Applications of heavy-to-light currents at NNLO in SCET

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PHYSICS AT THE TERA SCALE
Helmholtz Alliance

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Based on NPB 843 (2011), 143 [1007.3758]

Annual Helmholtz Alliance Meeting, Dresden, December 2nd, 2010

Outline

- Introduction and Motivation
- Matching heavy-to-light currents from QCD onto SCET at NNLO
- Some details on the two-loop computation
- Results and applications
  - Results for the matching coeffcients
  - Heavy-to-light form factor ratios
  - Exclusive radiative decays
  - Semi-inclusive $\bar{B} \to X_s \ell^+ \ell^-$
  - Implications on inclusive V_{ub}

Introduction and motivation

- Heavy-to-light currents

$$\bar{q}\,\Gamma_i\,b \qquad \text{with} \qquad \Gamma_i = \{1,\, \gamma_5,\, \gamma^\mu,\, \gamma_5\gamma^\mu,\, i\sigma^{\mu\nu}\}$$

  govern many semi-leptonic and radiative B decays
  - $\bar{B} \to X_u\,\ell\,\nu$
  - Exclusive radiative decays
  - Semi-inclusive $\bar{B} \to X_s \ell^+ \ell^-$
  - Matrix elements of heavy-to-light currents (transition form factors) are inputs to factorization formulae in non-leptonic B decays *[Beneke,Buchalla,Neubert,Sachrajda'99,'00]*
- Experimental cuts (to eliminate backgrounds) put us in kinematic region where the hadronic final state has large energy ($E \sim m_b$) but small invariant mass ($m_X \ll m_b$)
- Appropriate Theoretical framework: SCET *[Bauer et. al.'00,'01; Beneke et. al.'02]*
- Many of these decays require precision beyond NLO
- Goal: Two-loop $\mathcal{O}(\alpha_s^2)$ matching coefficients for heavy-to-light currents from QCD onto SCET

Matching QCD onto SCET I

- Generic heavy-to-light current $\bar{q}\,\Gamma_i\,b$ in SCET

$$[\bar{q}\,\Gamma_i\,b](0) = \sum_j \int ds\, \tilde{C}_i^j(s)\, \left[\bar{\xi} W_{hc}\right](s n_+)\, \Gamma'_j\, h_v(0) + \underset{(1/m_b \text{ suppressed})}{\text{“three} - \text{body operators“}} + \ldots$$

- Adopt momentum space representation for matching coefficients C_i^j

$$C_i^j(n_+ p) = \int ds\, e^{i s n_+ p}\, \tilde{C}_i^j(s).$$

- Introduce $n_\pm^\mu = (1,0,0,\pm 1)$ and $v^\mu = (n_+^\mu + n_-^\mu)/2 = (1,0,0,0)$. Have $n_\pm^2 = 0$, $(n_+\, n_-) = 2$

- Momentum decomposition in SCET: $a^\mu = (n_+ \cdot a)\,\frac{n_-^\mu}{2} + (n_- \cdot a)\,\frac{n_+^\mu}{2} + a_\perp^\mu$

Γ_i	1	γ_5	γ^μ			$\gamma_5\gamma^\mu$			$i\sigma^{\mu\nu}$			
Γ'_j	1	γ_5	γ^μ	v^μ	n_-^μ	$\gamma_5\gamma^\mu$	$v^\mu\gamma_5$	$n_-^\mu\gamma_5$	$\gamma^{[\mu}\gamma^{\nu]}$	$v^{[\mu}\gamma^{\nu]}$	$n_-^{[\mu}\gamma^{\nu]}$	$n_-^{[\mu}v^{\nu]}$
C_i^j	C_S	C_P	C_V^1	C_V^2	C_V^3	C_A^1	C_A^2	C_A^3	C_T^1	C_T^2	C_T^3	C_T^4

- Constraints: $C_P = C_S$, $C_A^i = C_V^i$ (in NDR scheme w/ anti-commuting γ_5)

- Moreover, $C_T^2 = C_T^4 = 0$ since pseudo-tensor current is reducible in four dimensions

Matching QCD onto SCET II

- Perform matching with on-shell quarks
- Intermediate step: parameterize QCD result in terms of 12 form factors F_i^j

$$\langle q(p)|\,\bar{q}\,\Gamma_i\,b\,|b(p_b)\rangle = \sum_j\, F_i^j(q^2)\,\bar{u}(p)\,\Gamma'_j\,u(p_b)$$

- Momenta:

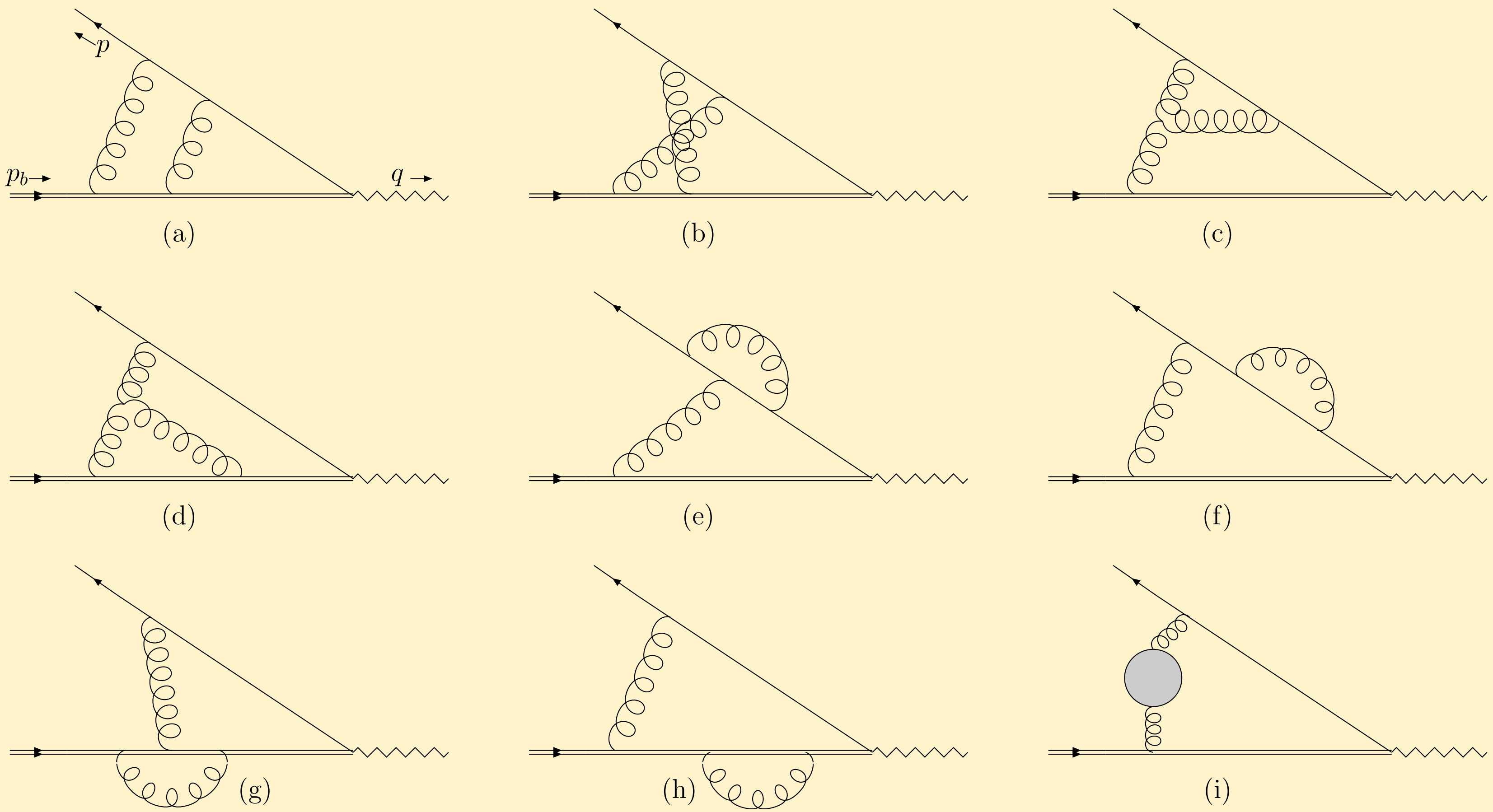
$$\begin{aligned} p_b &= m_b\,v \\ p &= \frac{u\,m_b\,n_-}{2} \\ q^2 &= (p_b - p)^2 = (1-u)\,m_b^2 \end{aligned}$$

