Gauge Coupling Unification at Three Loops in the MSSM framework

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Outline

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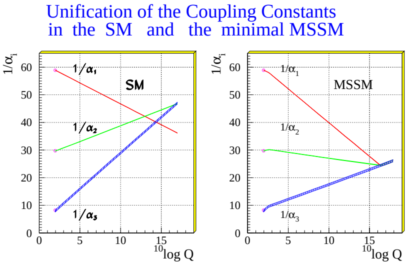
Studying Gauge Coupling Unification in the MSSM: Why?



[Amaldi, Furstenau, de Boer] [Langacker, Luo], [Ellis, Kelley, Nanopoulos]

Often heard interpretation:

- Left figure: SM and GUTs do not go together
- Right figure: Fruitful to study gauge coupling unification further to rule out or confirm GUTs

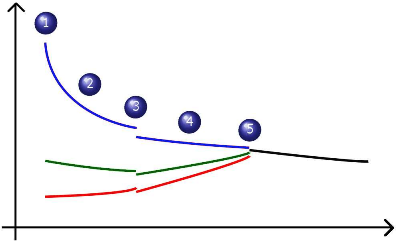
However...

Studying Gauge Coupling Unification in the MSSM: Why?

- ... there are a lot of ideas to remedy the problems with gauge coupling unification in the SM and other scenarios
- So why? Assuming a
  - concrete GUT scenario (in this talk minimal SUSY SU(5))
  - and a concrete MSSM scenario

  one is able to bound the phase space of the GUT parameters by exploiting gauge coupling unification.

Predicting GUT parameters by Exploiting Gauge Coupling Unification



1. Experimental determination of gauge couplings
2. Using RGE of SM
3. Decoupling of heavy SUSY partners
4. Using RGE of MSSM
5. Decoupling of heavy GUT particles

Predicting GUT parameters by Exploiting Gauge Coupling Unification

1. Experimental determination of gauge couplings
   - $\sin^2 \Theta^{\overline{\rm MS}} = 0.23119 \pm 0.00014$ [Particle Data Group '08]
   - $\alpha_{\rm QED} = 1/137.036$ and its hadronic contribution $\Delta\alpha_{\rm had}^{(5)} = 0.02761 \pm 0.00015$ [Teubner, Hagiwara, Liao, Martin, Nomura '10]
   - $\alpha_s(M_Z) = 0.1184 \pm 0.0010$ [Bethke '09]
2. Using RGE of SM
   - 2-loop for all gauge and Yukawa couplings in the SM [Ford,Jack,Jones '92;Machacek,Vaughn '83 and '84]
   - 3-loop in QCD [Tarasov,Vladimirov,Zharkov '80,Larin,Vermaseren '93]
3. Decoupling of heavy SUSY partners
   - 1-loop for all gauge and Yukawa couplings [Yamada '93;Pierce,Bagger,Matchev,Zhang '97]
   - 2-loop QCD contributions for α_3 and $y_{\rm bottom}$ [Harlander,Mihaila,Steinhauser '05;Bauer,Mihaila,J.S. '09]
4. Using RGE of MSSM
   - 3-loop for all gauge and Yukawa couplings in the MSSM [Ferreira,Jack,Jones '96;Harlander,Mihaila,Steinhauser '09]
5. Decoupling of heavy GUT particles
   - 1-loop for all gauge couplings [Hall '81;Weinberg '80;Einhorn,Jones '82]

Predicting GUT parameters with Knowledge of Gauge Couplings at the GUT Scale

After step 4: Knowledge of $\alpha_{\rm i}^{MSSM}(\mu_{GUT})$ $(\rm i = 1, 2, 3)$.
Solving

$$\begin{aligned}\alpha^{GUT}(\mu_{GUT}) &= \zeta_1^{-1}(\alpha_{1/2/3}^{MSSM}(\mu_{GUT}), \mu_{GUT}, M_{H_c}, M_X, M_\Sigma)\ \alpha_1^{MSSM}(\mu_{GUT}) \\ &= \zeta_2^{-1}(\alpha_{1/2/3}^{MSSM}(\mu_{GUT}), \mu_{GUT}, M_{H_c}, M_X, M_\Sigma)\ \alpha_2^{MSSM}(\mu_{GUT}) \\ &= \zeta_3^{-1}(\alpha_{1/2/3}^{MSSM}(\mu_{GUT}), \mu_{GUT}, M_{H_c}, M_X, M_\Sigma)\ \alpha_3^{MSSM}(\mu_{GUT})\end{aligned}$$

to first order results in

$$\begin{aligned}4\pi\left(-\frac{1}{\alpha_1^{MSSM}(\mu_{GUT})} + 3\frac{1}{\alpha_2^{MSSM}(\mu_{GUT})} - 2\frac{1}{\alpha_3^{MSSM}(\mu_{GUT})}\right) &= -\frac{12}{5}\ln\frac{\mu_{GUT}^2}{M_{H_c}^2}, \\ 4\pi\left(5\frac{1}{\alpha_1^{MSSM}(\mu_{GUT})} - 3\frac{1}{\alpha_2^{MSSM}(\mu_{GUT})} - 2\frac{1}{\alpha_3^{MSSM}(\mu_{GUT})}\right) &= -24\ln\frac{\mu_{GUT}^3}{M_X^2\cdot M_\Sigma}.\end{aligned}$$

So as a result one predicts M_{H_c} and $M_G := \sqrt[3]{M_X^2 \cdot M_\Sigma}$ in step 5.

