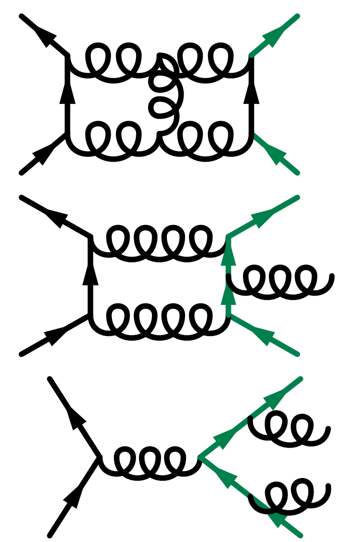
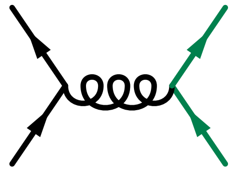
Building blocks for NNLO:



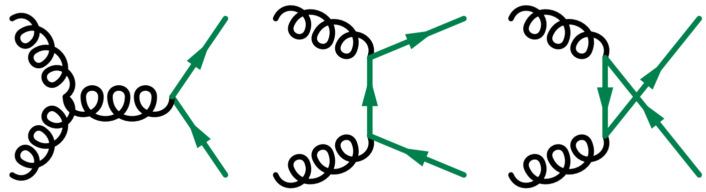
- two-loop $t\bar{t}$, ($m_t \to 0$: Czakon/Mitov/Moch 07;
  $q\bar{q}$: Czakon 08; Bonciani et.al. 08/09)
- one-loop $t\bar{t} + j$ (Dittmaier/Uwer/Weinzierl 07)
  $t\bar{t}$ squared
  (Körner et.al. 05-09, Anastasiou/Mert-Aybert 08)
- tree $t\bar{t} + jj$ (IR subtraction: Czakon 10)

Top-pair production: two LO subprocesses:

$$q\bar{q} \to t\bar{t}: \begin{cases} 90\% & \text{Tevatron} \\ 20-10\% & \text{LHC7}-14 \end{cases}$$



$$gg \to t\bar{t}: \begin{cases} 10\% & \text{Tevatron} \\ 80-90\% & \text{LHC7}-14 \end{cases}$$



Behaviour at production threshold $\hat{s} \sim 4m_t^2$: $\left(\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}\right)$

- $q\bar{q}$ channel: colour octet, spin triplet

$$\hat{\sigma}^{(8)}_{q\bar{q}} = \frac{\pi\beta}{9m_t^2}\left[1 + \frac{\alpha_s}{4\pi}\left(\frac{-2\pi^2}{2N_c}\frac{1}{\beta} + 8C_F\log^2 8\beta^2 - (32C_F + 4N_C)\log 8\beta^2\right) + \dots\right]$$

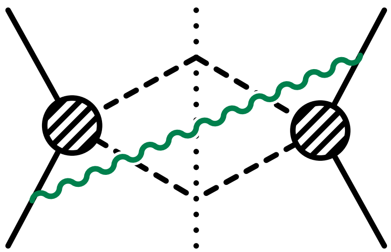
- gg channel: colour singlet/octet, spin singlet

$$\hat{\sigma}^{(1)}_{gg} = \frac{5\pi\beta}{192m_t^2}\left[1 + \frac{\alpha_s}{4\pi}\left(2C_F\pi^2\frac{1}{\beta} + 8N_C\log^2 8\beta^2 - 32N_C\log 8\beta^2\right) + \dots\right]$$

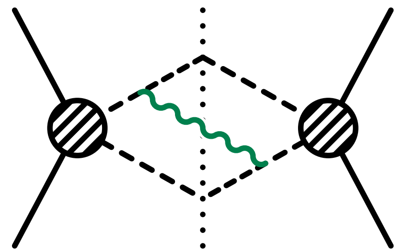
$$\hat{\sigma}^{(8)}_{gg} = \frac{\pi\beta}{96m_t^2}\left[1 + \frac{\alpha_s}{4\pi}\left(\frac{-2\pi^2}{2N_C}\frac{1}{\beta} + 8N_C\log^2 8\beta^2 - (32N_C + 4N_C)\log 8\beta^2\right) + \dots\right]$$

$\Rightarrow$ Universal behaviour depending on initial/final colour states

Soft corrections: (Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, ...)



