Quantum Computing at DESY (and beyond)

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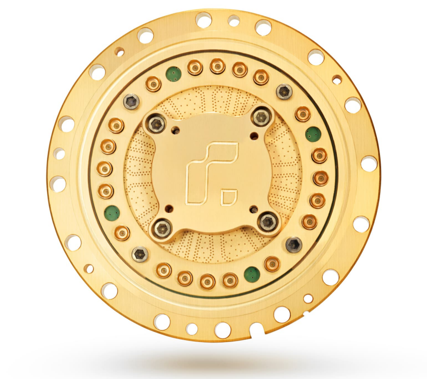
Quantum computing: why

- qubit = 2-level quantum system $|0\rangle$, $|1\rangle$

  → superpostion state: $|\mathrm{qubit}\rangle = a|0\rangle + b|1\rangle$, $a^2 + b^2 = 1$

  → 2-level atom, Josephson junction, polarized photons, ...



- 2 classical bits
  $(\mathrm{bit1}, \mathrm{bit2})$: 4 different states: (0,0),(1,0),(0,1),(1,1)

  → only one state realized at a given time

- 2 qubits
  $|\mathrm{qubit1}, \mathrm{qubit2}\rangle$: 4 different basis states: $|0,0\rangle, |1,0\rangle, |0,1\rangle, |1,1\rangle$

  → can be realized simultanenously:

  $|\Psi\rangle = \frac{1}{2}|0,0\rangle + \frac{1}{2}|1,0\rangle + \frac{1}{2}|0,1\rangle + \frac{1}{2}|1,1\rangle$

Quantum computing: why

- a general (2 qubit) quantum state is superposition

  $$|\Psi\rangle = \alpha|0,0\rangle + \beta|1,0\rangle + \gamma|0,1\rangle + \delta|1,1\rangle \ , \ |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

  – can store abritrary information
  – computation speed independent from number of qubits

- need to measure $\alpha, \beta, \gamma, \delta$

  $\rightarrow$ count number of states $|0,0\rangle, |1,0\rangle, |0,1\rangle|1,1\rangle$ in repeated experiment

- computational speed

  $\rightarrow$ quantum supremacy: 50 qubits $\rightarrow$ 2^{50} states

  $\rightarrow$ classical optimization problems (e.g. particle track reconstruction)

  $\rightarrow$ demonstrated by google experiment

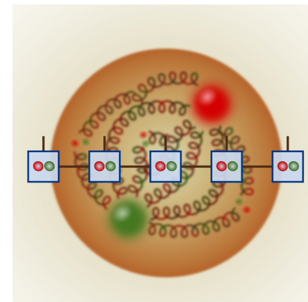
Entanglement

- entaglement $\rightarrow$ allows operations which are classically impossible
- example:
  - superdense coding (infamous Alice and Bob experiment)
- CNOT gate on $|\underbrace{\text{qubit1}}_{\text{control}}\underbrace{\text{qubit2}}_{\text{target}}\rangle$

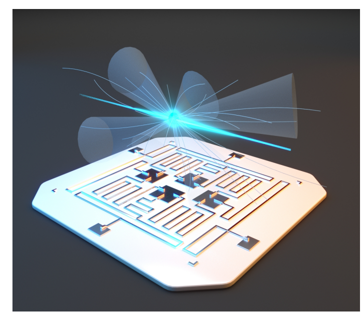
$$\begin{array}{ccc} |00\rangle & \rightarrow & |00\rangle \\ |01\rangle & \rightarrow & |01\rangle \\ |10\rangle & \rightarrow & |11\rangle \\ |11\rangle & \rightarrow & |10\rangle \end{array}$$

Quantum computing applications

- problems extremely hard or inaccessible classically
  - quantum field theories
    $\rightarrow$ very early universe
    $\rightarrow$ matter-antimatter asymmetry
    cooperation: DESY, IQOQI (Innsbruck)
    IQC& Perimeter (Waterloo)



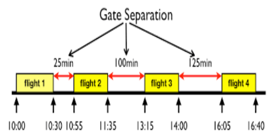
  - condensed matter physics:
    $\rightarrow$ superconductivity: Hubbard model away from half-filling
    $\rightarrow$ topological systems
  - chemistry
  - atom and molecule spectra
  - material science

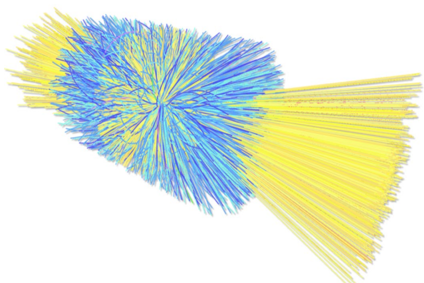


- problems in biology, chemistry, materials, ...