Outline

Gauge Coupling Unification at Three Loops: Why?

Sources of Error:

- Experimental uncertainties
- Uncertainties due to using finite order RGEs
- Uncertainties due to using finite order decoupling contants

The latter can be rearranged to yield

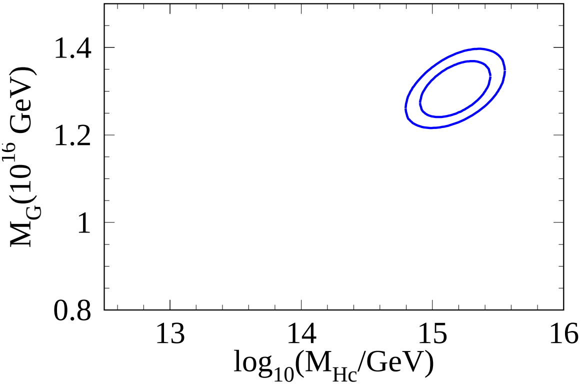
- scale uncertainties by varying μ_{SUSY} and μ_{GUT}
- additional finite shifts

Three-loop analysis makes sense if the experimental uncertainty is smaller than the theoretical uncertainty in two-loop analysis.

Some Definite Scenario to Discuss the Uncertainties

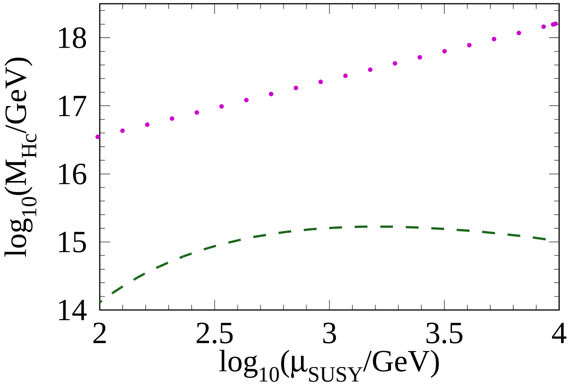
- Choose as an MSSM scenario:
  - mSUGRA with
  - $m_0 = m_{1/2} = -A_0 = 1000$ GeV
  - $\tan\beta = 3$
  - $\mathrm{sign}(\mu) > 0$
- If not indicated otherwise: $\mu_{SUSY} = 1000$ GeV
- If not indicated otherwise: $\mu_{GUT} = 10^{16}$ GeV

Estimating the Experimental Uncertainty



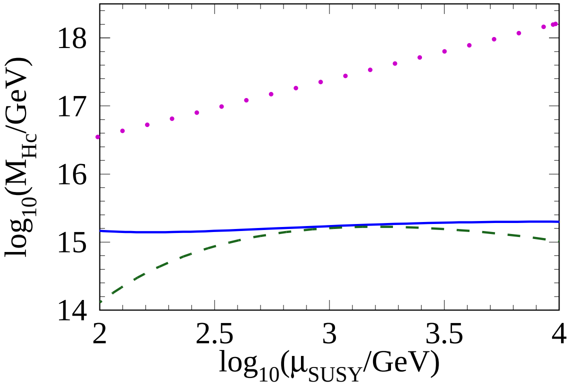
The experimental uncertainty in M_{H_c} and M_G is smaller than an order of magnitude.

Estimating the Theoretical Uncertainty



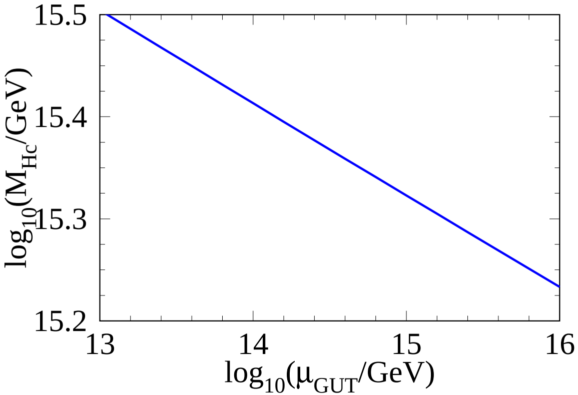
The shift between the one- and the two-loop analysis is huge, roughly two orders of magnitude.