- The form factors F_i^j are UV finite, but IR divergent
- Obtain C_i^j via

$$C_i^j = Z_J^{-1}\,F_i^j$$

- Z_J is the renormalization factor of the SCET current $\left[\bar{\xi}W_{hc}\right]\Gamma'_j h_v$. It is universal! It subtracts the IR divergences and yields finite C_i^j.

Two-loop diagrams



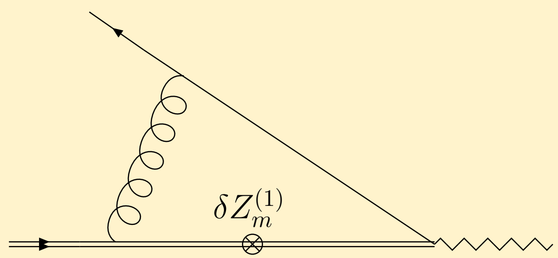
- Work with $n_l = 4$ massless and one massive flavour (m_b)
- Charm mass can also be implemented (see plots later on)

UV renormalization

- Bare two-loop result contains UV and IR (soft and collinear) divergences

- UV conterterms: Apply on-shell scheme for the heavy mass as well as for the heavy (h) and light (l) quark field
  - Z_m and Z_h are known *[Broadhurst,Gray,Grafe,Schilcher'90,'91; Melnikov,van Ritbergen'00]*
  - Z_l only starts at two loops.
    Derive all-order representation: $Z_l^{\mathrm{os}} = 1 + 2\,C_F\,t_f\,\frac{g_0^4 (m^2)^{D-4}}{(4\pi)^D}\,\frac{(D-1)\Gamma(4-\frac{D}{2})\Gamma(-\frac{D}{2})}{(D-5)(D-7)}$
  - Renormalize α_s in the $\overline{\mathrm{MS}}$ scheme
  - Non-vanishing anomalous dimension of scalar and tensor current:
    Additional counterterms Z_S and Z_T *[Nanopoulos,Ross'79; Tarrach'81; Broadhurst,Grozin'94]*

- All UV renormalizations are simple multiplications except the one-loop mass counterterm



Completing the matching

- The form factors F_i^j are UV finite, but IR divergent
- Obtain C_i^j via $\quad C_i^j = Z_J^{-1}\, F_i^j$
- Perurbative expansion

$$F_i^j \;=\; \sum_{k=0}^{\infty}\left(\frac{\alpha_s^{(5)}}{4\pi}\right)^k F_i^{j,(k)}\,, \qquad Z_J = 1 + \sum_{k=1}^{\infty}\left(\frac{\alpha_s^{(4)}}{4\pi}\right)^k Z_J^{(k)}\,, \qquad C_i^j = \sum_{k=0}^{\infty}\left(\frac{\alpha_s^{(4)}}{4\pi}\right)^k C_i^{j,(k)}$$

- Need D-dim. relation between four– and five – flavour α_s

$$\alpha_s^{(5)} \;=\; \alpha_s^{(4)}\left[1 + \frac{\alpha_s^{(4)}}{4\pi}\,\delta\alpha_s^{(1)} + \mathcal{O}(\alpha_s^2)\right]$$

$$\delta\alpha_s^{(1)} \;=\; T_F\left[\frac{4}{3}\ln\frac{\mu^2}{m_b^2} + \left(\frac{2}{3}\ln^2\frac{\mu^2}{m_b^2} + \frac{\pi^2}{9}\right)\epsilon + \left(\frac{2}{9}\ln^3\frac{\mu^2}{m_b^2} + \frac{\pi^2}{9}\ln\frac{\mu^2}{m_b^2} - \frac{4}{9}\zeta_3\right)\epsilon^2 + \mathcal{O}(\epsilon^3)\right].$$

- Yields finite matching coefficients C_i^j

$$C_i^{j,(0)} \;=\; F_i^{j,(0)}$$

$$C_i^{j,(1)} \;=\; F_i^{j,(1)} - Z_J^{(1)} F_i^{j,(0)}$$

[Bauer et. al.'00,'01; Beneke,Feldmann'00; Beneke,Kiyo,Yang'04]

$$C_i^{j,(2)} \;=\; F_i^{j,(2)} + \delta\alpha_s^{(1)} F_i^{j,(1)} - Z_J^{(1)}\left(F_i^{j,(1)} - Z_J^{(1)} F_i^{j,(0)}\right) - Z_J^{(2)} F_i^{j,(0)}$$

Checks

- Vector FFs done by several groups [Bonciani,Ferroglia'08; Asatrian,Greub,Pecjak'08; Beneke,Li,TH'08; Bell'08]
- Matching coefficients obey renormalization group equation

$$\frac{d}{d\ln\mu}C_i^j(u;\mu) = \left[\Gamma_{\text{cusp}}(\alpha_s^{(4)})\ln\frac{um_b}{\mu} + \gamma'(\alpha_s^{(4)}) + \gamma_i(\alpha_s^{(5)})\right]C_i^j(u;\mu)$$

  - Γ_{cusp} and γ' are related to the SCET current. They are again universal.
  - γ_i contains the anomalous dimension of the QCD current

  $\Longrightarrow$ Can distinguish a scale μ (governs the RGE in SCET) from a scale ν (governs the RGE in QCD). Write from now on

$$C_i^j \equiv C_i^j(u;\,\mu,\,\nu)$$

- Scalar coefficient C_S is not independent (EOM) [Hill,Becher,Lee,Neubert'04; Bonciani,Ferroglia'08]