$\Rightarrow \alpha_s \ \log^2(8\beta^2)$



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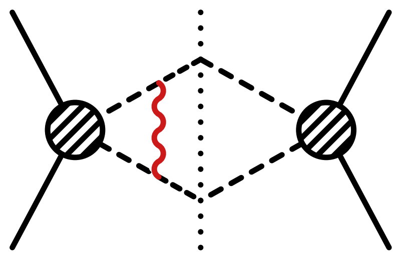
Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)



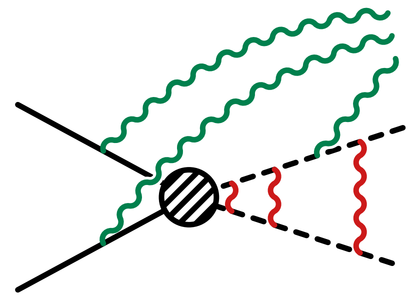
$\Rightarrow \alpha_s \ \frac{1}{\beta}$

Counting of threshold corrections:

$$\hat{\sigma}_{pp'} \quad \propto \quad \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \dots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \left\{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \dots\right\} :$$

Combination of Coulomb- and soft effects?



Heavy particles nonrelativistic near threshold:

$$E \sim m\beta^2 \,, \qquad |\vec{p}| \sim m\beta$$

soft gluon momenta of same order: $q_s \sim m\beta^2 \sim E$

$\Rightarrow$ heavy particles "feel" soft radiation

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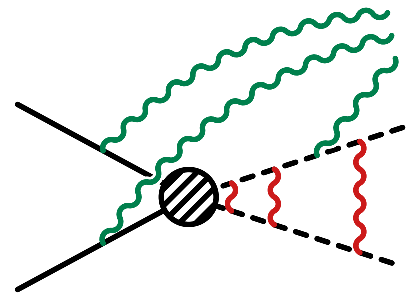
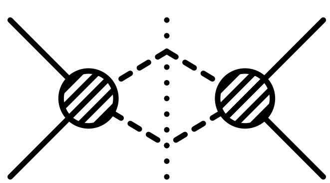
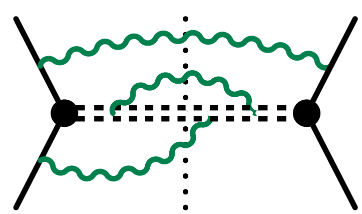
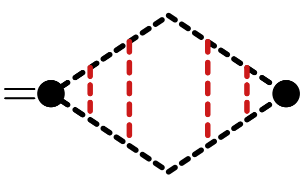


Factorization of cross section (Beneke, Falgari, CS 09/10)

$$\hat{\sigma}_{pp' \to HH'}|_{\hat{s}\to 4M^2} = \sum_{R,i} H_i W_i^R \otimes J^R$$

Hard, soft and Coulomb functions:

$H_i =$  , $W_i{}^R =$  , $J^R =$ 

Soft radiation "sees" only total colour charge R of heavy particles

(Singlet, octet,...)

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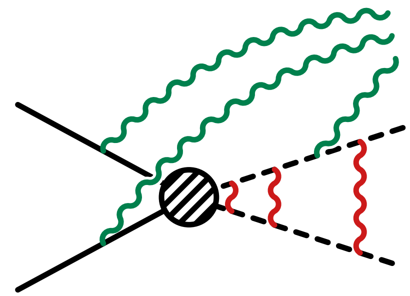

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$$\hat{\sigma}_{pp' \to HH'}\big|_{\hat{s} \to 4M^2} = \sum_{R,i} H_i W_i^R \otimes J^R$$

- disentangles hard, soft and Coulomb contribution
  (for S-wave production and up to NNLL)
- can perform simultaneous summation of threshold Logs and Coulomb corrections