Quantum computing applications

- classical optimization problems
  - aerospace, e.g. flight gate assignment
    DESY-DLR collaboration
  - logistics
  - traffic
  - Einstein telescope
  - particle tracking
    → Higgs tagging
    DESY-CERN Openlab cooperation
    → electron-positron identification in LUXE experiment





problems share the same or very
similar structure of cost function

- more examples (qgan, ...) → Kerstin Borras

Classical optimization problem: flight gate assignment

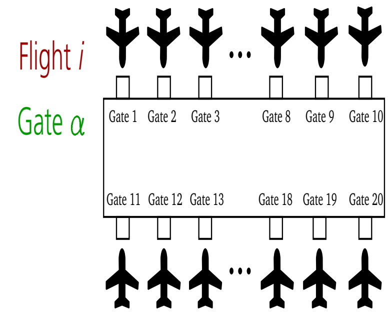
- Find shortest path between two connecting flights

$$x_{i\alpha} = \begin{cases} 1, & \text{if flight } i \in F \text{ is assigned to gate } \alpha \in G \\ 0, & \text{otherwise} \end{cases}$$

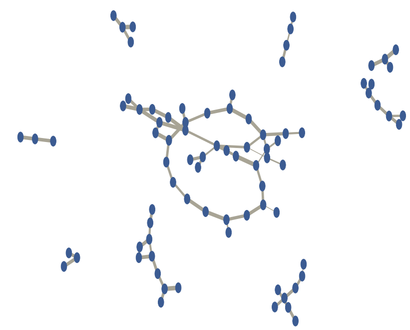
$x \in \{0,1\}^{F \otimes G} \rightarrow x$ binary variable $\rightarrow x \in \{-1,1\}^{F \otimes G}$

eigenstate of third Pauli matrix σ_z

$$H = \sum_{j=1}^{n} Q_{jj}\sigma_j^z + \sum_{\substack{j,k=1 \\ j<k}}^{n} Q_{jk}\sigma_j^z \otimes \sigma_k^z$$



- Q_{ij} coeffecients specific for a real given airport
- Goal: find ground state (shortest path)
- contraints:
  - every flight can only be assigned to a single gate
  - no aircraft can be at the same gate at the same time



Hamiltonian formulation

- Quantum computing uses Hamiltonian approach
- search for wavefunction $|\Psi>$

$$|\Psi>=\sum_{i_1,i_2,\cdots,i_N} C_{i_1,i_2,\cdots,i_N}|i_1 i_2 \cdots i_N>$$

$C_{i_1,i_2,\cdots,i_N}$ coefficient matrix with 2^N entries

$\Rightarrow$ exponential scaling $\rightarrow$ becomes impossible .

Solution: Variational Quantum Simulation

- start with some initial state $|\Psi_{\rm init}\rangle$
- apply succesive gate operations $\equiv$ unitary operations $e^{iS\theta}$
- examples for S: Pauli matrices σ_x, σ_y, σ_z, parametric CNOT

$$|\Psi(\vec{\theta})\rangle = e^{iS_{(n)}\theta_n} \dots e^{iS_{(1)}\theta_1}|\psi_{\rm init}\rangle$$

- with $R_j := e^{iS_{(j)}\theta_j}$ cost function evaluated on quantum computer

$$C := \left\langle \psi_{\rm init} \left| \left(\prod_{j=1}^{n} R_j\right)^\dagger H \prod_{j=1}^{n} R_j \right| \psi_{\rm init} \right\rangle$$

- goal: minimize C over the angles $\vec{\theta}$
  $\rightarrow$ obtain minimal energy, i.e. ground state