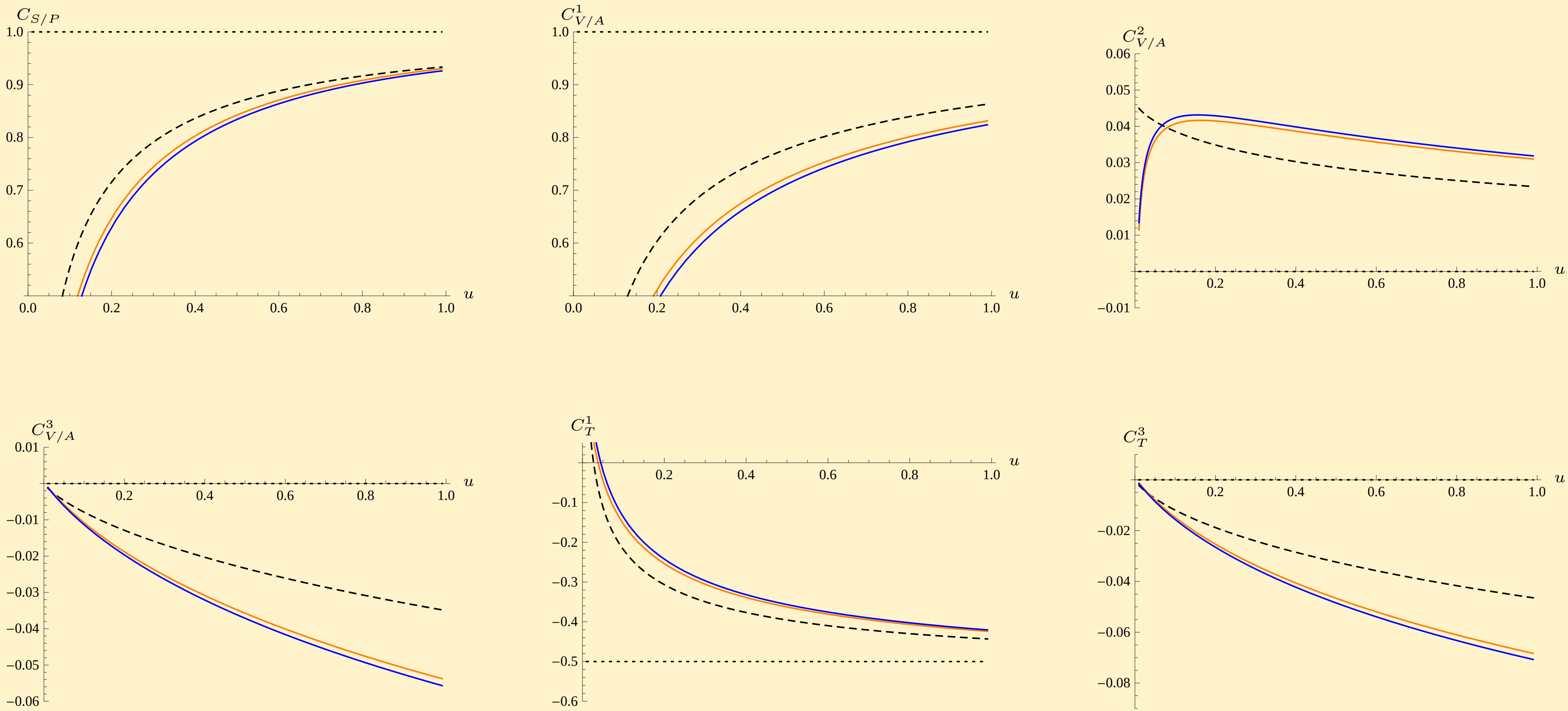
$$C_V^1(u;\mu) + \left(1-\frac{u}{2}\right)C_V^2(u;\mu) + C_V^3(u;\mu) = \frac{\overline{m}_b(\nu)}{m_b}C_S(u;\mu,\nu)$$

- Tensor coefficients in $u=1$ (or $q^2=0$) enter the $\bar{B}\to X_s\gamma$ process. Can check [Ali,Greub,Pecjak'07]

$$\begin{aligned} -2\,F_T^1(u=1) &+ \frac{1}{2}F_T^2(u=1) + F_T^3(u=1) \\ -2\,C_T^1(u=1;\mu,\nu) &+ C_T^3(u=1;\mu,\nu) \end{aligned}$$

Matching coefficients

- Matching coefficients as a function of u for $\mu = \nu = m_b$
- Dotted: Tree-level. Dashed: NLO.
- Orange: NNLO, $m_c = 0$. Blue: NNLO, $m_c = 0.3 m_b$.



Heavy-to-light form factor ratios

- Factorization formula for heavy-to-light form factors at large recoil

$$F_i^{B\to M}(E) = C_i(E)\,\xi_a(E) + \underbrace{\int_0^\infty \frac{d\omega}{\omega}\int_0^1 dv\; T_i(E;\ln\omega,v)\,\phi_{B+}(\omega)\,\phi_M(v)}_{\text{spectator scattering}}$$

[Beneke,Feldmann'00,'03]

- For $B\to P$, have three FFs $\{f_+,\ f_0,\ f_T\}$ and a single ξ_P
- For $B\to V$, have seven FFs $\{V,\ A_{0,1,2},\ T_{1,2,3}\}$ and two $\xi_{\perp,\parallel}$
- Five independent (A0-type) matching coefficients, now at NNLO. $(u = 2E/m_B)$

$$C_{f_+}^{(A0)} = C_V^1(u;\mu) + \tfrac{u}{2}C_V^2(u;\mu) + C_V^3(u;\mu)\,, \qquad C_V^{(A0)} = C_V^1(u;\mu)$$

$$C_{f_0}^{(A0)} = C_V^1(u;\mu) + \left(1-\tfrac{u}{2}\right)C_V^2(u;\mu) + C_V^3(u;\mu)\,, \quad C_{T_1}^{(A0)} = -2C_T^1(u;\mu,\nu) + C_T^3(u;\mu,\nu)$$

$$C_{f_T}^{(A0)} = -2C_T^1(u;\mu,\nu) - C_T^4(u;\mu,\nu)$$

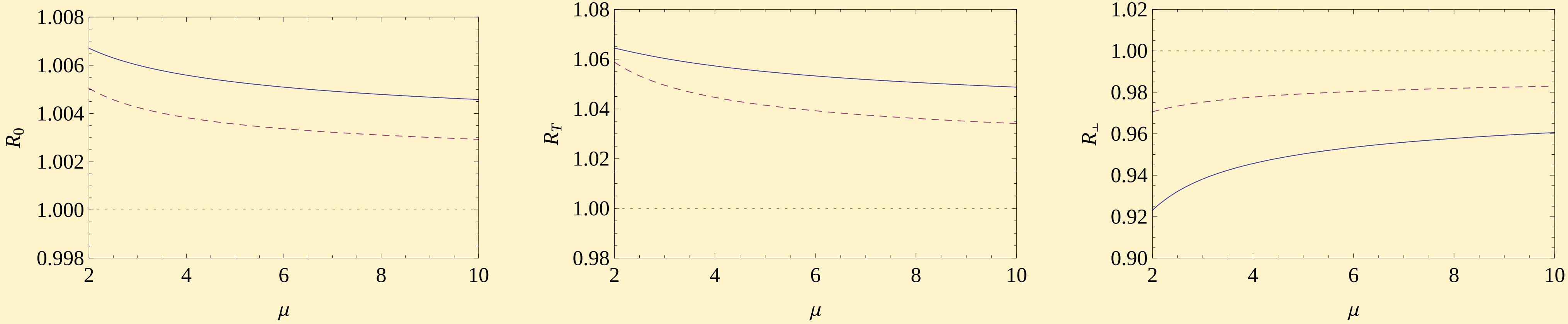
- In the physical form factor scheme, define to all orders in PT *[Beneke,Feldmann'00]*

$$\xi_P^{\rm FF} \equiv f_+, \qquad \xi_\perp^{\rm FF} \equiv \frac{m_B}{m_B+m_V}\,V, \qquad \xi_\parallel^{\rm FF} \equiv \frac{m_B+m_V}{2E}\,A_1 - \frac{m_B-m_V}{m_B}\,A_2$$