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH_i}{d\mu} \Rightarrow$ related to IR singlarities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

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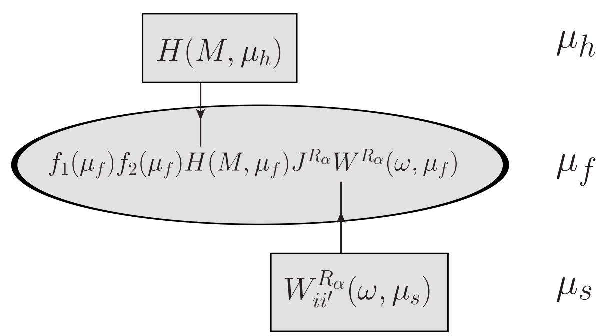
Resummation:

- evolve hard function from $\mu_h \sim 4m_t$ to μ_f
- evolve soft function from μ_s to μ_f (Mellin space: Korchemsky/Marchesini 92 momentum space: Becher/Neubert 06)



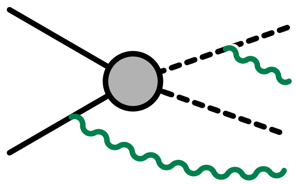
- (N)LO Coulomb-Green function (Fadin/Khoze 87; Beneke/Signer/Smirnov 99,. . .)

All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms (Beneke, Czakon, Falgari, Mitov, CS 09

Implemented in HATHOR, Aliev et.al. 10)

Pure soft corrections: (also Moch/Uwer+Langenfeld (08/09))

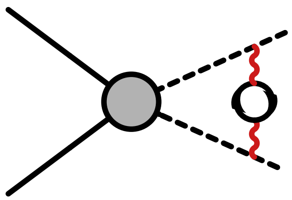
$$\Delta\sigma_{\mathsf{s}}^{(2)} \sim \alpha_s^2 (c_{\mathsf{LL}}^{(2)} \ln^4\beta + c_{\mathsf{NLL}}^{(2)} \ln^3\beta + c_{\mathsf{NNLL,2}}^{(2)} \ln^2\beta + \underbrace{c_{\mathsf{NNLL,1}}^{(2)} \ln\beta}_{\text{2-loop } \gamma_{H,s}})$$



Potential corrections: 2nd Coulomb, NLO potentials

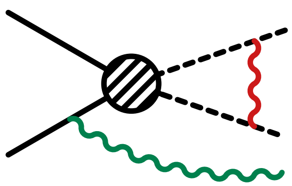
$$\Delta\sigma_{\mathsf{p}}^{(2)} \sim \alpha_s^2 \left(\frac{c_{C^2}}{\beta^2} + \frac{1}{\beta}(c_{\mathsf{C,0}}^{(2)} + c_{\mathsf{C,1}}^{(2)} \log\beta) + \underbrace{c_{\mathsf{n\text{-}C}}^{(2)} \ln\beta}_{\text{spin-dependent}}\right)$$

(using Beneke, Signer, Smirnov 99, Czarnecki/Melnikov 97/01)



mixed Coulomb/soft, hard corrections:

$$\Delta\sigma_{\mathsf{p}\otimes\,\mathsf{sh}}^{(2)} \sim\sim \frac{\alpha_s}{\beta}\alpha_s (c_{\mathsf{LL}}^{(1)} \ln\beta^2 + c_{\mathsf{NLL}}^{(1)} \ln\beta + c + \underbrace{H^{(1)}}_{\text{process dependent}})$$