- minimization performed classically (hybrid classical-quantum approach)
  $\leftarrow$ also possible on quantum computer itself

Finding ground state: Variational Quantum Simulation

(0) evaluate cost function for initial parameters $\vec{\theta}_{\text{init}}$ on *quantum computer*

$$C(\vec{\theta}_{\text{init}}) := \left\langle \psi_{\text{init}} \middle| \left(\prod_{j=1}^{n} R_j(\vec{\theta}_{\text{init}})\right)^\dagger H \prod_{j=1}^{n} R_j(\vec{\theta}_{\text{init}}) \middle| \psi_{\text{init}} \right\rangle$$

$\downarrow$

(1) give to *classical computer* $\rightarrow$ optimize over $\vec{\theta}_{\text{init}}$
e.g. gradient descent, baysean optimization, ...
$\rightarrow$ obtain new set of parameters $\vec{\theta}_{\text{new}}$

$\downarrow$

(2) give to *quantum computer* evaluate new cost function

$$C(\vec{\theta}_{\text{new}}) := \left\langle \psi_{\text{init}} \middle| \left(\prod_{j=1}^{n} R_j(\vec{\theta}_{\text{new}})\right)^\dagger H \prod_{j=1}^{n} R_j(\vec{\theta}_{\text{new}}) \middle| \psi_{\text{init}} \right\rangle$$

$\downarrow$

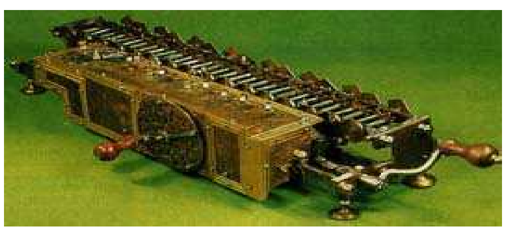
(3) give to *classical computer* $\rightarrow$ optimize over $\vec{\theta}_{\text{init}}$ and $\vec{\theta}_{\text{new}}$, ...
$\rightarrow$ obtain new set of parameters $\vec{\theta}_{\text{new}}$

(4) go to (2) until converge, i.e. find minimum

Variational quantum simulation

- evaluate cost function $\langle\Psi(\vec{\theta})|H|\Psi(\vec{\theta})\rangle$ on quantum device