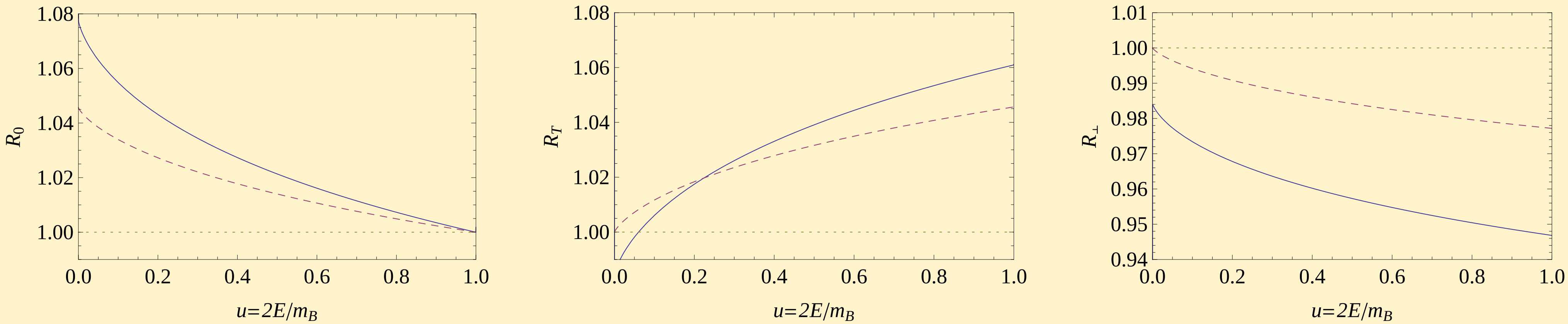
- In this scheme, only 3 non-trivial ratios: $R_0(u) \equiv \dfrac{C_{f_0}^{(A0)}}{C_{f_+}^{(A0)}}\,, \quad R_T(u) \equiv \dfrac{C_{f_T}^{(A0)}}{C_{f_+}^{(A0)}}\,, \quad R_\perp(u) \equiv \dfrac{C_{T_1}^{(A0)}}{C_V^{(A0)}}$

Heavy-to-light form factor ratios

- 3 indep. ratios: $R_0(u) \equiv C_{f_0}^{(A0)}/C_{f_+}^{(A0)}$, $R_T(u) \equiv C_{f_T}^{(A0)}/C_{f_+}^{(A0)}$, $R_\perp(u) \equiv C_{T_1}^{(A0)}/C_V^{(A0)}$
- $R_{0,T,\perp}$ as a function of μ for $u = 0.85$ and $\nu = m_b$



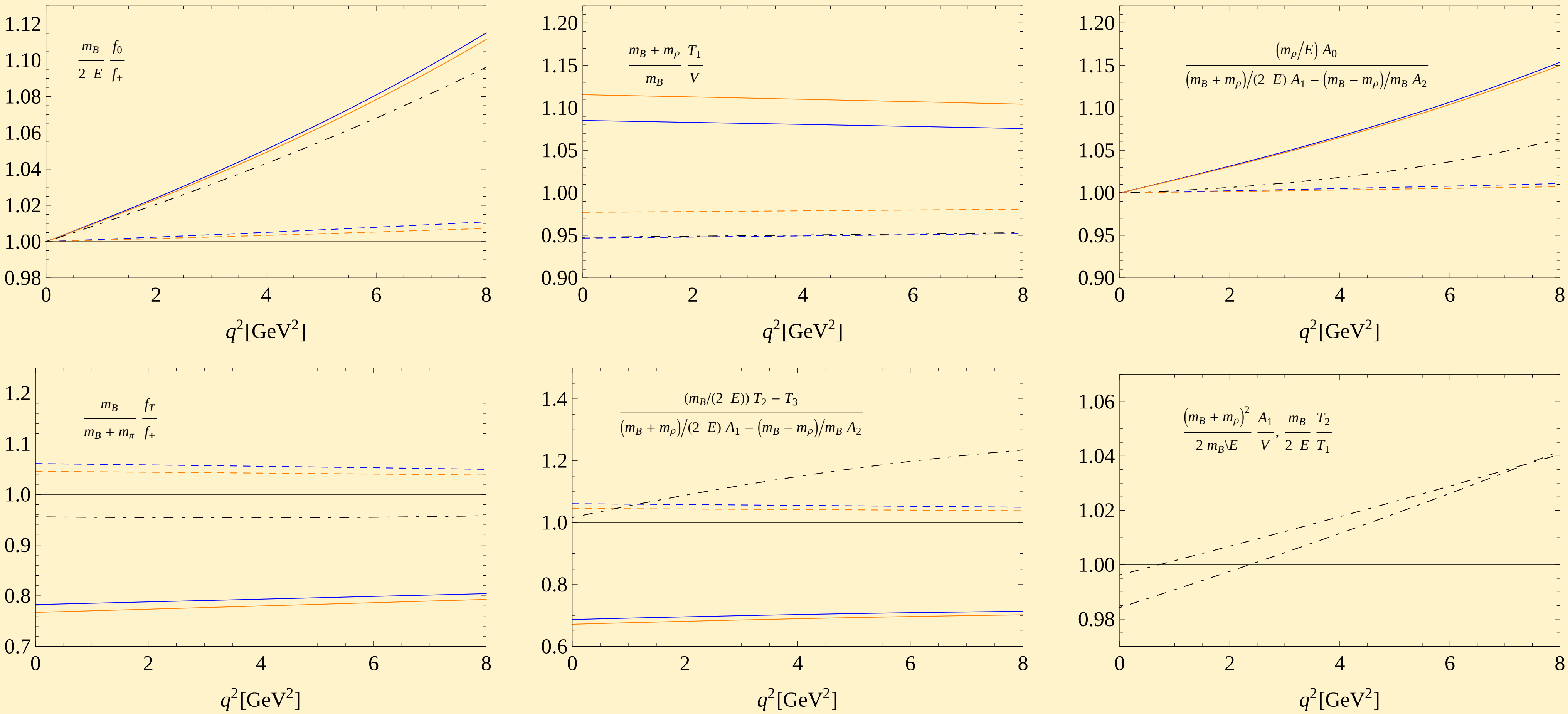
- $R_{0,T,\perp}$ as a function of u for $\mu = \nu = m_b$



Heavy-to-light form factor ratios

- Corrections to $B \to \pi$ and $B \to \rho$ form factor ratios as a function of q^2
  - Solid: Fulls result with R_X at NNLO (blue), and at NLO (orange)
  - Dashed: Same as above, but without spectator scattering
  - Dash-dotted: Result from QCD sum rules

[Ball,Zwicky'04]



- Radiative corrections to A0-coefficients smaller than impact of spectator scattering

Exclusive radiative decays

- Exclusive radiative decays make use of form factors at maximum recoil, i.e. $u = 1$, or $E = m_B/2$, or $q^2 = 0$.
- Consider the two ratios in the physical FF scheme

$$\begin{aligned}
\mathcal{R}_1(E) &\equiv \frac{m_B}{m_B + m_P}\frac{f_T(E)}{f_+(E)} = {\color{red}R_T(E)} + \int_0^1 d\tau\, C_{T+}^{(B1)}(\tau, E)\,\frac{\Xi_P(\tau, E)}{f_+(E)}, \\
\mathcal{R}_2(E) &\equiv \frac{m_B + m_V}{m_B}\frac{T_1(E)}{V(E)} = {\color{red}R_\perp(E)} + \frac{m_B + m_V}{m_B}\int_0^1 d\tau\, C_{T_1 V}^{(B1)}(\tau, E)\,\frac{\Xi_\perp(\tau, E)}{V(E)}
\end{aligned}$$