Total top-pair production cross-section

$\sigma_{t\bar{t}}$(pb)	Tevatron	LHC7	LHC10	LHC14
NLO	$6.50^{+0.32+0.33}_{-0.70-0.24}$	150^{+18+8}_{-19-8}	380^{+44+17}_{-46-17}	842^{+97+30}_{-97-32}
NLO+NLL	$6.57^{+0.52+0.33}_{-0.30-0.24}$	151^{+23+8}_{-12-9}	382^{+60+17}_{-32-18}	$848^{+136+30}_{-75-32}$
NLO+NNLL	$6.77^{+0.27+0.35}_{-0.48-0.25}$	155^{+4+8}_{-9-9}	390^{+14+17}_{-26-18}	858^{+35+31}_{-64-33}
NNLO$_{\rm app}(\beta)$	$7.10^{+0.0+0.36}_{-0.26,-0.26}$	162^{+2+9}_{-3-9}	407^{+9+17}_{-5-18}	895^{+24+31}_{-6-33}
NNLO$_{\rm app}(\beta)$ + NNLL	$7.13^{+0.22+0.36}_{-0.24-0.26}$	162^{+4+9}_{-1-9}	405^{+14+17}_{-2-18}	892^{+38+31}_{-3-33}
NNLO$_{\rm app}(\beta)$ + NNLL+BS	$7.14^{+0.14+0.36}_{-0.22-0.26}$	162^{+4+9}_{-1-9}	407^{+14+17}_{-2-18}	896^{+38+31}_{-3-33}

($m_t = 173.1$ GeV, $\tilde{\mu}_f = mt$, MSTW08NNLO) (Beneke, Falgari, Klein, CS preliminary)

- Resummation in momentum space using fixed μ_s from minimising $\Delta\sigma^{\rm NLO}_{\rm soft}(\mu_s)$
  $\Rightarrow \tilde{\mu}_s = 85/146$ GeV for Tevatron/LHC7: no big scale hierarchy
- vary μ_s, μ_h, μ_f from $0.5\tilde{\mu} < \mu < 2\tilde{\mu}$, add uncertainties in quadrature
- (N)NLL includes (N)LO Coulomb resummation
- BS: include bound-state contributions below threshold
- Preliminary estimate of uncertainty from $\alpha_s^2 C^{(2)}$ terms: $\sim 3\%$

Alternative threshold expansions

Pair invariant mass cross sections (Kidonakis,Sterman 97, Ahrens et.al. 10)

$$\frac{d\sigma(t\bar{t})}{dM_{t\bar{t}}} \Rightarrow \left[\frac{\log^n(1-z)}{1-z}\right]_+ , \; z = \frac{M_{t\bar{t}}^2}{\hat{s}}$$

One particle inclusive cross sections: (Laenen, Oderda, Sterman 98)

$$\frac{d\sigma(t+X)}{ds_4} \Rightarrow \left[\frac{\log^n(s_4/m^2)}{s_4}\right] , \; s_4 = p_X^2 - m_t^2$$

$\sigma_{t\bar{t}}$(pb)	Tevatron	LHC7	LHC10	LHC14
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NNLO$_{\mathrm{app}}(\beta)$	$7.10^{+0.0+0.36}_{-0.26,-0.26}$	162^{+2+9}_{-3-9}	407^{+9+17}_{-5-18}	895^{+24+31}_{-6-33}
NLO + NNLL ($M_{t\bar{t}}$) (Ahrens et.al. 10)	$6.48^{+0.17+0.32}_{-0.21-0.25}$	146^{+7+8}_{-7-8}	368^{+20+19}_{-14-15}	813^{+50+30}_{-36-35}
NNLO$_{\mathrm{app}}(s_4)$ (mt=173; Kidonakis 10)	$7.08^{+0.00+0.36}_{-0.24-0.27}$	163^{+7+9}_{-5-9}	415^{+17+18}_{-21-19}	920^{+50+33}_{-39-35}

($m_t = 173.1$ GeV, $\mu_f = mt$, MSTW08NNLO)

Squark -antisquarks at LHC

- Two production channels:

$$q_i \bar{q}_j \to \tilde{q}_k \overline{\tilde{q}_l} \quad , \qquad gg \to \tilde{q}_k \overline{\tilde{q}_l}$$

- Simplified setup: equal squark masses, no stop
- Matching to NLO result (Beenakker et.al. 96, PROSPINO)