- feedback loop

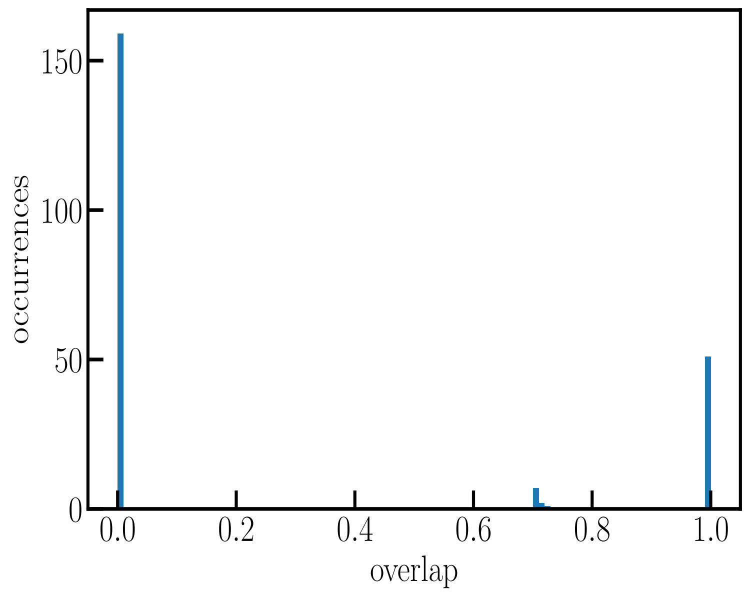


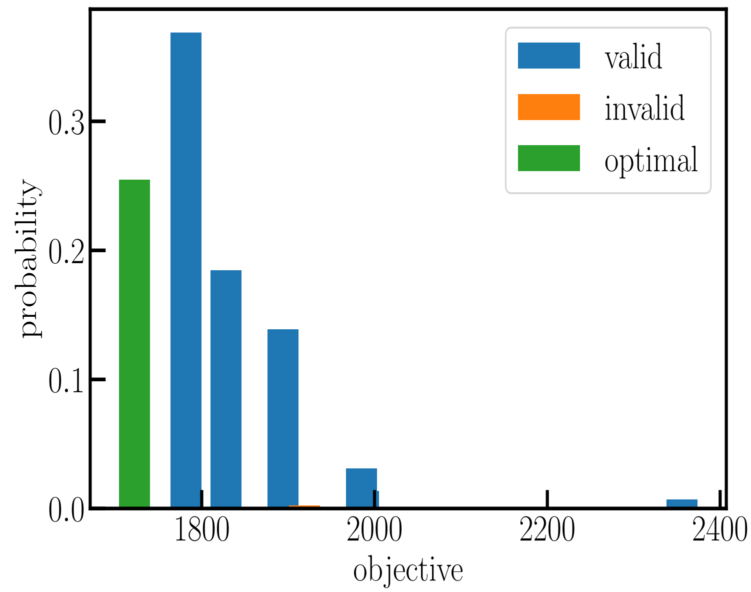
- optimize over parameters $\vec{\theta}$ on classical computer
  $\rightarrow$ give back new set of $\vec{\theta}$

VQS for FGA

(L. Funcke, T. Hartung, S. Kühn, T. Stollenwerk, P. Stornati, K.J.)

- use variational quantum simulation to find ground state
  - use 6 qubits on simulator
  - overlap: $\langle \Psi_{VQS} | \Psi_{\text{exact}} \rangle$





- Remarks:
  - Hamiltonian is diagonal $\rightarrow$ classical optimization
  - QC helpful through principles of superposition and entanglement?
  - the same Hamiltonian can be used in particle track reconstruction

A condensed matter physical model

- 1-dimensional Heisenberg model

$$H = \sum_{i=1}^{N} \beta \left[\sigma_x(i)\sigma_x(i+1) + \sigma_y(i)\sigma_y(i+1) + \sigma_z(i)\sigma_z(i+1)\right] + J\sigma_z(i)$$

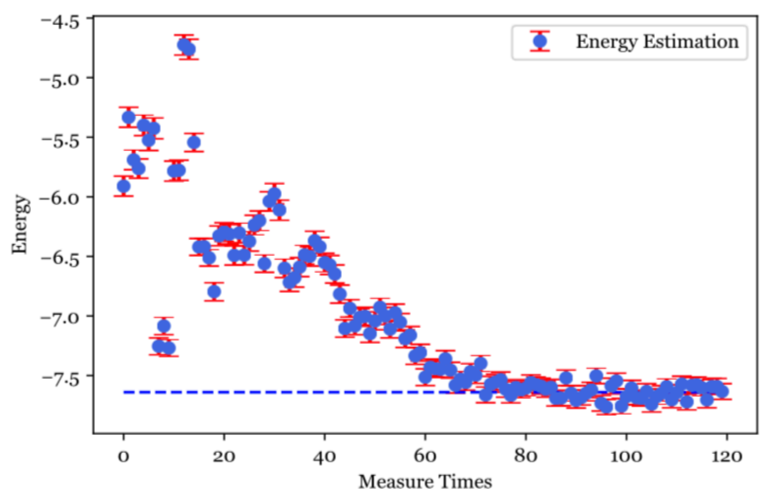
- Pauli matrices

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- nearest neighbour interaction, tensor products
- Hamiltonian expressed in Pauli matrices $\rightarrow$ suitable for quantum computer
- shows phase transitions, critical behaviour, non-trivial spectrum

Using the simulator

- Simulator with no noise
- IBM's *Simultaneous Perturbation Stochastic Approximation* (SPSA) algorithm
- dashed line exact result