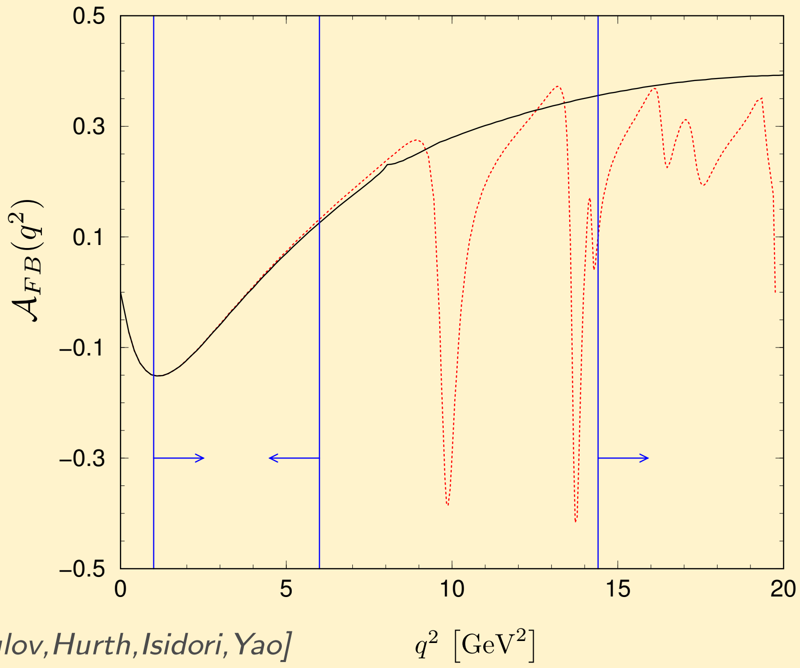
- Specifying to the π ($\mathcal{R}_1$) meson and the ρ ($\mathcal{R}_2$) meson, numerically have

$$\begin{aligned}
\mathcal{R}_1(E_{\max}) &= 1 + \Big[0.046\,(\text{NLO}) + 0.015\,(\text{NNLO})\Big]\,(R_T) \\
&\quad -0.160\left\{1 + 0.524\,(\text{NLO spec.}) - 0.002\,(\delta^{\|}_{\log})\right\} = 0.817, \\
\mathcal{R}_2(E_{\max}) &= 1 - \Big[0.023\,(\text{NLO}) + 0.030\,(\text{NNLO})\Big]\,(R_\perp) \\
&\quad +0.084\left\{1 + 0.406\,(\text{NLO spec.}) + 0.032\,(\delta^{\|}_{\log})\right\} = 1.067.
\end{aligned}$$

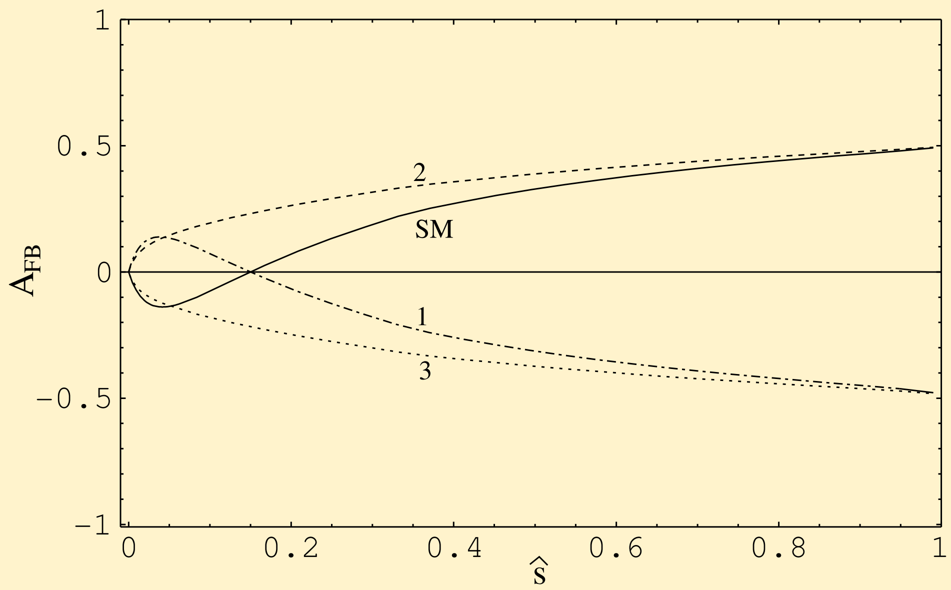
- A0-type and spectator scattering: Opposite sign, latter are larger
- Sum rule results: $\mathcal{R}_1 = 0.955$ and $\mathcal{R}_2 = 0.947$. *[Ball,Zwicky'04]*

Semi-inclusive $\bar{B} \to X_s\,\ell^+\ell^-$

- $\bar{B} \to X_s\,\ell^+\ell^-$ is a FCNC process, sensititve to NP, complementary to $\bar{B} \to X_s\,\gamma$
- Need cut on m_X to discriminate background from
  $b \;\to\; c\,\ell^-\bar{\nu}_\ell \to s\,\ell^+\ell^-\bar{\nu}_\ell\nu_\ell \;=\; b \;\to\; s\,\ell^+\ell^- + \not{E}$
- $m_X \leq m_X^{\rm cut} = 1.8\ldots2.0\,\text{GeV}$ and $1\,\text{GeV}^2 \leq q^2 \leq 6\,\text{GeV}^2 \quad\Rightarrow\quad$ "shape function region"
- Forward-backward asymmetry



[Ghinculov, Hurth, Isidori, Yao]



[Ali, Greub, Hiller, Lunghi, $\hat{s} \equiv q^2/m_b^2$]

Semi-inclusive $\bar{B} \to X_s\, \ell^+ \ell^-$

- In the shape function region and at leading power in $\Lambda_{\rm QCD}/m_b$, have

$$\mathrm{d}\Gamma^{[0]} = h^{[0]} \times J \otimes S$$

- $h^{[0]}$: process-dependent hard function. J, S: Universal jet- and shape-function
- For $h^{[0]}$, match first on two QCD currents with coefficients $C^{\rm incl}_{9/7}$

$$J_9^\mu = \bar{s}\,\gamma^\mu P_L b\,, \qquad J_7^\mu = \frac{2\,m_b}{q^2}\,\bar{s}\,iq_\rho\sigma^{\rho\mu}P_R b\Big|_{\nu=m_b}$$

- Then match QCD onto SCET

$$J_9^\mu = \sum_{i=1,2,3} c_i^9(u,\mu)\,[\bar{\xi}W_{hc}]\,\Gamma^\mu_{9,i}\,h_v\,, \qquad J_7^\mu = \frac{2m_b}{q^2}\sum_{i=1,2} c_i^7(u,\mu)\,[\bar{\xi}W_{hc}]\,\Gamma^\mu_{7,i}\,h_v$$

- Neede here: $c_1^9(u,\mu) = C_V^1(u;\mu)$ and $c_1^7(u,\mu) = -2\,C_T^1(u;\mu,\nu=m_b) + C_T^3(u;\mu,\nu=m_b)$
- Differential decay rate

$$\frac{d^3\Gamma}{dq^2dp_X^+d\cos\theta} = \frac{3}{8}\Big[(1{+}\cos^2\theta)H_T(q^2,p_X^+){+}2\,(1{-}\cos^2\theta)H_L(q^2,p_X^+){+}\,2\cos\theta\,H_A(q^2,p_X^+)\Big]$$