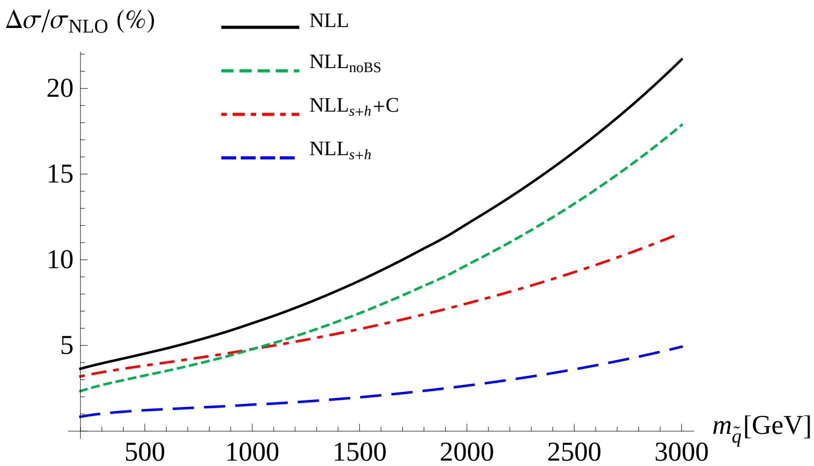
Resumed Results:

NLL: full Coulomb $\otimes$ res. soft

noBS:
NLL without bound states

NLL$_{s+h}$:
resummation of H and W

C: Coulomb resummation



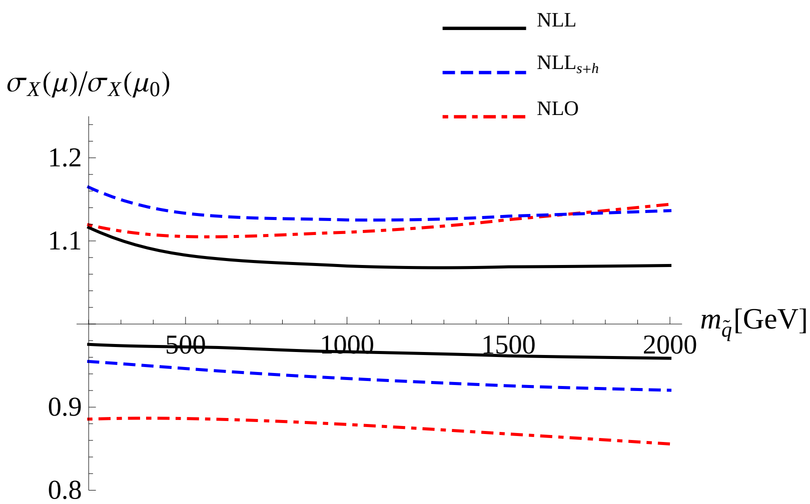
($\sqrt{s} = 14$ TeV, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$MSTW08NLO)

Scale uncertainty reduced by combined resummation

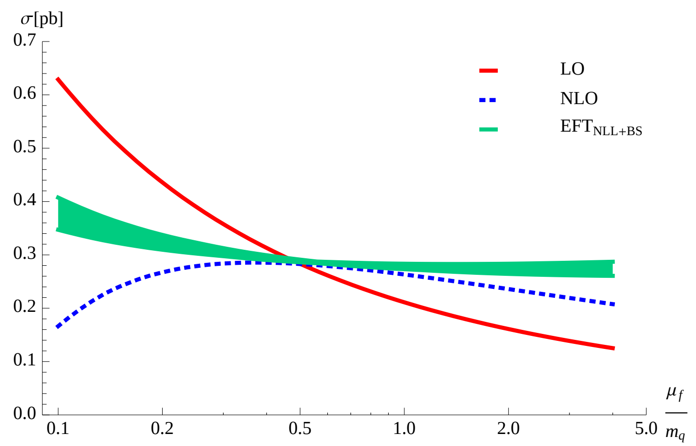
NLO $\frac{m_{\tilde{q}}}{2} < \mu_f < m_{\tilde{q}}$

NLL: vary all scales $\frac{\tilde{\mu}_i}{2} < \mu_i < 2\tilde{\mu}_i$, add in quadrature

$\Rightarrow$ significant reduction for combined resummation!