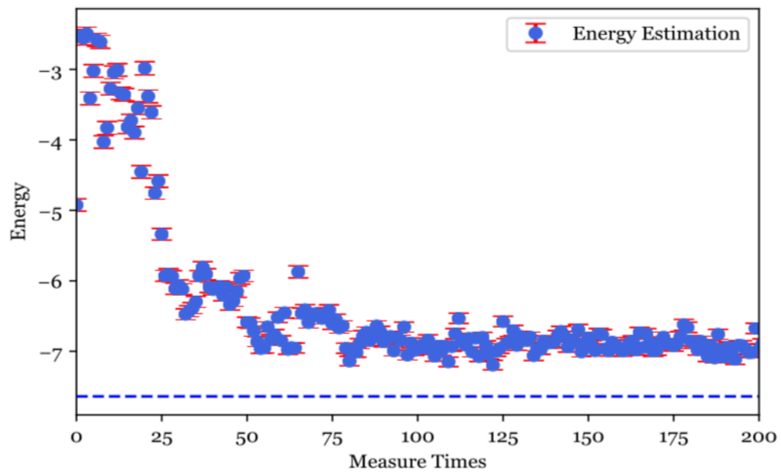


(figure by Xiaoyang Wang)

- 3 qubits

Switching on noise

- Simulator with noise



(figure by Xiaoyang Wang)

- 3 qubits
- measurement error → cannot find ground state
- need of error mitigation/correction
  correcting readout error
  → L. Funcke, T. Hartung, S. Kühn, P. Stornati, X. Wang, K.J.,arXiv:2007.03663

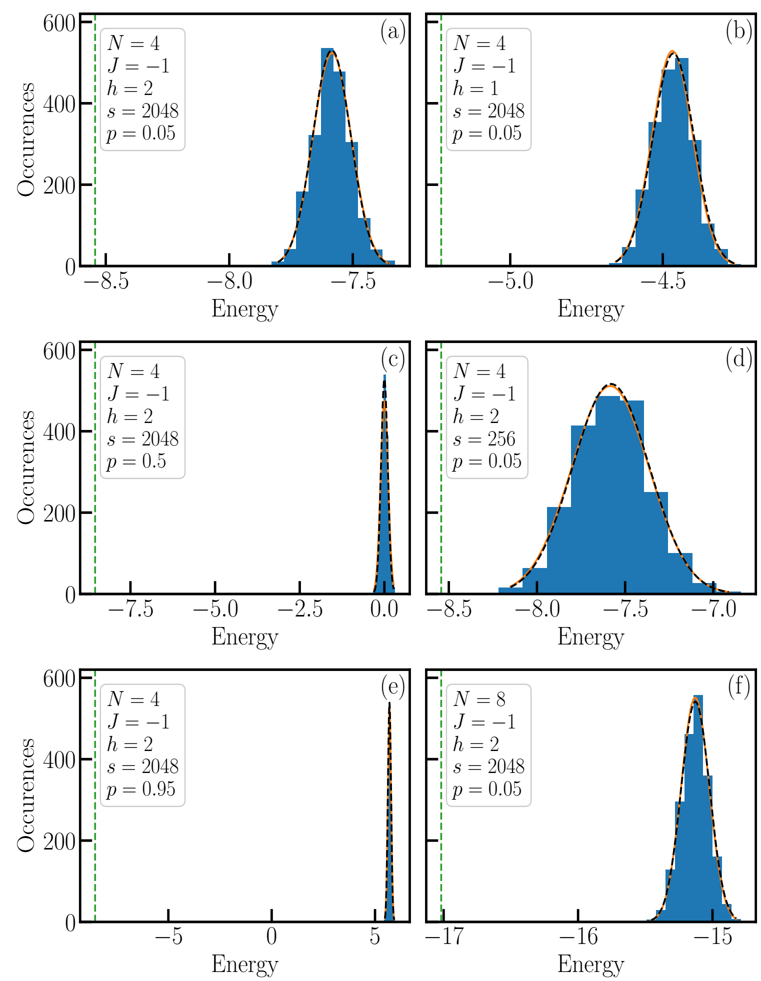
General measurement error mitigation in NISQ area

(L. Funcke, T. Hartung, S. Kühn, P. Stornati, X. Wang, K.J.,arXiv:2007.03663)