Semi-inclusive $\bar{B} \to X_s \ell^+\ell^-$

- Position of the FBA zero occurs at q_0^2 with

$$0 = \text{const} \times \int_0^{p_X^{+\text{cut}}} dp_X^+ \, h_A^{[0]}(q_0^2, p_X^+) \, \frac{(q_{0+} - q_{0-})^2}{q_{0+}} \, q_0^2 \int d\omega \, p^- J(p^-\omega) \, S(p_X^+ - \omega)$$

$$h_A^{[0]}(q^2, p_X^+) = 2\mathcal{C}_{10}\, c_1^9(u) \, \mathrm{Re}\left[C_9^{\text{incl}}(q^2) c_1^9(u) + \frac{2m_b}{q_-} C_7^{\text{incl}}(q^2) c_1^7(u) \right]$$

- $h_A^{[0]}(q_0^2, p_X^+)$ hardly varies with p_X^+. Pull in front of integral. Zero at $h_A^{[0]}(q_0^2, \langle p_X^+ \rangle) = 0$.
  Zero independent of J and S!

$$\frac{q_0^2}{2m_b(m_B - \langle p_X^+ \rangle)} = -\frac{\mathrm{Re}\,[C_7^{\text{incl}}(q_0^2)]}{\mathrm{Re}\,[C_9^{\text{incl}}(q_0^2)]} \, \underbrace{\frac{c_1^7(u_0)}{c_1^9(u_0)}}_{=R_\perp}$$

- For $m_X^{\text{cut}} = (2.0 \ldots 1.8)\,$GeV, have [pert. NLO impact is -2.2%, NNLO another -3%]

$$\begin{aligned}
q_0^2\Big|_{R_\perp = 1} &= (3.62 \,\ldots\, 3.69)\,\text{GeV}^2 \\
q_0^2\Big|_{R_\perp\ \text{NLO}} &= (3.55 \,\ldots\, 3.61)\,\text{GeV}^2 \\
q_0^2\Big|_{R_\perp\ \text{NNLO}} &= \left[(3.34 \,\ldots\, 3.40)\,{}^{+0.04}_{-0.13}\,{}_{\mu} \pm 0.08\,{}_{m_b}\,{}^{+0.05}_{-0.04}\,{}_{m_c} \pm 0.14\,{}_{\text{SF}} \pm 0.14\,{}_{\langle p_X^+ \rangle}\right]\text{GeV}^2 \\
&= \left[(3.34 \,\ldots\, 3.40)^{+0.22}_{-0.25}\right]\text{GeV}^2, \quad \text{includes} - 0.1\ \text{GeV}^2 \text{ as estimate from subleading SF}
\end{aligned}$$

[Lee, Tackmann'08]

Semi-leptonic $\bar{B} \to X_u \, \ell \, \nu_\ell$ decays and V_{ub}

- V_{ub} is an important parameter in quark flavour physics,
  - governs strength of $b \to u$ transition
  - determines the side of the unitarity triangle opposite to β

- Determination of $|V_{ub}|$ from inclusive vs. exclusive semi-leptonic $b \to u \, \ell \, \nu_\ell$ modes:

[M. Antonelli et. al., flavour review, 07/2009]

$$|V_{ub}|^{\text{incl.}} = (4.11^{+0.27}_{-0.28}) \, 10^{-3}$$ *[BLNP,GGOU,DGE]*

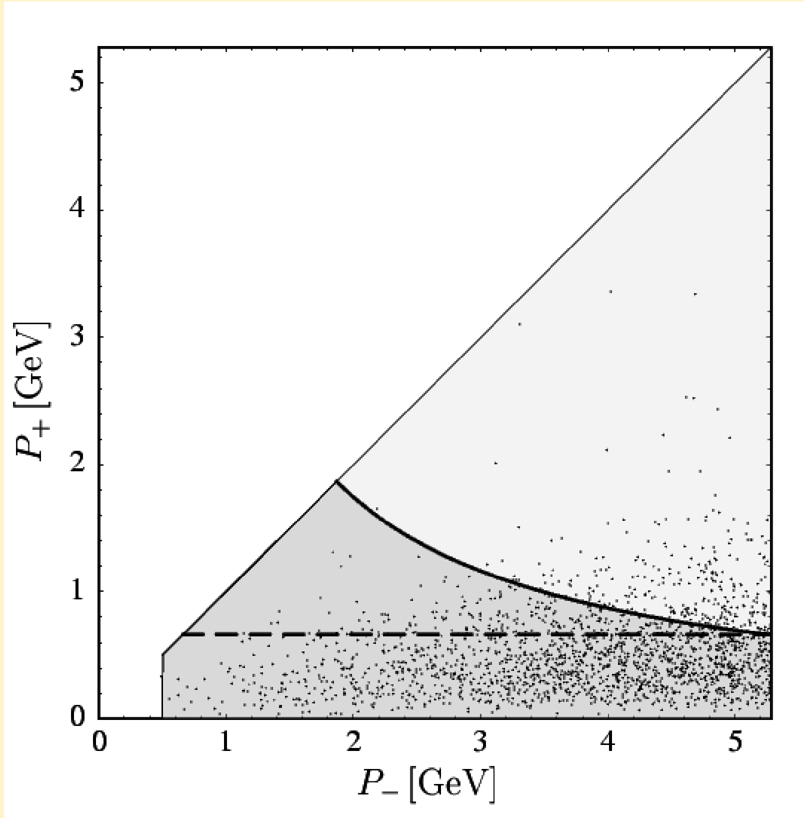
$$|V_{ub}|^{\text{excl.}} = (3.38 \pm 0.36) \, 10^{-3}$$ *[$B \to \pi \, \ell \, \nu_\ell$, lattice. J. Bailey et. al.]*

- Inclusive $\bar{B} \to X_u \, \ell \, \nu_\ell$ transition,
  kinematic regions require different theoretical treatment.
  - kinematic variables $p_X^\pm = E_X \mp |\vec{p}_X|$
  - Local OPE region: $\Lambda_{\text{QCD}} \ll p_X^+ \sim p_X^-$
  - SCET region: $\Lambda_{\text{QCD}} \sim p_X^+ \ll p_X^-$
  - Shape-function OPE: $\Lambda_{\text{QCD}} \ll p_X^+ \ll p_X^-$

Semi-leptonic $\bar{B} \to X_u \, \ell \, \nu_\ell$ decays and V_{ub}

- Exptl. determination of $|V_{ub}|$ requires kinematic cuts (charm background)

  - $\bar{B} \to X_u \ell \bar{\nu}_\ell$ events distributed over PS
  - Shaded regions: $s_X = p_X^+ \, p_X^- \gtrless M_D^2$
  - Dashed line:
    $E_l \geq (M_B^2 - M_D^2)/(2\, M_B)$
    implies $p_X^+ \leq M_D^2/M_B$
  - Even after cuts still many events left in shape-function region of small p_X^+ and large p_X^-



[Bosch,Lange,Neubert,Paz'04,'05]