($\sqrt{s} = 14$ TeV, MSTW08NLO, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$)



($m_{\tilde{q}} = 1$ TeV, $\mu_s^0/2 < \mu_s < 2\mu_s^0$)

Threshold corrections $\sim \log^n \beta$, $\frac{1}{\beta^n}$

- Factorization of soft and Coulomb corrections
- $\log \beta$ resummation from momentum space solution to RGEs
- combined Soft and Coulomb resummation possible
- theoretical progress: now NNLL resummation feasible

Threshold expansion to $\mathcal{O}(\alpha_s^2)$ of $t\bar{t}$ cross section

NNLL resummation for $t\bar{t}$

- dominant higher-order corrections included in $\mathrm{NNLO}_{\mathrm{approx}}$
- discrepancy to NNLL from integrated $\frac{d\sigma}{dM_{tt}^2}$? (Ahrens et.al. 10)

Squark-antisquark production

- total corrections $4-10\%$ for $m_{\tilde{q}} = 300$ GeV-2 TeV
- reduced μ_f-dependence for combined soft/gluon resummation

Hadron collider cross sections from QCD factorization

(Collins, Soper, Sterman)

$$\sigma_{NN'}(s) = \sum_{pp'} \int dx_1 dx_2 \;\; f_{N/p}(x_1,\mu_f) f_{N'/p'}(x_2,\mu_f)\, \hat\sigma_{pp'}(s x_1 x_2, \mu_f)$$

- $\hat\sigma_{pp'}$: partonic cross section: compute in perturbation theory
- $f_{p/N}(x)$: Parton distribution function for parton p in hadron N : fitted to experiment

PDF uncertainties for top: (e.g. Guffanti/Rojo arXiv:1008.4671 [hep-ph])

	CTEQ6.6	MSTW2008	NNPDF2.0	ABKM09	HERAPDF1.0
$\sigma_{t\bar t}^{\text{NLO}}(7\text{TeV})[pb]$	147.7 ± 6.4	159.0 ± 4.7	160.0 ± 5.9	131.9 ± 4.8	136.4 ± 4.7

- Different α_s values
- Differences in gluon pdf at large x (impact of Tevatron jet-data)

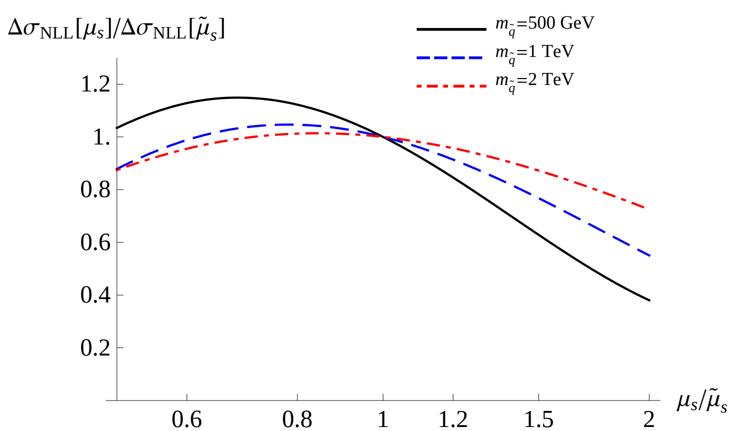
Choice of scales for resummation in momentum space