- generated state $|\Psi(\vec{\theta})\rangle$ is a bit string $|00110011100101\rangle$
- false measurement
  $|0\rangle \rightarrow |1\rangle$ with probability p_1
  $|1\rangle \rightarrow |0\rangle$ with probability p_2
- setting (for simplicity) $p_1 = p_2 = p$
- measuring s-times:
  get k correct and $s-k$
  incorrect results distributed as

$$f(k, s, 1-p) = \begin{pmatrix} s \\ k \end{pmatrix} (1-p)^k p^{s-k}$$

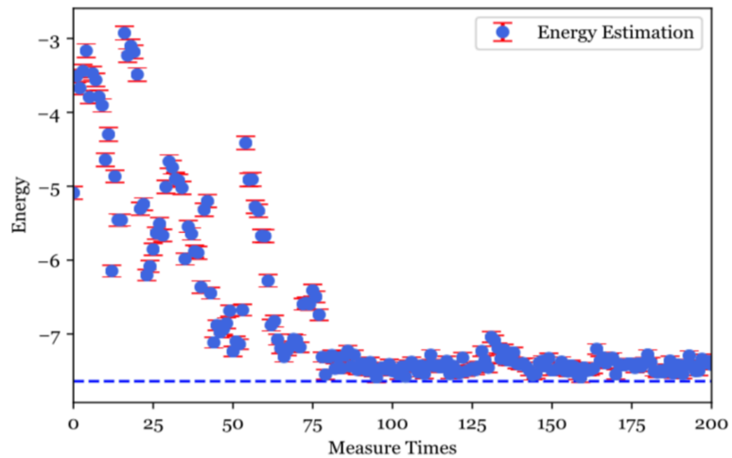
- recompute exact energy from noisy measurments
- can be generalized to arbitrary number of qubits
- only readout noise!



example: transverse ising model

Error mitigation

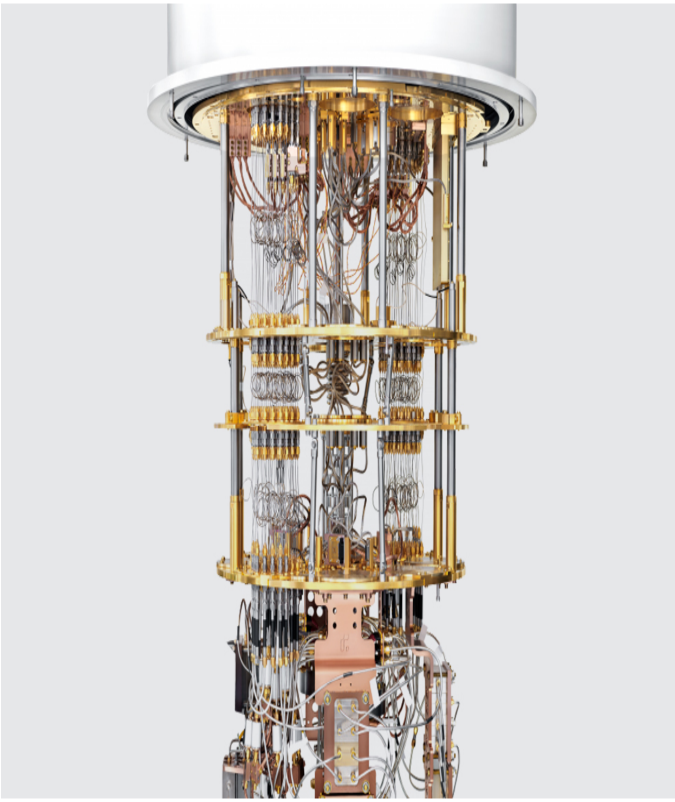
- Simulator with error mitigation



(figure by Xiaoyang Wang)

- 3 qubits
- works well → find ground state
- perspective for larger number of qubits

Inside a Quantum Computer



(Rigetti Aspen)

- Shielded to 50,000 times less than Earth's magnetic field
- In a high vacuum: pressure is 10 billion times lower than atmospheric pressure
- Cooled 180 times colder than interstellar space (0.015 Kelvin)

$\rightarrow$ prevent decoherence

- qubits based based on Josephson junction
- application of unitary gate operations

$\rightarrow$ generate entanglement

- reaching 128 qubits this year
- IBMQ: Free 14 qubits, 400 qubits 2021, 1000 qubits 2023
- D-Wave: 2000 qubits $\rightarrow$ 5000 qubits

Quantum Computing: how?