- Triple differential decay rate in regions where $p_X^+ \ll p_X^-$

$$\frac{d^3\Gamma(\bar{B} \to X_u \ell \bar{\nu}_\ell)}{dE_\ell \, dp_X^+ \, p_X^-} = \Gamma_{0u} \, H_u(E_\ell, p_X^+, p_X^-, \mu_i) \int d\omega \, (n_- \cdot p) \, J((n_- \cdot p)\omega, \mu_i) \, S(p_X^+ - \omega, \mu_i)$$

[Korchemsky,Sterman;Bauer,Pirjol,Stewart]

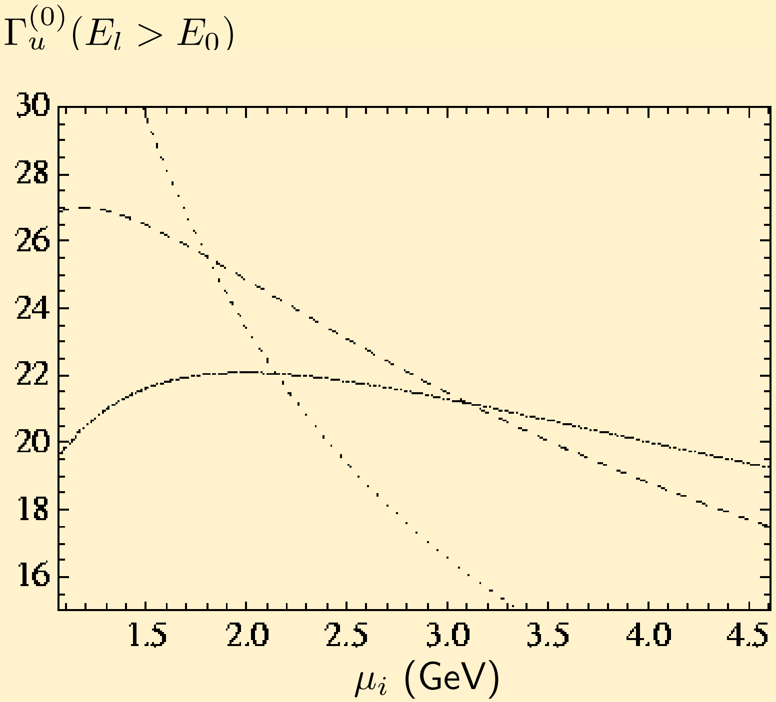
- Jet function J and perturbative part of soft function S known to two loops

[Becher,Neubert '05,'06]

Implications on $|V_{ub}|$

[Greub,Neubert,Pecjak'09]

- NNLO shift on NLO partial decay rates in the BLNP framework for the jet scale $\mu_i = 1.5$ GeV in resummed PT is -15% to -20%.
- For higher values of μ_i and in fixed order PT the shifts are more moderate
- Large dependence of NLO rates on μ_i reduced but still significant at NNLO
- NNLO corrections raise NLO value of $|V_{ub}|$ by $\lesssim 10\%$



Method	$\Delta\mathcal{B}^{\exp}$ $[10^{-4}]$	$\lvert V_{ub}\rvert$ $[10^{-3}]$ NLO	$\lvert V_{ub}\rvert$ $[10^{-3}]$ NNLO
$E_l > 2.1$ GeV	$3.3 \pm 0.2 \pm 0.7$	$3.56 \pm 0.40^{+0.48+0.31}_{-0.27-0.26}$	$3.81 \pm 0.43^{+0.33+0.31}_{-0.21-0.26}$
$E_l > 2.0$ GeV	$5.7 \pm 0.4 \pm 0.5$	$3.97 \pm 0.22^{+0.37+0.26}_{-0.23-0.25}$	$4.30 \pm 0.24^{+0.26+0.28}_{-0.20-0.27}$
$E_l > 1.9$ GeV	$8.5 \pm 0.4 \pm 1.5$	$4.27 \pm 0.39^{+0.32+0.25}_{-0.19-0.22}$	$4.65 \pm 0.43^{+0.27+0.27}_{-0.18-0.24}$
$M_X < 1.7$ GeV	$12.3 \pm 1.1 \pm 1.2$	$3.55 \pm 0.24^{+0.22+0.21}_{-0.13-0.19}$	$3.87 \pm 0.26^{+0.21+0.21}_{-0.13-0.19}$
$M_X < 1.55$ GeV	$11.7 \pm 0.9 \pm 0.7$	$3.67 \pm 0.18^{+0.29+0.26}_{-0.17-0.24}$	$3.96 \pm 0.19^{+0.20+0.26}_{-0.13-0.24}$
$P_+ < 0.66$ GeV	$11.0 \pm 1.0 \pm 1.6$	$3.56 \pm 0.31^{+0.30+0.27}_{-0.17-0.23}$	$3.84 \pm 0.33^{+0.21+0.26}_{-0.13-0.22}$
$P_+ < 0.66$ GeV	$9.4 \pm 1.0 \pm 0.8$	$3.30 \pm 0.23^{+0.27+0.25}_{-0.16-0.22}$	$3.55 \pm 0.24^{+0.19+0.24}_{-0.13-0.21}$

central values: $\mu_i = 2.0$ (GeV), $\mu_h = 4.25$ (GeV)

[Greub,Neubert,Pecjak'09]

Conclusion

- We computed the hard matching coefficients from QCD onto SCET at NNLO for all Dirac structures

- NNLO corrections are moderate and add contructively to NLO contributions

- We discussed Heavy-to-light form factor ratios and exclusive radiative decays. Only $R_\perp$ receives large NNLO corrections

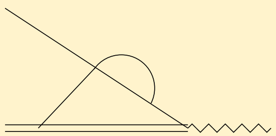
- The perturb. NNLO shift on the FBA zero in semi-inclusive $\bar{B} \to X_s\,\ell^+\ell^-$ is -3%. Final result for the zero: $q_0^2 = \left[(3.34\,\ldots\,3.40)^{+0.22}_{-0.25}\right]$ GeV^2 for $m_X^{\rm cut} = (2.0\ldots 1.8)$ GeV

- NNLO Corrections tend to increase the difference between $|V_{ub}|$ determined from inclusive vs. exclusive decay modes.

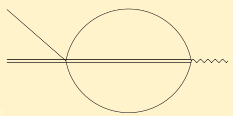
Backup slides

Reduction methods

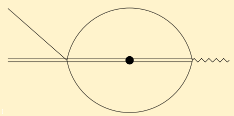
- Dimensional regularisation with $D = 4 - 2\epsilon$ regulates UV and IR. Poles up to $1/\epsilon^4$.
- Passarino-Veltman reduction to scalar integrals
  (in general with irreducible scalar products in the numerator) *[Passarino,Veltman'79]*
- Integration-by-parts (IBP) identities, 8 per diagram *[Tkachov'81; Chetyrkin,Tkachov'81]*
- Lorentz-Invarianz (LI) identities, 1 per diagram *[Gehrmann,Remiddi'99]*
- Solve system of equations with Laporta algorithm *[Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08]*
- Obtain scalar integrals as a linear combination of master integrals