Estimating the Theoretical Uncertainty



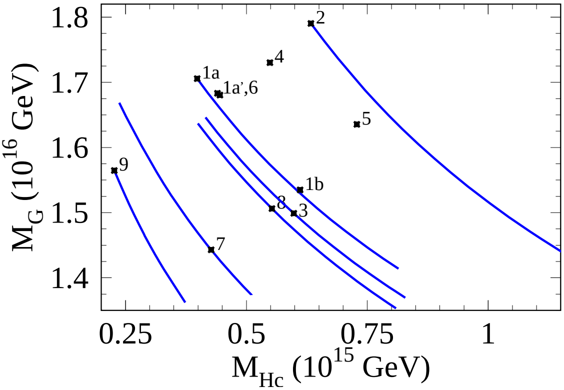
The three loop curve leads to a stabilisation of pertubation theory in the finite shift as well as in the scale uncertainty.

Estimating the Theoretical Uncertainty due to the GUT Decoupling Constants



The variation of μ_{GUT} leads to a comparatively small variation in M_{H_c}. However it is hard to estimate the finite shift if one used two-loop instead of one-loop decoupling at the GUT scale. . .

Application: Predicting GUT Parameters for SPS Scenarios

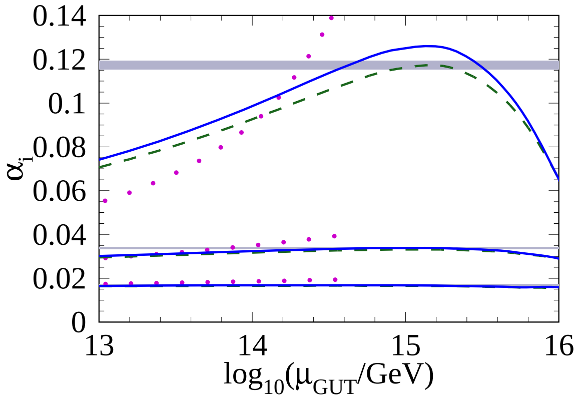


Summary

- We put together all pieces at highest known precision to discuss gauge coupling unification in the MSSM framework.
- Rather than excluding or confirming the very idea of GUTs, this can be used to predict GUT parameters.
- The resulting precision leads to theoretical errors smaller than the corresponding experimental ones. This is *cum grano salis*, as some theoretical uncertainties can only be rougly estimated (yet).

Hint: This talk is based on arXiv:1008.3070 (soon to be published in Phys. Rev. D).

Backup



0.14
0.12
0.1
0.08
0.06
0.04
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0
α_i
13
14
15
16
$\log_{10}(\mu_{GUT}/\mathrm{GeV})$

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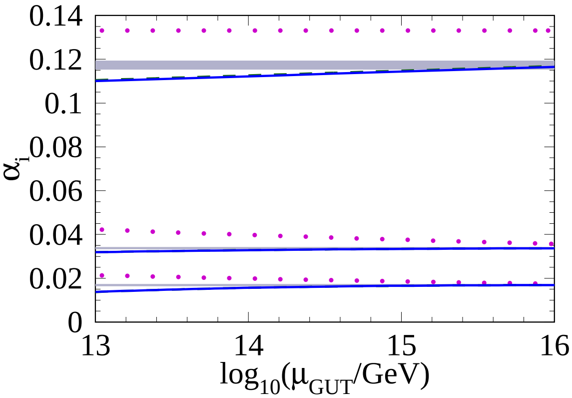
0.14
0.12
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α_i
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16
$\log_{10}(\mu_{GUT}/\mathrm{GeV})$

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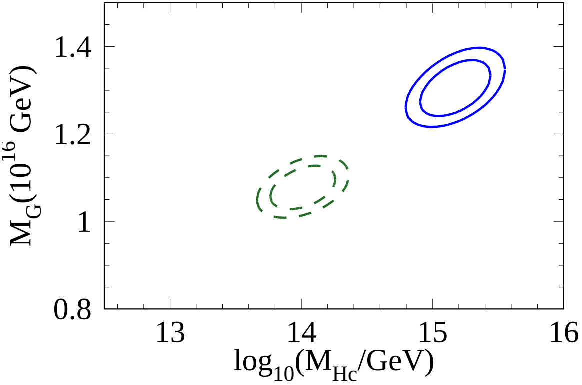