- python programming language

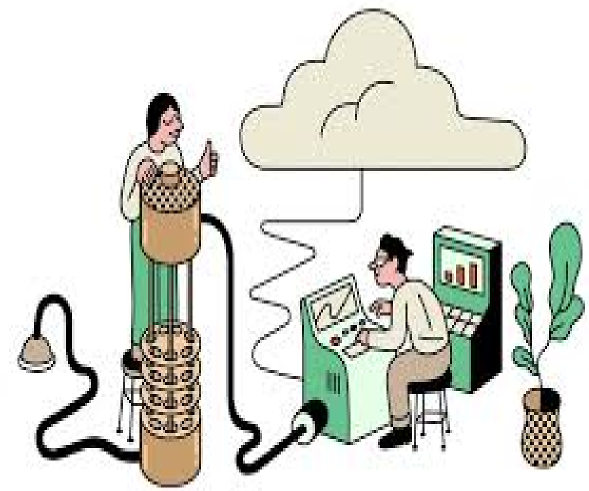
  → company provides quantum libraries

- very convenient setup

  → simulator runs on your local machine

  → hardware usable through quantum cloud service

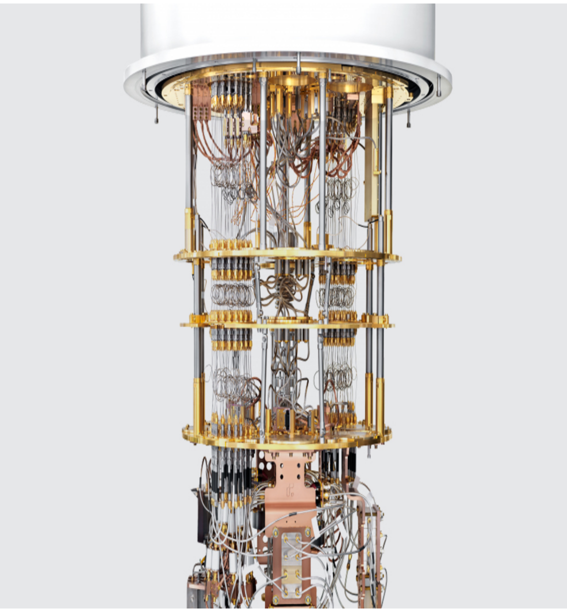
  → build on reservation system



- documentation, tutorials and examples availabe on website

  → you can start now

Some publications



- *Zeta-regularized vacuum expectation values from quantum computing simulations*
  T. Hartung and K.J., **J.Math.Phys.** 60 (2019) 9, 093504
- *Measurement Error Mitigation in Quantum Computers Through Classical Bit-Flip Correction*
  L. Funcke, T. Hartung, S.Kühn, P. Stornati, K.J., arxiv:2007.03663
- *A resource efficient approach for quantum and classical simulations of gauge theories in particle physics*
  J.F. Haase, L. Dellantonio, A.Celi, D.Paulson, A. Kan, K.J., C.A. Muschik, **Quantum** 5 (2021) 393
- *Towards simulating 2D effects in lattice gauge theories on a quantum computer*
  D. Paulson, L. Dellantonio, J.F. Haase, A. Celi, A. Kan, A. Jena, C. Kokail, R. van Bijnen, K.J., P. Zoller, C. A. Muschik, accepted in **PRX Quantum**
- *Simulating Lattice Gauge Theories within Quantum Technologies*
  M.C. Bañuls et.al., **Eur.Phys.J.D** 74 (2020) 8, 165
- *Dimensional Expressivity Analysis of Quantum Circuits*
  L. Funcke, T. Hartung, S.Kühn, P. Stornati, K.J., **Quantum** 5 (2021) 422
- *Flight gate assignment with variational quantum simulations*
  L. Funcke, T. Hartung, S.Kühn, P. Stornati, T. Stollenwerk, K.J., in preparation
- *A measurement-based variational quantum eigensolver*
  R. Ferguson, L. Dellantonio, A. Al Balushi, W.Dür, C. Muschik, K.J., **Phys.Rev.Lett.** 126
- *Investigating a 3+1D Topological θ-Term in the Hamiltonian Formulation of Lattice Gauge Theories for Quantum and Classical Simulations*
  A. Kan, L. Funcke. S. Kühn, L. Dellantonio, C.A. Muschik, K.J., **to appear in PRD**