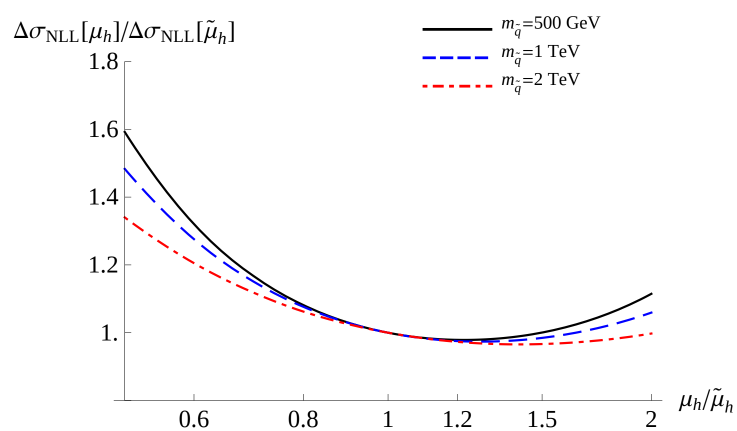
Soft scale $\tilde{\mu}_s$ that minimizes hadronic $\Delta\sigma^{\text{NLO}}_{\text{soft}}$ (Becher, Neubert, Xu 07)

$$\tilde{\mu}_s/m_{\tilde{q}} \approx 0.5\ldots 0.2 \qquad \text{for } m_{\tilde{q}} = 0.5,\ldots 2 \text{ TeV}$$

Hard scale: $\tilde{\mu}_h = 2m_{\tilde{q}}$

Dependence on scale choices:





($\sqrt{s} = 14$ TeV, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$)

Coulomb scale: $\mu_C = \max\{2m_{\tilde{q}}\beta, C_F m_{\tilde{q}}\alpha_s(\mu_C)\}$

Comparison to Mellin-approach: (Kulesza, Motyka 08/09, Beenakker et.al. 09)

Good agreement for appropriate choice of scales ($\mu_h = \mu_f$: NLL$_s$):

$m_{\tilde{q}}$[GeV]	NLO[pb]	NLL$_{\text{Mellin}}$[pb]	NLL$_s$ [pb]	NLL [pb]
200	1.3×10^3	1.31×10^3 (1%)	1.31×10^3 (1%)	1.34×10^3 (3.4%)
500	1.6×10^1	1.61×10^1 (1.2%)	1.62×10^1 (1.3%)	1.67×10^1 (4.2%)
1000	2.89×10^{-1}	2.93×10^{-1}(1.7%)	2.94×10^{-1}(1.7%)	3.06×10^{-1}(5.8%)
2000	1.11×10^{-3}	1.14×10^{-3}(3.4%)	1.14×10^{-3}(3.1%)	1.24×10^{-3} (11%)
3000	7.13×10^{-6}	7.59×10^{-6}(6.4)%	7.54×10^{-6}(5.8%)	8.61×10^{-6} (21)%

(LHC 14 TeV, $m_{\tilde{g}} = m_{\tilde{q}}$)

Potential corrections:

- 2nd Coulomb correction
- NLO Coulomb potentials:

$$\tilde{V}_{\mathrm{C}}^{(1)}(\boldsymbol{p},\boldsymbol{q}) = \frac{D_{R_\alpha}\alpha_s^2}{\boldsymbol{q}^2}\left(a_1 - \beta_0 \ln\frac{\boldsymbol{q}^2}{\mu^2}\right)$$

- Non-Coulomb potential:

$$\tilde{V}_{\mathrm{nC}}^{(1)}(\boldsymbol{p},\boldsymbol{q}) = \frac{4\pi D_{R_\alpha}\alpha_s}{\boldsymbol{q}^2}\left[\frac{\pi\alpha_s|\boldsymbol{q}|}{4m}\left(\frac{D_{R_\alpha}}{2}+C_A\right)+\frac{\boldsymbol{p}^2}{m^2}+\frac{\boldsymbol{q}^2}{m^2}\,v_{\mathrm{spin}}\right],$$

($v_{\mathrm{spin}} = 0$ (singlet); $-2/3$ (triplet))

Corrections to cross section:

$$\Delta\hat{\sigma}_{\mathrm{nC}} = \hat{\sigma}^{(0)}\,\alpha_s^2\,\ln\beta\,\left[-2D_{R_\alpha}^2\,(1+v_{\mathrm{spin}})+D_{R_\alpha}C_A\right]$$

(extracted from Beneke, Signer, Smirnov 99, Pineda, Signer 06)