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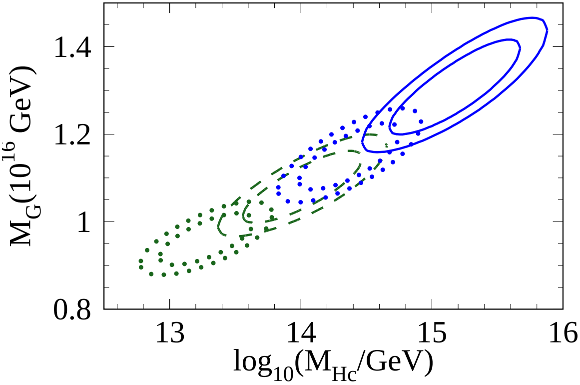

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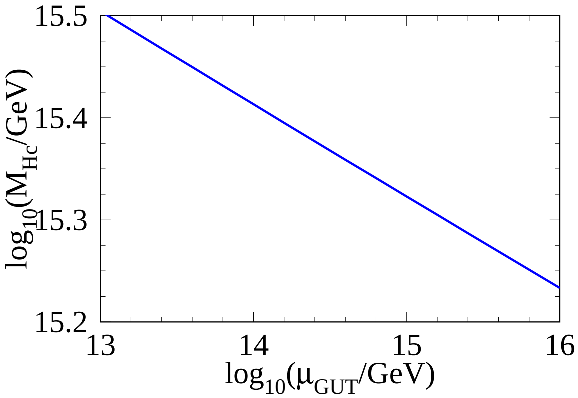
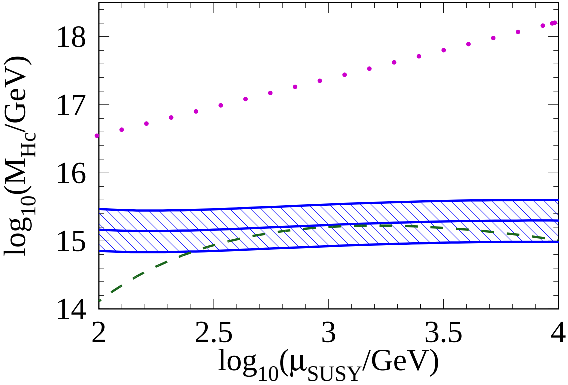

15.5
15.4
15.3
15.2
13
14
15
16
$\log_{10}(M_{Hc}/\mathrm{GeV})$
$\log_{10}(\mu_{GUT}/\mathrm{GeV})$

Backup



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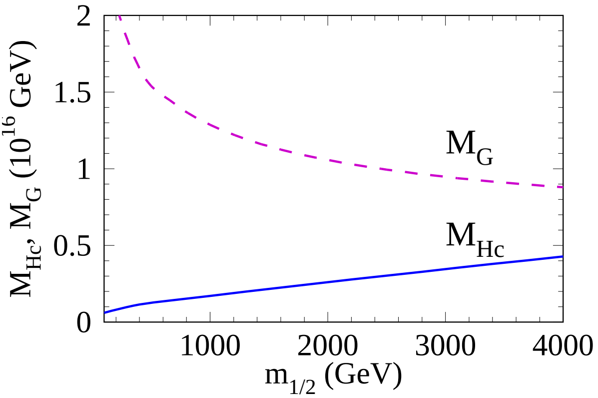

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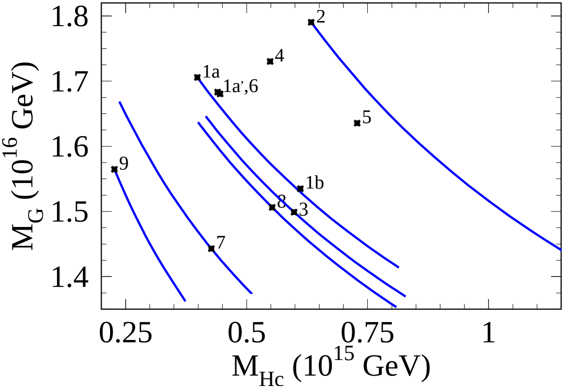

1.8
1.7
1.6
1.5
1.4
M_G (10^{16} GeV)
0.25
0.5
0.75
1
M_{Hc} (10^{15} GeV)
2
4
1a
1a',6
5
9
1b
8
3
7

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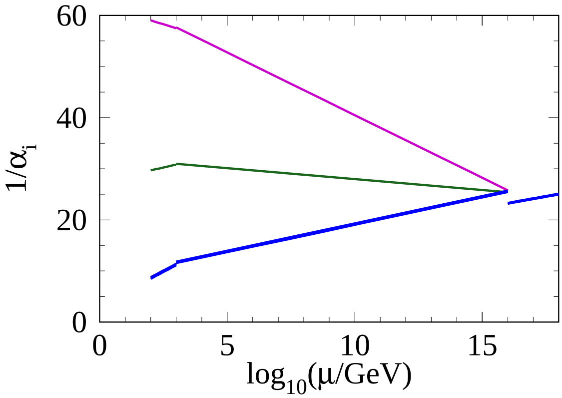

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