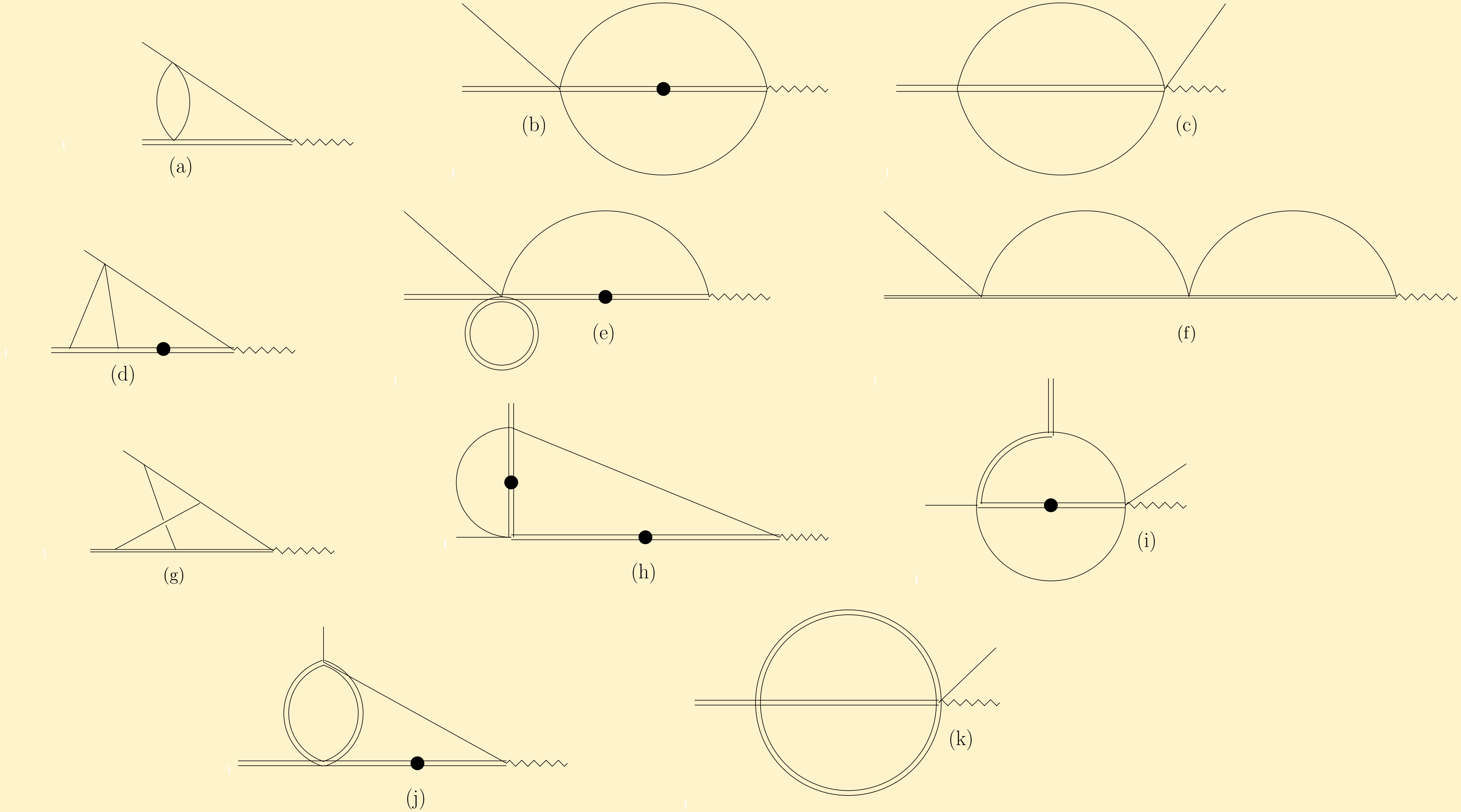
$$= \frac{(8-3D)(7uD-8D-24u+28)}{3(D-4)^2\, m_b^4\, u^3}$$



$$- \frac{2[u^2(D-4)+(16D-56)(1-u)]}{3(D-4)^2\, m_b^2\, u^3}$$



Master integrals



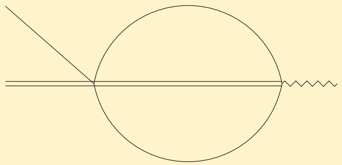
- Double lines are massive, single lines are massless
- Dots on lines denote squared propagators

Master Integrals

- Reduction yields 18 master integrals with poles up to $1/\epsilon^4$. Analytic calculation of coefficient functions yields harmonic polylogarithms up to weight 4 of argument u or $1-u$. *[Remiddi,Vermaseren'99]*

- Several calculations in agreement *[Bell'07; Bonciani,Ferroglia'08; Asatrian,Greub,Pecjak'08; Beneke,Li,TH'08; Bell'08]*

- Applied techniques
  - Hypergeometric functions, use `HypExp` or XSummer for ϵ-expansion *[Moch,Uwer'05; Maitre,TH'05,'07]*



$$= \frac{(m_b^2)^{1-2\epsilon}}{(4\pi)^{4-2\epsilon}} \frac{\Gamma^2(1-\epsilon)\Gamma(\epsilon)\Gamma(2\,\epsilon-1)}{\Gamma(2-\epsilon)} \, {}_2F_1(\epsilon, 2\,\epsilon-1\,;\,2-\epsilon\,;\,1-u)$$

  - Differential equations *[Kotikov'91; Remiddi'97]*

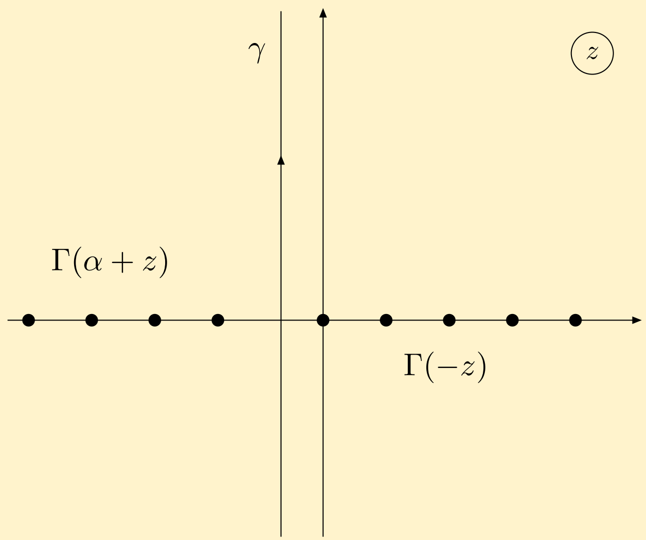
$$\frac{\partial}{\partial u}\mathrm{MI}_i(u) = f(u,\epsilon)\,\mathrm{MI}_i(u) + \sum_{j\neq i} g_j(u,\epsilon)\,\mathrm{MI}_j(u)$$

    - ∗ Requires result of Laporta reduction.
    - ∗ Boundary condition in $u=0$ or $u=1$ from Mellin-Barnes representation

Master Integrals (cont'd.)

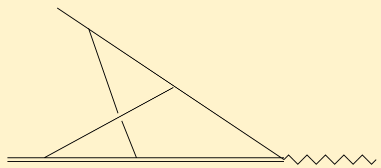
- Applied techniques (cont'd.)
  - Mellin-Barnes representation *[Smirnov'99; Tausk'99]*

$$\frac{1}{(A_1+A_2)^\alpha} = \int\limits_\gamma \frac{dz}{2\pi i}\; A_1^z\, A_2^{-\alpha-z}\, \frac{\Gamma(-z)\,\Gamma(\alpha+z)}{\Gamma(\alpha)}$$



    - ∗ partially automated
    - ∗ Numerical cross checks possible

    [Czakon'05; Gluza,Kajda,Riemann'07]

- Most difficult master integral:  *[TH'09]*
  - Solved with differential equations technique
  - Possesses a three-fold Mellin-Barnes integral at $u = 1$